Computer algebra independent integration tests

1-Algebraic-functions/1.3-Miscellaneous/1.3.2-Algebraic-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1025]. This is test number [52].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 98.24 (1007)	% 1.76 (18)
Mathematica	% 97.27 (997)	% 2.73 (28)
Maple	% 82.05 (841)	% 17.95 (184)
Maxima	% 37.56 (385)	% 62.44 (640)
Fricas	% 68.00 (697)	% 32.00 (328)
Sympy	% 24.98 (256)	% 75.02 (769)
Giac	% 47.22 (484)	% 52.78 (541)
Mupad	% 44.39 (455)	% 55.61 (570)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

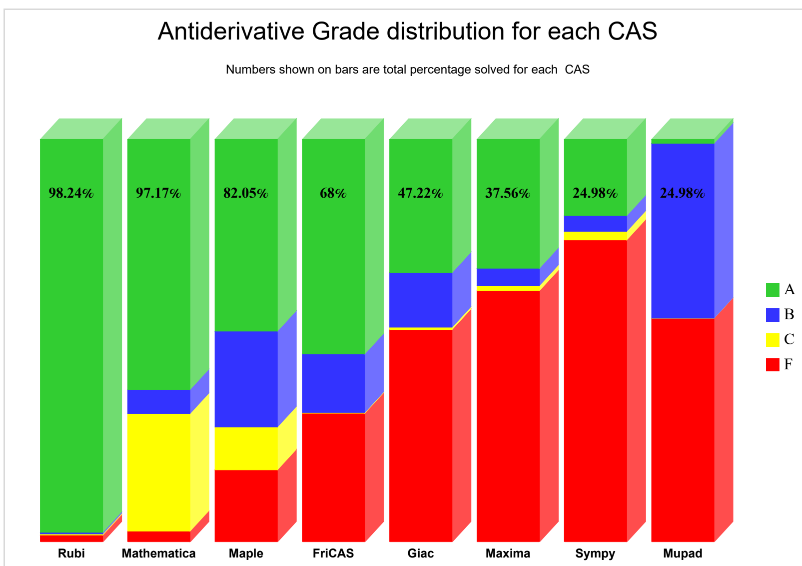
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

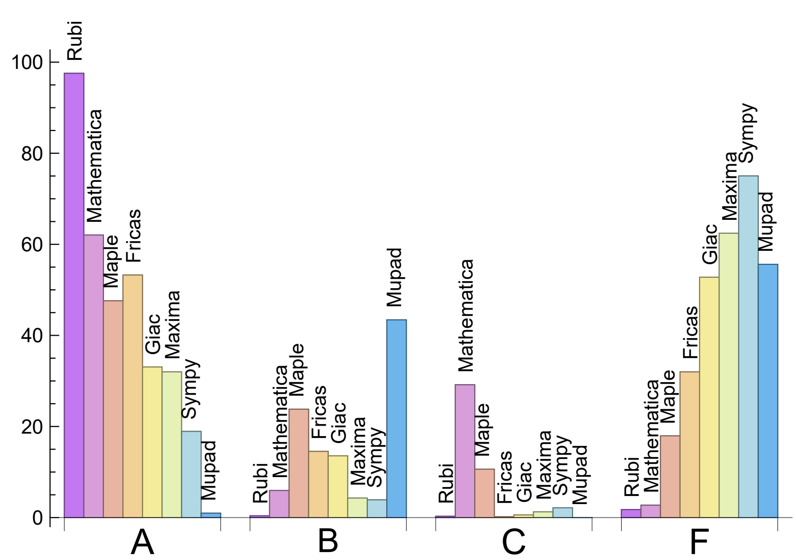
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.56	0.39	0.29	1.76
Mathematica	62.05	5.95	29.17	2.73
Maple	47.61	23.80	10.63	17.95
Maxima	32.00	4.29	1.27	62.44
Fricas	53.27	14.54	0.20	32.00
Sympy	18.93	3.90	2.15	75.02
Giac	33.07	13.56	0.59	52.78
Mupad	0.98	43.41	0.00	55.61

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	18	100.00 %	0.00 %	0.00 %
Mathematica	28	100.00 %	0.00 %	0.00 %
Maple	184	98.91 %	0.00 %	1.09 %
Maxima	640	97.03 %	0.16 %	2.81 %
Fricas	328	71.34 %	24.39 %	4.27 %
Sympy	769	83.62 %	16.38 %	0.00 %
Giac	541	71.35 %	12.01 %	16.64 %
Mupad	570	91.75 %	8.25 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

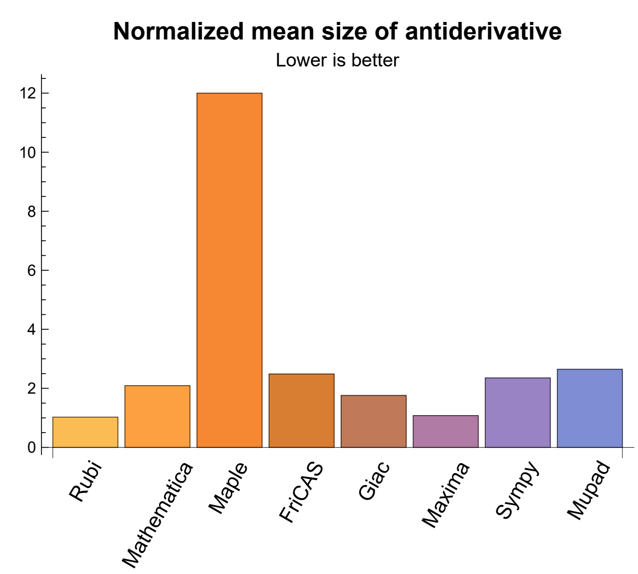
1.3 Performance

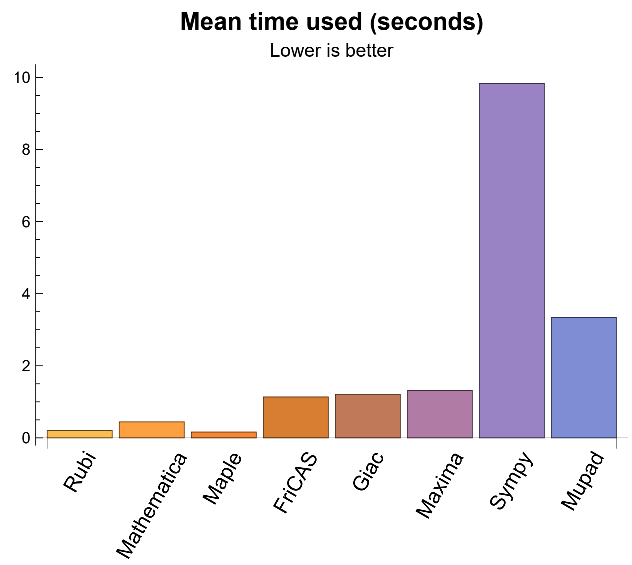
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.20	142.30	1.03	84.00	1.00
Mathematica	0.44	300.70	2.09	86.00	1.00
Maple	0.16	1481.73	12.00	93.00	1.29
Maxima	1.31	95.05	1.08	43.00	0.91
Fricas	1.14	229.05	2.49	75.00	1.40
Sympy	9.83	196.39	2.35	56.00	0.96
Giac	1.21	199.70	1.76	51.00	1.11
Mupad	3.35	228.86	2.65	50.00	1.05

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{901, 902, 903, 904, 905, 906, 907, 908, 909, 910}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {726}

Mathematica {1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 160, 161, 162, 163, 164, 165, 166, 167, 196, 202, 203, 204, 205, 226, 355, 451, 452, 560, 561, 562, 754, 755, 764, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 778, 780, 785, 786, 790, 791, 795, 797, 798, 799, 801, 802, 804, 885, 1009, 1010, 1018, 1024, 1025}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

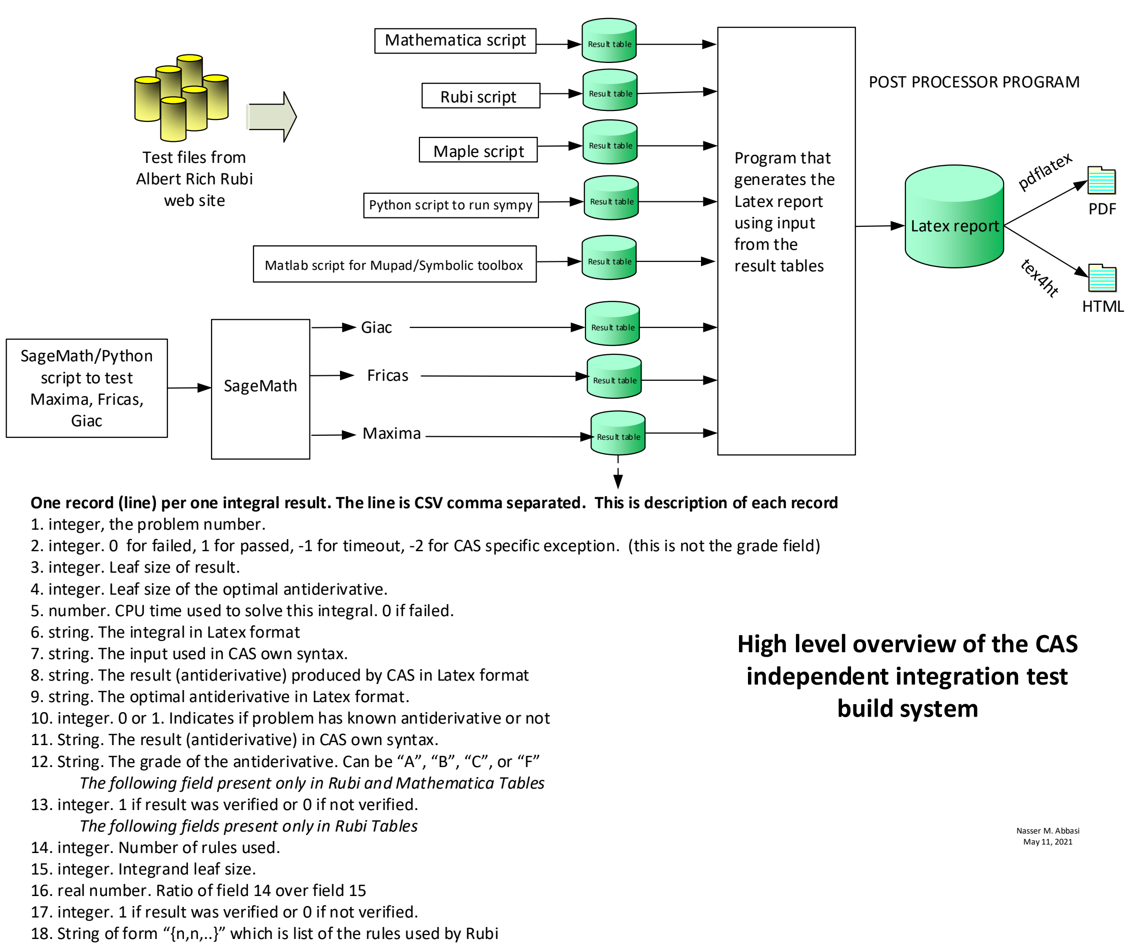
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 29, 30, 31, 32, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1019, 1020, 1021, 1022, 1024 }

B grade: { 413, 726, 997, 1025 }

C grade: { 396, 941, 1018 }

F grade: { 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 197, 616, 617, 995, 996, 1017, 1023 }

2.1.2 Mathematica

A grade: { 23, 24, 25, 26, 29, 30, 31, 32, 36, 37, 38, 39, 74, 75, 76, 77, 78, 79, 80, 81, 82, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 243, 244, 245, 247, 249, 250, 251, 252, 253, 259, 263, 264, 265, 266, 267, 268, 269, 272, 273, 276, 277, 279, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 304, 305, 307, 308, 310, 311, 312, 318, 319, 320, 321, 323, 324, 327, 331, 332, 333, 334, 335, 336, 337, 344, 345, 346, 347, 348, 349, 352, 353, 356, 358, 359, 360, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 388, 390, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 530, 531, 534, 535, 538, 539, 540, 541, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 560, 561, 563, 564, 565, 566, 567, 568, 569, 570, 572, 573, 575, 576, 578, 579, 581, 582, 583, 584, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 664, 665, 666, 667, 670, 671, 672, 673, 674, 678, 679, 680, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 738, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 762, 763, 805, 806, 807, 808, 809, 810, 811, 812, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 848, 849, 850, 851, 854, 856, 858, 859, 860, 862, 863, 864, 866, 868, 870, 872, 874, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 915, 920, 921, 922, 923, 924, 925, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 991, 992, 993, 994, 999, 1000, 1001, 1007, 1008, 1016, 1020, 1023 }

B grade: { 261, 322, 408, 413, 524, 562, 623, 624, 681, 682, 737, 739, 740, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 846, 847, 852, 853, 855, 857, 861, 869, 871, 873, 875, 918, 919, 926, 927, 961, 985, 986, 987, 988, 989, 990, 996, 1011, 1012, 1013, 1014, 1015, 1021, 1022 }

C grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 241, 242, 246, 248, 254, 255, 256, 257, 258, 260, 262, 270, 271, 274, 275, 278, 283, 284, 285, 286, 287, 288, 302, 303, 306, 309, 313, 314, 315, 316, 317, 325, 326, 328, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 354, 355, 357, 361, 362, 363, 364, 365, 378, 380, 381, 383, 386, 387, 389, 391, 392, 397, 431, 442, 523, 525, 526, 527, 532, 533, 536, 537, 571, 574, 577, 580, 601, 603, 662, 663, 668, 669, 675, 676, 677, 694, 711, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 796, 797, 798, 799, 800, 801, 802, 803, 804, 813, 865, 867, 916, 917, 942, 984, 998, 1002, 1003, 1004, 1005, 1006, 1009, 1010, 1018, 1019, 1024, 1025 }

F grade: { 18, 19, 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 172, 173, 195, 197, 528, 529, 542, 543, 558, 559, 587, 913, 914, 995, 1017 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 10, 11, 12, 13, 14, 15, 16, 17, 53, 54, 55, 57, 58, 59, 66, 68, 84, 85, 86, 93, 127, 128, 129, 130, 137, 138, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 176, 196, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 259, 261, 270, 271, 272, 273, 274, 275, 285, 286, 287, 289, 290, 291, 292, 294, 295, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 340, 350, 351, 352, 353, 354, 362, 363, 364, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 387, 390, 392, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 423, 425, 426, 427, 428, 429, 430, 431, 438, 439, 440, 441, 444, 445, 446, 448, 450, 451, 454, 455, 456, 474, 475, 530, 531, 538, 539, 565, 566, 567, 569, 570, 571, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 595, 596, 597, 598, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 630, 632, 633, 634, 636, 637, 639, 640, 641, 643, 644, 646, 647, 648, 649, 650, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 682, 685, 686, 687, 689, 691, 692, 693, 694, 695, 696, 698, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 733, 735, 736, 738, 741, 742, 743, 744, 748, 749, 750, 751, 752, 756, 757, 758, 762, 763, 805, 806, 807, 808, 809, 810, 813, 814, 815, 817, 818, 819, 820, 821, 822, 823, 824, 825, 828, 829, 830, 833, 834, 835, 836, 837, 839, 841, 842, 844, 845, 847, 848, 849, 850, 851, 852, 853, 854, 856, 858, 860, 864, 865, 867, 868, 870, 872, 874, 877, 879, 882, 883, 884, 885, 886, 887, 888, 889, 890, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 967, 969, 970, 971, 972, 975, 979, 980, 981, 982, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1006, 1007, 1020, 1023 }

B grade: { 9, 52, 56, 64, 65, 67, 73, 83, 91, 92, 94, 95, 100, 125, 126, 135, 136, 139, 174, 175, 178, 179, 180, 182, 183, 184, 232, 260, 262, 263, 264, 265, 266, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 283, 284, 288, 296, 297, 298, 299, 300, 301, 307, 308, 309, 310, 311, 312, 318, 319, 320, 321, 322, 323, 324, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 355, 356, 357, 358, 359, 360, 361, 365, 369, 408, 413, 422, 424, 432, 433, 442, 443, 447, 449, 452, 457, 458, 459, 473, 476, 477, 478, 487, 523, 524, 525, 528, 529, 534, 535, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 568, 572, 601, 629, 631, 635, 638, 642, 645, 651, 652, 681, 683, 684, 688, 690, 697, 699, 700, 709, 726, 729, 730, 731, 732, 734, 737, 739, 740, 745, 746, 747, 753, 754, 755, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 811, 812, 816, 826, 827, 831, 832, 840, 843, 846, 855, 857, 859, 861, 862, 863, 869, 871, 873, 875, 878, 880, 881, 891, 918, 919, 920, 921, 965, 966, 968, 973, 974, 976, 977, 978, 983, 984, 997, 998, 1015, 1018, 1021, 1022 }

C grade: { 21, 22, 43, 44, 45, 46, 51, 74, 75, 76, 77, 82, 101, 102, 103, 104, 113, 114, 115, 116, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 226, 293, 378, 380, 381, 383, 386, 388, 389, 391, 406, 407, 409, 410, 434, 435, 436, 437, 468, 469, 470, 485, 486, 526, 527, 532, 533, 536, 537, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 796, 797, 798, 799, 800, 801, 802, 803, 804, 838, 866, 930, 954, 1002, 1003, 1004, 1005, 1009, 1010, 1016, 1017, 1019, 1024, 1025 }

F grade: { 5, 6, 7, 8, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 87, 88, 89, 90, 96, 97, 98, 99, 105, 106, 107, 108, 109, 110, 111, 112, 117, 118, 119, 120, 121, 122, 123, 124, 131, 132, 133, 134, 140, 141, 142, 143, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 224, 225, 253, 254, 255, 256, 257, 258, 393, 453, 460, 461, 462, 463, 464, 465, 466, 467, 471, 472, 479, 480, 481, 482, 483, 484, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 562, 563, 564, 587, 588, 589, 590, 591, 592, 593, 594, 653, 654, 655, 656, 657, 876, 911, 912, 913, 914, 942, 995, 996, 999, 1000, 1001, 1008, 1011, 1012, 1013, 1014 }

2.1.4 Maxima

A grade: { 174, 175, 176, 179, 180, 227, 228, 229, 230, 231, 233, 234, 235, 236, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 263, 264, 265, 266, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 290, 293, 296, 297, 298, 299, 300, 301, 307, 308, 309, 310, 311, 312, 318, 319, 321, 322, 331, 332, 333, 334, 335, 336, 344, 345, 347, 348, 355, 356, 357, 358, 359, 370, 371, 372, 373, 374, 375, 376, 379, 384, 385, 396, 398, 399, 400, 419, 420, 421, 422, 423, 424, 425, 443, 444, 445, 446, 447, 448, 449, 450, 454, 455, 456, 530, 531, 538, 539, 544, 545, 546, 552, 553, 554, 565, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 646, 647, 648, 649, 654, 655, 656, 670, 676, 677, 679, 680, 681, 683, 684, 685, 686, 693, 694, 695, 696, 697, 698, 699, 700, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 724, 725, 733, 734, 735, 737, 738, 739, 740, 741, 742, 743, 744, 745, 747, 748, 749, 762, 763, 805, 808, 809, 810, 814, 815, 816, 817, 818, 819, 820, 822, 824, 825, 833, 835, 836, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 852, 854, 856, 857, 858, 859, 860, 861, 862, 864, 865, 866, 867, 868, 870, 872, 874, 883, 884, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 935, 936, 937, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 952, 953, 954, 959, 960, 961, 963, 964, 971, 972, 975, 977, 979, 980, 981, 982, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1007, 1020, 1023 }

B grade: { 178, 182, 183, 184, 232, 238, 320, 323, 324, 337, 346, 349, 360, 369, 564, 566, 574, 580, 612, 613, 614, 615, 616, 617, 645, 653, 675, 678, 729, 730, 731, 732, 736, 746, 750, 829, 880, 948, 957, 958, 974, 976, 978, 1015 }

C grade: { 581, 582, 583, 584, 585, 586, 595, 596, 597, 598, 599, 600, 855 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 237, 239, 240, 241, 242, 243, 245, 255, 256, 257, 258, 259, 260, 261, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 289, 291, 292, 294, 295, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 352, 353, 354, 361, 362, 363, 364, 365, 366, 367, 368, 377, 378, 380, 381, 382, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 451, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 532, 533, 534, 535, 536, 537, 540, 541, 542, 543, 547, 548, 549, 550, 551, 555, 556, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 601, 629, 630, 631, 650, 651, 652, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 682, 687, 688, 689, 690, 691, 692, 701, 702, 703, 704, 705, 706, 710, 720, 721, 723, 726, 727, 728, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 806, 807, 811, 812, 813, 821, 823, 826, 827, 828, 830, 831, 832, 834, 837, 838, 850, 851, 853, 863, 869, 871, 873, 875, 876, 877, 878, 879, 881, 882, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 918, 919, 920, 921, 930, 931, 932, 933, 934, 938, 939, 940, 955, 956, 962, 965, 966, 967, 968, 969, 970, 973, 983, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1021, 1022, 1024, 1025 }

2.1.5 FriCAS

A grade: { 77, 103, 104, 105, 107, 109, 111, 113, 114, 117, 119, 121, 123, 198, 199, 200, 201, 202, 203, 204, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 259, 261, 263, 264, 265, 266, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 307, 308, 309, 311, 312, 318, 319, 320, 323, 324, 331, 332, 333, 334, 335, 336, 337, 344, 345, 346, 349, 355, 356, 359, 366, 369, 370, 371, 372, 373, 376, 377, 379, 382, 384, 385, 390, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 454, 455, 456, 457, 460, 461, 462, 463, 464, 466, 467, 468, 469, 470, 471, 473, 474, 475, 476, 479, 480, 481, 482, 485, 486, 487, 490, 491, 492, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 514, 515, 516, 518, 519, 520, 522, 524, 525, 528, 529, 530, 531, 534, 535, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 551, 552, 553, 555, 556, 564, 565, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 581, 582, 583, 584, 585, 586, 595, 596, 597, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 646, 647, 648, 649, 656, 658, 659, 660, 661, 662, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 685, 686, 691, 692, 693, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 723, 724, 725, 727, 728, 731, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 751, 752, 754, 755, 756, 757, 758, 760, 761, 762, 763, 805, 806, 807, 808, 809, 810, 811, 812, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 835, 836, 837, 838, 848, 849, 850, 851, 852, 853, 854, 856, 858, 860, 862, 867, 872, 874, 877, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 915, 916, 917, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 959, 960, 961, 962, 963, 964, 965, 966, 967, 969, 970, 971, 972, 975, 977, 979, 980, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 1004, 1005, 1007, 1009, 1010, 1011, 1012, 1013, 1014, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025 }

B grade: { 21, 22, 43, 44, 45, 46, 51, 74, 75, 76, 82, 101, 102, 106, 108, 110, 112, 115, 116, 118, 120, 122, 124, 174, 175, 176, 178, 179, 180, 182, 183, 184, 205, 232, 238, 310, 321, 322, 347, 348, 357, 358, 360, 367, 368, 374, 375, 388, 408, 413, 422, 447, 458, 459, 465, 477, 478, 483, 484, 526, 527, 532, 533, 536, 537, 545, 546, 550, 554, 566, 574, 580, 598, 629, 630, 631, 644, 645, 650, 651, 652, 653, 654, 655, 663, 681, 682, 683, 684, 687, 688, 689, 690, 694, 710, 720, 721, 726, 729, 730, 732, 750, 753, 759, 813, 832, 833, 834, 839, 840, 841, 842, 843, 844, 845, 846, 847, 859, 861, 863, 864, 865, 866, 868, 869, 870, 871, 873, 875, 878, 879, 918, 919, 935, 957, 958, 968, 973, 974, 976, 978, 981, 997, 998, 999, 1006, 1015, 1016, 1017 }

C grade: { 855, 857 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 253, 254, 255, 256, 257, 258, 260, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 352, 353, 354, 361, 362, 363, 364, 365, 378, 380, 381, 383, 386, 387, 389, 391, 392, 393, 394, 395, 396, 397, 453, 472, 488, 489, 493, 494, 499, 500, 505, 506, 510, 511, 513, 517, 521, 523, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 601, 613, 614, 615, 616, 617, 657, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 876, 913, 914, 930, 996, 1000, 1001, 1002, 1003, 1008 }

2.1.6 Sympy

A grade: { 23, 24, 25, 29, 30, 31, 36, 37, 38, 152, 153, 154, 155, 156, 157, 158, 159, 168, 169, 170, 171, 174, 175, 176, 180, 206, 212, 244, 245, 250, 251, 252, 289, 290, 379, 388, 403, 408, 412, 413, 421, 422, 443, 446, 447, 452, 454, 455, 456, 504, 524, 525, 527, 531, 537, 539, 545, 546, 547, 564, 565, 567, 569, 570, 571, 573, 575, 576, 577, 580, 602, 606, 607, 608, 609, 610, 611, 612, 618, 619, 620, 621, 622, 634, 635, 636, 641, 642, 643, 658, 659, 662, 663, 664, 665, 668, 669, 670, 676, 677, 678, 679, 680, 681, 682, 683, 684, 686, 687, 688, 689, 694, 696, 697, 698, 699, 700, 707, 708, 709, 711, 722, 724, 725, 726, 729, 731, 735, 806, 808, 809, 810, 811, 812, 814, 815, 817, 818, 819, 820, 829, 833, 835, 836, 839, 841, 848, 849, 850, 852, 854, 856, 858, 860, 862, 868, 870, 872, 874, 878, 879, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 917, 924, 928, 936, 937, 939, 943, 944, 945, 946, 948, 949, 950, 952, 953, 954, 966, 977, 984, 1020, 1023 }

B grade: { 177, 181, 197, 416, 417, 418, 468, 469, 470, 487, 498, 526, 530, 532, 533, 536, 538, 566, 603, 604, 605, 623, 685, 695, 714, 762, 763, 837, 840, 843, 846, 877, 933, 957, 958, 959, 963, 964, 967, 1007 }

C grade: { 26, 32, 39, 207, 208, 209, 213, 214, 215, 221, 222, 380, 588, 589, 590, 693, 941, 942, 951, 965, 971, 972 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 160, 161, 162, 163, 164, 165, 166, 167, 172, 173, 178, 179, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 210, 211, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 409, 410, 411, 414, 415, 419, 420, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 444, 445, 448, 449, 450, 451, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 528, 529, 534, 535, 540, 541, 542, 543, 544, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 568, 572, 574, 578, 579, 581, 582, 583, 584, 585, 586, 587, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 613, 614, 615, 616, 617, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 637, 638, 639, 640, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 660, 661, 666, 667, 671, 672, 673, 674, 675, 690, 691, 692, 701, 702, 703, 704, 705, 706, 710, 712, 713, 715, 716, 717, 718, 719, 720, 721, 723, 727, 728, 730, 732, 733, 734, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 807, 813, 816, 821, 822, 823, 824, 825, 826, 827, 828, 830, 831, 832, 834, 838, 842, 844, 845, 847, 851, 853, 855, 857, 859, 861, 863, 864, 865, 866, 867, 869, 871, 873, 875, 876, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 918, 919, 920, 921, 922, 923, 925, 926, 927, 929, 930, 931, 932, 934, 935, 938, 940, 947, 955, 956, 960, 961, 962, 968, 969, 970, 973, 974, 975, 976, 978, 979, 980, 981, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1021, 1022, 1024, 1025 }

2.1.7 Giac

A grade: { 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 259, 261, 263, 264, 265, 289, 290, 291, 293, 295, 318, 319, 344, 345, 367, 369, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 390, 398, 399, 400, 403, 409, 416, 417, 418, 452, 454, 455, 456, 473, 474, 475, 530, 531, 538, 539, 544, 545, 546, 547, 548, 550, 551, 552, 553, 554, 555, 556, 564, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 602, 603, 604, 605, 606, 607, 608, 609, 610, 618, 619, 620, 622, 623, 624, 629, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 670, 675, 676, 677, 681, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 716, 718, 720, 721, 722, 723, 724, 725, 727, 728, 731, 733, 734, 735, 736, 737, 738, 739, 741, 742, 743, 744, 745, 748, 749, 755, 756, 762, 763, 806, 807, 808, 809, 810, 814, 816, 817, 818, 819, 820, 821, 822, 823, 830, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 844, 845, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 865, 868, 872, 874, 877, 882, 883, 893, 894, 895, 896, 897, 898, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 959, 960, 961, 962, 963, 964, 965, 966, 967, 970, 971, 972, 973, 974, 975, 976, 977, 979, 980, 981, 982, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1007, 1020 }

B grade: { 174, 175, 176, 178, 179, 180, 183, 184, 227, 267, 268, 269, 320, 322, 323, 324, 331, 332, 333, 346, 348, 349, 355, 356, 357, 368, 388, 401, 402, 404, 405, 406, 407, 408, 410, 411, 412, 413, 414, 415, 419, 420, 421, 422, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 449, 566, 582, 583, 584, 585, 586, 621, 625, 626, 627, 628, 630, 631, 651, 652, 653, 654, 655, 656, 678, 679, 680, 682, 714, 715, 719, 726, 729, 730, 732, 740, 746, 747, 750, 751, 753, 754, 757, 758, 759, 760, 761, 813, 815, 826, 827, 828, 831, 832, 843, 846, 867, 870, 878, 879, 880, 881, 918, 919, 957, 958, 968, 969, 978, 983, 997, 998, 1006, 1015, 1021, 1022, 1023 }

C grade: { 294, 596, 600, 824, 825, 892 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 181, 182, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 255, 256, 257, 258, 260, 262, 266, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 321, 325, 326, 327, 328, 329, 330, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 347, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 370, 378, 380, 381, 383, 386, 387, 389, 391, 392, 393, 394, 395, 396, 397, 423, 425, 442, 448, 450, 451, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 532, 533, 534, 535, 536, 537, 540, 541, 542, 543, 549, 557, 558, 559, 560, 561, 562, 563, 581, 587, 588, 589, 590, 591, 592, 593, 594, 595, 597, 598, 599, 601, 611, 612, 613, 614, 615, 616, 617, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 717, 752, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 811, 812, 829, 863, 864, 866, 869, 871, 873, 875, 876, 884, 885, 886, 887, 888, 889, 890, 891, 899, 911, 912, 913, 914, 920, 921, 941, 942, 995, 996, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1024, 1025 }

2.1.8 Mupad

A grade: { 901, 902, 903, 904, 905, 906, 907, 908, 909, 910 }

B grade: { 14, 15, 16, 17, 43, 44, 45, 46, 47, 48, 49, 50, 51, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 92, 93, 94, 95, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 178, 179, 180, 182, 183, 184, 198, 199, 200, 201, 202, 203, 204, 205, 209, 215, 221, 222, 227, 230, 232, 238, 244, 245, 247, 249, 250, 251, 252, 289, 290, 291, 292, 293, 320, 333, 346, 357, 369, 370, 379, 384, 385, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 455, 456, 498, 504, 516, 518, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 536, 537, 538, 539, 544, 545, 546, 547, 548, 552, 553, 554, 555, 556, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 590, 595, 596, 597, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 656, 670, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 707, 708, 709, 711, 712, 722, 724, 725, 726, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 750, 751, 762, 763, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 824, 825, 826, 827, 828, 830, 831, 833, 835, 836, 837, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 854, 855, 856, 857, 858, 859, 860, 861, 862, 865, 867, 868, 870, 872, 874, 876, 877, 878, 879, 880, 881, 884, 885, 886, 887, 888, 889, 890, 900, 915, 916, 917, 918, 919, 922, 923, 924, 925, 928, 930, 931, 933, 935, 936, 937, 939, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 957, 958, 959, 960, 963, 964, 965, 966, 967, 968, 969, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 997, 1007, 1015, 1020, 1021, 1022, 1023 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 231, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 246, 248, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 371, 372, 373, 374, 375, 376, 377, 378, 380, 381, 382, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 453, 454, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 519, 520, 521, 522, 523, 534, 535, 540, 541, 542, 543, 549, 550, 551, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 591, 592, 593, 594, 598, 601, 613, 614, 625, 626, 627, 629, 630, 631, 646, 647, 648, 650, 651, 652, 653, 654, 655, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 701, 702, 703, 704, 705, 706, 710, 713, 714, 715, 716, 717, 718, 719, 720, 721, 723, 727, 728, 749, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 817, 818, 819, 820, 821, 822, 823, 829, 832, 834, 838, 850, 851, 852, 853, 863, 864, 866, 869, 871, 873, 875, 882, 883, 891, 892, 893, 894, 895, 896, 897, 898, 899, 911, 912, 913, 914, 920, 921, 926, 927, 929, 932, 934, 938, 940, 955, 956, 961, 962, 970, 995, 996, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1024, 1025 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	148	139	0	0	0	0	-1
normalized size	1	1.00	1.02	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.186	0.152	0.000	0.902	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	148	143	0	0	0	0	-1
normalized size	1	1.00	0.92	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.124	0.092	0.000	0.671	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	146	143	0	0	0	0	-1
normalized size	1	1.00	0.90	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.157	0.057	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	150	139	0	0	0	0	-1
normalized size	1	1.00	0.96	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.113	0.053	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	164	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	0.207	0.101	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	166	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	0.208	0.074	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	167	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.344	0.124	0.060	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	167	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.146	0.061	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	169	495	0	0	0	0	-1
normalized size	1	1.00	0.68	1.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	0.208	0.206	0.000	1.800	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	136	132	0	0	0	0	-1
normalized size	1	1.00	0.93	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.210	0.053	0.000	1.310	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	136	143	0	0	0	0	-1
normalized size	1	1.00	0.83	0.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.125	0.056	0.000	1.039	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	134	132	0	0	0	0	-1
normalized size	1	1.00	0.80	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.182	0.045	0.000	0.947	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	138	139	0	0	0	0	-1
normalized size	1	1.00	0.88	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.107	0.052	0.000	1.287	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	331	128	123	0	0	0	0	164
normalized size	1	1.01	0.39	0.37	0.00	0.00	0.00	0.00	0.50
time (sec)	N/A	0.663	0.070	0.020	0.000	1.248	0.000	0.000	0.216

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	382	128	133	0	0	0	0	180
normalized size	1	1.01	0.34	0.35	0.00	0.00	0.00	0.00	0.47
time (sec)	N/A	0.723	0.080	0.026	0.000	1.287	0.000	0.000	2.420

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	376	126	124	0	0	0	0	164
normalized size	1	1.01	0.34	0.33	0.00	0.00	0.00	0.00	0.44
time (sec)	N/A	0.577	0.073	0.021	0.000	1.146	0.000	0.000	0.050

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	340	342	130	133	0	0	0	0	179
normalized size	1	1.01	0.38	0.39	0.00	0.00	0.00	0.00	0.53
time (sec)	N/A	0.575	0.067	0.025	0.000	1.067	0.000	0.000	0.046

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.076	0.075	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.084	0.061	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.064	0.059	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	C	F	B	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	0	0	3064	0	712	0	0	-1
normalized size	1	0.00	0.00	20.84	0.00	4.84	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.073	42.609	0.000	11.821	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	C	F	B	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	0	0	3250	0	720	0	0	-1
normalized size	1	0.00	0.00	20.44	0.00	4.53	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.085	41.368	0.000	10.945	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	498	163	0	0	0	212	0	-1
normalized size	1	1.29	0.42	0.00	0.00	0.00	0.55	0.00	-0.00
time (sec)	N/A	0.397	0.196	0.079	0.000	0.000	4.952	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	297	142	0	0	0	160	0	-1
normalized size	1	1.23	0.59	0.00	0.00	0.00	0.66	0.00	-0.00
time (sec)	N/A	0.305	0.135	0.057	0.000	0.000	3.917	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	245	111	0	0	0	114	0	-1
normalized size	1	1.28	0.58	0.00	0.00	0.00	0.59	0.00	-0.01
time (sec)	N/A	0.204	0.080	0.061	0.000	130.281	3.238	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	207	75	0	0	0	82	0	-1
normalized size	1	1.34	0.48	0.00	0.00	0.00	0.53	0.00	-0.01
time (sec)	N/A	0.150	0.026	0.052	0.000	16.488	2.659	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	435	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.082	0.331	0.079	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	818	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.079	0.211	0.069	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	392	0	0	0	206	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.66	0.00	-0.00
time (sec)	N/A	0.178	0.464	0.061	0.000	0.000	5.166	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	287	0	0	0	155	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.61	0.00	-0.00
time (sec)	N/A	0.138	0.391	0.055	0.000	0.000	4.314	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	201	0	0	0	110	0	-1
normalized size	1	1.00	1.37	0.00	0.00	0.00	0.75	0.00	-0.01
time (sec)	N/A	0.102	0.168	0.061	0.000	78.715	3.330	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	163	0	0	0	78	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.63	0.00	-0.01
time (sec)	N/A	0.066	0.122	0.049	0.000	35.892	2.270	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.048	0.064	0.058	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	761	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.083	0.335	0.063	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1513	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.083	0.477	0.066	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	416	166	0	0	0	204	0	-1
normalized size	1	1.36	0.54	0.00	0.00	0.00	0.67	0.00	-0.00
time (sec)	N/A	0.268	0.144	0.062	0.000	0.000	5.155	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	239	145	0	0	0	153	0	-1
normalized size	1	1.28	0.78	0.00	0.00	0.00	0.82	0.00	-0.01
time (sec)	N/A	0.237	0.103	0.059	0.000	146.456	4.178	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	195	95	0	0	0	109	0	-1
normalized size	1	1.38	0.67	0.00	0.00	0.00	0.77	0.00	-0.01
time (sec)	N/A	0.148	0.045	0.057	0.000	54.679	3.333	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	172	78	0	0	0	78	0	-1
normalized size	1	1.42	0.64	0.00	0.00	0.00	0.64	0.00	-0.01
time (sec)	N/A	0.115	0.036	0.048	0.000	7.788	2.305	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	332	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.091	0.042	0.057	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	760	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.085	0.317	0.061	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1357	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.083	0.437	0.074	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	326	258	0	75	0	0	70
normalized size	1	1.00	8.81	6.97	0.00	2.03	0.00	0.00	1.89
time (sec)	N/A	0.105	0.439	0.069	0.000	1.565	0.000	0.000	3.593

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	327	253	0	76	0	0	74
normalized size	1	1.00	8.18	6.32	0.00	1.90	0.00	0.00	1.85
time (sec)	N/A	0.123	0.372	0.061	0.000	1.385	0.000	0.000	3.631

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	325	262	0	238	0	0	62
normalized size	1	1.00	8.55	6.89	0.00	6.26	0.00	0.00	1.63
time (sec)	N/A	0.112	0.304	0.059	0.000	1.388	0.000	0.000	2.861

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	328	249	0	241	0	0	63
normalized size	1	1.00	8.41	6.38	0.00	6.18	0.00	0.00	1.62
time (sec)	N/A	0.114	0.291	0.056	0.000	1.074	0.000	0.000	2.843

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	325	0	0	0	0	0	106
normalized size	1	1.00	5.16	0.00	0.00	0.00	0.00	0.00	1.68
time (sec)	N/A	0.179	1.116	0.368	0.000	0.000	0.000	0.000	5.812

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	336	0	0	0	0	0	107
normalized size	1	1.00	5.17	0.00	0.00	0.00	0.00	0.00	1.65
time (sec)	N/A	0.199	1.135	0.353	0.000	0.000	0.000	0.000	5.851

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	390	0	0	0	0	0	102
normalized size	1	1.00	5.91	0.00	0.00	0.00	0.00	0.00	1.55
time (sec)	N/A	0.201	0.473	0.153	0.000	0.000	0.000	0.000	3.677

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	375	0	0	0	0	0	103
normalized size	1	1.00	5.68	0.00	0.00	0.00	0.00	0.00	1.56
time (sec)	N/A	0.193	0.681	0.156	0.000	0.000	0.000	0.000	3.647

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	373	889	0	300	0	0	95
normalized size	1	1.00	7.61	18.14	0.00	6.12	0.00	0.00	1.94
time (sec)	N/A	0.123	1.097	0.073	0.000	1.042	0.000	0.000	4.731

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	336	262	0	0	0	0	-1
normalized size	1	1.00	2.13	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.518	0.044	0.000	0.990	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	335	257	0	0	0	0	-1
normalized size	1	1.00	1.94	1.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.480	0.045	0.000	0.546	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	333	266	0	0	0	0	-1
normalized size	1	1.00	1.89	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.331	0.036	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	338	253	0	0	0	0	-1
normalized size	1	1.00	2.00	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.229	0.341	0.036	0.000	0.511	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	340	264	0	0	0	0	-1
normalized size	1	1.00	2.14	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.475	0.039	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	340	261	0	0	0	0	-1
normalized size	1	1.00	1.94	1.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	0.499	0.040	0.000	0.702	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	338	270	0	0	0	0	-1
normalized size	1	1.00	1.90	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.353	0.036	0.000	0.692	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	342	255	0	0	0	0	-1
normalized size	1	1.00	2.01	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.461	0.039	0.000	0.709	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	336	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.409	1.541	0.113	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	399	0	0	0	0	0	-1
normalized size	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.430	1.284	0.108	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	400	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	0.430	0.099	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	387	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.398	1.028	0.105	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	380	900	0	0	0	0	-1
normalized size	1	1.00	1.43	3.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	1.495	0.013	0.000	1.341	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	207	258	0	0	0	0	-1
normalized size	1	1.00	1.43	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.357	0.037	0.000	0.688	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	209	253	0	0	0	0	-1
normalized size	1	1.00	1.31	1.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.241	0.326	0.038	0.000	0.645	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	207	262	0	0	0	0	-1
normalized size	1	1.00	1.27	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.189	0.032	0.000	0.940	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	209	249	0	0	0	0	-1
normalized size	1	1.00	1.34	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	0.233	0.032	0.000	0.870	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	324	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	1.078	0.095	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	388	0	0	0	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.888	0.102	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	389	0	0	0	0	0	-1
normalized size	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.387	0.318	0.094	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	375	0	0	0	0	0	-1
normalized size	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.370	0.713	0.093	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	372	892	0	0	0	0	-1
normalized size	1	1.00	1.51	3.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.271	1.069	0.013	0.000	1.107	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	240	0	44	0	0	205
normalized size	1	1.00	2.00	10.43	0.00	1.91	0.00	0.00	8.91
time (sec)	N/A	0.059	0.009	0.045	0.000	0.745	0.000	0.000	0.225

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	54	240	0	47	0	0	221
normalized size	1	1.00	2.00	8.89	0.00	1.74	0.00	0.00	8.19
time (sec)	N/A	0.065	0.012	0.038	0.000	0.875	0.000	0.000	0.184

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	240	0	40	0	0	205
normalized size	1	1.00	1.00	9.60	0.00	1.60	0.00	0.00	8.20
time (sec)	N/A	0.059	0.008	0.042	0.000	0.501	0.000	0.000	2.551

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	240	0	38	0	0	221
normalized size	1	1.00	1.00	9.60	0.00	1.52	0.00	0.00	8.84
time (sec)	N/A	0.067	0.009	0.037	0.000	0.486	0.000	0.000	2.529

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	51	0	0	0	0	0	65
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	1.30
time (sec)	N/A	0.132	0.024	0.132	0.000	0.000	0.000	0.000	3.444

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	53	0	0	0	0	0	67
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	1.29
time (sec)	N/A	0.138	0.028	0.128	0.000	0.000	0.000	0.000	3.591

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0	74
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	1.40
time (sec)	N/A	0.142	0.027	0.119	0.000	0.000	0.000	0.000	5.453

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0	78
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	1.47
time (sec)	N/A	0.141	0.023	0.122	0.000	0.000	0.000	0.000	5.373

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	650	0	294	0	0	67
normalized size	1	1.00	1.00	14.13	0.00	6.39	0.00	0.00	1.46
time (sec)	N/A	0.116	0.038	0.195	0.000	0.866	0.000	0.000	3.126

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	273	246	0	0	0	0	327
normalized size	1	1.00	1.96	1.77	0.00	0.00	0.00	0.00	2.35
time (sec)	N/A	0.150	0.316	0.011	0.000	0.636	0.000	0.000	2.711

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	271	246	0	0	0	0	359
normalized size	1	1.00	1.77	1.61	0.00	0.00	0.00	0.00	2.35
time (sec)	N/A	0.161	0.287	0.011	0.000	0.518	0.000	0.000	0.190

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	269	246	0	0	0	0	327
normalized size	1	1.00	1.72	1.58	0.00	0.00	0.00	0.00	2.10
time (sec)	N/A	0.146	0.190	0.012	0.000	0.671	0.000	0.000	0.120

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	275	246	0	0	0	0	359
normalized size	1	1.00	1.83	1.64	0.00	0.00	0.00	0.00	2.39
time (sec)	N/A	0.163	0.197	0.010	0.000	0.672	0.000	0.000	2.543

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	419	0	0	0	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.315	1.385	0.097	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	447	0	0	0	0	0	-1
normalized size	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.318	1.336	0.094	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	448	0	0	0	0	0	-1
normalized size	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	0.917	0.086	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	422	0	0	0	0	0	-1
normalized size	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.355	0.095	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	384	661	0	0	0	0	-1
normalized size	1	1.00	1.74	2.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	1.177	0.013	0.000	1.669	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	193	240	0	0	0	0	207
normalized size	1	1.00	1.50	1.86	0.00	0.00	0.00	0.00	1.60
time (sec)	N/A	0.142	0.257	0.010	0.000	1.414	0.000	0.000	0.057

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	195	240	0	0	0	0	224
normalized size	1	1.00	1.34	1.66	0.00	0.00	0.00	0.00	1.54
time (sec)	N/A	0.149	0.243	0.008	0.000	1.487	0.000	0.000	0.066

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	193	240	0	0	0	0	208
normalized size	1	1.00	1.30	1.62	0.00	0.00	0.00	0.00	1.41
time (sec)	N/A	0.135	0.108	0.010	0.000	1.071	0.000	0.000	2.534

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	195	240	0	0	0	0	223
normalized size	1	1.00	1.39	1.71	0.00	0.00	0.00	0.00	1.59
time (sec)	N/A	0.155	0.191	0.010	0.000	1.269	0.000	0.000	0.064

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	407	0	0	0	0	0	-1
normalized size	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	1.440	0.086	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	371	0	0	0	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	0.711	0.086	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	372	0	0	0	0	0	-1
normalized size	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	0.232	0.078	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	410	0	0	0	0	0	-1
normalized size	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	0.300	0.086	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	295	653	0	0	0	0	-1
normalized size	1	1.00	1.46	3.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.675	0.012	0.000	0.501	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	267	245	0	205	0	0	-1
normalized size	1	1.00	6.36	5.83	0.00	4.88	0.00	0.00	-0.02
time (sec)	N/A	0.114	0.446	0.085	0.000	0.506	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	269	243	0	207	0	0	-1
normalized size	1	1.00	5.85	5.28	0.00	4.50	0.00	0.00	-0.02
time (sec)	N/A	0.114	0.489	0.075	0.000	0.508	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	267	245	0	50	0	0	-1
normalized size	1	1.00	6.07	5.57	0.00	1.14	0.00	0.00	-0.02
time (sec)	N/A	0.104	0.379	0.063	0.000	0.495	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	269	247	0	59	0	0	-1
normalized size	1	1.00	6.11	5.61	0.00	1.34	0.00	0.00	-0.02
time (sec)	N/A	0.094	0.377	0.070	0.000	0.495	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	322	0	0	1240	0	0	-1
normalized size	1	1.00	4.67	0.00	0.00	17.97	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.631	0.223	0.000	1.775	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	446	0	0	1294	0	0	-1
normalized size	1	1.00	6.28	0.00	0.00	18.23	0.00	0.00	-0.01
time (sec)	N/A	0.192	1.437	0.219	0.000	1.764	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	447	0	0	1245	0	0	-1
normalized size	1	1.00	6.21	0.00	0.00	17.29	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.432	0.138	0.000	1.750	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	325	0	0	1303	0	0	-1
normalized size	1	1.00	4.51	0.00	0.00	18.10	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.530	0.145	0.000	1.670	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	663	0	0	1273	0	0	-1
normalized size	1	1.00	9.08	0.00	0.00	17.44	0.00	0.00	-0.01
time (sec)	N/A	0.199	1.276	0.195	0.000	1.176	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	648	0	0	1330	0	0	-1
normalized size	1	1.00	8.64	0.00	0.00	17.73	0.00	0.00	-0.01
time (sec)	N/A	0.201	1.176	0.186	0.000	1.171	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	649	0	0	1278	0	0	-1
normalized size	1	1.00	8.54	0.00	0.00	16.82	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.594	0.127	0.000	1.154	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	666	0	0	1339	0	0	-1
normalized size	1	1.00	8.76	0.00	0.00	17.62	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.781	0.136	0.000	1.153	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	269	245	0	50	0	0	-1
normalized size	1	1.00	6.40	5.83	0.00	1.19	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.435	0.033	0.000	0.474	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	267	247	0	59	0	0	-1
normalized size	1	1.00	5.80	5.37	0.00	1.28	0.00	0.00	-0.02
time (sec)	N/A	0.102	0.487	0.037	0.000	0.495	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	265	245	0	204	0	0	-1
normalized size	1	1.00	6.02	5.57	0.00	4.64	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.298	0.034	0.000	0.488	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	271	243	0	206	0	0	-1
normalized size	1	1.00	6.16	5.52	0.00	4.68	0.00	0.00	-0.02
time (sec)	N/A	0.087	0.349	0.028	0.000	0.471	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	320	0	0	1236	0	0	-1
normalized size	1	1.00	4.64	0.00	0.00	17.91	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.594	0.187	0.000	1.739	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	329	0	0	1288	0	0	-1
normalized size	1	1.00	4.63	0.00	0.00	18.14	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.817	0.189	0.000	1.722	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	330	0	0	1239	0	0	-1
normalized size	1	1.00	4.58	0.00	0.00	17.21	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.360	0.129	0.000	1.763	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	323	0	0	1299	0	0	-1
normalized size	1	1.00	4.49	0.00	0.00	18.04	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.489	0.136	0.000	1.673	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	667	0	0	1270	0	0	-1
normalized size	1	1.00	9.14	0.00	0.00	17.40	0.00	0.00	-0.01
time (sec)	N/A	0.176	1.215	0.174	0.000	1.171	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	649	0	0	1324	0	0	-1
normalized size	1	1.00	8.65	0.00	0.00	17.65	0.00	0.00	-0.01
time (sec)	N/A	0.185	1.153	0.179	0.000	1.172	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	650	0	0	1273	0	0	-1
normalized size	1	1.00	8.55	0.00	0.00	16.75	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.643	0.125	0.000	1.137	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	670	0	0	1335	0	0	-1
normalized size	1	1.00	8.82	0.00	0.00	17.57	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.716	0.129	0.000	1.143	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	269	245	0	0	0	0	-1
normalized size	1	1.00	1.86	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.416	0.036	0.000	0.482	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	267	245	0	0	0	0	-1
normalized size	1	1.00	1.84	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	0.416	0.038	0.000	0.491	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	291	260	0	0	0	0	-1
normalized size	1	1.00	1.68	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	0.629	0.039	0.000	0.559	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	291	264	0	0	0	0	-1
normalized size	1	1.00	1.56	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.277	0.579	0.041	0.000	0.553	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	289	262	0	0	0	0	-1
normalized size	1	1.00	1.52	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.474	0.040	0.000	0.564	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	293	258	0	0	0	0	-1
normalized size	1	1.00	1.60	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.236	0.495	0.039	0.000	0.562	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	438	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.540	1.829	0.116	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	466	0	0	0	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.568	1.671	0.115	0.000	32.479	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	467	0	0	0	0	0	-1
normalized size	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.491	0.577	0.102	0.000	32.385	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	441	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.475	0.928	0.112	0.000	32.578	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	209	255	0	0	0	0	-1
normalized size	1	1.00	1.54	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.495	0.036	0.000	0.493	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	232	257	0	0	0	0	-1
normalized size	1	1.00	1.53	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.658	0.037	0.000	0.487	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	230	255	0	0	0	0	-1
normalized size	1	1.00	1.40	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.269	0.034	0.000	0.479	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	211	253	0	0	0	0	-1
normalized size	1	1.00	1.35	1.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.183	0.033	0.000	0.487	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	225	255	0	0	0	0	-1
normalized size	1	1.00	1.53	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.574	0.036	0.000	0.502	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	427	0	0	0	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.407	1.253	0.115	0.000	1.266	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	454	0	0	0	0	0	-1
normalized size	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	1.349	0.105	0.000	1.258	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	455	0	0	0	0	0	-1
normalized size	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.441	0.431	0.092	0.000	1.243	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	430	0	0	0	0	0	-1
normalized size	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.414	0.983	0.102	0.000	1.247	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	319	214	275	0	0	0	0	-1
normalized size	1	1.01	0.68	0.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.240	0.641	0.069	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	331	235	264	0	0	0	0	-1
normalized size	1	1.01	0.71	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.298	0.764	0.066	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	327	233	273	0	0	0	0	-1
normalized size	1	1.01	0.72	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.783	0.257	0.068	0.000	3.906	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	323	233	266	0	0	0	0	-1
normalized size	1	1.01	0.73	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.854	0.744	0.063	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	358	360	213	275	0	0	0	0	-1
normalized size	1	1.01	0.59	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.004	0.530	0.033	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	346	348	235	268	0	0	0	0	-1
normalized size	1	1.01	0.68	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.985	0.697	0.031	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	344	233	277	0	0	0	0	-1
normalized size	1	1.01	0.68	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.696	0.195	0.031	0.000	4.016	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	364	233	266	0	0	0	0	-1
normalized size	1	1.01	0.64	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.792	0.609	0.033	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	39	132	0	0	56	0	334
normalized size	1	1.00	0.31	1.06	0.00	0.00	0.45	0.00	2.67
time (sec)	N/A	0.046	0.028	0.061	0.000	0.669	5.246	0.000	0.139

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	40	125	0	0	99	0	373
normalized size	1	1.00	0.29	0.90	0.00	0.00	0.71	0.00	2.68
time (sec)	N/A	0.053	0.028	0.056	0.000	0.985	8.930	0.000	3.637

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	58	140	0	0	94	0	334
normalized size	1	1.00	0.41	0.99	0.00	0.00	0.66	0.00	2.35
time (sec)	N/A	0.050	0.037	0.053	0.000	0.975	8.928	0.000	2.716

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	61	135	0	0	61	0	376
normalized size	1	1.00	0.45	0.99	0.00	0.00	0.45	0.00	2.76
time (sec)	N/A	0.050	0.037	0.049	0.000	0.883	5.426	0.000	4.451

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	41	132	0	0	56	0	334
normalized size	1	1.00	0.32	1.04	0.00	0.00	0.44	0.00	2.63
time (sec)	N/A	0.046	0.028	0.031	0.000	0.752	5.196	0.000	2.647

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	42	125	0	0	99	0	373
normalized size	1	1.00	0.30	0.89	0.00	0.00	0.70	0.00	2.65
time (sec)	N/A	0.051	0.025	0.027	0.000	0.954	9.003	0.000	3.242

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	60	140	0	0	94	0	334
normalized size	1	1.00	0.42	0.97	0.00	0.00	0.65	0.00	2.32
time (sec)	N/A	0.047	0.045	0.023	0.000	0.910	8.978	0.000	2.702

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	63	135	0	0	61	0	376
normalized size	1	1.00	0.46	0.98	0.00	0.00	0.44	0.00	2.72
time (sec)	N/A	0.048	0.034	0.026	0.000	0.934	5.513	0.000	4.099

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	332	334	194	240	0	0	0	0	207
normalized size	1	1.01	0.58	0.72	0.00	0.00	0.00	0.00	0.62
time (sec)	N/A	0.640	0.222	0.009	0.000	0.630	0.000	0.000	0.225

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	377	379	195	240	0	0	0	0	224
normalized size	1	1.01	0.52	0.64	0.00	0.00	0.00	0.00	0.59
time (sec)	N/A	0.700	0.226	0.010	0.000	0.635	0.000	0.000	2.742

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	373	375	193	240	0	0	0	0	208
normalized size	1	1.01	0.52	0.64	0.00	0.00	0.00	0.00	0.56
time (sec)	N/A	0.597	0.116	0.010	0.000	0.780	0.000	0.000	2.647

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	343	196	240	0	0	0	0	223
normalized size	1	1.01	0.57	0.70	0.00	0.00	0.00	0.00	0.65
time (sec)	N/A	0.604	0.208	0.009	0.000	0.786	0.000	0.000	2.611

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	450	452	211	274	0	0	0	0	356
normalized size	1	1.00	0.47	0.61	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	1.060	0.563	0.013	0.000	0.000	0.000	0.000	0.128

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	474	476	233	265	0	0	0	0	387
normalized size	1	1.00	0.49	0.56	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	1.106	0.702	0.012	0.000	0.000	0.000	0.000	0.093

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	475	477	231	274	0	0	0	0	355
normalized size	1	1.00	0.49	0.58	0.00	0.00	0.00	0.00	0.75
time (sec)	N/A	0.924	0.236	0.011	0.000	0.000	0.000	0.000	2.675

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	463	465	213	265	0	0	0	0	388
normalized size	1	1.00	0.46	0.57	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	1.034	0.353	0.011	0.000	0.000	0.000	0.000	0.097

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	34	129	0	0	42	0	207
normalized size	1	1.00	0.28	1.08	0.00	0.00	0.35	0.00	1.72
time (sec)	N/A	0.049	0.018	0.009	0.000	0.864	2.960	0.000	2.637

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	34	122	0	0	65	0	223
normalized size	1	1.00	0.25	0.91	0.00	0.00	0.49	0.00	1.66
time (sec)	N/A	0.059	0.023	0.009	0.000	1.062	3.334	0.000	0.065

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	52	129	0	0	60	0	207
normalized size	1	1.00	0.38	0.94	0.00	0.00	0.44	0.00	1.51
time (sec)	N/A	0.051	0.027	0.009	0.000	1.165	3.086	0.000	2.617

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	56	122	0	0	46	0	223
normalized size	1	1.00	0.43	0.93	0.00	0.00	0.35	0.00	1.70
time (sec)	N/A	0.054	0.029	0.009	0.000	1.112	3.287	0.000	2.656

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.143	0.070	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	0.184	0.066	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	133	451	253	490	6397	835	495
normalized size	1	1.00	0.83	2.82	1.58	3.06	39.98	5.22	3.09
time (sec)	N/A	0.106	0.124	0.010	0.984	0.703	7.628	0.398	3.204

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	104	283	184	348	3704	577	363
normalized size	1	1.00	0.83	2.25	1.46	2.76	29.40	4.58	2.88
time (sec)	N/A	0.068	0.080	0.007	1.020	0.724	4.856	0.379	2.950

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	167	122	222	1906	361	247
normalized size	1	1.00	1.00	1.78	1.30	2.36	20.28	3.84	2.63
time (sec)	N/A	0.046	0.071	0.004	0.956	0.679	2.786	0.327	2.946

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	94	0	0	0	741	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	7.48	0.00	-0.01
time (sec)	N/A	0.058	0.057	0.038	0.000	0.703	6.433	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	252	1565	601	1565	0	2660	1410
normalized size	1	1.00	0.86	5.32	2.04	5.32	0.00	9.05	4.80
time (sec)	N/A	0.202	0.270	0.021	0.810	0.700	0.000	0.613	3.734

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	211	1142	474	1216	0	2034	1136
normalized size	1	1.00	0.85	4.60	1.91	4.90	0.00	8.20	4.58
time (sec)	N/A	0.148	0.205	0.018	0.984	0.529	0.000	0.614	3.392

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	172	793	359	893	11851	1477	878
normalized size	1	1.00	0.85	3.91	1.77	4.40	58.38	7.28	4.33
time (sec)	N/A	0.115	0.170	0.013	0.663	0.444	14.440	0.498	3.192

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	188	0	0	0	4760	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	22.78	0.00	-0.00
time (sec)	N/A	0.127	0.167	0.043	0.000	0.435	12.992	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	402	3780	1153	3564	0	0	2896
normalized size	1	1.00	0.88	8.24	2.51	7.76	0.00	0.00	6.31
time (sec)	N/A	0.317	0.471	0.056	0.901	0.535	0.000	0.000	7.140

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	345	2972	953	2919	0	4934	2436
normalized size	1	1.00	0.87	7.51	2.41	7.37	0.00	12.46	6.15
time (sec)	N/A	0.264	0.381	0.037	0.853	0.514	0.000	0.717	5.598

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	290	2280	770	2313	0	3874	2001
normalized size	1	1.00	0.86	6.77	2.28	6.86	0.00	11.50	5.94
time (sec)	N/A	0.209	0.359	0.028	0.785	0.491	0.000	0.665	4.338

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	332	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	0.347	0.043	0.000	0.473	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	284	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.865	0.614	0.072	0.000	0.448	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	292	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.862	0.683	0.063	0.000	0.449	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	239	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.240	0.058	0.000	0.454	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	0.156	0.053	0.000	0.440	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	237	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	0.187	0.056	0.000	0.448	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	222	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.158	0.121	0.055	0.000	0.451	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	244	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.559	0.244	0.049	0.000	0.465	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	273	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.611	0.269	0.067	0.000	0.454	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.585	0.385	0.059	0.000	0.442	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.463	0.172	0.061	0.000	0.444	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1480	1482	820	1126	0	0	0	0	-1
normalized size	1	1.00	0.55	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.815	2.149	0.225	0.000	22.811	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	B	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	636	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	4.71	0.00	-0.01
time (sec)	N/A	0.086	0.048	0.066	0.000	0.462	30.803	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	296	1640	0	19	0	0	273
normalized size	1	1.00	18.50	102.50	0.00	1.19	0.00	0.00	17.06
time (sec)	N/A	0.077	0.869	0.072	0.000	0.438	0.000	0.000	0.201

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	280	732	0	28	0	0	292
normalized size	1	1.00	14.00	36.60	0.00	1.40	0.00	0.00	14.60
time (sec)	N/A	0.085	0.727	0.062	0.000	0.490	0.000	0.000	2.831

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	278	1656	0	25	0	0	276
normalized size	1	1.00	15.44	92.00	0.00	1.39	0.00	0.00	15.33
time (sec)	N/A	0.079	0.234	0.063	0.000	0.473	0.000	0.000	2.771

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	298	724	0	28	0	0	289
normalized size	1	1.00	16.56	40.22	0.00	1.56	0.00	0.00	16.06
time (sec)	N/A	0.083	0.559	0.061	0.000	0.468	0.000	0.000	0.106

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	424	4397	0	181	0	0	632
normalized size	1	1.00	14.13	146.57	0.00	6.03	0.00	0.00	21.07
time (sec)	N/A	0.093	1.356	0.071	0.000	0.480	0.000	0.000	2.822

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	427	1908	0	191	0	0	677
normalized size	1	1.00	11.24	50.21	0.00	5.03	0.00	0.00	17.82
time (sec)	N/A	0.108	1.485	0.071	0.000	0.481	0.000	0.000	0.141

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	425	4437	0	187	0	0	629
normalized size	1	1.00	11.81	123.25	0.00	5.19	0.00	0.00	17.47
time (sec)	N/A	0.093	0.457	0.052	0.000	0.488	0.000	0.000	2.801

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	426	1888	0	185	0	0	680
normalized size	1	1.00	13.31	59.00	0.00	5.78	0.00	0.00	21.25
time (sec)	N/A	0.093	0.546	0.059	0.000	0.480	0.000	0.000	0.120

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	186	334	0	0	175	0	-1
normalized size	1	1.00	0.52	0.94	0.00	0.00	0.49	0.00	-0.00
time (sec)	N/A	0.233	0.138	0.064	0.000	0.547	4.654	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	146	310	0	0	138	0	-1
normalized size	1	1.00	0.45	0.95	0.00	0.00	0.42	0.00	-0.00
time (sec)	N/A	0.189	0.139	0.016	0.000	0.533	4.088	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	109	127	0	0	88	0	-1
normalized size	1	1.00	0.69	0.80	0.00	0.00	0.56	0.00	-0.01
time (sec)	N/A	0.089	0.066	0.010	0.000	0.520	3.454	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	85	0	0	37	0	37
normalized size	1	1.00	0.85	0.81	0.00	0.00	0.35	0.00	0.35
time (sec)	N/A	0.019	0.107	0.003	0.000	0.437	0.789	0.000	2.644

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	405	565	0	0	0	0	-1
normalized size	1	1.00	0.55	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.725	0.836	0.025	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1221	1221	382	402	0	0	0	0	-1
normalized size	1	1.00	0.31	0.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.800	1.931	0.029	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	157	218	0	0	141	0	-1
normalized size	1	1.00	0.53	0.74	0.00	0.00	0.48	0.00	-0.00
time (sec)	N/A	0.160	0.136	0.025	0.000	1.228	4.188	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	133	197	0	0	105	0	-1
normalized size	1	1.00	0.51	0.75	0.00	0.00	0.40	0.00	-0.00
time (sec)	N/A	0.124	0.147	0.015	0.000	1.268	3.438	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	0	61	0	-1
normalized size	1	1.00	0.65	0.79	0.00	0.00	0.50	0.00	-0.01
time (sec)	N/A	0.062	0.041	0.010	0.000	1.121	2.322	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	0	36	0	37
normalized size	1	1.00	0.84	0.80	0.00	0.00	0.41	0.00	0.42
time (sec)	N/A	0.010	0.032	0.003	0.000	0.958	0.734	0.000	2.630

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	200	169	0	0	0	0	-1
normalized size	1	1.00	0.49	0.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	0.262	0.016	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	425	421	0	0	0	0	-1
normalized size	1	1.00	0.70	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.760	1.106	0.026	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	513	483	0	0	0	0	-1
normalized size	1	1.00	0.78	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.157	2.516	0.026	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	126	261	0	0	0	0	-1
normalized size	1	1.00	0.42	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.136	0.067	0.033	0.000	1.129	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	108	239	0	0	0	0	-1
normalized size	1	1.00	0.40	0.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.117	0.063	0.016	0.000	0.525	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	59	115	0	0	61	0	57
normalized size	1	1.00	0.52	1.01	0.00	0.00	0.54	0.00	0.50
time (sec)	N/A	0.047	0.031	0.011	0.000	0.694	8.094	0.000	2.877

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	94	0	0	36	0	37
normalized size	1	1.00	0.51	0.87	0.00	0.00	0.33	0.00	0.34
time (sec)	N/A	0.019	0.011	0.005	0.000	0.561	0.809	0.000	2.659

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	818	818	434	496	0	0	0	0	-1
normalized size	1	1.00	0.53	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.601	0.885	0.022	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.729	0.427	0.061	0.000	0.745	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.571	0.246	0.059	0.000	0.463	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1605	1605	1416	1153	0	0	0	0	-1
normalized size	1	1.00	0.88	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	9.680	7.374	0.102	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	205	132	200	119	233	0	355	234
normalized size	1	1.27	0.82	1.24	0.74	1.45	0.00	2.20	1.45
time (sec)	N/A	0.128	0.104	0.009	0.985	0.906	0.000	0.382	2.973

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	134	63	60	136	74	0	72	-1
normalized size	1	0.94	0.44	0.42	0.95	0.52	0.00	0.50	-0.01
time (sec)	N/A	0.162	0.024	0.009	1.033	0.773	0.000	0.299	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	187	63	60	47	74	0	72	-1
normalized size	1	1.31	0.44	0.42	0.33	0.52	0.00	0.50	-0.01
time (sec)	N/A	0.105	0.023	0.008	1.216	0.722	0.000	0.287	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	78	63	60	98	74	0	48	50
normalized size	1	1.18	0.95	0.91	1.48	1.12	0.00	0.73	0.76
time (sec)	N/A	0.113	0.023	0.006	1.042	0.640	0.000	0.300	2.831

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	187	63	60	47	74	0	72	-1
normalized size	1	1.31	0.44	0.42	0.33	0.52	0.00	0.50	-0.01
time (sec)	N/A	0.102	0.020	0.008	1.197	0.711	0.000	0.268	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	59	60	73	0	25	40
normalized size	1	1.00	0.91	1.84	1.88	2.28	0.00	0.78	1.25
time (sec)	N/A	0.022	0.013	0.007	0.932	0.626	0.000	0.248	2.835

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	175	61	58	43	72	0	46	-1
normalized size	1	1.30	0.45	0.43	0.32	0.53	0.00	0.34	-0.01
time (sec)	N/A	0.053	0.016	0.003	0.960	0.411	0.000	0.327	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	183	62	59	171	73	0	73	-1
normalized size	1	1.32	0.45	0.42	1.23	0.53	0.00	0.53	-0.01
time (sec)	N/A	0.103	0.024	0.023	0.977	0.443	0.000	0.346	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	178	62	60	48	72	0	69	-1
normalized size	1	1.33	0.46	0.45	0.36	0.54	0.00	0.51	-0.01
time (sec)	N/A	0.093	0.026	0.008	0.970	0.427	0.000	0.244	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	184	65	61	176	76	0	91	-1
normalized size	1	1.31	0.46	0.44	1.26	0.54	0.00	0.65	-0.01
time (sec)	N/A	0.105	0.027	0.017	0.986	0.439	0.000	0.294	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	254	143	236	0	433	0	177	-1
normalized size	1	1.00	0.57	0.93	0.00	1.71	0.00	0.70	-0.00
time (sec)	N/A	0.246	0.153	0.036	0.000	0.590	0.000	0.507	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	26	70	87	0	28	62
normalized size	1	1.00	0.91	0.81	2.19	2.72	0.00	0.88	1.94
time (sec)	N/A	0.027	0.017	0.006	1.081	0.480	0.000	0.306	2.708

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	208	132	205	0	402	0	153	-1
normalized size	1	1.00	0.64	0.99	0.00	1.94	0.00	0.74	-0.00
time (sec)	N/A	0.073	0.113	0.018	0.000	0.495	0.000	0.330	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	194	111	221	0	391	0	185	-1
normalized size	1	1.01	0.58	1.15	0.00	2.04	0.00	0.96	-0.01
time (sec)	N/A	0.217	0.076	0.019	0.000	0.492	0.000	0.357	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	209	65	215	0	396	0	185	-1
normalized size	1	1.00	0.31	1.03	0.00	1.90	0.00	0.89	-0.00
time (sec)	N/A	0.189	0.017	0.019	0.000	0.494	0.000	0.544	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	204	48	238	0	411	0	204	-1
normalized size	1	1.01	0.24	1.18	0.00	2.03	0.00	1.01	-0.00
time (sec)	N/A	0.216	0.019	0.021	0.000	0.473	0.000	0.394	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	89	60	0	141	0	71	-1
normalized size	1	0.97	1.16	0.78	0.00	1.83	0.00	0.92	-0.01
time (sec)	N/A	0.143	0.067	0.013	0.000	0.459	0.000	0.459	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	19	19	53	28	19
normalized size	1	1.00	1.00	1.24	0.90	0.90	2.52	1.33	0.90
time (sec)	N/A	0.018	0.005	0.005	1.063	0.440	1.507	0.391	2.644

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	0	19	66	28	19
normalized size	1	1.00	1.00	1.24	0.00	0.90	3.14	1.33	0.90
time (sec)	N/A	0.017	0.008	0.002	0.000	0.455	1.463	0.483	2.739

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	38	64	80	138	0	59	-1
normalized size	1	1.03	0.54	0.90	1.13	1.94	0.00	0.83	-0.01
time (sec)	N/A	0.136	0.009	0.010	1.953	0.517	0.000	0.246	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	32	37	46	32	0	81	54
normalized size	1	1.00	0.67	0.77	0.96	0.67	0.00	1.69	1.12
time (sec)	N/A	0.114	0.009	0.006	0.965	0.448	0.000	0.343	2.830

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	40	81	121	175	0	103	-1
normalized size	1	1.08	0.38	0.78	1.16	1.68	0.00	0.99	-0.01
time (sec)	N/A	0.153	0.011	0.010	1.990	0.484	0.000	0.328	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	152	63	58	85	75	0	137	109
normalized size	1	1.10	0.46	0.42	0.62	0.54	0.00	0.99	0.79
time (sec)	N/A	0.188	0.038	0.008	0.984	0.496	0.000	0.336	2.965

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	113	52	47	64	63	116	109	88
normalized size	1	1.11	0.51	0.46	0.63	0.62	1.14	1.07	0.86
time (sec)	N/A	0.157	0.026	0.007	0.932	0.485	88.605	0.242	2.901

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	74	41	36	43	51	87	81	67
normalized size	1	1.12	0.62	0.55	0.65	0.77	1.32	1.23	1.02
time (sec)	N/A	0.137	0.021	0.007	0.932	0.492	38.304	0.301	2.885

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	26	25	37	58	17	28
normalized size	1	1.00	0.86	0.72	0.69	1.03	1.61	0.47	0.78
time (sec)	N/A	0.020	0.007	0.004	0.909	0.474	14.889	0.270	2.769

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	141	96	0	118	0	0	190	-1
normalized size	1	1.21	0.82	0.00	1.01	0.00	0.00	1.62	-0.01
time (sec)	N/A	0.158	0.038	0.033	1.968	0.000	0.000	0.407	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	151	50	0	138	0	0	212	-1
normalized size	1	1.14	0.38	0.00	1.04	0.00	0.00	1.59	-0.01
time (sec)	N/A	0.163	0.013	0.015	2.404	0.000	0.000	0.398	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	191	68	0	0	0	0	0	-1
normalized size	1	1.26	0.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.058	0.012	0.000	0.966	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	146	52	0	0	0	0	0	-1
normalized size	1	1.23	0.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.007	0.008	0.000	0.940	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	142	55	0	0	0	0	0	-1
normalized size	1	1.23	0.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.010	0.013	0.000	1.064	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	193	57	0	0	0	0	0	-1
normalized size	1	1.25	0.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.013	0.013	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	106	122	0	80	0	61	-1
normalized size	1	1.00	1.49	1.72	0.00	1.13	0.00	0.86	-0.01
time (sec)	N/A	0.025	0.162	0.015	0.000	0.914	0.000	0.287	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	59	114	0	0	0	0	-1
normalized size	1	1.00	1.23	2.38	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.060	0.022	0.000	0.805	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	72	28	0	43	0	22	-1
normalized size	1	1.00	2.25	0.88	0.00	1.34	0.00	0.69	-0.03
time (sec)	N/A	0.013	0.027	0.007	0.000	0.756	0.000	0.426	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	0	0	0	-1
normalized size	1	1.00	1.50	3.58	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.013	0.017	0.010	0.000	0.884	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	198	527	414	541	0	243	-1
normalized size	1	1.00	0.81	2.16	1.70	2.22	0.00	1.00	-0.00
time (sec)	N/A	0.333	0.523	0.067	2.279	0.736	0.000	0.726	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	149	342	269	407	0	185	-1
normalized size	1	1.00	0.93	2.12	1.67	2.53	0.00	1.15	-0.01
time (sec)	N/A	0.163	0.381	0.038	2.246	0.494	0.000	0.817	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	143	200	145	313	0	149	-1
normalized size	1	1.00	1.39	1.94	1.41	3.04	0.00	1.45	-0.01
time (sec)	N/A	0.070	0.284	0.025	2.254	0.498	0.000	0.821	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	173	179	149	865	0	0	-1
normalized size	1	1.00	1.54	1.60	1.33	7.72	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.179	0.042	2.242	0.700	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	133	326	145	333	0	269	-1
normalized size	1	1.00	1.05	2.57	1.14	2.62	0.00	2.12	-0.01
time (sec)	N/A	0.087	0.102	0.053	2.234	0.639	0.000	0.676	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	174	558	265	427	0	598	-1
normalized size	1	1.00	0.84	2.68	1.27	2.05	0.00	2.88	-0.00
time (sec)	N/A	0.170	0.114	0.066	2.171	1.191	0.000	0.823	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	222	849	410	561	0	1076	-1
normalized size	1	1.00	0.70	2.67	1.29	1.76	0.00	3.38	-0.00
time (sec)	N/A	0.312	0.176	0.077	2.291	3.200	0.000	0.976	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	255	552	0	0	0	0	-1
normalized size	1	1.00	0.71	1.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.520	0.480	0.046	0.000	0.446	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	208	356	0	0	0	0	-1
normalized size	1	1.00	0.78	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	0.314	0.020	0.000	0.460	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	86	184	0	0	0	0	-1
normalized size	1	1.00	0.44	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.055	0.016	0.000	0.451	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	111	192	0	0	0	0	-1
normalized size	1	1.00	0.46	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.254	0.034	0.000	0.498	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	238	444	0	0	0	0	-1
normalized size	1	1.00	0.74	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.445	0.656	0.036	0.000	0.480	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	302	708	0	0	0	0	-1
normalized size	1	1.00	0.71	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.634	0.573	0.044	0.000	0.458	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	294	1027	454	553	0	0	-1
normalized size	1	1.00	1.04	3.64	1.61	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.383	0.541	0.076	2.118	1.855	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	191	679	303	417	0	0	-1
normalized size	1	1.00	0.96	3.41	1.52	2.10	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.616	0.057	2.289	0.978	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	96	432	189	328	0	0	-1
normalized size	1	1.00	0.68	3.06	1.34	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.059	0.048	2.210	0.729	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	193	401	197	1049	0	0	-1
normalized size	1	1.00	1.28	2.66	1.30	6.95	0.00	0.00	-0.01
time (sec)	N/A	0.192	1.278	0.060	2.271	1.295	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	146	641	189	350	0	0	-1
normalized size	1	1.00	0.88	3.88	1.15	2.12	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.091	0.076	2.265	1.361	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	186	1042	303	435	0	0	-1
normalized size	1	1.00	0.73	4.07	1.18	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.218	0.129	0.066	2.477	3.446	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	245	1498	453	573	0	0	-1
normalized size	1	1.00	0.67	4.09	1.24	1.57	0.00	0.00	-0.00
time (sec)	N/A	0.369	0.191	0.095	3.038	14.423	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	290	933	0	0	0	0	-1
normalized size	1	1.00	0.74	2.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	0.597	0.053	0.000	0.473	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	235	734	0	0	0	0	-1
normalized size	1	1.00	0.76	2.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.438	0.418	0.033	0.000	0.466	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	206	527	0	0	0	0	-1
normalized size	1	1.00	0.79	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	0.317	0.029	0.000	0.470	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	228	670	0	0	0	0	-1
normalized size	1	1.00	0.74	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	0.358	0.033	0.000	0.449	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	275	791	0	0	0	0	-1
normalized size	1	1.00	0.72	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.632	0.467	0.036	0.000	0.453	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	357	1197	0	0	0	0	-1
normalized size	1	1.00	0.74	2.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.809	0.668	0.038	0.000	0.455	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	86	52	0	55	39	18	55
normalized size	1	1.00	1.69	1.02	0.00	1.08	0.76	0.35	1.08
time (sec)	N/A	0.023	0.032	0.025	0.000	0.417	21.541	0.285	2.666

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	104	78	76	77	66	30	88
normalized size	1	1.00	1.44	1.08	1.06	1.07	0.92	0.42	1.22
time (sec)	N/A	0.032	0.048	0.028	2.055	0.433	66.857	0.353	0.210

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	86	68	0	55	0	22	56
normalized size	1	1.00	1.62	1.28	0.00	1.04	0.00	0.42	1.06
time (sec)	N/A	0.029	0.031	0.101	0.000	0.433	0.000	0.291	2.674

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	98	80	0	65	0	0	101
normalized size	1	1.00	0.87	0.71	0.00	0.58	0.00	0.00	0.89
time (sec)	N/A	0.064	0.036	0.069	0.000	0.445	0.000	0.000	2.658

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	109	130	121	82	0	47	134
normalized size	1	1.00	1.03	1.23	1.14	0.77	0.00	0.44	1.26
time (sec)	N/A	0.056	0.050	0.282	1.993	0.460	0.000	0.366	2.994

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	42	0	32	0	40	-1
normalized size	1	1.00	0.94	0.81	0.00	0.62	0.00	0.77	-0.02
time (sec)	N/A	0.099	0.017	0.016	0.000	0.436	0.000	0.420	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	65	60	0	42	0	61	-1
normalized size	1	1.00	0.96	0.88	0.00	0.62	0.00	0.90	-0.01
time (sec)	N/A	0.191	0.025	0.040	0.000	0.433	0.000	0.453	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	224	527	413	545	0	0	-1
normalized size	1	1.00	0.80	1.88	1.47	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.290	0.386	0.052	2.269	0.483	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	172	342	268	413	0	0	-1
normalized size	1	1.00	1.02	2.02	1.59	2.44	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.342	0.040	2.231	0.460	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	152	200	153	313	0	0	-1
normalized size	1	1.00	1.43	1.89	1.44	2.95	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.132	0.026	2.180	0.438	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	190	179	155	881	0	0	-1
normalized size	1	1.00	1.70	1.60	1.38	7.87	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.235	0.043	1.960	0.661	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	133	326	153	333	0	0	-1
normalized size	1	1.00	1.02	2.51	1.18	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.123	0.053	1.520	0.660	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	173	558	271	443	0	0	-1
normalized size	1	1.00	0.79	2.56	1.24	2.03	0.00	0.00	-0.00
time (sec)	N/A	0.135	0.159	0.064	1.755	1.315	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	258	553	0	0	0	0	-1
normalized size	1	1.00	0.64	1.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.522	0.480	0.032	0.000	0.433	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	212	358	0	0	0	0	-1
normalized size	1	1.00	0.68	1.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	0.280	0.020	0.000	0.467	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	86	127	0	0	0	0	-1
normalized size	1	1.00	0.34	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.056	0.015	0.000	0.445	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	111	297	0	0	0	0	-1
normalized size	1	1.00	0.38	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.312	0.262	0.023	0.000	0.442	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	372	238	444	0	0	0	0	-1
normalized size	1	0.99	0.63	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	0.406	0.028	0.000	0.441	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	348	247	1027	465	781	0	0	-1
normalized size	1	0.98	0.70	2.90	1.31	2.21	0.00	0.00	-0.00
time (sec)	N/A	0.379	0.475	0.074	1.831	1.812	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	190	679	311	585	0	0	-1
normalized size	1	1.00	0.94	3.36	1.54	2.90	0.00	0.00	-0.00
time (sec)	N/A	0.239	0.317	0.062	1.813	1.093	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	86	432	199	443	0	0	-1
normalized size	1	1.00	0.59	2.96	1.36	3.03	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.067	0.050	1.739	0.857	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	253	401	204	1293	0	0	-1
normalized size	1	1.00	1.66	2.64	1.34	8.51	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.349	0.062	1.915	1.600	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	148	641	197	469	0	0	-1
normalized size	1	1.00	0.87	3.77	1.16	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.087	0.072	1.972	2.180	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	189	1042	311	613	0	0	-1
normalized size	1	1.00	0.74	4.09	1.22	2.40	0.00	0.00	-0.00
time (sec)	N/A	0.228	0.120	0.081	1.956	7.604	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	271	935	0	0	0	0	-1
normalized size	1	1.00	0.60	2.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.675	0.502	0.055	0.000	0.739	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	219	643	0	0	0	0	-1
normalized size	1	1.00	0.58	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.442	0.410	0.034	0.000	0.557	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	203	514	0	0	0	0	-1
normalized size	1	1.00	0.62	1.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	0.472	0.027	0.000	0.631	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	223	650	0	0	0	0	-1
normalized size	1	1.00	0.59	1.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.470	0.369	0.035	0.000	0.770	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	266	866	0	0	0	0	-1
normalized size	1	1.00	0.60	1.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.649	0.479	0.038	0.000	0.609	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	259	137	533	328	423	0	219	-1
normalized size	1	1.20	0.63	2.47	1.52	1.96	0.00	1.01	-0.00
time (sec)	N/A	0.620	0.319	0.063	1.724	0.780	0.000	0.489	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	181	97	354	218	325	0	159	-1
normalized size	1	1.28	0.69	2.51	1.55	2.30	0.00	1.13	-0.01
time (sec)	N/A	0.467	0.180	0.036	1.752	0.815	0.000	0.438	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	77	180	126	267	0	127	120
normalized size	1	1.00	1.12	2.61	1.83	3.87	0.00	1.84	1.74
time (sec)	N/A	0.054	0.089	0.024	1.644	0.640	0.000	0.511	3.007

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	184	80	235	159	927	0	0	-1
normalized size	1	1.92	0.83	2.45	1.66	9.66	0.00	0.00	-0.01
time (sec)	N/A	0.433	0.136	0.042	1.459	0.839	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	140	212	454	156	433	0	281	-1
normalized size	1	1.35	2.04	4.37	1.50	4.16	0.00	2.70	-0.01
time (sec)	N/A	0.386	0.469	0.050	1.484	0.844	0.000	0.465	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	218	278	923	322	577	0	713	-1
normalized size	1	1.25	1.60	5.30	1.85	3.32	0.00	4.10	-0.01
time (sec)	N/A	0.507	0.482	0.062	1.532	0.929	0.000	0.584	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	271	245	1518	557	755	0	1414	-1
normalized size	1	1.02	0.92	5.73	2.10	2.85	0.00	5.34	-0.00
time (sec)	N/A	0.607	0.501	0.076	1.910	1.406	0.000	0.772	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	478	293	662	0	0	0	0	-1
normalized size	1	1.30	0.80	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.718	0.926	0.044	0.000	0.797	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	370	250	406	0	0	0	0	-1
normalized size	1	1.31	0.89	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.517	0.576	0.023	0.000	0.829	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	279	98	199	0	0	0	0	-1
normalized size	1	1.31	0.46	0.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.063	0.017	0.000	0.762	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	353	141	272	0	0	0	0	-1
normalized size	1	1.33	0.53	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	0.574	0.033	0.000	0.810	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	472	314	571	0	0	0	0	-1
normalized size	1	1.30	0.87	1.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.630	0.992	0.039	0.000	0.542	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	598	402	955	0	0	0	0	-1
normalized size	1	1.28	0.86	2.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.812	1.089	0.047	0.000	0.602	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	311	142	1018	368	427	0	527	-1
normalized size	1	1.25	0.57	4.09	1.48	1.71	0.00	2.12	-0.00
time (sec)	N/A	0.730	0.409	0.069	1.709	0.985	0.000	2.104	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	222	104	593	247	335	0	438	-1
normalized size	1	1.29	0.60	3.45	1.44	1.95	0.00	2.55	-0.01
time (sec)	N/A	0.536	0.244	0.056	1.712	0.932	0.000	1.992	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	79	336	156	269	0	387	61
normalized size	1	1.00	0.84	3.57	1.66	2.86	0.00	4.12	0.65
time (sec)	N/A	0.066	0.102	0.046	2.238	0.855	0.000	1.820	3.739

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	206	118	652	201	1073	0	0	-1
normalized size	1	1.63	0.94	5.17	1.60	8.52	0.00	0.00	-0.01
time (sec)	N/A	0.490	0.226	0.061	2.111	1.015	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	170	256	820	202	404	0	0	-1
normalized size	1	1.23	1.86	5.94	1.46	2.93	0.00	0.00	-0.01
time (sec)	N/A	0.526	0.580	0.071	2.412	1.069	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	260	190	1653	313	557	0	0	-1
normalized size	1	1.27	0.93	8.06	1.53	2.72	0.00	0.00	-0.00
time (sec)	N/A	0.591	0.326	0.078	2.422	1.521	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	287	245	2605	534	733	0	0	-1
normalized size	1	0.98	0.84	8.92	1.83	2.51	0.00	0.00	-0.00
time (sec)	N/A	0.725	0.519	0.085	2.492	2.431	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	526	308	1098	0	0	0	0	-1
normalized size	1	1.30	0.76	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.876	0.854	0.055	0.000	0.733	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	430	270	820	0	0	0	0	-1
normalized size	1	1.30	0.82	2.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.635	0.777	0.038	0.000	0.840	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	348	243	515	0	0	0	0	-1
normalized size	1	1.34	0.93	1.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.291	0.548	0.030	0.000	0.684	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	422	278	873	0	0	0	0	-1
normalized size	1	1.35	0.89	2.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.664	0.764	0.037	0.000	0.800	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	520	329	1039	0	0	0	0	-1
normalized size	1	1.34	0.85	2.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.869	0.834	0.040	0.000	0.572	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	648	430	1666	0	0	0	0	-1
normalized size	1	1.31	0.87	3.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.075	1.110	0.044	0.000	0.792	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	267	140	533	340	425	0	274	-1
normalized size	1	1.19	0.62	2.37	1.51	1.89	0.00	1.22	-0.00
time (sec)	N/A	0.622	0.282	0.052	2.277	0.706	0.000	0.566	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	189	101	354	223	333	0	191	-1
normalized size	1	1.28	0.68	2.39	1.51	2.25	0.00	1.29	-0.01
time (sec)	N/A	0.465	0.168	0.038	2.490	0.665	0.000	0.524	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	70	184	129	267	0	138	111
normalized size	1	1.00	0.97	2.56	1.79	3.71	0.00	1.92	1.54
time (sec)	N/A	0.052	0.079	0.027	2.809	0.825	0.000	0.507	3.342

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	184	80	312	155	972	0	0	-1
normalized size	1	1.92	0.83	3.25	1.61	10.12	0.00	0.00	-0.01
time (sec)	N/A	0.414	0.122	0.042	3.338	0.811	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	148	210	452	173	451	0	315	-1
normalized size	1	1.37	1.94	4.19	1.60	4.18	0.00	2.92	-0.01
time (sec)	N/A	0.387	0.300	0.043	2.784	0.615	0.000	0.697	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	218	269	922	359	593	0	815	-1
normalized size	1	1.23	1.52	5.21	2.03	3.35	0.00	4.60	-0.01
time (sec)	N/A	0.472	0.479	0.054	2.371	0.943	0.000	10.392	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	498	297	665	0	0	0	0	-1
normalized size	1	1.12	0.67	1.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.709	0.794	0.035	0.000	0.804	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	398	253	409	0	0	0	0	-1
normalized size	1	1.12	0.71	1.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.521	0.607	0.024	0.000	0.754	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	319	107	164	0	0	0	0	-1
normalized size	1	1.12	0.37	0.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.209	0.112	0.019	0.000	0.796	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	387	151	345	0	0	0	0	-1
normalized size	1	1.13	0.44	1.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.483	0.179	0.028	0.000	0.803	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	486	314	596	0	0	0	0	-1
normalized size	1	1.12	0.72	1.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.637	1.016	0.030	0.000	0.693	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	323	1215	1240	389	675	0	597	-1
normalized size	1	1.04	3.92	4.00	1.25	2.18	0.00	1.93	-0.00
time (sec)	N/A	0.783	11.647	0.073	2.359	1.079	0.000	1.670	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	242	133	783	262	541	0	510	-1
normalized size	1	1.29	0.71	4.19	1.40	2.89	0.00	2.73	-0.01
time (sec)	N/A	0.574	0.266	0.063	2.309	0.955	0.000	1.574	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	104	50	478	161	395	0	449	61
normalized size	1	1.04	0.50	4.78	1.61	3.95	0.00	4.49	0.61
time (sec)	N/A	0.073	0.056	0.047	2.217	0.872	0.000	1.548	3.930

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	214	110	1015	201	1477	0	0	-1
normalized size	1	1.60	0.82	7.57	1.50	11.02	0.00	0.00	-0.01
time (sec)	N/A	0.514	0.465	0.059	2.234	1.218	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	174	229	1088	247	599	0	0	-1
normalized size	1	1.19	1.57	7.45	1.69	4.10	0.00	0.00	-0.01
time (sec)	N/A	0.449	0.418	0.068	2.342	0.669	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	246	281	1947	450	961	0	0	-1
normalized size	1	1.16	1.33	9.18	2.12	4.53	0.00	0.00	-0.00
time (sec)	N/A	0.584	0.568	0.081	2.347	1.255	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	559	296	1158	0	0	0	0	-1
normalized size	1	1.16	0.61	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.906	0.822	0.056	0.000	0.485	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	475	255	667	0	0	0	0	-1
normalized size	1	1.16	0.62	1.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.661	0.602	0.039	0.000	0.498	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	411	241	466	0	0	0	0	-1
normalized size	1	1.15	0.68	1.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	0.516	0.033	0.000	0.453	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	476	268	685	0	0	0	0	-1
normalized size	1	1.16	0.65	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.680	0.654	0.040	0.000	0.482	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	567	319	1082	0	0	0	0	-1
normalized size	1	1.16	0.65	2.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.867	0.904	0.042	0.000	0.484	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	49	64	0	169	0	0	-1
normalized size	1	1.00	0.65	0.85	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.021	0.072	0.000	0.580	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	57	0	153	0	68	-1
normalized size	1	1.00	0.84	1.14	0.00	3.06	0.00	1.36	-0.02
time (sec)	N/A	0.012	0.011	0.061	0.000	0.579	0.000	0.315	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	0	98	0	58	-1
normalized size	1	1.00	1.00	0.71	0.00	4.08	0.00	2.42	-0.04
time (sec)	N/A	0.008	0.006	0.051	0.000	0.547	0.000	0.191	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	41	17	0	28	17
normalized size	1	1.00	1.00	1.61	1.78	0.74	0.00	1.22	0.74
time (sec)	N/A	0.004	0.004	0.005	2.217	0.447	0.000	0.240	2.693

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	30	44	50	25	0	0	29
normalized size	1	1.00	0.61	0.90	1.02	0.51	0.00	0.00	0.59
time (sec)	N/A	0.009	0.006	0.004	2.867	0.448	0.000	0.000	2.672

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	26	29	0	29	-1
normalized size	1	1.00	0.89	0.76	0.70	0.78	0.00	0.78	-0.03
time (sec)	N/A	0.008	0.014	0.013	2.525	0.456	0.000	0.185	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	26	29	0	29	-1
normalized size	1	1.00	0.89	0.76	0.70	0.78	0.00	0.78	-0.03
time (sec)	N/A	0.013	0.005	0.012	2.077	0.455	0.000	0.197	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	44	38	40	41	0	42	-1
normalized size	1	1.00	0.62	0.54	0.56	0.58	0.00	0.59	-0.01
time (sec)	N/A	0.015	0.015	0.012	2.095	0.431	0.000	0.175	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	29	37	42	256	0	48	-1
normalized size	1	1.00	0.59	0.76	0.86	5.22	0.00	0.98	-0.02
time (sec)	N/A	0.013	0.061	0.008	2.704	0.454	0.000	0.182	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	29	37	42	256	0	48	-1
normalized size	1	1.00	0.59	0.76	0.86	5.22	0.00	0.98	-0.02
time (sec)	N/A	0.018	0.016	0.008	2.038	0.454	0.000	0.194	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	30	43	32	127	0	43	-1
normalized size	1	1.00	0.68	0.98	0.73	2.89	0.00	0.98	-0.02
time (sec)	N/A	0.015	0.010	0.015	2.175	0.450	0.000	0.212	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	32	27	0	42	0	27	-1
normalized size	1	1.00	0.73	0.61	0.00	0.95	0.00	0.61	-0.02
time (sec)	N/A	0.006	0.009	0.008	0.000	0.442	0.000	0.194	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	43	76	0	0	0	0	-1
normalized size	1	1.00	0.52	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.008	0.034	0.000	0.448	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	19	18	20	19	19
normalized size	1	1.00	1.00	0.86	0.86	0.82	0.91	0.86	0.86
time (sec)	N/A	0.003	0.005	0.003	1.903	0.416	0.482	0.158	2.674

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	27	81	0	0	36	0	-1
normalized size	1	1.00	0.21	0.62	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.082	0.006	0.028	0.000	0.463	1.016	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	27	62	0	0	0	0	-1
normalized size	1	1.00	0.50	1.15	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.005	0.038	0.000	0.437	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	0	76	0	30	-1
normalized size	1	1.00	1.00	0.86	0.00	3.45	0.00	1.36	-0.05
time (sec)	N/A	0.009	0.004	0.008	0.000	0.441	0.000	0.212	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	27	116	0	0	0	0	-1
normalized size	1	1.00	0.17	0.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.006	0.043	0.000	0.447	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	30	0	22	18
normalized size	1	1.00	1.00	0.86	1.10	1.43	0.00	1.05	0.86
time (sec)	N/A	0.004	0.004	0.004	1.970	0.431	0.000	0.179	2.870

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	31	28	19	0	12	20
normalized size	1	1.00	1.00	1.24	1.12	0.76	0.00	0.48	0.80
time (sec)	N/A	0.004	0.004	0.006	2.260	0.428	0.000	0.168	2.912

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	29	1521	0	0	0	0	-1
normalized size	1	1.00	0.10	5.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.005	0.275	0.000	0.433	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	29	270	0	0	0	0	-1
normalized size	1	1.00	0.11	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.061	0.004	0.065	0.000	0.447	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	321	0	85	14	35	-1
normalized size	1	1.00	0.96	13.96	0.00	3.70	0.61	1.52	-0.04
time (sec)	N/A	0.015	0.005	0.137	0.000	0.496	1.152	0.189	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	27	232	0	0	0	0	-1
normalized size	1	1.00	0.23	2.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.005	0.176	0.000	0.428	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	68	0	31	-1
normalized size	1	1.00	1.00	0.79	0.00	2.83	0.00	1.29	-0.04
time (sec)	N/A	0.009	0.004	0.008	0.000	0.479	0.000	0.201	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	27	1784	0	0	0	0	-1
normalized size	1	1.00	0.09	5.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.228	0.006	0.139	0.000	0.442	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	27	353	0	0	0	0	-1
normalized size	1	1.00	0.10	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.073	0.005	0.073	0.000	0.465	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.012	0.097	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	35	0	0	0	0	-1
normalized size	1	1.00	0.83	0.73	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.010	0.079	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	37	0	0	0	0	-1
normalized size	1	1.00	0.85	0.71	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.011	0.092	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	80	33	30	18	0	0	0	43
normalized size	1	2.35	0.97	0.88	0.53	0.00	0.00	0.00	1.26
time (sec)	N/A	0.030	0.029	0.046	1.513	0.000	0.000	0.000	2.888

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	106	191	0	0	0	0	-1
normalized size	1	1.00	0.93	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.208	0.066	0.000	0.560	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	20	0	20	16
normalized size	1	1.00	1.00	1.06	1.06	1.25	0.00	1.25	1.00
time (sec)	N/A	0.005	0.003	0.002	1.480	0.451	0.000	0.208	3.400

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	32	32	0	32	26
normalized size	1	1.00	1.00	1.04	1.23	1.23	0.00	1.23	1.00
time (sec)	N/A	0.009	0.007	0.002	1.080	0.501	0.000	0.236	2.918

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	44	44	0	44	36
normalized size	1	1.00	1.00	1.03	1.22	1.22	0.00	1.22	1.00
time (sec)	N/A	0.016	0.010	0.003	1.266	0.478	0.000	0.245	3.103

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	140	90	0	94	0	390	179
normalized size	1	1.00	0.95	0.61	0.00	0.64	0.00	2.65	1.22
time (sec)	N/A	0.132	0.166	0.005	0.000	0.438	0.000	0.268	2.954

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	66	0	70	0	206	129
normalized size	1	1.00	1.00	0.69	0.00	0.74	0.00	2.17	1.36
time (sec)	N/A	0.081	0.098	0.005	0.000	0.427	0.000	0.208	2.703

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	40	0	29	136	75	79
normalized size	1	1.00	0.74	0.85	0.00	0.62	2.89	1.60	1.68
time (sec)	N/A	0.047	0.049	0.003	0.000	0.436	0.712	0.208	2.706

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	73	0	318	0	1016	2983
normalized size	1	1.00	0.77	0.75	0.00	3.28	0.00	10.47	30.75
time (sec)	N/A	0.104	0.074	0.006	0.000	0.517	0.000	0.859	18.080

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	99	88	0	399	0	1190	2642
normalized size	1	1.00	0.96	0.85	0.00	3.87	0.00	11.55	25.65
time (sec)	N/A	0.102	0.268	0.016	0.000	0.513	0.000	10.214	18.884

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	361	604	0	196	0	797	1358
normalized size	1	1.00	1.58	2.65	0.00	0.86	0.00	3.50	5.96
time (sec)	N/A	0.373	1.291	0.023	0.000	0.461	0.000	0.389	81.167

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	229	431	0	149	0	445	1012
normalized size	1	1.00	1.39	2.61	0.00	0.90	0.00	2.70	6.13
time (sec)	N/A	0.210	0.876	0.016	0.000	0.449	0.000	0.271	37.521

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	114	179	377	0	103	388	189	110
normalized size	1	1.81	2.84	5.98	0.00	1.63	6.16	3.00	1.75
time (sec)	N/A	0.099	0.559	0.009	0.000	0.465	1.042	0.238	0.239

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	195	258	0	290	0	194	524
normalized size	1	1.00	1.47	1.94	0.00	2.18	0.00	1.46	3.94
time (sec)	N/A	0.230	1.068	0.020	0.000	0.501	0.000	0.673	11.141

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	205	274	0	367	0	311	7637
normalized size	1	1.00	1.45	1.94	0.00	2.60	0.00	2.21	54.16
time (sec)	N/A	0.216	0.960	0.017	0.000	0.510	0.000	1.890	28.822

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	282	294	0	208	0	1447	529
normalized size	1	1.00	0.75	0.78	0.00	0.55	0.00	3.86	1.41
time (sec)	N/A	0.372	0.405	0.006	0.000	0.426	0.000	0.807	3.303

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	214	222	0	159	942	866	385
normalized size	1	1.00	0.82	0.85	0.00	0.61	3.61	3.32	1.48
time (sec)	N/A	0.236	0.260	0.004	0.000	0.452	2.701	0.378	3.211

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	151	151	146	0	106	384	427	252
normalized size	1	2.36	2.36	2.28	0.00	1.66	6.00	6.67	3.94
time (sec)	N/A	0.095	0.151	0.006	0.000	0.442	1.821	0.302	2.995

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	142	181	0	516	0	2652	4060
normalized size	1	1.00	0.90	1.15	0.00	3.29	0.00	16.89	25.86
time (sec)	N/A	0.233	0.194	0.006	0.000	0.496	0.000	2.909	27.722

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	223	187	252	0	675	0	2594	4681
normalized size	1	1.38	1.15	1.56	0.00	4.17	0.00	16.01	28.90
time (sec)	N/A	0.273	0.591	0.018	0.000	0.514	0.000	39.981	33.215

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	21
normalized size	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.006	0.019	0.003	0.000	0.453	0.940	0.230	2.968

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	21
normalized size	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.007	0.017	0.003	0.000	0.429	0.388	0.193	2.937

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	16	0	15	51	15	15
normalized size	1	1.00	1.00	0.70	0.00	0.65	2.22	0.65	0.65
time (sec)	N/A	0.023	0.020	0.003	0.000	0.448	0.406	0.190	2.838

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	31	32	0	77	45
normalized size	1	1.00	1.00	0.87	0.82	0.84	0.00	2.03	1.18
time (sec)	N/A	0.114	0.053	0.006	1.306	0.477	0.000	0.223	3.021

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	59	34	51	0	76	563
normalized size	1	1.00	0.88	1.23	0.71	1.06	0.00	1.58	11.73
time (sec)	N/A	0.090	0.042	0.008	1.395	0.441	0.000	0.232	14.086

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	24	15	23	110	51	33
normalized size	1	1.00	1.00	1.26	0.79	1.21	5.79	2.68	1.74
time (sec)	N/A	0.054	0.022	0.003	1.342	0.494	106.392	0.250	2.977

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	58	17	40	44	48	206
normalized size	1	1.00	0.95	3.05	0.89	2.11	2.32	2.53	10.84
time (sec)	N/A	0.025	0.014	0.006	1.418	0.441	31.434	0.204	7.725

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	51	41	41	0	0	122
normalized size	1	1.00	1.00	1.59	1.28	1.28	0.00	0.00	3.81
time (sec)	N/A	0.089	0.040	0.010	1.526	0.459	0.000	0.000	4.098

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	50	24	44	0	149	120
normalized size	1	1.00	0.85	1.92	0.92	1.69	0.00	5.73	4.62
time (sec)	N/A	0.080	0.028	0.015	1.261	0.454	0.000	0.264	3.795

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	45	58	54	44	0	0	189
normalized size	1	1.00	1.32	1.71	1.59	1.29	0.00	0.00	5.56
time (sec)	N/A	0.091	0.042	0.017	1.522	0.439	0.000	0.000	4.879

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	147	90	0	122	0	451	179
normalized size	1	1.00	1.00	0.61	0.00	0.83	0.00	3.07	1.22
time (sec)	N/A	0.122	0.223	0.004	0.000	0.448	0.000	0.430	2.920

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	70	66	0	92	0	255	129
normalized size	1	1.00	0.74	0.69	0.00	0.97	0.00	2.68	1.36
time (sec)	N/A	0.101	0.194	0.005	0.000	0.425	0.000	0.390	2.862

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	40	0	50	0	107	79
normalized size	1	1.00	0.83	0.85	0.00	1.06	0.00	2.28	1.68
time (sec)	N/A	0.055	0.087	0.004	0.000	0.437	0.000	0.316	2.906

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	73	0	158	0	1093	213
normalized size	1	1.00	0.77	0.75	0.00	1.63	0.00	11.27	2.20
time (sec)	N/A	0.071	0.047	0.006	0.000	0.514	0.000	0.998	4.330

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	135	88	0	182	0	1402	1637
normalized size	1	1.00	1.31	0.85	0.00	1.77	0.00	13.61	15.89
time (sec)	N/A	0.095	0.211	0.010	0.000	0.488	0.000	12.671	10.926

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	75	120	0	243	0	1895	1610
normalized size	1	1.00	0.44	0.70	0.00	1.42	0.00	11.08	9.42
time (sec)	N/A	0.112	0.090	0.013	0.000	0.487	0.000	37.261	11.847

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	238	517	0	479	0	511	1107
normalized size	1	1.00	1.22	2.65	0.00	2.46	0.00	2.62	5.68
time (sec)	N/A	0.349	0.666	0.021	0.000	0.518	0.000	3.472	18.151

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	177	385	0	372	0	272	129
normalized size	1	1.00	1.25	2.71	0.00	2.62	0.00	1.92	0.91
time (sec)	N/A	0.228	0.561	0.008	0.000	0.472	0.000	3.277	0.246

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	195	266	0	346	0	306	5098
normalized size	1	1.00	1.44	1.97	0.00	2.56	0.00	2.27	37.76
time (sec)	N/A	0.181	0.859	0.021	0.000	0.631	0.000	3.526	19.761

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	178	272	0	317	0	438	4285
normalized size	1	1.00	1.29	1.97	0.00	2.30	0.00	3.17	31.05
time (sec)	N/A	0.114	0.713	0.017	0.000	0.636	0.000	4.497	17.444

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	109	313	0	126	0	532	787
normalized size	1	1.00	0.89	2.54	0.00	1.02	0.00	4.33	6.40
time (sec)	N/A	0.201	0.145	0.014	0.000	0.525	0.000	12.251	12.320

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	153	457	0	182	0	802	1290
normalized size	1	1.00	0.88	2.63	0.00	1.05	0.00	4.61	7.41
time (sec)	N/A	0.223	0.181	0.018	0.000	0.528	0.000	16.937	18.735

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	271	246	0	225	0	932	429
normalized size	1	1.00	0.98	0.89	0.00	0.81	0.00	3.36	1.55
time (sec)	N/A	0.319	0.393	0.006	0.000	0.479	0.000	5.577	3.342

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	120	172	0	167	0	480	268
normalized size	1	1.00	0.74	1.06	0.00	1.02	0.00	2.94	1.64
time (sec)	N/A	0.218	0.443	0.004	0.000	0.504	0.000	5.226	3.360

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	119	148	0	321	0	2374	762
normalized size	1	1.00	0.77	0.95	0.00	2.07	0.00	15.32	4.92
time (sec)	N/A	0.202	0.271	0.005	0.000	0.459	0.000	10.122	7.023

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	223	192	237	0	260	0	2318	559
normalized size	1	1.42	1.22	1.51	0.00	1.66	0.00	14.76	3.56
time (sec)	N/A	0.217	0.883	0.014	0.000	0.482	0.000	78.698	7.488

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	275	182	300	0	297	0	0	287
normalized size	1	1.68	1.11	1.83	0.00	1.81	0.00	0.00	1.75
time (sec)	N/A	0.178	0.266	0.018	0.000	0.472	0.000	0.000	5.743

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	63	23	44	48	54	209
normalized size	1	1.00	1.00	2.03	0.74	1.42	1.55	1.74	6.74
time (sec)	N/A	0.046	0.015	0.005	1.494	0.435	3.073	0.613	8.122

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	31	32	0	77	42
normalized size	1	1.00	1.00	0.87	0.82	0.84	0.00	2.03	1.11
time (sec)	N/A	0.325	0.040	0.006	1.385	0.432	0.000	0.539	3.065

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	43	59	34	51	0	76	381
normalized size	1	1.00	0.90	1.23	0.71	1.06	0.00	1.58	7.94
time (sec)	N/A	0.242	0.037	0.004	1.496	0.424	0.000	0.469	10.394

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	17	25	110	54	25
normalized size	1	1.00	1.00	1.24	0.81	1.19	5.24	2.57	1.19
time (sec)	N/A	0.113	0.020	0.003	1.472	0.427	101.264	0.391	3.061

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	59	20	41	46	49	205
normalized size	1	1.00	0.95	2.68	0.91	1.86	2.09	2.23	9.32
time (sec)	N/A	0.055	0.014	0.005	1.389	0.448	37.873	0.368	3.714

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	51	41	41	0	0	122
normalized size	1	1.00	1.00	1.59	1.28	1.28	0.00	0.00	3.81
time (sec)	N/A	0.196	0.029	0.007	1.953	0.451	0.000	0.000	4.127

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	50	24	44	0	149	118
normalized size	1	1.00	0.85	1.92	0.92	1.69	0.00	5.73	4.54
time (sec)	N/A	0.205	0.026	0.013	1.896	0.486	0.000	0.461	3.787

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	46	57	51	43	0	0	186
normalized size	1	1.00	1.39	1.73	1.55	1.30	0.00	0.00	5.64
time (sec)	N/A	0.215	0.039	0.015	2.017	0.447	0.000	0.000	4.777

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	48	48	0	36	0	0	93
normalized size	1	1.00	1.71	1.71	0.00	1.29	0.00	0.00	3.32
time (sec)	N/A	0.321	0.156	0.007	0.000	0.439	0.000	0.000	4.493

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	58	62	0	37	226	41	200
normalized size	1	1.00	1.76	1.88	0.00	1.12	6.85	1.24	6.06
time (sec)	N/A	0.143	0.162	0.007	0.000	0.430	31.387	0.381	10.853

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	86	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.116	0.014	0.000	0.619	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	158	175	266	161	279	163	-1
normalized size	1	1.00	0.90	1.00	1.52	0.92	1.59	0.93	-0.01
time (sec)	N/A	0.133	0.320	0.013	0.936	0.424	10.464	0.450	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	128	126	99	114	116	103	210
normalized size	1	1.00	0.94	0.93	0.73	0.84	0.85	0.76	1.54
time (sec)	N/A	0.103	0.198	0.007	0.889	0.418	4.481	0.521	4.657

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	81	75	46	74	54	65	136
normalized size	1	1.00	1.19	1.10	0.68	1.09	0.79	0.96	2.00
time (sec)	N/A	0.034	0.047	0.004	0.873	0.414	2.205	0.345	3.888

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	109	1325	0	187	0	0	-1
normalized size	1	1.00	0.93	11.32	0.00	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.145	0.039	0.000	0.449	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	141	4136	0	284	0	0	-1
normalized size	1	1.00	0.93	27.39	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.290	0.038	0.000	0.498	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	180	9721	0	536	0	0	-1
normalized size	1	1.00	0.93	50.37	0.00	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.575	0.052	0.000	0.779	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	213	0	0	416	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	1.85	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.338	0.050	0.000	0.566	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	175	0	0	337	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	1.84	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.235	0.021	0.000	0.547	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	139	0	0	301	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.334	0.011	0.000	0.601	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	143	0	0	298	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.250	0.020	0.000	0.555	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	167	0	0	487	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.425	0.013	0.000	0.521	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	186	0	0	812	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	4.08	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.602	0.015	0.000	0.596	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	0	0	26	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.63	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.015	0.079	0.000	0.447	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	67	0	0	59	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.058	0.045	0.000	0.428	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	60	0	34	418	0	-1
normalized size	1	1.00	0.78	1.33	0.00	0.76	9.29	0.00	-0.02
time (sec)	N/A	0.010	0.102	0.039	0.000	0.456	1.277	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	55	0	28	197	0	-1
normalized size	1	1.00	1.07	1.34	0.00	0.68	4.80	0.00	-0.02
time (sec)	N/A	0.007	0.060	0.028	0.000	0.455	1.180	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	64	0	30	197	0	-1
normalized size	1	1.00	1.07	1.56	0.00	0.73	4.80	0.00	-0.02
time (sec)	N/A	0.007	0.062	0.025	0.000	0.476	1.222	0.000	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	70	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	1.06	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.226	0.044	0.000	0.568	0.000	0.000	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	134	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.316	0.019	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	276	685	0	345	0	373	-1
normalized size	1	1.00	0.91	2.26	0.00	1.14	0.00	1.23	-0.00
time (sec)	N/A	0.384	0.552	0.015	0.000	0.483	0.000	0.370	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	213	409	0	219	0	224	-1
normalized size	1	1.00	0.90	1.73	0.00	0.92	0.00	0.95	-0.00
time (sec)	N/A	0.239	0.318	0.009	0.000	0.563	0.000	0.296	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	120	173	0	123	0	111	-1
normalized size	1	1.00	1.02	1.47	0.00	1.04	0.00	0.94	-0.01
time (sec)	N/A	0.063	0.207	0.006	0.000	0.569	0.000	0.217	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	187	4918	0	371	0	0	-1
normalized size	1	1.00	0.87	22.87	0.00	1.73	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.205	0.065	0.000	4.676	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	237	58067	0	826	0	0	-1
normalized size	1	1.00	0.89	218.30	0.00	3.11	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.321	0.052	0.000	2.559	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	300	295147	0	1954	0	0	-1
normalized size	1	1.00	0.91	894.38	0.00	5.92	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.689	0.164	0.000	19.384	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	357	0	0	923	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	2.49	0.00	0.00	-0.00
time (sec)	N/A	0.600	1.061	0.054	0.000	0.697	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	291	0	0	657	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	2.18	0.00	0.00	-0.00
time (sec)	N/A	0.415	0.568	0.022	0.000	0.693	0.000	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	223	0	0	692	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	2.97	0.00	0.00	-0.00
time (sec)	N/A	0.305	0.393	0.012	0.000	0.708	0.000	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	238	0	0	716	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	2.93	0.00	0.00	-0.00
time (sec)	N/A	0.292	0.572	0.021	0.000	0.711	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	257	0	0	1456	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	5.41	0.00	0.00	-0.00
time (sec)	N/A	0.368	0.486	0.014	0.000	0.863	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	315	0	0	2514	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	7.50	0.00	0.00	-0.00
time (sec)	N/A	0.500	0.861	0.015	0.000	1.379	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	138	216	0	158	0	0	-1
normalized size	1	1.00	0.84	1.32	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.383	0.086	0.000	0.481	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	92	167	0	78	0	0	-1
normalized size	1	1.00	0.85	1.55	0.00	0.72	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.140	0.020	0.000	0.468	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	120	0	32	2147	0	-1
normalized size	1	1.00	0.83	2.31	0.00	0.62	41.29	0.00	-0.02
time (sec)	N/A	0.021	0.034	0.017	0.000	0.464	2.749	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.025	0.057	0.000	0.503	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.029	0.048	0.000	0.500	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	150	0	0	159	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.392	0.056	0.000	0.499	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	100	0	0	79	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.135	0.049	0.000	0.463	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	0	0	33	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.59	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.061	0.049	0.000	0.479	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.027	0.057	0.000	0.467	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.027	0.058	0.000	0.471	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	157	0	0	201	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.379	0.052	0.000	0.458	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	111	0	0	110	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.231	0.051	0.000	0.459	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	48	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.64	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.073	0.052	0.000	0.464	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	15	311	0	15
normalized size	1	1.00	1.00	0.00	0.00	0.88	18.29	0.00	0.88
time (sec)	N/A	0.054	0.007	0.050	0.000	0.473	2.641	0.000	3.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.028	0.048	0.000	0.463	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.033	0.052	0.000	0.498	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	173	0	0	204	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	1.01	0.00	0.00	-0.00
time (sec)	N/A	0.112	0.486	0.060	0.000	0.481	0.000	0.000	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	123	0	0	113	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.253	0.059	0.000	0.482	0.000	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	51	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.073	0.061	0.000	0.489	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	18	36	0	18
normalized size	1	1.00	1.00	0.00	0.00	0.90	1.80	0.00	0.90
time (sec)	N/A	0.057	0.008	0.059	0.000	0.430	1.568	0.000	3.193

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.025	0.058	0.000	0.525	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.029	0.060	0.000	0.492	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	280	0	0	654	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	1.79	0.00	0.00	-0.00
time (sec)	N/A	0.474	2.913	0.112	0.000	0.527	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	186	0	0	239	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	1.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	0.612	0.011	0.000	0.482	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	89	0	0	80	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.392	0.010	0.000	0.474	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.293	0.143	0.153	0.000	0.454	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.192	0.109	0.000	0.475	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	89	0	0	80	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.041	0.010	0.000	0.483	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.498	0.035	0.116	0.000	0.463	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	228	0	0	377	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	1.27	0.00	0.00	-0.00
time (sec)	N/A	0.419	1.200	0.109	0.000	0.479	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	135	0	0	122	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.325	0.351	0.109	0.000	0.451	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	36	0	0	41	0	0	41
normalized size	1	1.00	0.88	0.00	0.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.254	0.075	0.107	0.000	0.462	0.000	0.000	3.084

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.302	0.150	0.115	0.000	0.450	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	36	0	0	41	0	0	39
normalized size	1	1.00	0.88	0.00	0.00	1.00	0.00	0.00	0.95
time (sec)	N/A	0.444	0.034	0.108	0.000	0.426	0.000	0.000	3.076

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	175	0	0	231	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.71	0.00	0.00	-0.00
time (sec)	N/A	0.617	0.234	0.110	0.000	0.484	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	76	0	0	117	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.529	0.100	0.107	0.000	0.417	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	152	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.586	0.191	0.106	0.000	0.562	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	76	0	0	117	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.735	0.038	0.107	0.000	0.447	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	772	353	0	0	0	0	-1
normalized size	1	1.00	4.04	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.513	1.619	0.077	0.000	0.000	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	191	394	0	141	70	0	50
normalized size	1	1.00	2.36	4.86	0.00	1.74	0.86	0.00	0.62
time (sec)	N/A	0.058	0.117	0.053	0.000	0.750	0.561	0.000	3.113

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	233	394	0	146	63	0	28
normalized size	1	1.00	3.19	5.40	0.00	2.00	0.86	0.00	0.38
time (sec)	N/A	0.045	0.136	0.054	0.000	0.750	0.579	0.000	3.141

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	87	70	0	153	70	0	54
normalized size	1	1.00	2.29	1.84	0.00	4.03	1.84	0.00	1.42
time (sec)	N/A	0.063	0.060	0.010	0.000	0.740	0.747	0.000	3.315

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	87	70	0	155	63	0	67
normalized size	1	1.00	2.29	1.84	0.00	4.08	1.66	0.00	1.76
time (sec)	N/A	0.062	0.059	0.010	0.000	0.710	0.740	0.000	3.118

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	A	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	0	78	0	144	0	0	196
normalized size	1	1.00	0.00	2.05	0.00	3.79	0.00	0.00	5.16
time (sec)	N/A	0.096	0.267	0.069	0.000	0.736	0.000	0.000	3.419

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	A	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	0	72	0	144	0	0	139
normalized size	1	1.00	0.00	1.89	0.00	3.79	0.00	0.00	3.66
time (sec)	N/A	0.095	0.271	0.068	0.000	0.737	0.000	0.000	5.302

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	42	36	155	78	38	42
normalized size	1	1.00	1.00	1.00	0.86	3.69	1.86	0.90	1.00
time (sec)	N/A	0.066	0.020	0.005	2.171	0.485	0.623	10.576	3.100

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	42	67	168	75	41	199
normalized size	1	1.00	1.05	0.95	1.52	3.82	1.70	0.93	4.52
time (sec)	N/A	0.067	0.022	0.003	1.872	0.437	0.659	10.053	3.107

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	85	74	0	208	90	0	278
normalized size	1	1.00	2.12	1.85	0.00	5.20	2.25	0.00	6.95
time (sec)	N/A	0.130	0.049	0.152	0.000	0.419	1.133	0.000	3.209

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	85	77	0	213	80	0	30
normalized size	1	1.00	2.12	1.92	0.00	5.32	2.00	0.00	0.75
time (sec)	N/A	0.128	0.052	0.141	0.000	0.420	1.124	0.000	3.143

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	61	42	78	0	146	0	0	-1
normalized size	1	1.45	1.00	1.86	0.00	3.48	0.00	0.00	-0.02
time (sec)	N/A	0.219	0.315	0.088	0.000	0.446	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	61	42	74	0	146	0	0	-1
normalized size	1	1.45	1.00	1.76	0.00	3.48	0.00	0.00	-0.02
time (sec)	N/A	0.218	0.042	0.088	0.000	0.467	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	153	73	0	233
normalized size	1	1.00	2.15	1.85	0.00	3.82	1.82	0.00	5.82
time (sec)	N/A	0.089	0.049	0.010	0.000	0.427	1.117	0.000	3.488

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	155	66	0	67
normalized size	1	1.00	2.15	1.85	0.00	3.88	1.65	0.00	1.68
time (sec)	N/A	0.089	0.047	0.011	0.000	0.425	1.124	0.000	3.381

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	42	36	155	78	38	42
normalized size	1	1.00	1.00	1.00	0.86	3.69	1.86	0.90	1.00
time (sec)	N/A	0.059	0.020	0.004	1.421	0.412	0.793	10.128	3.433

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	42	67	168	75	41	923
normalized size	1	1.00	1.05	0.95	1.52	3.82	1.70	0.93	20.98
time (sec)	N/A	0.062	0.022	0.005	1.537	0.451	0.846	9.217	3.527

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	78	0	146	0	0	-1
normalized size	1	1.00	1.00	1.86	0.00	3.48	0.00	0.00	-0.02
time (sec)	N/A	0.223	0.317	0.052	0.000	0.461	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	74	0	146	0	0	-1
normalized size	1	1.00	1.00	1.76	0.00	3.48	0.00	0.00	-0.02
time (sec)	N/A	0.216	0.045	0.055	0.000	0.506	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	84	0	165	0	0	-1
normalized size	1	1.00	0.00	2.00	0.00	3.93	0.00	0.00	-0.02
time (sec)	N/A	0.249	0.474	0.089	0.000	0.449	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	78	0	165	0	0	-1
normalized size	1	1.00	0.00	1.86	0.00	3.93	0.00	0.00	-0.02
time (sec)	N/A	0.243	0.503	0.084	0.000	0.465	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	126	4947	125	233	0	155	167
normalized size	1	1.00	0.94	36.92	0.93	1.74	0.00	1.16	1.25
time (sec)	N/A	0.373	0.215	0.092	0.652	0.489	0.000	0.350	3.644

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	65	3410	62	161	88	72	123
normalized size	1	1.00	0.94	49.42	0.90	2.33	1.28	1.04	1.78
time (sec)	N/A	0.215	0.091	0.040	0.686	0.443	6.510	0.405	3.498

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	1931	21	105	29	22	45
normalized size	1	1.00	1.00	83.96	0.91	4.57	1.26	0.96	1.96
time (sec)	N/A	0.086	0.024	0.037	0.707	0.423	4.479	0.323	3.469

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	107	2175	0	316	88	94	1270
normalized size	1	1.00	1.22	24.72	0.00	3.59	1.00	1.07	14.43
time (sec)	N/A	0.247	0.129	0.042	0.000	0.521	10.498	0.336	3.948

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	291	2459	0	530	0	210	4602
normalized size	1	1.00	1.93	16.28	0.00	3.51	0.00	1.39	30.48
time (sec)	N/A	0.352	1.082	0.054	0.000	1.103	0.000	0.372	5.563

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	157	3485	0	1168	0	0	-1
normalized size	1	1.00	1.07	23.71	0.00	7.95	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.266	0.044	0.000	0.650	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	83	1995	0	510	0	107	-1
normalized size	1	1.00	0.81	19.37	0.00	4.95	0.00	1.04	-0.01
time (sec)	N/A	0.067	0.127	0.040	0.000	0.483	0.000	0.364	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	139	2289	0	581	0	211	-1
normalized size	1	1.00	0.87	14.31	0.00	3.63	0.00	1.32	-0.01
time (sec)	N/A	0.240	0.338	0.048	0.000	0.559	0.000	0.461	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	126	1473	125	191	0	156	200
normalized size	1	1.00	0.90	10.52	0.89	1.36	0.00	1.11	1.43
time (sec)	N/A	0.299	0.198	0.182	0.782	0.416	0.000	0.391	3.740

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	943	62	118	0	72	119
normalized size	1	1.00	0.86	12.92	0.85	1.62	0.00	0.99	1.63
time (sec)	N/A	0.202	0.070	0.026	0.582	0.413	0.000	0.331	3.564

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	455	22	61	0	23	60
normalized size	1	1.00	1.00	17.50	0.85	2.35	0.00	0.88	2.31
time (sec)	N/A	0.111	0.029	0.027	0.559	0.427	0.000	0.320	3.508

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	107	636	0	232	0	94	156
normalized size	1	1.00	1.15	6.84	0.00	2.49	0.00	1.01	1.68
time (sec)	N/A	0.223	0.111	0.059	0.000	0.473	0.000	0.354	4.308

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	307	863	0	445	0	211	248
normalized size	1	1.00	1.99	5.60	0.00	2.89	0.00	1.37	1.61
time (sec)	N/A	0.298	0.673	0.073	0.000	0.546	0.000	0.345	5.463

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	296	1544	0	0	0	0	-1
normalized size	1	1.00	0.95	4.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.504	0.547	0.076	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	0	619	0	0	0	0	-1
normalized size	1	1.00	0.00	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.160	0.080	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	0	619	0	0	0	0	-1
normalized size	1	1.00	0.00	2.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	0.140	0.028	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	496	3560	0	0	0	0	-1
normalized size	1	1.00	1.55	11.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.408	0.688	0.066	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	604	1789	0	0	0	0	-1
normalized size	1	1.00	1.86	5.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	5.290	0.063	0.000	0.000	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	320	0	0	0	0	0	-1
normalized size	1	1.00	2.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.601	0.019	0.000	0.842	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	156	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.290	0.023	0.000	0.557	0.000	0.000	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	61	25	32	26	25
normalized size	1	1.00	1.00	0.00	2.26	0.93	1.19	0.96	0.93
time (sec)	N/A	0.110	0.064	0.024	0.646	0.451	33.632	0.370	3.346

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.006	0.003	0.005	1.282	0.413	0.222	0.318	0.047

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	21	21	26	22	9
normalized size	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.008	0.004	0.007	1.408	0.442	0.397	0.345	3.080

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	19	19	22	20	19
normalized size	1	1.00	1.00	0.74	0.70	0.70	0.81	0.74	0.70
time (sec)	N/A	0.012	0.010	0.014	0.647	0.445	0.241	0.340	3.103

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	92	24	24	0	24	24
normalized size	1	1.00	1.00	2.88	0.75	0.75	0.00	0.75	0.75
time (sec)	N/A	0.014	0.012	0.030	0.699	0.427	0.000	0.326	0.030

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	22	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.011	0.009	0.013	0.695	0.418	0.238	0.344	0.032

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	14	14	15	15	14
normalized size	1	1.00	0.90	0.75	0.70	0.70	0.75	0.75	0.70
time (sec)	N/A	0.009	0.006	0.002	0.699	0.420	0.162	0.427	0.077

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	24	46	45	47	68	45	73
normalized size	1	1.00	0.39	0.74	0.73	0.76	1.10	0.73	1.18
time (sec)	N/A	0.037	0.007	0.010	1.533	0.438	0.641	0.335	3.076

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	173	49	49	0	49	49
normalized size	1	1.00	1.00	2.37	0.67	0.67	0.00	0.67	0.67
time (sec)	N/A	0.028	0.024	0.122	0.703	0.430	0.000	0.321	0.036

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	83	82	76	121	82	82
normalized size	1	1.00	1.00	0.64	0.63	0.58	0.93	0.63	0.63
time (sec)	N/A	0.046	0.035	0.004	0.734	0.439	3.081	0.356	0.153

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	22	175	272	638	0	139	223
normalized size	1	1.00	0.11	0.88	1.36	3.19	0.00	0.70	1.12
time (sec)	N/A	0.396	0.007	0.045	1.390	1.523	0.000	1.118	0.122

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.005	0.002	0.003	1.997	0.461	0.310	0.286	0.139

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	17	15	15
normalized size	1	1.00	1.00	0.84	0.79	0.79	0.89	0.79	0.79
time (sec)	N/A	0.014	0.009	0.003	0.869	0.433	0.168	0.386	0.042

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	24	71	83	120	110	83	42
normalized size	1	1.00	0.22	0.66	0.77	1.11	1.02	0.77	0.39
time (sec)	N/A	0.083	0.007	0.005	1.956	0.482	2.182	0.438	0.085

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	83	61	60	62	0	60	78
normalized size	1	1.00	1.09	0.80	0.79	0.82	0.00	0.79	1.03
time (sec)	N/A	0.034	0.018	0.010	1.357	0.460	0.000	0.340	3.080

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	76	75	71	0	75	75
normalized size	1	1.00	1.00	0.64	0.63	0.60	0.00	0.63	0.63
time (sec)	N/A	0.049	0.032	0.004	0.527	0.483	0.000	0.464	0.152

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	29	242	293	547	311	140	208
normalized size	1	1.00	0.14	1.20	1.46	2.72	1.55	0.70	1.03
time (sec)	N/A	0.224	0.006	0.030	1.519	1.384	24.147	1.204	0.061

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	36	15	44	0	0	32
normalized size	1	1.00	0.97	1.00	0.42	1.22	0.00	0.00	0.89
time (sec)	N/A	0.037	0.022	0.006	0.706	0.426	0.000	0.000	3.257

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	5	35	0	76	23
normalized size	1	1.00	1.00	0.93	0.17	1.21	0.00	2.62	0.79
time (sec)	N/A	0.034	0.014	0.004	0.814	0.408	0.000	0.376	3.096

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	5	35	0	56	23
normalized size	1	1.00	1.00	0.93	0.17	1.21	0.00	1.93	0.79
time (sec)	N/A	0.024	0.012	0.001	0.734	0.400	0.000	0.492	3.064

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	5	33	0	51	21
normalized size	1	1.00	1.00	1.00	0.20	1.32	0.00	2.04	0.84
time (sec)	N/A	0.014	0.011	0.004	0.670	0.403	0.000	0.400	3.044

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	24	5	32	0	42	20
normalized size	1	1.00	1.00	1.00	0.21	1.33	0.00	1.75	0.83
time (sec)	N/A	0.034	0.013	0.003	0.858	0.403	0.000	0.431	3.079

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	5	35	0	60	23
normalized size	1	1.00	1.00	0.93	0.17	1.21	0.00	2.07	0.79
time (sec)	N/A	0.033	0.011	0.001	0.836	0.412	0.000	0.506	3.102

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.065	0.106	0.000	0.441	0.000	0.000	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	112	0	0	0	121	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.88	0.00	-0.01
time (sec)	N/A	0.117	0.076	0.049	0.000	0.420	6.144	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	0	0	0	75	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.95	0.00	-0.01
time (sec)	N/A	0.038	0.028	0.023	0.000	0.420	4.098	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	0	0	0	34	0	51
normalized size	1	1.00	1.25	0.00	0.00	0.00	0.85	0.00	1.28
time (sec)	N/A	0.010	0.017	0.016	0.000	0.414	1.470	0.000	3.135

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.043	0.060	0.000	0.423	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.024	0.056	0.000	0.412	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.076	0.067	0.000	0.426	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	155	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.146	0.105	0.000	0.423	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	8	44	0	0	29
normalized size	1	1.00	1.00	1.06	0.24	1.33	0.00	0.00	0.88
time (sec)	N/A	0.031	0.018	0.005	1.068	0.418	0.000	0.000	3.358

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	5	41	0	15	25
normalized size	1	1.00	1.00	1.00	0.16	1.32	0.00	0.48	0.81
time (sec)	N/A	0.035	0.010	0.004	1.019	0.422	0.000	0.328	3.137

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	42	7	41	0	0	27
normalized size	1	1.00	1.00	1.50	0.25	1.46	0.00	0.00	0.96
time (sec)	N/A	0.026	0.008	0.014	1.124	0.405	0.000	0.000	3.105

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	42	4	51	0	0	-1
normalized size	1	1.00	1.00	1.50	0.14	1.82	0.00	0.00	-0.04
time (sec)	N/A	0.019	0.008	0.014	1.039	0.425	0.000	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	7	41	0	0	22
normalized size	1	1.00	1.00	1.08	0.27	1.58	0.00	0.00	0.85
time (sec)	N/A	0.032	0.008	0.006	1.115	0.404	0.000	0.000	3.345

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	5	44	0	5	25
normalized size	1	1.00	1.00	1.00	0.16	1.42	0.00	0.16	0.81
time (sec)	N/A	0.034	0.007	0.005	1.058	0.399	0.000	0.366	3.465

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	540	1145	0	0	0	0	-1
normalized size	1	1.00	1.33	2.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.464	3.031	0.123	0.000	0.440	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	11	11	12	11	11
normalized size	1	1.00	1.00	1.20	0.73	0.73	0.80	0.73	0.73
time (sec)	N/A	0.009	0.025	0.007	0.888	0.384	0.203	0.319	3.519

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	72	20	13	20	31	13	19
normalized size	1	1.00	4.24	1.18	0.76	1.18	1.82	0.76	1.12
time (sec)	N/A	0.009	0.045	0.004	0.880	0.395	0.266	0.326	3.163

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	11	11	31	11	11
normalized size	1	1.00	1.00	1.20	0.73	0.73	2.07	0.73	0.73
time (sec)	N/A	0.008	0.028	0.006	0.869	0.383	0.263	0.398	3.098

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	11	11	31	11	11
normalized size	1	1.00	1.00	1.20	0.73	0.73	2.07	0.73	0.73
time (sec)	N/A	0.008	0.034	0.005	0.870	0.393	0.264	0.405	3.108

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	21	11	11	10	11	10
normalized size	1	1.00	1.00	1.62	0.85	0.85	0.77	0.85	0.77
time (sec)	N/A	0.010	0.009	0.003	0.884	0.403	0.160	0.359	3.232

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	34	33	32	39	33	32
normalized size	1	1.00	1.00	0.77	0.75	0.73	0.89	0.75	0.73
time (sec)	N/A	0.024	0.021	0.003	0.842	0.405	0.198	0.482	0.111

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	20	16	15	15	17	16	15
normalized size	1	1.00	0.95	0.76	0.71	0.71	0.81	0.76	0.71
time (sec)	N/A	0.013	0.008	0.003	0.884	0.412	0.152	0.307	3.005

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	25	27	26	25
normalized size	1	1.00	1.00	0.82	0.79	0.76	0.82	0.79	0.76
time (sec)	N/A	0.022	0.013	0.006	0.882	0.404	0.170	0.333	3.095

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	30	29	42	38	29
normalized size	1	1.00	1.00	1.06	0.91	0.88	1.27	1.15	0.88
time (sec)	N/A	0.021	0.015	0.004	1.049	0.387	0.436	0.361	3.024

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	17	17	13	10	10	32	0	10
normalized size	1	1.70	1.70	1.30	1.00	1.00	3.20	0.00	1.00
time (sec)	N/A	0.052	0.028	0.017	0.868	0.421	6.963	0.000	3.273

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	86	10	8	0	39
normalized size	1	1.00	1.00	0.76	5.06	0.59	0.47	0.00	2.29
time (sec)	N/A	0.038	0.016	0.027	0.947	0.406	53.072	0.000	3.368

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	38	97	0	0	0	-1
normalized size	1	1.00	0.92	1.03	2.62	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.092	0.349	0.031	1.760	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	36	95	0	0	0	-1
normalized size	1	1.00	0.91	1.03	2.71	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.097	0.331	0.030	1.731	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	35	92	0	0	0	148
normalized size	1	1.00	0.91	1.03	2.71	0.00	0.00	0.00	4.35
time (sec)	N/A	0.117	0.313	0.031	1.705	0.000	0.000	0.000	9.778

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	B	F(-1)	F(-1)	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	95	0	0	0	37
normalized size	1	0.00	0.92	1.03	2.57	0.00	0.00	0.00	1.00
time (sec)	N/A	3.374	0.808	0.033	1.758	0.000	0.000	0.000	9.680

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	95	0	0	0	146
normalized size	1	0.00	0.92	1.03	2.57	0.00	0.00	0.00	3.95
time (sec)	N/A	2.998	1.154	0.032	1.810	0.000	0.000	0.000	9.216

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	88	78	151	94	139	151	167
normalized size	1	1.00	0.48	0.42	0.82	0.51	0.75	0.82	0.90
time (sec)	N/A	0.255	0.307	0.003	0.879	0.888	7.020	0.407	3.370

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	77	66	112	81	110	127	124
normalized size	1	1.00	0.56	0.48	0.81	0.59	0.80	0.92	0.90
time (sec)	N/A	0.166	0.187	0.004	0.878	0.849	5.966	0.428	0.063

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	63	54	72	67	94	131	79
normalized size	1	1.00	0.71	0.61	0.81	0.75	1.06	1.47	0.89
time (sec)	N/A	0.090	0.105	0.003	0.876	0.554	4.767	0.512	0.030

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	35	49	68	82	36
normalized size	1	1.00	0.98	0.85	0.85	1.20	1.66	2.00	0.88
time (sec)	N/A	0.030	0.026	0.001	0.826	0.515	0.210	0.357	0.046

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	79	51	70	118	65	59	130
normalized size	1	1.00	1.39	0.89	1.23	2.07	1.14	1.04	2.28
time (sec)	N/A	0.066	0.162	0.006	1.931	0.452	27.295	0.424	0.088

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	161	60	73	147	139	80	131
normalized size	1	1.00	2.98	1.11	1.35	2.72	2.57	1.48	2.43
time (sec)	N/A	0.066	0.200	0.015	1.968	0.483	87.775	0.437	0.124

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	221	81	113	181	0	105	80
normalized size	1	1.00	2.76	1.01	1.41	2.26	0.00	1.31	1.00
time (sec)	N/A	0.074	0.348	0.016	1.958	0.489	0.000	0.363	3.400

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	232	383	268	286	0	915	-1
normalized size	1	1.00	0.71	1.17	0.82	0.88	0.00	2.81	-0.00
time (sec)	N/A	0.243	0.417	0.006	0.961	0.541	0.000	0.630	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	147	183	167	184	0	549	-1
normalized size	1	1.00	0.66	0.82	0.75	0.82	0.00	2.45	-0.00
time (sec)	N/A	0.156	0.237	0.003	0.944	0.519	0.000	0.463	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	84	94	93	103	0	279	-1
normalized size	1	1.00	0.63	0.71	0.70	0.77	0.00	2.10	-0.01
time (sec)	N/A	0.096	0.139	0.004	0.941	0.565	0.000	0.368	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	41	43	50	0	99	44
normalized size	1	1.00	0.77	0.73	0.77	0.89	0.00	1.77	0.79
time (sec)	N/A	0.033	0.028	0.003	0.907	0.523	0.000	0.356	3.395

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	116	221	0	194	0	150	-1
normalized size	1	1.00	1.00	1.91	0.00	1.67	0.00	1.29	-0.01
time (sec)	N/A	0.156	0.131	0.037	0.000	0.537	0.000	0.494	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	181	151	0	1003	0	232	-1
normalized size	1	1.00	1.32	1.10	0.00	7.32	0.00	1.69	-0.01
time (sec)	N/A	0.169	0.176	0.026	0.000	0.605	0.000	0.656	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	258	784	0	2856	0	895	-1
normalized size	1	1.00	1.15	3.50	0.00	12.75	0.00	4.00	-0.00
time (sec)	N/A	0.429	0.422	0.037	0.000	0.810	0.000	1.103	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	213	394	243	228	0	341	317
normalized size	1	1.00	0.93	1.71	1.06	0.99	0.00	1.48	1.38
time (sec)	N/A	0.259	0.201	0.007	0.904	0.455	0.000	0.374	0.069

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	138	235	148	138	0	198	184
normalized size	1	1.00	0.91	1.56	0.98	0.91	0.00	1.31	1.22
time (sec)	N/A	0.157	0.133	0.006	0.882	0.438	0.000	0.411	3.213

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	82	116	81	71	109	105	89
normalized size	1	1.00	0.91	1.29	0.90	0.79	1.21	1.17	0.99
time (sec)	N/A	0.080	0.067	0.004	0.880	0.459	4.722	0.332	0.054

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	87	35	33	49	38	33
normalized size	1	1.00	0.95	2.12	0.85	0.80	1.20	0.93	0.80
time (sec)	N/A	0.024	0.019	0.010	0.886	0.449	0.549	0.351	0.053

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	61	77	95	125	85	88	181
normalized size	1	1.00	0.74	0.94	1.16	1.52	1.04	1.07	2.21
time (sec)	N/A	0.079	0.090	0.010	2.026	0.522	13.006	0.343	3.282

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	144	216	191	283	0	191	220
normalized size	1	1.00	1.11	1.66	1.47	2.18	0.00	1.47	1.69
time (sec)	N/A	0.178	0.194	0.016	2.035	0.609	0.000	0.456	3.591

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	228	459	367	534	0	375	1094
normalized size	1	1.00	1.12	2.25	1.80	2.62	0.00	1.84	5.36
time (sec)	N/A	0.278	0.429	0.018	2.081	1.117	0.000	0.458	5.009

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	273	416	251	392	0	324	461
normalized size	1	1.00	1.14	1.73	1.05	1.63	0.00	1.35	1.92
time (sec)	N/A	0.280	0.272	0.013	0.930	0.463	0.000	0.484	0.094

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	185	253	158	269	0	191	197
normalized size	1	1.00	1.11	1.52	0.95	1.62	0.00	1.15	1.19
time (sec)	N/A	0.172	0.169	0.013	0.929	0.474	0.000	0.372	3.197

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	112	125	90	163	131	102	98
normalized size	1	1.00	1.18	1.32	0.95	1.72	1.38	1.07	1.03
time (sec)	N/A	0.090	0.088	0.010	1.049	0.466	42.382	0.406	0.062

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	142	43	75	124	44	43
normalized size	1	1.00	0.85	3.02	0.91	1.60	2.64	0.94	0.91
time (sec)	N/A	0.033	0.034	0.020	0.873	0.465	1.091	0.335	0.047

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	164	161	176	444	153	174	125
normalized size	1	1.00	1.27	1.25	1.36	3.44	1.19	1.35	0.97
time (sec)	N/A	0.119	0.277	0.014	1.990	0.545	50.510	0.410	3.511

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	230	312	367	854	0	311	275
normalized size	1	1.00	1.14	1.54	1.82	4.23	0.00	1.54	1.36
time (sec)	N/A	0.245	0.758	0.018	2.014	0.862	0.000	0.413	0.731

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	401	610	659	1252	0	521	1441
normalized size	1	1.00	1.31	1.99	2.15	4.09	0.00	1.70	4.71
time (sec)	N/A	0.403	0.826	0.023	2.086	2.020	0.000	0.505	5.964

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	232	383	268	231	0	409	-1
normalized size	1	1.00	0.72	1.18	0.83	0.71	0.00	1.26	-0.00
time (sec)	N/A	0.230	0.349	0.003	0.942	0.538	0.000	0.417	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	147	183	167	140	0	238	-1
normalized size	1	1.00	0.66	0.82	0.75	0.63	0.00	1.07	-0.00
time (sec)	N/A	0.158	0.170	0.003	0.922	0.548	0.000	0.325	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	84	94	93	71	0	115	-1
normalized size	1	1.00	0.64	0.72	0.71	0.54	0.00	0.88	-0.01
time (sec)	N/A	0.094	0.098	0.004	0.927	0.583	0.000	0.366	0.000

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	41	42	34	0	38	44
normalized size	1	1.00	0.78	0.76	0.78	0.63	0.00	0.70	0.81
time (sec)	N/A	0.031	0.022	0.006	0.856	0.541	0.000	0.438	3.258

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	92	0	743	0	140	-1
normalized size	1	1.00	1.00	0.95	0.00	7.66	0.00	1.44	-0.01
time (sec)	N/A	0.085	0.103	0.018	0.000	0.558	0.000	0.502	0.000

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	216	265	0	2493	0	654	-1
normalized size	1	1.00	1.33	1.63	0.00	15.29	0.00	4.01	-0.01
time (sec)	N/A	0.203	0.253	0.027	0.000	0.635	0.000	0.757	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	281	840	0	4390	0	1303	-1
normalized size	1	1.00	1.08	3.22	0.00	16.82	0.00	4.99	-0.00
time (sec)	N/A	0.481	0.750	0.102	0.000	1.277	0.000	1.271	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	555	0	728	1416	0	5699	-1
normalized size	1	1.00	1.59	0.00	2.08	4.05	0.00	16.28	-0.00
time (sec)	N/A	0.279	0.852	0.007	1.126	0.673	0.000	0.968	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	284	0	402	712	0	2511	-1
normalized size	1	1.00	1.17	0.00	1.66	2.94	0.00	10.38	-0.00
time (sec)	N/A	0.182	0.374	0.005	1.063	0.571	0.000	0.587	0.000

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	128	0	187	294	0	806	-1
normalized size	1	1.00	0.88	0.00	1.29	2.03	0.00	5.56	-0.01
time (sec)	N/A	0.108	0.163	0.005	0.988	0.539	0.000	0.444	0.000

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	53	0	60	81	0	129	146
normalized size	1	1.00	0.85	0.00	0.97	1.31	0.00	2.08	2.35
time (sec)	N/A	0.040	0.036	0.006	0.933	0.534	0.000	0.439	3.583

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	136	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.106	0.005	0.000	0.577	0.000	0.000	0.000

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	77	70	0	164	122	0	-1
normalized size	1	1.00	0.83	0.75	0.00	1.76	1.31	0.00	-0.01
time (sec)	N/A	0.075	0.075	0.008	0.000	0.479	92.656	0.000	0.000

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	54	0	130	102	0	-1
normalized size	1	1.00	0.87	0.77	0.00	1.86	1.46	0.00	-0.01
time (sec)	N/A	0.057	0.040	0.003	0.000	0.499	66.154	0.000	0.000

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	40	0	103	0	0	-1
normalized size	1	1.00	0.96	0.82	0.00	2.10	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.022	0.003	0.000	0.500	0.000	0.000	0.000

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	78	0	0	-1
normalized size	1	1.00	1.00	0.83	0.00	2.60	0.00	0.00	-0.03
time (sec)	N/A	0.032	0.018	0.006	0.000	0.477	0.000	0.000	0.000

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	43	0	164	48	0	-1
normalized size	1	1.00	0.79	0.83	0.00	3.15	0.92	0.00	-0.02
time (sec)	N/A	0.046	0.028	0.008	0.000	0.490	11.425	0.000	0.000

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	43	59	0	262	70	0	-1
normalized size	1	1.00	0.57	0.79	0.00	3.49	0.93	0.00	-0.01
time (sec)	N/A	0.064	0.034	0.011	0.000	0.495	16.790	0.000	0.000

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	81	86	0	169	114	0	-1
normalized size	1	1.00	0.80	0.85	0.00	1.67	1.13	0.00	-0.01
time (sec)	N/A	0.073	0.079	0.010	0.000	0.480	85.592	0.000	0.000

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	66	65	0	135	95	0	-1
normalized size	1	1.00	0.87	0.86	0.00	1.78	1.25	0.00	-0.01
time (sec)	N/A	0.057	0.047	0.007	0.000	0.463	70.064	0.000	0.000

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	46	0	110	0	0	-1
normalized size	1	1.00	0.96	0.87	0.00	2.08	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.023	0.005	0.000	0.477	0.000	0.000	0.000

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	80	0	0	-1
normalized size	1	1.00	1.00	0.84	0.00	2.50	0.00	0.00	-0.03
time (sec)	N/A	0.031	0.021	0.007	0.000	0.501	0.000	0.000	0.000

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	44	49	0	175	44	0	-1
normalized size	1	1.00	0.79	0.88	0.00	3.12	0.79	0.00	-0.02
time (sec)	N/A	0.044	0.030	0.011	0.000	0.526	15.942	0.000	0.000

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	46	70	0	277	63	0	-1
normalized size	1	1.00	0.57	0.86	0.00	3.42	0.78	0.00	-0.01
time (sec)	N/A	0.061	0.037	0.010	0.000	0.506	15.629	0.000	0.000

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	21	17
normalized size	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	0.74
time (sec)	N/A	0.007	0.005	0.004	1.968	0.484	1.012	0.407	3.108

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	78	0	0	-1
normalized size	1	1.00	1.00	0.83	0.00	2.60	0.00	0.00	-0.03
time (sec)	N/A	0.031	0.032	0.007	0.000	0.496	0.000	0.000	0.000

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	0	116	0	0	-1
normalized size	1	1.00	1.00	0.86	0.00	3.14	0.00	0.00	-0.03
time (sec)	N/A	0.167	0.065	0.012	0.000	0.492	0.000	0.000	0.000

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	147	0	0	-1
normalized size	1	1.00	1.00	0.89	0.00	3.34	0.00	0.00	-0.02
time (sec)	N/A	0.373	0.146	0.019	0.000	0.477	0.000	0.000	0.000

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	46	0	182	0	0	-1
normalized size	1	1.00	1.00	0.90	0.00	3.57	0.00	0.00	-0.02
time (sec)	N/A	0.656	0.260	0.024	0.000	0.481	0.000	0.000	0.000

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	34	83	120	55	0	77	54
normalized size	1	1.00	0.45	1.09	1.58	0.72	0.00	1.01	0.71
time (sec)	N/A	0.026	0.010	0.019	2.074	0.479	0.000	0.391	3.502

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	35	69	67	50	60	67	44
normalized size	1	1.00	0.58	1.15	1.12	0.83	1.00	1.12	0.73
time (sec)	N/A	0.020	0.007	0.014	2.001	0.446	118.561	0.425	3.355

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	34	55	30	43	39	57	30
normalized size	1	1.00	0.77	1.25	0.68	0.98	0.89	1.30	0.68
time (sec)	N/A	0.014	0.007	0.013	1.988	0.448	43.654	0.384	3.575

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	13	16	12	8	37	14
normalized size	1	1.00	1.00	1.44	1.78	1.33	0.89	4.11	1.56
time (sec)	N/A	0.004	0.003	0.005	0.579	0.451	2.246	0.400	3.176

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	29	30	28	20	58	25
normalized size	1	1.00	1.14	1.38	1.43	1.33	0.95	2.76	1.19
time (sec)	N/A	0.011	0.006	0.007	0.727	0.450	3.847	0.382	3.107

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	34	38	38	34	68	28
normalized size	1	1.00	0.94	1.00	1.12	1.12	1.00	2.00	0.82
time (sec)	N/A	0.014	0.007	0.005	0.641	0.459	5.124	0.436	3.093

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	23	11	28	8	11	18
normalized size	1	1.00	2.22	2.56	1.22	3.11	0.89	1.22	2.00
time (sec)	N/A	0.004	0.006	0.006	1.397	0.460	2.831	0.337	3.108

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	12	0	28	10	20	18
normalized size	1	1.00	2.22	1.33	0.00	3.11	1.11	2.22	2.00
time (sec)	N/A	0.004	0.004	0.004	0.000	0.440	3.171	0.333	3.097

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	1059	16	67	14	16	26
normalized size	1	1.00	1.00	58.83	0.89	3.72	0.78	0.89	1.44
time (sec)	N/A	0.072	0.042	0.056	0.571	0.470	3.387	0.339	3.381

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	70	21	32	19	16	10
normalized size	1	1.00	1.00	4.38	1.31	2.00	1.19	1.00	0.62
time (sec)	N/A	0.061	0.026	0.046	0.638	0.450	0.216	0.468	3.345

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	30	27	49	36	94	29	26
normalized size	1	1.00	0.68	0.61	1.11	0.82	2.14	0.66	0.59
time (sec)	N/A	0.031	0.013	0.007	1.469	0.433	0.652	0.345	0.090

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	57	58	89	53	133	138	124
normalized size	1	1.00	0.47	0.48	0.74	0.44	1.10	1.14	1.02
time (sec)	N/A	0.111	0.043	0.013	1.545	0.447	12.715	0.368	3.273

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	71	42	0	268	49	41	42
normalized size	1	1.00	1.42	0.84	0.00	5.36	0.98	0.82	0.84
time (sec)	N/A	0.043	0.030	0.028	0.000	0.492	3.711	0.355	3.761

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	71	133	0	268	37	41	96
normalized size	1	1.00	1.42	2.66	0.00	5.36	0.74	0.82	1.92
time (sec)	N/A	0.064	0.010	0.023	0.000	0.483	79.828	0.381	4.516

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	56	0	382	97	55	56
normalized size	1	1.00	0.93	0.82	0.00	5.62	1.43	0.81	0.82
time (sec)	N/A	0.042	0.190	0.023	0.000	0.490	4.393	0.347	3.688

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	314	0	382	0	52	50
normalized size	1	1.00	0.93	4.62	0.00	5.62	0.00	0.76	0.74
time (sec)	N/A	0.509	0.072	0.027	0.000	0.503	0.000	0.459	3.899

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	39	44	0	39	0	27	39
normalized size	1	1.00	1.15	1.29	0.00	1.15	0.00	0.79	1.15
time (sec)	N/A	0.031	0.033	0.012	0.000	0.809	0.000	0.368	3.329

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	51	42	0	49	0	43	27
normalized size	1	1.00	0.69	0.57	0.00	0.66	0.00	0.58	0.36
time (sec)	N/A	0.045	0.041	0.005	0.000	0.857	0.000	0.452	3.162

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	16	14	16	13
normalized size	1	1.00	1.00	0.74	0.68	0.84	0.74	0.84	0.68
time (sec)	N/A	0.005	0.008	0.002	0.675	0.442	0.193	0.307	0.030

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	107	52	67	86	100	55	162
normalized size	1	1.00	1.98	0.96	1.24	1.59	1.85	1.02	3.00
time (sec)	N/A	0.083	0.075	0.005	1.331	0.468	1.453	0.485	0.095

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	18	12	12	32	12	12
normalized size	1	1.00	1.00	1.29	0.86	0.86	2.29	0.86	0.86
time (sec)	N/A	0.019	0.005	0.013	0.657	0.438	4.230	0.306	0.188

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	58	91	46	63	94	50	71
normalized size	1	1.00	0.95	1.49	0.75	1.03	1.54	0.82	1.16
time (sec)	N/A	0.044	0.042	0.009	1.483	0.435	2.554	0.328	3.238

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	93	30	32	39	30	32
normalized size	1	1.00	1.00	2.51	0.81	0.86	1.05	0.81	0.86
time (sec)	N/A	0.038	0.015	0.013	1.726	0.475	2.305	0.373	0.066

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	58	91	45	64	92	49	71
normalized size	1	1.00	0.95	1.49	0.74	1.05	1.51	0.80	1.16
time (sec)	N/A	0.031	0.024	0.007	1.511	0.477	2.213	0.364	0.119

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	54	21	21	36	22	25
normalized size	1	1.00	1.00	1.74	0.68	0.68	1.16	0.71	0.81
time (sec)	N/A	0.021	0.010	0.016	0.591	0.452	2.428	0.387	3.083

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	101	51	65	92	54	79
normalized size	1	1.00	0.95	1.55	0.78	1.00	1.42	0.83	1.22
time (sec)	N/A	0.037	0.027	0.007	1.318	0.435	2.307	0.353	3.158

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	42	0	51	0	45	-1
normalized size	1	1.00	0.79	0.68	0.00	0.82	0.00	0.73	-0.02
time (sec)	N/A	0.025	0.018	0.007	0.000	0.900	0.000	0.404	0.000

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	62	55	0	61	0	55	-1
normalized size	1	1.00	0.83	0.73	0.00	0.81	0.00	0.73	-0.01
time (sec)	N/A	0.044	0.053	0.007	0.000	0.872	0.000	0.408	0.000

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	54	48	0	59	0	53	-1
normalized size	1	1.00	0.79	0.71	0.00	0.87	0.00	0.78	-0.01
time (sec)	N/A	0.042	0.025	0.007	0.000	0.925	0.000	0.396	0.000

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	62	60	0	73	0	67	-1
normalized size	1	1.00	0.78	0.75	0.00	0.91	0.00	0.84	-0.01
time (sec)	N/A	0.039	0.030	0.007	0.000	0.931	0.000	0.447	0.000

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	65	67	0	101	0	88	-1
normalized size	1	1.00	0.60	0.61	0.00	0.93	0.00	0.81	-0.01
time (sec)	N/A	0.070	0.051	0.010	0.000	1.791	0.000	0.471	0.000

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	32	0	47	0	33	-1
normalized size	1	1.00	1.00	0.68	0.00	1.00	0.00	0.70	-0.02
time (sec)	N/A	0.030	0.013	0.010	0.000	0.808	0.000	0.428	0.000

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	52	49	48	58	49	102
normalized size	1	1.00	0.84	0.78	0.73	0.72	0.87	0.73	1.52
time (sec)	N/A	0.130	0.070	0.007	2.239	0.466	38.422	0.376	3.090

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	60	54	51	59	60	51	118
normalized size	1	1.00	0.85	0.76	0.72	0.83	0.85	0.72	1.66
time (sec)	N/A	0.110	0.038	0.005	2.077	0.454	73.416	0.328	0.033

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	97	46	23	44	46	43
normalized size	1	1.00	1.00	1.87	0.88	0.44	0.85	0.88	0.83
time (sec)	N/A	0.053	0.024	0.010	1.811	0.450	0.770	0.309	3.170

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	14	0	37	0	25	-1
normalized size	1	1.00	0.90	0.70	0.00	1.85	0.00	1.25	-0.05
time (sec)	N/A	0.105	0.037	0.008	0.000	0.798	0.000	0.374	0.000

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	31	67	34	29	29	36	30	53
normalized size	1	0.72	1.56	0.79	0.67	0.67	0.84	0.70	1.23
time (sec)	N/A	0.068	0.029	0.008	1.804	0.459	2.431	0.340	3.105

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	127	90	89	76	0	89	88
normalized size	1	1.00	1.09	0.78	0.77	0.66	0.00	0.77	0.76
time (sec)	N/A	0.138	0.079	0.005	0.899	0.459	0.000	0.390	0.127

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	54	53	35	0	82	-1
normalized size	1	1.00	0.70	0.65	0.64	0.42	0.00	0.99	-0.01
time (sec)	N/A	0.059	0.055	0.011	0.896	0.460	0.000	3.457	0.000

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	43	41	40	39	216	268	-1
normalized size	1	1.00	0.67	0.64	0.62	0.61	3.38	4.19	-0.02
time (sec)	N/A	0.049	0.029	0.010	0.883	0.456	2.510	7.653	0.000

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	57	59	58	57	0	474	-1
normalized size	1	1.00	0.70	0.72	0.71	0.70	0.00	5.78	-0.01
time (sec)	N/A	0.080	0.047	0.013	0.874	0.465	0.000	7.170	0.000

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	54	53	35	0	82	-1
normalized size	1	1.00	0.70	0.65	0.64	0.42	0.00	0.99	-0.01
time (sec)	N/A	0.047	0.021	0.006	0.884	0.449	0.000	3.180	0.000

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	135	121	120	76	0	0	-1
normalized size	1	1.00	0.71	0.64	0.63	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.366	0.102	0.014	0.919	0.465	0.000	0.000	0.000

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	183	154	153	85	0	271	-1
normalized size	1	1.00	0.79	0.66	0.66	0.36	0.00	1.16	-0.00
time (sec)	N/A	0.381	0.131	0.020	0.935	0.457	0.000	51.246	0.000

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	103	107	106	62	0	859	-1
normalized size	1	1.00	0.64	0.67	0.66	0.39	0.00	5.37	-0.01
time (sec)	N/A	0.276	0.088	0.013	0.889	0.468	0.000	15.544	0.000

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	0	35	0	25	-1
normalized size	1	1.00	1.00	0.80	0.00	1.75	0.00	1.25	-0.05
time (sec)	N/A	0.078	0.019	0.005	0.000	0.797	0.000	0.318	0.000

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	52	44	38	0	85	0	49	-1
normalized size	1	1.18	1.00	0.86	0.00	1.93	0.00	1.11	-0.02
time (sec)	N/A	0.035	0.018	0.010	0.000	1.084	0.000	0.304	0.000

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	80	58	45	99	61	50
normalized size	1	1.00	0.93	1.48	1.07	0.83	1.83	1.13	0.93
time (sec)	N/A	0.381	0.081	0.004	0.881	0.472	32.898	0.324	3.100

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	53	50	0	84	0	44	-1
normalized size	1	1.00	0.76	0.71	0.00	1.20	0.00	0.63	-0.01
time (sec)	N/A	0.035	0.028	0.006	0.000	1.354	0.000	0.346	0.000

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	17	15	15
normalized size	1	1.00	1.00	0.84	0.79	0.79	0.89	0.79	0.79
time (sec)	N/A	0.021	0.010	0.002	0.876	0.451	0.166	0.392	3.043

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	50	49	49	71	50	47
normalized size	1	1.00	0.86	0.65	0.64	0.64	0.92	0.65	0.61
time (sec)	N/A	0.066	0.052	0.005	0.885	0.466	2.323	0.353	0.048

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	A	B	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	224	68	443	0	302	56	171	255
normalized size	1	2.80	0.85	5.54	0.00	3.78	0.70	2.14	3.19
time (sec)	N/A	0.286	0.103	0.049	0.000	0.517	12.153	1.711	0.104

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	130	0	87	0	81	-1
normalized size	1	1.00	1.00	1.46	0.00	0.98	0.00	0.91	-0.01
time (sec)	N/A	0.263	0.059	0.017	0.000	4.139	0.000	1.240	0.000

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	68	0	62	0	65	-1
normalized size	1	1.00	1.00	1.11	0.00	1.02	0.00	1.07	-0.02
time (sec)	N/A	0.513	0.048	0.019	0.000	2.396	0.000	1.163	0.000

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	28	27	18	26	14	14
normalized size	1	1.00	1.00	3.50	3.38	2.25	3.25	1.75	1.75
time (sec)	N/A	0.002	0.002	0.005	0.872	0.436	0.953	0.252	0.158

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	32	27	27	0	22	12
normalized size	1	1.00	1.00	4.00	3.38	3.38	0.00	2.75	1.50
time (sec)	N/A	0.011	0.005	0.016	0.884	0.434	0.000	0.402	0.064

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	42	39	49	28	60	22	26
normalized size	1	1.00	1.91	1.77	2.23	1.27	2.73	1.00	1.18
time (sec)	N/A	0.003	0.016	0.004	0.867	0.436	1.484	0.385	3.719

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	42	45	51	42	0	35	35
normalized size	1	1.00	1.91	2.05	2.32	1.91	0.00	1.59	1.59
time (sec)	N/A	0.005	0.002	0.007	0.823	0.423	0.000	0.439	3.124

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	50	43	20	39	0	42	138
normalized size	1	1.00	1.39	1.19	0.56	1.08	0.00	1.17	3.83
time (sec)	N/A	0.004	0.018	0.017	1.975	0.424	0.000	0.334	5.090

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	50	59	41	36	0	51	41
normalized size	1	1.00	1.39	1.64	1.14	1.00	0.00	1.42	1.14
time (sec)	N/A	0.014	0.005	0.019	1.946	0.440	0.000	0.410	0.060

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	76	76	55	46	83	47	473
normalized size	1	1.00	1.10	1.10	0.80	0.67	1.20	0.68	6.86
time (sec)	N/A	0.013	0.071	0.013	0.883	0.423	13.845	0.330	12.863

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	76	79	138	64	0	62	119
normalized size	1	1.00	1.10	1.14	2.00	0.93	0.00	0.90	1.72
time (sec)	N/A	0.025	0.037	0.012	0.846	0.434	0.000	0.341	0.050

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	32	33	13	13	0	20	13
normalized size	1	1.00	2.13	2.20	0.87	0.87	0.00	1.33	0.87
time (sec)	N/A	0.012	0.018	0.016	1.948	0.436	0.000	0.301	0.179

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	34	30	15	15	0	16	15
normalized size	1	1.00	1.89	1.67	0.83	0.83	0.00	0.89	0.83
time (sec)	N/A	0.021	0.019	0.015	2.538	0.408	0.000	0.312	3.139

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	93	85	24	24	0	41	36
normalized size	1	1.00	3.88	3.54	1.00	1.00	0.00	1.71	1.50
time (sec)	N/A	0.060	0.229	0.033	1.936	0.418	0.000	0.497	0.180

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	97	80	59	105	0	74	31
normalized size	1	1.00	2.37	1.95	1.44	2.56	0.00	1.80	0.76
time (sec)	N/A	0.067	0.075	0.026	1.733	0.420	0.000	0.493	0.204

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	43	46	37	28	0	36	37
normalized size	1	1.00	1.34	1.44	1.16	0.88	0.00	1.12	1.16
time (sec)	N/A	0.012	0.013	0.007	1.800	0.406	0.000	0.354	3.127

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	67	39	43	32	0	29	43
normalized size	1	1.00	1.76	1.03	1.13	0.84	0.00	0.76	1.13
time (sec)	N/A	0.014	0.021	0.008	1.851	0.397	0.000	0.396	0.030

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	83	64	49	38	0	36	49
normalized size	1	1.00	1.98	1.52	1.17	0.90	0.00	0.86	1.17
time (sec)	N/A	0.016	0.052	0.017	2.015	0.407	0.000	0.446	3.171

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	78	60	70	58	0	40	51
normalized size	1	1.00	1.90	1.46	1.71	1.41	0.00	0.98	1.24
time (sec)	N/A	0.019	0.068	0.013	0.906	0.404	0.000	0.310	0.050

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	123	152	118	180	0	119	90
normalized size	1	1.00	1.62	2.00	1.55	2.37	0.00	1.57	1.18
time (sec)	N/A	0.039	0.272	0.010	1.235	0.413	0.000	0.570	0.273

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	76	76	80	54	0	74	57
normalized size	1	1.00	1.55	1.55	1.63	1.10	0.00	1.51	1.16
time (sec)	N/A	0.012	0.041	0.013	1.395	0.406	0.000	0.398	3.164

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	79	103	53	46	0	114	74
normalized size	1	1.00	1.72	2.24	1.15	1.00	0.00	2.48	1.61
time (sec)	N/A	0.023	0.034	0.025	1.425	0.398	0.000	0.404	3.239

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	17	27	17	0	29	17
normalized size	1	1.00	0.95	0.85	1.35	0.85	0.00	1.45	0.85
time (sec)	N/A	0.056	0.009	0.004	0.611	0.395	0.000	0.415	0.057

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	17	16	17	0	29	-1
normalized size	1	1.00	0.94	0.94	0.89	0.94	0.00	1.61	-0.06
time (sec)	N/A	0.031	0.006	0.003	0.610	0.384	0.000	0.259	0.000

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	106	81	103	83	0	107	62
normalized size	1	1.00	1.96	1.50	1.91	1.54	0.00	1.98	1.15
time (sec)	N/A	0.057	0.073	0.023	1.219	0.408	0.000	0.298	0.089

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	18	0	17	0	23	15
normalized size	1	1.00	1.00	1.64	0.00	1.55	0.00	2.09	1.36
time (sec)	N/A	0.003	0.005	0.006	0.000	0.394	0.000	0.293	3.125

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	41	40	0	34	0	0	-1
normalized size	1	1.00	1.41	1.38	0.00	1.17	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.227	0.023	0.000	0.430	0.000	0.000	0.000

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	197	551	0	372	0	287	-1
normalized size	1	1.00	1.09	3.06	0.00	2.07	0.00	1.59	-0.01
time (sec)	N/A	0.195	0.384	0.072	0.000	0.461	0.000	0.517	0.000

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	306	1066	0	171	0	350	-1
normalized size	1	1.00	1.78	6.20	0.00	0.99	0.00	2.03	-0.01
time (sec)	N/A	0.144	0.452	0.049	0.000	0.447	0.000	0.477	0.000

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	333	5984	0	223	0	452	-1
normalized size	1	1.00	1.08	19.49	0.00	0.73	0.00	1.47	-0.00
time (sec)	N/A	0.251	1.109	0.067	0.000	0.430	0.000	0.559	0.000

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	59	71	0	77	0	81	-1
normalized size	1	1.00	0.91	1.09	0.00	1.18	0.00	1.25	-0.02
time (sec)	N/A	0.030	0.028	0.006	0.000	0.432	0.000	0.370	0.000

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	79	118	0	97	0	143	-1
normalized size	1	1.00	0.95	1.42	0.00	1.17	0.00	1.72	-0.01
time (sec)	N/A	0.033	0.034	0.021	0.000	0.402	0.000	0.512	0.000

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	146	0	129	0	184	-1
normalized size	1	1.00	0.96	1.45	0.00	1.28	0.00	1.82	-0.01
time (sec)	N/A	0.037	0.051	0.027	0.000	0.397	0.000	0.423	0.000

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	187	370	0	187	0	197	-1
normalized size	1	1.00	1.73	3.43	0.00	1.73	0.00	1.82	-0.01
time (sec)	N/A	0.102	0.422	0.031	0.000	0.450	0.000	0.453	0.000

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	881	2407	0	121	0	263	-1
normalized size	1	1.00	10.13	27.67	0.00	1.39	0.00	3.02	-0.01
time (sec)	N/A	0.065	1.666	0.089	0.000	0.420	0.000	0.463	0.000

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	914	14529	0	171	0	367	-1
normalized size	1	1.00	6.13	97.51	0.00	1.15	0.00	2.46	-0.01
time (sec)	N/A	0.095	2.462	0.278	0.000	0.414	0.000	0.453	0.000

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	59	62	51	59	58	182	58	38
normalized size	1	1.40	1.48	1.21	1.40	1.38	4.33	1.38	0.90
time (sec)	N/A	0.215	0.368	0.009	0.832	0.400	0.702	0.329	0.183

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	59	62	51	59	58	182	58	51
normalized size	1	1.40	1.48	1.21	1.40	1.38	4.33	1.38	1.21
time (sec)	N/A	0.239	0.369	0.007	1.039	0.399	10.179	0.387	3.421

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	278	1050	0	0	0	0	-1
normalized size	1	1.00	2.73	10.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.768	0.147	0.000	0.438	0.000	0.000	0.000

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	256	946	0	0	0	0	-1
normalized size	1	1.00	4.13	15.26	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.633	0.075	0.000	0.424	0.000	0.000	0.000

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	156	200	0	0	0	0	-1
normalized size	1	1.00	9.18	11.76	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.014	0.153	0.074	0.000	0.428	0.000	0.000	0.000

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	261	963	0	0	0	0	-1
normalized size	1	1.00	3.58	13.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.913	0.072	0.000	0.426	0.000	0.000	0.000

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	298	1039	0	0	0	0	-1
normalized size	1	1.00	2.73	9.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	1.079	0.076	0.000	0.417	0.000	0.000	0.000

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	278	1050	0	0	0	0	-1
normalized size	1	1.00	2.73	10.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	1.025	0.073	0.000	0.427	0.000	0.000	0.000

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	256	946	0	0	0	0	-1
normalized size	1	1.00	4.13	15.26	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.845	0.078	0.000	0.418	0.000	0.000	0.000

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	100	200	0	0	0	0	-1
normalized size	1	1.00	5.88	11.76	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.012	0.270	0.073	0.000	0.423	0.000	0.000	0.000

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	298	963	0	0	0	0	-1
normalized size	1	1.00	4.08	13.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.987	0.075	0.000	0.415	0.000	0.000	0.000

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	327	1039	0	0	0	0	-1
normalized size	1	1.00	3.00	9.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	1.091	0.074	0.000	0.409	0.000	0.000	0.000

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	10468	5229	0	0	0	0	-1
normalized size	1	1.00	14.34	7.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.903	6.203	0.199	0.000	0.424	0.000	0.000	0.000

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	622	622	5218	4890	0	0	0	0	-1
normalized size	1	1.00	8.39	7.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.660	6.095	0.066	0.000	0.416	0.000	0.000	0.000

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	822	1056	0	0	0	0	-1
normalized size	1	1.00	3.62	4.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.173	2.248	0.066	0.000	0.421	0.000	0.000	0.000

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	5276	5024	0	0	0	0	-1
normalized size	1	1.00	7.83	7.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.683	6.134	0.081	0.000	0.431	0.000	0.000	0.000

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	7543	7887	0	0	0	0	-1
normalized size	1	1.00	11.38	11.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.810	6.124	0.267	0.000	0.418	0.000	0.000	0.000

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	1065	1704	0	0	0	0	-1
normalized size	1	1.00	4.53	7.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.177	2.284	0.062	0.000	0.429	0.000	0.000	0.000

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	748	748	7629	8103	0	0	0	0	-1
normalized size	1	1.00	10.20	10.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.786	6.154	0.080	0.000	0.430	0.000	0.000	0.000

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	6287	2655	0	0	0	0	-1
normalized size	1	1.00	13.91	5.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.579	6.127	0.076	0.000	0.422	0.000	0.000	0.000

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	3470	2519	0	0	0	0	-1
normalized size	1	1.00	8.74	6.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.401	6.061	0.029	0.000	0.403	0.000	0.000	0.000

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	540	530	0	0	0	0	-1
normalized size	1	1.00	3.75	3.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	1.487	0.029	0.000	0.418	0.000	0.000	0.000

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	3526	2601	0	0	0	0	-1
normalized size	1	1.00	8.07	5.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.416	6.088	0.032	0.000	0.426	0.000	0.000	0.000

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	6386	2757	0	0	0	0	-1
normalized size	1	1.00	12.35	5.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	6.224	0.040	0.000	0.425	0.000	0.000	0.000

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	7235	2694	0	0	0	0	-1
normalized size	1	1.00	12.97	4.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.565	6.143	0.031	0.000	0.471	0.000	0.000	0.000

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	4389	2551	0	0	0	0	-1
normalized size	1	1.00	9.42	5.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	6.091	0.029	0.000	0.473	0.000	0.000	0.000

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	813	788	0	0	0	0	-1
normalized size	1	1.00	4.54	4.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	2.931	0.028	0.000	0.461	0.000	0.000	0.000

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	3593	2616	0	0	0	0	-1
normalized size	1	1.00	7.58	5.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	6.084	0.033	0.000	0.429	0.000	0.000	0.000

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	6452	2777	0	0	0	0	-1
normalized size	1	1.00	10.92	4.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.584	6.130	0.043	0.000	0.425	0.000	0.000	0.000

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	8500	2733	0	0	0	0	-1
normalized size	1	1.00	14.53	4.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.679	6.170	0.030	0.000	0.476	0.000	0.000	0.000

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	5647	2582	0	0	0	0	-1
normalized size	1	1.00	11.64	5.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.530	6.122	0.031	0.000	0.467	0.000	0.000	0.000

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	1247	1147	0	0	0	0	-1
normalized size	1	1.00	3.21	2.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.387	6.056	0.031	0.000	0.451	0.000	0.000	0.000

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	2941	2607	0	0	0	0	-1
normalized size	1	1.00	9.46	8.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	6.125	0.034	0.000	0.427	0.000	0.000	0.000

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	582	582	5812	2780	0	0	0	0	-1
normalized size	1	1.00	9.99	4.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.675	6.167	0.043	0.000	0.438	0.000	0.000	0.000

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	927	965	0	0	0	0	-1
normalized size	1	1.00	7.19	7.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.302	0.843	1.130	0.000	0.431	0.000	0.000	0.000

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	4865	4426	0	0	0	0	-1
normalized size	1	1.00	11.29	10.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.528	6.046	0.118	0.000	0.412	0.000	0.000	0.000

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	249	961	0	0	0	0	-1
normalized size	1	1.00	2.31	8.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.593	0.849	0.000	0.423	0.000	0.000	0.000

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	602	2564	0	0	0	0	-1
normalized size	1	1.00	1.64	6.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	4.758	0.110	0.000	0.423	0.000	0.000	0.000

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	1148	1180	0	0	0	0	-1
normalized size	1	1.00	9.11	9.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.326	0.977	1.681	0.000	0.410	0.000	0.000	0.000

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	6019	5421	0	0	0	0	-1
normalized size	1	1.00	13.87	12.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.528	6.077	0.118	0.000	0.423	0.000	0.000	0.000

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	6084	5477	0	0	0	0	-1
normalized size	1	1.00	10.54	9.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.688	6.058	0.137	0.000	0.423	0.000	0.000	0.000

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	826	1182	0	0	0	0	-1
normalized size	1	1.00	6.35	9.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	0.121	0.495	0.000	0.410	0.000	0.000	0.000

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	5428	5427	0	0	0	0	-1
normalized size	1	1.00	12.23	12.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.468	6.051	0.071	0.000	0.420	0.000	0.000	0.000

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	76	57	81	0	0	158
normalized size	1	1.00	1.02	1.36	1.02	1.45	0.00	0.00	2.82
time (sec)	N/A	0.211	0.108	0.021	0.964	0.414	0.000	0.000	7.906

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	0	84	63	89	55
normalized size	1	1.00	1.00	0.86	0.00	1.29	0.97	1.37	0.85
time (sec)	N/A	0.159	0.076	0.007	0.000	0.399	4.510	0.420	0.043

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	101	88	58	0	78	0	80	52
normalized size	1	1.91	1.66	1.09	0.00	1.47	0.00	1.51	0.98
time (sec)	N/A	0.215	0.150	0.006	0.000	0.415	0.000	0.439	0.045

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	12	12	12
normalized size	1	1.00	1.00	0.93	0.86	0.86	0.86	0.86	0.86
time (sec)	N/A	0.117	0.006	0.003	0.982	0.400	0.157	0.396	3.387

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	46	45	43	60	43	43
normalized size	1	1.00	1.00	0.75	0.74	0.70	0.98	0.70	0.70
time (sec)	N/A	0.166	0.048	0.001	0.438	0.403	2.419	0.430	3.360

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	86	85	91	116	84	84
normalized size	1	1.00	1.00	0.75	0.75	0.80	1.02	0.74	0.74
time (sec)	N/A	0.193	0.087	0.003	0.438	0.417	3.453	0.345	0.060

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	100	102	0	180	70	0	46
normalized size	1	1.00	1.72	1.76	0.00	3.10	1.21	0.00	0.79
time (sec)	N/A	0.057	0.063	0.024	0.000	0.436	3.183	0.000	4.083

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	100	102	0	180	70	0	62
normalized size	1	1.00	1.72	1.76	0.00	3.10	1.21	0.00	1.07
time (sec)	N/A	0.137	0.007	0.017	0.000	0.440	7.026	0.000	4.045

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	64	39	0	182	0	123	67
normalized size	1	1.00	1.21	0.74	0.00	3.43	0.00	2.32	1.26
time (sec)	N/A	0.031	0.044	0.023	0.000	0.439	0.000	0.436	3.588

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	22	27	18	23
normalized size	1	1.00	1.00	0.78	0.74	0.96	1.17	0.78	1.00
time (sec)	N/A	0.007	0.011	0.002	0.429	0.410	0.194	0.329	0.030

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	17	25	105	54	35
normalized size	1	1.00	1.00	1.13	0.74	1.09	4.57	2.35	1.52
time (sec)	N/A	0.008	0.004	0.003	0.966	0.413	92.595	0.451	3.695

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	58	36	37	0	39	180
normalized size	1	1.00	1.00	1.76	1.09	1.12	0.00	1.18	5.45
time (sec)	N/A	0.015	0.012	0.014	0.432	0.403	0.000	0.385	7.557

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	27	35	2	2	2	31	-1
normalized size	1	1.00	0.60	0.78	0.04	0.04	0.04	0.69	-0.02
time (sec)	N/A	0.158	0.071	0.009	0.963	0.407	0.106	0.399	0.000

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	27	35	2	2	2	31	-1
normalized size	1	1.00	0.60	0.78	0.04	0.04	0.04	0.69	-0.02
time (sec)	N/A	0.055	0.015	0.008	0.969	0.410	0.102	0.390	0.000

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	27	35	2	2	2	31	-1
normalized size	1	1.00	0.60	0.78	0.04	0.04	0.04	0.69	-0.02
time (sec)	N/A	0.098	0.013	0.010	0.980	0.404	0.102	0.363	0.000

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	27	27	2	2	2	12	-1
normalized size	1	1.00	0.60	0.60	0.04	0.04	0.04	0.27	-0.02
time (sec)	N/A	0.133	0.035	0.008	0.958	0.402	0.099	0.370	0.000

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	41	0	6	0	6	-1
normalized size	1	1.00	0.94	0.79	0.00	0.12	0.00	0.12	-0.02
time (sec)	N/A	0.183	0.062	0.010	0.000	0.411	0.000	0.319	0.000

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	30	6	6	0	6	-1
normalized size	1	1.00	0.94	0.58	0.12	0.12	0.00	0.12	-0.02
time (sec)	N/A	0.126	0.029	0.009	0.987	0.407	0.000	0.318	0.000

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	64	40	0	8	0	8	-1
normalized size	1	1.00	0.94	0.59	0.00	0.12	0.00	0.12	-0.01
time (sec)	N/A	0.155	0.165	0.012	0.000	0.417	0.000	0.414	0.000

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	27	25	0	54	19
normalized size	1	1.00	0.74	1.03	0.87	0.81	0.00	1.74	0.61
time (sec)	N/A	0.015	0.189	0.012	0.964	0.417	0.000	0.443	3.327

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	25	25	0	54	19
normalized size	1	1.00	0.74	1.03	0.81	0.81	0.00	1.74	0.61
time (sec)	N/A	0.288	0.146	0.010	0.606	0.424	0.000	0.589	0.036

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	81	0	25	0	68	19
normalized size	1	1.00	0.74	2.61	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.140	0.092	0.035	0.000	0.392	0.000	0.500	0.065

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	81	0	25	0	68	19
normalized size	1	1.00	0.74	2.61	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.125	0.062	0.036	0.000	0.404	0.000	0.366	0.053

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	0	25	0	68	19
normalized size	1	1.00	0.74	1.03	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.654	0.184	0.013	0.000	0.413	0.000	0.616	3.324

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	73	85	44	212	0	-1
normalized size	1	1.00	0.88	1.70	1.98	1.02	4.93	0.00	-0.02
time (sec)	N/A	0.067	0.041	0.018	0.450	0.416	10.184	0.000	0.000

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	45	0	44	0	57	31
normalized size	1	1.00	1.00	1.07	0.00	1.05	0.00	1.36	0.74
time (sec)	N/A	0.130	0.039	0.008	0.000	0.405	0.000	0.456	3.388

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	262	0	80	0	133	60
normalized size	1	1.00	0.83	4.03	0.00	1.23	0.00	2.05	0.92
time (sec)	N/A	0.133	0.051	0.048	0.000	0.413	0.000	0.567	3.444

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	240	1407	0	1174	0	536	-1
normalized size	1	1.00	1.23	7.22	0.00	6.02	0.00	2.75	-0.01
time (sec)	N/A	0.207	0.395	0.033	0.000	0.457	0.000	0.492	0.000

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	34	52	37	34	20
normalized size	1	1.00	1.00	1.04	1.21	1.86	1.32	1.21	0.71
time (sec)	N/A	0.014	0.017	0.032	0.964	0.410	22.797	0.382	3.850

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	0	52	0	34	-1
normalized size	1	1.00	1.00	1.04	0.00	1.86	0.00	1.21	-0.04
time (sec)	N/A	0.058	0.006	0.034	0.000	0.418	0.000	0.495	0.000

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	18	18
normalized size	1	1.00	1.00	0.86	0.82	0.82	0.77	0.82	0.82
time (sec)	N/A	0.005	0.010	0.003	0.581	0.410	0.148	0.340	0.044

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	18	18
normalized size	1	1.00	1.00	0.86	0.82	0.82	0.77	0.82	0.82
time (sec)	N/A	0.082	0.003	0.004	0.816	0.397	1.170	0.567	0.029

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	0	11	36	11	11
normalized size	1	1.00	1.00	0.71	0.00	0.65	2.12	0.65	0.65
time (sec)	N/A	0.061	0.023	0.008	0.000	0.460	1.273	0.371	3.700

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	125	0	11	0	11	-1
normalized size	1	1.00	1.00	7.35	0.00	0.65	0.00	0.65	-0.06
time (sec)	N/A	0.047	0.035	0.112	0.000	0.457	0.000	0.326	0.000

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	19	7	6	6
normalized size	1	1.00	1.00	0.70	0.60	1.90	0.70	0.60	0.60
time (sec)	N/A	0.001	0.005	0.003	1.957	0.388	0.150	0.364	0.010

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	34	6	25	51	12	32
normalized size	1	1.00	1.00	3.40	0.60	2.50	5.10	1.20	3.20
time (sec)	N/A	0.002	0.006	0.007	1.931	0.446	1.042	0.366	0.148

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	19	7	6	6
normalized size	1	1.00	1.00	0.70	0.60	1.90	0.70	0.60	0.60
time (sec)	N/A	0.004	0.005	0.009	1.956	0.417	1.393	0.395	0.012

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	7	8	29	0	6	6
normalized size	1	1.00	1.00	0.58	0.67	2.42	0.00	0.50	0.50
time (sec)	N/A	0.007	0.008	0.004	1.970	0.442	0.000	0.420	3.125

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	21	31	8	29	41	13	30
normalized size	1	1.00	1.75	2.58	0.67	2.42	3.42	1.08	2.50
time (sec)	N/A	0.007	0.018	0.006	1.962	0.412	1.023	0.326	3.428

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	21	7	8	29	0	6	6
normalized size	1	1.00	1.75	0.58	0.67	2.42	0.00	0.50	0.50
time (sec)	N/A	0.009	0.003	0.008	1.971	0.407	0.000	0.345	3.370

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	8	29	0	4	4
normalized size	1	1.00	1.00	1.25	2.00	7.25	0.00	1.00	1.00
time (sec)	N/A	0.005	0.007	0.004	1.989	0.406	0.000	0.342	3.185

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	18	29	8	29	41	13	33
normalized size	1	1.00	4.50	7.25	2.00	7.25	10.25	3.25	8.25
time (sec)	N/A	0.006	0.014	0.007	1.972	0.412	1.022	0.316	0.082

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	18	5	8	29	0	4	4
normalized size	1	1.00	4.50	1.25	2.00	7.25	0.00	1.00	1.00
time (sec)	N/A	0.007	0.003	0.007	1.950	0.423	0.000	0.388	3.362

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
normalized size	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.002	0.002	0.898	0.407	0.059	0.361	0.026

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
normalized size	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.008	0.001	0.002	0.913	0.396	0.151	0.332	0.026

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	30	0	26	7	10	-1
normalized size	1	1.00	1.00	1.11	0.00	0.96	0.26	0.37	-0.04
time (sec)	N/A	0.014	0.008	0.006	0.000	0.450	1.014	0.422	0.000

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	30	0	26	0	10	-1
normalized size	1	1.00	1.00	1.11	0.00	0.96	0.00	0.37	-0.04
time (sec)	N/A	0.021	0.006	0.007	0.000	0.415	0.000	0.337	0.000

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	33	56	28	14	14	15	15	-1
normalized size	1	1.32	2.24	1.12	0.56	0.56	0.60	0.60	-0.04
time (sec)	N/A	0.014	0.026	0.007	0.881	0.411	1.519	0.360	0.000

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	33	56	28	0	14	0	21	-1
normalized size	1	1.32	2.24	1.12	0.00	0.56	0.00	0.84	-0.04
time (sec)	N/A	0.022	0.003	0.007	0.000	0.412	0.000	0.317	0.000

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	9
normalized size	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82
time (sec)	N/A	0.001	0.003	0.003	0.890	0.404	0.061	0.334	0.193

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	C	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	20	12	16	0	15	16
normalized size	1	1.00	2.09	1.82	1.09	1.45	0.00	1.36	1.45
time (sec)	N/A	0.001	0.023	0.002	0.920	0.410	0.000	0.321	3.557

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	7	7
normalized size	1	1.00	1.00	0.89	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.001	0.002	0.003	0.884	0.403	0.056	0.341	0.095

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	25	22	7	23	0	13	18
normalized size	1	1.00	2.78	2.44	0.78	2.56	0.00	1.44	2.00
time (sec)	N/A	0.001	0.023	0.003	0.899	0.406	0.000	0.426	3.635

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	12	10	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.92	0.77	0.69	0.69
time (sec)	N/A	0.001	0.003	0.004	0.808	0.404	0.058	0.345	3.507

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	20	12	19	0	15	20
normalized size	1	1.00	1.92	1.54	0.92	1.46	0.00	1.15	1.54
time (sec)	N/A	0.001	0.021	0.002	0.831	0.407	0.000	0.334	3.523

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
normalized size	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.002	0.003	0.882	0.401	0.057	0.318	3.419

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	27	22	7	26	0	13	22
normalized size	1	1.00	2.45	2.00	0.64	2.36	0.00	1.18	2.00
time (sec)	N/A	0.001	0.022	0.002	0.919	0.407	0.000	0.334	3.491

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	49	67	41	52	97	39	172
normalized size	1	1.00	1.40	1.91	1.17	1.49	2.77	1.11	4.91
time (sec)	N/A	0.007	0.019	0.007	1.968	0.416	1.619	0.352	6.136

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	49	86	0	96	0	0	-1
normalized size	1	1.00	1.40	2.46	0.00	2.74	0.00	0.00	-0.03
time (sec)	N/A	0.007	0.008	0.013	0.000	0.449	0.000	0.000	0.000

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	61	70	53	74	0	0	-1
normalized size	1	1.00	1.42	1.63	1.23	1.72	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.039	0.018	1.987	0.412	0.000	0.000	0.000

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	47	41	53	63	0	44	37
normalized size	1	1.00	1.34	1.17	1.51	1.80	0.00	1.26	1.06
time (sec)	N/A	0.061	0.023	0.010	1.987	0.413	0.000	0.395	3.204

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	72	134	65	93	0	0	-1
normalized size	1	1.00	1.41	2.63	1.27	1.82	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.060	0.041	2.504	0.423	0.000	0.000	0.000

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	59	75	65	82	0	87	82
normalized size	1	1.00	1.31	1.67	1.44	1.82	0.00	1.93	1.82
time (sec)	N/A	0.091	0.038	0.014	2.076	0.461	0.000	0.543	3.491

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	18	2	2	2
normalized size	1	1.00	1.00	1.50	1.00	9.00	1.00	1.00	1.00
time (sec)	N/A	0.001	0.004	0.002	1.675	0.441	0.141	0.344	0.006

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	32	29	0	27	0	0	-1
normalized size	1	1.00	16.00	14.50	0.00	13.50	0.00	0.00	-0.50
time (sec)	N/A	0.001	0.026	0.018	0.000	0.426	0.000	0.000	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	14	2	14	2
normalized size	1	1.00	1.00	1.50	1.00	7.00	1.00	7.00	1.00
time (sec)	N/A	0.001	0.003	0.004	2.000	0.425	0.137	0.428	0.028

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	42	29	0	81	0	0	-1
normalized size	1	1.00	21.00	14.50	0.00	40.50	0.00	0.00	-0.50
time (sec)	N/A	0.001	0.024	0.011	0.000	0.436	0.000	0.000	0.000

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	18	17	31	15	17	17
normalized size	1	1.00	0.87	0.78	0.74	1.35	0.65	0.74	0.74
time (sec)	N/A	0.003	0.006	0.004	1.933	0.447	0.195	0.357	0.031

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	50	42	0	60	0	0	-1
normalized size	1	1.00	2.17	1.83	0.00	2.61	0.00	0.00	-0.04
time (sec)	N/A	0.003	0.041	0.011	0.000	0.446	0.000	0.000	0.000

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	16	15	25	15	25	15
normalized size	1	1.00	0.86	0.76	0.71	1.19	0.71	1.19	0.71
time (sec)	N/A	0.002	0.004	0.003	1.950	0.424	0.194	0.372	0.028

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	70	47	0	120	0	0	-1
normalized size	1	1.00	3.33	2.24	0.00	5.71	0.00	0.00	-0.05
time (sec)	N/A	0.003	0.056	0.010	0.000	0.428	0.000	0.000	0.000

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	57	53	0	0	0	0	0	54
normalized size	1	1.16	1.08	0.00	0.00	0.00	0.00	0.00	1.10
time (sec)	N/A	0.017	0.013	0.071	0.000	0.457	0.000	0.000	4.673

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	38	21	0	30	58	30	20
normalized size	1	1.00	1.36	0.75	0.00	1.07	2.07	1.07	0.71
time (sec)	N/A	0.009	0.012	0.004	0.000	0.417	0.354	0.336	0.034

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	175	0	84	17	140	105
normalized size	1	1.00	1.00	4.73	0.00	2.27	0.46	3.78	2.84
time (sec)	N/A	0.042	0.023	0.046	0.000	0.444	0.169	0.438	0.143

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	74	33	0	90	27	88	57
normalized size	1	1.00	1.85	0.82	0.00	2.25	0.68	2.20	1.42
time (sec)	N/A	0.043	0.040	0.011	0.000	0.441	0.208	0.409	3.501

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	77	111	94	67	0	117	86
normalized size	1	1.00	1.43	2.06	1.74	1.24	0.00	2.17	1.59
time (sec)	N/A	0.128	0.057	0.015	2.029	0.420	0.000	0.441	3.328

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	111	0	67	0	117	86
normalized size	1	1.00	1.28	1.85	0.00	1.12	0.00	1.95	1.43
time (sec)	N/A	0.300	0.087	0.017	0.000	0.442	0.000	0.514	3.377

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	38	0	66	0	50	-1
normalized size	1	1.00	0.76	0.75	0.00	1.29	0.00	0.98	-0.02
time (sec)	N/A	0.111	0.052	0.005	0.000	0.434	0.000	0.426	0.000

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	38	54	66	0	50	-1
normalized size	1	1.00	0.76	0.75	1.06	1.29	0.00	0.98	-0.02
time (sec)	N/A	0.097	0.046	0.006	1.984	0.473	0.000	0.484	0.000

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	51	54	64	0	0	56
normalized size	1	1.00	0.76	1.00	1.06	1.25	0.00	0.00	1.10
time (sec)	N/A	0.148	0.023	0.011	1.973	0.430	0.000	0.000	4.782

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	82	51	0	64	0	0	56
normalized size	1	1.00	1.52	0.94	0.00	1.19	0.00	0.00	1.04
time (sec)	N/A	0.049	0.142	0.010	0.000	0.420	0.000	0.000	0.061

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	0	20	0	0	20
normalized size	1	1.00	0.85	0.74	0.00	0.74	0.00	0.00	0.74
time (sec)	N/A	0.030	0.011	0.003	0.000	0.416	0.000	0.000	3.419

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	13	0	16	0	0	138
normalized size	1	1.00	0.86	0.93	0.00	1.14	0.00	0.00	9.86
time (sec)	N/A	0.149	0.024	0.007	0.000	0.416	0.000	0.000	3.380

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	16	0	0	10
normalized size	1	1.00	1.00	0.92	0.00	1.33	0.00	0.00	0.83
time (sec)	N/A	0.070	0.011	0.004	0.000	0.419	0.000	0.000	0.051

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	29	34	0	30	0	0	48
normalized size	1	1.00	0.81	0.94	0.00	0.83	0.00	0.00	1.33
time (sec)	N/A	0.140	0.019	0.007	0.000	0.424	0.000	0.000	3.492

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	0	30	0	0	43
normalized size	1	1.00	0.88	1.03	0.00	0.91	0.00	0.00	1.30
time (sec)	N/A	0.193	0.012	0.006	0.000	0.409	0.000	0.000	3.494

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	105	117	0	208	0	0	-1
normalized size	1	1.00	1.50	1.67	0.00	2.97	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.131	0.053	0.000	0.443	0.000	0.000	0.000

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	52	63	0	38	0	46	-1
normalized size	1	1.00	0.63	0.76	0.00	0.46	0.00	0.55	-0.01
time (sec)	N/A	0.154	0.064	0.032	0.000	0.429	0.000	0.359	0.000

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	73	91	63	0	36	0	18	-1
normalized size	1	1.55	1.94	1.34	0.00	0.77	0.00	0.38	-0.02
time (sec)	N/A	0.460	0.118	0.026	0.000	0.416	0.000	0.340	0.000

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	70	75	0	74	0	39	-1
normalized size	1	1.00	0.57	0.61	0.00	0.60	0.00	0.32	-0.01
time (sec)	N/A	0.286	0.025	0.028	0.000	0.420	0.000	0.414	0.000

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	64	62	0	117	0	86	-1
normalized size	1	1.00	0.48	0.47	0.00	0.88	0.00	0.65	-0.01
time (sec)	N/A	0.074	0.100	0.023	0.000	0.493	0.000	0.469	0.000

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	44	49	0	83	0	67	-1
normalized size	1	1.00	0.49	0.54	0.00	0.92	0.00	0.74	-0.01
time (sec)	N/A	0.048	0.039	0.016	0.000	0.410	0.000	0.372	0.000

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	40	42	0	75	0	49	-1
normalized size	1	1.00	0.66	0.69	0.00	1.23	0.00	0.80	-0.02
time (sec)	N/A	0.030	0.018	0.010	0.000	0.414	0.000	0.480	0.000

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	72	79	0	142	0	88	-1
normalized size	1	1.00	0.66	0.72	0.00	1.30	0.00	0.81	-0.01
time (sec)	N/A	0.065	0.032	0.039	0.000	0.423	0.000	0.571	0.000

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	95	218	0	205	0	0	-1
normalized size	1	1.00	0.66	1.51	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.115	0.016	0.000	0.427	0.000	0.000	0.000

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	28	0	31	0	30	23
normalized size	1	1.00	0.93	1.00	0.00	1.11	0.00	1.07	0.82
time (sec)	N/A	0.116	0.027	0.006	0.000	0.412	0.000	0.391	3.515

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.034	0.101	0.068	0.000	0.413	0.000	0.000	0.000

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.038	0.179	0.066	0.000	0.415	0.000	0.000	0.000

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.104	0.027	0.068	0.000	0.426	0.000	0.000	0.000

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.113	0.022	0.064	0.000	0.418	0.000	0.000	0.000

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.012	0.006	0.020	0.000	0.442	0.000	0.000	0.000

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.013	0.006	0.019	0.000	0.409	0.000	0.000	0.000

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.008	0.007	0.015	0.000	0.412	0.000	0.000	0.000

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	0.009	0.010	0.000	0.414	0.000	0.000	0.000

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.008	0.012	0.018	0.000	0.403	0.000	0.000	0.000

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	0.012	0.016	0.000	0.410	0.000	0.000	0.000

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	135	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	2.87	0.00	0.00	-0.02
time (sec)	N/A	0.109	0.023	180.000	0.000	1.812	0.000	0.000	0.000

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	146	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	3.04	0.00	0.00	-0.02
time (sec)	N/A	0.108	0.020	180.000	0.000	1.796	0.000	0.000	0.000

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.266	0.114	0.089	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.100	0.073	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	28	27	25	37	27	27
normalized size	1	1.00	1.00	0.68	0.66	0.61	0.90	0.66	0.66
time (sec)	N/A	0.045	0.026	0.004	2.342	0.948	11.870	0.397	3.367

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	38	21	20	20	24	20	22
normalized size	1	1.00	1.46	0.81	0.77	0.77	0.92	0.77	0.85
time (sec)	N/A	0.040	0.025	0.006	2.291	1.052	3.662	0.350	3.361

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	54	48	30	30	39	30	42
normalized size	1	1.00	1.29	1.14	0.71	0.71	0.93	0.71	1.00
time (sec)	N/A	0.154	0.023	0.015	2.390	0.626	23.130	0.339	0.026

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	48	50	0	75	0	44	17
normalized size	1	1.00	2.40	2.50	0.00	3.75	0.00	2.20	0.85
time (sec)	N/A	0.008	0.014	0.013	0.000	0.852	0.000	0.351	3.460

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	52	62	0	84	0	40	21
normalized size	1	1.00	2.60	3.10	0.00	4.20	0.00	2.00	1.05
time (sec)	N/A	0.009	0.014	0.012	0.000	0.729	0.000	0.416	3.470

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	136	244	0	1532	0	0	-1
normalized size	1	1.00	1.12	2.02	0.00	12.66	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.102	0.033	0.000	2.052	0.000	0.000	0.000

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	189	397	0	2411	0	0	-1
normalized size	1	1.00	1.04	2.19	0.00	13.32	0.00	0.00	-0.01
time (sec)	N/A	0.273	0.281	0.041	0.000	103.950	0.000	0.000	0.000

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	17	16	16	0	11	16
normalized size	1	1.00	1.00	0.65	0.62	0.62	0.00	0.42	0.62
time (sec)	N/A	0.006	0.013	0.004	0.964	0.848	0.000	0.511	3.541

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	27	23	22	32	0	22	22
normalized size	1	1.00	1.04	0.88	0.85	1.23	0.00	0.85	0.85
time (sec)	N/A	0.011	0.031	0.007	1.932	0.747	0.000	0.614	3.529

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	16	11	11	12	11	9
normalized size	1	1.00	0.87	1.07	0.73	0.73	0.80	0.73	0.60
time (sec)	N/A	0.003	0.012	0.005	0.863	0.646	0.158	0.427	3.565

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	19	23	28	27	0	27	18
normalized size	1	1.00	0.86	1.05	1.27	1.23	0.00	1.23	0.82
time (sec)	N/A	0.005	0.013	0.004	0.884	0.710	0.000	0.522	3.504

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	37	13	9	17	0	17	-1
normalized size	1	1.00	3.08	1.08	0.75	1.42	0.00	1.42	-0.08
time (sec)	N/A	0.007	0.015	0.006	1.945	0.554	0.000	0.589	0.000

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	37	15	11	17	0	17	-1
normalized size	1	1.00	3.08	1.25	0.92	1.42	0.00	1.42	-0.08
time (sec)	N/A	0.008	0.017	0.007	1.930	0.648	0.000	0.616	0.000

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	12	17	13	13	10	13	10
normalized size	1	1.00	0.80	1.13	0.87	0.87	0.67	0.87	0.67
time (sec)	N/A	0.004	0.008	0.003	0.874	0.614	0.140	0.597	3.640

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	95	54	42	49	0	53	-1
normalized size	1	1.00	1.76	1.00	0.78	0.91	0.00	0.98	-0.02
time (sec)	N/A	0.040	0.078	0.008	1.907	0.613	0.000	0.505	0.000

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	342	0	0	0	45	27
normalized size	1	1.00	0.97	5.80	0.00	0.00	0.00	0.76	0.46
time (sec)	N/A	0.069	0.036	0.102	0.000	0.000	0.000	4.566	3.532

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	28	0	30	0	33	27
normalized size	1	1.00	0.66	0.47	0.00	0.51	0.00	0.56	0.46
time (sec)	N/A	0.052	0.020	0.008	0.000	0.773	0.000	0.395	3.544

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	51	38	0	40	0	51	-1
normalized size	1	1.00	0.54	0.40	0.00	0.43	0.00	0.54	-0.01
time (sec)	N/A	0.091	0.030	0.003	0.000	0.700	0.000	0.383	0.000

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	0	20	26	18	19
normalized size	1	1.00	1.00	1.11	0.00	1.11	1.44	1.00	1.06
time (sec)	N/A	0.049	0.010	0.007	0.000	0.787	0.497	0.480	3.499

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	84	81	0	62	0	60	-1
normalized size	1	1.00	0.79	0.76	0.00	0.58	0.00	0.56	-0.01
time (sec)	N/A	0.029	0.028	0.007	0.000	0.710	0.000	0.430	0.000

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	14	64	0	14	25
normalized size	1	1.00	1.00	0.96	0.56	2.56	0.00	0.56	1.00
time (sec)	N/A	0.016	0.019	0.005	0.897	0.734	0.000	0.424	3.508

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	35	49	33	35
normalized size	1	1.00	1.00	0.81	0.79	0.83	1.17	0.79	0.83
time (sec)	N/A	0.029	0.011	0.004	2.011	0.945	0.247	0.391	3.448

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	25	27	37	25	27
normalized size	1	1.00	1.00	0.81	0.78	0.84	1.16	0.78	0.84
time (sec)	N/A	0.023	0.010	0.004	2.130	0.724	0.230	0.363	0.035

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	49	53	0	95	0	45	-1
normalized size	1	1.00	0.64	0.70	0.00	1.25	0.00	0.59	-0.01
time (sec)	N/A	0.023	0.016	0.003	0.000	0.824	0.000	0.427	0.000

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	53	0	64	48	79	46
normalized size	1	1.00	1.00	1.15	0.00	1.39	1.04	1.72	1.00
time (sec)	N/A	0.086	0.034	0.004	0.000	0.691	2.984	0.432	0.036

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	23	18	0	14	0	13	-1
normalized size	1	1.00	1.15	0.90	0.00	0.70	0.00	0.65	-0.05
time (sec)	N/A	0.006	0.027	0.004	0.000	0.457	0.000	0.360	0.000

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	36	18	35	38	19	68	0	14
normalized size	1	1.03	0.51	1.00	1.09	0.54	1.94	0.00	0.40
time (sec)	N/A	0.011	0.007	0.007	2.153	0.511	1.137	0.000	3.591

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	111	0	13	16	100	0	13
normalized size	1	1.00	7.40	0.00	0.87	1.07	6.67	0.00	0.87
time (sec)	N/A	0.018	0.179	0.092	1.128	0.673	98.052	0.000	3.684

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	28	25	37	24	46	37	37
normalized size	1	1.00	0.53	0.47	0.70	0.45	0.87	0.70	0.70
time (sec)	N/A	0.021	0.026	0.004	0.578	0.506	11.728	0.337	0.043

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	27	25	25
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.87	0.81	0.81
time (sec)	N/A	0.015	0.012	0.015	0.654	0.594	0.240	0.321	0.069

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	9	9	8	9	9
normalized size	1	1.00	1.00	1.27	0.82	0.82	0.73	0.82	0.82
time (sec)	N/A	0.003	0.006	0.004	0.625	0.529	0.139	0.365	3.516

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	4	4	5	4	4
normalized size	1	1.00	1.00	0.83	0.67	0.67	0.83	0.67	0.67
time (sec)	N/A	0.002	0.003	0.004	1.388	0.686	0.211	0.332	0.143

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	17	16	16	0	25	16
normalized size	1	1.00	0.85	0.85	0.80	0.80	0.00	1.25	0.80
time (sec)	N/A	0.005	0.006	0.002	1.432	0.685	0.000	0.414	3.508

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	26	11	12	11	11
normalized size	1	1.00	1.00	0.71	1.53	0.65	0.71	0.65	0.65
time (sec)	N/A	0.003	0.002	0.002	0.730	0.568	0.138	0.349	0.024

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	11	11	15	11	12
normalized size	1	1.00	1.00	0.63	0.58	0.58	0.79	0.58	0.63
time (sec)	N/A	0.003	0.004	0.001	0.483	0.625	1.529	0.366	0.025

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	28	27	25	37	27	27
normalized size	1	1.00	1.00	0.68	0.66	0.61	0.90	0.66	0.66
time (sec)	N/A	0.010	0.008	0.004	1.625	0.672	3.363	0.380	0.027

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	88	64	63	65	39	64	73
normalized size	1	1.00	1.31	0.96	0.94	0.97	0.58	0.96	1.09
time (sec)	N/A	0.030	0.020	0.010	1.723	0.984	1.055	0.632	3.832

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
normalized size	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.001	0.001	0.884	0.667	0.058	0.331	0.002

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
normalized size	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.002	0.002	0.623	1.280	0.058	0.347	0.023

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	44	7	7	8	7	7
normalized size	1	1.00	1.00	4.00	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.002	0.001	0.013	0.607	0.762	4.355	0.354	0.026

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	95	49	0	192	0	68	-1
normalized size	1	1.00	1.56	0.80	0.00	3.15	0.00	1.11	-0.02
time (sec)	N/A	0.025	0.065	0.008	0.000	0.714	0.000	0.721	0.000

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	94	55	0	202	0	59	-1
normalized size	1	1.00	1.45	0.85	0.00	3.11	0.00	0.91	-0.02
time (sec)	N/A	0.025	0.077	0.013	0.000	0.598	0.000	0.740	0.000

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	21	21	26	22	9
normalized size	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.007	0.004	0.007	1.011	0.569	0.386	0.392	3.339

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	21	21	26	22	9
normalized size	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.011	0.003	0.006	1.006	0.581	0.535	0.303	0.027

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	82	77	100	202	80	233
normalized size	1	1.00	1.00	1.14	1.07	1.39	2.81	1.11	3.24
time (sec)	N/A	0.104	0.096	0.022	0.997	0.620	1.547	0.428	4.234

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	23	15	22	0	48	22
normalized size	1	1.00	0.62	0.62	0.41	0.59	0.00	1.30	0.59
time (sec)	N/A	0.027	0.009	0.003	0.451	0.580	0.000	0.383	3.513

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	37	13	9	17	0	17	-1
normalized size	1	1.00	3.08	1.08	0.75	1.42	0.00	1.42	-0.08
time (sec)	N/A	0.007	0.008	0.004	0.974	0.597	0.000	0.465	0.000

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	60	67	0	89	0	51	-1
normalized size	1	1.00	0.63	0.71	0.00	0.94	0.00	0.54	-0.01
time (sec)	N/A	0.054	0.037	0.007	0.000	2.324	0.000	0.386	0.000

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	24	23	21	184	23	16
normalized size	1	1.00	0.80	0.69	0.66	0.60	5.26	0.66	0.46
time (sec)	N/A	0.011	0.011	0.002	0.434	0.594	1.166	0.324	3.506

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	28	27	22	265	27	24
normalized size	1	1.00	0.81	0.76	0.73	0.59	7.16	0.73	0.65
time (sec)	N/A	0.013	0.011	0.005	0.432	0.781	1.155	0.341	3.578

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	48	0	33	32	29	39
normalized size	1	1.00	0.90	1.66	0.00	1.14	1.10	1.00	1.34
time (sec)	N/A	0.026	0.028	0.008	0.000	0.645	1.795	0.408	3.897

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	26	48	0	36	87	32	40
normalized size	1	1.00	1.04	1.92	0.00	1.44	3.48	1.28	1.60
time (sec)	N/A	0.025	0.031	0.007	0.000	0.666	3.773	0.402	3.648

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	0	15	56	15	22
normalized size	1	1.00	1.00	0.76	0.00	0.71	2.67	0.71	1.05
time (sec)	N/A	0.023	0.025	0.003	0.000	0.618	0.370	0.341	0.041

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	175	0	97	0	105	127
normalized size	1	1.00	0.83	2.69	0.00	1.49	0.00	1.62	1.95
time (sec)	N/A	0.054	0.054	0.019	0.000	0.672	0.000	0.572	3.499

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	0	41	0	63	64
normalized size	1	1.00	1.00	0.90	0.00	1.32	0.00	2.03	2.06
time (sec)	N/A	0.042	0.036	0.010	0.000	0.755	0.000	0.464	0.188

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	58	54	0	54	0	50	-1
normalized size	1	1.00	0.71	0.66	0.00	0.66	0.00	0.61	-0.01
time (sec)	N/A	0.044	0.067	0.004	0.000	2.095	0.000	0.442	0.000

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	56	55	57	155	55	95
normalized size	1	1.00	1.00	0.76	0.74	0.77	2.09	0.74	1.28
time (sec)	N/A	0.111	0.038	0.011	0.970	0.936	3.777	0.464	3.412

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	81	80	80	221	80	130
normalized size	1	1.00	1.07	0.70	0.70	0.70	1.92	0.70	1.13
time (sec)	N/A	0.150	0.087	0.011	0.960	0.948	5.432	0.431	0.087

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	51	0	20	0	4	4
normalized size	1	1.00	1.00	12.75	0.00	5.00	0.00	1.00	1.00
time (sec)	N/A	0.043	0.063	0.016	0.000	0.677	0.000	0.333	0.032

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	41	50	40	0	31	18
normalized size	1	1.00	1.00	1.86	2.27	1.82	0.00	1.41	0.82
time (sec)	N/A	0.010	0.010	0.005	0.439	0.894	0.000	0.390	3.387

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	40	37	26	0	28	20
normalized size	1	1.00	1.00	1.67	1.54	1.08	0.00	1.17	0.83
time (sec)	N/A	0.009	0.008	0.006	0.972	0.958	0.000	0.396	0.037

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	28	38	45	51	40	0	35	24
normalized size	1	1.17	1.58	1.88	2.12	1.67	0.00	1.46	1.00
time (sec)	N/A	0.011	0.016	0.007	0.433	0.671	0.000	0.380	0.030

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	60	38	38	32	38	20
normalized size	1	1.00	1.00	2.50	1.58	1.58	1.33	1.58	0.83
time (sec)	N/A	0.017	0.007	0.008	0.435	0.425	3.228	0.380	0.037

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	42	43	51	42	0	35	35
normalized size	1	1.00	1.91	1.95	2.32	1.91	0.00	1.59	1.59
time (sec)	N/A	0.005	0.020	0.008	0.441	0.434	0.000	0.386	0.040

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	43	44	35	25	0	35	23
normalized size	1	1.00	1.48	1.52	1.21	0.86	0.00	1.21	0.79
time (sec)	N/A	0.012	0.013	0.007	0.974	0.442	0.000	0.459	3.367

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	36	35	0	25	26
normalized size	1	1.00	0.97	0.85	1.09	1.06	0.00	0.76	0.79
time (sec)	N/A	0.012	0.051	0.008	1.017	0.439	0.000	0.386	3.468

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	12	7	6	16	0	6	6
normalized size	1	1.00	1.50	0.88	0.75	2.00	0.00	0.75	0.75
time (sec)	N/A	0.005	0.008	0.006	0.963	0.447	0.000	0.388	0.014

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	15	11	18	0	16	13
normalized size	1	1.00	0.85	1.15	0.85	1.38	0.00	1.23	1.00
time (sec)	N/A	0.013	0.004	0.005	0.426	0.430	0.000	0.328	3.518

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	41	0	33	0	73	17
normalized size	1	1.00	1.00	1.86	0.00	1.50	0.00	3.32	0.77
time (sec)	N/A	0.032	0.028	0.019	0.000	0.445	0.000	0.534	3.583

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	39	32	11	11	15	13	45
normalized size	1	1.00	1.62	1.33	0.46	0.46	0.62	0.54	1.88
time (sec)	N/A	0.027	0.024	0.027	0.976	0.479	0.579	0.361	0.099

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	27	62	0	35	28
normalized size	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00
time (sec)	N/A	0.010	0.022	0.004	0.439	0.460	0.000	0.466	3.472

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	27	62	0	35	28
normalized size	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00
time (sec)	N/A	0.011	0.004	0.007	0.434	0.453	0.000	0.415	3.529

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	27	62	0	35	28
normalized size	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00
time (sec)	N/A	0.012	0.004	0.007	0.446	0.474	0.000	0.422	3.593

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	27	62	0	35	28
normalized size	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00
time (sec)	N/A	0.013	0.004	0.006	0.455	0.460	0.000	0.497	3.529

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	27	62	0	35	28
normalized size	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00
time (sec)	N/A	0.012	0.004	0.006	0.440	0.468	0.000	0.560	3.524

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	57	29	27	62	0	35	28
normalized size	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00
time (sec)	N/A	0.013	0.004	0.007	0.436	0.481	0.000	0.474	3.515

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	58	37	36	87	0	50	33
normalized size	1	1.00	1.45	0.92	0.90	2.18	0.00	1.25	0.82
time (sec)	N/A	0.017	0.017	0.006	0.443	0.471	0.000	0.496	3.903

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	58	37	36	87	0	50	33
normalized size	1	1.00	1.45	0.92	0.90	2.18	0.00	1.25	0.82
time (sec)	N/A	0.017	0.004	0.009	0.445	0.463	0.000	0.570	3.557

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	58	37	36	87	0	50	33
normalized size	1	1.00	1.45	0.92	0.90	2.18	0.00	1.25	0.82
time (sec)	N/A	0.017	0.004	0.006	0.456	0.442	0.000	0.491	3.475

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	58	37	36	87	0	50	33
normalized size	1	1.00	1.45	0.92	0.90	2.18	0.00	1.25	0.82
time (sec)	N/A	0.019	0.004	0.006	0.449	0.449	0.000	0.515	3.654

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	A	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	68	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	1.08	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.027	0.024	0.000	1.028	0.000	0.000	0.000

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F(-1)	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	180	0	0	0	0	0	-1
normalized size	1	0.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.139	0.525	0.089	0.000	0.000	0.000	0.000	0.000

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	F	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	319	34	438	0	383	0	218	649
normalized size	1	4.09	0.44	5.62	0.00	4.91	0.00	2.79	8.32
time (sec)	N/A	0.568	0.427	0.199	0.000	1.434	0.000	0.862	4.380

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	87	753	0	770	0	444	-1
normalized size	1	1.00	0.69	5.98	0.00	6.11	0.00	3.52	-0.01
time (sec)	N/A	0.157	0.036	0.106	0.000	0.843	0.000	0.720	0.000

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	0	0	62	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	2.82	0.00	0.00	-0.05
time (sec)	N/A	0.062	1.058	0.084	0.000	1.794	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	0	0	0	0	0	-1
normalized size	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.135	0.489	0.090	0.000	0.000	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	47	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.142	0.564	0.083	0.000	0.000	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	330	1528	0	0	0	0	-1
normalized size	1	1.00	1.79	8.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.583	0.425	0.000	1.287	0.000	0.000	0.000

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	90	1036	0	0	0	0	-1
normalized size	1	1.00	0.69	7.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.059	0.065	0.000	1.017	0.000	0.000	0.000

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	419	514	0	305	0	0	-1
normalized size	1	1.00	7.76	9.52	0.00	5.65	0.00	0.00	-0.02
time (sec)	N/A	0.253	1.964	0.093	0.000	23.149	0.000	0.000	0.000

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	416	517	0	304	0	0	-1
normalized size	1	1.00	7.85	9.75	0.00	5.74	0.00	0.00	-0.02
time (sec)	N/A	0.260	1.450	0.076	0.000	19.134	0.000	0.000	0.000

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	160	69	0	154	0	164	-1
normalized size	1	1.00	1.90	0.82	0.00	1.83	0.00	1.95	-0.01
time (sec)	N/A	0.122	0.332	0.032	0.000	0.686	0.000	0.591	0.000

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	23	7	22	36	15	13
normalized size	1	1.00	0.85	1.15	0.35	1.10	1.80	0.75	0.65
time (sec)	N/A	0.004	0.005	0.004	1.084	0.657	0.442	0.438	3.413

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.012	0.030	0.000	0.000	0.000	0.000	0.000

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	13884	242984	0	324	0	0	-1
normalized size	1	1.00	157.77	2761.18	0.00	3.68	0.00	0.00	-0.01
time (sec)	N/A	0.249	6.518	0.165	0.000	5.823	0.000	0.000	0.000

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	15147	269221	0	331	0	0	-1
normalized size	1	1.00	172.12	3059.33	0.00	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.329	6.546	0.164	0.000	6.032	0.000	0.000	0.000

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	148	0	0	161	0	0	-1
normalized size	1	1.00	3.22	0.00	0.00	3.50	0.00	0.00	-0.02
time (sec)	N/A	0.622	1.118	0.089	0.000	16.050	0.000	0.000	0.000

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	161	0	0	161	0	0	-1
normalized size	1	1.00	3.50	0.00	0.00	3.50	0.00	0.00	-0.02
time (sec)	N/A	0.624	1.186	0.089	0.000	12.882	0.000	0.000	0.000

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	148	0	0	161	0	0	-1
normalized size	1	1.00	3.22	0.00	0.00	3.50	0.00	0.00	-0.02
time (sec)	N/A	1.171	0.139	0.068	0.000	14.396	0.000	0.000	0.000

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	161	0	0	161	0	0	-1
normalized size	1	1.00	3.50	0.00	0.00	3.50	0.00	0.00	-0.02
time (sec)	N/A	1.168	0.188	0.068	0.000	12.962	0.000	0.000	0.000

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	75	147	94	96	0	58	132
normalized size	1	1.00	3.95	7.74	4.95	5.05	0.00	3.05	6.95
time (sec)	N/A	0.557	1.376	0.056	0.654	0.968	0.000	0.622	6.098

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	123	120	2515	0	458	0	0	-1
normalized size	1	1.37	1.33	27.94	0.00	5.09	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.159	13.667	0.000	10.815	0.000	0.000	0.000

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	C	F	B	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	0	1026	0	289	0	0	-1
normalized size	1	0.00	0.00	9.96	0.00	2.81	0.00	0.00	-0.01
time (sec)	N/A	0.534	0.186	8.030	0.000	8.809	0.000	0.000	0.000

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	47	46	88	0	51	0	0	-1
normalized size	1	0.96	0.94	1.80	0.00	1.04	0.00	0.00	-0.02
time (sec)	N/A	0.120	0.021	0.026	0.000	0.658	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	383	555	0	472	0	0	-1
normalized size	1	1.00	4.79	6.94	0.00	5.90	0.00	0.00	-0.01
time (sec)	N/A	0.449	0.914	0.086	0.000	67.599	0.000	0.000	0.000

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
normalized size	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.001	0.000	0.001	0.460	1.538	0.058	0.352	0.005

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	1932	234	0	73	0	193	549
normalized size	1	1.00	15.84	1.92	0.00	0.60	0.00	1.58	4.50
time (sec)	N/A	0.154	4.388	0.042	0.000	0.654	0.000	0.417	3.925

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	149	1910	234	0	73	0	193	549
normalized size	1	1.22	15.66	1.92	0.00	0.60	0.00	1.58	4.50
time (sec)	N/A	0.388	1.371	0.064	0.000	0.613	0.000	0.583	3.895

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	A	A	A	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	31	32	16	31	73	81	31
normalized size	1	0.00	0.91	0.94	0.47	0.91	2.15	2.38	0.91
time (sec)	N/A	0.065	0.040	0.003	1.086	0.618	72.393	0.457	3.452

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	1671	1597	0	164	0	0	-1
normalized size	1	1.00	9.44	9.02	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.089	6.130	0.743	0.000	0.929	0.000	0.000	0.000

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	243	2787	2992	0	97	0	0	-1
normalized size	1	2.43	27.87	29.92	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.139	6.102	1.123	0.000	0.762	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [23] had the largest ratio of [.7895]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	19	0.210
2	A	4	4	1.00	23	0.174
3	A	4	4	1.00	21	0.190
4	A	4	4	1.00	21	0.190
5	A	4	4	1.00	33	0.121
6	A	4	4	1.00	35	0.114
7	A	4	4	1.00	36	0.111
8	A	4	4	1.00	36	0.111
9	A	4	4	1.00	24	0.167
10	A	4	4	1.00	20	0.200
11	A	4	4	1.00	24	0.167
12	A	4	4	1.00	22	0.182
13	A	4	4	1.00	22	0.182
14	A	8	8	1.01	15	0.533
15	A	8	8	1.01	17	0.471
16	A	8	8	1.01	15	0.533
17	A	8	8	1.01	17	0.471
18	A	1	1	1.00	25	0.040
19	A	3	3	1.00	25	0.120
20	F	0	0	N/A	0	N/A
21	F	0	0	N/A	0	N/A
22	F	0	0	N/A	0	N/A
23	A	23	15	1.29	19	0.790
24	A	15	14	1.23	19	0.737
25	A	14	13	1.28	19	0.684
26	A	12	11	1.34	17	0.647
27	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	F	0	0	N/A	0	N/A
29	A	10	6	1.00	19	0.316
30	A	8	6	1.00	19	0.316
31	A	7	6	1.00	19	0.316
32	A	5	4	1.00	17	0.235
33	F	0	0	N/A	0	N/A
34	F	0	0	N/A	0	N/A
35	F	0	0	N/A	0	N/A
36	A	22	14	1.36	19	0.737
37	A	14	13	1.28	19	0.684
38	A	13	12	1.38	19	0.632
39	A	11	10	1.42	17	0.588
40	F	0	0	N/A	0	N/A
41	F	0	0	N/A	0	N/A
42	F	0	0	N/A	0	N/A
43	A	2	2	1.00	28	0.071
44	A	2	2	1.00	32	0.062
45	A	2	2	1.00	30	0.067
46	A	2	2	1.00	30	0.067
47	A	2	2	1.00	53	0.038
48	A	2	2	1.00	55	0.036
49	A	2	2	1.00	56	0.036
50	A	2	2	1.00	56	0.036
51	A	2	2	1.00	30	0.067
52	A	4	4	1.00	24	0.167
53	A	4	4	1.00	28	0.143
54	A	4	4	1.00	26	0.154
55	A	4	4	1.00	26	0.154
56	A	4	4	1.00	24	0.167
57	A	4	4	1.00	28	0.143
58	A	4	4	1.00	26	0.154
59	A	4	4	1.00	26	0.154
60	A	4	4	1.00	38	0.105
61	A	4	4	1.00	40	0.100
62	A	4	4	1.00	41	0.098
63	A	4	4	1.00	41	0.098

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	4	4	1.00	29	0.138
65	A	4	4	1.00	20	0.200
66	A	4	4	1.00	24	0.167
67	A	4	4	1.00	22	0.182
68	A	4	4	1.00	22	0.182
69	A	4	4	1.00	34	0.118
70	A	4	4	1.00	36	0.111
71	A	4	4	1.00	37	0.108
72	A	4	4	1.00	37	0.108
73	A	4	4	1.00	25	0.160
74	A	2	2	1.00	20	0.100
75	A	2	2	1.00	22	0.091
76	A	2	2	1.00	20	0.100
77	A	2	2	1.00	22	0.091
78	A	2	2	1.00	43	0.047
79	A	2	2	1.00	44	0.045
80	A	2	2	1.00	45	0.044
81	A	2	2	1.00	46	0.043
82	A	2	2	1.00	30	0.067
83	A	4	4	1.00	22	0.182
84	A	4	4	1.00	22	0.182
85	A	4	4	1.00	20	0.200
86	A	4	4	1.00	24	0.167
87	A	4	4	1.00	35	0.114
88	A	4	4	1.00	35	0.114
89	A	4	4	1.00	36	0.111
90	A	4	4	1.00	38	0.105
91	A	4	4	1.00	29	0.138
92	A	4	4	1.00	18	0.222
93	A	4	4	1.00	18	0.222
94	A	4	4	1.00	16	0.250
95	A	4	4	1.00	20	0.200
96	A	4	4	1.00	31	0.129
97	A	4	4	1.00	31	0.129
98	A	4	4	1.00	32	0.125
99	A	4	4	1.00	34	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	4	4	1.00	25	0.160
101	A	2	2	1.00	30	0.067
102	A	2	2	1.00	36	0.056
103	A	2	2	1.00	34	0.059
104	A	2	2	1.00	32	0.062
105	A	2	2	1.00	58	0.034
106	A	2	2	1.00	61	0.033
107	A	2	2	1.00	62	0.032
108	A	2	2	1.00	61	0.033
109	A	2	2	1.00	52	0.038
110	A	2	2	1.00	55	0.036
111	A	2	2	1.00	56	0.036
112	A	2	2	1.00	55	0.036
113	A	2	2	1.00	30	0.067
114	A	2	2	1.00	36	0.056
115	A	2	2	1.00	34	0.059
116	A	2	2	1.00	32	0.062
117	A	2	2	1.00	58	0.034
118	A	2	2	1.00	61	0.033
119	A	2	2	1.00	62	0.032
120	A	2	2	1.00	61	0.033
121	A	2	2	1.00	52	0.038
122	A	2	2	1.00	55	0.036
123	A	2	2	1.00	56	0.036
124	A	2	2	1.00	55	0.036
125	A	4	4	1.00	23	0.174
126	A	4	4	1.00	25	0.160
127	A	4	4	1.00	25	0.160
128	A	4	4	1.00	29	0.138
129	A	4	4	1.00	27	0.148
130	A	4	4	1.00	27	0.148
131	A	4	4	1.00	42	0.095
132	A	4	4	1.00	44	0.091
133	A	4	4	1.00	45	0.089
134	A	4	4	1.00	45	0.089
135	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	4	4	1.00	25	0.160
137	A	4	4	1.00	23	0.174
138	A	4	4	1.00	23	0.174
139	A	4	4	1.00	23	0.174
140	A	4	4	1.00	38	0.105
141	A	4	4	1.00	40	0.100
142	A	4	4	1.00	41	0.098
143	A	4	4	1.00	41	0.098
144	A	6	6	1.01	25	0.240
145	A	6	6	1.01	29	0.207
146	A	6	6	1.01	27	0.222
147	A	6	6	1.01	27	0.222
148	A	6	6	1.01	27	0.222
149	A	6	6	1.01	31	0.194
150	A	6	6	1.01	29	0.207
151	A	6	6	1.01	29	0.207
152	A	5	5	1.00	21	0.238
153	A	5	5	1.00	25	0.200
154	A	5	5	1.00	23	0.217
155	A	5	5	1.00	23	0.217
156	A	5	5	1.00	23	0.217
157	A	5	5	1.00	27	0.185
158	A	5	5	1.00	25	0.200
159	A	5	5	1.00	25	0.200
160	A	8	8	1.01	16	0.500
161	A	8	8	1.01	18	0.444
162	A	8	8	1.01	16	0.500
163	A	8	8	1.01	18	0.444
164	A	8	8	1.00	22	0.364
165	A	8	8	1.00	24	0.333
166	A	8	8	1.00	22	0.364
167	A	8	8	1.00	24	0.333
168	A	6	6	1.00	18	0.333
169	A	6	6	1.00	20	0.300
170	A	6	6	1.00	18	0.333
171	A	6	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	1	1	1.00	31	0.032
173	A	3	3	1.00	30	0.100
174	A	2	1	1.00	18	0.056
175	A	2	1	1.00	16	0.062
176	A	2	1	1.00	15	0.067
177	A	3	2	1.00	18	0.111
178	A	2	1	1.00	20	0.050
179	A	2	1	1.00	18	0.056
180	A	2	1	1.00	17	0.059
181	A	3	2	1.00	20	0.100
182	A	2	1	1.00	20	0.050
183	A	2	1	1.00	18	0.056
184	A	2	1	1.00	17	0.059
185	A	3	2	1.00	20	0.100
186	A	7	2	1.00	20	0.100
187	A	7	2	1.00	20	0.100
188	A	7	2	1.00	20	0.100
189	A	5	2	1.00	20	0.100
190	A	5	2	1.00	18	0.111
191	A	5	2	1.00	17	0.118
192	A	8	3	1.00	20	0.150
193	A	8	3	1.00	20	0.150
194	A	5	2	1.00	22	0.091
195	A	8	3	1.00	20	0.150
196	A	13	12	1.00	19	0.632
197	F	0	0	N/A	0	N/A
198	A	2	2	1.00	27	0.074
199	A	2	2	1.00	29	0.069
200	A	2	2	1.00	27	0.074
201	A	2	2	1.00	29	0.069
202	A	2	2	1.00	31	0.065
203	A	2	2	1.00	35	0.057
204	A	2	2	1.00	33	0.061
205	A	2	2	1.00	33	0.061
206	A	11	10	1.00	19	0.526
207	A	10	9	1.00	19	0.474

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	8	6	1.00	17	0.353
209	A	2	2	1.00	11	0.182
210	A	15	13	1.00	19	0.684
211	A	32	15	1.00	19	0.790
212	A	9	8	1.00	19	0.421
213	A	8	7	1.00	19	0.368
214	A	6	5	1.00	17	0.294
215	A	1	1	1.00	11	0.091
216	A	7	7	1.00	19	0.368
217	A	11	11	1.00	19	0.579
218	A	12	12	1.00	19	0.632
219	A	4	4	1.00	19	0.210
220	A	4	4	1.00	19	0.210
221	A	3	3	1.00	17	0.176
222	A	2	2	1.00	11	0.182
223	A	14	13	1.00	19	0.684
224	A	10	3	1.00	20	0.150
225	A	10	3	1.00	22	0.136
226	A	16	8	1.00	24	0.333
227	A	4	3	1.27	19	0.158
228	A	4	3	0.94	19	0.158
229	A	4	3	1.31	19	0.158
230	A	4	4	1.18	19	0.210
231	A	4	3	1.31	19	0.158
232	A	3	3	1.00	17	0.176
233	A	4	3	1.30	15	0.200
234	A	5	4	1.32	19	0.210
235	A	4	3	1.33	19	0.158
236	A	5	4	1.31	19	0.210
237	A	9	5	1.00	19	0.263
238	A	3	3	1.00	17	0.176
239	A	8	4	1.00	15	0.267
240	A	9	5	1.01	19	0.263
241	A	8	5	1.00	19	0.263
242	A	9	6	1.01	19	0.316
243	A	4	4	0.97	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
244	A	3	3	1.00	17	0.176
245	A	2	2	1.00	15	0.133
246	A	5	5	1.03	19	0.263
247	A	3	3	1.00	19	0.158
248	A	6	5	1.08	19	0.263
249	A	4	3	1.10	21	0.143
250	A	4	3	1.11	21	0.143
251	A	4	3	1.12	21	0.143
252	A	3	3	1.00	19	0.158
253	A	7	7	1.21	21	0.333
254	A	7	7	1.14	21	0.333
255	A	6	6	1.26	21	0.286
256	A	5	5	1.23	17	0.294
257	A	5	5	1.23	21	0.238
258	A	6	6	1.25	21	0.286
259	A	4	4	1.00	15	0.267
260	A	6	6	1.00	17	0.353
261	A	3	3	1.00	15	0.200
262	A	3	3	1.00	17	0.176
263	A	5	5	1.00	26	0.192
264	A	4	4	1.00	26	0.154
265	A	3	3	1.00	24	0.125
266	A	4	3	1.00	26	0.115
267	A	3	3	1.00	26	0.115
268	A	4	4	1.00	26	0.154
269	A	5	5	1.00	26	0.192
270	A	7	7	1.00	26	0.269
271	A	6	6	1.00	26	0.231
272	A	5	5	1.00	22	0.227
273	A	7	7	1.00	26	0.269
274	A	7	7	1.00	26	0.269
275	A	8	7	1.00	26	0.269
276	A	6	6	1.00	26	0.231
277	A	5	5	1.00	26	0.192
278	A	4	4	1.00	24	0.167
279	A	5	4	1.00	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	4	4	1.00	26	0.154
281	A	5	5	1.00	26	0.192
282	A	6	6	1.00	26	0.231
283	A	8	8	1.00	26	0.308
284	A	7	7	1.00	26	0.269
285	A	6	6	1.00	22	0.273
286	A	7	7	1.00	26	0.269
287	A	8	7	1.00	26	0.269
288	A	9	7	1.00	26	0.269
289	A	3	3	1.00	21	0.143
290	A	3	3	1.00	23	0.130
291	A	3	3	1.00	23	0.130
292	A	5	5	1.00	23	0.217
293	A	4	4	1.00	25	0.160
294	A	4	4	1.00	23	0.174
295	A	4	4	1.00	28	0.143
296	A	5	5	1.00	26	0.192
297	A	4	4	1.00	26	0.154
298	A	3	3	1.00	24	0.125
299	A	4	3	1.00	26	0.115
300	A	3	3	1.00	26	0.115
301	A	4	4	1.00	26	0.154
302	A	7	7	1.00	26	0.269
303	A	6	6	1.00	26	0.231
304	A	5	5	1.00	22	0.227
305	A	7	7	1.00	26	0.269
306	A	7	7	0.99	26	0.269
307	A	6	5	0.98	26	0.192
308	A	5	4	1.00	26	0.154
309	A	4	4	1.00	24	0.167
310	A	5	4	1.00	26	0.154
311	A	4	4	1.00	26	0.154
312	A	5	4	1.00	26	0.154
313	A	8	8	1.00	26	0.308
314	A	7	7	1.00	26	0.269
315	A	6	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
316	A	7	7	1.00	26	0.269
317	A	8	7	1.00	26	0.269
318	A	9	9	1.20	21	0.429
319	A	8	8	1.28	21	0.381
320	A	5	5	1.00	19	0.263
321	A	9	9	1.92	21	0.429
322	A	6	6	1.35	21	0.286
323	A	7	7	1.25	21	0.333
324	A	9	8	1.02	21	0.381
325	A	8	8	1.30	21	0.381
326	A	7	7	1.31	21	0.333
327	A	6	6	1.31	17	0.353
328	A	8	8	1.33	21	0.381
329	A	8	8	1.30	21	0.381
330	A	9	8	1.28	21	0.381
331	A	10	9	1.25	21	0.429
332	A	9	8	1.29	21	0.381
333	A	6	6	1.00	19	0.316
334	A	10	10	1.63	21	0.476
335	A	7	6	1.23	21	0.286
336	A	8	7	1.27	21	0.333
337	A	10	9	0.98	21	0.429
338	A	9	9	1.30	21	0.429
339	A	8	8	1.30	21	0.381
340	A	7	7	1.34	17	0.412
341	A	8	8	1.35	21	0.381
342	A	9	8	1.34	21	0.381
343	A	10	8	1.31	21	0.381
344	A	9	9	1.19	21	0.429
345	A	8	8	1.28	21	0.381
346	A	5	5	1.00	19	0.263
347	A	9	9	1.92	21	0.429
348	A	6	6	1.37	21	0.286
349	A	7	7	1.23	21	0.333
350	A	8	8	1.12	21	0.381
351	A	7	7	1.12	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
352	A	6	6	1.12	17	0.353
353	A	8	8	1.13	21	0.381
354	A	8	8	1.12	21	0.381
355	A	10	9	1.04	21	0.429
356	A	9	8	1.29	21	0.381
357	A	6	5	1.04	19	0.263
358	A	10	10	1.60	21	0.476
359	A	7	6	1.19	21	0.286
360	A	8	7	1.16	21	0.333
361	A	9	9	1.16	21	0.429
362	A	8	8	1.16	21	0.381
363	A	7	7	1.15	17	0.412
364	A	8	8	1.16	21	0.381
365	A	9	8	1.16	21	0.381
366	A	6	5	1.00	19	0.263
367	A	5	5	1.00	19	0.263
368	A	4	4	1.00	19	0.210
369	A	2	2	1.00	19	0.105
370	A	3	3	1.00	19	0.158
371	A	4	4	1.00	22	0.182
372	A	5	5	1.00	19	0.263
373	A	6	5	1.00	22	0.227
374	A	8	5	1.00	33	0.152
375	A	9	6	1.00	30	0.200
376	A	6	6	1.00	19	0.316
377	A	3	3	1.00	19	0.158
378	A	4	4	1.00	19	0.210
379	A	2	2	1.00	19	0.105
380	A	4	4	1.00	17	0.235
381	A	3	3	1.00	19	0.158
382	A	4	4	1.00	19	0.210
383	A	6	6	1.00	19	0.316
384	A	2	2	1.00	19	0.105
385	A	2	2	1.00	19	0.105
386	A	5	5	1.00	19	0.263
387	A	4	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
388	A	3	3	1.00	17	0.176
389	A	3	3	1.00	19	0.158
390	A	4	4	1.00	19	0.210
391	A	6	6	1.00	19	0.316
392	A	5	5	1.00	19	0.263
393	A	2	2	1.00	21	0.095
394	A	2	2	1.00	19	0.105
395	A	2	2	1.00	23	0.087
396	C	5	3	2.35	54	0.056
397	A	2	2	1.00	26	0.077
398	A	2	2	1.00	7	0.286
399	A	3	2	1.00	15	0.133
400	A	4	2	1.00	22	0.091
401	A	5	2	1.00	25	0.080
402	A	5	2	1.00	23	0.087
403	A	2	1	1.00	21	0.048
404	A	7	4	1.00	25	0.160
405	A	7	4	1.00	25	0.160
406	A	9	7	1.00	25	0.280
407	A	8	6	1.00	23	0.261
408	A	7	5	1.81	21	0.238
409	A	9	8	1.00	25	0.320
410	A	9	8	1.00	25	0.320
411	A	10	2	1.00	25	0.080
412	A	10	2	1.00	23	0.087
413	B	6	2	2.36	21	0.095
414	A	8	4	1.00	25	0.160
415	A	14	5	1.38	25	0.200
416	A	3	3	1.00	15	0.200
417	A	3	3	1.00	15	0.200
418	A	2	1	1.00	17	0.059
419	A	5	3	1.00	23	0.130
420	A	5	4	1.00	23	0.174
421	A	3	2	1.00	21	0.095
422	A	4	3	1.00	19	0.158
423	A	6	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
424	A	4	3	1.00	23	0.130
425	A	6	5	1.00	23	0.217
426	A	5	2	1.00	25	0.080
427	A	5	2	1.00	25	0.080
428	A	3	2	1.00	23	0.087
429	A	8	4	1.00	21	0.190
430	A	7	4	1.00	25	0.160
431	A	9	5	1.00	25	0.200
432	A	8	6	1.00	25	0.240
433	A	7	5	1.00	25	0.200
434	A	9	8	1.00	23	0.348
435	A	9	8	1.00	21	0.381
436	A	6	4	1.00	25	0.160
437	A	7	5	1.00	25	0.200
438	A	10	2	1.00	25	0.080
439	A	6	2	1.00	25	0.080
440	A	8	4	1.00	25	0.160
441	A	14	5	1.42	23	0.217
442	A	16	5	1.68	21	0.238
443	A	4	3	1.00	27	0.111
444	A	6	4	1.00	42	0.095
445	A	6	5	1.00	42	0.119
446	A	4	3	1.00	40	0.075
447	A	5	4	1.00	39	0.103
448	A	7	6	1.00	42	0.143
449	A	5	4	1.00	42	0.095
450	A	7	6	1.00	42	0.143
451	A	15	7	1.00	39	0.180
452	A	9	4	1.00	35	0.114
453	A	4	3	1.00	25	0.120
454	A	3	2	1.00	25	0.080
455	A	3	2	1.00	25	0.080
456	A	4	3	1.00	23	0.130
457	A	3	2	1.00	25	0.080
458	A	3	2	1.00	25	0.080
459	A	3	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
460	A	6	5	1.00	27	0.185
461	A	6	5	1.00	27	0.185
462	A	6	5	1.00	27	0.185
463	A	5	5	1.00	27	0.185
464	A	5	5	1.00	27	0.185
465	A	6	5	1.00	27	0.185
466	A	3	2	1.00	17	0.118
467	A	3	2	1.00	26	0.077
468	A	1	1	1.00	17	0.059
469	A	1	1	1.00	15	0.067
470	A	1	1	1.00	15	0.067
471	A	1	1	1.00	25	0.040
472	A	4	3	1.00	28	0.107
473	A	3	2	1.00	28	0.071
474	A	3	2	1.00	28	0.071
475	A	4	3	1.00	26	0.115
476	A	3	2	1.00	28	0.071
477	A	3	2	1.00	28	0.071
478	A	3	2	1.00	28	0.071
479	A	6	5	1.00	30	0.167
480	A	6	5	1.00	30	0.167
481	A	6	5	1.00	30	0.167
482	A	5	5	1.00	30	0.167
483	A	5	5	1.00	30	0.167
484	A	6	5	1.00	30	0.167
485	A	3	2	1.00	21	0.095
486	A	3	2	1.00	19	0.105
487	A	3	2	1.00	13	0.154
488	A	2	2	1.00	21	0.095
489	A	2	2	1.00	21	0.095
490	A	3	2	1.00	23	0.087
491	A	3	2	1.00	21	0.095
492	A	3	2	1.00	15	0.133
493	A	2	2	1.00	23	0.087
494	A	2	2	1.00	23	0.087
495	A	3	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
496	A	3	2	1.00	23	0.087
497	A	3	2	1.00	23	0.087
498	A	2	2	1.00	23	0.087
499	A	2	2	1.00	23	0.087
500	A	2	2	1.00	23	0.087
501	A	3	2	1.00	25	0.080
502	A	3	2	1.00	25	0.080
503	A	3	2	1.00	25	0.080
504	A	2	2	1.00	25	0.080
505	A	2	2	1.00	25	0.080
506	A	2	2	1.00	25	0.080
507	A	4	3	1.00	56	0.054
508	A	4	3	1.00	54	0.056
509	A	4	3	1.00	33	0.091
510	A	2	2	1.00	56	0.036
511	A	3	3	1.00	56	0.054
512	A	5	4	1.00	33	0.121
513	A	3	3	1.00	56	0.054
514	A	4	3	1.00	58	0.052
515	A	4	3	1.00	58	0.052
516	A	3	3	1.00	58	0.052
517	A	3	3	1.00	58	0.052
518	A	4	4	1.00	58	0.069
519	A	5	4	1.00	62	0.065
520	A	4	4	1.00	62	0.065
521	A	4	4	1.00	62	0.065
522	A	5	5	1.00	60	0.083
523	A	7	6	1.00	30	0.200
524	A	4	3	1.00	37	0.081
525	A	4	3	1.00	37	0.081
526	A	2	2	1.00	37	0.054
527	A	2	2	1.00	37	0.054
528	A	2	2	1.00	42	0.048
529	A	2	2	1.00	42	0.048
530	A	4	4	1.00	30	0.133
531	A	4	4	1.00	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
532	A	2	2	1.00	42	0.048
533	A	2	2	1.00	42	0.048
534	A	2	2	1.45	51	0.039
535	A	2	2	1.45	51	0.039
536	A	2	2	1.00	40	0.050
537	A	2	2	1.00	40	0.050
538	A	4	4	1.00	32	0.125
539	A	4	4	1.00	32	0.125
540	A	2	2	1.00	51	0.039
541	A	2	2	1.00	51	0.039
542	A	2	2	1.00	56	0.036
543	A	2	2	1.00	56	0.036
544	A	4	2	1.00	29	0.069
545	A	4	2	1.00	29	0.069
546	A	3	2	1.00	27	0.074
547	A	7	6	1.00	29	0.207
548	A	8	6	1.00	29	0.207
549	A	8	7	1.00	29	0.241
550	A	4	3	1.00	25	0.120
551	A	7	6	1.00	29	0.207
552	A	4	2	1.00	29	0.069
553	A	4	2	1.00	29	0.069
554	A	3	2	1.00	29	0.069
555	A	7	6	1.00	29	0.207
556	A	8	6	1.00	29	0.207
557	A	10	10	1.00	29	0.345
558	A	9	9	1.00	27	0.333
559	A	9	9	1.00	25	0.360
560	A	10	10	1.00	29	0.345
561	A	10	10	1.00	29	0.345
562	A	4	4	1.00	25	0.160
563	A	4	4	1.00	29	0.138
564	A	3	2	1.00	31	0.065
565	A	3	3	1.00	15	0.200
566	A	5	5	1.00	15	0.333
567	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
568	A	4	3	1.00	13	0.231
569	A	4	3	1.00	13	0.231
570	A	4	3	1.00	15	0.200
571	A	9	9	1.00	13	0.692
572	A	4	3	1.00	13	0.231
573	A	4	3	1.00	13	0.231
574	A	9	9	1.00	15	0.600
575	A	3	3	1.00	13	0.231
576	A	4	3	1.00	13	0.231
577	A	13	10	1.00	15	0.667
578	A	10	7	1.00	19	0.368
579	A	4	3	1.00	19	0.158
580	A	10	9	1.00	21	0.429
581	A	3	3	1.00	26	0.115
582	A	3	3	1.00	26	0.115
583	A	3	3	1.00	24	0.125
584	A	3	3	1.00	23	0.130
585	A	3	3	1.00	26	0.115
586	A	3	3	1.00	26	0.115
587	A	4	3	1.00	17	0.176
588	A	5	5	1.00	17	0.294
589	A	4	4	1.00	15	0.267
590	A	2	2	1.00	9	0.222
591	A	5	5	1.00	17	0.294
592	A	3	3	1.00	17	0.176
593	A	4	4	1.00	17	0.235
594	A	5	5	1.00	17	0.294
595	A	3	3	1.00	28	0.107
596	A	3	3	1.00	28	0.107
597	A	3	3	1.00	26	0.115
598	A	3	3	1.00	25	0.120
599	A	3	3	1.00	28	0.107
600	A	3	3	1.00	28	0.107
601	A	8	6	1.00	21	0.286
602	A	1	1	1.00	17	0.059
603	A	1	1	1.00	21	0.048

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
604	A	1	1	1.00	17	0.059
605	A	1	1	1.00	19	0.053
606	A	1	1	1.00	21	0.048
607	A	4	3	1.00	25	0.120
608	A	3	2	1.00	17	0.118
609	A	4	3	1.00	27	0.111
610	A	4	3	1.00	25	0.120
611	A	5	4	1.70	22	0.182
612	A	4	3	1.00	29	0.103
613	A	1	1	1.00	176	0.006
614	A	1	1	1.00	174	0.006
615	A	1	1	1.00	164	0.006
616	F	0	0	N/A	0	N/A
617	F	0	0	N/A	0	N/A
618	A	4	3	1.00	19	0.158
619	A	4	3	1.00	19	0.158
620	A	4	3	1.00	17	0.176
621	A	4	3	1.00	15	0.200
622	A	7	6	1.00	19	0.316
623	A	6	6	1.00	19	0.316
624	A	6	6	1.00	19	0.316
625	A	4	3	1.00	21	0.143
626	A	4	3	1.00	21	0.143
627	A	4	3	1.00	19	0.158
628	A	4	3	1.00	17	0.176
629	A	7	6	1.00	21	0.286
630	A	8	7	1.00	21	0.333
631	A	9	8	1.00	21	0.381
632	A	4	3	1.00	19	0.158
633	A	4	3	1.00	19	0.158
634	A	4	3	1.00	17	0.176
635	A	4	3	1.00	15	0.200
636	A	7	6	1.00	19	0.316
637	A	8	7	1.00	19	0.368
638	A	9	7	1.00	19	0.368
639	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
640	A	4	3	1.00	19	0.158
641	A	4	3	1.00	17	0.176
642	A	4	3	1.00	15	0.200
643	A	7	6	1.00	19	0.316
644	A	8	7	1.00	19	0.368
645	A	9	7	1.00	19	0.368
646	A	4	3	1.00	21	0.143
647	A	4	3	1.00	21	0.143
648	A	4	3	1.00	19	0.158
649	A	4	3	1.00	17	0.176
650	A	6	5	1.00	21	0.238
651	A	7	6	1.00	21	0.286
652	A	8	6	1.00	21	0.286
653	A	4	3	1.00	19	0.158
654	A	4	3	1.00	19	0.158
655	A	4	3	1.00	17	0.176
656	A	4	3	1.00	15	0.200
657	A	6	4	1.00	19	0.210
658	A	8	6	1.00	17	0.353
659	A	7	6	1.00	17	0.353
660	A	6	6	1.00	17	0.353
661	A	5	5	1.00	17	0.294
662	A	6	6	1.00	17	0.353
663	A	7	6	1.00	17	0.353
664	A	8	6	1.00	19	0.316
665	A	7	6	1.00	19	0.316
666	A	6	6	1.00	19	0.316
667	A	5	5	1.00	19	0.263
668	A	6	6	1.00	19	0.316
669	A	7	6	1.00	19	0.316
670	A	2	2	1.00	13	0.154
671	A	5	5	1.00	17	0.294
672	A	6	5	1.00	21	0.238
673	A	7	5	1.00	25	0.200
674	A	8	5	1.00	29	0.172
675	A	8	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
676	A	7	6	1.00	20	0.300
677	A	6	6	1.00	18	0.333
678	A	2	2	1.00	20	0.100
679	A	4	3	1.00	20	0.150
680	A	4	3	1.00	20	0.150
681	A	2	2	1.00	18	0.111
682	A	2	2	1.00	20	0.100
683	A	3	2	1.00	22	0.091
684	A	3	3	1.00	17	0.176
685	A	3	3	1.00	23	0.130
686	A	3	2	1.00	22	0.091
687	A	5	4	1.00	34	0.118
688	A	5	5	1.00	31	0.161
689	A	6	4	1.00	47	0.085
690	A	9	5	1.00	58	0.086
691	A	4	4	1.00	11	0.364
692	A	6	6	1.00	11	0.546
693	A	2	1	1.00	17	0.059
694	A	8	6	1.00	13	0.462
695	A	2	1	1.00	16	0.062
696	A	4	2	1.00	14	0.143
697	A	5	4	1.00	12	0.333
698	A	4	2	1.00	13	0.154
699	A	4	2	1.00	13	0.154
700	A	4	2	1.00	15	0.133
701	A	5	5	1.00	12	0.417
702	A	6	5	1.00	14	0.357
703	A	5	4	1.00	13	0.308
704	A	5	4	1.00	17	0.235
705	A	5	4	1.00	17	0.235
706	A	4	3	1.00	13	0.231
707	A	7	5	1.00	18	0.278
708	A	7	5	1.00	20	0.250
709	A	8	5	1.00	18	0.278
710	A	4	3	1.00	26	0.115
711	A	10	6	0.72	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
712	A	3	1	1.00	27	0.037
713	A	5	3	1.00	17	0.176
714	A	5	3	1.00	17	0.176
715	A	5	3	1.00	23	0.130
716	A	5	3	1.00	17	0.176
717	A	6	2	1.00	23	0.087
718	A	5	1	1.00	25	0.040
719	A	5	2	1.00	21	0.095
720	A	3	2	1.00	23	0.087
721	A	4	3	1.18	16	0.188
722	A	3	1	1.00	28	0.036
723	A	5	5	1.00	16	0.312
724	A	4	3	1.00	22	0.136
725	A	8	4	1.00	21	0.190
726	B	16	10	2.80	20	0.500
727	A	9	6	1.00	25	0.240
728	A	6	4	1.00	35	0.114
729	A	2	2	1.00	13	0.154
730	A	3	3	1.00	15	0.200
731	A	3	3	1.00	13	0.231
732	A	4	4	1.00	11	0.364
733	A	3	3	1.00	18	0.167
734	A	4	4	1.00	17	0.235
735	A	4	4	1.00	18	0.222
736	A	5	5	1.00	17	0.294
737	A	2	2	1.00	16	0.125
738	A	2	2	1.00	21	0.095
739	A	3	3	1.00	26	0.115
740	A	3	3	1.00	25	0.120
741	A	3	3	1.00	12	0.250
742	A	3	3	1.00	15	0.200
743	A	3	3	1.00	15	0.200
744	A	3	3	1.00	15	0.200
745	A	3	3	1.00	17	0.176
746	A	4	4	1.00	15	0.267
747	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
748	A	3	3	1.00	22	0.136
749	A	4	4	1.00	20	0.200
750	A	7	7	1.00	20	0.350
751	A	2	2	1.00	15	0.133
752	A	5	5	1.00	21	0.238
753	A	8	6	1.00	18	0.333
754	A	5	4	1.00	18	0.222
755	A	6	4	1.00	18	0.222
756	A	3	2	1.00	16	0.125
757	A	3	2	1.00	16	0.125
758	A	3	2	1.00	16	0.125
759	A	10	9	1.00	18	0.500
760	A	5	4	1.00	18	0.222
761	A	6	5	1.00	18	0.278
762	A	5	5	1.40	35	0.143
763	A	6	6	1.40	28	0.214
764	A	7	7	1.00	23	0.304
765	A	6	6	1.00	23	0.261
766	A	3	3	1.00	23	0.130
767	A	6	6	1.00	23	0.261
768	A	7	7	1.00	23	0.304
769	A	7	7	1.00	19	0.368
770	A	6	6	1.00	19	0.316
771	A	3	3	1.00	19	0.158
772	A	6	6	1.00	19	0.316
773	A	7	7	1.00	19	0.368
774	A	6	6	1.00	31	0.194
775	A	5	5	1.00	31	0.161
776	A	2	2	1.00	31	0.065
777	A	5	5	1.00	31	0.161
778	A	5	5	1.00	34	0.147
779	A	2	2	1.00	34	0.059
780	A	5	5	1.00	34	0.147
781	A	8	8	1.00	24	0.333
782	A	7	7	1.00	24	0.292
783	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
784	A	7	7	1.00	24	0.292
785	A	8	8	1.00	24	0.333
786	A	14	13	1.00	26	0.500
787	A	12	12	1.00	26	0.462
788	A	7	7	1.00	26	0.269
789	A	10	10	1.00	26	0.385
790	A	12	12	1.00	26	0.462
791	A	15	13	1.00	28	0.464
792	A	13	13	1.00	28	0.464
793	A	11	11	1.00	28	0.393
794	A	10	9	1.00	28	0.321
795	A	13	12	1.00	28	0.429
796	A	4	4	1.00	19	0.210
797	A	10	10	1.00	19	0.526
798	A	3	3	1.00	19	0.158
799	A	9	9	1.00	19	0.474
800	A	4	4	1.00	24	0.167
801	A	10	10	1.00	24	0.417
802	A	12	11	1.00	24	0.458
803	A	4	4	1.00	24	0.167
804	A	10	10	1.00	24	0.417
805	A	11	10	1.00	27	0.370
806	A	14	7	1.00	20	0.350
807	A	12	6	1.91	21	0.286
808	A	2	1	1.00	22	0.045
809	A	4	2	1.00	18	0.111
810	A	4	2	1.00	18	0.111
811	A	5	5	1.00	23	0.217
812	A	6	6	1.00	22	0.273
813	A	6	6	1.00	20	0.300
814	A	3	2	1.00	15	0.133
815	A	3	2	1.00	21	0.095
816	A	5	4	1.00	19	0.210
817	A	9	6	1.00	24	0.250
818	A	9	6	1.00	24	0.250
819	A	10	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
820	A	11	8	1.00	13	0.615
821	A	12	8	1.00	25	0.320
822	A	13	9	1.00	15	0.600
823	A	14	9	1.00	19	0.474
824	A	2	1	1.00	35	0.029
825	A	8	5	1.00	37	0.135
826	A	16	8	1.00	29	0.276
827	A	16	8	1.00	36	0.222
828	A	31	13	1.00	43	0.302
829	A	5	4	1.00	21	0.190
830	A	6	4	1.00	27	0.148
831	A	8	6	1.00	27	0.222
832	A	4	4	1.00	27	0.148
833	A	4	4	1.00	20	0.200
834	A	6	6	1.00	29	0.207
835	A	2	1	1.00	20	0.050
836	A	3	2	1.00	25	0.080
837	A	2	2	1.00	28	0.071
838	A	3	3	1.00	26	0.115
839	A	1	1	1.00	11	0.091
840	A	2	2	1.00	19	0.105
841	A	2	2	1.00	15	0.133
842	A	2	2	1.00	14	0.143
843	A	3	3	1.00	17	0.176
844	A	3	3	1.00	13	0.231
845	A	2	2	1.00	14	0.143
846	A	3	3	1.00	17	0.176
847	A	3	3	1.00	13	0.231
848	A	1	0	1.00	9	0.000
849	A	4	3	1.00	15	0.200
850	A	2	2	1.00	13	0.154
851	A	3	3	1.00	19	0.158
852	A	3	3	1.32	11	0.273
853	A	4	4	1.32	17	0.235
854	A	1	1	1.00	9	0.111
855	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
856	A	1	1	1.00	7	0.143
857	A	2	2	1.00	21	0.095
858	A	1	1	1.00	9	0.111
859	A	2	2	1.00	19	0.105
860	A	1	1	1.00	7	0.143
861	A	2	2	1.00	21	0.095
862	A	3	3	1.00	17	0.176
863	A	4	4	1.00	30	0.133
864	A	7	7	1.00	20	0.350
865	A	6	6	1.00	20	0.300
866	A	7	7	1.00	23	0.304
867	A	6	6	1.00	25	0.240
868	A	1	1	1.00	11	0.091
869	A	2	2	1.00	21	0.095
870	A	1	1	1.00	9	0.111
871	A	2	2	1.00	23	0.087
872	A	2	2	1.00	11	0.182
873	A	3	3	1.00	21	0.143
874	A	2	2	1.00	9	0.222
875	A	3	3	1.00	23	0.130
876	A	3	3	1.16	14	0.214
877	A	4	4	1.00	15	0.267
878	A	7	6	1.00	17	0.353
879	A	7	6	1.00	17	0.353
880	A	10	8	1.00	34	0.235
881	A	12	7	1.00	30	0.233
882	A	5	4	1.00	21	0.190
883	A	7	5	1.00	23	0.217
884	A	9	7	1.00	25	0.280
885	A	7	6	1.00	25	0.240
886	A	3	3	1.00	13	0.231
887	A	2	2	1.00	24	0.083
888	A	2	2	1.00	22	0.091
889	A	2	2	1.00	30	0.067
890	A	3	3	1.00	27	0.111
891	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
892	A	7	5	1.00	28	0.179
893	A	13	10	1.55	25	0.400
894	A	8	6	1.00	29	0.207
895	A	6	6	1.00	16	0.375
896	A	6	6	1.00	16	0.375
897	A	4	4	1.00	16	0.250
898	A	7	7	1.00	16	0.438
899	A	8	8	1.00	16	0.500
900	A	3	3	1.00	24	0.125
901	A	0	0	0.00	0	0.000
902	A	0	0	0.00	0	0.000
903	A	0	0	0.00	0	0.000
904	A	0	0	0.00	0	0.000
905	A	0	0	0.00	0	0.000
906	A	0	0	0.00	0	0.000
907	A	0	0	0.00	0	0.000
908	A	0	0	0.00	0	0.000
909	A	0	0	0.00	0	0.000
910	A	0	0	0.00	0	0.000
911	A	2	2	1.00	37	0.054
912	A	2	2	1.00	38	0.053
913	A	5	4	1.00	40	0.100
914	A	7	5	1.00	40	0.125
915	A	6	5	1.00	18	0.278
916	A	7	6	1.00	21	0.286
917	A	8	6	1.00	22	0.273
918	A	3	3	1.00	21	0.143
919	A	3	3	1.00	24	0.125
920	A	11	10	1.00	19	0.526
921	A	10	7	1.00	24	0.292
922	A	4	3	1.00	19	0.158
923	A	3	3	1.00	17	0.176
924	A	1	1	1.00	15	0.067
925	A	1	1	1.00	17	0.059
926	A	2	2	1.00	17	0.118
927	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
928	A	1	1	1.00	17	0.059
929	A	6	6	1.00	17	0.353
930	A	5	5	1.00	11	0.454
931	A	3	3	1.00	11	0.273
932	A	5	3	1.00	13	0.231
933	A	2	2	1.00	21	0.095
934	A	5	5	1.00	16	0.312
935	A	2	2	1.00	13	0.154
936	A	6	6	1.00	16	0.375
937	A	5	4	1.00	12	0.333
938	A	4	3	1.00	18	0.167
939	A	10	6	1.00	17	0.353
940	A	1	1	1.00	17	0.059
941	C	3	2	1.03	27	0.074
942	A	2	2	1.00	37	0.054
943	A	3	2	1.00	17	0.118
944	A	5	4	1.00	17	0.235
945	A	1	1	1.00	15	0.067
946	A	3	3	1.00	14	0.214
947	A	1	1	1.00	17	0.059
948	A	2	1	1.00	13	0.077
949	A	2	1	1.00	15	0.067
950	A	7	4	1.00	15	0.267
951	A	6	6	1.00	15	0.400
952	A	1	0	1.00	9	0.000
953	A	1	0	1.00	9	0.000
954	A	2	1	1.00	19	0.053
955	A	3	3	1.00	15	0.200
956	A	3	3	1.00	16	0.188
957	A	4	4	1.00	15	0.267
958	A	5	5	1.00	15	0.333
959	A	4	4	1.00	25	0.160
960	A	2	2	1.00	11	0.182
961	A	2	2	1.00	17	0.118
962	A	6	6	1.00	22	0.273
963	A	4	3	1.00	13	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
964	A	4	3	1.00	15	0.200
965	A	4	4	1.00	19	0.210
966	A	4	4	1.00	21	0.190
967	A	3	3	1.00	17	0.176
968	A	7	7	1.00	19	0.368
969	A	7	6	1.00	19	0.316
970	A	6	6	1.00	17	0.353
971	A	10	8	1.00	17	0.471
972	A	11	9	1.00	17	0.529
973	A	3	3	1.00	22	0.136
974	A	5	5	1.00	11	0.454
975	A	5	5	1.00	13	0.385
976	A	5	5	1.17	11	0.454
977	A	5	5	1.00	15	0.333
978	A	4	4	1.00	11	0.364
979	A	5	5	1.00	13	0.385
980	A	4	4	1.00	11	0.364
981	A	3	3	1.00	11	0.273
982	A	2	2	1.00	11	0.182
983	A	5	5	1.00	19	0.263
984	A	2	2	1.00	23	0.087
985	A	2	2	1.00	13	0.154
986	A	3	3	1.00	11	0.273
987	A	3	3	1.00	15	0.200
988	A	3	3	1.00	19	0.158
989	A	3	3	1.00	19	0.158
990	A	3	3	1.00	19	0.158
991	A	2	2	1.00	15	0.133
992	A	3	3	1.00	15	0.200
993	A	3	3	1.00	12	0.250
994	A	3	3	1.00	16	0.188
995	F	0	0	N/A	0	N/A
996	F	0	0	N/A	0	N/A
997	B	25	12	4.09	31	0.387
998	A	5	4	1.00	25	0.160
999	A	2	2	1.00	27	0.074

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1000	A	2	2	1.00	33	0.061
1001	A	2	2	1.00	34	0.059
1002	A	7	6	1.00	51	0.118
1003	A	2	2	1.00	49	0.041
1004	A	2	2	1.00	43	0.047
1005	A	2	2	1.00	44	0.045
1006	A	9	8	1.00	20	0.400
1007	A	3	3	1.00	15	0.200
1008	A	3	3	1.00	15	0.200
1009	A	1	1	1.00	52	0.019
1010	A	1	1	1.00	57	0.018
1011	A	2	2	1.00	59	0.034
1012	A	2	2	1.00	58	0.034
1013	A	3	3	1.00	58	0.052
1014	A	3	3	1.00	57	0.053
1015	A	3	3	1.00	66	0.045
1016	A	9	9	1.37	31	0.290
1017	F	0	0	N/A	0	N/A
1018	C	9	6	0.96	20	0.300
1019	A	2	2	1.00	46	0.043
1020	A	1	0	1.00	15	0.000
1021	A	12	9	1.00	17	0.529
1022	A	13	10	1.22	33	0.303
1023	F	0	0	N/A	0	N/A
1024	A	1	1	1.00	38	0.026
1025	B	2	2	2.43	30	0.067

Chapter 3

Listing of integrals

3.1 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] 2/9*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*2^(1/3)*(1+x)*E
llipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2
))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1
/2))^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2134, 218, 2137, 203}

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) + (2*2^(1/3
)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipti
cF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2}{3}\,\text{Subst}\left(\int \frac{}{1+}\right.$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.19, size = 148, normalized size = 1.02

$$\frac{4i\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*
I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3) - I
*Sqrt[3])*Sqrt[1 + x^3])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x^2-2^{\frac{2}{3}}x+2\cdot 2^{\frac{1}{3}}\right)}{x^6+5x^3+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x^2 - 2^(2/3)*x + 2*2^(1/3))/(x^6 + 5*x^3 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

maple [A] time = 0.15, size = 139, normalized size = 0.96

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}\,\left(2^{\frac{2}{3}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1
/2),(-3/2+1/2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x^3+1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] int(1/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x+1)\left(x^2-x+1\right)}\,\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.2 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=160

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/9*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)-2/9*2^(1/3)*(1-x)
*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1
/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3
^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2134, 218, 2137, 203}

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) - (2*2^(1/
3)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \frac{\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx}{3\,2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2}{3}\,\text{Subst}\left(\int \frac{}{1-}\right.$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.12, size = 148, normalized size = 0.92

$$-\frac{4i\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] ((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ell
ipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3]
+ (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3
) - I*Sqrt[3])*Sqrt[1 - x^3])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^3+1}\left(x^2+2^{\frac{2}{3}}x+2\cdot 2^{\frac{1}{3}}\right)}{x^6-5x^3+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x^2 + 2^(2/3)*x + 2*2^(1/3))/(x^6 - 5*x^3 + 4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
```

```
ror%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument V
alue
```

maple [A] time = 0.09, size = 143, normalized size = 0.89

$$\frac{2 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-\frac{1}{2}+\frac{i \sqrt{3}}{2}-2^{\frac{2}{3}}}, \sqrt{-\right.}{3 \sqrt{-x^{3}+1}\, \left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}-2^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2
+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3
^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{-x^{3}+1}\, \left(x-2^{\frac{2}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\sqrt{1-x^{3}}\, \left(x-2^{2/3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] -int(1/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x \sqrt{1-x^{3}}-2^{\frac{2}{3}} \sqrt{1-x^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

3.3 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=163

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] -2/9*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)-2/9*2^(1/3)*(1-x)
*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1
/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3
^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2134, 219, 2137, 206}

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3]) - (2*2^(
1/3)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx}{3\,2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2}{3}\,\text{Subst}\Bigg(\int$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.16, size = 146, normalized size = 0.90

$$-\frac{4i\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] ((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ell
ipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3]
+ (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]])/((1 + 2*2^(2/3
) - I*Sqrt[3])*Sqrt[-1 + x^3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2]%
%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument Value
```

maple [A] time = 0.06, size = 143, normalized size = 0.88

$$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}\,\left(-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))
^(1/2),(3/2+1/2*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{x^{3}-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] -int(1/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x\sqrt{x^{3}-1}-2^{\frac{2}{3}}\sqrt{x^{3}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

3.4 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 2/9*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9*2^(1/3)*(1+x)
*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1
/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-
3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2134, 219, 2137, 206}

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3]) + (2*2^(1
/3)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/
4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2}{3}\operatorname{Subst}\left(\int \frac{}{1}\right.$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.11, size = 150, normalized size = 0.96

$$\frac{4i\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*
I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3) - I
*Sqrt[3])*Sqrt[-1 - x^3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[1]%
%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument Value
```

maple [A] time = 0.05, size = 139, normalized size = 0.89

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}\,\left(2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2
/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1
/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] int(1/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x+1)\left(x^2-x+1\right)}\,\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.5 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=280

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\ \cdots\right.}{\sqrt{a+bx^3}}\right.}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] 2/9*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(1/2))/b^(
1/3)*3^(1/2)/a^(1/2)+2/9*2^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2134, 218, 2137, 203}

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\cdots\right.}{\cdots}\right.}{3\sqrt{3}\sqrt{a}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]])
/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
```

```
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{3\,2^{2/3}\sqrt[3]{a}} + \frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}}}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}}{\left(\left(1+\right.\right.}}}$$

Mathematica [C] time = 0.21, size = 164, normalized size = 0.59

$$-\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3
) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a + b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^{\frac{1}{3}} x + 2^{\frac{2}{3}} a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^{3}+a}\left(b^{\frac{1}{3}} x + 2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

```
[Out] int(1/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^{3}} \left(2^{\frac{2}{3}} \sqrt[3]{a} + \sqrt[3]{b} x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

3.6 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=288

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\right.}{\sqrt{a-b}}\right.}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] -2/9*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)^(1/2))/b
^(1/3)*3^(1/2)/a^(1/2)-2/9*2^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(1/3)/(-b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2134, 218, 2137, 203}

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}}{}\right.}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]]
)/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)
*x)^2]*Sqrt[a - b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
```

```
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{3\,2^{2/3}\sqrt[3]{a}} + \frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|\right.}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}\right.}}}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}(}{\left(\left(1+\sqrt{3}\right.\right.}}}$$

Mathematica [C] time = 0.21, size = 166, normalized size = 0.58

$$\frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3)
 + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a - b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b^{\frac{1}{3}} x + 2^{\frac{2}{3}} a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{-bx^{3}+a}\left(b^{\frac{1}{3}} x - 2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate(1/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int(1/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{a - bx^{3}} + \sqrt[3]{b} x \sqrt{a - bx^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(1/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x)
```

3.7 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=297

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{\ldots}\right.}{\sqrt{bx^3-a}}\right.}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] -2/9*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)^(1/2))/b
^(1/3)*3^(1/2)/a^(1/2)-2/9*2^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(1/3)/(b*x^3-a)^(1/2)/(-a^(
1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2134, 219, 2137, 206}

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),4\sqrt{3}-7\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\ldots\right.}{\ldots}\right.}{3\sqrt{3}\sqrt{a}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3
]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*
b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
```

```
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{3\,2^{2/3}\sqrt[3]{a}} + \frac{\sqrt[3]{2}\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}\right.\right.}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\cdots}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\cdots}}$$

Mathematica [C] time = 0.12, size = 167, normalized size = 0.56

$$\frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3)
 + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a + b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b^{\frac{1}{3}} x+2^{\frac{2}{3}} a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3-a)^(1/2),x)
```

```
[Out] int(1/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{bx^{3}-a}\left(b^{\frac{1}{3}} x-2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(1/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int(1/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{-a+bx^{3}}+\sqrt[3]{b} x \sqrt{-a+bx^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(1/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x)
```

3.8 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=293

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\right.}{\sqrt{-a-}}\right.}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] 2/9*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a)^(1/2))/b
^(1/3)*3^(1/2)/a^(1/2)+2/9*2^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(1/3)/(-b*x^3-a)^(1/2)/(-a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2134, 219, 2137, 206}

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),4\sqrt{3}-7\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{}}{}\right.}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]
])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b
^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
```

```
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{3\,2^{2/3}\sqrt[3]{a}} + \frac{\sqrt[3]{2}\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\cdots}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} + \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}}\cdots}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\cdots}{\left(\left(1-\cdots\right.\right.}}}$$

Mathematica [C] time = 0.15, size = 167, normalized size = 0.57

$$-\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3
) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a - b*x^3]
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^{\frac{1}{3}} x + 2^{\frac{2}{3}} a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(1/(b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^{3}-a}\left(b^{\frac{1}{3}} x + 2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

```
[Out] int(1/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a - bx^{3}} \left(2^{\frac{2}{3}} \sqrt[3]{a} + \sqrt[3]{b} x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

3.9 $\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=249

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\,c^{3/2}d}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

```
[Out] 2/9*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/c^(3/2)/d*3^(1/
2)+2/9*2^(1/3)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3)*d*x+c*(1-3^(1/2)))/(2^(2/
3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^2-2^(2/3
)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c/d
/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A] time = 0.29, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2134, 218, 2137, 203}

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\,c^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]*
c^(3/2)*d) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^(
2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
-7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*c
 + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2134

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
```

```
d^3, 0]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3c} + \frac{2\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3c}$$

$$= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-\right.}{3\sqrt[4]{3}\,cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\sin^{-1}\right.}{3\sqrt[4]{3}\,cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+}}$$

Mathematica [C] time = 0.21, size = 169, normalized size = 0.68

$$-\frac{i2^{5/6}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{\frac{4d^2x^2}{c^2}-\frac{2\sqrt[3]{2}dx}{c}+2^{2/3}}\,\Pi\left(\frac{i\sqrt[3]{2}\sqrt{3}}{2+\sqrt[3]{-2}};\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)|\sqrt[3]{-1}\right)}{\left(2+\sqrt[3]{-2}\right)d\sqrt{c^3+4d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] ((-I)*2^(5/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3) -
 (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1/3)*Sqrt[3])/(2 + (
-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)
]/2^(1/6)], (-1)^(1/3)])/((2 + (-2)^(1/3))*d*Sqrt[c^3 + 4*d^3*x^3])
```

fricas [F] time = 1.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4d^3x^3+c^3}}{4d^4x^4+4cd^3x^3+c^3dx+c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*d^3*x^3 + c^3)/(4*d^4*x^4 + 4*c*d^3*x^3 + c^3*d*x + c^4), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4\,d^3x^3 + c^3}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

maple [B] time = 0.21, size = 495, normalized size = 1.99

$$\frac{2\left(\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}\right)\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}}\sqrt{\frac{x+\frac{2^{\frac{1}{3}}c}{2d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}+\frac{2^{\frac{1}{3}}c}{2d}}}\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}}\ \mathrm{Ellipt}}{\sqrt{4d^3x^3+c^3}\left(\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(
1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1
/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3
+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(((x-(1
/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))
*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1
/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1
/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))
*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4\,d^3x^3 + c^3}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c^3 + 4\,d^3\,x^3}\,\left(c + d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] int(1/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
[Out] Integral(1/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

3.10 $\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=146

$$\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)/(x^3+1)^(1/2)/((1+x
)/(1+x+3^(1/2))^2)^(1/2)+arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(9
+6*3^(1/2))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2135, 218, 2140, 203}

$$\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}+\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[3])]
 + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\int \frac{6\left(1-\sqrt{3}\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{12\sqrt{3}}+\frac{\int \frac{1}{\sqrt{1+x^3}}\,dx}{2\sqrt{3}}$$

$$= \frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{\text{Subst}\left(\int \frac{}{1+(3}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-\right.}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.21, size = 136, normalized size = 0.93

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[3])
*Sqrt[1 + x^3])
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}{x^5+2x^4-2x^3+x^2+2x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x - sqrt(3) + 1)/(x^5 + 2*x^4 - 2*x^3 + x^2 + 2*x -
 2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2]%
%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Value
```

maple [A] time = 0.05, size = 132, normalized size = 0.90

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\ \mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2/3*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2
),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{3}+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x+1)\left(x^{2}-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.11 $\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=164

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2
)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3+1)^(1/2)/((1
-x)/(1-x+3^(1/2))^2)^(1/2)-arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))
/(9+6*3^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2135, 218, 2140, 203}

$$-\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3]
)]) - (Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3
/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{\int \frac{-6\left(1-\sqrt{3}\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{12\sqrt{3}} + \frac{\int \frac{1}{\sqrt{1-x^3}}\,dx}{2\sqrt{3}}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} - \frac{\text{Subst}\left(\int \frac{1}{1+(3+2}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\right.}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

Mathematica [C] time = 0.13, size = 136, normalized size = 0.83

$$\frac{4\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\,\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[1 - x^3])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}{x^5-2x^4-2x^3-x^2+2x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)/(x^5 - 2*x^4 - 2*x^3 - x^2 + 2*x
+ 2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2]%
%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Value
```

maple [A] time = 0.06, size = 143, normalized size = 0.87

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}-\sqrt{3}},\sqrt{-}\right.}{3\sqrt{-x^{3}+1}\,\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}-\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2
+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3
^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{-x^{3}+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x\sqrt{1-x^{3}}-\sqrt{3}\,\sqrt{1-x^{3}}-\sqrt{1-x^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
)
```

3.12 $\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=167

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2
)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(1/4)/(x^3-1)^(1/2)/((-1
+x)/(1-x-3^(1/2))^2)^(1/2)-arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))
/(9+6*3^(1/2))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2135, 219, 2140, 206}

$$-\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[
3])]) - (Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^
(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\int \frac{6\left(1-\sqrt{3}\right)-6x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx}{12\sqrt{3}} + \frac{\int \frac{1}{\sqrt{-1+x^3}}\,dx}{2\sqrt{3}}$$

$$= -\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}} - \frac{\mathrm{Subst}\left(\int \frac{}{1\text{-}}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-\right.}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.18, size = 134, normalized size = 0.80

$$\frac{4\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\,\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[-1 + x^3])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}{x^5-2x^4-2x^3-x^2+2x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 - 1)*(x + sqrt(3) - 1)/(x^5 - 2*x^4 - 2*x^3 - x^2 + 2*x
+ 2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.04, size = 132, normalized size = 0.79

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2/3*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/
2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)
))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
)
```

3.13 $\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=157

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)
-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3-1)^(1/2)/((-1
-x)/(1+x-3^(1/2))^2)^(1/2)+arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2)
)/(9+6*3^(1/2))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2135, 219, 2140, 206}

$$\frac{\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3]
)] + (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/
4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\int \frac{-6\left(1-\sqrt{3}\right)-6x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx}{12\sqrt{3}} + \frac{\int \frac{1}{\sqrt{-1-x^3}}\,dx}{2\sqrt{3}}$$

$$= \frac{\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(3+2\right.}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+\right.}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.11, size = 138, normalized size = 0.88

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\,\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[3])
*Sqrt[-1 - x^3])
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}{x^5+2x^4-2x^3+x^2+2x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(x - sqrt(3) + 1)/(x^5 + 2*x^4 - 2*x^3 + x^2 + 2*x
 - 2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.05, size = 139, normalized size = 0.89

$$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \text{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{\frac{3}{2}+\sqrt{3}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \cdots}{\frac{3}{2} \cdots}} \cdots\right.}{3 \sqrt{-x^{3}-1}\, \left(\frac{3}{2}+\sqrt{3}+\frac{i \sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+
3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1
/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^{3}-1}\, \left(x+\sqrt{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{- \left(x+1\right) \left(x^{2}-x+1\right)}\, \left(x+1+\sqrt{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.14 $\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=329

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{26+15\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$

```
[Out] 1/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1+
x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(x^3+1)^(1/
2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2))/
(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2
)/(x^3+1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^(1/2)+2/3
*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x
+3^(1/2))^2)^(1/2)*(3/2*6^(1/2)+5/2*2^(1/2))*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(
1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 331, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2136, 218, 2142, 2113, 537, 571, 93, 204}

$$\frac{2\sqrt{26+15\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(Sqrt
[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[26 + 15*Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 -
 Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{\int \frac{1}{\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1 \ldots\right.}{\ldots}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1 \ldots\right.}{\ldots}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+ \ldots}{\ldots}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+ \ldots}{\ldots}$$

$$= \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{26+15\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1} \ldots\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+ \ldots}}$$

Mathematica [C] time = 0.07, size = 128, normalized size = 0.39

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{7i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(\sqrt{3}+7i\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[1 + x^3])
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}}{x^4+3x^3+x+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)/(x^4 + 3*x^3 + x + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

maple [A] time = 0.02, size = 123, normalized size = 0.37

$$\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3}{4}+\frac{i\sqrt{3}}{4},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+3)/(x^3+1)^(1/2),x)
```

```
[Out] (3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^
(1/2)/(x^3+1)^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/4*I
*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

mupad [B] time = 0.22, size = 164, normalized size = 0.50

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 1)^(1/2)*(x + 3)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*ellipticPi(- (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
))/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

3.15 $\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=380

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{\cdots}\right.}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -1/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/((x^2+x+1)/(1
-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*7^(1/2)/(-x^3+1)^(1
/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)+4/13*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/
2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2
^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2
))^2)^(1/2)-2/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*
(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(4+3^(1
/2))/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 382, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2136, 218, 2142, 2113, 537, 571, 93, 206}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{\cdots}\right.}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] -((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
- x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(2
*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2 + Sqr
t[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 -
 Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*S
qrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[
3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*S
qrt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])
/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \frac{\int \frac{1}{\sqrt{1-x^3}}\,dx}{4+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{1-x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1\right.}{}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1\right.}{}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-}{}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-}{}$$

$$= -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\right.}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right.}}}$$

Mathematica [C] time = 0.08, size = 128, normalized size = 0.34

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(\sqrt{3}+5i\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*EllipticP
i[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((5*I + Sqrt[3])*Sqrt[1 - x^3])
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}}{x^4+3\,x^3-x-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)/(x^4 + 3*x^3 - x - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

maple [A] time = 0.03, size = 133, normalized size = 0.35

$$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{5}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right.}{3\sqrt{-x^3+1}\left(\frac{5}{2}+\frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+3)/(-x^3+1)^(1/2),x)
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(5/2
+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

mupad [B] time = 2.42, size = 180, normalized size = 0.47

$$-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\ \Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/2)*(x + 3)),x)
[Out] -(((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/8 +
 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/(2*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
 x^3)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

3.16 $\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=374

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\cdots\right.}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] -2/39*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(5/2*6^(1/
2)-7/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-
1+x)/(1-x-3^(1/2))^2)^(1/2)-1/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1
/2))^2)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2
)^(1/2)*7^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)+4/13*3^(1/4)*(1
-x)*EllipticPi((-1+x+3^(1/2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/
2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(x^3-1)
^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 376, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2136, 219, 2142, 2113, 537, 571, 93, 206}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\cdots\right.}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
- x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(2
*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) - (2*Sqrt[62 - 3
5*Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(13*3^(1/4)*Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]
]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sqr
t[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(
13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \frac{\int \frac{1}{\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\right.}{}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(\right.}{}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(}{}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(}{}$$

$$= -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\mathrm{si}\right.}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{}}$$

Mathematica [C] time = 0.07, size = 126, normalized size = 0.34

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(\sqrt{3}+5i\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*EllipticP
i[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((5*I + Sqrt[3])*Sqrt[-1 + x^3])
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^3-1}}{x^4+3x^3-x-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)/(x^4 + 3*x^3 - x - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

maple [A] time = 0.02, size = 124, normalized size = 0.33

$$\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{8}+\frac{i\sqrt{3}}{8},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+3)/(x^3-1)^(1/2),x)
```

```
[Out] 1/2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/8+1
/8*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

mupad [B] time = 0.05, size = 164, normalized size = 0.44

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 - 1)^(1/2)*(x + 3)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)))/(4*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

3.17 $\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=340

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{4\sqrt[4]{3}(x+1\cdots}{\cdots}$$

```
[Out] 2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(
1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((-1
-x)/(1+x-3^(1/2))^2)^(1/2)+1/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/
2))^2)^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)
^(1/2)*26^(1/2)/(-x^3-1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1+x
)*EllipticPi((-1-x+3^(1/2))/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^
2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+
x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 342, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2136, 219, 2142, 2113, 537, 571, 93, 204}

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{4\sqrt[4]{3}(x+1\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(Sqrt
[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) + (2*(1 + x)*Sqrt[(1
 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sq
rt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[-((1 + x)/(1
- Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/
(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + S
qrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = -\frac{\int \frac{1}{\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1}{(1+}}\right.}{}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1}{(1+}}\right.}{}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-}{(1+}}}{}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-}{(1+}}}{}$$

$$= \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right.\right.}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{\cdot}}$$

Mathematica [C] time = 0.07, size = 130, normalized size = 0.38

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{7i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(\sqrt{3}+7i\right)\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[-1 - x^3])
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}}{x^4+3x^3+x+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)/(x^4 + 3*x^3 + x + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

maple [A] time = 0.02, size = 133, normalized size = 0.39

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{7}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}\,\left(\frac{7}{2}+\frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+3)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(7/2+
1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

mupad [B] time = 0.05, size = 179, normalized size = 0.53

$$\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\;\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^3 - 1)^(1/2)*(x + 3)),x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/
2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i
)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(- (3^(1/2)*1i)/4 -
3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x+1)\left(x^2-x+1\right)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

3.18 $\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal. Leaf size=139

$$-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}$$

```
[Out] 1/8*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/c/d-3/8*ln(d*(-d*x+c)+2^(2/3)*d*(d^3*x^3
-c^3)^(1/3))*2^(2/3)/c/d+1/4*arctan(1/3*(1-2^(1/3)*(-d*x+c)/(d^3*x^3-c^3)^(
1/3))*3^(1/2))*3^(1/2)*2^(2/3)/c/d
```

Rubi [A] time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2148}

$$-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/
(2*2^(1/3)*c*d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) - (3*Log[d*(c
- d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}-\frac{3\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3\cdots}\right.}{4\sqrt[3]{2}cd}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^3x^3 - c^3\right)^{\frac{1}{3}}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)\left(d^3x^3 - c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
[Out] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^3x^3 - c^3\right)^{\frac{1}{3}}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d^3\,x^3 - c^3\right)^{1/3}\,(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
[Out] int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(-c + dx)\left(c^2 + cdx + d^2x^2\right)}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
[Out] Integral(1/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x)), x)
```

3.19 $\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal. Leaf size=186

$$-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}+\frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\,cd}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd}-\mathrm{lo}$$

```
[Out] -1/2*ln(d*x+c)/c/d-1/4*ln(-d*x+(d^3*x^3+2*c^3)^(1/3))/c/d+3/4*ln(d*(d*x+2*c
)-d*(d^3*x^3+2*c^3)^(1/3))/c/d+1/6*arctan(1/3*(1+2*d*x/(d^3*x^3+2*c^3)^(1/3
))*3^(1/2))/c/d*3^(1/2)-1/2*arctan(1/3*(1+2*(d*x+2*c)/(d^3*x^3+2*c^3)^(1/3)
)*3^(1/2))*3^(1/2)/c/d
```

Rubi [A] time = 0.20, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = $\{2149, 239, 2151\}$

$$-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}+\frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\,cd}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd}-\mathrm{lo}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]*c*d) - (Sq
rt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*c*d
) - Log[c + d*x]/(2*c*d) - Log[-(d*x) + (2*c^3 + d^3*x^3)^(1/3)]/(4*c*d) +
(3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c*d)
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2149

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[
1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c
+ d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3
- a*d^3, 0]
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \frac{\int \frac{1}{\sqrt[3]{2c^3+d^3x^3}}\,dx}{2c} + \frac{\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx}{2c}$$
$$= \frac{\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}\,cd} - \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2cd} - \frac{\log(c+dx)}{2cd} - \frac{\log\left(-dx+\sqrt[3]{2c\right.}}{4cd}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^3x^3+2\,c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(d^3x^3+2c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^3x^3+2\,c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(2\,c^3+d^3\,x^3\right)^{1/3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
```

```
[Out] int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

```
[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(1/3)), x)
```

3.20 $\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=187

$$-\frac{\log(c+dx)}{2c^2d}-\frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}-\frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\,c^2d}+\frac{\sqrt{3}\tan^{-1}}{2}$$

```
[Out] -1/2*ln(d*x+c)/c^2/d-1/4*ln(d*x-(d^3*x^3+2*c^3)^(1/3))/c^2/d+3/4*ln(d*(d*x+
2*c)-d*(d^3*x^3+2*c^3)^(1/3))/c^2/d-1/6*arctan(1/3*(1+2*d*x/(d^3*x^3+2*c^3)
^(1/3))*3^(1/2))/c^2/d*3^(1/2)+1/2*arctan(1/3*(1+2*(d*x+2*c)/(d^3*x^3+2*c^3
)^(1/3))*3^(1/2))*3^(1/2)/c^2/d
```

Rubi [F] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
[Out] Defer[Int][1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^3x^3+2\,c^3\right)^{\frac{2}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx+c\right)\left(d^{3}x^{3}+2c^{3}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

```
[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^{3}x^{3}+2\,c^{3}\right)^{\frac{2}{3}}\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(2\,c^3+d^3\,x^3\right)}^{2/3}\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)),x)
```

```
[Out] int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+dx\right)\left(2c^{3}+d^{3}x^{3}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(2/3),x)
```

```
[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(2/3)), x)
```

3.21 $\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=147

$$-\frac{\log\left(x-\sqrt[3]{x^3+1}\right)}{2\ 2^{2/3}}+\frac{3\log\left(-\sqrt[3]{2}\sqrt[3]{x^3+1}+\sqrt[3]{2}\,x+2\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\left(x+2^{2/3}\right)}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(\sqrt[3]{2}\,x\right.}{2^{2/3}}$$

```
[Out] -1/2*ln(1+2^(1/3)*x)*2^(1/3)-1/4*ln(x-(x^3+1)^(1/3))*2^(1/3)+3/4*ln(2+2^(1/
3)*x-2^(1/3)*(x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1+2*x/(x^3+1)^(1/3))*3^
(1/2))*2^(1/3)*3^(1/2)+1/2*arctan(1/3*(1+2*(2^(2/3)+x)/(x^3+1)^(1/3))*3^(1/
2))*3^(1/2)*2^(1/3)
```

Rubi [F] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
```

```
[Out] Defer[Int][1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx=\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
```

```
[Out] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]
```

fricas [B] time = 11.82, size = 712, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(-1/3*(13910019318573948542*sqrt(3)*(442971093109
30172741433829405399636654451725916403400759596345420183*x^16 + 46991175387
7577297266687493361266274298219751726156511748796788210304*x^13 - 168603219
036433260440647021325346295645242325246375460547582960409424*x^10 - 1978806
301182376573938292954227792627373330283397876582611558332893440*x^7 - 14400
```

```
90891687177581422918763089301968602581036872213084389912370301872*x^4 - 2^(
2/3)*(52271077453125107612995923977654758349394876922885552819209999866413*
x^15 + 59067454785454857729328582078834077849329928125521336059399799480517
2*x^12 + 306314261222931431619887382966630423064822217690279625339197857781
7900*x^9 + 7331049558697577809008352571597039403457968857066730277786114959
327080*x^6 + 77232448067562904437597705467808729717394447501735196355441861
14816064*x^3 + 291168089878390092195634857418355141558919044601510645260807
0501424800) + 6*2^(1/3)*(12601355996216322093314748679149120543302140685677
058235520929344665*x^14 - 5558690630019665139246271949192126784782079889001
9850227115938089718*x^11 - 450398920105320599307639536027883986131793624729
303407436233610788504*x^8 - 72188870588094826143251705267039410623833894384
4373553906510879866584*x^5 - 3386681580686843734363092730678494644056913607
51378507442472921774544*x^2) - 62367643045453979229021701235594440425380660
140976292433240780519680*x)*(x^3 + 1)^(2/3) - 13910019318573948542*sqrt(3)*
(20244151386762728582873176440916642276036913846721964342570319874272*x^17
+ 741146137078834990968958694956953525786968216162791369141561079231342*x^1
4 + 2179843197271775401147438396101666875537043663345199103065290718350660*
x^11 + 21110249350284448030276350331723739969986388702750815288350190294268
08*x^8 + 690583979302212649541846671752323578671762361564987198532372077617
072*x^5 + 42560446719395994043503690929493089250376947849898596094387069196
992*x^2 + 2^(2/3)*(58175953016441250552894129028785848895343146706912452780
410096144857*x^16 + 6033291234402259284595124428808463674980863404672105084
10170807919392*x^13 + 99321772442116051464080292497021614887213800679935641
7482692017634440*x^10 - 315373668616978600368729679828820826067145203897860
799345951918357208*x^7 - 15359897811758984549040097640804776981234391400095
23257833795294171024*x^4 - 774581653994506522185065060515457999562469670838
035710700279100960480*x) - 2*2^(1/3)*(4425033739586262364130843214610526559
1584981692216944246872622437586*x^15 + 937303319945530879145881930294041650
15738145719370012253256237142833*x^12 + 13218541316595455203956380934358348
61993288285254840631143087754453816*x^9 + 424770576770174688958921382572527
8162202431773760010908121531655858240*x^6 + 4593245463688643634993735851341
6218383598381701882855001517331855855040*x^3 + 16158837376147892971429107707
86922880950970969890530541101538638738800))*(x^3 + 1)^(1/3) + sqrt(3)*(5808
458566248141380585366589250357524223410237450426571441100184341339711713923
78653765*x^18 + 85128502112016585963203224235079794367450370616046622522881
06173984889011398391939493844*x^15 + 46037674634299399876464933353333393651
798714498861959697684952859181279514449172348801132*x^12 + 1000163483533668
12357999723948540966952435611836580420294833827058766585456463611215562912*
x^9 + 913977586253668076790534210688867294404951076896021210254557365342556
42370122935700628112*x^6 + 276792064712221478189323489147072714065541212161
41734785863966451139338545569046396842944*x^3 - 13910019318573948542*2^(2/3
)*(3844366680114123938578119587438413410802428820066154040455085354797*x^17
 - 493131971154919078063173195983280278594703770406004388326552124793591*x^
14 - 2263656329733750526575239788393341804272268328404078377386979655411628
*x^11 - 3603296088959643040065882606156977332942778368970867958841266275405
688*x^8 - 23751439241454624747907892976430825810233524575836444336983180902
72160*x^5 - 538527827084536759298395164308728360347336217790784309877024260
129712*x^2) + 166920231822887382504*2^(1/3)*(135958920440428283662759820067
08049395032909698880004129949511339226*x^16 + 13513338488515825037717904859
5991346450771199327236207956421113461903*x^13 + 402245899028058436823068109
521885840258775610614711826343657868879359*x^10 + 5472587101498793346918329
99834525308297790387563356879645468036532966*x^7 + 363674199703640963884960
012124387263106254909521640663154302302116404*x^4 + 97123895740704644005292
055222464498011501842944639406026020532340120*x) - 180077408083879446119265
3903259802591850188394016866170707655609076236167687893936558400))/(4912705
745775473374655778624996785809194682896822406415994000025418186301732995555
53387*x^18 + 10277776658535231887928963830517649364075160462302952752368573
529738604577075128345830496*x^15 + 3805307460404116495559861338250665758871
8033800015428848687354515819408113275280820067228*x^12 + 104552977375786496
```

```
056156515228686393360634250389206134816652347595105200990156089430013680*x^
9 + 19378977878621710892383256210067417618988113173228069450235805883107523
1461508817660387440*x^6 + 1762507736152141132702163647685459405315437312825
77338989973916134409349945587251955701568*x^3 + 587295173581501937080873224
84283950706773934182867349449322904141070201590185330889048000)) + 1/12*2^(
1/3)*log((6048*x^16 + 6048*x^13 - 9072*x^10 - 12204*x^7 - 2808*x^4 + 2^(2/3
)*(352*x^18 - 5136*x^15 - 10632*x^12 - 3224*x^9 + 3390*x^6 + 1434*x^3 - 35)
 + 3*(2032*x^14 + 752*x^11 - 3000*x^8 - 1576*x^5 + 172*x^2 + 2^(2/3)*(112*x
^16 - 1760*x^13 - 2228*x^10 + 356*x^7 + 707*x^4 - 22*x) - 2*2^(1/3)*(352*x^
15 - 728*x^12 - 1736*x^9 - 451*x^6 + 215*x^3 - 1))*(x^3 + 1)^(2/3) - 18*2^(
1/3)*(112*x^17 - 192*x^14 - 820*x^11 - 586*x^8 - 21*x^5 + 49*x^2) + 3*(2096
*x^15 + 1664*x^12 - 2680*x^9 - 2492*x^6 - 224*x^3 + 2^(2/3)*(112*x^17 - 176
0*x^14 - 2996*x^11 - 472*x^8 + 779*x^5 + 125*x^2) - 2*2^(1/3)*(336*x^16 - 6
64*x^13 - 2132*x^10 - 1107*x^7 + 55*x^4 + 29*x) + 16)*(x^3 + 1)^(1/3) + 324
*x)/(64*x^18 + 192*x^15 + 240*x^12 + 160*x^9 + 60*x^6 + 12*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

maple [C] time = 42.61, size = 3064, normalized size = 20.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x)
```

```
[Out] -1/6*ln(-(-15559137585059152-12498127505504256*2^(1/3)*(x^3+1)^(1/3)-160495
4020235328*2^(1/3)*x^4-1604954020235328*2^(2/3)*x^2-23004340956706368*2^(1/
3)*x-936223178470608*x^6+14712078518823840*x^3-4279877387294208*x^5*2^(2/3)
+840505690860402*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^
2-11688730639030284*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x
+6471910353179844*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^3
+1203809884289286*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^4
-3218487773589102*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^5
-72607968203490*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^4
-1560939140169318*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x
^5+3775614346581480*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)
*x^2-3842311729647552*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/
3)*x^3-10498622607665136*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1
/3)*x^4-7759251414704196*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2
/3)*x-3531674097632562*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3
)*x^2+2613886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1
/3)*x+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5+64346838756483
36*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2-5504178119120758*RootOf(2^(2/3)+2^
(1/3)*_Z+_Z^2)*2^(2/3)-10391133689698608*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x+
7959206999356368*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)-10107087250606332*(x^3
+1)^(2/3)*2^(2/3)-9125490357912936*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)-2960082830251008*(x^3+1)^(1/3)*x^5-32590199706036744*(x^3+1)^(2/3)*x-
12827025597754368*(x^3+1)^(1/3)*x^2+3712807561447224*(x^3+1)^(2/3)*x^4+7959
206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2+5665414413224496*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+2321435374812274*RootOf(2^(2/3)+
```

2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6+3532767618003008*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*2^(1/3)*x^3+10197714008127436*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*
x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+26138868
55325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^3+28842279448706
16*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^2-8123294120973864*(x^3+
1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+3217341937824168*RootOf(2^(2/3
)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4+2081809489180344*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)^2*2^(2/3)*x-11138422684341672*(x^3+1)^(2/3)*2^(1/3)*x^2+391907464819
4292*(x^3+1)^(2/3)*2^(2/3)*x^3-1315592369000448*(x^3+1)^(1/3)*2^(2/3)*x^4-6
964190009986188*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^4+3288980
922501120*(x^3+1)^(1/3)*2^(1/3)*x^3-20062783627256832*(x^3+1)^(1/3)*2^(2/3)
x-7115580883942020(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)+2
161300347926748*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x)/(1+2^(1/
3)*x)^6)*2^(1/3)-1/6*ln(-(-15559137585059152-12498127505504256*2^(1/3)*(x^3
+1)^(1/3)-1604954020235328*2^(1/3)*x^4-1604954020235328*2^(2/3)*x^2-2300434
0956706368*2^(1/3)*x-936223178470608*x^6+14712078518823840*x^3-427987738729
4208*x^5*2^(2/3)+840505690860402*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z
^2)^2*2^(1/3)*x^2-11688730639030284*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*2^(1/3)*x+6471910353179844*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)*2^(1/3)*x^3+1203809884289286*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)*2^(2/3)*x^4-3218487773589102*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)*2^(2/3)*x^5-72607968203490*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*2^(1/3)*x^4-1560939140169318*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)^2*2^(1/3)*x^5+3775614346581480*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)^2*2^(2/3)*x^2-3842311729647552*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*
_Z+_Z^2)^2*2^(2/3)*x^3-10498622607665136*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)*2^(1/3)*x^4-7759251414704196*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)*2^(2/3)*x-3531674097632562*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)*2^(2/3)*x^2+2613886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)^2*2^(1/3)*x+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^
5+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2-5504178119120758*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)-10391133689698608*RootOf(2^(2/3)+2^(
1/3)*_Z+_Z^2)*x+7959206999356368*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)-101070
87250606332*(x^3+1)^(2/3)*2^(2/3)-9125490357912936*(x^3+1)^(1/3)*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)-2960082830251008*(x^3+1)^(1/3)*x^5-32590199706036744*
(x^3+1)^(2/3)*x-12827025597754368*(x^3+1)^(1/3)*x^2+3712807561447224*(x^3+1
)^(2/3)*x^4+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2+56
65414413224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+2321435374812274
*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6+3532767618003008*RootOf(2^(2/3
)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3+10197714008127436*2^(2/3)*RootOf(2^(2/3)+2
^(1/3)*_Z+_Z^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1
/3)*x^6+2613886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^
3+2884227944870616*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^2-812329
4120973864*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+32173419378241
68*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4+2081809489180344*RootOf(2^
(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-11138422684341672*(x^3+1)^(2/3)*2^(1/3)*
x^2+3919074648194292*(x^3+1)^(2/3)*2^(2/3)*x^3-1315592369000448*(x^3+1)^(1/
3)*2^(2/3)*x^4-6964190009986188*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*x^4+3288980922501120*(x^3+1)^(1/3)*2^(1/3)*x^3-20062783627256832*(x^3+
1)^(1/3)*2^(2/3)*x-7115580883942020*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*2^(1/3)+2161300347926748*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*x)/(1+2^(1/3)*x)^6)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)+1/6*RootOf(2^(2/3)
+2^(1/3)*_Z+_Z^2)*ln(-(-4550781346817636-3372637147591320*2^(1/3)*(x^3+1)^(
1/3)-3129477143943360*2^(1/3)*x^4-3129477143943360*2^(2/3)*x^2-844958828864
7072*2^(1/3)*x-1825528333966960*x^6+1382185738574984*x^3-3794491037031324*x
^5*2^(2/3)+840505690860402*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*
2^(1/3)*x^2+16011331334883780*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
*2^(1/3)*x-1244136642528564*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2
^(1/3)*x^3-1349025820696266*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2

```
^(2/3)*x^4+96609493250466*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(
2/3)*x^5-72607968203490*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(
1/3)*x^4-1560939140169318*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2
^(1/3)*x^5+3775614346581480*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2
*2^(2/3)*x^2-3842311729647552*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
^2*2^(2/3)*x^3-3429757412307240*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)*2^(1/3)*x^4+12987025125355476*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z
^2)*2^(2/3)*x+5212685479353366*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2
)*2^(2/3)*x^2+2613886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2
)^2*2^(1/3)*x+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5+643468
3875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2+5504178119120758*RootOf(2^
(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)+18718371646419984*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*x+4910160751940304*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)-29915063666643
12*(x^3+1)^(2/3)*2^(2/3)+9125490357912936*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1
/3)*_Z+_Z^2)+355014436588560*(x^3+1)^(1/3)*x^5-11843923165977072*(x^3+1)^(2
/3)*x-4082666020768440*(x^3+1)^(1/3)*x^2+1159971856461672*(x^3+1)^(2/3)*x^4
+4910160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2+6636187113750
264*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+1432130219315922*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6+3532767618003008*RootOf(2^(2/3)+2^(1/3)*_
Z+_Z^2)^2*2^(1/3)*x^3-3132178772121420*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z
^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+2613
886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^3+1221822944
1455304*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^2-7245952797616344*
(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+3217341937824168*RootOf(2
^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4+2081809489180344*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)^2*2^(2/3)*x-6471421936049328*(x^3+1)^(2/3)*2^(1/3)*x^2+61051150
340088*(x^3+1)^(2/3)*2^(2/3)*x^3+2218840228678500*(x^3+1)^(1/3)*2^(2/3)*x^4
-6964190009986188*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^4+37276
51584179880*(x^3+1)^(1/3)*2^(1/3)*x^3-6212752640299800*(x^3+1)^(1/3)*2^(2/3
)*x+7115580883942020*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)+
2161300347926748*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x)/(1+2^(1
/3)*x)^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3+1\right)^{2/3}\,\left(2^{1/3}\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)),x)
```

```
[Out] int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x+1\right)\left(x^2-x+1\right)\right)^{\frac{2}{3}}\left(\sqrt[3]{2}x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2**(1/3)*x)/(x**3+1)**(2/3),x)
```

```
[Out] Integral(1/(((x + 1)*(x**2 - x + 1))**(2/3)*(2**(1/3)*x + 1)), x)
```

3.22 $\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=159

$$\frac{\log\left(-\sqrt[3]{1-x^3}-x\right)}{2\,2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x^3}+\sqrt[3]{2}x-2\right)}{2\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\,2^{2/3}-2x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1-\sqrt[3]{2}\right.}{2^{2/3}}$$

```
[Out] 1/2*ln(1-2^(1/3)*x)*2^(1/3)+1/4*ln(-x-(-x^3+1)^(1/3))*2^(1/3)-3/4*ln(-2+2^(
1/3)*x+2^(1/3)*(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1-2*x/(-x^3+1)^(1/3)
)*3^(1/2))*2^(1/3)*3^(1/2)-1/2*arctan(1/3*(1+(2*2^(2/3)-2*x)/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)*2^(1/3)
```

Rubi [F] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

```
[Out] Defer[Int][1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx=\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

```
[Out] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]
```

fricas [B] time = 10.95, size = 720, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/3*(13910019318573948542*sqrt(3)*(4429710931093
01727414338294053996366544517259164034007595963454201 83*x^16 - 469911753877
577297266687493361266274298219751726156511748796788210304*x^13 - 1686032190
364332604406470213253462956452423252463754605475829604094 24*x^10 + 19788063
011823765739382929542277926273733302833978765826115583328934 40*x^7 - 144009
08916871775814229187630893019686025810368722130843899123703018 72*x^4 + 2^(2
```

```
/3)*(52271077453125107612995923977654758349394876922885552819209999866413*x
^15 - 590674547854548577293285820788340778493299281255213360593997994805172
*x^12 + 3063142612229314316198873829666304230648222176902796253391978577817
900*x^9 - 73310495586975778090083525715970394034579688570667302777861149593
27080*x^6 + 772324480675629044375977054678087297173944475017351963554418611
4816064*x^3 - 2911680898783900921956348574183551415589190446015106452608070
501424800) + 6*2^(1/3)*(126013559962163220933147486791491205433021406856770
58235520929344665*x^14 + 55586906300196651392462719491921267847820798890019
850227115938089718*x^11 - 4503989201053205993076395360278839861317936247293
03407436233610788504*x^8 + 721888705880948261432517052670394106238338943844
373553906510879866584*x^5 - 33866815806868437343630927306784946440569136075
1378507442472921774544*x^2) + 623676430454539792290217012355944404253806601
40976292433240780519680*x)*(-x^3 + 1)^(2/3) + 13910019318573948542*sqrt(3)*
(20244151386762728582873176440916642276036913846721964342570319874272*x^17
- 741146137078834990968958694956953525786968216162791369141561079231342*x^1
4 + 2179843197271775401147438396101666875537043663345199103065290718350660*
x^11 - 21110249350284448030276350331723739969986388702750815288350190294268
08*x^8 + 690583979302212649541846671752323578671762361564987198532372077617
072*x^5 - 42560446719395994043503690929493089250376947849898596094387069196
992*x^2 - 2^(2/3)*(58175953016441250552894129028785848895343146706912452780
410096144857*x^16 - 6033291234402259284595124428808463674980863404672105084
10170807919392*x^13 + 99321772442116051464080292497021614887213800679935641
7482692017634440*x^10 + 315373668616978600368729679828820826067145203897860
799345951918357208*x^7 - 15359897811758984549040097640804776981234391400095
23257833795294171024*x^4 + 774581653994506522185065060515457999562469670838
035710700279100960480*x) - 2*2^(1/3)*(4425033739586262364130843214610526559
1584981692216944246872622437586*x^15 - 937303319945530879145881930294041650
15738145719370012253256237142833*x^12 + 13218541316595455203956380934358348
61993288285254840631143087754453816*x^9 - 424770576770174688958921382572527
8162202431773760010908121531655858240*x^6 + 4593245463688643634993735851341
621838359838170188285500151733185855040*x^3 - 16158837376147892971429107707
86922880950970969890530541101538638738800))*(-x^3 + 1)^(1/3) + sqrt(3)*(580
845856624814138058536658925035752422341023745042657144110018434133971171392
378653765*x^18 - 8512850211201658596320322423507979436745037061604662252288
106173984889011398391939493844*x^15 + 4603767463429939987646493335333339365
1798714498861959697684952859181279514449172348801132*x^12 - 100016348353366
812357999723948540966952435611836580420294833827058766585456463611215562912
*x^9 + 91397758625366807679053421068886729440495107689602121025455736534255
642370122935700628112*x^6 - 27679206471222147818932348914707271406554121216
141734785863966451139338545569046396842944*x^3 + 13910019318573948542*2^(2/
3)*(3844366680114123938578119587438413410802428820066154040455085354797*x^1
7 + 493131971154919078063173195983280278594703770406004388326552124793591*x
^14 - 226365632973375052657523978839334180427226832840407837738697965541162
8*x^11 + 360329608895964304006588260615697733294277836897086795884126627540
5688*x^8 - 2375143924145462474790789297643082581023352457583644433698318090
272160*x^5 + 53852782708453675929839516430872836034733621779078430987702426
0129712*x^2) + 166920231822887382504*2^(1/3)*(13595892044042828366275982006
708049395032909698880004129949511339226*x^16 - 1351333848851582503771790485
95991346450771199327236207956421113461903*x^13 + 40224589902805843682306810
9521885840258775610614711826343657868879359*x^10 - 547258710149879334691832
999834525308297790387563356879645468036532966*x^7 + 36367419970364096388496
0012124387263106254909521640663154302302116404*x^4 - 9712389574070464400529
2055222464498011501842944639406026020532340120*x) - 18007740808387944611926
53903259802591850188394016866170707655609076236167687893936558400))/(491270
574577547337465577862499678580919468289682240641599400002541818630173299555
553387*x^18 - 1027777665853523188792896383051764936407516046230295275236857
3529738604577075128345830496*x^15 + 380530746040411649555986133825066575887
18033800015428848687354515819408113275280820067228*x^12 - 10455297737578649
6056156515228686393360634250389206134816652347595105200990156089430013680*x
```

```
^9 + 19378977878621710892383256210067417618988113173228069450235805883107523
14615088176603874400*x^6 - 176250773615214113270216364768545940531543731282
5773389899739161344093499455872519557015680*x^3 + 587295173581501937080873224
84283950706773934182867349449322904141070201590185330889048000)) - 1/12*2^
(1/3)*log((6048*x^16 - 6048*x^13 - 9072*x^10 + 12204*x^7 - 2808*x^4 + 2^(2/
3)*(352*x^18 + 5136*x^15 - 10632*x^12 + 3224*x^9 + 3390*x^6 - 1434*x^3 - 35
) + 3*(2032*x^14 - 752*x^11 - 3000*x^8 + 1576*x^5 + 172*x^2 + 2^(2/3)*(112*
x^16 + 1760*x^13 - 2228*x^10 - 356*x^7 + 707*x^4 + 22*x) + 2*2^(1/3)*(352*x
^15 + 728*x^12 - 1736*x^9 + 451*x^6 + 215*x^3 + 1))*(-x^3 + 1)^(2/3) + 18*2
^(1/3)*(112*x^17 + 192*x^14 - 820*x^11 + 586*x^8 - 21*x^5 - 49*x^2) - 3*(20
96*x^15 - 1664*x^12 - 2680*x^9 + 2492*x^6 - 224*x^3 + 2^(2/3)*(112*x^17 + 1
760*x^14 - 2996*x^11 + 472*x^8 + 779*x^5 - 125*x^2) + 2*2^(1/3)*(336*x^16 +
 664*x^13 - 2132*x^10 + 1107*x^7 + 55*x^4 - 29*x) - 16)*(-x^3 + 1)^(1/3) -
324*x)/(64*x^18 - 192*x^15 + 240*x^12 - 160*x^9 + 60*x^6 - 12*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

maple [C] time = 41.37, size = 3250, normalized size = 20.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x)
```

```
[Out] -1/6*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*ln(-(-4550781346817636-337263714759132
0*2^(1/3)*(-x^3+1)^(1/3)-3129477143943360*2^(1/3)*x^4-3129477143943360*2^(2
/3)*x^2+8449588288647072*2^(1/3)*x-1825528333966960*x^6-1382185738574984*x^
3+3794491037031324*x^5*2^(2/3)-6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)^2*x^5+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2+550417811
9120758*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)-1871837164641998 4*RootOf(2^
(2/3)+2^(1/3)*_Z+_Z^2)*x+4910160751940304*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)+4910160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2-66361871137
50264*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+1432130219315922*RootOf(2
^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6-3532767618003008*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)^2*2^(1/3)*x^3+3132178772121420*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+32
17341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4-20818094891803
44*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-2991506366664312*2^(2/3)*(-x
^3+1)^(2/3)+9125490357912936*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)
+1159971856461672*(-x^3+1)^(2/3)*x^4-355014436588560*(-x^3+1)^(1/3)*x^5+118
43923165977072*(-x^3+1)^(2/3)*x-4082666020768440*(-x^3+1)^(1/3)*x^2-6471421
936049328*2^(1/3)*(-x^3+1)^(2/3)*x^2-61051150340088*2^(2/3)*(-x^3+1)^(2/3)*
x^3+2218840228678500*2^(2/3)*(-x^3+1)^(1/3)*x^4-6964190009986188*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x^4-3727651584179880*2^(1/3)*(-x^3+1
)^(1/3)*x^3+7245952797616344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)
*x^3+6212752640299800*2^(2/3)*(-x^3+1)^(1/3)*x+7115580883942020*RootOf(2^(2
/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)-216130034792674 8*RootOf(2^(2/3)
+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x-2613886855325640*RootOf(2^(2/3)+2^(1/3
)*_Z+_Z^2)^2*(-x^3+1)^(2/3)*x^3+12218229441455304*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*(-x^3+1)^(2/3)*x^2+3775614346581480*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^
```

```
2*2^(2/3)*(-x^3+1)^(2/3)*x^2+3842311729647552*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3-3429757412307240*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x^4-12987025125355476*RootOf(2^(2/3)+2^(1/3)*_
Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x+5212685479353366*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^2-2613886855325640*RootOf(2^(2/3)+2^(1/3)*_
Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x+840505690860402*RootOf(2^(2/3)+2^(1/3)*_
Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^2-1601133133488378O*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x+1244136642528564*RootOf(2^(2/3)+2^(1/3
)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)*x^3-1349025820696266*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x^4-96609493250466*RootOf(2^(2/3)+2^(1/3
)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^5-72607968203490*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x^4+1560939140169318*RootOf(2^(2/3)+2^(1
/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^5)/(2^(1/3)*x-1)^6)+1/6*ln(-(-15559
137585059152-12498127505504256*2^(1/3)*(-x^3+1)^(1/3)-1604954020235328*2^(1
/3)*x^4-1604954020235328*2^(2/3)*x^2+23004340956706368*2^(1/3)*x-9362231784
70608*x^6-14712078518823840*x^3+4279877387294208*x^5*2^(2/3)-61508007634873
80*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5+6434683875648336*RootOf(2^(2/3)+2^
(1/3)*_Z+_Z^2)^2*x^2-5504178119120758*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/
3)+10391133689698608*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x+7959206999356368*x^4
*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*2^(1/3)*x^2-5665414413224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)
*x^5+2321435374812274*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6-353276761
8003008*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3-10197714008127436*2^(
2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^
(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+3217341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*2^(2/3)*x^4-2081809489180344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)
*x-10107087250606332*2^(2/3)*(-x^3+1)^(2/3)-9125490357912936*RootOf(2^(2/3)
+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)+3712807561447224*(-x^3+1)^(2/3)*x^4+296008
2830251008*(-x^3+1)^(1/3)*x^5+32590199706036744*(-x^3+1)^(2/3)*x-1282702559
7754368*(-x^3+1)^(1/3)*x^2-11138422684341672*2^(1/3)*(-x^3+1)^(2/3)*x^2-391
9074648194292*2^(2/3)*(-x^3+1)^(2/3)*x^3-1315592369000448*2^(2/3)*(-x^3+1)^
(1/3)*x^4-6964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)
*x^4-3288980922501120*2^(1/3)*(-x^3+1)^(1/3)*x^3+8123294120973864*RootOf(2^
(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)*x^3+20062783627256832*2^(2/3)*(-x^3+1
)^(1/3)*x-7115580883942020*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)
^(2/3)-2161300347926748*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x-
2613886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(2/3)*x^3+28842
27944870616*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(2/3)*x^2+377561434658
1480*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(2/3)*x^2+384231172
9647552*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3-104986
22607665136*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x^4+7759
251414704196*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x-35316
74097632562*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^2-2613
886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x+840
505690860402*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^2+1
1688730639030284*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x-6
471910353179844*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)*x^3+
1203809884289286*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x^4
+3218487773589102*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^
5-72607968203490*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x
^4+1560939140169318*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3
)*x^5)/(2^(1/3)*x-1)^6)*2^(1/3)+1/6*ln(-(-15559137585059152-124981275055042
56*2^(1/3)*(-x^3+1)^(1/3)-1604954020235328*2^(1/3)*x^4-1604954020235328*2^(
2/3)*x^2+23004340956706368*2^(1/3)*x-936223178470608*x^6-14712078518823840*
x^3+4279877387294208*x^5*2^(2/3)-6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)^2*x^5+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2-5504178
119120758*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)+10391133689698608*RootOf(
2^(2/3)+2^(1/3)*_Z+_Z^2)*x+7959206999356368*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2-566541441
```

```
3224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+2321435374812274*RootOf
(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6-3532767618003008*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)^2*2^(1/3)*x^3-10197714008127436*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*
_Z+_Z^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6
+3217341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4-20818094891
80344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-10107087250606332*2^(2/3)
*(-x^3+1)^(2/3)-9125490357912936*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(
1/3)+3712807561447224*(-x^3+1)^(2/3)*x^4+2960082830251008*(-x^3+1)^(1/3)*x^
5+32590199706036744*(-x^3+1)^(2/3)*x-12827025597754368*(-x^3+1)^(1/3)*x^2-1
1138422684341672*2^(1/3)*(-x^3+1)^(2/3)*x^2-3919074648194292*2^(2/3)*(-x^3+
1)^(2/3)*x^3-1315592369000448*2^(2/3)*(-x^3+1)^(1/3)*x^4-6964190009986188*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x^4-3288980922501120*2^(1/3
)*(-x^3+1)^(1/3)*x^3+8123294120973864*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3
+1)^(1/3)*x^3+20062783627256832*2^(2/3)*(-x^3+1)^(1/3)*x-7115580883942020*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)-2161300347926748*Root
Of(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x-2613886855325640*RootOf(2^(2
/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(2/3)*x^3+2884227944870616*RootOf(2^(2/3)+2
^(1/3)*_Z+_Z^2)*(-x^3+1)^(2/3)*x^2+3775614346581480*RootOf(2^(2/3)+2^(1/3)*
_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(2/3)*x^2+3842311729647552*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3-10498622607665136*RootOf(2^(2/3)+2
^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x^4+7759251414704196*RootOf(2^(2/3)+
2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x-3531674097632562*RootOf(2^(2/3)+2
^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^2-2613886855325640*RootOf(2^(2/3)+
2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x+840505690860402*RootOf(2^(2/3)+
2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^2+11688730639030284*RootOf(2^(2
/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x-6471910353179844*RootOf(2^(2/
3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)*x^3+1203809884289286*RootOf(2^(2
/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x^4+3218487773589102*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^5-72607968203490*RootOf(2^(2
/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x^4+1560939140169318*RootOf(2
^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^5)/(2^(1/3)*x-1)^6)*Root
Of(2^(2/3)+2^(1/3)*_Z+_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="maxima")
[Out] -integrate(1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(1-x^3\right)}^{2/3}\,\left(2^{1/3}\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)),x)
[Out] -int(1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt[3]{2}x\left(1-x^3\right)^{\frac{2}{3}}-\left(1-x^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2**(1/3)*x)/(-x**3+1)**(2/3),x)
```

```
[Out] -Integral(1/(2**(1/3)*x*(1 - x**3)**(2/3) - (1 - x**3)**(2/3)), x)
```

3.23 $\int (c+dx)^4 \sqrt[3]{a+bx^3}\, dx$

Optimal. Leaf size=387

$$\frac{a^2d^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}}+\frac{a^2d^4\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}}-\frac{2ac^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}-\frac{4ac^3d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+$$

```
[Out] 3/2*a*c^2*d^2*(b*x^3+a)^(1/3)/b+1/18*a*d^4*x^2*(b*x^3+a)^(1/3)/b+1/30*(b*x^
3+a)^(1/3)*(5*d^4*x^5+24*c*d^3*x^4+45*c^2*d^2*x^3+40*c^3*d*x^2+15*c^4*x)+1/
2*a*c^4*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^
(2/3)+1/5*a*c*d^3*x^4*(1+b*x^3/a)^(2/3)*hypergeom([2/3, 4/3],[7/3],-b*x^3/a
)/(b*x^3+a)^(2/3)-2/3*a*c^3*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)+1/18*a^
2*d^4*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)-4/9*a*c^3*d*arctan(1/3*(1+2*b^(
1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)+1/27*a^2*d^4*arctan(1/3*(1
+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 498, normalized size of antiderivative = 1.29, number of steps used = 23, number of rules used = 15, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.790, Rules used = {1853, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628, 261, 365, 364, 321}

$$\frac{a^2d^4\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{27b^{5/3}}-\frac{a^2d^4\log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{54b^{5/3}}+\frac{a^2d^4\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}}-\frac{4ac^3d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}}+\frac{2\cdots}{\cdots}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

```
[Out] (3*a*c^2*d^2*(a + b*x^3)^(1/3))/(2*b) + (a*d^4*x^2*(a + b*x^3)^(1/3))/(18*b
) + ((a + b*x^3)^(1/3)*(15*c^4*x + 40*c^3*d*x^2 + 45*c^2*d^2*x^3 + 24*c*d^3
*x^4 + 5*d^4*x^5))/30 - (4*a*c^3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) + (a^2*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(5/3)) + (a*c^4*x*(1 + (b*x^3)/a)^(2
/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) +
 (a*c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x
^3)/a)])/(5*(a + b*x^3)^(2/3)) - (4*a*c^3*d*Log[1 - (b^(1/3)*x)/(a + b*x^3)
^(1/3)])/(9*b^(2/3)) + (a^2*d^4*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(27
*b^(5/3)) + (2*a*c^3*d*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x
)/(a + b*x^3)^(1/3)])/(9*b^(2/3)) - (a^2*d^4*Log[1 + (b^(2/3)*x^2)/(a + b*x
^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(54*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx = \frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)+a\int \frac{\frac{c^4}{2}+\frac{4}{3}c^3dx+\frac{3}{2}c}{(a}$$

$$=\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)+a\int\left(\frac{c^4}{2\left(a+bx^3\right)^{2/3}}\right.$$

$$=\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)+\frac{1}{2}\left(ac^4\right)\int\frac{1}{\left(a+bx^3\right)}$$

$$=\frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24\right.$$

$$=\frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24\right.$$

$$=\frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24\right.$$

$$=\frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24\right.$$

$$=\frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24\right.$$

$$=\frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24\right.$$

Mathematica [A] time = 0.20, size = 163, normalized size = 0.42

$$\frac{\sqrt[3]{a+bx^3}\left(6bc^4x\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx^2\left(12bc^3-ad^3\right){}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+d^2\left(\left(a+bx^3\right)\sqrt[3]{\frac{bx^3}{a}+1}\left(9c^2+d^2\right.\right.\right.}{6b\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(6*b*c^4*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(12*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a
)] + d^2*((9*c^2 + d^2*x^2)*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 6*b*c*d*x^4
*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)])))/(6*b*(1 + (b*x^3)/a)^(1
/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{1}{3}} (dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx + c)^4 \left(b\,x^3 + a\right)^{\frac{1}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{1}{3}} (dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^3 + a\right)^{1/3} \, (c + d\,x)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x)^4,x)
```

```
[Out] int((a + b*x^3)^(1/3)*(c + d*x)^4, x)
```

sympy [A] time = 4.95, size = 212, normalized size = 0.55

$$\frac{\sqrt[3]{a}\, c^4 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{4\sqrt[3]{a}\, c^3 d x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{4\sqrt[3]{a}\, c d^3 x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \sqrt[3]{a}\, d^4 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**4*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 4*a**(1/3)*c**3*d*x**2*gamma(2/3)*hyper((-1/3, 2/3),
(5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 4*a**(1/3)*c*d**3*x**4*g
amma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3
)) + a**(1/3)*d**4*x**5*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*gamma(8/3)) + 6*c**2*d**2*Piecewise((a**(1/3)*x**3/3, Eq(b,
 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

3.24 $\int (c+dx)^3 \sqrt[3]{a+bx^3}\, dx$

Optimal. Leaf size=242

$$-\frac{ac^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}-\frac{ac^2d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{ax\left(\frac{bx^3}{a}+1\right)^{2/3}\left(5bc^3-ad^3\right)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{10b\left(a+bx^3\right)^{2/3}}+\frac{1}{20}\sqrt[3]{a}$$

```
[Out] 3/4*a*c*d^2*(b*x^3+a)^(1/3)/b+1/10*a*d^3*x*(b*x^3+a)^(1/3)/b+1/20*(b*x^3+a)
^(1/3)*(4*d^3*x^4+15*c*d^2*x^3+20*c^2*d*x^2+10*c^3*x)+1/10*a*(-a*d^3+5*b*c^
3)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/
3)-1/2*a*c^2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-1/3*a*c^2*d*arctan(1/3
*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 297, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {1853, 1888, 1886, 261, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{ac^2d\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}}+\frac{ac^2d\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{6b^{2/3}}-\frac{ac^2d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{ax\left(\frac{bx^3}{a}+1\right)^{2/3}\left(5bc^3-a\right.}{10b\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

```
[Out] (3*a*c*d^2*(a + b*x^3)^(1/3))/(4*b) + (a*d^3*x*(a + b*x^3)^(1/3))/(10*b) +
((a + b*x^3)^(1/3)*(10*c^3*x + 20*c^2*d*x^2 + 15*c*d^2*x^3 + 4*d^3*x^4))/20
 - (a*c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]
*b^(2/3)) + (a*(5*b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(10*b*(a + b*x^3)^(2/3)) - (a*c^2*d*Log[1 - (
b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(2/3)) + (a*c^2*d*Log[1 + (b^(2/3)*x^2)
/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int (c+dx)^3 \sqrt[3]{a+bx^3}\,dx = \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+a\int \frac{\frac{c^3}{2}+c^2dx+\frac{3}{4}cd^2x^2+\frac{d^3}{5}\cdots}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\frac{a\int \frac{\frac{1}{5}\left(5bc^3-a\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\frac{a\int \frac{\frac{1}{5}\left(5bc^3-a\cdots\right.}{(a+\cdots}}{2\cdots}$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d\cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d\cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d\cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d\cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d\cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d\cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d\cdots\right.$$

Mathematica [A] time = 0.13, size = 142, normalized size = 0.59

$$\frac{\sqrt[3]{a+bx^3}\left(4bc^3x\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+d\left(6bc^2x^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+d\left(3c\left(a+bx^3\right)\sqrt[3]{\frac{bx^3}{a}+1}+bdx^4\,{}_2F_1\left(-\frac{1}{3},\right.\right.\right.\right.}{4b\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(4*b*c^3*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(6*b*c^2*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)] + d*(3*
c*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + b*d*x^4*Hypergeometric2F1[-1/3, 4/3,
7/3, -((b*x^3)/a)]))))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (dx+c)^3\left(b\,x^3+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^3+a\right)^{1/3}\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x)^3,x)
```

```
[Out] int((a + b*x^3)^(1/3)*(c + d*x)^3, x)
```

sympy [A] time = 3.92, size = 160, normalized size = 0.66

$$\frac{\sqrt[3]{a}\,c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}\,c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt[3]{a}\,d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+3cd^2\left(\left\{\begin{matrix}\frac{\sqrt[3]{a}}{3} \\ \frac{(a+}{}\end{matrix}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**3*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + a**(1/3)*c**2*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5
/3,), b*x**3*exp_polar(I*pi)/a)/gamma(5/3) + a**(1/3)*d**3*x**4*gamma(4/3)*
hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 3*c*d
**2*Piecewise((a**(1/3)*x**3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True
))
```

3.25 $\int (c+dx)^2 \sqrt[3]{a+bx^3}\, dx$

Optimal. Leaf size=192

$$-\frac{acd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}-\frac{2acd\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F}{2\left(a+bx\right.}$$

```
[Out] 1/4*a*d^2*(b*x^3+a)^(1/3)/b+1/12*(b*x^3+a)^(1/3)*(3*d^2*x^3+8*c*d*x^2+6*c^2
*x)+1/2*a*c^2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x
^3+a)^(2/3)-1/3*a*c*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-2/9*a*c*d*arcta
n(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 245, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1853, 1886, 261, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{2acd\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}}+\frac{acd\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{9b^{2/3}}-\frac{2acd\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (a*d^2*(a + b*x^3)^(1/3))/(4*b) + ((a + b*x^3)^(1/3)*(6*c^2*x + 8*c*d*x^2 +
 3*d^2*x^3))/12 - (2*a*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]])/(3*Sqrt[3]*b^(2/3)) + (a*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2
F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) - (2*a*c*d*Log[1 - (
b^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*b^(2/3)) + (a*c*d*Log[1 + (b^(2/3)*x^2)/(
a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx = \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+a\int\frac{\frac{c^2}{2}+\frac{2cdx}{3}+\frac{d^2x^2}{4}}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+a\int\frac{\frac{c^2}{2}+\frac{2cdx}{3}}{\left(a+bx^3\right)^{2/3}}\,dx+\frac{1}{4}\left(ad^2\right)\int\frac{x^2}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+a\int\left(\frac{c^2}{2\left(a+bx^3\right)^{2/3}}+\frac{2cd}{3\left(a+b\right.}\right.$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{1}{2}\left(ac^2\right)\int\frac{1}{\left(a+bx^3\right)^{2/3}}\,dx+\frac{1}{3}$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{1}{3}(2acd)\,\text{Subst}\left(\int\frac{x}{1-bx^3}\,dx\right.$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\right.}{2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\right.}{2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\right.}{2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)-\frac{2acd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{ac^2x}{}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 0.58

$$\frac{\sqrt[3]{a+bx^3}\left(4bc^2x\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+d\left(4bcx^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+d\left(a+bx^3\right)\sqrt[3]{\frac{bx^3}{a}+1}\right)\right)}{4b\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(4*b*c^2*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(d*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 4*b*c*x^2*Hypergeometric2F1[-
1/3, 2/3, 5/3, -((b*x^3)/a)])))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

fricas [F] time = 130.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^2 + 2\,cdx + c^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{1}{3}} (dx + c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \left(b\,x^3 + a\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(b*x^3+a)^(1/3),x)
[Out] int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{1}{3}} (dx + c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3 + a\right)^{1/3} (c + d\,x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x)^2,x)
[Out] int((a + b*x^3)^(1/3)*(c + d*x)^2, x)
```

sympy [A] time = 3.24, size = 114, normalized size = 0.59

$$\frac{\sqrt[3]{a}\,c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2\sqrt[3]{a}\,cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + d^2\left(\begin{cases}\frac{\sqrt[3]{a}\,x^3}{3} & \text{for } b = 0\\ \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + d**2*Piecewise((a**(1/3)*x*
*3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

3.26 $\int (c+dx)\sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=155

$$-\frac{ad\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}-\frac{ad\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{1}{6}\sqrt[3]{a+bx^3}\left(3cx+2dx^2\right)+\frac{acx\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{b}{}\right.}{2\left(a+bx^3\right)^{2/3}}$$

```
[Out] 1/6*(2*d*x^2+3*c*x)*(b*x^3+a)^(1/3)+1/2*a*c*x*(1+b*x^3/a)^(2/3)*hypergeom([
1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-1/6*a*d*ln(b^(1/3)*x-(b*x^3+a)^(1
/3))/b^(2/3)-1/9*a*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^
(2/3)*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 207, normalized size of antiderivative = 1.34, number of steps used = 12, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1853, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{ad\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}}+\frac{ad\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{18b^{2/3}}-\frac{ad\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{1}{6}\sqrt[3]{a+bx^3}\left(3cx+2dx^2\right)+\frac{acx\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*x^3)^(1/3),x]
```

```
[Out] ((3*c*x + 2*d*x^2)*(a + b*x^3)^(1/3))/6 - (a*d*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) + (a*c*x*(1 + (b*x^3)/a)^(2/
3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) -
(a*d*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*b^(2/3)) + (a*d*Log[1 + (b^
(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + a\int \frac{\frac{c}{2}+\frac{dx}{3}}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + a\int \left(\frac{c}{2\left(a+bx^3\right)^{2/3}} + \frac{dx}{3\left(a+bx^3\right)^{2/3}}\right)dx$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + \frac{1}{2}(ac)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx + \frac{1}{3}(ad)\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + \frac{1}{3}(ad)\,\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{\left(ac\left(1+\frac{bx}{a}\right.\right.}{2}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + \frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} + \frac{(ad)\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{}}\right.}{9\sqrt[3]{}}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + \frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} - \frac{ad\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right.}{9b^{2/3}}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + \frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} - \frac{ad\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right.}{9b^{2/3}}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} - \frac{ad\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}} + \frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right.}{2\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 0.48

$$\frac{x\sqrt[3]{a+bx^3}\left(2c\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*x^3)^(1/3),x]
[Out] (x*(a + b*x^3)^(1/3)*(2*c*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)] +
 d*x*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)]))/(2*(1 + (b*x^3)/a)^(
1/3))
```

fricas [F] time = 16.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{1}{3}}(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(1/3)*(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (dx+c)\left(b\,x^3+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(b*x^3+a)^(1/3),x)
[Out] int((d*x+c)*(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{1/3}\,\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x),x)
[Out] int((a + b*x^3)^(1/3)*(c + d*x), x)
```

sympy [C] time = 2.66, size = 82, normalized size = 0.53

$$\frac{\sqrt[3]{a}\,cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}\,dx^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x**3+a)**(1/3),x)
[Out] a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(1/3)*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

3.27 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$

Optimal. Leaf size=435

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{\sqrt[3]{bc^3-ad^3}\,\log\left(c^3+d^3x^3\right)}{3d^2}-\frac{\sqrt[3]{bc^3-ad^3}\,\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2}$$

```
[Out] (b*x^3+a)^(1/3)/d+x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d^3*x
^3/c^3)/c/(1+b*x^3/a)^(1/3)+1/3*(-a*d^3+b*c^3)^(1/3)*ln(d^3*x^3+c^3)/d^2+1/
2*b^(1/3)*c*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d^3+b*c^3)^(1/3)*ln((
-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d^3+b*c^3)^(1/3)*ln((-
a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/d^2+1/3*b^(1/3)*c*arctan(1/3*(1+2*b^(
1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)-1/3*(-a*d^3+b*c^3)^(1/3)*arcta
n(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)+1
/3*(-a*d^3+b*c^3)^(1/3)*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1
/3))*3^(1/2))/d^2*3^(1/2)
```

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x),x]
```

```
[Out] Defer[Int][(a + b*x^3)^(1/3)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x),x]
```

```
[Out] Integrate[(a + b*x^3)^(1/3)/(c + d*x), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x+c),x)
[Out] int((b*x^3+a)^(1/3)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x),x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x), x)
```

3.28 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$

Optimal. Leaf size=818

$$-\frac{d^3\sqrt[3]{bx^3+a}\,F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x^4}{2c^5\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{d\sqrt[3]{bx^3+a}\,x^2}{c^3+d^3x^3}+\frac{\sqrt[3]{bx^3+a}\,F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{\sqrt[3]{b}\tan^{-1}}{\cdots}$$

```
[Out] -c^2*(b*x^3+a)^(1/3)/d/(d^3*x^3+c^3)-d*x^2*(b*x^3+a)^(1/3)/(d^3*x^3+c^3)+x*
(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d^3*x^3/c^3)/c^2/(1+b*x^3
/a)^(1/3)-1/2*d^3*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-1/3,2,7/3,-b*x^3/a,-d^3
*x^3/c^3)/c^5/(1+b*x^3/a)^(1/3)-1/6*b*c^2*ln(d^3*x^3+c^3)/d^2/(-a*d^3+b*c^3
)^(2/3)-1/9*a*d*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(2/3)-1/18*(-2*a*d^3+3*b*c
^3)*ln(d^3*x^3+c^3)/c/d^2/(-a*d^3+b*c^3)^(2/3)-1/2*b^(1/3)*ln(b^(1/3)*x-(b*
x^3+a)^(1/3))/d^2+1/3*a*d*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-
a*d^3+b*c^3)^(2/3)+1/6*(-2*a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^
3+a)^(1/3))/c/d^2/(-a*d^3+b*c^3)^(2/3)+1/2*b*c^2*ln((-a*d^3+b*c^3)^(1/3)+d*
(b*x^3+a)^(1/3))/d^2/(-a*d^3+b*c^3)^(2/3)-1/3*b^(1/3)*arctan(1/3*(1+2*b^(1/
3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)+2/9*a*d*arctan(1/3*(1+2*(-a*d^3+
b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(2/3)*3^(1/2)+1
/9*(-2*a*d^3+3*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1
/3))*3^(1/2))/c/d^2/(-a*d^3+b*c^3)^(2/3)*3^(1/2)-1/3*b*c^2*arctan(1/3*(1-2*
d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/d^2/(-a*d^3+b*c^3)^(2/3)*3
^(1/2)
```

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
[Out] Defer[Int][(a + b*x^3)^(1/3)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

Mathematica [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
[Out] Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x)^2,x)
```

```
[Out] int((a + b*x^3)^(1/3)/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x)**2, x)
```

3.29 $\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}} dx$

Optimal. Leaf size=310

$$\frac{2acd^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3b^{4/3}}-\frac{4acd^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}-\frac{c^4\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{c^4\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{2c^3 d}{}$$

```
[Out] 3*c^2*d^2*(b*x^3+a)^(2/3)/b+4/3*c*d^3*x*(b*x^3+a)^(2/3)/b+2*c^3*d*x^2*(1+b*
x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^(1/3)+1/5*d^4*x
^5*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 5/3],[8/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1
/2*c^4*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+2/3*a*c*d^3*ln(-b^(1/3)*x+(b*
x^3+a)^(1/3))/b^(4/3)+1/3*c^4*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^
(1/2))/b^(1/3)*3^(1/2)-4/9*a*c*d^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3
))*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1893, 239, 365, 364, 261, 321}

$$\frac{2acd^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3b^{4/3}}-\frac{4acd^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{3c^2d^2\left(a+bx^3\right)^{2/3}}{b}+\frac{2c^3dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```

```
[Out] (3*c^2*d^2*(a + b*x^3)^(2/3))/b + (4*c*d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^
4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
- (4*a*c*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt
[3]*b^(4/3)) + (2*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/
3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) + (d^4*x^5*(1 + (b*x^3)/a)^(1/3)*H
ypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(1/3)) - (c^4
*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3)) + (2*a*c*d^3*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(4/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \left(\frac{c^4}{\sqrt[3]{a+bx^3}} + \frac{4c^3dx}{\sqrt[3]{a+bx^3}} + \frac{6c^2d^2x^2}{\sqrt[3]{a+bx^3}} + \frac{4cd^3x^3}{\sqrt[3]{a+bx^3}} + \frac{d^4x^4}{\sqrt[3]{a+bx^3}}\right)dx$$

$$= c^4\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx + \left(4c^3d\right)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx + \left(6c^2d^2\right)\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx + \left(4cd^3\right)\int \frac{x^3}{\sqrt[3]{a+bx^3}}$$

$$= \frac{3c^2d^2\left(a+bx^3\right)^{2/3}}{b} + \frac{4cd^3x\left(a+bx^3\right)^{2/3}}{3b} + \frac{c^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{c^4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

$$= \frac{3c^2d^2\left(a+bx^3\right)^{2/3}}{b} + \frac{4cd^3x\left(a+bx^3\right)^{2/3}}{3b} + \frac{c^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{4acd^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}} + \frac{2c}{}$$

Mathematica [A] time = 0.46, size = 392, normalized size = 1.26

$$5c\left(3bc^3\sqrt[3]{a+bx^3}\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right) - 4ad^3\sqrt[3]{a+bx^3}\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right) + 2\sqrt[3]{a+bx^3}\left(4ad^3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```

```
[Out] (180*b^(4/3)*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/
3, -((b*x^3)/a)] + 18*b^(4/3)*d^4*x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2
F1[1/3, 5/3, 8/3, -((b*x^3)/a)] + 5*c*(54*a*b^(1/3)*c*d^2 + 24*a*b^(1/3)*d^
3*x + 54*b^(4/3)*c*d^2*x^3 + 24*b^(4/3)*d^3*x^4 + 2*Sqrt[3]*(3*b*c^3 - 4*a*
d^3)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]
] + 2*(-3*b*c^3 + 4*a*d^3)*(a + b*x^3)^(1/3)*Log[1 - (b^(1/3)*x)/(a + b*x^3
)^(1/3)] + 3*b*c^3*(a + b*x^3)^(1/3)*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3
) + (b^(1/3)*x)/(a + b*x^3)^(1/3)] - 4*a*d^3*(a + b*x^3)^(1/3)*Log[1 + (b^(
```

```
2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(90*b^(4/3)*
(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^4}{(b\,x^3+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)c^4+\int\frac{d^4x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^4 + integra
te((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x)/(b*x^3 + a)^(1/3), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^4}{(b\,x^3+a)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x)^4/(a + b*x^3)^(1/3), x)
```

sympy **[A]** time = 5.17, size = 206, normalized size = 0.66

$$6c^2d^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right)+\frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}+\frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\right.}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4/(b*x**3+a)**(1/3),x)
```

```
[Out] 6*c**2*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(
2*b), True)) + c**4*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar
(I*pi)/a)/(3*a**(1/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((1/3, 2/
3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3)) + 4*c*d**3*x*
*4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/
3)*gamma(7/3)) + d**4*x**5*gamma(5/3)*hyper((1/3, 5/3), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(8/3))
```

3.30 $\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}} \, dx$

Optimal. Leaf size=255

$$\frac{ad^3 \log \left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}}-\frac{ad^3 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}-\frac{c^3 \log \left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}+\frac{c^3 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{3c^2dx^2 \sqrt[3]{\ldots}}{\ldots}$$

```
[Out] 3/2*c*d^2*(b*x^3+a)^(2/3)/b+1/3*d^3*x*(b*x^3+a)^(2/3)/b+3/2*c^2*d*x^2*(1+b*
x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1/2*c^3*l
n(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+1/6*a*d^3*ln(-b^(1/3)*x+(b*x^3+a)^(1/
3))/b^(4/3)+1/3*c^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
1/3)*3^(1/2)-1/9*a*d^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/
b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1893, 239, 365, 364, 261, 321}

$$\frac{ad^3 \log \left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}}-\frac{ad^3 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{3c^2dx^2 \sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}-\frac{c^3 \log \left(\sqrt[3]{a+bx^3}-\ldots\right)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```

```
[Out] (3*c*d^2*(a + b*x^3)^(2/3))/(2*b) + (d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^3*
ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) -
(a*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^
(4/3)) + (3*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3
, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) - (c^3*Log[-(b^(1/3)*x) + (a + b*x^3
)^(1/3)])/(2*b^(1/3)) + (a*d^3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*b^
(4/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \int \left(\frac{c^3}{\sqrt[3]{a+bx^3}} + \frac{3c^2dx}{\sqrt[3]{a+bx^3}} + \frac{3cd^2x^2}{\sqrt[3]{a+bx^3}} + \frac{d^3x^3}{\sqrt[3]{a+bx^3}}\right) dx$$

$$= c^3 \int \frac{1}{\sqrt[3]{a+bx^3}}\,dx + \left(3c^2d\right) \int \frac{x}{\sqrt[3]{a+bx^3}}\,dx + \left(3cd^2\right) \int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx + d^3 \int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{3cd^2\left(a+bx^3\right)^{2/3}}{2b} + \frac{d^3x\left(a+bx^3\right)^{2/3}}{3b} + \frac{c^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{c^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} - \frac{(\ldots}{\ldots}$$

$$= \frac{3cd^2\left(a+bx^3\right)^{2/3}}{2b} + \frac{d^3x\left(a+bx^3\right)^{2/3}}{3b} + \frac{c^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{ad^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}} + \frac{3c^2dx^2\ldots}{\ldots}$$

Mathematica [A] time = 0.39, size = 287, normalized size = 1.13

$$\frac{1}{18}\left(\frac{3bc^3\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right) - ad^3\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right) + \left(2ad^3-6bc^3\right)\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right) + 2\sqrt{\ldots}}{b^{4/3}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```

```
[Out] ((27*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*
x^3)/a)])/(a + b*x^3)^(1/3) + (27*b^(1/3)*c*d^2*(a + b*x^3)^(2/3) + 6*b^(1/
3)*d^3*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] + (-6*b*c^3 + 2*a*d^3)*Log[1 - (b^(1/3)
*x)/(a + b*x^3)^(1/3)] + 3*b*c^3*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) +
(b^(1/3)*x)/(a + b*x^3)^(1/3)] - a*d^3*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2
/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/b^(4/3))/18
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x^3+a)^(1/3),x)
[Out] int((d*x+c)^3/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)c^3+\int \frac{d^3x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^3 + integra
te((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x)/(b*x^3 + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x)^3/(a + b*x^3)^(1/3), x)
```

sympy [A] time = 4.31, size = 155, normalized size = 0.61

$$3cd^2\left(\begin{cases} \frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0 \\ \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{2b} & \text{otherwise} \end{cases}\right)+\frac{c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x**3+a)**(1/3),x)
```

```
[Out] 3*c*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*b
), True)) + c**3*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*
pi)/a)/(3*a**(1/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((1/3, 2/3), (
5/3,), b*x**3*exp_polar(I*pi)/a)/(a**(1/3)*gamma(5/3)) + d**3*x**4*gamma(4/
3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/
3))
```

3.31 $\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}} dx$

Optimal. Leaf size=147

$$-\frac{c^2 \log \left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\, x\right)}{2\sqrt[3]{b}}+\frac{c^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{cdx^2 \sqrt[3]{\frac{bx^3}{a}+1}\; {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{d^2 \left(a+bx^3\right)^{2/3}}{2b}$$

```
[Out] 1/2*d^2*(b*x^3+a)^(2/3)/b+c*d*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5
/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1/2*c^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3
)+1/3*c^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/
2)
```

Rubi [A] time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1886, 261, 1893, 239, 365, 364}

$$-\frac{c^2 \log \left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\, x\right)}{2\sqrt[3]{b}}+\frac{c^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{cdx^2 \sqrt[3]{\frac{bx^3}{a}+1}\; {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{d^2 \left(a+bx^3\right)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyperge
ometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[-(b^(
1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = d^2\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx + \int \frac{c^2+2cdx}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b} + \int \left(\frac{c^2}{\sqrt[3]{a+bx^3}} + \frac{2cdx}{\sqrt[3]{a+bx^3}}\right)dx$$
$$= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b} + c^2\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx + (2cd)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b} + \frac{c^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{c^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} + \frac{\left(2cd\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+}}}{\sqrt[3]{a+bx^3}}$$
$$= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b} + \frac{c^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} - \frac{c^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{\right.}}{2\sqrt[3]{b}}$$

Mathematica [A] time = 0.17, size = 201, normalized size = 1.37

$$\frac{c^2\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{6\sqrt[3]{b}} - \frac{c^2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{c^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{cdx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{d^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyperge
ometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[1 - (
b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (c^2*Log[1 + (b^(2/3)*x^2)/(a
+ b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b^(1/3))
```

fricas [F] time = 78.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2x^2+2\,cdx+c^2}{\left(bx^3+a\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)/(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((d*x + c)^2/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x^3+a)^(1/3),x)
[Out] int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)c^2+\int\frac{d^2x^2}{\left(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + integra
te((d^2*x^2 + 2*c*d*x)/(b*x^3 + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x^3)^(1/3),x)
[Out] int((c + d*x)^2/(a + b*x^3)^(1/3), x)
```

sympy [A] time = 3.33, size = 110, normalized size = 0.75

$$d^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right)+\frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{2cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*b), T
rue)) + c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*a**(1/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

3.32 $\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=124

$$-\frac{c\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}+\frac{c\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

```
[Out] 1/2*d*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^
(1/3)-1/2*c*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+1/3*c*arctan(1/3*(1+2*b^
(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1893, 239, 365, 364}

$$-\frac{c\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}+\frac{c\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^(1/3),x]
```

```
[Out] (c*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
 + (d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(2*(a + b*x^3)^(1/3)) - (c*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b
^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \left(\frac{c}{\sqrt[3]{a+bx^3}} + \frac{dx}{\sqrt[3]{a+bx^3}}\right) dx$$
$$= c\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx + d\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{c\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} + \frac{\left(d\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}}$$
$$= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{c\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

Mathematica [A] time = 0.12, size = 163, normalized size = 1.31

$$\frac{1}{6}\left(\frac{c\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{b}}+\frac{3dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\right.}{\sqrt[3]{a+bx^3}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^(1/3),x]
```

```
[Out] ((3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(a + b*x^3)^(1/3) + (c*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3
)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(
2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/b^(1/3))/6
```

fricas [F] time = 35.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dx+c}{\left(bx^3+a\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx+c}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{dx+c}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^(1/3),x)
[Out] int((d*x+c)/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)c+d\int \frac{x}{\left(bx^{3}+a\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + d*integra
te(x/(b*x^3 + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{{\left(b\,x^3+a\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3)^(1/3),x)
[Out] int((c + d*x)/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 2.27, size = 78, normalized size = 0.63

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**(1/3),x)
[Out] c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1
/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

3.33 $\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=333

$$-\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}}+\frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}}-\frac{\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}-\frac{\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a}\right.}{2\sqrt[3]{bc^3-ad^3}}$$

```
[Out] -1/2*d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,1,5/3,-b*x^3/a,-d^3*x^3/c^3)/
c^2/(b*x^3+a)^(1/3)+1/3*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(1/3)-1/2*ln((-a*d^3
+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(1/3)-1/2*ln((-a*d^3+b*c^
3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d^
3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(1/3)*3^(1/2)-1
/3*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3
+b*c^3)^(1/3)*3^(1/2)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
[Out] Defer[Int][1/((c + d*x)*(a + b*x^3)^(1/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
[Out] Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx+c\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
[Out] int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{1}{3}}\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x)),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^{3}}\,\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x**3+a)**(1/3),x)
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)), x)
```

3.34 $\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=761

$$-\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},2;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}}+\frac{d^4x^5\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{1}{3},2;\frac{8}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}}-\frac{cd^3x\left(a+bx^3\right)^{2/3}}{\left(c^3+d^3x^3\right)\left(bc^3-ad^3\right)}+\frac{ad^3}{9c}$$

```
[Out] c^2*d^2*(b*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)-c*d^3*x*(b*x^3+a)^(2/3
)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)-d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,2,5
/3,-b*x^3/a,-d^3*x^3/c^3)/c^3/(b*x^3+a)^(1/3)+1/5*d^4*x^5*(1+b*x^3/a)^(1/3)
*AppellF1(5/3,1/3,2,8/3,-b*x^3/a,-d^3*x^3/c^3)/c^6/(b*x^3+a)^(1/3)+1/6*b*c^
2*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(4/3)+1/9*a*d^3*ln(d^3*x^3+c^3)/c/(-a*d^3+
b*c^3)^(4/3)+1/18*(-2*a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(4/3)
-1/3*a*d^3*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3)^(4
/3)-1/6*(-2*a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(
-a*d^3+b*c^3)^(4/3)-1/2*b*c^2*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-
a*d^3+b*c^3)^(4/3)+2/9*a*d^3*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^
3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(4/3)*3^(1/2)+1/9*(-2*a*d^3+3*b*c^3)*
arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^
3+b*c^3)^(4/3)*3^(1/2)-1/3*b*c^2*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+
b*c^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(4/3)*3^(1/2)
```

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*x^3)^(1/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx$$

Mathematica [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
[Out] int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{1/3}\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x)^2),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(b*x**3+a)**(1/3),x)
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)**2), x)
```

3.35 $\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=1513

$$\frac{2a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)d^6}{9\sqrt{3}\,c^2\left(bc^3-ad^3\right)^{7/3}}+\frac{a^2\log\left(c^3+d^3x^3\right)d^6}{27c^2\left(bc^3-ad^3\right)^{7/3}}-\frac{a^2\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c}-\sqrt[3]{bx^3+a}\right)d^6}{9c^2\left(bc^3-ad^3\right)^{7/3}}+\frac{6x^5\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{1}{3},3;\frac{8}{3};\cdots\right.}{5c^7\sqrt[3]{bx^3+\cdots}}$$

```
[Out] 3/2*c^4*d^2*(b*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2-3/2*c^3*d^3*x*(b
*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+4/3*b*c^4*d^2*(b*x^3+a)^(2/3)/
(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/3*c*d^2*(-3*a*d^3+b*c^3)*(b*x^3+a)^(2/3)/(
-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/18*d^3*(-7*a*d^3+3*b*c^3)*x*(b*x^3+a)^(2/3)
/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/18*d^3*(-5*a*d^3+9*b*c^3)*x*(b*x^3+a)^(2/
3)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-7/18*d^3*(a*d^3+3*b*c^3)*x*(b*x^3+a)^(2/3
)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-3/2*d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1
/3,3,5/3,-b*x^3/a,-d^3*x^3/c^3)/c^4/(b*x^3+a)^(1/3)+6/5*d^4*x^5*(1+b*x^3/a)
^(1/3)*AppellF1(5/3,1/3,3,8/3,-b*x^3/a,-d^3*x^3/c^3)/c^7/(b*x^3+a)^(1/3)+2/
9*b^2*c^4*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(7/3)+1/27*a^2*d^6*ln(d^3*x^3+c^3)
/c^2/(-a*d^3+b*c^3)^(7/3)-1/18*b*c*(-3*a*d^3+b*c^3)*ln(d^3*x^3+c^3)/(-a*d^3
+b*c^3)^(7/3)+7/54*a*d^3*(-a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c^2/(-a*d^3+b*c^3
)^(7/3)+1/54*(5*a^2*d^6-12*a*b*c^3*d^3+9*b^2*c^6)*ln(d^3*x^3+c^3)/c^2/(-a*d
^3+b*c^3)^(7/3)-1/9*a^2*d^6*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c^
2/(-a*d^3+b*c^3)^(7/3)-7/18*a*d^3*(-a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*
x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c^3)^(7/3)-1/18*(5*a^2*d^6-12*a*b*c^3*d^
3+9*b^2*c^6)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c^3
)^(7/3)-2/3*b^2*c^4*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^
3)^(7/3)+1/6*b*c*(-3*a*d^3+b*c^3)*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3)
)/(-a*d^3+b*c^3)^(7/3)+2/27*a^2*d^6*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/
c/(b*x^3+a)^(1/3))*3^(1/2))/c^2/(-a*d^3+b*c^3)^(7/3)*3^(1/2)+7/27*a*d^3*(-a
*d^3+3*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(
1/2))/c^2/(-a*d^3+b*c^3)^(7/3)*3^(1/2)+1/27*(5*a^2*d^6-12*a*b*c^3*d^3+9*b^2
*c^6)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c^
2/(-a*d^3+b*c^3)^(7/3)*3^(1/2)-4/9*b^2*c^4*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3
)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(7/3)*3^(1/2)+1/9*b*c*(-3*a
*d^3+b*c^3)*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2)
)/(-a*d^3+b*c^3)^(7/3)*3^(1/2)
```

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^3*(a + b*x^3)^(1/3)),x]
```

```
[Out] Defer[Int][1/((c + d*x)^3*(a + b*x^3)^(1/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx$$

Mathematica [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^3*(a + b*x^3)^(1/3)),x]
[Out] Integrate[1/((c + d*x)^3*(a + b*x^3)^(1/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^3), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^3\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(b*x^3+a)^(1/3),x)
[Out] int(1/(d*x+c)^3/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{1/3}\,\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(b*x**3+a)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)**3), x)
```

3.36 $\int \frac{(c+dx)^4}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=306

$$-\frac{2c^3 d\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}-\frac{4c^3 d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{ad^4\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}}+\frac{2ad^4\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}}+\cdots$$

```
[Out] 6*c^2*d^2*(b*x^3+a)^(1/3)/b+1/3*d^4*x^2*(b*x^3+a)^(1/3)/b+c^4*x*(1+b*x^3/a)
^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)+c*d^3*x^4*(1+b*
x^3/a)^(2/3)*hypergeom([2/3, 4/3],[7/3],-b*x^3/a)/(b*x^3+a)^(2/3)-2*c^3*d*l
n(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)+1/3*a*d^4*ln(b^(1/3)*x-(b*x^3+a)^(1/3)
)/b^(5/3)-4/3*c^3*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
2/3)*3^(1/2)+2/9*a*d^4*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/
b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 416, normalized size of antiderivative = 1.36, number of steps used = 22, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {1893, 246, 245, 331, 292, 31, 634, 617, 204, 628, 261, 365, 364, 321}

$$-\frac{4c^3 d\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}}+\frac{2c^3 d\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{2/3}}-\frac{4c^3 d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{2ad^4\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

```
[Out] (6*c^2*d^2*(a + b*x^3)^(1/3))/b + (d^4*x^2*(a + b*x^3)^(1/3))/(3*b) - (4*c^
3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)
) + (2*a*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt
[3]*b^(5/3)) + (c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3
, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + (c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hyper
geometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (4*c^3*d*Log
[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(2/3)) + (2*a*d^4*Log[1 - (b^(1/3
)*x)/(a + b*x^3)^(1/3)])/(9*b^(5/3)) + (2*c^3*d*Log[1 + (b^(2/3)*x^2)/(a +
b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(2/3)) - (a*d^4*Log[1 +
 (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*b^(5/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{(c+dx)^4}{\left(a+bx^3\right)^{2/3}}\,dx = \int \left(\frac{c^4}{\left(a+bx^3\right)^{2/3}} + \frac{4c^3dx}{\left(a+bx^3\right)^{2/3}} + \frac{6c^2d^2x^2}{\left(a+bx^3\right)^{2/3}} + \frac{4cd^3x^3}{\left(a+bx^3\right)^{2/3}} + \frac{d^4x^4}{\left(a+bx^3\right)^{2/3}}\right) dx$$

$$= c^4\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx + \left(4c^3d\right)\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx + \left(6c^2d^2\right)\int \frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx + \left(4cd^3\right)\int$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \left(4c^3d\right)\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) - \frac{\left(2ad^4\right)}{}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{cd^3x^4\left(1+\frac{bx^3}{a}\right)}{(a}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{cd^3x^4\left(1+\frac{bx^3}{a}\right)}{(a}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{cd^3x^4\left(1+\frac{bx^3}{a}\right)}{(a}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} - \frac{4c^3d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3}\right.}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} - \frac{4c^3d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}} + \frac{2ad^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}} + \frac{c^4}{}$$

Mathematica [A] time = 0.14, size = 166, normalized size = 0.54

$$\frac{3bc^4x\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+d\left(x^2\left(6bc^3-ad^3\right) {}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{bx^3+a}\right)+d\left(\left(a+bx^3\right)\left(18c^2+d^2x^2\right)+3bcd\right.\right.}{3b\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

```
[Out] (3*b*c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)] + d*((6*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a +
 b*x^3)] + d*((18*c^2 + d^2*x^2)*(a + b*x^3) + 3*b*c*d*x^4*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])))/(3*b*(a + b*x^3)^(2
/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^4}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^4}{\left(b\,x^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^4}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+dx)^4}{\left(bx^3+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(a + b*x^3)^(2/3),x)
[Out] int((c + d*x)^4/(a + b*x^3)^(2/3), x)
```

sympy [A] time = 5.16, size = 204, normalized size = 0.67

$$6c^2d^2\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)+\frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}+\frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\\ \frac{7}{3}\end{matrix}\right.}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4/(b*x**3+a)**(2/3),x)
[Out] 6*c**2*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b
, True)) + c**4*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*a**(2/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((2/3, 2/3),
(5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3)) + 4*c*d**3*x**4*g
amma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*g
amma(7/3)) + d**4*x**5*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*a**(2/3)*gamma(8/3))
```

3.37 $\int \frac{(c+dx)^3}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=187

$$-\frac{3c^2d\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}-\frac{\sqrt{3}\,c^2d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{b^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2bc^3-ad^3\right)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}+\frac{3cd^2\sqrt[3]{a}}{b}$$

```
[Out] 3*c*d^2*(b*x^3+a)^(1/3)/b+1/2*d^3*x*(b*x^3+a)^(1/3)/b+1/2*(-a*d^3+2*b*c^3)*
x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)-
3/2*c^2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-c^2*d*arctan(1/3*(1+2*b^(1/
3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A] time = 0.24, antiderivative size = 239, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1888, 1886, 261, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{c^2d\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{b^{2/3}}+\frac{c^2d\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{2b^{2/3}}-\frac{\sqrt{3}\,c^2d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{b^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2bc^3-ad^3\right)\,{}_2}{2b\left(a+bx^3\right)^{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x^3)^(2/3),x]
```

```
[Out] (3*c*d^2*(a + b*x^3)^(1/3))/b + (d^3*x*(a + b*x^3)^(1/3))/(2*b) - (Sqrt[3]*
c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/b^(2/3) + ((2*
b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((
b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3)) - (c^2*d*Log[1 - (b^(1/3)*x)/(a + b*x^3
)^(1/3)])/b^(2/3) + (c^2*d*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/
3)*x)/(a + b*x^3)^(1/3)])/(2*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{(c+dx)^3}{\left(a+bx^3\right)^{2/3}}\,dx = \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\int \frac{2bc^3-ad^3+6bc^2dx+6bcd^2x^2}{\left(a+bx^3\right)^{2/3}}\,dx}{2b}$$

$$= \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\int \frac{2bc^3-ad^3+6bc^2dx}{\left(a+bx^3\right)^{2/3}}\,dx}{2b} + \left(3cd^2\right)\int \frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\int \left(\frac{2bc^3\left(1-\frac{ad^3}{2bc^3}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{6bc^2dx}{\left(a+bx^3\right)^{2/3}}\right)dx}{2b}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \left(3c^2d\right)\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx + \frac{\left(2bc^3-ad^3\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{2b}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \left(3c^2d\right)\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{\left(\left(2bc^3-ad^3\right.\right.}{}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\left(2bc^3-ad^3\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}} + \frac{\left(c^2d\right)\text{Sub}}{}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\left(2bc^3-ad^3\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}} - \frac{c^2d\log\left(1\right.}{b}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\left(2bc^3-ad^3\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}} - \frac{c^2d\log\left(1\right.}{b}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} - \frac{\sqrt{3}\,c^2d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{\left(2bc^3-ad^3\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1}{2b\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 0.10, size = 145, normalized size = 0.78

$$\frac{4bc^3x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right) + d\left(6bc^2x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{bx^3+a}\right) + d\left(12c\left(a+bx^3\right) + bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3}\right.\right.\right.}{4b\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x^3)^(2/3),x]
```

```
[Out] (4*b*c^3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)] + d*(6*b*c^2*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)] +
 d*(12*c*(a + b*x^3) + b*d*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3,
 4/3, 7/3, -((b*x^3)/a)]))))/(4*b*(a + b*x^3)^(2/3))
```

fricas [F] time = 146.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^3x^3 + 3\,cd^2x^2 + 3\,c^2dx + c^3}{\left(bx^3 + a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)/(b*x^3 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^3}{\left(bx^3 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^3}{\left(b\,x^3 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x+c)^3/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^3}{\left(bx^3 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x)^3}{\left(b\,x^3 + a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x^3)^(2/3),x)
```

[Out] int((c + d*x)^3/(a + b*x^3)^(2/3), x)

sympy [A] time = 4.18, size = 153, normalized size = 0.82

$$3cd^2 \left(\begin{cases} \frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b = 0 \\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise} \end{cases} \right) + \frac{c^3 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{2}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{c^2 d x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{d^3 x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3/(b*x**3+a)**(2/3),x)

[Out] 3*c*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b, T
rue)) + c**3*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*a**(2/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,
), b*x**3*exp_polar(I*pi)/a)/(a**(2/3)*gamma(5/3)) + d**3*x**4*gamma(4/3)*h
yper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))

3.38 $\int \frac{(c+dx)^2}{\left(a+bx^3\right)^{2/3}} \, dx$

Optimal. Leaf size=141

$$-\frac{cd\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}-\frac{2cd\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{c^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\,_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}+\frac{d^2\sqrt[3]{a+bx^3}}{b}$$

```
[Out] d^2*(b*x^3+a)^(1/3)/b+c^2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b
*x^3/a)/(b*x^3+a)^(2/3)-c*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-2/3*c*d*a
rctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 195, normalized size of antiderivative = 1.38, number of steps used = 13, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1886, 261, 1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{2cd\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}}+\frac{cd\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{2/3}}-\frac{2cd\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{c^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\,_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\right.}{\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(1/3))/b - (2*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeomet
ric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (2*c*d*Log[1 - (b^
(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(2/3)) + (c*d*Log[1 + (b^(2/3)*x^2)/(a +
b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{(c+dx)^2}{\left(a+bx^3\right)^{2/3}}\,dx = d^2\int \frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx + \int \frac{c^2+2cdx}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \int \left(\frac{c^2}{\left(a+bx^3\right)^{2/3}} + \frac{2cdx}{\left(a+bx^3\right)^{2/3}}\right)dx$$

$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + c^2\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx + (2cd)\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + (2cd)\,\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{\left(c^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,d}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{(2cd)\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{2cd\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{(cd)\,\text{Subst}\left(\right.}{}$$

$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{2cd\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{cd\log\left(1+\right.}{}$$

$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} - \frac{2cd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{2cd\log\left(1-\right.}{3b^{2/3}}$$

Mathematica [A] time = 0.04, size = 95, normalized size = 0.67

$$\frac{bc^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+d\left(bcx^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{bx^3+a}\right)+d\left(a+bx^3\right)\right)}{b\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (b*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a
)] + d*(d*(a + b*x^3) + b*c*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a +
 b*x^3)]))/(b*(a + b*x^3)^(2/3))
```

fricas [F] time = 54.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2x^2+2\,cdx+c^2}{\left(bx^3+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

[Out] integral((d^2*x^2 + 2*c*d*x + c^2)/(b*x^3 + a)^(2/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")

[Out] integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{(b\,x^3+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2/(b*x^3+a)^(2/3),x)

[Out] int((d*x+c)^2/(b*x^3+a)^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")

[Out] integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{(b\,x^3+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^2/(a + b*x^3)^(2/3),x)

[Out] int((c + d*x)^2/(a + b*x^3)^(2/3), x)

sympy [A] time = 3.33, size = 109, normalized size = 0.77

$$d^2\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for}\ b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)+\frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{2cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2/(b*x**3+a)**(2/3),x)

[Out] d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b, True)
) + c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*a**(2/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))

3.39 $\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx$

Optimal. Leaf size=121

$$-\frac{d\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{cx\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}$$

```
[Out] c*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-
1/2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-1/3*d*arctan(1/3*(1+2*b^(1/3)*x
/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 172, normalized size of antiderivative = 1.42, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1893, 246, 245, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{d\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}}+\frac{d\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{6b^{2/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{cx\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^(2/3),x]
```

```
[Out] -((d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3
))) + (c*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)])/(a + b*x^3)^(2/3) - (d*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(
2/3)) + (d*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3
)^(1/3)])/(6*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^3\right)^{2/3}}\,dx = \int \left(\frac{c}{\left(a+bx^3\right)^{2/3}}+\frac{dx}{\left(a+bx^3\right)^{2/3}}\right)dx$$

$$= c\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx + d\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= d\,\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{\left(c\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} + \frac{d\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} - \frac{d\,\text{Subst}\left(\int \frac{1-}{1+\sqrt[3]{b}\,x}\right.}{3}$$

$$= \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{d\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \right.}{6b^{2/3}}$$

$$= \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{d\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{d\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6b^{2/3}}$$

$$= -\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}} + \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}} - \frac{d\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{d\log\left(1\right.}{}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 0.64

$$\frac{x\left(2c\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{bx^3+a}\right)\right)}{2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(2*c*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)
] + d*x*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)]))/(2*(a + b*x^3
)^(2/3))
```

fricas [F] time = 7.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dx+c}{\left(bx^3+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/(b*x^3 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{\left(bx^3 + a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{\left(b\,x^3 + a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x+c)/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{\left(bx^3 + a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + d\,x}{\left(b\,x^3 + a\right)^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3)^(2/3),x)
```

```
[Out] int((c + d*x)/(a + b*x^3)^(2/3), x)
```

sympy [C] time = 2.31, size = 78, normalized size = 0.64

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{2}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**(2/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2
/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))
```

3.40 $\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=332

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\left(a+bx^3\right)^{2/3}}-\frac{d\log\left(c^3+d^3x^3\right)}{3\left(bc^3-ad^3\right)^{2/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}}+\frac{d\log\left(\sqrt[3]{bc^3-ad^3}\right.}{2\left(bc^3-\right.}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d^3*x^3/c^3)/c/(b*x^3+
a)^(2/3)-1/3*d*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(2/3)+1/2*d*ln((-a*d^3+b*c^3)
^(1/3)*x/c-(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(2/3)+1/2*d*ln((-a*d^3+b*c^3)^(1
/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(2/3)+1/3*d*arctan(1/3*(1+2*(-a*d^3+b
*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(2/3)*3^(1/2)-1/3*
d*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3+
b*c^3)^(2/3)*3^(1/2)
```

Rubi [F] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
[Out] Defer[Int][1/((c + d*x)*(a + b*x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
[Out] Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(b\,x^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
[Out] int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{2/3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x)),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x**3+a)**(2/3),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)), x)
```

3.41 $\int \frac{1}{(c+dx)^2\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=760

$$-\frac{d^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\left(a+bx^3\right)^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\left(a+bx^3\right)^{2/3}}-\frac{d\left(3bc^3-ad^3\right)\log\left(c^3\right.}{9c\left(bc^3-ad^3\right)^{5/}}$$

```
[Out] c^2*d^2*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)+d^4*x^2*(b*x^3+a)^(1/3
)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)+x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,2,4/3,-
b*x^3/a,-d^3*x^3/c^3)/c^2/(b*x^3+a)^(2/3)-1/2*d^3*x^4*(1+b*x^3/a)^(2/3)*App
ellF1(4/3,2/3,2,7/3,-b*x^3/a,-d^3*x^3/c^3)/c^5/(b*x^3+a)^(2/3)-1/3*b*c^2*d*
ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(5/3)-1/9*a*d^4*ln(d^3*x^3+c^3)/c/(-a*d^3+b*
c^3)^(5/3)-1/9*d*(-a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(5/3)+1/
3*a*d^4*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3)^(5/3)
+1/3*d*(-a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*
d^3+b*c^3)^(5/3)+b*c^2*d*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3
+b*c^3)^(5/3)+2/9*a*d^4*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^
(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(5/3)*3^(1/2)+2/9*d*(-a*d^3+3*b*c^3)*arcta
n(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c
^3)^(5/3)*3^(1/2)-2/3*b*c^2*d*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c
^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(5/3)*3^(1/2)
```

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+dx)^2\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)^2\left(a+bx^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(b\,x^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
[Out] int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{2/3}\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x)^2),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(b*x**3+a)**(2/3),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)**2), x)
```

3.42 $\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=1357

$$\frac{d^6\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{7}{3};\frac{2}{3},3;\frac{10}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x^7}{7c^9\left(bx^3+a\right)^{2/3}}-\frac{7d^3\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},3;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x^4}{4c^6\left(bx^3+a\right)^{2/3}}+\frac{d^4\left(3bc^3+2ad^3\right)}{3c\left(bc^3-ad^3\right)^2\left(\right.}$$

```
[Out] 3/2*c^4*d^2*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+3/2*c^2*d^4*x^2*
(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+5/3*b*c^4*d^2*(b*x^3+a)^(1/3
)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/6*c*d^2*(-6*a*d^3+b*c^3)*(b*x^3+a)^(1/3)
/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/6*d^4*(-4*a*d^3+9*b*c^3)*x^2*(b*x^3+a)^(1
/3)/c/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/3*d^4*(2*a*d^3+3*b*c^3)*x^2*(b*x^3+a
)^(1/3)/c/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2
/3,3,4/3,-b*x^3/a,-d^3*x^3/c^3)/c^3/(b*x^3+a)^(2/3)-7/4*d^3*x^4*(1+b*x^3/a)
^(2/3)*AppellF1(4/3,2/3,3,7/3,-b*x^3/a,-d^3*x^3/c^3)/c^6/(b*x^3+a)^(2/3)+1/
7*d^6*x^7*(1+b*x^3/a)^(2/3)*AppellF1(7/3,2/3,3,10/3,-b*x^3/a,-d^3*x^3/c^3)/
c^9/(b*x^3+a)^(2/3)-5/9*b^2*c^4*d*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(8/3)+1/18
*b*c*d*(-6*a*d^3+b*c^3)*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(8/3)-1/9*a*d^4*(-a*
d^3+6*b*c^3)*ln(d^3*x^3+c^3)/c^2/(-a*d^3+b*c^3)^(8/3)-1/18*d*(2*a^2*d^6-6*a
*b*c^3*d^3+9*b^2*c^6)*ln(d^3*x^3+c^3)/c^2/(-a*d^3+b*c^3)^(8/3)+1/3*a*d^4*(-
a*d^3+6*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c
^3)^(8/3)+1/6*d*(2*a^2*d^6-6*a*b*c^3*d^3+9*b^2*c^6)*ln((-a*d^3+b*c^3)^(1/3)
*x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c^3)^(8/3)+5/3*b^2*c^4*d*ln((-a*d^3+b*c
^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(8/3)-1/6*b*c*d*(-6*a*d^3+b*c^3
)*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(8/3)+2/9*a*d^4
*(-a*d^3+6*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))
*3^(1/2))/c^2/(-a*d^3+b*c^3)^(8/3)*3^(1/2)+1/9*d*(2*a^2*d^6-6*a*b*c^3*d^3+9
*b^2*c^6)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2)
)/c^2/(-a*d^3+b*c^3)^(8/3)*3^(1/2)-10/9*b^2*c^4*d*arctan(1/3*(1-2*d*(b*x^3+
a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(8/3)*3^(1/2)+1/9*b*
c*d*(-6*a*d^3+b*c^3)*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3)
)*3^(1/2))/(-a*d^3+b*c^3)^(8/3)*3^(1/2)
```

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^3*(a + b*x^3)^(2/3)),x]
```

```
[Out] Defer[Int][1/((c + d*x)^3*(a + b*x^3)^(2/3)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^3*(a + b*x^3)^(2/3)),x]
[Out] Integrate[1/((c + d*x)^3*(a + b*x^3)^(2/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{2}{3}} (dx + c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^3), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)^3 \left(b\,x^3 + a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(b*x^3+a)^(2/3),x)
[Out] int(1/(d*x+c)^3/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{2}{3}} (dx + c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3 + a\right)^{2/3}\,(c + d\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x)^3),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(b*x**3+a)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)**3), x)
```

3.43 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=37

$$\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*2^(2/3)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2137, 203}

$$\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx &= \left(2\,2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{1+x^3}}\right)\\ &= \frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.44, size = 326, normalized size = 8.81

$$-\frac{4\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}}}{\sqrt{2}\sqrt[4]{3}}\right.\right.\right.}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3) + 4*S
qrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I + Sq
rt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3
) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt
[3] - (2*I)*x]*Sqrt[1 + x^3])
```

fricas [B] time = 1.57, size = 75, normalized size = 2.03

$$\frac{1}{3}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{6}\,2^{\frac{1}{6}}\left(2x^5+2x^2-2^{\frac{2}{3}}\left(7x^4+4x\right)-2^{\frac{1}{3}}\left(5x^3+2\right)\right)\sqrt{x^3+1}}{12\left(2x^6+3x^3+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(6)*2^(1/6)*arctan(-1/12*sqrt(6)*2^(1/6)*(2*x^5 + 2*x^2 - 2^(2/3)*(
7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument
 Value
```

maple [C] time = 0.07, size = 258, normalized size = 6.97

$$-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] -4*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*2^(2/3)*(3/2-1/2*I*3^(1/2))*((
x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2
^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x-2^{\frac{2}{3}}}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

mupad [B] time = 3.59, size = 70, normalized size = 1.89

$$\frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,1\mathrm{i}+\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,1\mathrm{i}-\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x+2^{2/3}\right)^6}\right)1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2^(2/3))/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)
*(3^(1/2)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6)*
1i)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2^{\frac{2}{3}}}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx-\int\frac{2x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Inte
gral(2*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

3.44 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

```
[Out] -2/3*2^(2/3)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2137, 203}

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/Sqrt[3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx &= -\left(\left(2\,2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)\right)\\ &= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.37, size = 327, normalized size = 8.18

$$-\frac{4\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(6i\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{-2ix+\sqrt{3}\cdots}\right.}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{2ix+\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] - (2*I
)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3)
+ 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)
*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (6*I)*Sqrt[3]*Sqrt[I
 + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2
^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (
2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I +
 Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

fricas [B] time = 1.38, size = 76, normalized size = 1.90

$$-\frac{1}{3}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{\sqrt{6}\,2^{\frac{1}{6}}\left(2x^5-2x^2+2^{\frac{2}{3}}\left(7x^4-4x\right)-2^{\frac{1}{3}}\left(5x^3-2\right)\right)\sqrt{-x^3+1}}{12\left(2x^6-3x^3+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(6)*2^(1/6)*arctan(1/12*sqrt(6)*2^(1/6)*(2*x^5 - 2*x^2 + 2^(2/3)*(
7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument V
alue
```

maple [C] time = 0.06, size = 253, normalized size = 6.32

$$\frac{4i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+2*I*2^(2/3)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))
^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/
3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x+2^{\frac{2}{3}}}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

mupad [B] time = 3.63, size = 74, normalized size = 1.85

$$\frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-\sqrt{3}\,1\mathrm{i}+2^{1/3}\sqrt{3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,1\mathrm{i}+\sqrt{1-x^3}-2^{1/3}\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x-2^{2/3}\right)^6}\right)1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 2^(2/3))/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log((((1 - x^3)^(1/2) - 3^(1/2)*1i + 2^(1/3)*3^(1/2)*x*1i)
*(3^(1/2)*1i + (1 - x^3)^(1/2) - 2^(1/3)*3^(1/2)*x*1i)^3)/(x - 2^(2/3))^6)*
1i)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2^{\frac{2}{3}}}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx-\int\frac{2x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integ
ral(2*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

3.45 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

```
[Out] -2/3*2^(2/3)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2137, 206\}$

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx &= -\left(\left(2\,2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1-\sqrt[3]{2}x}{\sqrt{-1+x^3}}\right)\right)\\ &= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.30, size = 325, normalized size = 8.55

$$-\frac{4\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(6i\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{-2ix+\sqrt{3}-i}\right.}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{2ix+\sqrt{\cdots}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] - (2*I
)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3)
+ 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)
*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (6*I)*Sqrt[3]*Sqrt[I
 + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2
^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (
2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I +
 Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

fricas [B] time = 1.39, size = 238, normalized size = 6.26

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\log\left(\frac{x^{18}+1440\,x^{15}+17400\,x^{12}-21056\,x^{9}-10368\,x^{6}+15360\,x^{3}+2\sqrt{6}\,2^{\frac{1}{6}}\left(126\,x^{14}+2664\,x^{11}-46\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 10368*
x^6 + 15360*x^3 + 2*sqrt(6)*2^(1/6)*(126*x^14 + 2664*x^11 - 4608*x^5 + 2304
*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 512*x) +
 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512))*sqrt
(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^5 - 6
4*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4 - 128*
x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 40
96))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument V
alue
```

maple [C] time = 0.06, size = 262, normalized size = 6.89

$$-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{6\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-6*2^(2/3)*(-3/2-1/2*I*3^(1/2))*((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2
^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))
/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,x + 2^{\frac{2}{3}}}{\sqrt{x^3-1}\left(x - 2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

mupad [B] time = 2.86, size = 62, normalized size = 1.63

$$\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{x^3-1}-\sqrt{3}+2^{1/3}\,\sqrt{3}\,x\right)^3\left(\sqrt{3}+\sqrt{x^3-1}-2^{1/3}\,\sqrt{3}\,x\right)}{\left(x-2^{2/3}\right)^6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 2^(2/3))/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log((((x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(3^
(1/2) + (x^3 - 1)^(1/2) - 2^(1/3)*3^(1/2)*x))/(x - 2^(2/3))^6))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{\frac{2}{3}}}{x\sqrt{x^3-1} - 2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx - \int \frac{2x}{x\sqrt{x^3-1} - 2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integ
ral(2*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

3.46 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=39

$$\frac{2\, 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*2^(2/3)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2137, 206}

$$\frac{2\, 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx &= \left(2\, 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{-1-x^3}}\right)\\ &= \frac{2\, 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [C] time = 0.29, size = 328, normalized size = 8.41

$$-\frac{4\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+}}{\sqrt{2}}\right.\right.\right.}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3) + 4*S
qrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I + Sq
rt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3
) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt
[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

fricas [B] time = 1.07, size = 241, normalized size = 6.18

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\log\left(\frac{x^{18} - 1440\,x^{15} + 17400\,x^{12} + 21056\,x^{9} - 10368\,x^{6} - 15360\,x^{3} - 2\sqrt{6}\,2^{\frac{1}{6}}\left(126\,x^{14} - 2664\,x^{11} + 4608\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 10368*
x^6 - 15360*x^3 - 2*sqrt(6)*2^(1/6)*(126*x^14 - 2664*x^11 + 4608*x^5 + 2304
*x^2 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 512*x) -
 2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512))*sqrt
(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 +
64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 128
*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4
096))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument
 Value
```

maple [C] time = 0.06, size = 249, normalized size = 6.38

$$\frac{4i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2i2^{\frac{2}{3}}\sqrt{3}\ \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] 4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))-2*I*2^(2/3)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
```

```
x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2)),(
I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,x - 2^{\frac{2}{3}}}{\sqrt{-x^3 - 1}\left(x + 2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

mupad [B] time = 2.84, size = 63, normalized size = 1.62

$$\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}+\sqrt{-x^3-1}+2^{1/3}\,\sqrt{3}\,x\right)^3\,\left(\sqrt{3}-\sqrt{-x^3-1}+2^{1/3}\,\sqrt{3}\,x\right)}{\left(x+2^{2/3}\right)^6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2^(2/3))/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2) + (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(
3^(1/2) - (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x))/(x + 2^(2/3))^6))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{2^{\frac{2}{3}}}{x\sqrt{-x^3-1} + 2^{\frac{2}{3}}\sqrt{-x^3-1}}\right)dx - \int \frac{2x}{x\sqrt{-x^3-1} + 2^{\frac{2}{3}}\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x) - In
tegral(2*x/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x)
```

3.47 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=63

$$\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2/3*2^(2/3)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(1
/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2137, 203}

$$\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a +
 b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx &= \frac{\left(2\,2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+3ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}\\ &= \frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 1.12, size = 325, normalized size = 5.16

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{2\sqrt[4]{3}\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}-\frac{3\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\ldots\right.}{\sqrt[3]{-1}+2\ldots}\right.}{\sqrt{3}\sqrt[3]{b}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sq
rt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((2*3^(1/4)*((-1)
^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*Ellipt
icF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))
]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^
(1/3))] - (3*(-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*
x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^
(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/(Sqrt[3]*b^(1/3)*Sqrt[a + b*x
^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{-2b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3+a)^(1/
2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3+a)^(1/
2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^
(2/3)*a^(1/3))), x)
```

mupad [B] time = 5.81, size = 106, normalized size = 1.68

$$\frac{2^{2/3}\sqrt{3}\ \ln\left(\frac{\left(\sqrt{3}\ \sqrt{a}\ 1\mathrm{i}-\sqrt{b\,x^3+a}+2^{1/3}\,\sqrt{3}\ a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)^3\left(\sqrt{3}\ \sqrt{a}\ 1\mathrm{i}+\sqrt{b\,x^3+a}+2^{1/3}\,\sqrt{3}\ a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\right)1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b
^(1/3)*x)),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2)*a^(1/2)*1i - (a + b*x^3)^(1/2) + 2^(1/3)*3^(
1/2)*a^(1/6)*b^(1/3)*x*1i)^3*(3^(1/2)*a^(1/2)*1i + (a + b*x^3)^(1/2) + 2^(1
/3)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6)*1i)/(3*
a^(1/6)*b^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}\,x\sqrt{a+bx^3}}\right)dx-\int\frac{2\sqrt[3]{b}\,x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}\,x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(
b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)
*x*sqrt(a + b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(a
+ b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x)
```

3.48 $\int \frac{2^{2/3} \sqrt[3]{a}+2 \sqrt[3]{b} x}{\left(2^{2/3} \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx$

Optimal. Leaf size=65

$$-\frac{2\, 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{2} \sqrt[3]{b} x\right)}{\sqrt{a-b x^3}}\right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

```
[Out] -2/3*2^(2/3)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)^
(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2137, 203}

$$-\frac{2\, 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{2} \sqrt[3]{b} x\right)}{\sqrt{a-b x^3}}\right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a -
 b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a -
 b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2^{2/3} \sqrt[3]{a}+2 \sqrt[3]{b} x}{\left(2^{2/3} \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx &= -\frac{\left(2\, 2^{2/3} \sqrt[3]{a}\right) \text{Subst}\left(\int \frac{1}{1+3 a x^2} \, dx, x, \frac{1-\frac{\sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{a}}}{\sqrt{a-b x^3}}\right)}{\sqrt[3]{b}}\\
&= -\frac{2\, 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{2} \sqrt[3]{b} x\right)}{\sqrt{a-b x^3}}\right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}
\end{aligned}$$

Mathematica [C] time = 1.13, size = 336, normalized size = 5.17

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{2\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}+\frac{\sqrt[3]{-1}\,2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{3b^{2/3}x^2}{a^{2/3}}+\frac{3\sqrt[3]{b}x}{\sqrt[3]{a}}+3\ldots}}{\sqrt[3]{-1}\ldots}\right.}{\sqrt[3]{b}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2*((-1)^(1/3)*
a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1
 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3
))*a^(1/3)*Sqrt[3 + (3*b^(1/3)*x)/a^(1/3) + (3*b^(2/3)*x^2)/a^(2/3)]*Ellipt
icPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3)
)))/(b^(1/3)*Sqrt[a - b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{2b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{-b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3+a)^(
1/2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3+a)^(
1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2
^(2/3)*a^(1/3))), x)
```

mupad [B] time = 5.85, size = 107, normalized size = 1.65

$$\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+\sqrt{a-b\,x^3}-2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)^3}{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)^6}\right)1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b
^(1/3)*x)),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log((((a - b*x^3)^(1/2) - 3^(1/2)*a^(1/2)*1i + 2^(1/3)*3^(
1/2)*a^(1/6)*b^(1/3)*x*1i)*(3^(1/2)*a^(1/2)*1i + (a - b*x^3)^(1/2) - 2^(1/3
)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6)*1i)/(3*
a^(1/6)*b^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx-\int \frac{2\sqrt[3]{b}x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(
-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)
*x*sqrt(a - b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt(a
 - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

3.49 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] -2/3*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)^
(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2137, 206}

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a
+ b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx &= -\frac{\left(2\,2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx,x,\frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}\\ &= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.47, size = 390, normalized size = 5.91

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(2\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\cdots}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[-a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(2*((-1)^(1/3) +
 2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)
^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/
3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - (-1)
^(1/3)*2^(2/3)*Sqrt[3]*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2
/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqr
t[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)
]))/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{2b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b^(1/3)*x+2^(2/3)*a^(1/3))/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3-a)^(1
/2),x)
```

```
[Out] int((2*b^(1/3)*x+2^(2/3)*a^(1/3))/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3-a)^(1
/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\, b^{\frac{1}{3}} x + 2^{\frac{2}{3}} a^{\frac{1}{3}}}{\sqrt{bx^3 - a}\left(b^{\frac{1}{3}} x - 2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^
(2/3)*a^(1/3))), x)
```

mupad [B] time = 3.68, size = 102, normalized size = 1.55

$$\frac{\sqrt{3}\, 4^{1/3} \ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{3}\,\sqrt{a}-2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)\left(\sqrt{b\,x^3-a}-\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)^3}{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)^6}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b
^(1/3)*x)),x)
```

```
[Out] (3^(1/2)*4^(1/3)*log((((b*x^3 - a)^(1/2) + 3^(1/2)*a^(1/2) - 2^(1/3)*3^(1/2
)*a^(1/6)*b^(1/3)*x)*((b*x^3 - a)^(1/2) - 3^(1/2)*a^(1/2) + 2^(1/3)*3^(1/2)
*a^(1/6)*b^(1/3)*x)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6))/(3*a^(1/6)*b^(1/3)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a + bx^3} + \sqrt[3]{b}\,x\sqrt{-a + bx^3}}\, dx - \int \frac{2\sqrt[3]{b}\,x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a + bx^3} + \sqrt[3]{b}\,x\sqrt{-a + bx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(
b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3
)*x*sqrt(-a + b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt
(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

3.50 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2/3*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a)^
(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2137, 206}

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a
- b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
- b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx &= \frac{\left(2\,2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}\\ &= \frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.68, size = 375, normalized size = 5.68

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\sqrt[3]{-1}\,2^{2/3}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\cdots}{\left(1+\sqrt[3]{-1}\right)\cdots}}\cdots\right.\right.\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sq
rt[-a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2*((-1)^(1/3)
 + 2^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x
)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]], (-1)^(1/3)])/3^(1/4) + (-1)^(1/3)*2^(2/3)*Sqrt[3]*(1 +
 (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticP
i[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]))/(((-1)^(1/3) + 2^(2/3))
*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*
Sqrt[-a - b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{-2b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{-b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*b^(1/3)*x+2^(2/3)*a^(1/3))/(b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3-a)^(
1/2),x)
```

```
[Out] int((-2*b^(1/3)*x+2^(2/3)*a^(1/3))/(b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3-a)^(
1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,b^{\frac{1}{3}}x - 2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3 - a}\left(b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2
^(2/3)*a^(1/3))), x)
```

mupad [B] time = 3.65, size = 103, normalized size = 1.56

$$\frac{\sqrt{3}\,4^{1/3}\,\ln\left(\frac{\left(\sqrt{-b\,x^3-a}+\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)^3\left(\sqrt{3}\,\sqrt{a}-\sqrt{-b\,x^3-a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) +
 b^(1/3)*x)),x)
```

```
[Out] (3^(1/2)*4^(1/3)*log((((- a - b*x^3)^(1/2) + 3^(1/2)*a^(1/2) + 2^(1/3)*3^(1
/2)*a^(1/6)*b^(1/3)*x)^3*(3^(1/2)*a^(1/2) - (- a - b*x^3)^(1/2) + 2^(1/3)*3
^(1/2)*a^(1/6)*b^(1/3)*x))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6))/(3*a^(1/6)*b^(
1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}\,x\sqrt{-a-bx^3}}\right)dx-\int\frac{2\sqrt[3]{b}\,x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}\,x\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(
-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3
)*x*sqrt(-a - b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(
-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x)
```

3.51 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=49

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}\,d}$$

```
[Out] 2/3*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/d*3^(1/2)/c^(1/
2)
```

Rubi [A] time = 0.12, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2137, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(Sqrt[3]*Sq
rt[c]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx &= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{1+3c^3x^2}\,dx, x, \frac{1+\frac{2dx}{c}}{\sqrt{c^3+4d^3x^3}}\right)}{d}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}\,d}\end{aligned}$$

Mathematica [C] time = 1.10, size = 373, normalized size = 7.61

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(2\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)\middle|\sqrt[3]{-1}\right)-\sqrt[3]{\cdots}\right.}{\left(2+\sqrt[3]{-2}\right)d\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/\cdots}}{\left(1+\sqrt[3]{-1}\right)\cdots}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(2*Sqrt[((-2)^(1/3)
*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))*c
 - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^
(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)] - (-1)^(1/3)*2^(2/3)
*Sqrt[3]*(1 + (-1)^(1/3))*c*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^
(1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(
I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*
d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]))/((2 + (-2)^(1/3))*d*Sqrt
[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 + 4*d^3*x^3]
)
```

fricas [B] time = 1.04, size = 300, normalized size = 6.12

$$\left[\frac{\sqrt{3}\sqrt{-\frac{1}{c}}\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6-\sqrt{3}\left(4\,cd^4x^4-10\,c^2d^3x^3-18\,c^3d^2x^2-8\,c^4dx-c^5\right)\sqrt{4\,d^3x^3+c^3}\,\sqrt{-\frac{1}{c}}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{6\,d}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(3)*sqrt(-1/c)*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4 + 28
*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*c*d^4*x^4 - 10*c^2*d^3*x^3
 - 18*c^3*d^2*x^2 - 8*c^4*d*x - c^5)*sqrt(4*d^3*x^3 + c^3)*sqrt(-1/c))/(d^6
*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c
^5*d*x + c^6))/d, -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*
d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)/((8*d^4*x^4 + 4*c*d^3*x^3 + 2*c^3*
d*x + c^4)*sqrt(c)))/(sqrt(c)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,dx - c}{\sqrt{4\,d^3x^3 + c^3}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

maple [C] time = 0.07, size = 889, normalized size = 18.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] -4*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1
/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*
3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1/
2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(
1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/
2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+
c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(
```

```
1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2
^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/
2))+6*c/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/
2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)
-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)
*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*
c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*
I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d
^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(
((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^
(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)+1/
4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*
c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^
(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,dx-c}{\sqrt{4\,d^3x^3+c^3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

mupad [B] time = 4.73, size = 95, normalized size = 1.94

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(-\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2\mathrm{i}\right)^3\,\left(\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2\mathrm{i}\right)}{(c+d\,x)^6}\right)\,1\mathrm{i}}{3\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - 2*d*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] (3^(1/2)*log(((3^(1/2)*c^(3/2)*1i - (c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(1/
2)*d*x*2i)^3*((c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(3/2)*1i + 3^(1/2)*c^(1/2
)*d*x*2i))/(c + d*x)^6)*1i)/(3*c^(1/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{c}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\right)dx-\int\frac{2dx}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] -Integral(-c/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**3)), x
) - Integral(2*d*x/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**
3)), x)
```

3.52 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=158

$$\frac{2\left(2-3\,2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] 2/9*(2-3*2^(2/3))*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*(
3+2*2^(1/3))*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/
2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1
/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2139, 218, 2137, 203}

$$\frac{2\left(2-3\,2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(2 - 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt
[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{3}\left(-3+\sqrt[3]{2}\right)\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx+\frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{1}{3}\left(2\left(2\right.\right.$$

$$= \frac{2\left(2-3\,2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.52, size = 336, normalized size = 2.13

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(3\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right.\right.\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2ix}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^(1/3
))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I
+ (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqrt[3
])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(3\,x^{3}+2\,x^{2}-2^{\frac{2}{3}}\left(3\,x^{2}+2\,x\right)+2\cdot 2^{\frac{1}{3}}(3\,x+2)\right)\sqrt{x^{3}+1}}{x^{6}+5\,x^{3}+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^3 + 2*x^2 - 2^(2/3)*(3*x^2 + 2*x) + 2*2^(1/3)*(3*x + 2))*sqrt
(x^3 + 1)/(x^6 + 5*x^3 + 4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument
 Value
```

maple [B] time = 0.04, size = 262, normalized size = 1.66

$$\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(2-3\,2^{\frac{2}{3}}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] 6*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(2-3*2^(2/3))*(3/2-1/2*I*3^(1/2
))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/
2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(
1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x+2}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x+2}{\sqrt{x^3+1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] int((3*x + 2)/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x+2}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((3*x + 2)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.53 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=173

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\left(2+3\,2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/9*(2+3*2^(2/3))*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)+2/9
*(3-2*2^(1/3))*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)
^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2139, 218, 2137, 203}

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\left(2+3\,2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqr
t[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\left(\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$$

$$= \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}\left(2\left(\right.\right.$$

$$= -\frac{2\left(2+3\,2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1}}$$

Mathematica [C] time = 0.48, size = 335, normalized size = 1.94

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(4\sqrt{3}\left(3+\sqrt[3]{2}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-3i\sqrt{-\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*(3 +
 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[
3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(3\,x^3+2\,x^2+2^{\frac{2}{3}}\left(3\,x^2+2\,x\right)+2\cdot 2^{\frac{1}{3}}(3\,x+2)\right)\sqrt{-x^3+1}}{x^6-5\,x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^3 + 2*x^2 + 2^(2/3)*(3*x^2 + 2*x) + 2*2^(1/3)*(3*x + 2))*sqrt
(-x^3 + 1)/(x^6 - 5*x^3 + 4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument V
alue
```

maple [A] time = 0.04, size = 257, normalized size = 1.49

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}+1}}-\frac{2i\left(-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1
/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2
*I*3^(1/2)))^(1/2))-2/3*I*(-2-3*2^(2/3))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2
)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3\,x+2}{\sqrt{-x^{3}+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{3\,x+2}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + 2)/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] int(-(3*x + 2)/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3x}{x\sqrt{1-x^{3}}-2^{\frac{2}{3}}\sqrt{1-x^{3}}}\,dx-\int \frac{2}{x\sqrt{1-x^{3}}-2^{\frac{2}{3}}\sqrt{1-x^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(3*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(2
/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

3.54 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=176

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\left(2+3\,2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right.}{\sqrt{x^3-1}}\right.}{3\sqrt{3}}$$

```
[Out] -2/9*(2+3*2^(2/3))*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)+2/9
*(3-2*2^(1/3))*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^
(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2139, 219, 2137, 206}

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\left(2+3\,2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right.}{\sqrt{x^3-1}}\right.}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*S
qrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\left(\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}-\frac{1}{3}\left(2\left(2\right.\right.$$

$$= -\frac{2\left(2+3\,2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}+\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-}}$$

Mathematica [C] time = 0.33, size = 333, normalized size = 1.89

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(4\sqrt{3}\left(3+\sqrt[3]{2}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-3i\sqrt{-2ix}\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{2ix}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*(3 +
 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[
3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument V
alue
```

maple [A] time = 0.04, size = 266, normalized size = 1.51

$$-\frac{6\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-2-3\,2^{\frac{2}{3}}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -6*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-2-3*2^(2/3))*(-3/2-1/2*I*3^(1
/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1
/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3\,x+2}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{3\,x+2}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + 2)/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] int(-(3*x + 2)/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx-\int \frac{2}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(3*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(2
/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

3.55 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=169

$$\frac{2\left(2-3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 2/9*(2-3*2^(2/3))*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9
*(3+2*2^(1/3))*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)
^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2139, 219, 2137, 206}

$$\frac{2\left(2-3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(2 - 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sq
rt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x
^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{1}{3}\left(-3+\sqrt[3]{2}\right)\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx + \frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{1}{3}(\ldots$$

$$= \frac{2\left(2-3\,2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\ldots}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\ldots}$$

Mathematica [C] time = 0.34, size = 338, normalized size = 2.00

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(3\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\ldots}}{\sqrt{2}\sqrt[4]{3}}\ldots\right.\right.\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2i\ldots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^(1/3
))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I
+ (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqrt[3
])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(3\,x^3+2\,x^2-2^{\frac{2}{3}}\left(3\,x^2+2\,x\right)+2\cdot 2^{\frac{1}{3}}(3\,x+2)\right)\sqrt{-x^3-1}}{x^6+5\,x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*x^3 + 2*x^2 - 2^(2/3)*(3*x^2 + 2*x) + 2*2^(1/3)*(3*x + 2))*sqr
t(-x^3 - 1)/(x^6 + 5*x^3 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

maple [A] time = 0.04, size = 253, normalized size = 1.50

$$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}-1}}-\frac{2 i \left(2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1
/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*
I*3^(1/2)))^(1/2))-2/3*I*(2-3*2^(2/3))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(
1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/
2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(
1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x+2}{\sqrt{-x^{3}-1}\,\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x+2}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] int((3*x + 2)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x+2}{\sqrt{-\left(x+1\right)\left(x^{2}-x+1\right)}\,\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((3*x + 2)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.56 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=159

$$\frac{2\left(e-2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] 2/9*(e-2^(2/3)*f)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*(
2^(1/3)*e+f)*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/
2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1
/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2139, 218, 2137, 203}

$$\frac{2\left(e-2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(e - 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt
[3]) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx + \frac{1}{3}\left(\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(2\left(e\right.\right.$$

$$= \frac{2\left(e-2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.47, size = 340, normalized size = 2.14

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(f\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+}}{\sqrt{2}\sqrt[4]{3}}\right.\right.\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2ix}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)*e -
 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/
(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqr
t[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

maple [B] time = 0.04, size = 264, normalized size = 1.66

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, f \operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-2^{\frac{2}{3}}f\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(e-2^(2/3)*f)*(3/2-1/2*I*3^(1
/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(
1/2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3
^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e+f\,x}{\sqrt{x^3+1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int((e + f*x)/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.57 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=175

$$-\frac{2\left(e+2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/9*(e+2^(2/3)*f)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)-2/9
*(2^(1/3)*e-f)*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)
^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2139, 218, 2137, 203}

$$-\frac{2\left(e+2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqr
t[3]) - (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\left(\frac{1}{3}\left(-\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{6}\left(\sqrt[3]{2}e+2f\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{1}{3}\left(2\right.$$

$$= -\frac{2\left(e+2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\mathrm{s}\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-}$$

Mathematica [C] time = 0.50, size = 340, normalized size = 1.94

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\sqrt[3]{2}e+2f\right)\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-if\sqrt{\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*(2^(
1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*S
qrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/
3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(fx^3+ex^2+2^{\frac{2}{3}}\left(fx^2+ex\right)+2\cdot 2^{\frac{1}{3}}\left(fx+e\right)\right)\sqrt{-x^3+1}}{x^6-5x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + 2^(2/3)*(f*x^2 + e*x) + 2*2^(1/3)*(f*x + e))*sqrt
(-x^3 + 1)/(x^6 - 5*x^3 + 4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument V
alue
```

maple [A] time = 0.04, size = 261, normalized size = 1.49

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,f\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2i\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-e-2^(2/3)*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^
(1/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{-x^{3}+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{e+f\,x}{\sqrt{1-x^{3}}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] int(-(e + f*x)/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{x\sqrt{1-x^{3}}-2^{\frac{2}{3}}\sqrt{1-x^{3}}}\,dx-\int \frac{fx}{x\sqrt{1-x^{3}}-2^{\frac{2}{3}}\sqrt{1-x^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(f*x
/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

3.58 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=178

$$-\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] -2/9*(e+2^(2/3)*f)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)-2/9
*(2^(1/3)*e-f)*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^
(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2139, 219, 2137, 206}

$$-\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*S
qrt[3]) - (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\left(\frac{1}{3}\left(-\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{6}\left(\sqrt[3]{2}e+2f\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{1}{3}\left(2\left(\right.\right.$$

$$= -\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\mathrm{s}\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1}}$$

Mathematica [C] time = 0.35, size = 338, normalized size = 1.90

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\sqrt[3]{2}e+2f\right)\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-if\sqrt{-2}\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{2ix}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*(2^(
1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*S
qrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/
3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(fx^3+ex^2+2^{\frac{2}{3}}\left(fx^2+ex\right)+2\cdot 2^{\frac{1}{3}}\left(fx+e\right)\right)\sqrt{x^3-1}}{x^6-5\,x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(f*x^3 + e*x^2 + 2^(2/3)*(f*x^2 + e*x) + 2*2^(1/3)*(f*x + e))*sqr
t(x^3 - 1)/(x^6 - 5*x^3 + 4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument V
alue
```

maple [A] time = 0.04, size = 270, normalized size = 1.52

$$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, f\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-e-2^{\frac{2}{3}}f\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-e-2^(2/3)*f)*(-3/2-1/2*I*3^
(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I
*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^
(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{e+f\,x}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] int(-(e + f*x)/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx-\int \frac{fx}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(f*x
/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

3.59 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=170

$$\frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 2/9*(e-2^(2/3)*f)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9
*(2^(1/3)*e+f)*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)
^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2139, 219, 2137, 206}

$$\frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(e - 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sq
rt[3]) + (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x
^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx+\frac{1}{3}\left(\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{1}{3}($$

$$= \frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-}}$$

Mathematica [C] time = 0.46, size = 342, normalized size = 2.01

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(f\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{}}}{\sqrt{2}\sqrt[4]{}}\right.\right.\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)*e -
 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/
(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqr
t[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(fx^3+ex^2-2^{\frac{2}{3}}\left(fx^2+ex\right)+2\cdot 2^{\frac{1}{3}}\left(fx+e\right)\right)\sqrt{-x^3-1}}{x^6+5x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(f*x^3 + e*x^2 - 2^(2/3)*(f*x^2 + e*x) + 2*2^(1/3)*(f*x + e))*sqr
t(-x^3 - 1)/(x^6 + 5*x^3 + 4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument
 Value
```

maple [A] time = 0.04, size = 255, normalized size = 1.50

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,f\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2i\left(e-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2^(2/3)*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))
*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3
^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I
*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^{3}-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e+f\,x}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{-\left(x+1\right)\left(x^{2}-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.60 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=316

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(\sqrt[3]{\cdots}\right.}{}$$

```
[Out] 2/9*(b^(1/3)*e-2^(2/3)*a^(1/3)*f)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x
)*3^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)+2/9*(2^(1/3)*b^(1/3)*e+a
^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)*3^(3/4)/a^(1/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2139, 218, 2137, 203}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(\sqrt[3]{\cdots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/
3)*b^(1/3)*x))/Sqrt[a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 +
Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx+\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{}{\sqrt{a-}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$= \frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{}\right.}{}$$

Mathematica [C] time = 1.54, size = 336, normalized size = 1.06

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2^{2/3}\sqrt[3]{a}f-\sqrt[3]{b}e\right)\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}-\frac{\sqrt[4]{3}f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{}}{}\right.}{\sqrt{3}b^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((3^(1/4)*f*((-
1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*Elli
pticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]) + ((-1)^(1/3)*(1 + (-1)^(1/3))*(-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f
)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[
3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((
1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/(Sqrt[3]*
b^(2/3)*Sqrt[a + b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{bx^{3}+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{b\,x^{3}+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

```
[Out] int((e + f*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{a+bx^{3}}\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

3.61 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=324

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(2^{2/3}\ldots\right.}{}$$

```
[Out] -2/9*(b^(1/3)*e+2^(2/3)*a^(1/3)*f)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*
x)*3^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)-2/9*(2^(1/3)*b^(1/3)*e
-a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(
(a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x
)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2139, 218, 2137, 203}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(2^{2/3}\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1
/3)*b^(1/3)*x))/Sqrt[a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 +
 Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/
3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)
*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x
^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\left(\frac{1}{3}\left(-\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx\right)+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}\ \cdots}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\cdots\right.}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\cdots}{\left(1+\sqrt{3}\right)\cdots}\right.\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$= -\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\,b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\cdots\right.}{\cdots}$$

Mathematica [C] time = 1.28, size = 399, normalized size = 1.23

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)+\cdots\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\cdots}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1
)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]) + ((
-1)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) -
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1
/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]],
 (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}} x+2^{\frac{2}{3}} a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{-bx^{3}+a}\left(b^{\frac{1}{3}} x-2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int((e + f*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{a-bx^{3}}+\sqrt[3]{b} x \sqrt{a-bx^{3}}}\, dx-\int \frac{fx}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{a-bx^{3}}+\sqrt[3]{b} x \sqrt{a-bx^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*s
qrt(a - b*x**3)), x)
```

3.62 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=333

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(2^{2/3}\right.}{}$$

```
[Out] -2/9*(b^(1/3)*e+2^(2/3)*a^(1/3)*f)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)
*x)*3^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)-2/9*(2^(1/3)*b^(1/3)*e
-a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/
(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^
(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x
)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {2139, 219, 2137, 206}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(2^{2/3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(
1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) - (2*Sqrt[2
 - Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(
1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
 + b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\left(\frac{1}{3}\left(-\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{-a+bx^3}}\,dx\right)+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int\frac{2^{2/3}}{\left(2^{2/3}\sqrt[3]{a}-\right.}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1+}{(1-}\right.\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$= -\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\,b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\right.}{}$$

Mathematica [C] time = 0.43, size = 400, normalized size = 1.20

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)+\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1
)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]) + ((
-1)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) -
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1
/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]],
 (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{bx^{3}-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int((e + f*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^{3}}+\sqrt[3]{b}\,x\sqrt{-a+bx^{3}}}\,dx-\int \frac{fx}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^{3}}+\sqrt[3]{b}\,x\sqrt{-a+bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(e/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*
x*sqrt(-a + b*x**3)), x)
```

3.63 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=329

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(\sqrt[3]{b}e\right.}{}$$

```
[Out] 2/9*(b^(1/3)*e-2^(2/3)*a^(1/3)*f)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*
x)*3^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)+2/9*(2^(1/3)*b^(1/3)*e
+a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/
2))*3^(3/4)/a^(1/3)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2139, 219, 2137, 206}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(\sqrt[3]{b}e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1
/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) + (2*Sqrt[2
- Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a
- b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int\frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx+\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int$$

$$=\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{}\right.}{\left(1-\sqrt{}\right.}\right.\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$=\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\right.}{}$$

Mathematica [C] time = 1.03, size = 387, normalized size = 1.18

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2^{2/3}\sqrt[3]{a}f-\sqrt[3]{b}e\right)\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{3}}-\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3) +
 2^(2/3))*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x
)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]], (-1)^(1/3)])/3^(1/4)) + ((-1)^(1/3)*(1 + (-1)^(1/3))*(
-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/
3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (
-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/(
((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{-b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-bx^{3}-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

```
[Out] int((e + f*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{-a-bx^{3}}\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

3.64 $\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=265

$$\frac{2(de-cf)\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

```
[Out] 2/9*(-c*f+d*e)*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/c^(3
/2)/d^2*3^(1/2)+1/9*2^(1/3)*(c*f+2*d*e)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3)*
d*x+c*(1-3^(1/2)))/(2^(2/3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((c^2-2^(2/3)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/c/d^2/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(2^(2/
3)*d*x+c*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2139, 218, 2137, 203}

$$\frac{2(de-cf)\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*(d*e - c*f)*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])
/(3*Sqrt[3]*c^(3/2)*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c + 2^
(2/3)*d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c
+ 2^(2/3)*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sq
rt[3])*c + 2^(2/3)*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d^2*Sqrt[(c*(c + 2^
(2/3)*d*x))/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
```

```
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3cd} + \frac{(2de+cf)\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3cd}$$

$$= \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(2de+cf)\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|\right.}{3\sqrt[4]{3}\,cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$= \frac{2(de-cf)\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\,c^{3/2}d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(2de+cf)\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+}{\left(\left(1+\sqrt{3}\right)c+\right.}}}{3\sqrt[4]{3}\,cd^2\sqrt{\frac{c\left(c+2^{2/3}d\right.}{\left(\left(1+\sqrt{3}\right)c+2^2\right.}}}$$

Mathematica [C] time = 1.49, size = 380, normalized size = 1.43

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-f\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)|\sqrt[3]{-1}\right)+\right.}{\left(2+\sqrt[3]{-2}\right)d^2\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3\ldots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(f*Sqrt[((-2)^(1/
3)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))
*c - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1
)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2^(
2/3)*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((
1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*Ell
ipticPi[(I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-
1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3]))/((2 +
(-2)^(1/3))*d^2*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*S
qrt[c^3 + 4*d^3*x^3])
```

fricas [F] time = 1.34, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4\,d^3x^3+c^3}\,\left(fx+e\right)}{4\,d^4x^4+4\,cd^3x^3+c^3dx+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*d^3*x^3 + c^3)*(f*x + e)/(4*d^4*x^4 + 4*c*d^3*x^3 + c^3*d*x
 + c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

maple [B] time = 0.01, size = 900, normalized size = 3.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2*f/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2
^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x
+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d)
)^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x
^3+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*
2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
)^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2
)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2))+2*(-c*f+d*e)/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/
3)+1/4*I*3^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3
))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/
d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/
4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/
d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/
d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/
4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((
1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*
c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(
1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/
4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{c^3+4\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] int((e + f*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral((e + f*x)/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

3.65 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

```
[Out] -2/9*2^(2/3)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*(1+x)*
EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/
2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(
1/2))^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2139, 218, 2137, 203}

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1+x^3}}\,dx - \frac{1}{3}\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{1}{3}\left(2\,2^{2/3}\right)\mathrm{Subst}\left(\int \right.$$

$$= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.36, size = 207, normalized size = 1.43

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}} + \frac{i2^{2/3}\sqrt{x^2-x+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[1 + x^3]
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(x^3-2^{\frac{2}{3}}x^2+2\cdot 2^{\frac{1}{3}}x\right)\sqrt{x^3+1}}{x^6+5x^3+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^3 - 2^(2/3)*x^2 + 2*2^(1/3)*x)*sqrt(x^3 + 1)/(x^6 + 5*x^3 + 4),
 x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument
 Value
```

maple [B] time = 0.04, size = 258, normalized size = 1.78

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}\cdots}{-\frac{3}{2}\cdots}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(3/2-1/2*I*3^(1/2))*((x
+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^
(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/
(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{x^3+1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] int(x/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.66 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=160

$$\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/9*2^(2/3)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)+2/9*(1-x)
*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1
/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3
^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2139, 218, 2137, 203}

$$\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{3}\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$$
$$= \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{1}{3}\left(2\,2^{2/3}\right)\text{Subst}\Big($$
$$= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.33, size = 209, normalized size = 1.31

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{x^2+x+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[1 - x^3]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^3+2^{\frac{2}{3}}x^2+2\cdot 2^{\frac{1}{3}}x\right)\sqrt{-x^3+1}}{x^6-5x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^3 + 2^(2/3)*x^2 + 2*2^(1/3)*x)*sqrt(-x^3 + 1)/(x^6 - 5*x^3 + 4)
, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument V
alue
```

maple [A] time = 0.04, size = 253, normalized size = 1.58

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{\ }}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/
2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(
2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{-x^{3}+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] -int(x/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{1-x^{3}}-2^{\frac{2}{3}}\sqrt{1-x^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

3.67 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=163

$$\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

```
[Out] -2/9*2^(2/3)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)+2/9*(1-x)
*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1
/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3
^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2139, 219, 2137, 206}

$$\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3])
+ (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{3}\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}-\frac{1}{3}\left(2\,2^{2/3}\right)\mathrm{Subst}\Bigg(\int$$

$$= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.19, size = 207, normalized size = 1.27

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{x^2+x+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[-1 + x^3]
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(x^3+2^{\frac{2}{3}}x^2+2\cdot 2^{\frac{1}{3}}x\right)\sqrt{x^3-1}}{x^6-5x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(x^3 + 2^(2/3)*x^2 + 2*2^(1/3)*x)*sqrt(x^3 - 1)/(x^6 - 5*x^3 + 4)
, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument
 Value
```

maple [B] time = 0.03, size = 262, normalized size = 1.61

$$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(-3/2-1/2*I*3^(1/2))*((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2
^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))
/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] -int(x/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

3.68 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

```
[Out] -2/9*2^(2/3)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9*(1+x
)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(
1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x
-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2139, 219, 2137, 206}

$$\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3])
+ (2*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{-1-x^3}}\,dx - \frac{1}{3}\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{1}{3}\left(2\,2^{2/3}\right)\text{Subs}$$

$$= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.23, size = 209, normalized size = 1.34

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{x^2-x+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[-1 - x^3]
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(x^3-2^{\frac{2}{3}}x^2+2\cdot 2^{\frac{1}{3}}x\right)\sqrt{-x^3-1}}{x^6+5\,x^3+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(x^3 - 2^(2/3)*x^2 + 2*2^(1/3)*x)*sqrt(-x^3 - 1)/(x^6 + 5*x^3 + 4
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

maple [A] time = 0.03, size = 249, normalized size = 1.60

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2)
)^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(
I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2)
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] int(x/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.69 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=275

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\cdots\right.}{\sqrt{a+b\cdots}}\right.}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[Out] -2/9*2^(2/3)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(
1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)
+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2139, 218, 2137, 203}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\cdots\right.}{\sqrt{a+b\cdots}}\right.}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
 b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(
1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}} - \frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7\ldots\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}\,b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\ldots\right.}}\ldots}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\ldots}{\left(\left(1+\sqrt{\ldots}\right.\right.}}\ldots}$$

Mathematica [C] time = 1.08, size = 324, normalized size = 1.18

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}\,2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\;\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}} - \frac{\sqrt[4]{3}\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\ldots}{\sqrt{\frac{\sqrt[3]{a}+(-\ldots}{\left(1+\sqrt[3]{\ldots}\right.}}}\right)}{\sqrt{3}\,b^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((3^(1/4)*((-1)
^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*Ellipt
icF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))
]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^
(1/3))]) + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x
)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(
2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/
3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/(Sqrt[3]*b^(2/3)*Sqrt[a + b*x^
3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b^{\frac{1}{3}} x+2^{\frac{2}{3}} a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^{3}+a}\left(b^{\frac{1}{3}} x+2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

```
[Out] int(x/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^{3}}\left(2^{\frac{2}{3}} \sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

3.70 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=283

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\right.}{\sqrt{a-bx^3}}\right.}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[Out] -2/9*2^(2/3)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)^
(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2139, 218, 2137, 203}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\right.}{\sqrt{a-bx^3}}\right.}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a -
 b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*S
qrt[a - b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}}+\frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$=\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-\right.}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$=-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{}{\left(\left(1\right.\right.}}}$$

Mathematica [C] time = 0.89, size = 388, normalized size = 1.37

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2
^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(
1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3)
 - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^
(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/
a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3
) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3
]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-b^{\frac{1}{3}} x+2^{\frac{2}{3}} a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{-bx^{3}+a}\left(b^{\frac{1}{3}} x-2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int(x/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{a-bx^{3}}+\sqrt[3]{b} x \sqrt{a-bx^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x)
```

3.71 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=292

$$\frac{2\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{bx^3-a}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}\cdots\right.}{\sqrt{bx^3\cdots}}\right.}{3\sqrt{3}\,\sqrt[6]{a}\,b^{2/3}}$$

```
[Out] -2/9*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)^
(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^
(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2139, 219, 2137, 206}

$$\frac{2\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{bx^3-a}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}\cdots\right.}{\sqrt{bx^3\cdots}}\right.}{3\sqrt{3}\,\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}}+\frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7 \ldots}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}\,b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\ldots}{\left(\left(1-\sqrt{3}\right)\ldots\right.}}}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\ldots}}$$

Mathematica [C] time = 0.32, size = 389, normalized size = 1.33

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\frac{\sqrt[3]{-1}\,2^{\ldots}}{\ldots}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3\ldots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2
^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(
1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3)
 - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^
(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/
a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3
) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3
]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-b^{\frac{1}{3}} x+2^{\frac{2}{3}} a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{bx^{3}-a}\left(b^{\frac{1}{3}} x-2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int(x/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{-a+bx^{3}}+\sqrt[3]{b} x \sqrt{-a+bx^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x)
```

3.72 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=288

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\cdots\right.}{\sqrt{-a-bx^3}}\right.}{3\sqrt{3}\sqrt[6]{a}\,b^{2/3}}$$

```
[Out] -2/9*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a)
^(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2139, 219, 2137, 206}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\cdots\right.}{\sqrt{-a-bx^3}}\right.}{3\sqrt{3}\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2)]*Sqrt[-a - b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}} - \frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\cdots\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}\,b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{\cdots}}{\left(\left(1-\sqrt{\cdots}\right.\right.}}}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\cdots}}$$

Mathematica [C] time = 0.71, size = 375, normalized size = 1.30

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}\,2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{3}}-\frac{\left(\sqrt[3]{-1}+2^{2/3}\right)\cdots}{\cdots}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3) +
 2^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/
a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))]], (-1)^(1/3)])/3^(1/4)) + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1
/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqr
t[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/
((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3)
)*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]
*Sqrt[-a - b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b^{\frac{1}{3}} x + 2^{\frac{2}{3}} a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-bx^{3}-a}\left(b^{\frac{1}{3}} x + 2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

```
[Out] int(x/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a-bx^{3}}\left(2^{\frac{2}{3}} \sqrt[3]{a} + \sqrt[3]{b} x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

3.73 $\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2}$$

```
[Out] -2/9*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/d^2*3^(1/2)/c^
(1/2)+1/9*2^(1/3)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3)*d*x+c*(1-3^(1/2)))/(2^
(2/3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^2-2^(
2/3)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
d^2/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2139, 218, 2137, 203}

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]
*Sqrt[c]*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^
(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
 -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*
c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
```

```
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3d} - \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3d}$$

$$= \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right.}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}\,d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\sin^{-1}\left(\right.\right.}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4}}$$

Mathematica [C] time = 1.07, size = 372, normalized size = 1.51

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)|\sqrt[3]{-1}\right)+\right.}{\left(2+\sqrt[3]{-2}\right)d^2\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3+4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(Sqrt[((-2)^(1/3)
*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))*c
 - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^
(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2^(2/
3)*(1 + (-1)^(1/3))*c*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))
*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1
/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/(
(1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3]))/((2 + (-2)^(1/3))*d^2
*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 + 4*d^3
*x^3])
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{4\,d^3x^3+c^3}\,x}{4\,d^4x^4+4\,cd^3x^3+c^3dx+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*d^3*x^3 + c^3)*x/(4*d^4*x^4 + 4*c*d^3*x^3 + c^3*d*x + c^4),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{4\,d^3x^3+c^3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

maple [B] time = 0.01, size = 892, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(
1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1
/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3
+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^
(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^
(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*
2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1
/2))-2*c/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1
/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1
/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/
3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1
/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(
4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*Elliptic
Pi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)
*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)
+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*
2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3
))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)
*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{4\,d^3x^3+c^3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{c^3+4\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
[Out] int(x/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
[Out] Integral(x/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

3.74 $\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

```
[Out] 2/3*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2138, 206}

$$\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
[Out] (2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right)\\ &= \frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 2.00

$$\frac{1}{3}\log\left(\frac{(x+1)^2}{\sqrt{x^3+1}}+3\right)-\frac{1}{3}\log\left(3-\frac{(x+1)^2}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
[Out] -1/3*Log[3 - (1 + x)^2/Sqrt[1 + x^3]] + Log[3 + (1 + x)^2/Sqrt[1 + x^3]]/3
```

fricas [B] time = 0.74, size = 44, normalized size = 1.91

$$\frac{1}{3}\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}\,(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x+1}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

maple [C] time = 0.04, size = 240, normalized size = 10.43

$$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(2-x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/
2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*
((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi
(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x+1}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

mupad [B] time = 0.23, size = 205, normalized size = 8.91

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x\cdots}{\frac{3}{2}+\cdots}}}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
```

```
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x
 + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1
/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx-\int \frac{1}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(1/(x*sqrt(
x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

3.75 $\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=27

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

```
[Out] -2/3*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2138, 206}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
[Out] (-2*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{1-x^3}}\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 2.00

$$\frac{1}{3}\log\left(3-\frac{(1-x)^2}{\sqrt{1-x^3}}\right)-\frac{1}{3}\log\left(\frac{(1-x)^2}{\sqrt{1-x^3}}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
[Out] Log[3 - (1 - x)^2/Sqrt[1 - x^3]]/3 - Log[3 + (1 - x)^2/Sqrt[1 - x^3]]/3
```

fricas [B] time = 0.88, size = 47, normalized size = 1.74

$$\frac{1}{3}\log\left(-\frac{x^3-12\,x^2-6\,\sqrt{-x^3+1}\,(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(-(x^3 - 12*x^2 - 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2
+ 12*x + 8))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-1}{\sqrt{-x^3+1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

maple [C] time = 0.04, size = 240, normalized size = 8.89

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - 2i\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x+2)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
(x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I
*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-1}{\sqrt{-x^3+1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

mupad [B] time = 0.18, size = 221, normalized size = 8.19

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-}\right.\right.\right.}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((1 - x^3)^(1/2)*(x + 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2)))*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((1 - x^3)^(1/2)*(((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int \left(-\frac{1}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-1/(x*sqrt
(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

3.76 $\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=25

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

```
[Out] -2/3*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2138, 203}

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{-1+x^3}}\right)\right)\\ &= -\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

fricas [B] time = 0.50, size = 40, normalized size = 1.60

$$-\frac{1}{3}\arctan\left(\frac{\left(x^3-12\,x^2-6\,x-10\right)\sqrt{x^3-1}}{6\left(x^4-x^3-x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-1}{\sqrt{x^3-1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

maple [C] time = 0.04, size = 240, normalized size = 9.60

$$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\cdots}{\cdots}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x+2)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3
/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*
((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(
((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-1}{\sqrt{x^3-1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

mupad [B] time = 2.55, size = 205, normalized size = 8.20

$$\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\cdots\right)\cdots\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticF(asin
```

```
((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*(-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1
/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int \left(-\frac{1}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-1/(x*sqrt
(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

3.77 $\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=25

$$\frac{2}{3}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

```
[Out] 2/3*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2138, 203}

$$\frac{2}{3}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{-1-x^3}}\right)\\ &= \frac{2}{3}\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

fricas [A] time = 0.49, size = 38, normalized size = 1.52

$$-\frac{1}{3}\arctan\left(\frac{\left(x^3+12\,x^2-6\,x+10\right)\sqrt{-x^3-1}}{6\left(x^4+x^3+x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x + 1
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x+1}{\sqrt{-x^3-1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

maple [C] time = 0.04, size = 240, normalized size = 9.60

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(2-x)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x
+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x
^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3
^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x+1}{\sqrt{-x^3-1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

mupad [B] time = 2.53, size = 221, normalized size = 8.84

$$-\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+}{-\frac{3}{2}+}\right)\right.}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 1)/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)),
-((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6
 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/
2)/((3^(1/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/
2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1/2)*(x^3
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx-\int \frac{1}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(1/(x*sqr
t(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

3.78 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b} x}{\left(2 \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a+b x^3}} \, dx$

Optimal. Leaf size=50

$$\frac{2 \tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b} x\right)^2}{3 \sqrt[6]{a} \sqrt{a+b x^3}}\right)}{3 \sqrt[6]{a} \sqrt[3]{b}}$$

```
[Out] 2/3*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^3+a)^(1/2))/a^(1/6)/b^(1
/3)
```

Rubi [A] time = 0.13, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2138, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b} x\right)^2}{3 \sqrt[6]{a} \sqrt{a+b x^3}}\right)}{3 \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[3]{a}+\sqrt[3]{b} x}{\left(2 \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a+b x^3}} \, dx &= \frac{\left(2 \sqrt[3]{a}\right) \text{Subst}\left(\int \frac{1}{9-a x^2} \, dx, x, \frac{\left(1+\frac{\sqrt[3]{b} x}{\sqrt[3]{a}}\right)^2}{\sqrt{a+b x^3}}\right)}{\sqrt[3]{b}}\\ &= \frac{2 \tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b} x\right)^2}{3 \sqrt[6]{a} \sqrt{a+b x^3}}\right)}{3 \sqrt[6]{a} \sqrt[3]{b}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.02

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^2}{3\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x
]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*(1 + (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[a + b*x^3])])/(3*a
^(1/6)*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x+2a^{\frac{1}{3}}\right)\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\sqrt{bx^{3}+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),
 x)
```

mupad [B] time = 3.44, size = 65, normalized size = 1.30

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)\left(\sqrt{b\,x^3+a}-\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)^3}{x^3\,\left(b^{1/3}\,x-2\,a^{1/3}\right)^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

```
[Out] log((((a + b*x^3)^(1/2) + a^(1/2))*((a + b*x^3)^(1/2) - a^(1/2) + 2*a^(1/6)
*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}\,x\sqrt{a+bx^3}}\,dx-\int \frac{\sqrt[3]{b}\,x}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}\,x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x
)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x*
*3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*s
qrt(a + b*x**3)), x)
```

3.79 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=52

$$-\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] -2/3*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*x^3+a)^(1/2))/a^(1/6)/b^
(1/3)
```

Rubi [A] time = 0.14, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2138, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx &= -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9-ax^2}\,dx, x, \frac{\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)^2}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}\\ &= -\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.02

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{3\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x
]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*(1 - (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[a - b*x^3])])/(3*
a^(1/6)*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x+2a^{\frac{1}{3}}\right)\sqrt{-b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3)))
, x)
```

mupad [B] time = 3.59, size = 67, normalized size = 1.29

$$\frac{\ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{a}\right)\left(\sqrt{a-b\,x^3}+\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)^3}{x^3\,\left(b^{1/3}\,x+2\,a^{1/3}\right)^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
```

```
[Out] log((((a - b*x^3)^(1/2) - a^(1/2))*((a - b*x^3)^(1/2) + a^(1/2) + 2*a^(1/6)
*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\right)dx-\int\frac{\sqrt[3]{b}\,x}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),
x)
```

```
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sq
rt(a - b*x**3)), x)
```

3.80 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\, x}{\left(2 \sqrt[3]{a}+\sqrt[3]{b}\, x\right)\sqrt{-a+bx^3}}\, dx$

Optimal. Leaf size=53

$$-\frac{2 \tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\, x\right)^2}{3 \sqrt[6]{a}\, \sqrt{bx^3-a}}\right)}{3 \sqrt[6]{a}\, \sqrt[3]{b}}$$

```
[Out] -2/3*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^3-a)^(1/2))/a^(1/6)/b^(1
/3)
```

Rubi [A] time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2138, 203}

$$-\frac{2 \tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\, x\right)^2}{3 \sqrt[6]{a}\, \sqrt{bx^3-a}}\right)}{3 \sqrt[6]{a}\, \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\, x}{\left(2 \sqrt[3]{a}+\sqrt[3]{b}\, x\right)\sqrt{-a+bx^3}}\, dx &= -\frac{\left(2 \sqrt[3]{a}\right) \text{Subst}\left(\int \frac{1}{9+ax^2}\, dx, x, \frac{\left(1-\frac{\sqrt[3]{b}\, x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}\\
&= -\frac{2 \tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\, x\right)^2}{3 \sqrt[6]{a}\, \sqrt{-a+bx^3}}\right)}{3 \sqrt[6]{a}\, \sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 1.02

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{3\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),
x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*(1 - (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[-a + b*x^3])])/(3*
a^(1/6)*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x + 2a^{\frac{1}{3}}\right)\sqrt{b\,x^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3))/(b^(1/3)*x+2*a^(1/3))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3))/(b^(1/3)*x+2*a^(1/3))/(b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}}{\sqrt{bx^3 - a}\left(b^{\frac{1}{3}}x + 2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),
 x)
```

mupad [B] time = 5.45, size = 74, normalized size = 1.40

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{a}\,1\mathrm{i}\right)\left(\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x+\sqrt{b\,x^3-a}\,1\mathrm{i}\right)^3}{x^3\,\left(b^{1/3}\,x+2\,a^{1/3}\right)^3}\right)\,1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

```
[Out] (log((((b*x^3 - a)^(1/2) + a^(1/2)*1i)*((b*x^3 - a)^(1/2)*1i + a^(1/2) + 2*
a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))*1i)/(3*a^(1/6)*b^(1/
3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\right)dx-\int\frac{\sqrt[3]{b}\,x}{2\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x
)
```

```
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x
*sqrt(-a + b*x**3)), x)
```

3.81 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=53

$$\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2/3*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^3-a)^(1/2))/a^(1/6)/b^(1
/3)
```

Rubi [A] time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2138, 203}

$$\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx &= \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9+ax^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}\\
&= \frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.02

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^2}{3\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),
x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*(1 + (b^(1/3)*x)/a^(1/3))^2)/(3*Sqrt[-a - b*x^3])])/(3*a
^(1/6)*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x + 2a^{\frac{1}{3}}\right)\sqrt{-b\,x^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3))/(-b^(1/3)*x+2*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3))/(-b^(1/3)*x+2*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}}{\sqrt{-bx^3 - a}\left(b^{\frac{1}{3}}x - 2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3)))
, x)
```

mupad [B] time = 5.37, size = 78, normalized size = 1.47

$$\frac{\ln\left(\frac{\left(\sqrt{-b\,x^3-a}-\sqrt{a}\,1\mathrm{i}\right)\left(2\,a^{1/6}\,b^{1/3}\,x-\sqrt{a}+\sqrt{-b\,x^3-a}\,1\mathrm{i}\right)^3}{x^3\,\left(b^{1/3}\,x-2\,a^{1/3}\right)^3}\right)1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
```

```
[Out] (log((((- a - b*x^3)^(1/2) - a^(1/2)*1i)*((- a - b*x^3)^(1/2)*1i - a^(1/2)
+ 2*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))*1i)/(3*a^(1/6)*b
^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\,\sqrt{-a-bx^3}+\sqrt[3]{b}\,x\sqrt{-a-bx^3}}\,dx-\int \frac{\sqrt[3]{b}\,x}{-2\sqrt[3]{a}\,\sqrt{-a-bx^3}+\sqrt[3]{b}\,x\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),
x)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*
x**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*
x*sqrt(-a - b*x**3)), x)
```

3.82 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}\,d}$$

```
[Out] -2/3*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/d/c^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2138, 206}

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx &= -\frac{(2c)\,\text{Subst}\left(\int \frac{1}{9-c^3x^2}\,dx, x, \frac{\left(1-\frac{2dx}{c}\right)^2}{\sqrt{c^3-8d^3x^3}}\right)}{d}\\ &= -\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}\,d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d)

fricas [B] time = 0.87, size = 294, normalized size = 6.39

$$\left[\frac{\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6-3\left(8\,d^4x^4-52\,cd^3x^3+12\,c^2d^2x^2-4\,c^3dx+5\,c^4\right)\sqrt{-8\,d^3x^3+c^3}\,\sqrt{c}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{6\,\sqrt{c}\,d}, -\frac{\sqrt{-}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 15
6*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 - 3*(8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^2*
d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c))/(d^6*x^6 + 6*c
*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c
^6))/(sqrt(c)*d), -1/3*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 - 6*c^
2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*d^3*x^
3 - 2*c^4*d*x + c^5))/(c*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,dx - c}{\sqrt{-8\,d^3x^3 + c^3}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

maple [C] time = 0.20, size = 650, normalized size = 14.13

$$\frac{6\left(\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}\right)\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,c\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\ldots}{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\ldots}}\ldots\right.}{\sqrt{-8d^3x^3+c^3}\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}+\frac{c}{d}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] -4*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/
2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2)
)*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((
x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2
*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/2*(-1/2-1/2*
I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)
)^(1/2),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(
-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))+6*c/d*(1/2*(-1/2+1/2*I*3^(1/2))*c/
d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/
2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*c/d)/(1/2
*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)
/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3
*x^3+c^3)^(1/2)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d)*EllipticPi(((x-1/2*(-1/2
-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))
*c/d))^(1/2),(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1
```

```
/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1
/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,dx - c}{\sqrt{-8\,d^3x^3 + c^3}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

mupad [B] time = 3.13, size = 67, normalized size = 1.46

$$\frac{\ln\left(\frac{\left(\sqrt{c^3-8\,d^3\,x^3}-c^{3/2}\right)\left(\sqrt{c^3-8\,d^3\,x^3}+c^{3/2}+4\,\sqrt{c}\,d\,x\right)^3}{x^3\,(c+d\,x)^3}\right)}{3\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - 2*d*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] log((((c^3 - 8*d^3*x^3)^(1/2) - c^(3/2))*((c^3 - 8*d^3*x^3)^(1/2) + c^(3/2)
 + 4*c^(1/2)*d*x)^3)/(x^3*(c + d*x)^3))/(3*c^(1/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{c}{c\sqrt{c^3 - 8d^3x^3} + dx\sqrt{c^3 - 8d^3x^3}}\right) dx - \int \frac{2dx}{c\sqrt{c^3 - 8d^3x^3} + dx\sqrt{c^3 - 8d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] -Integral(-c/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**3)), x
) - Integral(2*d*x/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**
3)), x)
```

3.83 $\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=139

$$\frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)+\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(e-f)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}$$

```
[Out] 2/9*(e+2*f)*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))+2/9*(e-f)*(1+x)*EllipticF((1
+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/
2)
```

Rubi [A] time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2139, 218, 2138, 206}

$$\frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)+\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(e-f)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(e + 2*f)*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 + (2*Sqrt[2 + Sqrt[3]]
*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 +
x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{1}{3}(e-f)\int \frac{1}{\sqrt{1+x^3}}\,dx + \frac{1}{6}(e+2f)\int \frac{2+2x}{(2-x)\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}(2(e+2f))$$

$$= \frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right) + \frac{2\sqrt{2+\sqrt{3}}\,(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right.\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.32, size = 273, normalized size = 1.96

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x+1)}{\sqrt{3}-3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}-i}\sqrt{x^2-x+1}\,(e+2f)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-3if\sqrt{-2ix+}\right.}{\left(\sqrt{3}+3i\right)\sqrt{2ix+\sqrt{3}-i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt[
3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2*Sq
rt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(
2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*Sqrt[1 + x^3])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{x^3+1}\left(fx+e\right)}{x^4-2x^3+x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 + 1)*(f*x + e)/(x^4 - 2*x^3 + x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{fx+e}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

maple [B] time = 0.01, size = 246, normalized size = 1.77

$$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, f\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e+2f\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2-x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e+2*f)*(3/2-1/2*I*3^(1/2)
)*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2
)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

mupad [B] time = 2.71, size = 327, normalized size = 2.35

$$\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(e+2f\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2f\left(\frac{3}{2}+\cdots\right.}{\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*(e + 2*f)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
- x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, a
sin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)))/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*f*((3^
(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx-\int \frac{fx}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(f*x/(x*sqr
t(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

3.84 $\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=153

$$-\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(e+f)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/9*(e-2*f)*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))-2/9*(e+f)*(1-x)*EllipticF(
(1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+
x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^
(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2139, 218, 2138, 206}

$$-\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(e+f)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(e - 2*f)*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]
]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[
(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = \frac{1}{6}(e-2f)\int \frac{2-2x}{(2+x)\sqrt{1-x^3}}\,dx + \frac{1}{3}(e+f)\int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}(2(e-2f)$$

$$= -\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}\,(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{}}{1+\sqrt{}}\right.\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.29, size = 271, normalized size = 1.77

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(3f\sqrt{2ix+\sqrt{3}+i}\left(i\sqrt{3}x+x+i\sqrt{3}-1\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-2\sqrt{3}\sqrt{-2ix+\sqrt{3}-}\right.}{\left(\sqrt{3}+3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (2
*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt[3
] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[3]*
(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt
[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*
I)*x]*Sqrt[1 - x^3])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}\left(fx+e\right)}{x^4+2x^3-x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*(f*x + e)/(x^4 + 2*x^3 - x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^3+1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

maple [A] time = 0.01, size = 246, normalized size = 1.61

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,f\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2i(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(x+2)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2
)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^3+1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

mupad [B] time = 0.19, size = 359, normalized size = 2.35

$$-\frac{2f\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((1 - x^3)^(1/2)*(x + 2)),x)
```

```
[Out] - (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/
((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2*((
3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)
*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(e - 2*f)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1
i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
+ 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{-(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

3.85 $\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=156

$$-\frac{2}{9}(e-2f)\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,(e+f)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] -2/9*(e-2*f)*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))-2/9*(e+f)*(1-x)*EllipticF((1
-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+
1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1
/2)
```

Rubi [A] time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2139, 219, 2138, 203}

$$-\frac{2}{9}(e-2f)\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,(e+f)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(e - 2*f)*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]
]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[
(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1
 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{1}{6}(e-2f)\int \frac{2-2x}{(2+x)\sqrt{-1+x^3}}\,dx + \frac{1}{3}(e+f)\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{1}{3}(2(e-2f$$

$$= -\frac{2}{9}(e-2f)\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}\,(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+}{1-}\right.\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.19, size = 269, normalized size = 1.72

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(3f\sqrt{2ix+\sqrt{3}+i}\left(i\sqrt{3}x+x+i\sqrt{3}-1\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-2\sqrt{3}\sqrt{-2ix+\sqrt{3}-}\right.}{\left(\sqrt{3}+3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (2
*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt[3
] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[3]*
(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt
[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*
I)*x]*Sqrt[-1 + x^3])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3-1}\left(fx+e\right)}{x^4+2x^3-x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)*(f*x + e)/(x^4 + 2*x^3 - x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3-1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

maple [A] time = 0.01, size = 246, normalized size = 1.58

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, f\, \mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} + \frac{2\left(e-2f\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\cdots}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(x+2)/(x^3-1)^(1/2),x)
```

```
[Out] 2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-2*f)*(-3/2-1/2*I*3^(1/2))
*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*
EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3-1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

mupad [B] time = 0.12, size = 327, normalized size = 2.10

$$-\frac{2f\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\cdots}{-\frac{3}{2}\cdots}}}{3\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

```
[Out] - (2*f*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
- 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/
2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)*(e - 2*f)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

3.86 $\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=150

$$\frac{2}{9}(e+2f)\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(e-f)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 2/9*(e+2*f)*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))+2/9*(e-f)*(1+x)*EllipticF((1
+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+
1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(
1/2)
```

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2139, 219, 2138, 203}

$$\frac{2}{9}(e+2f)\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(e-f)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(e + 2*f)*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 + (2*Sqrt[2 - Sqrt[3]]
*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1
+ x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{1}{3}(e-f)\int \frac{1}{\sqrt{-1-x^3}}\,dx + \frac{1}{6}(e+2f)\int \frac{2+2x}{(2-x)\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{1}{3}(2(e+2f)$$

$$= \frac{2}{9}(e+2f)\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right) + \frac{2\sqrt{2-\sqrt{3}}\,(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{}}{1-\sqrt{}}\right.\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.20, size = 275, normalized size = 1.83

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x+1)}{\sqrt{3}-3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}-i}\sqrt{x^2-x+1}\,(e+2f)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-3if\sqrt{-2ix+\right.}{\left(\sqrt{3}+3i\right)\sqrt{2ix+\sqrt{3}-i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt[
3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2*Sq
rt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(
2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*Sqrt[-1 - x^3])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^3-1}\left(fx+e\right)}{x^4-2x^3+x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 - 1)*(f*x + e)/(x^4 - 2*x^3 + x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{fx+e}{\sqrt{-x^3-1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

maple [A] time = 0.01, size = 246, normalized size = 1.64

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,f\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\left(e+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2-x)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1
/2*I*3^(1/2)))^(1/2))+2/3*I*(e+2*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2
))^(1/2)/(-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{-x^3-1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

mupad [B] time = 2.54, size = 359, normalized size = 2.39

$$-\frac{2f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x\cdots}}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(
1/2)*1i)/2 - 3/2))^(1/2)*(e + 2*f)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2
)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2))^(1/2)) - (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x +
(3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*e
llipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx-\int \frac{fx}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(f*x/(x*s
qrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

3.87 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=297

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\right.}{}$$

```
[Out] 2/9*(b^(1/3)*e+2*a^(1/3)*f)*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^
3+a)^(1/2))/b^(2/3)/a^(1/2)+2/9*(b^(1/3)*e-a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*E
llipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/(b*x^
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2139, 218, 2138, 206}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqr
t[a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - a^(
1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\left(\frac{1}{6}\left(-\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx\right)-\frac{1}{3}\left(-\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$= \frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3\sqrt[4]{3}\ldots}$$

Mathematica [C] time = 1.38, size = 419, normalized size = 1.41

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\ldots\right.\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-1/2*(3^(1/4)*f*
((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I
)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/Sqrt[2] + I*(
b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((
-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2
/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) +
(I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))
/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}}x+2a^{\frac{1}{3}}\right)\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(-b^(1/3)*x+2*a^(1/3))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(-b^(1/3)*x+2*a^(1/3))/(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{bx^{3}+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{e+f\,x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

```
[Out] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-2\sqrt[3]{a}\,\sqrt{a+bx^{3}}+\sqrt[3]{b}\,x\sqrt{a+bx^{3}}}\,dx-\int \frac{fx}{-2\sqrt[3]{a}\,\sqrt{a+bx^{3}}+\sqrt[3]{b}\,x\sqrt{a+bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x
) - Integral(f*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3
)), x)
```

3.88 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=304

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\sqrt[3]{b}e-2\right.}{}$$

```
[Out] -2/9*(b^(1/3)*e-2*a^(1/3)*f)*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*
x^3+a)^(1/2))/b^(2/3)/a^(1/2)-2/9*(b^(1/3)*e+a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)
*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/
(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2139, 218, 2138, 206}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\sqrt[3]{b}e-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sq
rt[a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + a^
(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}$$

$$=-\frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$=-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{}\right.}{3\sqrt[4]{3}}$$

Mathematica [C] time = 1.34, size = 447, normalized size = 1.47

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-i\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+\right.}{\left(-3i+\sqrt{3}\right)\sqrt[3]{}}}\right.\right.\right.}{\left(\sqrt[3]{-1}-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-1/2*I)*f*Sqrt[
((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3
))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin
[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1
/3))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2*a^(1
/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt
[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + S
qrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a
^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac
")
[Out] Timed out
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+2a^{\frac{1}{3}}\right)\sqrt{-b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(b^(1/3)*x+2*a^(1/3))/(-b*x^3+a)^(1/2),x)
[Out] int((f*x+e)/(b^(1/3)*x+2*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-bx^{3}+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
[Out] integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{a-b\,x^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
[Out] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
[Out] Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

3.89 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=313

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(\sqrt[3]{b}e\right.}{}$$

```
[Out] -2/9*(b^(1/3)*e-2*a^(1/3)*f)*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^
3-a)^(1/2))/b^(2/3)/a^(1/2)-2/9*(b^(1/3)*e+a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*E
llipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),
2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(b
*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2139, 219, 2138, 203}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(\sqrt[3]{b}e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqr
t[-a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + a^
(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]
)/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - S
qrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a+b\cdots}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}\right.\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$= -\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\cdots\right.}{3\sqrt[4]{3}\sqrt[3]{b}\cdots}$$

Mathematica [C] time = 0.92, size = 448, normalized size = 1.43

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-i\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{\cdots}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right.\right.\right.}{\left(\sqrt[3]{-1}-2\right)b\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-1/2*I)*f*Sqrt[
((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3
))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin
[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1
/3))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2*a^(1
/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt
[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + S
qrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a
^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3]
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+2a^{\frac{1}{3}}\right)\sqrt{b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(b^(1/3)*x+2*a^(1/3))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(b^(1/3)*x+2*a^(1/3))/(b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{bx^{3}-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

```
[Out] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

3.90 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=310

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\right.}{}$$

```
[Out] 2/9*(b^(1/3)*e+2*a^(1/3)*f)*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^
3-a)^(1/2))/b^(2/3)/a^(1/2)+2/9*(b^(1/3)*e-a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*E
llipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*
I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(-b*x
^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2139, 219, 2138, 203}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt
[-a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - a^(
1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])
/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\left(\frac{1}{6}\left(-\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx\right)-\frac{1}{3}\left(-\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$= \frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{3\sqrt[4]{3}\sqrt[3]{\cdots}}$$

Mathematica [C] time = 0.36, size = 422, normalized size = 1.36

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\cdots\right.\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-1/2*(3^(1/4)*f*
((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I
)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/Sqrt[2] + I*(
b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((
-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2
/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) +
(I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))
/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}}x+2a^{\frac{1}{3}}\right)\sqrt{-b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(-b^(1/3)*x+2*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(-b^(1/3)*x+2*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{-bx^{3}-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{e+f\,x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
```

```
[Out] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-2\sqrt[3]{a}\,\sqrt{-a-bx^{3}}+\sqrt[3]{b}\,x\sqrt{-a-bx^{3}}}\,dx-\int \frac{fx}{-2\sqrt[3]{a}\,\sqrt{-a-bx^{3}}+\sqrt[3]{b}\,x\sqrt{-a-bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(e/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)),
 x) - Integral(f*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*
x**3)), x)
```

3.91 $\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal. Leaf size=221

$$-\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2}-\frac{\sqrt{2+\sqrt{3}}\,(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\right.}{3\sqrt[4]{3}\,cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

```
[Out] -2/9*(-c*f+d*e)*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/c^
(3/2)/d^2-1/9*(c*f+2*d*e)*(-2*d*x+c)*EllipticF((-2*d*x+c*(1-3^(1/2)))/(-2*d
*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((4*d^2*x^2+2*c*
d*x+c^2)/(-2*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c/d^2/(-8*d^3*x^3+c^3)^(1/
2)/(c*(-2*d*x+c)/(-2*d*x+c*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2139, 218, 2138, 206}

$$-\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2}-\frac{\sqrt{2+\sqrt{3}}\,(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\right.}{3\sqrt[4]{3}\,cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (-2*(d*e - c*f)*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(
9*c^(3/2)*d^2) - (Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c - 2*d*x)*Sqrt[(c^2 + 2
*c*d*x + 4*d^2*x^2)/((1 + Sqrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*c - 2*d*x)/((1 + Sqrt[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*
d^2*Sqrt[(c*(c - 2*d*x))/((1 + Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3]
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx}{3cd} + \frac{(2de+cf)\int \frac{1}{\sqrt{c^3-8d^3x^3}}\,dx}{3cd}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\,(2de+cf)(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{}\right.}{3\sqrt[4]{3}\,cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\sqrt{c^3-8d^3x^3}}$$

$$= -\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\,\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2} - \frac{\sqrt{2+\sqrt{3}}\,(2de+cf)(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2\right.}}}{3\sqrt[4]{3}\,cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-\right.}}}$$

Mathematica [C] time = 1.18, size = 384, normalized size = 1.74

$$-\frac{i\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(4\sqrt{2}\sqrt{\frac{ic+\sqrt{3}\,dx+idx}{-\sqrt{3}\,c+3ic}}\sqrt{\frac{c^2+2cdx+4d^2x^2}{c^2}}\,(de-cf)\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{2}\sqrt{\frac{ic+\sqrt{3}\,dx+idx}{3ic-\sqrt{3}\,c}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+\right.}{2\left(\sqrt[3]{-1}-2\right)d^2\sqrt{\frac{c-2(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] ((-1/2*I)*Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*(f*Sqrt[((-I + Sqrt[3])*c
+ 2*(I + Sqrt[3])*d*x)/((-3*I + Sqrt[3])*c)]*((-3*I + Sqrt[3])*c - 2*(3*I +
 Sqrt[3])*d*x)*EllipticF[ArcSin[Sqrt[2]*Sqrt[(I*c + I*d*x + Sqrt[3]*d*x)/((
3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2] + 4*Sqrt[2]*(d*e - c*f)*Sqrt[(I*c
 + I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]*Sqrt[(c^2 + 2*c*d*x + 4*d^2*
x^2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[2]*Sqrt[(I*c
+ I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2]))/((-2 +
 (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3
 - 8*d^3*x^3])
```

fricas [F] time = 1.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-8\,d^3x^3+c^3}\left(fx+e\right)}{8\,d^4x^4+8\,cd^3x^3-c^3dx-c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-8*d^3*x^3 + c^3)*(f*x + e)/(8*d^4*x^4 + 8*c*d^3*x^3 - c^3*d
*x - c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

maple [B] time = 0.01, size = 661, normalized size = 2.99

$$\frac{2\left(\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}\right)\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,f\,\mathrm{EllipticF}\left(\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i}{\cdots}\right.}{2d}}{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\cdots}}\right.\cdots}{\sqrt{-8d^3x^3+c^3}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2*f/d*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(
-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1
/2))*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)
*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+
1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/2*(-1/2-1
/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c
/d))^(1/2),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/
2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))+2*(-c*f+d*e)/d^2*(1/2*(-1/2+1/2
*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/2-1/2*I*3^(1/2))*
c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)*((x
-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I
*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))
^(1/2)/(-8*d^3*x^3+c^3)^(1/2)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d)*EllipticPi
(((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-
1/2*I*3^(1/2))*c/d))^(1/2),(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^
(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d),((1/2*(-1/2-1/2*I*3^(1/2))*c
/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{c^3-8\,d^3\,x^3}\;(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] int((e + f*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{-(-c + 2dx)\left(c^2 + 2cdx + 4d^2x^2\right)}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c +
d*x)), x)
```

3.92 $\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=129

$$\frac{4}{9}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] 4/9*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))-2/9*(1+x)*EllipticF((1+x-3^(1/2))/(1
+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2
))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2139, 218, 2138, 206}

$$\frac{4}{9}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (4*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{1+x^3}}\,dx\right)+\frac{1}{3}\int \frac{2+2x}{(2-x)\sqrt{1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{4}{3}\text{Subst}\left(\int \frac{1}{9-x}\right.$$

$$= \frac{4}{9}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.26, size = 193, normalized size = 1.50

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[1 + x^3]
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{x^3+1}\,x}{x^4-2\,x^3+x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 + 1)*x/(x^4 - 2*x^3 + x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

[Out] integrate(-x/(sqrt(x^3 + 1)*(x - 2)), x)

maple [B] time = 0.01, size = 240, normalized size = 1.86

$$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2-x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+4/3*(3/2-1/2*I*3^(1/2))*((x+1)/(
3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2
)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Elliptic
Pi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 + 1)*(x - 2)), x)
```

mupad [B] time = 0.06, size = 207, normalized size = 1.60

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\cdots}\right.\right.\right.}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2*ellipticPi((3^(
1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*
1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

3.93 $\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] 4/9*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))-2/9*(1-x)*EllipticF((1-x-3^(1/2))/(
1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/
2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2139, 218, 2138, 206}

$$\frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (4*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] -
x)^2]*Sqrt[1 - x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}\,dx - \frac{1}{3}\int \frac{2-2x}{(2+x)\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{9-x^2}\,dx$$

$$= \frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.24, size = 195, normalized size = 1.34

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{2i\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[1 - x^3]
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-x^3+1}\,x}{x^4+2x^3-x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*x/(x^4 + 2*x^3 - x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3+1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

maple [A] time = 0.01, size = 240, normalized size = 1.66

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+2)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/
2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3+1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

mupad [B] time = 0.07, size = 224, normalized size = 1.54

$$-\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\cdots\right.\right.}{3\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/2)*(x + 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
- 2*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(1 - x^3)^(1/
2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

3.94 $\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=148

$$\frac{4}{9}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] 4/9*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))-2/9*(1-x)*EllipticF((1-x+3^(1/2))/(1-
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2)
)^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2139, 219, 2138, 203}

$$\frac{4}{9}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (4*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3]
- x)^2)]*Sqrt[-1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{-1+x^3}}\,dx - \frac{1}{3}\int \frac{2-2x}{(2+x)\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4}{3}\text{Subst}\left(\int \frac{1}{9+x^2}\right.$$

$$= \frac{4}{9}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.11, size = 193, normalized size = 1.30

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{2i\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[-1 + x^3]
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3-1}\,x}{x^4+2\,x^3-x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)*x/(x^4 + 2*x^3 - x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3-1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

[Out] integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)

maple [B] time = 0.01, size = 240, normalized size = 1.62

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x+2)/(x^3-1)^(1/2),x)

[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4/3*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^3-1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)

mupad [B] time = 2.53, size = 208, normalized size = 1.41

$$-\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(3\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{6};\mathrm{as}\cdots\right.\right.}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((x^3 - 1)^(1/2)*(x + 2)),x)

[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2+x)/(x**3-1)**(1/2),x)

[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)

3.95 $\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=140

$$\frac{4}{9}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 4/9*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))-2/9*(1+x)*EllipticF((1+x+3^(1/2))/(1
+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2
))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2139, 219, 2138, 203}

$$\frac{4}{9}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (4*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3]
+ x)^2)]*Sqrt[-1 - x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{-1-x^3}}\,dx\right)+\frac{1}{3}\int \frac{2+2x}{(2-x)\sqrt{-1-x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{9+$$

$$= \frac{4}{9}\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.19, size = 195, normalized size = 1.39

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[-1 - x^3]
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^3-1}\,x}{x^4-2\,x^3+x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 - 1)*x/(x^4 - 2*x^3 + x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\sqrt{-x^3-1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

maple [B] time = 0.01, size = 240, normalized size = 1.71

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{3}\,\sqrt{i}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2-x)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
(x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I
*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{-x^3-1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

mupad [B] time = 0.06, size = 223, normalized size = 1.59

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{a}\right.\right.}{3\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
 x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin(((x + 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2
*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(- x^3 - 1)^(1/2)
*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

3.96 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=260

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 4/9*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^3+a)^(1/2))/a^(1/6)/b^(2
/3)-2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2139, 218, 2138, 206}

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (4*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}}+\frac{\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\cdots}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$= \frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\cdots\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a\cdots}}$$

Mathematica [C] time = 1.44, size = 407, normalized size = 1.57

$$\frac{\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(8i\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\sqrt{\cdots}\right.}{2\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2\cdots}}{\left(1+\sqrt[3]{-1}\cdots\right.}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(Sqrt[2]*3^(1/4)*
((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I
)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]) + (8*I)*a^(1/
3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3
))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqr
t[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*
x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/(2*(-2 + (-1)^(1/3))*
b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*S
qrt[a + b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-b^{\frac{1}{3}} x + 2 a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b^(1/3)*x+2*a^(1/3))/(b*x^3+a)^(1/2),x)
[Out] int(x/(-b^(1/3)*x+2*a^(1/3))/(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{bx^{3}+a}\left(b^{\frac{1}{3}} x - 2\,a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x}{\left(b^{1/3}\,x - 2\,a^{1/3}\right)\,\sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
[Out] -int(x/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-2\sqrt[3]{a}\,\sqrt{a+bx^{3}} + \sqrt[3]{b}\,x\sqrt{a+bx^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
[Out] -Integral(x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x
)
```

3.97 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=268

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] 4/9*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*x^3+a)^(1/2))/a^(1/6)/b^(
2/3)-2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2139, 218, 2138, 206}

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (4*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}} - \frac{\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\middle|-7\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$= \frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}}$$

Mathematica [C] time = 0.71, size = 371, normalized size = 1.38

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)+\frac{2\sqrt[3]{-1}\left(1\right.}{}\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3)
)*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b
^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3
)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ell
ipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/
3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[a - b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b^{\frac{1}{3}}x+2a^{\frac{1}{3}}\right)\sqrt{-b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^(1/3)*x+2*a^(1/3))/(-b*x^3+a)^(1/2),x)
[Out] int(x/(b^(1/3)*x+2*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-bx^{3}+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{a-b\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
[Out] int(x/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
[Out] Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

3.98 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=277

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

```
[Out] 4/9*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^3-a)^(1/2))/a^(1/6)/b^(2/
3)-2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(
1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2)
)*3^(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2139, 219, 2138, 203}

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (4*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(
1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx = \frac{\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}} - \frac{\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7\ldots}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{-a+bx^3}}$$

$$= \frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\ldots\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\ldots}$$

Mathematica [C] time = 0.23, size = 372, normalized size = 1.34

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)+\frac{2\sqrt[3]{-1}\left(1+\sqrt[3]{\ldots}\right.}{\ldots}\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3)
)*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b
^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3
)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ell
ipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/
3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[-a + b*x^3])
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [**F**] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b^{\frac{1}{3}} x + 2 a^{\frac{1}{3}}\right) \sqrt{b\,x^{3} - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^(1/3)*x+2*a^(1/3))/(b*x^3-a)^(1/2),x)
[Out] int(x/(b^(1/3)*x+2*a^(1/3))/(b*x^3-a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b x^{3} - a} \left(b^{\frac{1}{3}} x + 2\, a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(b^{1/3}\, x + 2\, a^{1/3}\right) \sqrt{b\, x^3 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
[Out] int(x/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2 \sqrt[3]{a} + \sqrt[3]{b} x\right) \sqrt{- a + b x^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
[Out] Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

3.99 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=273

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] 4/9*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^3-a)^(1/2))/a^(1/6)/b^(2
/3)-2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1
/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*
3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2139, 219, 2138, 203}

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (4*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}} + \frac{\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$= \frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\mathrm{si}\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}$$

Mathematica [C] time = 0.30, size = 410, normalized size = 1.50

$$\frac{\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(8i\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\right.}{2\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-}{(1+\sqrt[3]{}}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(Sqrt[2]*3^(1/4)*
((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I
)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]) + (8*I)*a^(1/
3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3
))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqr
t[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*
x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/(2*(-2 + (-1)^(1/3))*
b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*S
qrt[-a - b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-b^{\frac{1}{3}} x + 2 a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b^(1/3)*x+2*a^(1/3))/(-b*x^3-a)^(1/2),x)
[Out] int(x/(-b^(1/3)*x+2*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{-b x^{3}-a}\left(b^{\frac{1}{3}} x - 2\, a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x}{\left(b^{1/3}\, x - 2\, a^{1/3}\right)\, \sqrt{-b\, x^3 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
[Out] -int(x/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-2\sqrt[3]{a}\,\sqrt{-a - bx^{3}} + \sqrt[3]{b}\,x\sqrt{-a - bx^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
[Out] -Integral(x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)),
 x)
```

3.100 $\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal. Leaf size=202

$$\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}\,d^2}-\frac{\sqrt{2+\sqrt{3}}\,(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

```
[Out] 2/9*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/d^2/c^(1/2)-1/
9*(-2*d*x+c)*EllipticF((-2*d*x+c*(1-3^(1/2)))/(-2*d*x+c*(1+3^(1/2))),I*3^(1
/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((4*d^2*x^2+2*c*d*x+c^2)/(-2*d*x+c*(1+3^
(1/2)))^2)^(1/2)*3^(3/4)/d^2/(-8*d^3*x^3+c^3)^(1/2)/(c*(-2*d*x+c)/(-2*d*x+c
*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2139, 218, 2138, 206}

$$\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}\,d^2}-\frac{\sqrt{2+\sqrt{3}}\,(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(9*Sqrt[c]*d^2
) - (Sqrt[2 + Sqrt[3]]*(c - 2*d*x)*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/((1 + S
qrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c - 2*d*x)/((1 + Sqrt
[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c - 2*d*x))/((1
+ Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \frac{\int \frac{1}{\sqrt{c^3-8d^3x^3}}\,dx}{3d} - \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx}{3d}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\,(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}} + \text{(2c) Su}$$

$$= \frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}\,d^2} - \frac{\sqrt{2+\sqrt{3}}\,(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2d}{\left(1+\sqrt{3}\right)c-2d}\right.\right.}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

Mathematica [C] time = 0.67, size = 295, normalized size = 1.46

$$\frac{\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}\,c+2dx\right)\sqrt{\frac{\sqrt[3]{-1}\left(c+2\sqrt[3]{-1}\,dx\right)}{\left(1+\sqrt[3]{-1}\right)c}}\,F\left(\sin^{-1}\left(\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\right)|\sqrt[3]{-1}\right)+\frac{2\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)c\sqrt{\frac{c-2(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right.}}}{\cdots}\right.}{\left(\sqrt[3]{-1}-2\right)d^2\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3-8d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*((-2 + (-1)^(1/3))*((-1)^(1/3)*c +
2*d*x)*Sqrt[((-1)^(1/3)*(c + 2*(-1)^(1/3)*d*x))/((1 + (-1)^(1/3))*c)]*Ellip
ticF[ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]], (-1)^(1/3)]
 + (2*(-1)^(1/3)*(1 + (-1)^(1/3))*c*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^
(1/3))*c)]*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*
I + Sqrt[3]), ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]], (-
1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1
 + (-1)^(1/3))*c)]*Sqrt[c^3 - 8*d^3*x^3])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-8\,d^3x^3+c^3}\,x}{8\,d^4x^4+8\,cd^3x^3-c^3dx-c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-8*d^3*x^3 + c^3)*x/(8*d^4*x^4 + 8*c*d^3*x^3 - c^3*d*x - c^4
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-8\,d^3x^3+c^3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

maple [B] time = 0.01, size = 653, normalized size = 3.23

$$-\frac{2\left(\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}\right)\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,c\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-}{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{}}{2}\right.}{2d}}}\right.}{\sqrt{-8d^3x^3+c^3}\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}+\frac{c}{d}\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2/d*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1
/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2
))*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*(
(x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/
2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/2*(-1/2-1/2
*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d
))^(1/2),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*
(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))-2*c/d^2*(1/2*(-1/2+1/2*I*3^(1/2))
*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(
-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*c/d)/(
1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c
/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)/(-8*
d^3*x^3+c^3)^(1/2)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d)*EllipticPi(((x-1/2*(-
1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/
2))*c/d))^(1/2),(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)
/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/
2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-8\,d^3x^3+c^3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{c^3-8\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] int(x/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(-c+2dx\right)\left(c^2+2cdx+4d^2x^2\right)}\,\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c + d*x)),
x)
```

3.101 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] -2*arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.45, size = 267, normalized size = 6.36

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\,\left(\right.}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (4*I
)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/
(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3]
 - (2*I)*x]*Sqrt[1 + x^3])
```

fricas [B] time = 0.51, size = 205, normalized size = 4.88

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4\left(2x^6-18x^5+42x^4-8x^3-\sqrt{3}\right.}{x^8+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*
x^4 + 224*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 - sqrt(3)*(x^6
- 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(x^3 + 1)*sqrt(2*s
qrt(3) + 3) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x
 + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*
x^2 - 64*x + 16))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%%%} E
rror: Bad Argument Value
```

maple [C] time = 0.08, size = 245, normalized size = 5.83

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-4*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^3 + 1}\left(x - \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 1 + \sqrt{3}}{\sqrt{(x + 1)\left(x^2 - x + 1\right)}\left(x - \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1))
, x)
```

3.102 $\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=46

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] 2*arctanh((1-x)*(-3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[-3 + 2*Sqrt[
3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx,x,\frac{1-x}{\sqrt{1-x^3}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.49, size = 269, normalized size = 5.85

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(4\sqrt{-2ix+\sqrt{3}-i}\,\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}+i}\,\left(\left(\left(1\right.\right.\right.}{\left((1+2i)\sqrt{3}-3i\right)\sqrt{-2ix+\sqrt{3}-i}\,\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((2 + I) - Sqrt[3] + ((1 + 2*I) - I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 4*S
qrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-I + Sq
rt[3] - (2*I)*x]*Sqrt[1 - x^3])
```

fricas [B] time = 0.51, size = 207, normalized size = 4.50

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2+4\left(2x^6+18x^5+42x^4+8x^3-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*
x^4 - 224*x^3 + 64*x^2 + 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 - sqrt(3)*(x^6
+ 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(-x^3 + 1)*sqrt(2*
sqrt(3) + 3) - 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*
x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64
*x^2 + 64*x + 16))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

maple [C] time = 0.08, size = 243, normalized size = 5.28

$$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+4*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(
-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1
)^(1/2)/(-3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/
(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(x + 3^(1/2) - 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}-1}{\sqrt{-\left(x-1\right)\left(x^2+x+1\right)}\left(x-1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) - 1)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3))
), x)
```

3.103 $\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=44

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] 2*arctan((1-x)*(-3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[-3 + 2*Sqrt[
3]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.38, size = 267, normalized size = 6.07

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(4\sqrt{-2ix+\sqrt{3}-i}\,\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}+i}\,\left(\right.}{\left((1+2i)\sqrt{3}-3i\right)\sqrt{-2ix+\sqrt{3}-i}\,\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((2 + I) - Sqrt[3] + ((1 + 2*I) - I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 4*S
qrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-I + Sq
rt[3] - (2*I)*x]*Sqrt[-1 + x^3])
```

fricas [A] time = 0.49, size = 50, normalized size = 1.14

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{\left(\sqrt{3}\left(x^2+4x-2\right)-6x+6\right)\sqrt{2\sqrt{3}+3}}{6\sqrt{x^3-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*(sqrt(3)*(x^2 + 4*x - 2) - 6*x +
 6)*sqrt(2*sqrt(3) + 3)/sqrt(x^3 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

maple [C] time = 0.06, size = 245, normalized size = 5.57

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\cdots}{\frac{3}{2}-\cdots}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi((
(x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2
*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(x + 3^(1/2) - 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}-1}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x-1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) - 1)/(sqrt((x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3)))
, x)
```

3.104 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] -2*arctan((1+x)*(-3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}} \end{aligned}$$

Mathematica [C] time = 0.38, size = 269, normalized size = 6.11

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\,\left(\left(\sqrt{\right.\right.}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (4*I
)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/
(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3]
 - (2*I)*x]*Sqrt[-1 - x^3])
```

fricas [A] time = 0.49, size = 59, normalized size = 1.34

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{\sqrt{-x^3-1}\left(\sqrt{3}\left(x^2-4x-2\right)+6x+6\right)\sqrt{2\sqrt{3}+3}}{6\left(x^3+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*sqrt(-x^3 - 1)*(sqrt(3)*(x^2 - 4
*x - 2) + 6*x + 6)*sqrt(2*sqrt(3) + 3)/(x^3 + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%%%} E
rror: Bad Argument Value
```

maple [C] time = 0.07, size = 247, normalized size = 5.61

$$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{4i\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-4*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(
1/2)/(3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1+\sqrt{3}}{\sqrt{-(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)
), x)
```

3.105 $\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{a+b x^3}} \, dx$

Optimal. Leaf size=69

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{2 \sqrt{3}-3} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt{a+b x^3}}\right)}{\sqrt{2 \sqrt{3}-3} \sqrt[6]{a} \sqrt[3]{b}}$$

```
[Out] -2*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2)
)/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2140, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{2 \sqrt{3}-3} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt{a+b x^3}}\right)}{\sqrt{2 \sqrt{3}-3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{a+b x^3}} \, dx &= -\frac{\left(2 \sqrt[3]{a}\right) \text{Subst}\left(\int \frac{1}{1+\left(3-2 \sqrt{3}\right) a x^2} \, dx,x,\frac{1+\frac{\sqrt[3]{b} x}{\sqrt[3]{a}}}{\sqrt{a+b x^3}}\right)}{\sqrt[3]{b}}\\ &= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{-3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt{a+b x^3}}\right)}{\sqrt{-3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}} \end{aligned}$$

Mathematica [C] time = 0.63, size = 322, normalized size = 4.67

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4(-1)^{5/6}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\ldots}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\ldots}}\ldots\right)}{\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(5/6)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
2*I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3
))*a^(1/3))]], (-1)^(1/3)])/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3))))/Sqrt[a +
 b*x^3]
```

fricas [A] time = 1.77, size = 1240, normalized size = 17.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*
a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^
2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 -
 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 16
860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^2
*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 -
4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3
*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*
a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 +
37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*
b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 23
3856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 29
20*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x
^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1/3))*sqrt((2*sqr
t(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^1
6 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*
b*x^4 + 4608*a^7*x - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^
16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6
*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 1
3320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b
^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6
*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024
*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 32*sqrt(
3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^
12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8))/(b
^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a
^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4
```

```
096*a^8)), sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(1/2*
sqrt(1/3)*(a^(1/3)*b*x^2 + 2*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*(sqrt(3)
*a - a)*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))/sqrt(b*x^3 + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x+\left(1+\sqrt{3}\right) a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}} x+\left(1-\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^{3}+a}\left(b^{\frac{1}{3}} x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x -
 a^(1/3)*(sqrt(3) - 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^{3}}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/
2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(-sqr
t(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

3.106 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2)
)/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [C] time = 1.44, size = 446, normalized size = 6.28

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(4\sqrt{3}\sqrt[3]{a}\sqrt{-\frac{2i\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\right.\right.}{\left((1+2i)\sqrt{\ }\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(Sqrt[((-I + Sqrt
[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*((-3 +
(2 + I)*Sqrt[3])*a^(1/3) + (-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*EllipticF[
ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3]
)*a^(1/3))]], (1 + I*Sqrt[3])/2] + 4*Sqrt[3]*a^(1/3)*Sqrt[-(((2*I)*a^(1/3)
+ (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3)))]*Sqrt[1 + (b^(1/3)*x
)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)
*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*
I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-3*I + (1 + 2*I)*Sqrt[3])*b
^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sq
rt[a - b*x^3])
```

fricas [B] time = 1.76, size = 1294, normalized size = 18.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*
a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^
13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x
- sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x
^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x
))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 +
 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 9318
4*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 +
2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x
^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*
x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^3
 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824
*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 4464
0*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^
6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18
 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6
*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt((2*s
qrt(3) + 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 +
```

```
2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x
^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x
^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 15155
2*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*s
qrt(3) + 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*
x^2 - 2*sqrt(-b*x^3 + a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3
+ a)*(sqrt(3)*a - a)*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 - a
))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}} x+\left(1+\sqrt{3}\right) a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}} x+\left(1-\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x
^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x
^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-b x^{3}+a}\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) - 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3}\,\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a-bx^{3}}\left(-\sqrt[3]{a}+\sqrt{3}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(
1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-a*
*(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

3.107 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=72

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] 2*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/
a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx &= \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}
\end{aligned}$$

Mathematica [C] time = 0.43, size = 447, normalized size = 6.21

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(4\sqrt{3}\sqrt[3]{a}\sqrt{-\frac{2i\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\right.\right.}{\left((1+2i)\sqrt{\ }\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(Sqrt[((-I + Sqrt
[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*((-3 +
(2 + I)*Sqrt[3])*a^(1/3) + (-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*EllipticF[
ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3]
)*a^(1/3))]], (1 + I*Sqrt[3])/2] + 4*Sqrt[3]*a^(1/3)*Sqrt[-(((2*I)*a^(1/3)
+ (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3)))]*Sqrt[1 + (b^(1/3)*x
)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)
*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*
I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-3*I + (1 + 2*I)*Sqrt[3])*b
^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sq
rt[-a + b*x^3])
```

fricas [A] time = 1.75, size = 1245, normalized size = 17.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672
*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a
^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7
- 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 1
6860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^
2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 +
 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^
3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724
*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 -
 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2
*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 2
33856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2
920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*
x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1/3))*sqrt(-(2*s
qrt(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x
^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^
6*b*x^4 - 4608*a^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*
x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a
^6*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 +
 13320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5
*b^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqr
t(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*
```

```
x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/
(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 +
 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*(sqrt
(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/sqrt(b*x^3 - a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}} x+\left(1+\sqrt{3}\right) a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}} x+\left(1-\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^
3-a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^
3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^{3}-a}\left(b^{\frac{1}{3}} x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x +
 a^(1/3)*(sqrt(3) - 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3}\,\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{-a+bx^{3}}\left(-\sqrt[3]{a}+\sqrt{3}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(
1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-a
**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

3.108 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] -2*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2)
)/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx &= -\frac{\left(2\sqrt[3]{a}\right)\operatorname{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.53, size = 325, normalized size = 4.51

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4(-1)^{5/6}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}\cdots}{\sqrt[4]{3}\sqrt[3]{b}\cdots}\right)}{\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(5/6)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
2*I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3
))*a^(1/3))]], (-1)^(1/3)])/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3))))/Sqrt[-a
- b*x^3]
```

fricas [B] time = 1.67, size = 1303, normalized size = 18.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672
*a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x
^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x
 - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*
x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*
x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17
- 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^
6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 -
 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*
x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7
*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^
4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^
3 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b
^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 1382
4*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^1
4 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*
x^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 446
40*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x
^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^1
8 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^
6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt(-(2
*sqrt(3) + 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18
+ 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2
*x^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6
*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151
```

```
552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*s
qrt(3) + 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*x
^2 + 2*sqrt(-b*x^3 - a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3 -
 a)*(sqrt(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 + a))
]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x+\left(1+\sqrt{3}\right) a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}} x+\left(1-\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x+a^{\frac{1}{3}} \left(\sqrt{3}+1\right)}{\sqrt{-b x^{3}-a} \left(b^{\frac{1}{3}} x-a^{\frac{1}{3}} \left(\sqrt{3}-1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
 - a^(1/3)*(sqrt(3) - 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a
^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a}+\sqrt{3}\,\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{-a-bx^{3}}\,\left(-\sqrt{3}\,\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/
2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(-sq
rt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

3.109 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctanh((1+(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.28, size = 663, normalized size = 9.08

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)+bx^3\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-1\ldots}\right.\right.}{a\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\ldots\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[a + b*x^3]),x]
```

```
[Out] (x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sq
rt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a + 6
*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), (b*x
^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3])*a -
 b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), (b*x^3
)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[4/3, 1/2,
 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*A
ppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))
)/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2
, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF
1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3
*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

fricas [A] time = 1.18, size = 1273, normalized size = 17.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 145
152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x^
2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4*
x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*a
^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 1979
```

```
2*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
+ 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 +
 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*
x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^
5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 8601
6*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864
*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6
 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt((2*sqrt(3
) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b^5
*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 328
704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20 +
4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b^2
*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7
*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a
^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x
^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*
b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a - a)
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.41index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1/a*b)^(1/3)*x+3^(1/2))/(1+(1/a*b)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2)
,x)
```

```
[Out] int((1+(1/a*b)^(1/3)*x+3^(1/2))/(1+(1/a*b)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{bx^{3}+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) - s
qrt(3) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3+a}\,\left(x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/
2) + 1)),x)
```

```
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/
2) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}}{\sqrt{a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) -
 sqrt(3) + 1)), x)
```

3.110 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctanh((1-(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a-bx^3}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.18, size = 648, normalized size = 8.64

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-bx^3\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\left(\cdots\right.\right.}{a\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\left(8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\cdots\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[a - b*x^3]),x]
```

```
[Out] (x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[
3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
- 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*Sqrt[1
- (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(
10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^
3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(
8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a -
6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[a - b*x^3])
```

fricas [B] time = 1.17, size = 1330, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504
*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
+ 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 + 5
010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*
```

```
x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x
^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a
^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3
+ a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^1
4 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 + a))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(
a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*
b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
- 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 - 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*b*x^2 - 2*sq
rt(-b*x^3 + a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 + a)*(sqrt
(3)*a - a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.42index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}\right)\sqrt{-b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(1/a*b)^(1/3)*x+3^(1/2))/(1-(1/a*b)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2
),x)
```

```
[Out] int((1-(1/a*b)^(1/3)*x+3^(1/2))/(1-(1/a*b)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2
),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{-bx^{3}+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) +
sqrt(3) - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{a-b\,x^3}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)),x)
```

```
[Out] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1}{\sqrt{a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) -
 1 + sqrt(3))), x)
```

3.111 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=76

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctan((1-(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.59, size = 649, normalized size = 8.54

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-bx^3\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)\left(\cdots\right.}{a\left(2\left(3\sqrt{3}-5\right)a+bx^3\right)\left(8\left(3\sqrt{3}-5\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\cdots\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[-a + b*x^3]),x]
```

```
[Out] (x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[
3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
- 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*Sqrt[1
- (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(
10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^
3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(
8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a -
6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[-a + b*x^3])
```

fricas [A] time = 1.15, size = 1278, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
 4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 + 14
5152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7*b*x
^2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4
*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*
a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 197
92*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
```

```
 - 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2
*x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b
^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 860
16*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt(-(2*sqrt
(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520
*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7
*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x - sqrt(3)*(5*
a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^
(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 +
 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*
b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a -
 a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.44index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}\right) \sqrt{b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1/a*b)^(1/3)*x+1+3^(1/2))/(-(1/a*b)^(1/3)*x+1-3^(1/2))/(b*x^3-a)^(1/
2),x)
```

```
[Out] int((-(1/a*b)^(1/3)*x+1+3^(1/2))/(-(1/a*b)^(1/3)*x+1-3^(1/2))/(b*x^3-a)^(1/
2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{b x^{3} - a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) + s
qrt(3) - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)),x)
```

```
[Out] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1}{\sqrt{-a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
 - 1 + sqrt(3))), x)
```

3.112 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=76

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctan((1+(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.78, size = 666, normalized size = 8.76

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)-18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-10a}\right)+bx^3\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a-1}\right.\right.}{a\left(2\left(3\sqrt{3}-5\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)+\left(5-3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[-a - b*x^3]),x]
```

```
[Out] (x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sq
rt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a + 6
*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), (b*x
^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3])*a -
 b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), (b*x^3
)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[4/3, 1/2,
 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*A
ppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))
)/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2
, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF
1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3
*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

fricas [B] time = 1.15, size = 1339, normalized size = 17.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
 4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 9350
4*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
 - 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x
^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 -
```

```
5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3
*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*
x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*
a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3
 - a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^
14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 - a))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 -
 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)
*(a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^
7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 + 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 +
 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 + 2*sqr
t(-b*x^3 - a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 - a)*(sqrt(
3)*a - a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.45index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}\right) \sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/a*b)^(1/3)*x+1+3^(1/2))/((1/a*b)^(1/3)*x+1-3^(1/2))/(-b*x^3-a)^(1/2
),x)
```

```
[Out] int(((1/a*b)^(1/3)*x+1+3^(1/2))/((1/a*b)^(1/3)*x+1-3^(1/2))/(-b*x^3-a)^(1/2
),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{-b x^{3} - a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) -
sqrt(3) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3}+x\,{\left(\frac{b}{a}\right)}^{1/3}+1}{\sqrt{-b\,x^3-a}\,\left(x\,{\left(\frac{b}{a}\right)}^{1/3}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(
1/2) + 1)),x)
```

```
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(
1/2) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}}{\sqrt{-a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) + 1)), x)
```

3.113 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] -2*arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[3 + 2*Sqrt[3]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.44, size = 269, normalized size = 6.40

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left((1+\right.\right.\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*
((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqr
t[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2
*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 + x^3])
```

fricas [A] time = 0.47, size = 50, normalized size = 1.19

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}\left(x^2-4x-2\right)-6x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x -
 6)*sqrt(2*sqrt(3) - 3)/sqrt(x^3 + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

maple [C] time = 0.03, size = 245, normalized size = 5.83

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-4*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{(x+1)\left(x^2 - x + 1\right)}\left(x + 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3)))
, x)
```

3.114 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=46

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] 2*arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[3 + 2*Sqrt[3]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.49, size = 267, normalized size = 5.80

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(\sqrt{2ix+\sqrt{3}+i}\left(\left(\sqrt{3}+(2+i)\right)x+i\sqrt{3}+(1+2i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-4i\sqrt{-2ix\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) + I*Sqrt[3] + ((2 + I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - (4*
I)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3]
)/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/
4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-I + Sqrt
[3] - (2*I)*x]*Sqrt[1 - x^3])
```

fricas [A] time = 0.50, size = 59, normalized size = 1.28

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\sqrt{-x^3+1}\left(\sqrt{3}\left(x^2+4x-2\right)+6x-6\right)\sqrt{2\sqrt{3}-3}}{6\left(x^3-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*sqrt(-x^3 + 1)*(sqrt(3)*(x^2 + 4
*x - 2) + 6*x - 6)*sqrt(2*sqrt(3) - 3)/(x^3 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

maple [C] time = 0.04, size = 247, normalized size = 5.37

$$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-4i\sqrt{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))-4*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(
-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1
)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/
(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - 1 + \sqrt{3}}{\sqrt{-(x - 1)\left(x^2 + x + 1\right)}\left(x - \sqrt{3} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((x - 1 + sqrt(3))/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1)
), x)
```

3.115 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=44

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] 2*arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[3 + 2*Sqrt[3
]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.30, size = 265, normalized size = 6.02

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(\sqrt{2ix+\sqrt{3}+i}\left(\left(\sqrt{3}+(2+i)\right)x+i\sqrt{3}+(1+2i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-4i\sqrt{-2ix+\cdots}\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) + I*Sqrt[3] + ((2 + I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - (4*
I)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3]
)/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/
4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-I + Sqrt
[3] - (2*I)*x]*Sqrt[-1 + x^3])
```

fricas [B] time = 0.49, size = 204, normalized size = 4.64

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2-4\left(2x^6+18x^5+42x^4+8x^3+\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*
x^4 - 224*x^3 + 64*x^2 - 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 + sqrt(3)*(x^6
+ 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(x^3 - 1)*sqrt(2*s
qrt(3) - 3) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x
 - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*
x^2 + 64*x + 16))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

maple [C] time = 0.03, size = 245, normalized size = 5.57

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x\cdots}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi((
(x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1+\sqrt{3}}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
[Out] Integral((x - 1 + sqrt(3))/(sqrt((x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1))
, x)
```

3.116 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] -2*arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[3 + 2*Sqrt[
3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.35, size = 271, normalized size = 6.16

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\,(((}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*
((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqr
t[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2
*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[-1 - x^3])
```

fricas [B] time = 0.47, size = 206, normalized size = 4.68

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2+4\left(2x^6-18x^5+42x^4-8x^3+\sqrt{3}\right.}{x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*
x^4 + 224*x^3 + 64*x^2 + 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 + sqrt(3)*(x^6
- 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(-x^3 - 1)*sqrt(2*
sqrt(3) - 3) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*
x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64
*x^2 - 64*x + 16))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Err
or: Bad Argument Value
```

maple [C] time = 0.03, size = 243, normalized size = 5.52

$$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{i\left(x\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+4*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(
1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}\left(x + \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-\left(x + 1\right)\left(x^{2} - x + 1\right)}\left(x + 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))
), x)
```

3.117 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=69

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] -2*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))/
a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx &= -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.59, size = 320, normalized size = 4.64

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\ldots\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\ldots}}\right.}{\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/((3 + (2 + I)*Sqrt[3])*b^(1/3))))/Sqrt[a + b*x^3
]
```

fricas [A] time = 1.74, size = 1236, normalized size = 17.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672
*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a
^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7
- 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 1
6860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^
2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 -
 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^
3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724
*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 +
 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2
*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 2
33856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2
920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*
x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1/3))*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x
^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^
6*b*x^4 + 4608*a^7*x + sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*
x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a
^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 +
 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5
*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqr
t(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*
x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8))/
(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 +
```

```
 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*(sqrt
(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/sqrt(b*x^3 + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x + \left(1-\sqrt{3}\right) a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}} x + \left(1+\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^3+
a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^3+
a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}} \left(\sqrt{3}-1\right)}{\sqrt{bx^{3}+a} \left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(\sqrt{3}+1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x +
 a^(1/3)*(sqrt(3) + 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{a+bx^{3}}\,\left(\sqrt[3]{a} + \sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/
2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(a**
(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

3.118 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] 2*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))/
a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx &= \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.82, size = 329, normalized size = 4.63

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}-\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt[3]{b}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((((-1)^(1/3)*a^(
1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 +
(-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] - (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3
)*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*S
qrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)
/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3 + (2 + I)*Sqrt[3])))/(b^(1/3
)*Sqrt[a - b*x^3])
```

fricas [B] time = 1.72, size = 1288, normalized size = 18.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672
*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x
^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x
 + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*
x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*
x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17
+ 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^
6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 +
 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*
x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7
*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^
4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^
3 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b
^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 1382
4*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^1
4 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*
x^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 446
40*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x
^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^1
8 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^
6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt(-(2
*sqrt(3) - 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18
+ 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2
*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6
```

```
*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151
552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*s
qrt(3) - 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*x
^2 + 2*sqrt(-b*x^3 + a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3 +
 a)*(sqrt(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 - a))
]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}} x + \left(1-\sqrt{3}\right) a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}} x + \left(1+\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x
^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x
^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(\sqrt{3}-1\right)}{\sqrt{-bx^{3}+a} \left(b^{\frac{1}{3}} x - a^{\frac{1}{3}} \left(\sqrt{3}+1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) + 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt[3]{a} + \sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{a-bx^{3}} \left(-\sqrt{3}\,\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(
1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-sq
rt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

3.119 $\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{-a+b x^3}} \, dx$

Optimal. Leaf size=72

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)}{\sqrt{b x^3-a}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

```
[Out] 2*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/
a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)}{\sqrt{b x^3-a}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{-a+b x^3}} \, dx &= \frac{\left(2 \sqrt[3]{a}\right) \text{Subst}\left(\int \frac{1}{1-\left(3+2 \sqrt{3}\right) a x^2} \, dx,x,\frac{1-\frac{\sqrt[3]{b} x}{\sqrt[3]{a}}}{\sqrt{-a+b x^3}}\right)}{\sqrt[3]{b}}\\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)}{\sqrt{-a+b x^3}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}} \end{aligned}$$

Mathematica [C] time = 0.36, size = 330, normalized size = 4.58

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}-\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}}; \ldots\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt[3]{b}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((((-1)^(1/3)*a^(
1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 +
(-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] - (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3
)*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*S
qrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)
/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3 + (2 + I)*Sqrt[3])))/(b^(1/3
)*Sqrt[-a + b*x^3])
```

fricas [A] time = 1.76, size = 1239, normalized size = 17.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*
a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^
2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 -
 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16
860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2
*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 +
4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3
*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*
a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 -
37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*
b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 23
3856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 29
20*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x
^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1/3))*sqrt((2*sqr
t(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^1
6 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*
b*x^4 - 4608*a^7*x + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^
16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6
*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 1
3320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b
^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6
*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024
*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 32*sqrt(
3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^
12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b
^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a
```

```
^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4
096*a^8)), sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(1/2*
sqrt(1/3)*(a^(1/3)*b*x^2 + 2*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*(sqrt(3)
*a + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))/sqrt(b*x^3 - a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}} x+\left(1-\sqrt{3}\right) a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}} x+\left(1+\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^
3-a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^
3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x+a^{\frac{1}{3}} \left(\sqrt{3}-1\right)}{\sqrt{bx^{3}-a} \left(b^{\frac{1}{3}} x-a^{\frac{1}{3}} \left(\sqrt{3}+1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x -
 a^(1/3)*(sqrt(3) + 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt[3]{a}+\sqrt{3}\,\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{-a+bx^{3}}\,\left(-\sqrt{3}\,\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(
1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-s
qrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

3.120 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] -2*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2)
)/a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx &= -\frac{\left(2\sqrt[3]{a}\right)\operatorname{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.49, size = 323, normalized size = 4.49

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right)}{\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a
^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcS
in[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))])) + (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/((3 + (2 + I)*Sqrt[3])*b^(1/3))))/Sqrt[-a - b*x^
3]
```

fricas [B] time = 1.67, size = 1299, normalized size = 18.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*
a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^
13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x
+ sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x
^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x
))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 -
 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 9318
4*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 -
2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x
^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*
x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^3
 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824
*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 4464
0*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^
6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18
 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6
*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt((2*s
qrt(3) - 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 +
2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x
^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x
^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 15155
```

```
2*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*
x^2 - 2*sqrt(-b*x^3 - a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3
- a)*(sqrt(3)*a + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 + a
))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x + \left(1-\sqrt{3}\right) a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}} x + \left(1+\sqrt{3}\right) a^{\frac{1}{3}}\right) \sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}} x - a^{\frac{1}{3}} \left(\sqrt{3} - 1\right)}{\sqrt{-b x^{3} - a} \left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(\sqrt{3} + 1\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) + 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x + a
^(1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{- \sqrt{3} \sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{- a - b x^{3}} \left(\sqrt[3]{a} + \sqrt{3} \sqrt[3]{a} + \sqrt[3]{b} x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/
2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(a*
*(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

3.121 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\, x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\, x\right)\sqrt{a+bx^3}}\, dx$

Optimal. Leaf size=73

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\ \sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\ \sqrt{a}\ \sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctan((1+(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2140, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\ \sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\ \sqrt{a}\ \sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x^3
]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.22, size = 667, normalized size = 9.14

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-bx^3\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+\cdots}\right.\right.}{a\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\left(8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},\cdots\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[a + b*x^3]),x]
```

```
[Out] (x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1
, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2*S
qrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a +
 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -(
(b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/
3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*Sqrt[3]
)*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -
((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1,
4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3
, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt
[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a
))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5
 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*
Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

fricas [A] time = 1.17, size = 1270, normalized size = 17.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 14
5152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x
^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4
*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*
a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 197
```

```
92*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
 + 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2
*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b
^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 860
16*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt(-(2*sqrt
(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520
*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7
*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x + sqrt(3)*(5*
a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^
(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 +
 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*
b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a +
 a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument Valueindex.cc ind
ex_m operator + Error: Bad Argument Value
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2))/(b*x^3+a)^(1/2)
,x)
```

```
[Out] int(((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2))/(b*x^3+a)^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{b x^{3} + a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) + s
qrt(3) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{b\,x^3+a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/
3) + 1)),x)
```

```
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/
3) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) +
 1 + sqrt(3))), x)
```

3.122 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctan((1-(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^3]
])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\,\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$=\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.15, size = 649, normalized size = 8.65

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+bx^3\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\left(3bx\right.\right.}{a\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{}{6}\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[a - b*x^3]),x]
```

```
[Out] (x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3
 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]
*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a +
6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*
x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 - (
b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a
)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
 + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(8*(5
+ 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*
Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)
/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a - b*x^3])
```

fricas [B] time = 1.17, size = 1324, normalized size = 17.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
 4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 9350
4*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
 + 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x
^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 +
5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3
```

```
*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*
x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*
a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3
 + a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^
14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 + a))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 +
 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)
*(a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^
7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 - 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 +
 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*b*x^2 + 2*sqr
t(-b*x^3 + a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 + a)*(sqrt(
3)*a + a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument Valueindex.cc ind
ex_m operator + Error: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}\right) \sqrt{-b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1/a*b)^(1/3)*x+1-3^(1/2))/(-(1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3+a)^(1
/2),x)
```

```
[Out] int((-(1/a*b)^(1/3)*x+1-3^(1/2))/(-(1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3+a)^(1
/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{-b x^{3} + a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) -
sqrt(3) - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{a-b\,x^3}\,\left(\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)),x)
```

```
[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}}{\sqrt{a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) -
 sqrt(3) - 1)), x)
```

3.123 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=76

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctanh((1-(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2140, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.64, size = 650, normalized size = 8.55

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+bx^3\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\left(3bx\right.\right.}{a\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)\left(3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\frac{bx^3}{a},\frac{}{6}\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[-a + b*x^3]),x]
```

```
[Out] (x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3
 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]
*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a +
6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*
x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 - (
b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a
)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
 + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(8*(5
+ 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*
Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)
/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a + b*x^3])
```

fricas [A] time = 1.14, size = 1273, normalized size = 16.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 + 145
152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7*b*x^
2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4*
x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*a
^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 1979
2*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
```

```
- 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 +
 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*
x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^
5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 8601
6*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864
*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6
 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt((2*sqrt(3
) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b^5
*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 328
704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20 +
4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b^2
*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7
*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a
^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x
^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*
b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a + a)
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument Valueindex.cc ind
ex_m operator + Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1-\sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+1+\sqrt{3}\right)\sqrt{b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1/a*b)^(1/3)*x+1-3^(1/2))/(-(1/a*b)^(1/3)*x+1+3^(1/2))/(b*x^3-a)^(1/
2),x)
```

```
[Out] int((-(1/a*b)^(1/3)*x+1-3^(1/2))/(-(1/a*b)^(1/3)*x+1+3^(1/2))/(b*x^3-a)^(1/
2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^{3}-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) - s
qrt(3) - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)),x)
```

```
[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}}{\sqrt{-a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) - 1)), x)
```

3.124 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=76

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctanh((1+(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2140, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.72, size = 670, normalized size = 8.82

$$x\left(-\frac{3\left(10496\sqrt{3}a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)+18176a^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-bx^3\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+\cdots}\right.\right.}{a\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)\left(8\left(5+3\sqrt{3}\right)aF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)-3bx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},\cdots\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[-a - b*x^3]),x]
```

```
[Out] (x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1
, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2*S
qrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a +
 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -(
(b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/
3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*Sqrt[3]
)*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -
((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1,
4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3
, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt
[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a
))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5
 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*
Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

fricas [B] time = 1.14, size = 1335, normalized size = 17.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504
*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
- 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 - 5
```

```
010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*
x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x
^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a
^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3
- a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^1
4 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 - a))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(
a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*
b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
+ 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 + 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 - 2*sq
rt(-b*x^3 - a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 - a)*(sqrt
(3)*a + a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.43index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 - \sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 + \sqrt{3}\right) \sqrt{-b\,x^{3} - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3-a)^(1/2
),x)
```

```
[Out] int(((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3-a)^(1/2
),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{-b x^{3} - a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) +
sqrt(3) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{-b\,x^3-a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) + 1)),x)
```

```
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{-a-bx^3}\,\left(x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
+ 1 + sqrt(3))), x)
```

3.125 $\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x
)/(1+x+3^(1/2))^2)^(1/2)-arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(3
+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2141, 218, 2140, 203}

$$\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3 + 2*Sqrt[3]])
+ (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{12}\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \text{Subst}\left(\int \frac{}{1+(3}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}} + \frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.42, size = 269, normalized size = 1.86

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left((1+\right.\right.\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*
((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqr
t[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2
*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 + x^3])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}{x^4+x^3-3\,x^2+4\,x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x - sqrt(3) + 1)/(x^4 + x^3 - 3*x^2 + 4*x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

maple [B] time = 0.04, size = 245, normalized size = 1.69

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.126 $\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x
)/(1+x+3^(1/2))^2)^(1/2)-arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/
(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2141, 218, 2140, 206}

$$\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[-3 + 2*Sqrt[3]
]) + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{12}\int \frac{\left(1-\sqrt{3}\right)\left(-22+\left(1-\sqrt{3}\right)^3\right)+6x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \text{Subst}\left(\int \frac{}{1+}\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}} + \frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.42, size = 267, normalized size = 1.84

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2i\sqrt{-2ix+\sqrt{3}+i}\,\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\,\left(\right.\right.}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\,\sqrt{x^3+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (2*I
)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/
(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3]
 - (2*I)*x]*Sqrt[1 + x^3])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}{x^4+x^3-3\,x^2+4\,x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x + sqrt(3) + 1)/(x^4 + x^3 - 3*x^2 + 4*x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

maple [B] time = 0.04, size = 245, normalized size = 1.69

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

3.127 $\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=173

$$\frac{\left(e-\sqrt{3}f-f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-7-\right.}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(e-f*(1-3^(1
/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)/(
x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+arctan((1+x)*(3+2*3^(1/2))^(1/2)
/(x^3+1)^(1/2))*(e-f-f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2141, 218, 2140, 203}

$$\frac{\left(e-\sqrt{3}f-f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-7-\right.}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((e - f - Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/S
qrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 + Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{2\sqrt{3}} + \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)}$$

$$= \frac{\sqrt{2+\sqrt{3}}\,\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}} -$$

$$= \frac{\left(e-f-\sqrt{3}f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2+\sqrt{3}}\,\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x\cdots}{(1+\sqrt{3}\cdots}}}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}$$

Mathematica [C] time = 0.63, size = 291, normalized size = 1.68

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2\sqrt{-2ix+\sqrt{3}+i}\,\sqrt{x^2-x+1}\,\left(\left(3+\sqrt{3}\right)f-\sqrt{3}e\right)\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\cdots}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2*
I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqr
t[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] +
 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x
+ x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sq
rt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I +
 (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx^2+\left(e+f\right)x-\sqrt{3}\left(fx+e\right)+e\right)\sqrt{x^3+1}}{x^5+2\,x^4-2\,x^3+x^2+2\,x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^2 + (e + f)*x - sqrt(3)*(f*x + e) + e)*sqrt(x^3 + 1)/(x^5 + 2
*x^4 - 2*x^3 + x^2 + 2*x - 2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.04, size = 260, normalized size = 1.50

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, f \,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-f-\sqrt{3}\, f\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-f-f*3^(1/2))*(3/2-1/2*I*
3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+
1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2
*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.128 $\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=187

$$-\frac{\left(e+\sqrt{3}f+f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7\right.}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(e+f*(1-3^(
1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4)/
(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-arctan((1-x)*(3+2*3^(1/2))^(1/
2)/(-x^3+1)^(1/2))*(e+f+f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2141, 218, 2140, 203}

$$-\frac{\left(e+\sqrt{3}f+f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7\right.}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])
/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 + Sqrt[3]]*(e + (1 - Sqrt[3])*f)*(1 - x
)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] -
 x)^2]*Sqrt[1 - x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{\left(-e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(28-\left(1+\sqrt{3}\right)^3\right)} + \frac{\left(6e-\left(1+\sqrt{3}\right)\left(22-\left(1+\right.\right.\right.}{\left(1+\sqrt{3}\right)\left(28-\left(1\right.\right.}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right.}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{\left(e+f+\sqrt{3}f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2+\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\ldots}}{3^{3/4}\sqrt{\frac{1-\ldots}{(1+\sqrt{\ldots}}}}$$

Mathematica [C] time = 0.58, size = 291, normalized size = 1.56

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\sqrt{3}e+\left(3+\sqrt{3}\right)f\right)\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{2ix+\sqrt{3}\ldots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqrt
[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*EllipticF
[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I +
Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqr
t[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))
/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx^2+\left(e-f\right)x+\sqrt{3}\left(fx+e\right)-e\right)\sqrt{-x^3+1}}{x^5-2\,x^4-2\,x^3-x^2+2\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^2 + (e - f)*x + sqrt(3)*(f*x + e) - e)*sqrt(-x^3 + 1)/(x^5 -
2*x^4 - 2*x^3 - x^2 + 2*x + 2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.04, size = 264, normalized size = 1.41

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,f\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2i\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-e-f-3^(1/2)*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*
3^(1/2)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx-\int \frac{fx}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
) - Integral(f*x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3
)), x)
```

3.129 $\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=190

$$-\frac{\left(e+\sqrt{3}f+f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7\right.}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(e+f*(1-3^(
1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(1/4)/
(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-arctanh((1-x)*(3+2*3^(1/2))^(1
/2)/(x^3-1)^(1/2))*(e+f+f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2141, 219, 2140, 206}

$$-\frac{\left(e+\sqrt{3}f+f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7\right.}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]
])/Sqrt[3*(3 + 2*Sqrt[3])]]) - (Sqrt[2 - Sqrt[3]]*(e + (1 - Sqrt[3])*f)*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[
3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\left(-e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)} + \frac{\left(-6e-\left(1+\sqrt{3}\right)\left(-22+\right.\right.}{\left(1+\sqrt{3}\right)\left(-2\right.}$$

$$= -\frac{\sqrt{2-\sqrt{3}}\,\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{\right.}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

$$= -\frac{\left(e+f+\sqrt{3}f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}\,\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)}{3^{3/4}\sqrt{-\frac{}{(1}}}$$

Mathematica [C] time = 0.47, size = 289, normalized size = 1.52

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{2ix+\sqrt{3}+i}\,\sqrt{x^2+x+1}\,\left(\sqrt{3}e+\left(3+\sqrt{3}\right)f\right)\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{2ix+\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqrt
[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*EllipticF
[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I +
Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqr
t[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))
/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(fx^2+\left(e-f\right)x+\sqrt{3}\left(fx+e\right)-e\right)\sqrt{x^3-1}}{x^5-2x^4-2x^3-x^2+2x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(f*x^2 + (e - f)*x + sqrt(3)*(f*x + e) - e)*sqrt(x^3 - 1)/(x^5 -
2*x^4 - 2*x^3 - x^2 + 2*x + 2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.04, size = 262, normalized size = 1.38

$$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, f \operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(-e-f-\sqrt{3}\,f\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(-e-f-3^(1/2)*f)*(-3/2-1/2*
I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1
/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3
-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1
/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx-\int \frac{fx}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
) - Integral(f*x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1
)), x)
```

3.130 $\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=183

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-\right.}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(e-f*(1-3^(1
/2)))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(1/4)/(
-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+arctanh((1+x)*(3+2*3^(1/2))^(1
/2)/(-x^3-1)^(1/2))*(e-f*(1+3^(1/2)))/(9+6*3^(1/2))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2141, 219, 2140, 206}

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-\right.}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((e - (1 + Sqrt[3])*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]
])/Sqrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 - Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 +
x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] +
x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{2\sqrt{3}} + \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx}{12\sqrt{3}}$$

$$= \frac{\sqrt{2-\sqrt{3}}\,\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}\,\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\cdots}}{3^{3/4}\sqrt{-\frac{1}{\left(1-\cdots\right.}}}$$

Mathematica [C] time = 0.49, size = 293, normalized size = 1.60

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2\sqrt{-2ix+\sqrt{3}+i}\,\sqrt{x^2-x+1}\,\left(\left(3+\sqrt{3}\right)f-\sqrt{3}e\right)\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\cdots}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2*
I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqr
t[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] +
 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x
+ x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sq
rt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I +
 (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(fx^2+\left(e+f\right)x-\sqrt{3}\left(fx+e\right)+e\right)\sqrt{-x^3-1}}{x^5+2\,x^4-2\,x^3+x^2+2\,x-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(f*x^2 + (e + f)*x - sqrt(3)*(f*x + e) + e)*sqrt(-x^3 - 1)/(x^5 +
 2*x^4 - 2*x^3 + x^2 + 2*x - 2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.04, size = 258, normalized size = 1.41

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,f\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2i(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-f-3^(1/2)*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2
*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^{3}-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{-(x+1)\left(x^{2}-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.131 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=332

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{b}\,e-\left(1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\ (\cdots}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\cdots$$

```
[Out] -arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))*
(b^(1/3)*e-a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)-1/3*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*e-a^(1/3)*f*(1+3^(1/2)))*(1/2*6^
(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(1/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2141, 218, 2140, 206}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{b}\,e-\left(1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\ (\cdots}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1
/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3))) - (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*
a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3 ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{12\sqrt{3}\,a^{4/3}b^{4/3}} - \frac{(6\cdots}{\cdots}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\cdots\right.}}}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{\cdots}}$$

$$= -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}\,b^{2/3}} - \frac{\sqrt{2+\cdots}}{\cdots}$$

Mathematica [C] time = 1.83, size = 438, normalized size = 1.32

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\left(\sqrt{3}-1\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(i+\sqrt{\cdots}}{(-\cdots}}\right.\right.\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x
]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1/2*I)*3^(1/
4)*f*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqr
t[I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^
(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[
```

```
3])/2])/Sqrt[2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a^(1/3)*f)*Sqrt[((-2*I)*a^(
1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*
I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3
)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a
+ b*x^3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: catde
f: division by zero
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+\left(1-\sqrt{3}\right)a^{\frac{1}{3}}\right)\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{bx^{3}+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{b\,x^{3}+a}\,\left(b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] int((e + f*x)/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/
3)*x)), x)
```

3.132 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=336

$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+ \left(\left(1 \ldots\right.\right.$$

```
[Out] arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))*
(b^(1/3)*e+a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)+1/3*
(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+
a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*e+a^(1/3)*f*(1+3^(1/2)))*(1/2*
6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(1/3)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{2141, 218, 2140, 206\}$

$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+ \left(\left(1 \ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6
)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[a
]*b^(2/3)) + (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(
1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = -\frac{\left(\sqrt[3]{b}\,e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3 ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx}{12\sqrt{3}\,a^{4/3}b^{4/3}} + \frac{(6}{}$$

$$= \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}\,e+\left(1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{a}}$$

$$= \frac{\left(\sqrt[3]{b}\,e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}\,b^{2/3}} + \frac{\sqrt{2+\sqrt{3}}}{}$$

Mathematica [C] time = 1.67, size = 466, normalized size = 1.39

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{\frac{\left(\sqrt{3}-i\right)\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}\,x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{-\frac{i((1-}{}}\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x
]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
+ I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF
[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3
])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a^(1/3
```

```
)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[
(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/
((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

fricas [F] time = 32.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{2\left(2\,bfx^4+2\,bex^3-2\,afx-2\,ae-\sqrt{3}\left(bfx^4+bex^3+2\,afx+2\,ae\right)\right)\sqrt{-bx^3+a}\,a^{\frac{2}{3}}+\left(bfx^5+bex^4+8\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral((2*(2*b*f*x^4 + 2*b*e*x^3 - 2*a*f*x - 2*a*e - sqrt(3)*(b*f*x^4 + b
*e*x^3 + 2*a*f*x + 2*a*e))*sqrt(-b*x^3 + a)*a^(2/3) + (b*f*x^5 + b*e*x^4 +
8*a*f*x^2 + 8*a*e*x - sqrt(3)*(b*f*x^5 + b*e*x^4 - 4*a*f*x^2 - 4*a*e*x))*sq
rt(-b*x^3 + a)*a^(1/3)*b^(1/3) + (b*f*x^6 + b*e*x^5 - 10*a*f*x^3 - 10*a*e*x
^2 - 6*sqrt(3)*(a*f*x^3 + a*e*x^2))*sqrt(-b*x^3 + a)*b^(2/3))/(b^3*x^9 - 21
*a*b^2*x^6 + 12*a^2*b*x^3 + 8*a^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}}x+\left(1-\sqrt{3}\right)a^{\frac{1}{3}}\right)\sqrt{-b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1)))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{e+f\,x}{\sqrt{a-b\,x^3}\,\left(b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}\,-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] int(-(e + f*x)/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-\sqrt[3]{a}\,\sqrt{a-bx^3}\,+\sqrt{3}\,\sqrt[3]{a}\,\sqrt{a-bx^3}\,+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\,dx-\int \frac{fx}{-\sqrt[3]{a}\,\sqrt{a-bx^3}\,+\sqrt{3}\,\sqrt[3]{a}\,\sqrt{a-bx^3}\,+\sqrt[3]{b}\,x\sqrt{a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3)
 + b**(1/3)*x*sqrt(a - b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(a - b*x*
*3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

3.133 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=345

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\left(\left(1\right.\right.}{}$$

```
[Out] 1/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3
)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*e+a^(1/3)*f*(1+3^(1/2)))*(
(a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(1/4)/a^(1/3)/b^(2/3)/(b*x^3-a)^(1/2)/(-
a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+arcta
n(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))*(b^(1/3
)*e+a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2141, 219, 2140, 203}

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\left(\left(1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)
*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[a
]*b^(2/3)) + (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*a^(
1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3)
- b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3 ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{12\sqrt{3}\,a^{4/3}b^{4/3}} - \cdots$$

$$= \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\cdots}}\cdots}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{\cdots}}$$

$$= \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}\,b^{2/3}} + \frac{\sqrt{2-\sqrt{3}}\cdots}{\cdots}$$

Mathematica [C] time = 0.58, size = 467, normalized size = 1.35

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(\sqrt{3}-i\right)\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(1-\cdots\right.}{\cdots}}\right.\right.\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),
x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
+ I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF
[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3
])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a^(1/3
```

```
)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[
(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/
((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

fricas [F] time = 32.38, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{bx^3-a}\left(2\left(2\,bfx^4+2\,bex^3-2\,afx-2\,ae-\sqrt{3}\left(bfx^4+bex^3+2\,afx+2\,ae\right)\right)a^{\frac{2}{3}}+\left(bfx^5+bex^4+8\right.\right.}{b^3x^9-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(b*x^3 - a)*(2*(2*b*f*x^4 + 2*b*e*x^3 - 2*a*f*x - 2*a*e - sqr
t(3)*(b*f*x^4 + b*e*x^3 + 2*a*f*x + 2*a*e))*a^(2/3) + (b*f*x^5 + b*e*x^4 +
8*a*f*x^2 + 8*a*e*x - sqrt(3)*(b*f*x^5 + b*e*x^4 - 4*a*f*x^2 - 4*a*e*x))*a^
(1/3)*b^(1/3) + (b*f*x^6 + b*e*x^5 - 10*a*f*x^3 - 10*a*e*x^2 - 6*sqrt(3)*(a
*f*x^3 + a*e*x^2))*b^(2/3))/(b^3*x^9 - 21*a*b^2*x^6 + 12*a^2*b*x^3 + 8*a^3)
, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}}x+\left(1-\sqrt{3}\right)a^{\frac{1}{3}}\right)\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),
 x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt{3}\,\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx-\int \frac{fx}{-\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt{3}\,\sqrt[3]{a}\,\sqrt{-a+bx^3}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(e/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**
3) + b**(1/3)*x*sqrt(-a + b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(-a +
b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)
), x)
```

3.134 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=345

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\cdots$$

```
[Out] -1/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)
*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*e-a^(1/3)*f*(1+3^(1/2)))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(
1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(1/4)/a^(1/3)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-arctan(
a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))*(b^(1/3)
*e-a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2141, 219, 2140, 203}

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/
6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3))) - (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*
a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\left(\sqrt[3]{b}\,e-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx}{12\sqrt{3}\,a^{4/3}b^{4/3}} + \cdots$$

$$= -\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}\,e-\left(1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}}\cdots}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}\cdots}}$$

$$= -\frac{\left(\sqrt[3]{b}\,e-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}\,b^{2/3}} - \frac{\sqrt{2-\cdots}}{\cdots}$$

Mathematica [C] time = 0.93, size = 441, normalized size = 1.28

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}\,x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}\left(\left(\sqrt{3}-1\right)\sqrt[3]{a}\,f+\sqrt[3]{b}\,e\right)\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(i+\cdots}{(-\cdots}}\right.\right.\cdots\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),
x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1/2*I)*3^(1/
4)*f*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqr
t[I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^
(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[
```

```
3])/2])/Sqrt[2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a^(1/3)*f)*Sqrt[((-2*I)*a^(
1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*
I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3
)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a
 - b*x^3])
```

fricas [F] time = 32.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{2\left(2\,bfx^4+2\,bex^3+2\,afx+2\,ae-\sqrt{3}\left(bfx^4+bex^3-2\,afx-2\,ae\right)\right)\sqrt{-bx^3-a}\,a^{\frac{2}{3}}-\left(bfx^5+bex^4-8\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(2*(2*b*f*x^4 + 2*b*e*x^3 + 2*a*f*x + 2*a*e - sqrt(3)*(b*f*x^4 +
b*e*x^3 - 2*a*f*x - 2*a*e))*sqrt(-b*x^3 - a)*a^(2/3) - (b*f*x^5 + b*e*x^4 -
 8*a*f*x^2 - 8*a*e*x - sqrt(3)*(b*f*x^5 + b*e*x^4 + 4*a*f*x^2 + 4*a*e*x))*s
qrt(-b*x^3 - a)*a^(1/3)*b^(1/3) + (b*f*x^6 + b*e*x^5 + 10*a*f*x^3 + 10*a*e*
x^2 + 6*sqrt(3)*(a*f*x^3 + a*e*x^2))*sqrt(-b*x^3 - a)*b^(2/3))/(b^3*x^9 + 2
1*a*b^2*x^6 + 12*a^2*b*x^3 - 8*a^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+\left(1-\sqrt{3}\right)a^{\frac{1}{3}}\right)\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),
 x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{-a - bx^3} \left(-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1
/3)*x)), x)
```

3.135 $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}-\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

```
[Out] -1/3*arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))*2^(1/2)*3^(1/4)+1/3*(1
+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2-x+1)
/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2141, 218, 2140, 203}

$$\frac{\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}-\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4)) +
(Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)
/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\left(-1-\sqrt{3}\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)} + \frac{\left(-22+\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{-28+\left(1+\sqrt{3}\right)^3}$$

$$= \frac{\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(12\left(-1-\sqrt{3}\right)\right)\text{Sub}}{\left(1+\sqrt{3}\right)}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-\right.}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.50, size = 209, normalized size = 1.54

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}} + \frac{2i\left(1+\sqrt{3}\right)\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(3 + (2 + I)*Sqrt[3])))/Sqrt[1 + x^3]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x^2-\sqrt{3}\,x+x\right)}{x^5+2\,x^4-2\,x^3+x^2+2\,x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x^2 - sqrt(3)*x + x)/(x^5 + 2*x^4 - 2*x^3 + x^2 + 2
*x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

maple [B] time = 0.04, size = 255, normalized size = 1.88

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(-1-3^(1/2))*(3/2-1/2*I*3^(1/
2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1
/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^
(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.136 $\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}}$$

```
[Out] -1/3*arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))*2^(1/2)*3^(1/4)+1/3*(
1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2+x+1
)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/
2)
```

Rubi [A] time = 0.22, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2141, 218, 2140, 203}

$$\frac{\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/3^(3/4)) +
(Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/((((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{6\left(3-\sqrt{3}\right)} - \frac{\left(22-\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{1-x^3}}\,dx}{28-\left(1+\sqrt{3}\right)^3}$$

$$= \frac{\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,c\right.}{3-\sqrt{3}}$$

$$= -\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right.}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.66, size = 232, normalized size = 1.53

$$\frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(2\left(1+\sqrt{3}\right)\sqrt{x^2+x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)+\frac{i\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\left(\left(3+(2+i)\sqrt{3}\right)x+(1+2i)\sqrt{\right.}{\sqrt{\frac{1-(-1)^{2/3}}{1+\sqrt[3]{-1}}}}\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]*Ell
ipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[1 - x^3])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^3+1}\left(x^2+\sqrt{3}\,x-x\right)}{x^5-2\,x^4-2\,x^3-x^2+2\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x^2 + sqrt(3)*x - x)/(x^5 - 2*x^4 - 2*x^3 - x^2 +
2*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

maple [B] time = 0.04, size = 257, normalized size = 1.69

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2i\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2/3*I*(-1-3^(1/2))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2
)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{1-x^3}-\sqrt{3}\,\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
)
```

3.137 $\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=164

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{3^{3/4}}$$

```
[Out] -1/3*arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))*2^(1/2)*3^(1/4)+2/3*(
1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/3*6^(1/2)-1/2*
2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1
-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2141, 219, 2140, 206}

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/3^(3/4))
+ (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\left(-1-\sqrt{3}\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)} - \frac{\left(-22+\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{-28+\left(1+\sqrt{3}\right)^3}$$

$$= \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(12\left(-1-\sqrt{\right.\right.}}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-}{1-\sqrt{3}-}\right.\right.}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.27, size = 230, normalized size = 1.40

$$\frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(2\left(1+\sqrt{3}\right)\sqrt{x^2+x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)+\frac{i\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\left(\left(3+(2+i)\sqrt{3}\right)x+(1+2\right.}{\sqrt{\frac{1-(}{1+}}}\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]*Ell
ipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-1 + x^3])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{x^3-1}\left(x^2+\sqrt{3}\,x-x\right)}{x^5-2\,x^4-2\,x^3-x^2+2\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 - 1)*(x^2 + sqrt(3)*x - x)/(x^5 - 2*x^4 - 2*x^3 - x^2 +
2*x + 2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.03, size = 255, normalized size = 1.55

$$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{-\frac{3}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(-1-3^(1/2))*(-3/2-1/2*I*3^(1
/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1
/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3
^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
)
```

3.138 $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{3^{3/4}}$$

```
[Out] -1/3*arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))*2^(1/2)*3^(1/4)+2/3*
(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/3*6^(1/2)-1/2
*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/
(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2141, 219, 2140, 206}

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/3^(3/4))
+ (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx}{6\left(3-\sqrt{3}\right)} + \frac{\left(22-\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{28-\left(1+\sqrt{3}\right)^3}$$

$$= \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{2\,\text{Subst}\left(\int \frac{}{1-(} \right.}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\right.}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.18, size = 211, normalized size = 1.35

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}} + \frac{2i\left(1+\sqrt{3}\right)\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(3 + (2 + I)*Sqrt[3])))/Sqrt[-1 - x^3]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\left(x^2-\sqrt{3}\,x+x\right)}{x^5+2x^4-2x^3+x^2+2x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(x^2 - sqrt(3)*x + x)/(x^5 + 2*x^4 - 2*x^3 + x^2 +
 2*x - 2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.03, size = 253, normalized size = 1.62

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2i\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-2/3*I*(-1-3^(1/2))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1
/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2*I*3^
(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^{3}-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(x+1\right)\left(x^{2}-x+1\right)}\,\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.139 $\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=147

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

```
[Out] -1/3*arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))*2^(1/2)*3^(1/4)+2/3*
(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/3*6^(1/2)+1/2
*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1
+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2141, 218, 2140, 206}

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4))
+ (2*Sqrt[7/6 + 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\int \frac{\left(1-\sqrt{3}\right)\left(-22+\left(1-\sqrt{3}\right)^3\right)+6x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{6\left(3+\sqrt{3}\right)} + \frac{\left(-22+\left(1-\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{-28+\left(1-\sqrt{3}\right)^3}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{2\,\text{Subst}\left(\int \frac{}{1}\right.}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.57, size = 225, normalized size = 1.53

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\frac{\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(\left((1+2i)\sqrt{3}-3i\right)x-(2+i)\sqrt{3}+3\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}} - 2\left(\sqrt{3}-1\right)\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)}\right.\right.}{\left((1+2i)\sqrt{3}-3i\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 + (
-1)^(1/3))]*(3 - (2 + I)*Sqrt[3] + (-3*I + (1 + 2*I)*Sqrt[3])*x)*EllipticF[
ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))] - 2*(-1 + Sqrt[3])*Sqrt[1 - x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[1 + x^3]
)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x^2+\sqrt{3}\,x+x\right)}{x^5+2\,x^4-2\,x^3+x^2+2\,x-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x^2 + sqrt(3)*x + x)/(x^5 + 2*x^4 - 2*x^3 + x^2 + 2
*x - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

maple [B] time = 0.04, size = 255, normalized size = 1.73

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(1-3^(1/2))*(3/2-1/2*I*3^(1/2
))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/
2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^
(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

3.140 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=278

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}(}{\sqrt{a+b}}\right.}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}$$

```
[Out] -1/3*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/
2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
1/3*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2141, 218, 2140, 206}

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}(}{\sqrt{a+b}}\right.}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqr
t[a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6 + 2/Sqrt[3]]*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3 ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} + \frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}\ldots}{\left((1\ldots\right.}}}{\sqrt[4]{3}\,b^{2/3}\ldots}$$

Mathematica [C] time = 1.25, size = 427, normalized size = 1.54

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\right|\ldots\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2\ldots}}{\left(1+\sqrt[3]{-1}\ldots\right.}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1/2*I)*3^(1/
4)*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqrt[
I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1
/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3]
)/2])/Sqrt[2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3
])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b
^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), Arc
```

```
Sin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/
3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) +
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\left(2\left(2\,bx^4+2\,ax-\sqrt{3}\left(bx^4-2\,ax\right)\right)a^{\frac{2}{3}}-\left(bx^5-8\,ax^2-\sqrt{3}\left(bx^5+4\,ax^2\right)\right)a^{\frac{1}{3}}b^{\frac{1}{3}}+\left(bx^6+6\right.\right.}{b^3x^9+21\,ab^2x^6+12\,a^2bx^3-8\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*(2*(2*b*x^4 + 2*a*x - sqrt(3)*(b*x^4 - 2*a*x))*a^(
2/3) - (b*x^5 - 8*a*x^2 - sqrt(3)*(b*x^5 + 4*a*x^2))*a^(1/3)*b^(1/3) + (b*x
^6 + 6*sqrt(3)*a*x^3 + 10*a*x^3)*b^(2/3))/(b^3*x^9 + 21*a*b^2*x^6 + 12*a^2*
b*x^3 - 8*a^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b^{\frac{1}{3}}x+\left(1-\sqrt{3}\right)a^{\frac{1}{3}}\right)\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)),
x)
```

3.141 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=286

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,(}{\sqrt{a-b}}\right.}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}$$

```
[Out] -1/3*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1
/2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+2/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I
)*(1/3*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2141, 218, 2140, 206}

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,(}{\sqrt{a-b}}\right.}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqr
t[a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6 + 2/Sqrt[3]]*(a^(1/
3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)^2]*Sqrt[a - b*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} - \frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)\right.}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{a-bx^3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}\cdots}{\left(\left(1\cdots\right.\right.}}}{\sqrt[4]{3}\,b^{2/3}\cdots}$$

Mathematica [C] time = 1.35, size = 454, normalized size = 1.59

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\cdots\right.}{\left(-3i+\sqrt{3}\right)\cdots}}\right.\right.\right.}{\left(3-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])
*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[A
rcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])
*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 +
 (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3
```

```
)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt
[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[a - b*x^3])
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{2\left(2\,bx^4-2\,ax-\sqrt{3}\left(bx^4+2\,ax\right)\right)\sqrt{-bx^3+a}\,a^{\frac{2}{3}}+\left(bx^5+8\,ax^2-\sqrt{3}\left(bx^5-4\,ax^2\right)\right)\sqrt{-bx^3+a}\,a^{\frac{1}{3}}b^{\frac{1}{3}}}{b^3x^9-21\,ab^2x^6+12\,a^2bx^3+8\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral((2*(2*b*x^4 - 2*a*x - sqrt(3)*(b*x^4 + 2*a*x))*sqrt(-b*x^3 + a)*a^
(2/3) + (b*x^5 + 8*a*x^2 - sqrt(3)*(b*x^5 - 4*a*x^2))*sqrt(-b*x^3 + a)*a^(1
/3)*b^(1/3) + (b*x^6 - 6*sqrt(3)*a*x^3 - 10*a*x^3)*sqrt(-b*x^3 + a)*b^(2/3)
)/(b^3*x^9 - 21*a*b^2*x^6 + 12*a^2*b*x^3 + 8*a^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-b^{\frac{1}{3}}x+\left(1-\sqrt{3}\right)a^{\frac{1}{3}}\right)\sqrt{-b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"maxima")
```

```
[Out] -integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3)
 + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

3.142 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=282

$$\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}$$

```
[Out] -1/3*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2
))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+1/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*
2^(1/2)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)*3^(1/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*
x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2141, 219, 2140, 203}

$$\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt
[-a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
+ b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx = -\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3 ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} - \frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)\middle|-7+\ldots\right.}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{-a+bx^3}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}} + \frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x\ldots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\ldots\right.}}\ldots}{3^{3/4}b^{2/3}\sqrt{-\ldots}}$$

Mathematica [C] time = 0.43, size = 455, normalized size = 1.61

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\ldots\right.}{\left(-3i+\sqrt{3}\right)\ldots}}\right.\right.\right.}{\left(3-\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])
*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[A
rcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])
*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 +
 (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3
```

```
)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt
[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[-a + b*x^3])
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{bx^3-a}\left(2\left(2\,bx^4-2\,ax-\sqrt{3}\left(bx^4+2\,ax\right)\right)a^{\frac{2}{3}}+\left(bx^5+8\,ax^2-\sqrt{3}\left(bx^5-4\,ax^2\right)\right)a^{\frac{1}{3}}b^{\frac{1}{3}}+\left(bx^6-6\right.\right.}{b^3x^9-21\,ab^2x^6+12\,a^2bx^3+8\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(b*x^3 - a)*(2*(2*b*x^4 - 2*a*x - sqrt(3)*(b*x^4 + 2*a*x))*a^
(2/3) + (b*x^5 + 8*a*x^2 - sqrt(3)*(b*x^5 - 4*a*x^2))*a^(1/3)*b^(1/3) + (b*
x^6 - 6*sqrt(3)*a*x^3 - 10*a*x^3)*b^(2/3))/(b^3*x^9 - 21*a*b^2*x^6 + 12*a^2
*b*x^3 + 8*a^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-b^{\frac{1}{3}}x+\left(1-\sqrt{3}\right)a^{\frac{1}{3}}\right)\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="
maxima")
```

```
[Out] -integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt{3}\,\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**
3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

3.143 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}$$

```
[Out] -1/3*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/
2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+1/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2
^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)*3^(1/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2141, 219, 2140, 203}

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt
[-a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a
- b*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2141

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> -Dist[(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3 ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} + \frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+\cdots\right.}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}} + \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}}}{3^{3/4}b^{2/3}\sqrt{-\cdots}}$$

Mathematica [C] time = 0.98, size = 430, normalized size = 1.55

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\cdots\right.\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\cdots}{\left(1+\sqrt[3]{-1}\right)\cdots}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1/2*I)*3^(1/
4)*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqrt[
I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1
/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3]
)/2])/Sqrt[2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3
])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b
^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), Arc
```

```
Sin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/
3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) +
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{2\left(2\,bx^4+2\,ax-\sqrt{3}\left(bx^4-2\,ax\right)\right)\sqrt{-bx^3-a}\,a^{\frac{2}{3}}-\left(bx^5-8\,ax^2-\sqrt{3}\left(bx^5+4\,ax^2\right)\right)\sqrt{-bx^3-a}\,a^{\frac{1}{3}}b^{\frac{1}{3}}}{b^3x^9+21\,ab^2x^6+12\,a^2bx^3-8\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-(2*(2*b*x^4 + 2*a*x - sqrt(3)*(b*x^4 - 2*a*x))*sqrt(-b*x^3 - a)*a
^(2/3) - (b*x^5 - 8*a*x^2 - sqrt(3)*(b*x^5 + 4*a*x^2))*sqrt(-b*x^3 - a)*a^(
1/3)*b^(1/3) + (b*x^6 + 6*sqrt(3)*a*x^3 + 10*a*x^3)*sqrt(-b*x^3 - a)*b^(2/3
))/(b^3*x^9 + 21*a*b^2*x^6 + 12*a^2*b*x^3 - 8*a^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b^{\frac{1}{3}}x+\left(1-\sqrt{3}\right)a^{\frac{1}{3}}\right)\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)),
 x)
```

3.144 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=317

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1-\sqrt{3}\right)d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\right.\right.}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x}}$$

```
[Out] -(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)
/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*(c-d*(1+3^(1/2)))*((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/2)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)-4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1+
x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6^(1
/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c-d*(1-3^(1/2)))/(x^3+1
)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 319, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2142, 2113, 537, 571, 93, 205}

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{(c-}{(c-}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ar
cTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sq
rt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])) - (4*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*E
llipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-c+\left(1-\sqrt{3}\right)d+\left(-c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{}}}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+d+\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-\right.\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{\left(c-d-\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1}{}$$

Mathematica [C] time = 0.64, size = 214, normalized size = 0.68

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}\left(c-\left(1+\sqrt{3}\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*(c - (1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-
1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]
)/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

maple [A] time = 0.07, size = 275, normalized size = 0.87

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}\,d}+\frac{2\left(-c+\sqrt{3}\,d+d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x}{\frac{3}{2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x)
```

```
[Out] 2/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(d*3^(1/2)-c+d)/d^2*(3/2-1/2*
I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3+1)^(1/2)/(-1+c/d)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*
I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.145 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=329

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\Big(}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-}}$$

```
[Out] -(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c
+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*(c+d+d*3^(1/2))*((x^2+x+1)/(1-
x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/2)/
((1-x)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-
x+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c+d-d*3^(1/2))/(-x^3+1)^(1/
2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 1.30, antiderivative size = 331, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2142, 2113, 537, 571, 93, 208}

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c-\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\right.}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcT
anh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt
[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])) + (4*3
^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*El
lipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3]
- x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1-\sqrt{3}\right)d+\left(c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\,\sqrt{7-4\sqrt{3}+}}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+(1-\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} - \ldots$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} - \ldots$$

$$= -\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\ldots}{\ldots}$$

Mathematica [C] time = 0.76, size = 235, normalized size = 0.71

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(3+\sqrt{3}\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\ldots}\right.\right.}{c-\sqrt[3]{-1}d}\right.}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x + x^2
]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^3
])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   catde
f: division by zero
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x - \sqrt{3} - 1}{\sqrt{-x^3 + 1}\,(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

maple [A] time = 0.07, size = 264, normalized size = 0.80

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}\, d} - \frac{2i\left(c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))-2/3*I*(c+d+3^(1/2)*d)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3
^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x - \sqrt{3} - 1}{\sqrt{-x^3 + 1}\,(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{3}}{c\sqrt{1-x^3} + dx\sqrt{1-x^3}}\right) dx - \int \frac{x}{c\sqrt{1-x^3} + dx\sqrt{1-x^3}}\, dx - \int \left(-\frac{1}{c\sqrt{1-x^3} + dx\sqrt{1-x^3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(x
/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x**3
) + d*x*sqrt(1 - x**3)), x)
```

3.146 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=325

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}\,d+d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\Pi\Bigg(\frac{(c+\sqrt{}}{(c-\sqrt{}}}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] -(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c
+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*(c+d+d*3^(1/2))*((x^2+x+1)/(1-
x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2)/(
(1-x)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-x
+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1
/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c+d-d*3^(1/2))/(x^3-1)^(1/2)
/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 327, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2142, 2113, 537, 571, 93, 208}

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c-\sqrt{3}\,d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}\right.}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcT
anh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt
[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) + (4*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*E
llipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3]
 - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1 - x
)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1-\sqrt{3}\right)d+\left(c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\cdots}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+(1-\cdots\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(2\cdots\right.}{\cdots}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(4\cdots\right.}{\cdots}$$

$$= -\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1\cdots}{\cdots}$$

Mathematica [C] time = 0.26, size = 233, normalized size = 0.72

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(3+\sqrt{3}\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(\cdots}{1+\cdots}}\right.\right.}{c-\sqrt[3]{-1}d}\cdots\right.}{3d\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x + x^2
]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x^
3])
```

fricas [F] time = 3.91, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}{dx^4+cx^3-dx-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(x^3 - 1)*(x - sqrt(3) - 1)/(d*x^4 + c*x^3 - d*x - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x - \sqrt{3} - 1}{\sqrt{x^3 - 1}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

maple [A] time = 0.07, size = 273, normalized size = 0.84

$$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}\,d}+\frac{2\left(c+d+\sqrt{3}\,d\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x)
[Out] -2/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(c+d+3^(1/2)*d)/d^2*(-3/2-1/2
*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-
1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3-1)^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x - \sqrt{3} - 1}{\sqrt{x^3 - 1}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx-\int\frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int\left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(x
/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 - 1
) + d*x*sqrt(x**3 - 1)), x)
```

3.147 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=321

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1-\sqrt{3}\right)d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\right.\right.}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-}}$$

```
[Out] -(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)
/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*(c-d*(1+3^(1/2)))*((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1/2)
/((1+x)/(1+x+3^(1/2))^2)^(1/2)-4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1
+x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c-d*(1-3^(1/2)))/(-x^3
-1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 323, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2142, 2113, 537, 571, 93, 205}

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{(c-}{(c-}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ar
cTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sq
rt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])) - (4
*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sq
rt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-c+\left(1-\sqrt{3}\right)d+\left(-c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\,\sqrt{7-4}}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\left(-c+d+\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}\,\sqrt{7-4\sqrt{3}+x^2}\,(\left(}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right.}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right.}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= -\frac{\left(c-d-\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{c^2+cd+d^2}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}}{}$$

Mathematica [C] time = 0.74, size = 233, normalized size = 0.73

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2-x+1}\left(\sqrt{3}c-\left(3+\sqrt{3}\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\right.\right.}{c+\sqrt[3]{-1}d}\right.}{3d\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c - (3 + Sqrt[3])*d)*Sqrt[1 - x + x^2
]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 - x^3
])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{-x^3 - 1}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

maple [A] time = 0.06, size = 266, normalized size = 0.83

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}\,d} - \frac{2i\left(-c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c+3^(1/2)*d+d)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/
3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1
/2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{-x^3 - 1}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 1 + \sqrt{3}}{\sqrt{-(x+1)\left(x^2 - x + 1\right)}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.148 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=358

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+}{\left(x-\sqrt{3}+\right.}}}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}\right.}}}$$

```
[Out] -(1+x)*arctanh(2*(c^2+c*d+d^2)^(1/2)*((-1-x)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^
(1/2)+1/2*2^(1/2))/(c-d)^(1/2)/d^(1/2)/(7+4*3^(1/2)+(1+x+3^(1/2))^2/(1+x-3^
(1/2))^2)^(1/2))*(c-d*(1-3^(1/2)))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)/(c-d)^
(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1
/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x-3^(1/2))/(1+x-3^(1/2)),(c-d*(1-3^(1/2)
))^2/(c-d*(1+3^(1/2)))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1
)/(1+x-3^(1/2))^2)^(1/2)/(-d*3^(1/2)+c-d)/(x^3+1)^(1/2)/((-1-x)/(1+x-3^(1/2
))^2)^(1/2)
```

Rubi [A] time = 1.00, antiderivative size = 360, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2143, 2113, 537, 571, 93, 208}

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\sqrt{3}d-d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1\right.\right.}{\sqrt{d}\sqrt{-\frac{x+}{\left(x-\sqrt{3}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ar
cTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*Sqrt[3] + (1 + Sqrt[3] + x)^2/(
1 - Sqrt[3] + x)^2])]))/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1
 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[1 + x^3])) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*
(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[3
])*d)^2/(c - (1 + Sqrt[3])*d)^2, -ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x
)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[1 + x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2143

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((-1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[
2 + Sqrt[3]]*f*(1 - q*x)*Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2])/(
q*Sqrt[a + b*x^3]*Sqrt[-((1 - q*x)/(1 - Sqrt[3] - q*x)^2)]), Subst[Int[1/((
(1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4
*Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*S
qrt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-c+\left(1+\sqrt{3}\right)d+\left(-c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\,\sqrt{7+4}}\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\left(-c+d+\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}\,\sqrt{7+4\sqrt{3}+x^2}\left((-\right.}\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}} +$$

$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}} +$$

$$= -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{c^2+cd+d^2}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{c^2+cd+d^2}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}} + 4\sqrt[4]{3}$$

Mathematica [C] time = 0.53, size = 213, normalized size = 0.59

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}\left(c+\left(\sqrt{3}-1\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*(c + (-1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (
-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

maple [A] time = 0.03, size = 275, normalized size = 0.77

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}\,d}-\frac{2\left(c+\sqrt{3}\,d-d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x)
```

```
[Out] 2/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3^(1/2)*d+c-d)/d^2*(3/2-1/2*
I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3+1)^(1/2)/(c/d-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I
*3^(1/2))/(c/d-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.149 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=346

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2};\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}\right.}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}}$$

```
[Out] -(1-x)*arctan((c^2-c*d+d^2)^(1/2)*((-1+x)/(1-x-3^(1/2))^2)^(1/2)/d^(1/2)/(c
+d)^(1/2)/((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2))*(c+d-d*3^(1/2))*((x^2+x+1)/(1-
x-3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/2)/
((-1+x)/(1-x-3^(1/2))^2)^(1/2)-4*3^(1/4)*(1-x)*EllipticPi((-1+x-3^(1/2))/(1
-x-3^(1/2)),(c+d-d*3^(1/2))^2/(c+d+d*3^(1/2))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)
-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)/(c+d+d*3^(1/2))/(-x^3+1)^(1
/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.98, antiderivative size = 348, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2143, 2113, 537, 571, 93, 205}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\tan^{-1}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{(c\cdots}{(c\cdots}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*ArcT
an[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)])/(Sqrt[d]*Sq
rt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*S
qrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])) -
 (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, -ArcSin[(1 + Sqr
t[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2143

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((-1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[
2 + Sqrt[3]]*f*(1 - q*x)*Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2])/(
q*Sqrt[a + b*x^3]*Sqrt[-((1 - q*x)/(1 - Sqrt[3] - q*x)^2)]), Subst[Int[1/((
(1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4
*Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*S
qrt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1+\sqrt{3}\right)d+\left(c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7+4\sqrt{3}}}\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(c+(1+\right.\right.}\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} +$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} +$$

$$= -\frac{\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(}{}$$

Mathematica [C] time = 0.70, size = 235, normalized size = 0.68

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(\sqrt{3}-3\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{1\right.\right.}{c-\sqrt[3]{-1}d}\right.}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^
3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x + \sqrt{3} - 1}{\sqrt{-x^3 + 1}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

maple [A] time = 0.03, size = 268, normalized size = 0.77

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{\frac{i\sqrt{3}}{}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}\;d}+\frac{2i\left(-c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))+2/3*I*(3^(1/2)*d-c-d)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(c/d-1/2+1/2*I*3^(1/2))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(c/d-1/2+1/2*I
*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x + \sqrt{3} - 1}{\sqrt{-x^3 + 1}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{3}}{c\sqrt{1-x^3} + dx\sqrt{1-x^3}}\,dx - \int \frac{x}{c\sqrt{1-x^3} + dx\sqrt{1-x^3}}\,dx - \int \left(-\frac{1}{c\sqrt{1-x^3} + dx\sqrt{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(x/
(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x**3)
 + d*x*sqrt(1 - x**3)), x)
```

3.150 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=342

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2};\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d\right.}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^{\cdots}}}$$

```
[Out] -(1-x)*arctan((c^2-c*d+d^2)^(1/2)*((-1+x)/(1-x-3^(1/2))^2)^(1/2)/d^(1/2)/(c
+d)^(1/2)/((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2))*(c+d-d*3^(1/2))*((x^2+x+1)/(1-
x-3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2)/(
(-1+x)/(1-x-3^(1/2))^2)^(1/2)-4*3^(1/4)*(1-x)*EllipticPi((-1+x-3^(1/2))/(1-
x-3^(1/2)),(c+d-d*3^(1/2))^2/(c+d+d*3^(1/2))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)-
1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)/(c+d+d*3^(1/2))/(x^3-1)^(1/2
)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 344, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2143, 2113, 537, 571, 93, 205}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\tan^{-1}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{(c-\sqrt{3}\cdots}{(c+\sqrt{3}\cdots}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*ArcT
an[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)])/(Sqrt[d]*Sq
rt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*S
qrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]))
- (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)
^2]*EllipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, -ArcSin[(1 + Sq
rt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sqrt[-
((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2143

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((-1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[
2 + Sqrt[3]]*f*(1 - q*x)*Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2])/(
q*Sqrt[a + b*x^3]*Sqrt[-((1 - q*x)/(1 - Sqrt[3] - q*x)^2)]), Subst[Int[1/((
(1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4
*Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*S
qrt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1+\sqrt{3}\right)d+\left(c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\,\sqrt{7+4\sqrt{3}+\cdots}}\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\left(c+d-\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\,\sqrt{7+4\sqrt{3}+x^2}\left(\left(c+\left(1+\cdots\right.\right.\right.}\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d-\sqrt{3}\,d\right)^2}{\left(c+d+\sqrt{3}\,d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}\,d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}} + \cdots$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d-\sqrt{3}\,d\right)^2}{\left(c+d+\sqrt{3}\,d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}\,d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}} + \cdots$$

$$= -\frac{\left(c+d-\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\,\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\,\sqrt{c+d}\,\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1\cdots}{\cdots}$$

Mathematica [C] time = 0.19, size = 233, normalized size = 0.68

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(\sqrt{3}-3\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1\cdots}{1+\sqrt[3]{\cdots}}}\right.\right.}{c-\sqrt[3]{-1}\,d}\right.}{3d\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x
^3])
```

fricas [F] time = 4.02, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}{dx^4+cx^3-dx-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(x^3 - 1)*(x + sqrt(3) - 1)/(d*x^4 + c*x^3 - d*x - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

maple [A] time = 0.03, size = 277, normalized size = 0.81

$$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}\,d}-\frac{2\left(-c+\sqrt{3}\,d-d\right)\left(-\frac{3}{2}-\frac{i}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x)
[Out] -2/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*(-c+3^(1/2)*d-d)/d^2*(-3/2-1/
2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x
^3-1)^(1/2)/(c/d+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*
I*3^(1/2))/(c/d+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int \frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int \left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(x/
(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 - 1)
 + d*x*sqrt(x**3 - 1)), x)
```

3.151 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=362

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+}{\left(x-\sqrt{3}+\right.}}}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+\right.}}}$$

```
[Out] -(1+x)*arctanh(2*(c^2+c*d+d^2)^(1/2)*((-1-x)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^
(1/2)+1/2*2^(1/2))/(c-d)^(1/2)/d^(1/2)/(7+4*3^(1/2)+(1+x+3^(1/2))^2/(1+x-3^
(1/2))^2)^(1/2))*(c-d*(1-3^(1/2)))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)/(c-d)^
(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(
1/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x-3^(1/2))/(1+x-3^(1/2)),(c-d*(1-3^(1/2
)))^2/(c-d*(1+3^(1/2)))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+
1)/(1+x-3^(1/2))^2)^(1/2)/(-d*3^(1/2)+c-d)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1
/2))^2)^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 364, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2143, 2113, 537, 571, 93, 208}

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\sqrt{3}d-d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-(1\right.}{\sqrt{d}\sqrt{-\frac{x+}{\left(x-\sqrt{3}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ar
cTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*Sqrt[3] + (1 + Sqrt[3] + x)^2/(
1 - Sqrt[3] + x)^2)])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1
 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[
3])*d)^2/(c - (1 + Sqrt[3])*d)^2, -ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] +
x)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)
^2)]*Sqrt[-1 - x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2143

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((-1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[
2 + Sqrt[3]]*f*(1 - q*x)*Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2])/(
q*Sqrt[a + b*x^3]*Sqrt[-((1 - q*x)/(1 - Sqrt[3] - q*x)^2)]), Subst[Int[1/((
(1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4
*Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*S
qrt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-c+\left(1+\sqrt{3}\right)d+\left(-c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\,\sqrt{7+}}\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\left(-c+d+\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}\,\sqrt{7+4\sqrt{3}+x^2}\,(}\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{c^2+cd+d^2}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{c^2+cd+d^2}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}} + \frac{4\sqrt[4]{}}{}$$

Mathematica [C] time = 0.61, size = 233, normalized size = 0.64

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2-x+1}\left(\sqrt{3}c-\left(\sqrt{3}-3\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(}{}}\right.\right.}{c+\sqrt[3]{-1}d}\right.}{3d\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c - (-3 + Sqrt[3])*d)*Sqrt[1 - x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 - x^
3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

maple [A] time = 0.03, size = 266, normalized size = 0.73

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}\,d}+\frac{2i\left(c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+2/3*I*(c+3^(1/2)*d-d)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(c/d+1/2+1/2*I*3^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(c/d+1/2+1/2*I*3
^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-(x+1)\left(x^2-x+1\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.152 $\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=125

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(1+3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1/2))/(
1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1832, 266, 63, 207, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx + \int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(1+\sqrt{3}\right)\text{Subst}\Bigg(\int \frac{}{x}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\text{Subst}\Bigg($$

$$= -\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.31

$$x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right) - \frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + x*Hypergeometric2F1[1/3, 1/2,
 4/3, -x^3]
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}{x^4+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x + sqrt(3) + 1)/(x^4 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

maple [A] time = 0.06, size = 132, normalized size = 1.06

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(1+\sqrt{3}\right)\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/x/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2/3*arctanh((x^3+1)^(1/2))*(1+3^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

mupad [B] time = 0.14, size = 334, normalized size = 2.67

$$-\frac{2\sqrt{3}\,\mathrm{atanh}\left(\sqrt{x^3+1}\right)}{3}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-2\left(\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/(x*(x^3 + 1)^(1/2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2) - (2*3^(1/2)*atanh((x^3 + 1)^(1/2)))/3 - (2*((3^
(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
)/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [A] time = 5.25, size = 56, normalized size = 0.45

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\sqrt{3}\,\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
 - 2*sqrt(3)*asinh(x**(-3/2))/3 - 2*asinh(x**(-3/2))/3
```

3.153 $\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=139

$$\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

```
[Out] -2/3*arctanh((-x^3+1)^(1/2))*(1+3^(1/2))+2/3*(1-x)*EllipticF((1-x-3^(1/2))/
(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1
/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1832, 266, 63, 206, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}\,dx - \int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{1}{3}\left(1+\sqrt{3}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\text{Subst}\left(\right.$$

$$= -\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-\right.}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.03, size = 40, normalized size = 0.29

$$-x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right) - \frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 - x*Hypergeometric2F1[1/3, 1/2,
 4/3, x^3]
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}{x^4-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)/(x^4 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

maple [A] time = 0.06, size = 125, normalized size = 0.90

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2\left(1+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2/3*arctanh((-x^3+1)^(1/2))*(1+3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

mupad [B] time = 3.64, size = 373, normalized size = 2.68

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-1\right)^3\left(\sqrt{1-x^3}+1\right)}{x^6}\right)}{3}+\frac{\sqrt{x^3-1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\ \mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\cdots\right)}{\sqrt{1\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) - x + 1)/(x*(1 - x^3)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((((1 - x^3)^(1/2) - 1)^3*((1 - x^3)^(1/2) + 1))/x^6))/3 + ((x^
3 - 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(
x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/
2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)))/(1 - x^3)^(1/2)
```

sympy [A] time = 8.93, size = 99, normalized size = 0.71

$$-\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}+\sqrt{3}\left(\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/x/(-x**3+1)**(1/2),x)
```

```
[Out] -x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/
2))/3, True)) + sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1),
 (2*I*asin(x**(-3/2))/3, True))
```

3.154 $\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=142

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] 2/3*arctan((x^3-1)^(1/2))*(1+3^(1/2))+2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2)
)^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1832, 266, 63, 203, 219}

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-1+x^3}}\,dx - \int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\left(1+\sqrt{3}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\text{Subst}\left(\right.$$

$$= \frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+\right.}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.04, size = 58, normalized size = 0.41

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right) - \frac{x\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 - (x*Sqrt[1 - x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}{x^4-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 - 1)*(x - sqrt(3) - 1)/(x^4 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

maple [A] time = 0.05, size = 140, normalized size = 0.99

$$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\sqrt{3}\,\arctan\left(\sqrt{x^3-1}\right)}{3}+2\,\mathrm{ar}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/x/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*arctan((x^3-1)^(1/2))*3^(1/2)
+2/3*arctan((x^3-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

mupad [B] time = 2.72, size = 334, normalized size = 2.35

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{x^3-1}\right)}{3}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) - x + 1)/(x*(x^3 - 1)^(1/2)),x)
```

```
[Out] (2*3^(1/2)*atan((x^3 - 1)^(1/2)))/3 + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*elli
pticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
 x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((
3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(
1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [A] time = 8.93, size = 94, normalized size = 0.66

$$\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}+\begin{cases}\frac{2i\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\,\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}+\sqrt{3}\left(\begin{cases}\frac{2i\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\,\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/x/(x**3-1)**(1/2),x)
```

```
[Out] I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + Piecewise((
2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3, True)) + s
qrt(3)*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3
/2))/3, True))
```

3.155 $\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=136

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 2/3*arctan((-x^3-1)^(1/2))*(1+3^(1/2))+2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1
+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2
))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1832, 266, 63, 204, 219}

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
x)^2)]*Sqrt[-1 - x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx &= \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-1-x^3}}\,dx+\int \frac{1}{\sqrt{-1-x^3}}\,dx\\
&= \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{1}{3}\left(1+\sqrt{3}\right)\text{Subst}\left(\int \right.\\
&= \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}-\frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\text{Subst}\left(\right.\\
&= \frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1-x^3}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+\right.}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 61, normalized size = 0.45

$$
\frac{x\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)}{\sqrt{-x^3-1}}+\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (x*Sqrt[1 + x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$
\text{integral}\left(-\frac{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}{x^4+x},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(x + sqrt(3) + 1)/(x^4 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}\,x}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

maple [A] time = 0.05, size = 135, normalized size = 0.99

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}+\frac{2\sqrt{3}\ \mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+2/3*arctan((-x^3-1)^(1/2))*3^(1/2)+2/3*arctan((-x^3-1)
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^{3}-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

mupad [B] time = 4.45, size = 376, normalized size = 2.76

$$\frac{\sqrt{3}\ \ln\left(\frac{\left(\sqrt{-x^3-1}-\mathrm{i}\right)\left(\sqrt{-x^3-1}+1\mathrm{i}\right)^3}{x^6}\right)1\mathrm{i}}{3}+\frac{\sqrt{x^3+1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\ \mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1}{2}}\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\right.}{\sqrt{\cdot}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/(x*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((((- x^3 - 1)^(1/2) - 1i)*((- x^3 - 1)^(1/2) + 1i)^3)/x^6)*1i)
/3 + ((x^3 + 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3
^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x +
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/
2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2)))/(- x^3 - 1)^(1/2)
```

sympy [**A**] time = 5.43, size = 61, normalized size = 0.45

$$-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2\sqrt{3}\,i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/x/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3)) + 2*I*asinh(x**(-3/2))/3 + 2*sqrt(3)*I*asinh(x**(-3/2))/3
```

3.156 $\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=127

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(1-3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1/2))/(
1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1832, 266, 63, 207, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx + \int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(1-\sqrt{3}\right)\operatorname{Subst}\left(\int \frac{}{x}\right.$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\operatorname{Subst}\left(\right.$$

$$= -\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 0.32

$$x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right) - \frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + x*Hypergeometric2F1[1/3, 1/2,
 4/3, -x^3]
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}{x^4+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x - sqrt(3) + 1)/(x^4 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

maple [A] time = 0.03, size = 132, normalized size = 1.04

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\sqrt{3}-1\right)\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/x/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(3^(1/2)-1)*arctanh((x^3+1)^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

mupad [B] time = 2.65, size = 334, normalized size = 2.63

$$\frac{2\sqrt{3}\,\mathrm{atanh}\left(\sqrt{x^3+1}\right)}{3}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\left(\frac{3}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/(x*(x^3 + 1)^(1/2)),x)
```

```
[Out] (2*3^(1/2)*atanh((x^3 + 1)^(1/2)))/3 + (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(
1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellip
ticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^
(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
)/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [**A**] time = 5.20, size = 56, normalized size = 0.44

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2\sqrt{3}\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
 - 2*asinh(x**(-3/2))/3 + 2*sqrt(3)*asinh(x**(-3/2))/3
```

3.157 $\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

```
[Out] -2/3*arctanh((-x^3+1)^(1/2))*(1-3^(1/2))+2/3*(1-x)*EllipticF((1-x-3^(1/2))/
(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1
/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1832, 266, 63, 206, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}\,dx - \int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{1}{3}\left(1-\sqrt{3}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\text{Subst}\left(\int\right.$$

$$= -\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-\right.}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.30

$$-x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right) - \frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 - x*Hypergeometric2F1[1/3, 1/2,
 4/3, x^3]
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}{x^4-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)/(x^4 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

maple [A] time = 0.03, size = 125, normalized size = 0.89

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}+\frac{2\left(\sqrt{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+2/3*(3^(1/2)-1)*arctanh((-x^3+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{-x^{3}+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

mupad [B] time = 3.24, size = 373, normalized size = 2.65

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^{3}}-1\right)\left(\sqrt{1-x^{3}}+1\right)^{3}}{x^{6}}\right)}{3}+\frac{\sqrt{x^{3}-1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^{3}+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\right.}{\sqrt{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/(x*(1 - x^3)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((((1 - x^3)^(1/2) - 1)*((1 - x^3)^(1/2) + 1)^3)/x^6))/3 + ((x^
3 - 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(
x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/
2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)))/(1 - x^3)^(1/2)
```

sympy [A] time = 9.00, size = 99, normalized size = 0.70

$$-\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\sqrt{3}\left(\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/x/(-x**3+1)**(1/2),x)
```

```
[Out] -x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3)) - sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin
(x**(-3/2))/3, True)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1),
 (2*I*asin(x**(-3/2))/3, True))
```

3.158 $\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=144

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}$$

```
[Out] 2/3*arctan((x^3-1)^(1/2))*(1-3^(1/2))+2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2)
)^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1832, 266, 63, 203, 219}

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-1+x^3}}\,dx - \int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\left(1-\sqrt{3}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\text{Subst}\left(\int\right.$$

$$= \frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+\right.}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.04, size = 60, normalized size = 0.42

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right) - \frac{x\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 - (x*Sqrt[1 - x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}{x^4-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^3 - 1)*(x + sqrt(3) - 1)/(x^4 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

maple [A] time = 0.02, size = 140, normalized size = 0.97

$$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\sqrt{3}\ \arctan\left(\sqrt{x^3-1}\right)}{3}+\frac{2\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/x/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*3^(1/2)*arctan((x^3-1)^(1/2))
+2/3*arctan((x^3-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

mupad [B] time = 2.70, size = 334, normalized size = 2.32

$$-\frac{2\sqrt{3}\ \mathrm{atan}\left(\sqrt{x^3-1}\right)}{3}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\ \mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/(x*(x^3 - 1)^(1/2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*3^(1/2)*atan((x^3 - 1)^(1/2)))/3 - (2*((
3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(
1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [A] time = 8.98, size = 94, normalized size = 0.65

$$\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}-\sqrt{3}\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)+\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/x/(x**3-1)**(1/2),x)
```

```
[Out] I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - sqrt(3)*Pie
cewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3, Tr
ue)) + Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3
/2))/3, True))
```

3.159 $\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=138

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 2/3*arctan((-x^3-1)^(1/2))*(1-3^(1/2))+2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1
+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2
))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1832, 266, 63, 204, 219}

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
x)^2)]*Sqrt[-1 - x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-1-x^3}}\,dx + \int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{1}{3}\left(1-\sqrt{3}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\text{Subst}\left(\right.$$

$$= \frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+\right.}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.03, size = 63, normalized size = 0.46

$$\frac{x\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)}{\sqrt{-x^3-1}} + \frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (x*Sqrt[1 + x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}{x^4+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(x - sqrt(3) + 1)/(x^4 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

maple [A] time = 0.03, size = 135, normalized size = 0.98

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2\sqrt{3}\ \mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-2/3*3^(1/2)*arctan((-x^3-1)^(1/2))+2/3*arctan((-x^3-1)
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^{3}-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

mupad [B] time = 4.10, size = 376, normalized size = 2.72

$$\frac{\sqrt{3}\ \ln\left(\frac{\left(\sqrt{-x^{3}-1}-i\right)^{3}\left(\sqrt{-x^{3}-1}+1i\right)}{x^{6}}\right)1i}{3}+\frac{\sqrt{x^{3}+1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\ \mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1}{2}}\right.}{\sqrt{x^{3}+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\right.}{\sqrt{\cdot}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/(x*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((((- x^3 - 1)^(1/2) - 1i)^3*((- x^3 - 1)^(1/2) + 1i))/x^6)*1i)
/3 + ((x^3 + 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3
^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x +
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/
2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2)))/(- x^3 - 1)^(1/2)
```

sympy [A] time = 5.51, size = 61, normalized size = 0.44

$$-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\sqrt{3}\,i\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2i\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/x/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3)) - 2*sqrt(3)*I*asinh(x**(-3/2))/3 + 2*I*asinh(x**(-3/2))/3
```

3.160 $\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx$

<u>**Optimal**</u>. Leaf size=332

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\right.}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] -3/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(x^3+1)^(1
/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-12*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2)
)/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1
/2)/(x^3+1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^(1/2)-2
/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2)*(7*2^(1/2)+4*6^(1/2))*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+
x+3^(1/2))^2)^(1/2)
```

<u>**Rubi**</u> **[A]** time = 0.64, antiderivative size = 334, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2144, 218, 2142, 2113, 537, 571, 93, 204}

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\right.}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(S
qrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*Sqrt[2*(97 +
56*Sqrt[3])]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcS
in[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (12*3^(1/4)*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)
/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

<u>Rule 93</u>

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

<u>Rule 204</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\right.}{}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2-\right.}{}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+}}{}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+}}{}$$

$$= -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}$$

Mathematica [C] time = 0.22, size = 194, normalized size = 0.58

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}} + \frac{3i\sqrt{x^2-x+1}\,\Pi\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{3+\sqrt[3]{-1}}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3))))/S
qrt[1 + x^3]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}\,x}{x^4+3\,x^3+x+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3+1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

maple [A] time = 0.01, size = 240, normalized size = 0.72

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+3)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/4*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3+1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

mupad [B] time = 0.23, size = 207, normalized size = 0.62

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\left(\right.\right.\right.}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x + 3)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(2*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(
```

```
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*ellipticPi(- (3^
(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2
))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

3.161 $\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=377

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)\right| -}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] 3/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/((x^2+x+1)/(1-
x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*7^(1/2)/(-x^3+1)^(1/
2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-12/13*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/
2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2
^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2
))^2)^(1/2)-2/39*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)
*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*(5*2^(1/2)+2*6^(1/2))*3^(3/4)/(-x^3+1)^(
1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 379, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2144, 218, 2142, 2113, 537, 571, 93, 206}

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)\right| -}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])]])/(
2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2*(37
+ 20*Sqrt[3])]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[Ar
cSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(13*3^(1/4)*Sqr
t[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (12*3^(1/4)*Sqrt[2 + Sqrt[3
]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sq
rt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/
(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{1-x^3}}\,dx}{4+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2\right.}{}}{}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2\right.}{}}{}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+}}{}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+}}{}$$

$$= \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}$$

Mathematica [C] time = 0.23, size = 195, normalized size = 0.52

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{3i\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-3}\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((3
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))/Sqrt
[1 - x^3]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-x^3+1}\,x}{x^4+3\,x^3-x-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3+1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

maple [A] time = 0.01, size = 240, normalized size = 0.64

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+2i\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+3)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3+1)^(1/2)/(5/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*
3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*
I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3+1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

mupad [B] time = 2.74, size = 224, normalized size = 0.59

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(4\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}}{8}\right.\right.}{4\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/2)*(x + 3)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
```

```
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(4*ellipticF(asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
- 3*ellipticPi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(4*(1 - x^3)^(1/
2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

3.162 $\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=373

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{12\sqrt[4]{3}\cdots}{\cdots}$$

```
[Out] -2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((x
^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(4+3^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1
-x-3^(1/2))^2)^(1/2)+3/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))^2)
^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)
*7^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-12/13*3^(1/4)*(1-x)*El
lipticPi((-1+x+3^(1/2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/2)+2*I
)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(x^3-1)^(1/2)
/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 375, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2144, 219, 2142, 2113, 537, 571, 93, 206}

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{12\sqrt[4]{3}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(
2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) - (2*Sqrt[2]*(1
 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3]
 - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[-((1
 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (12*3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sqrt
[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(1
3*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\right.}{}$$

$$= -\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\right.}{}$$

$$= -\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{}}{}$$

$$= -\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{}}{}$$

$$= \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-}{1-\sqrt{3}-}\right.\right.}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-}}$$

Mathematica [C] time = 0.12, size = 193, normalized size = 0.52

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-3}\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((3
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))/Sqrt
[-1 + x^3]
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^3-1}\,x}{x^4+3\,x^3-x-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 - 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3-1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

maple [A] time = 0.01, size = 240, normalized size = 0.64

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{3\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x\cdots}{\cdots}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+3)/(x^3-1)^(1/2),x)
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3/2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-
3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)
*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi
(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/8+1/8*I*3^(1/2),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3-1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

mupad [B] time = 2.65, size = 208, normalized size = 0.56

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(4\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{as}\cdots\right.\right.}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(x + 3)),x)
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(4*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
```

```
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*ellipticPi((
3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1
/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(4*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
 x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

3.163 $\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=341

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{14+8\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{}\right.}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] -3/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(-x^3-1)^(
1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-12*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2
))/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2)^(
1/2)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^(1/2)
-2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/
(1+x-3^(1/2))^2)^(1/2)*(2*2^(1/2)+6^(1/2))*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(
1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 343, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2144, 219, 2142, 2113, 537, 571, 93, 204}

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{14+8\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{}\right.}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(S
qrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) - (2*Sqrt[14 + 8*
Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 +
x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (12*3^(1/4)*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)
/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\cdots}\right.}{\cdots}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2-\cdots}\right.}{\cdots}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\cdots}}{\cdots}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\cdots}}{\cdots}$$

$$= -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{2\sqrt{14+8\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\cdots}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\cdots}$$

Mathematica [C] time = 0.21, size = 196, normalized size = 0.57

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}} + \frac{3i\sqrt{x^2-x+1}\,\Pi\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{3+\sqrt[3]{-1}}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3))))/S
qrt[-1 - x^3]
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-x^3-1}\,x}{x^4+3\,x^3+x+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3-1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

maple [A] time = 0.01, size = 240, normalized size = 0.70

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+3)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((
x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-
x^3-1)^(1/2)/(7/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^
(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3-1}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

mupad [B] time = 2.61, size = 223, normalized size = 0.65

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{a}\right.\right.}{2\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- x^3 - 1)^(1/2)*(x + 3)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
```

```
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(2*ellipticF(asin(((x + 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*
ellipticPi(- (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(2*(- x^3 - 1)^(1/2
)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(x+1\right)\left(x^{2}-x+1\right)}\left(x+3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

3.164 $\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=450

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(de-cf)\Pi\left(\frac{\left(c-(1+\sqrt{\right.}}{\left(c-(1-\sqrt{\right.}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c^2-2\right.}$$

```
[Out] (-c*f+d*e)*(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(
c-d)^(1/2)/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(
1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1+x)*EllipticPi((-1-x+3^(1/2))/
(1+x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6
^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c^2-2*c*d-2*d^2)/(x^
3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+2/3*(1+x)*EllipticF((1+x-3^(1/2))/
(1+x+3^(1/2)),I*3^(1/2)+2*I)*(e-f-f*3^(1/2))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^
2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(-d*3^(1/2)+c-d)/(x^3+1)^(1/2)/((1+x)
/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 1.06, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2144, 218, 2142, 2113, 537, 571, 93, 205}

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(de-cf)\Pi\left(\frac{\left(c-(1+\right.}{\left(c-(1-\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[c
^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d
 + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt
[3]]*(e - f - Sqrt[3]*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*(c - d - Sqrt[3]*d)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) +
(4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c^2 - 2*c*d
 - 2*d^2)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d} - \frac{(de-cf)\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(4\sqrt[4]{3}\right.}{}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\left(4\sqrt[4]{3}\right.}{}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}}{}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}}{}$$

$$= \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)}{\sqrt[4]{3}\left(c-\right.}$$

Mathematica [C] time = 0.56, size = 211, normalized size = 0.47

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}\,(cf-de)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
+ (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1
/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c
+ (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3+1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

maple [A] time = 0.01, size = 274, normalized size = 0.61

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,f\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}\;d}+\frac{2\left(-cf+de\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+}{\frac{3}{2}-i}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x)
```

```
[Out] 2*f/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(-c*f+d*e)/d^2*(3/2-1/2*I*3
^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*
I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1
)^(1/2)/(c/d-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^
(1/2))/(c/d-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3+1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

mupad [B] time = 0.13, size = 356, normalized size = 0.79

$$\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\;\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1}{2}}}}{d^2\,\left(\frac{c}{d}-1\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 + 1)^(1/2)*(c + d*x)),x)
```

```
[Out] (2*f*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
```

```
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d*(x^3 - x*
(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)
*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(c*f - d*e)*((x + 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(c/d - 1), asin(((x + 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(
c/d - 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{(x+1)\left(x^2 - x + 1\right)}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.165 $\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=474

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(de-cf)\Pi}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -(-c*f+d*e)*(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)
/d^(1/2)/(c+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(
1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/2)/((1-x
)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1-x)*EllipticPi((-1+x+3^(1/2
))/(1-x+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^
(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c^2+2*c*d-2*d^2)/(-x^
3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-2/3*(1-x)*EllipticF((1-x-3^(1/2))/
(1-x+3^(1/2)),I*3^(1/2)+2*I)*(e+f+f*3^(1/2))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^
2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(c+d+d*3^(1/2))/(-x^3+1)^(1/2)/((1-x)
/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2144, 218, 2142, 2113, 537, 571, 93, 208}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(de-cf)\Gamma}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt[c + d]*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 -
c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])) - (2*Sqrt[2 +
Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]
) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -
ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c^2 + 2*c*d
 - 2*d^2)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = \frac{\left(e+f+\sqrt{3}f\right)\int \frac{1}{\sqrt{1-x^3}}\,dx}{c+d+\sqrt{3}d} + \frac{(de-cf)\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx}{c+d+\sqrt{3}d}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} + \ldots$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} - \ldots$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} + \ldots$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} + \ldots$$

$$= -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}\,\left(e+f+\ldots\right.}{\sqrt[4]{3}\ldots}$$

Mathematica [**C**] time = 0.70, size = 233, normalized size = 0.49

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\,(cf-de)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}\ldots}{1+\sqrt[3]{-1}}}\right.\right.}{\sqrt[3]{-1}d-c}\ldots\right.}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
 ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*Elli
pticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^3])
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^3+1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

maple [A] time = 0.01, size = 265, normalized size = 0.56

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,f\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}\,d}-\frac{2i\big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*f/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2
*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-
3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c*f+d*e)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(c/d-1/2+1/2*I*3^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(c/d-1/2+1/2*I*3
^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^3+1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

mupad [B] time = 0.09, size = 387, normalized size = 0.82

$$-\frac{2f\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\;\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{d\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((1 - x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(c*f - d*e)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3
^(1/2)*1i)/2 + 3/2)/(c/d + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(1 - x^3)^(1/2)*(c
/d + 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2*f*((3^(1/2)*1i)/2 +
3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(
1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d*(1 - x^3)^(1/
2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{-(x-1)\left(x^2+x+1\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

3.166 $\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=475

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(de-cf)\Pi\left(\cdots\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\,\left(\cdots\right.}$$

```
[Out] -2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(e+f+f*3^(1
/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(c
+d+d*3^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-(-c*f+d*e)*(1-x)
*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1
/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/d^
(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)
^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-x+3^(1/2)),(
c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))
*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c^2+2*c*d-2*d^2)/(x^3-1)^(1/2)/((1-x)/(
1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2144, 219, 2142, 2113, 537, 571, 93, 208}

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(de-cf)\Pi\left(\cdots\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\,\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt[c + d]*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 -
c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) - (2*Sqrt[2 -
 Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/
(3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
 x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2
)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)
^2, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c^2 +
2*c*d - 2*d^2)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = \frac{\left(e+f+\sqrt{3}f\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{c+d+\sqrt{3}d} + \frac{(de-cf)\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx}{c+d+\sqrt{3}d}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\left(4\cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(4\cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{\cdots}}{\cdots}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{\cdots}}{\cdots}$$

$$= -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{\cdots}\right.}{\sqrt[4]{3}\left(\cdots\right.}$$

Mathematica [C] time = 0.24, size = 231, normalized size = 0.49

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\,(cf-de)\Pi\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\cdots\right.}{\sqrt[3]{-1}d-c}\right.}{3d\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
 ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*Elli
pticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3-1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

maple [A] time = 0.01, size = 274, normalized size = 0.58

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,f\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}\,d}+\frac{2\left(-cf+de\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x)
```

```
[Out] 2*f/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-c*f+d*e)/d^2*(-3/2-1/2*I*3
^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*
I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)
^(1/2)/(c/d+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(
1/2))/(c/d+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3-1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

mupad [B] time = 2.67, size = 355, normalized size = 0.75

$$-\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\cdots}}{d^2\left(\frac{c}{d}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 - 1)^(1/2)*(c + d*x)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(c*f -
```

```
d*e)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3
/2)/(c/d + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(c/d + 1)*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
 1) + x^3)^(1/2)) - (2*f*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)))/(d*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

3.167 $\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=463

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(de-cf)\Pi\left(\frac{(c-(\cdots}{(c-(\cdots}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\,\left(c^2\cdots\right.}$$

```
[Out] 2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(e-f-f*3^(1/
2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-d
*3^(1/2)+c-d)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+(-c*f+d*e)*(1+x
)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1
/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c
-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1/2)/((1+x)/(1+x+3^(1/2))^2
)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1+x+3^(1/2)),
(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c^2-2*c*d-2*d^2)/(-x^3-1)^(1/2)/((
1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 1.03, antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2144, 219, 2142, 2113, 537, 571, 93, 205}

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(de-cf)\Pi\left(\frac{(c-(\cdots}{(c-(\cdots}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\,\left(c\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[c
^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d
 + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - Sqr
t[3]]*(e - f - Sqrt[3]*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*E
llipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(
1/4)*(c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3
]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)
^2, -ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c^2 -
2*c*d - 2*d^2)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2144

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d} - \frac{(de-cf)\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \cdots$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \cdots$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \cdots$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \cdots$$

$$= \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{\cdots}\right.}{\sqrt[4]{3}\left(\cdots\right.}$$

Mathematica [C] time = 0.35, size = 213, normalized size = 0.46

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}(cf-de)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
+ (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1
/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c
+ (-1)^(1/3)*d)))/(d*Sqrt[-1 - x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^3-1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

maple [A] time = 0.01, size = 265, normalized size = 0.57

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,f\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}\,d}-\frac{2i\left(-c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*f/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c*f+d*e)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(c/d+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^
(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(c/d+1/2+1/2*I*3^(1
/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^3-1}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

mupad [B] time = 0.10, size = 388, normalized size = 0.84

$$\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3}}{d^2\,\sqrt{-x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
```

```
[Out] (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/(d*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3
^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*(c*f - d*e)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^
(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*
1i)/2 + 3/2)/(c/d - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(- x^3 - 1)^(1/2)*(c/d - 1
)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{-(x+1)\left(x^2 - x + 1\right)}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.168 $\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=120

$$\frac{2\sqrt{2+\sqrt{3}}\,f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}e\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] -2/3*e*arctanh((x^3+1)^(1/2))+2/3*f*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1
/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(
1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1832, 266, 63, 207, 12, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\,f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}e\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*f*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx &= e\int \frac{1}{x\sqrt{1+x^3}}\,dx + \int \frac{f}{\sqrt{1+x^3}}\,dx \\
&= \frac{1}{3}e\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right) + f\int \frac{1}{\sqrt{1+x^3}}\,dx \\
&= \frac{2\sqrt{2+\sqrt{3}}\,f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}(2e)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\right. \\
&= -\frac{2}{3}e\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}\,f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 34, normalized size = 0.28

$$
fx\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right) - \frac{2}{3}e\tanh^{-1}\left(\sqrt{x^3+1}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[1 + x^3]),x]
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + f*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3
]
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\sqrt{x^3+1}\left(fx+e\right)}{x^4+x}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 + 1)*(f*x + e)/(x^4 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

maple [A] time = 0.01, size = 129, normalized size = 1.08

$$-\frac{2e\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,f\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(x^3+1)^(1/2),x)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2/3*e*arctanh((x^3+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

mupad [B] time = 2.64, size = 207, normalized size = 1.72

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\cdots}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(x*(x^3 + 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*(f*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/
2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*
1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1
/2)
```

sympy [A] time = 2.96, size = 42, normalized size = 0.35

$$-\frac{2e\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x**3+1)**(1/2),x)
```

```
[Out] -2*e*asinh(x**(-3/2))/3 + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp
_polar(I*pi))/(3*gamma(4/3))
```

3.169 $\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=134

$$-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{2\sqrt{2+\sqrt{3}}\,f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/3*e*arctanh((-x^3+1)^(1/2))-2/3*f*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(
1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^
(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1832, 266, 63, 206, 12, 218}

$$-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{2\sqrt{2+\sqrt{3}}\,f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 - (2*Sqrt[2 + Sqrt[3]]*f*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx &= e\int \frac{1}{x\sqrt{1-x^3}}\,dx+\int \frac{f}{\sqrt{1-x^3}}\,dx\\
&= \frac{1}{3}e\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^3\right)+f\int \frac{1}{\sqrt{1-x^3}}\,dx\\
&= -\frac{2\sqrt{2+\sqrt{3}}\,f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{1}{3}(2e)\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.\\
&= -\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{2\sqrt{2+\sqrt{3}}\,f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 34, normalized size = 0.25

$$fx\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 + f*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3]
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}\left(fx+e\right)}{x^4-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*(f*x + e)/(x^4 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^3+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

maple [A] time = 0.01, size = 122, normalized size = 0.91

$$-\frac{2e\,\mathrm{arctanh}\left(\sqrt{-x^3+1}\right)}{3}-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,f\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\right.}}{3}\right.}{3\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*e*arctanh((-x^3+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^3+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

mupad [B] time = 0.06, size = 223, normalized size = 1.66

$$-\frac{\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\right.\right.}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(x*(1 - x^3)^(1/2)),x)
```

```
[Out] -((x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(f*ellipticF(as
in((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)) + e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))
*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) - 3i)*1i)/((1 - x^3)^(1/2
)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 3.33, size = 65, normalized size = 0.49

$$e\left(\begin{cases} -\frac{2\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{\left|x^3\right|}>1 \\ \frac{2i\,\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}\right)+\frac{fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x**3+1)**(1/2),x)
```

```
[Out] e*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/2))/
3, True)) + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi)
)/(3*gamma(4/3))
```

3.170 $\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{2-\sqrt{3}}\,f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] 2/3*e*arctan((x^3-1)^(1/2))-2/3*f*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2
)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/
2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1832, 266, 63, 203, 12, 219}

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{2-\sqrt{3}}\,f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 - (2*Sqrt[2 - Sqrt[3]]*f*(1 - x)*Sqrt[(1 + x
 + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3
] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqr
t[-1 + x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx &= e\int \frac{1}{x\sqrt{-1+x^3}}\,dx+\int \frac{f}{\sqrt{-1+x^3}}\,dx\\
&= \frac{1}{3}e\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^3\right)+f\int \frac{1}{\sqrt{-1+x^3}}\,dx\\
&= -\frac{2\sqrt{2-\sqrt{3}}\,f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}+\frac{1}{3}(2e)\,\text{Subst}\left(\int \frac{1}{1+x\cdots}\right.\\
&= \frac{2}{3}e\tan^{-1}\left(\sqrt{-1+x^3}\right)-\frac{2\sqrt{2-\sqrt{3}}\,f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.38

$$
\frac{2}{3}e\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{fx\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{x^3-1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 + (f*x*Sqrt[1 - x^3]*Hypergeometric2F1[1/3,
1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\sqrt{x^3-1}\,(fx+e)}{x^4-x},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)*(f*x + e)/(x^4 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

maple [A] time = 0.01, size = 129, normalized size = 0.94

$$\frac{2e\arctan\left(\sqrt{x^3-1}\right)}{3}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,f\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(x^3-1)^(1/2),x)
```

```
[Out] 2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*e*arctan((x^3-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{x^3-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

mupad [B] time = 2.62, size = 207, normalized size = 1.51

$$-\frac{\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(x*(x^3 - 1)^(1/2)),x)
```

```
[Out] -((-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)
*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(f*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
) + e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*(-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) - 3i)*1i)/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x
^3)^(1/2)
```

sympy [A] time = 3.09, size = 60, normalized size = 0.44

$$e\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)-\frac{ifx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x**3-1)**(1/2),x)
```

```
[Out] e*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/
3, True)) - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

3.171 $\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=131

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}\,f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 2/3*e*arctan((-x^3-1)^(1/2))+2/3*f*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/
2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1
/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1832, 266, 63, 204, 12, 219}

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}\,f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*f*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3
] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqr
t[-1 - x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = e\int \frac{1}{x\sqrt{-1-x^3}}\,dx + \int \frac{f}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{1}{3}e\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x}\,dx, x, x^3\right) + f\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{1}{3}(2e)\,\text{Subst}\left(\int \frac{1}{-1-x\ldots}\right.$$

$$= \frac{2}{3}e\tan^{-1}\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}\,f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.03, size = 56, normalized size = 0.43

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-x^3-1}\right) + \frac{fx\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)}{\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (f*x*Sqrt[1 + x^3]*Hypergeometric2F1[1/3,
1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\left(fx+e\right)}{x^4+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(f*x + e)/(x^4 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^3-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

maple [A] time = 0.01, size = 122, normalized size = 0.93

$$\frac{2e\arctan\left(\sqrt{-x^3-1}\right)}{3}-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,f\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{3}\right.}{3\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+2/3*e*arctan((-x^3-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\sqrt{-x^3-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

mupad [B] time = 2.66, size = 223, normalized size = 1.70

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}}{2}}\right.\right.}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(x*(- x^3 - 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*(f*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)),
 -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - e*ellipticPi((3^(1/2)*1i
)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^
(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1/2)*(x
^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [**A**] time = 3.29, size = 46, normalized size = 0.35

$$\frac{2ie\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{ifx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x**3-1)**(1/2),x)
```

```
[Out] 2*I*e*asinh(x**(-3/2))/3 - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*
exp_polar(I*pi))/(3*gamma(4/3))
```

3.172 $\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal. Leaf size=95

$$\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}$$

```
[Out] -ln(d*x+c)/d+3/2*ln(d*(d*x+2*c)-d*(d^3*x^3+2*c^3)^(1/3))/d-arctan(1/3*(1+2*
(d*x+2*c)/(d^3*x^3+2*c^3)^(1/3))*3^(1/2))*3^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2151}

$$\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/d
) - Log[c + d*x]/d + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(2*
d)
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{dx-c}{\left(d^3x^3+2\,c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{-dx+c}{(dx+c)\left(d^3x^3+2c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

```
[Out] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{dx-c}{\left(d^3x^3+2\,c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c-d\,x}{\left(2\,c^3+d^3\,x^3\right)^{1/3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
```

```
[Out] int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{c}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\right)dx-\int \frac{dx}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

```
[Out] -Integral(-c/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*x**3)**(1/
3)), x) - Integral(d*x/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*
x**3)**(1/3)), x)
```

3.173 $\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal. Leaf size=234

$$-\frac{3(de-cf)\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}\,cd^2}+\frac{\sqrt{3}\,(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd^2}-\frac{f\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d^2}+\frac{f\,\mathrm{ta}}{}$$

```
[Out] 1/8*(-c*f+d*e)*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/c/d^2-1/2*f*ln(-d*x+(d^3*x^3-
c^3)^(1/3))/d^2-3/8*(-c*f+d*e)*ln(d*(-d*x+c)+2^(2/3)*d*(d^3*x^3-c^3)^(1/3))
*2^(2/3)/c/d^2+1/3*f*arctan(1/3*(1+2*d*x/(d^3*x^3-c^3)^(1/3))*3^(1/2))/d^2*
3^(1/2)+1/4*(-c*f+d*e)*arctan(1/3*(1-2^(1/3)*(-d*x+c)/(d^3*x^3-c^3)^(1/3))*
3^(1/2))*3^(1/2)*2^(2/3)/c/d^2
```

Rubi [A] time = 0.22, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2152, 239, 2148}

$$-\frac{3(de-cf)\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}\,cd^2}+\frac{\sqrt{3}\,(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd^2}-\frac{f\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d^2}+\frac{f\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (f*ArcTan[(1 + (2*d*x)/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + (S
qrt[3]*(d*e - c*f)*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/
Sqrt[3]])/(2*2^(1/3)*c*d^2) + ((d*e - c*f)*Log[(c - d*x)*(c + d*x)^2])/(4*2
^(1/3)*c*d^2) - (f*Log[-(d*x) + (-c^3 + d^3*x^3)^(1/3)])/(2*d^2) - (3*(d*e
- c*f)*Log[d*(c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d^
2)
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2152

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{f\int \frac{1}{\sqrt[3]{-c^3+d^3x^3}}\,dx}{d} + \frac{(de-cf)\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx}{d}$$

$$= \frac{f\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2} + \frac{\sqrt{3}\,(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd^2} + \frac{(de-cf)\log\left((c-\right.}{4\sqrt[3]{2}\,cd}$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
[Out] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(d^3x^3-c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{(dx+c)\left(d^3x^3-c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
[Out] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(d^3x^3-c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e + f\,x}{\left(d^3\,x^3 - c^3\right)^{1/3}\,\left(c + d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
```

```
[Out] int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt[3]{(-c + dx)\left(c^2 + cdx + d^2x^2\right)}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

```
[Out] Integral((e + f*x)/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x
)), x)
```

3.174 $\int x^2(a+bx)^n\left(c+dx^3\right)dx$

Optimal. Leaf size=160

$$-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad(}{b^6}$$

```
[Out] a^2*(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^6/(1+n)-a*(-5*a^3*d+2*b^3*c)*(b*x+a)^(2+
n)/b^6/(2+n)+(-10*a^3*d+b^3*c)*(b*x+a)^(3+n)/b^6/(3+n)+10*a^2*d*(b*x+a)^(4+
n)/b^6/(4+n)-5*a*d*(b*x+a)^(5+n)/b^6/(5+n)+d*(b*x+a)^(6+n)/b^6/(6+n)
```

Rubi [A] time = 0.11, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1620}

$$\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)}-\frac{5ad(a}{b^6(}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] (a^2*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*(2*b^3*c - 5*a^3
*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + ((b^3*c - 10*a^3*d)*(a + b*x)^(3 + n
))/(b^6*(3 + n)) + (10*a^2*d*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d*(a +
 b*x)^(5 + n))/(b^6*(5 + n)) + (d*(a + b*x)^(6 + n))/(b^6*(6 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^3\right)dx &= \int\left(\frac{\left(a^2b^3c-a^5d\right)(a+bx)^n}{b^5}+\frac{a\left(-2b^3c+5a^3d\right)(a+bx)^{1+n}}{b^5}+\frac{\left(b^3c-10a^3d\right)(a}{b^5}\right.\\ &= \frac{a^2\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^6(1+n)}-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{2+n}}{b^6(2+n)}+\frac{\left(b^3c-10a^3d\right)(a+b}{b^6(3+n)}\end{aligned}$$

Mathematica [A] time = 0.12, size = 133, normalized size = 0.83

$$\frac{(a+bx)^{n+1}\left(\frac{(a+bx)^2\left(b^3c-10a^3d\right)}{n+3}+\frac{a(a+bx)\left(5a^3d-2b^3c\right)}{n+2}+\frac{10a^2d(a+bx)^3}{n+4}+\frac{a^2b^3c-a^5d}{n+1}+\frac{d(a+bx)^5}{n+6}-\frac{5ad(a+bx)^4}{n+5}\right)}{b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*b^3*c - a^5*d)/(1 + n) + (a*(-2*b^3*c + 5*a^3*d)*(
a + b*x))/(2 + n) + ((b^3*c - 10*a^3*d)*(a + b*x)^2)/(3 + n) + (10*a^2*d*(a
 + b*x)^3)/(4 + n) - (5*a*d*(a + b*x)^4)/(5 + n) + (d*(a + b*x)^5)/(6 + n))
)/b^6
```

fricas [B] time = 0.70, size = 490, normalized size = 3.06

$$\frac{\left(2\,a^3b^3cn^3 + 30\,a^3b^3cn^2 + 148\,a^3b^3cn + 240\,a^3b^3c - 120\,a^6d + \left(b^6dn^5 + 15\,b^6dn^4 + 85\,b^6dn^3 + 225\,b^6dn^2 + 274\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] (2*a^3*b^3*c*n^3 + 30*a^3*b^3*c*n^2 + 148*a^3*b^3*c*n + 240*a^3*b^3*c - 120
*a^6*d + (b^6*d*n^5 + 15*b^6*d*n^4 + 85*b^6*d*n^3 + 225*b^6*d*n^2 + 274*b^6
*d*n + 120*b^6*d)*x^6 + (a*b^5*d*n^5 + 10*a*b^5*d*n^4 + 35*a*b^5*d*n^3 + 50
*a*b^5*d*n^2 + 24*a*b^5*d*n)*x^5 - 5*(a^2*b^4*d*n^4 + 6*a^2*b^4*d*n^3 + 11*
a^2*b^4*d*n^2 + 6*a^2*b^4*d*n)*x^4 + (b^6*c*n^5 + 18*b^6*c*n^4 + 240*b^6*c
+ (121*b^6*c + 20*a^3*b^3*d)*n^3 + 12*(31*b^6*c + 5*a^3*b^3*d)*n^2 + 4*(127
*b^6*c + 10*a^3*b^3*d)*n)*x^3 + (a*b^5*c*n^5 + 16*a*b^5*c*n^4 + 89*a*b^5*c*
n^3 + 2*(97*a*b^5*c - 30*a^4*b^2*d)*n^2 + 60*(2*a*b^5*c - a^4*b^2*d)*n)*x^2
 - 2*(a^2*b^4*c*n^4 + 15*a^2*b^4*c*n^3 + 74*a^2*b^4*c*n^2 + 60*(2*a^2*b^4*c
 - a^5*b*d)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6
*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

giac [B] time = 0.40, size = 835, normalized size = 5.22

$$\frac{(bx+a)^n b^6dn^5x^6 + (bx+a)^n ab^5dn^5x^5 + 15\,(bx+a)^n b^6dn^4x^6 + 10\,(bx+a)^n ab^5dn^4x^5 + 85\,(bx+a)^n b^6dn^3x^6 + (b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^6*d*n^5*x^6 + (b*x + a)^n*a*b^5*d*n^5*x^5 + 15*(b*x + a)^n*b
^6*d*n^4*x^6 + 10*(b*x + a)^n*a*b^5*d*n^4*x^5 + 85*(b*x + a)^n*b^6*d*n^3*x^
6 + (b*x + a)^n*b^6*c*n^5*x^3 - 5*(b*x + a)^n*a^2*b^4*d*n^4*x^4 + 35*(b*x +
 a)^n*a*b^5*d*n^3*x^5 + 225*(b*x + a)^n*b^6*d*n^2*x^6 + (b*x + a)^n*a*b^5*c
*n^5*x^2 + 18*(b*x + a)^n*b^6*c*n^4*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n^3*x^4
+ 50*(b*x + a)^n*a*b^5*d*n^2*x^5 + 274*(b*x + a)^n*b^6*d*n*x^6 + 16*(b*x +
a)^n*a*b^5*c*n^4*x^2 + 121*(b*x + a)^n*b^6*c*n^3*x^3 + 20*(b*x + a)^n*a^3*b
^3*d*n^3*x^3 - 55*(b*x + a)^n*a^2*b^4*d*n^2*x^4 + 24*(b*x + a)^n*a*b^5*d*n*
x^5 + 120*(b*x + a)^n*b^6*d*x^6 - 2*(b*x + a)^n*a^2*b^4*c*n^4*x + 89*(b*x +
 a)^n*a*b^5*c*n^3*x^2 + 372*(b*x + a)^n*b^6*c*n^2*x^3 + 60*(b*x + a)^n*a^3*
b^3*d*n^2*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*c*n
^3*x + 194*(b*x + a)^n*a*b^5*c*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n^2*x^2 +
 508*(b*x + a)^n*b^6*c*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d*n*x^3 + 2*(b*x + a)
^n*a^3*b^3*c*n^3 - 148*(b*x + a)^n*a^2*b^4*c*n^2*x + 120*(b*x + a)^n*a*b^5*
c*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n*x^2 + 240*(b*x + a)^n*b^6*c*x^3 + 30*(
b*x + a)^n*a^3*b^3*c*n^2 - 240*(b*x + a)^n*a^2*b^4*c*n*x + 120*(b*x + a)^n*
a^5*b*d*n*x + 148*(b*x + a)^n*a^3*b^3*c*n + 240*(b*x + a)^n*a^3*b^3*c - 120
*(b*x + a)^n*a^6*d)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 162
4*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

maple [B] time = 0.01, size = 451, normalized size = 2.82

$$-\frac{\left(-b^5d\,n^5x^5 - 15b^5d\,n^4x^5 + 5a\,b^4d\,n^4x^4 - 85b^5d\,n^3x^5 + 50a\,b^4d\,n^3x^4 - b^5c\,n^5x^2 - 225b^5d\,n^2x^5 - 20a^2b^3d\,n^3x^3 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c),x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^5*d*n^5*x^5-15*b^5*d*n^4*x^5+5*a*b^4*d*n^4*x^4-85*b^5*d*
n^3*x^5+50*a*b^4*d*n^3*x^4-b^5*c*n^5*x^2-225*b^5*d*n^2*x^5-20*a^2*b^3*d*n^3
```

```
*x^3+175*a*b^4*d*n^2*x^4-18*b^5*c*n^4*x^2-274*b^5*d*n*x^5-120*a^2*b^3*d*n^2
*x^3+2*a*b^4*c*n^4*x+250*a*b^4*d*n*x^4-121*b^5*c*n^3*x^2-120*b^5*d*x^5+60*a
^3*b^2*d*n^2*x^2-220*a^2*b^3*d*n*x^3+32*a*b^4*c*n^3*x+120*a*b^4*d*x^4-372*b
^5*c*n^2*x^2+180*a^3*b^2*d*n*x^2-2*a^2*b^3*c*n^3-120*a^2*b^3*d*x^3+178*a*b^
4*c*n^2*x-508*b^5*c*n*x^2-120*a^4*b*d*n*x+120*a^3*b^2*d*x^2-30*a^2*b^3*c*n^
2+388*a*b^4*c*n*x-240*b^5*c*x^2-120*a^4*b*d*x-148*a^2*b^3*c*n+240*a*b^4*c*x
+120*a^5*d-240*a^2*b^3*c)/b^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+7
20)
```

maxima [A] time = 0.98, size = 253, normalized size = 1.58

$$\frac{\left(\left(n^2+3n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^nc}{\left(n^3+6n^2+11n+6\right)b^3}+\frac{\left(\left(n^5+15n^4+85n^3+225n^2+274n+12\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2
 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5
 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*
b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d
/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

mupad [B] time = 3.20, size = 495, normalized size = 3.09

$$(a+b\,x)^n\left(\frac{d\,x^6\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720}+\frac{2\,a^3\,\left(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+\right.}{b^6\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+162\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^6*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(1764*
n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) + (2*a^3*(120*b^3*c
- 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^6*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (x^3*(3*n + n^2 + 2)*(120*b^
3*c + 15*b^3*c*n^2 + b^3*c*n^3 + 20*a^3*d*n + 74*b^3*c*n))/(b^3*(1764*n + 1
624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (2*a^2*n*x*(120*b^3*c
- 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^5*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d*n*x^5*(50*n + 35*n^2 +
10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^
6 + 720)) + (a*n*x^2*(n + 1)*(120*b^3*c - 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n
^3 + 74*b^3*c*n))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^
6 + 720)) - (5*a^2*d*n*x^4*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)))
```

sympy [A] time = 7.63, size = 6397, normalized size = 39.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**6/6), Eq(b, 0)), (60*a**5*d*log(a/b + x)/(
60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d/(60*a**5*b**6 + 300*a**4*b**
7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11
*x**5) + 300*a**4*b*d*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*
```

```
a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 6
25*a**4*b*d*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a*
*2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d*x**2*log
(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b
**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d*x**2/(60*a*
*5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a
*b**10*x**4 + 60*b**11*x**5) - 2*a**2*b**3*c/(60*a**5*b**6 + 300*a**4*b**7*
x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x
**5) + 600*a**2*b**3*d*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x +
600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5)
 + 900*a**2*b**3*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**
2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 10*a*b**4*c*x/
(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4*log(a/b + x)/(60*a**
5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*
b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4/(60*a**5*b**6 + 300*a**4*b*
*7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**1
1*x**5) - 20*b**5*c*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x*
*2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*d*x**
5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a
**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*d*
log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9
*x**3 + 12*b**10*x**4) - 125*a**5*d/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**
2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*d*x*log(a/b + x)
/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b
**10*x**4) - 440*a**4*b*d*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x
**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*d*x**2*log(a/b + x)/(
12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**
10*x**4) - 540*a**3*b**2*d*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b*
*8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - a**2*b**3*c/(12*a**4*b**6 + 48*
a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**
2*b**3*d*x**3*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x*
*2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d*x**3/(12*a**4*b**6 +
 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 4*a
*b**4*c*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x*
*3 + 12*b**10*x**4) - 60*a*b**4*d*x**4*log(a/b + x)/(12*a**4*b**6 + 48*a**3
*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 6*b**5*c*x*
*2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12
*b**10*x**4) + 12*b**5*d*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8
*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*d*log(a/b + x
)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5*
d/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*
b*d*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*
x**3) + 270*a**4*b*d*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b
**9*x**3) + 180*a**3*b**2*d*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x
 + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d*x**2/(6*a**3*b**6 + 18*a
**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 2*a**2*b**3*c/(6*a**3*b**6 + 1
8*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*d*x**3*log(a/b
 + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 6*a*b
**4*c*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*
a*b**4*d*x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3)
 - 6*b**5*c*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x*
*3) + 3*b**5*d*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9
*x**3), Eq(n, -4)), (-60*a**5*d*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6
*b**8*x**2) - 90*a**5*d/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**
4*b*d*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b
*d*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*d*x**2*log(a/
b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*a**2*b**3*c*log(a/b +
x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 9*a**2*b**3*c/(6*a**2*b**6 +
```

```
12*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*d*x**3/(6*a**2*b**6 + 12*a*b**7*
x + 6*b**8*x**2) + 12*a*b**4*c*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x +
6*b**8*x**2) + 12*a*b**4*c*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5*
a*b**4*d*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*b**5*c*x**2*log
(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*d*x**5/(6*a**2
*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*d*log(a/b + x)/(12
*a*b**6 + 12*b**7*x) + 60*a**5*d/(12*a*b**6 + 12*b**7*x) + 60*a**4*b*d*x*lo
g(a/b + x)/(12*a*b**6 + 12*b**7*x) - 30*a**3*b**2*d*x**2/(12*a*b**6 + 12*b*
*7*x) - 24*a**2*b**3*c*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 24*a**2*b**3*
c/(12*a*b**6 + 12*b**7*x) + 10*a**2*b**3*d*x**3/(12*a*b**6 + 12*b**7*x) - 2
4*a*b**4*c*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*d*x**4/(12*a*b
**6 + 12*b**7*x) + 12*b**5*c*x**2/(12*a*b**6 + 12*b**7*x) + 3*b**5*d*x**5/(
12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*d*log(a/b + x)/b**6 + a**4*d*x/b
**5 - a**3*d*x**2/(2*b**4) + a**2*c*log(a/b + x)/b**3 + a**2*d*x**3/(3*b**3
) - a*c*x/b**2 - a*d*x**4/(4*b**2) + c*x**2/(2*b) + d*x**5/(5*b), Eq(n, -1)
), (-120*a**6*d*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a**5*b*d*n*x*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d*n**2*x**2*(a + b*x)*
*n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n*
*2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d*n*x**2*(a + b*x)**n/(b**6*n**
6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b*
*6*n + 720*b**6) + 2*a**3*b**3*c*n**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**
5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6
) + 30*a**3*b**3*c*n**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 148*a**3*b
**3*c*n*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 240*a**3*b**3*c*(a + b*x)*
*n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n*
*2 + 1764*b**6*n + 720*b**6) + 20*a**3*b**3*d*n**3*x**3*(a + b*x)**n/(b**6*
n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) + 60*a**3*b**3*d*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21
*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
 720*b**6) + 40*a**3*b**3*d*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) -
 2*a**2*b**4*c*n**4*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4
*c*n**3*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 148*a**2*b**4*c*n**2*x*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) - 240*a**2*b**4*c*n*x*(a + b*x)**n/(b
**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
1764*b**6*n + 720*b**6) - 5*a**2*b**4*d*n**4*x**4*(a + b*x)**n/(b**6*n**6 +
 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*
n + 720*b**6) - 30*a**2*b**4*d*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*
n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b
**6) - 55*a**2*b**4*d*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30
*a**2*b**4*d*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4
+ 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*c*n**5*
x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 16*a*b**5*c*n**4*x**2*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 89*a*b**5*c*n**3*x**2*(a + b*x)**n/(b**6*
n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) + 194*a*b**5*c*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b
**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 7
20*b**6) + 120*a*b**5*c*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175
*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b
```

```
**5*d*n**5*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 10*a*b**5*d*n**4*x
**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
+ 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 35*a*b**5*d*n**3*x**5*(a + b*x
)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + 50*a*b**5*d*n**2*x**5*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 24*a*b**5*d*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
*6) + b**6*c*n**5*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
*4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 18*b**6*c*n
**4*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*
n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 121*b**6*c*n**3*x**3*(a +
 b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b
**6*n**2 + 1764*b**6*n + 720*b**6) + 372*b**6*c*n**2*x**3*(a + b*x)**n/(b**
6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 17
64*b**6*n + 720*b**6) + 508*b**6*c*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) + 240*b**6*c*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*d*n**
5*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*d*n**4*x**6*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 85*b**6*d*n**3*x**6*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b
**6*n + 720*b**6) + 225*b**6*d*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*
n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b
**6) + 274*b**6*d*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*
n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*
d*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True))
```

3.175 $\int x(a+bx)^n\left(c+dx^3\right)\,dx$

Optimal. Leaf size=126

$$-\frac{a\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^5(n+1)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

```
[Out] -a*(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^5/(1+n)+(-4*a^3*d+b^3*c)*(b*x+a)^(2+n)/b^
5/(2+n)+6*a^2*d*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d*(b*x+a)^(4+n)/b^5/(4+n)+d*(b*
x+a)^(5+n)/b^5/(5+n)
```

Rubi [A] time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1620}

$$-\frac{a\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^5(n+1)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] -((a*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^5*(1 + n))) + ((b^3*c - 4*a^3*d)
*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (6*a^2*d*(a + b*x)^(3 + n))/(b^5*(3 + n
)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5 + n))/(b^5*(
5 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x(a+bx)^n\left(c+dx^3\right)dx=\int\left(\frac{a\left(-b^3c+a^3d\right)(a+bx)^n}{b^4}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{1+n}}{b^4}+\frac{6a^2d(a+bx)^{2+n}}{b^4}-\frac{4a\ldots}{\ldots}\right.$$

$$=-\frac{a\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^5(1+n)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{2+n}}{b^5(2+n)}+\frac{6a^2d(a+bx)^{3+n}}{b^5(3+n)}-\frac{4ad(\ldots}{b^5\ldots}$$

Mathematica [A] time = 0.08, size = 104, normalized size = 0.83

$$\frac{(a+bx)^{n+1}\left(\frac{(a+bx)\left(b^3c-4a^3d\right)}{n+2}+\frac{a\left(a^3d-b^3c\right)}{n+1}+\frac{6a^2d(a+bx)^2}{n+3}+\frac{d(a+bx)^4}{n+5}-\frac{4ad(a+bx)^3}{n+4}\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d))/(1 + n) + ((b^3*c - 4*a^3*d)*(a
+ b*x))/(2 + n) + (6*a^2*d*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*x)^3)/(4 +
n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

fricas [B] time = 0.72, size = 348, normalized size = 2.76

$$-\frac{\left(a^2b^3cn^3 + 12\,a^2b^3cn^2 + 47\,a^2b^3cn + 60\,a^2b^3c - 24\,a^5d - \left(b^5dn^4 + 10\,b^5dn^3 + 35\,b^5dn^2 + 50\,b^5dn + 24\,b^5d\right)x^5 - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -(a^2*b^3*c*n^3 + 12*a^2*b^3*c*n^2 + 47*a^2*b^3*c*n + 60*a^2*b^3*c - 24*a^5
*d - (b^5*d*n^4 + 10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^5
- (a*b^4*d*n^4 + 6*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + 4*(a^2
*b^3*d*n^3 + 3*a^2*b^3*d*n^2 + 2*a^2*b^3*d*n)*x^3 - (b^5*c*n^4 + 13*b^5*c*n
^3 + 60*b^5*c + (59*b^5*c + 12*a^3*b^2*d)*n^2 + (107*b^5*c + 12*a^3*b^2*d)*
n)*x^2 - (a*b^4*c*n^4 + 12*a*b^4*c*n^3 + 47*a*b^4*c*n^2 + 12*(5*a*b^4*c - 2
*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^
2 + 274*b^5*n + 120*b^5)
```

giac [B] time = 0.38, size = 577, normalized size = 4.58

$$\frac{(bx+a)^n b^5dn^4x^5 + (bx+a)^n ab^4dn^4x^4 + 10\,(bx+a)^n b^5dn^3x^5 + 6\,(bx+a)^n ab^4dn^3x^4 + 35\,(bx+a)^n b^5dn^2x^5 + (bx \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*b
^5*d*n^3*x^5 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 + 35*(b*x + a)^n*b^5*d*n^2*x^5
 + (b*x + a)^n*b^5*c*n^4*x^2 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x +
a)^n*a*b^4*d*n^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + (b*x + a)^n*a*b^4*c*n^4
*x + 13*(b*x + a)^n*b^5*c*n^3*x^2 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 + 6*(b
*x + a)^n*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 + 12*(b*x + a)^n*a*b^4*c
*n^3*x + 59*(b*x + a)^n*b^5*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2*x^2 -
8*(b*x + a)^n*a^2*b^3*d*n*x^3 - (b*x + a)^n*a^2*b^3*c*n^3 + 47*(b*x + a)^n*
a*b^4*c*n^2*x + 107*(b*x + a)^n*b^5*c*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n*x^
2 - 12*(b*x + a)^n*a^2*b^3*c*n^2 + 60*(b*x + a)^n*a*b^4*c*n*x - 24*(b*x + a
)^n*a^4*b*d*n*x + 60*(b*x + a)^n*b^5*c*x^2 - 47*(b*x + a)^n*a^2*b^3*c*n - 6
0*(b*x + a)^n*a^2*b^3*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b^5*n^4 + 85*
b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

maple [B] time = 0.01, size = 283, normalized size = 2.25

$$\frac{\left(b^4d\,n^4x^4 + 10b^4d\,n^3x^4 - 4a\,b^3d\,n^3x^3 + 35b^4d\,n^2x^4 - 24a\,b^3d\,n^2x^3 + b^4c\,n^4x + 50b^4dn\,x^4 + 12a^2b^2d\,n^2x^2 - 44a\,b \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c),x)
```

```
[Out] (b*x+a)^(n+1)*(b^4*d*n^4*x^4+10*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+35*b^4*d*n^
2*x^4-24*a*b^3*d*n^2*x^3+b^4*c*n^4*x+50*b^4*d*n*x^4+12*a^2*b^2*d*n^2*x^2-44
*a*b^3*d*n*x^3+13*b^4*c*n^3*x+24*b^4*d*x^4+36*a^2*b^2*d*n*x^2-a*b^3*c*n^3-2
4*a*b^3*d*x^3+59*b^4*c*n^2*x-24*a^3*b*d*n*x+24*a^2*b^2*d*x^2-12*a*b^3*c*n^2
+107*b^4*c*n*x-24*a^3*b*d*x-47*a*b^3*c*n+60*b^4*c*x+24*a^4*d-60*a*b^3*c)/b^
5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

maxima [A] time = 1.02, size = 184, normalized size = 1.46

$$\frac{\left(b^2(n+1)x^2 + abnx - a^2\right)(bx+a)^n c}{\left(n^2 + 3\,n + 2\right)b^2} + \frac{\left(\left(n^4 + 10\,n^3 + 35\,n^2 + 50\,n + 24\right)b^5x^5 + \left(n^4 + 6\,n^3 + 11\,n^2 + 6\,n\right)ab^4x^4 - \cdots}{\left(n^5 + 15\,n^4 + 85\,n^3 + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((n
^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*
b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24
*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*
n + 120)*b^5)
```

mupad [B] time = 2.95, size = 363, normalized size = 2.88

$$(a+b\,x)^n\left(\frac{d\,x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}-\frac{a^2\,\left(-24\,d\,a^3+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b\right.}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2 +
 85*n^3 + 15*n^4 + n^5 + 120) - (a^2*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2 +
b^3*c*n^3 + 47*b^3*c*n))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (x^2*(n + 1)*(60*b^3*c + 12*b^3*c*n^2 + b^3*c*n^3 + 12*a^3*d*n + 47*b
^3*c*n))/(b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*n*x*(60
*b^3*c - 24*a^3*d + 12*b^3*c*n^2 + b^3*c*n^3 + 47*b^3*c*n))/(b^4*(274*n + 2
25*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n + 6*n^2 + n^3 + 6
))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*d*n*x^3*(3*
n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

sympy [A] time = 4.86, size = 3704, normalized size = 29.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x**2/2 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/(
12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**
9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48
*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5 + 4
8*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 88*a**
3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3
*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*a**2*b**
2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**
3 + 12*b**9*x**4) - a*b**3*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*
x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3*log(a/b + x)/(12*a
**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x*
*4) + 48*a*b**3*d*x**3/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 +
 48*a*b**8*x**3 + 12*b**9*x**4) - 4*b**4*c*x/(12*a**4*b**5 + 48*a**3*b**6*x
 + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 12*b**4*d*x**4*log(
a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**
3 + 12*b**9*x**4), Eq(n, -5)), (-24*a**4*d*log(a/b + x)/(6*a**3*b**5 + 18*a
**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 44*a**4*d/(6*a**3*b**5 + 18*a*
*2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**3*b*d*x*log(a/b + x)/(6*a
**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 108*a**3*b*d*x/
(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**2*b**
2*d*x**2*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b*
*8*x**3) - 72*a**2*b**2*d*x**2/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x*
*2 + 6*b**8*x**3) - a*b**3*c/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2
```

```
 + 6*b**8*x**3) - 24*a*b**3*d*x**3*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6
*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 3*b**4*c*x/(6*a**3*b**5 + 18*a**2*b**6
*x + 18*a*b**7*x**2 + 6*b**8*x**3) + 6*b**4*d*x**4/(6*a**3*b**5 + 18*a**2*b
**6*x + 18*a*b**7*x**2 + 6*b**8*x**3), Eq(n, -4)), (12*a**4*d*log(a/b + x)/
(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*a**2*b**5 + 4*a*b**
6*x + 2*b**7*x**2) + 24*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x +
 2*b**7*x**2) + 24*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12
*a**2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - a
*b**3*c/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) - 2*b**4*c*x/(2*a**2*b**5 + 4*a*b**6*x + 2
*b**7*x**2) + b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2), Eq(n, -
3)), (-12*a**4*d*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d/(3*a*b**5 +
 3*b**6*x) - 12*a**3*b*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2
*d*x**2/(3*a*b**5 + 3*b**6*x) + 3*a*b**3*c*log(a/b + x)/(3*a*b**5 + 3*b**6*
x) + 3*a*b**3*c/(3*a*b**5 + 3*b**6*x) - 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*
x) + 3*b**4*c*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + b**4*d*x**4/(3*a*b**5
+ 3*b**6*x), Eq(n, -2)), (a**4*d*log(a/b + x)/b**5 - a**3*d*x/b**4 + a**2*d
*x**2/(2*b**3) - a*c*log(a/b + x)/b**2 - a*d*x**3/(3*b**2) + c*x/b + d*x**4
/(4*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 12*a**3*b**2*d*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2
*d*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*
n**2 + 274*b**5*n + 120*b**5) - a**2*b**3*c*n**3*(a + b*x)**n/(b**5*n**5 +
15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a
**2*b**3*c*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225
*b**5*n**2 + 274*b**5*n + 120*b**5) - 47*a**2*b**3*c*n*(a + b*x)**n/(b**5*n
**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
- 60*a**2*b**3*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 22
5*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**3*x**3*(a + b*x)**n
/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 12
0*b**5) - 12*a**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 8*a**2*b**3*d*n*x*
*3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 +
274*b**5*n + 120*b**5) + a*b**4*c*n**4*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a*b**4*c*
n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**
2 + 274*b**5*n + 120*b**5) + 47*a*b**4*c*n**2*x*(a + b*x)**n/(b**5*n**5 + 1
5*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*a*
b**4*c*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5
*n**2 + 274*b**5*n + 120*b**5) + a*b**4*d*n**4*x**4*(a + b*x)**n/(b**5*n**5
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6
*a*b**4*d*n**3*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 11*a*b**4*d*n**2*x**4*(a + b*x)**
n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 1
20*b**5) + 6*a*b**4*d*n*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b*
*5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*c*n**4*x**2*(a + b*
x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
 + 120*b**5) + 13*b**5*c*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 59*b**5*c*n**2*x**
2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 2
74*b**5*n + 120*b**5) + 107*b**5*c*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5
*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*b**5*c*x
**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 +
 274*b**5*n + 120*b**5) + b**5*d*n**4*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 10*b**5*d*
n**3*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*
n**2 + 274*b**5*n + 120*b**5) + 35*b**5*d*n**2*x**5*(a + b*x)**n/(b**5*n**5
```

```
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 5
0*b**5*d*n*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225
*b**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*d*x**5*(a + b*x)**n/(b**5*n**
5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5), T
rue))
```

3.176 $\int (a+bx)^n \left(c+dx^3\right) dx$

Optimal. Leaf size=94

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

```
[Out] (-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^4/(1+n)+3*a^2*d*(b*x+a)^(2+n)/b^4/(2+n)-3*a*
d*(b*x+a)^(3+n)/b^4/(3+n)+d*(b*x+a)^(4+n)/b^4/(4+n)
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1850}

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 +
 n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)
^(4 + n))/(b^4*(4 + n))
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n\left(c+dx^3\right)dx&=\int\left(\frac{\left(b^3c-a^3d\right)(a+bx)^n}{b^3}+\frac{3a^2d(a+bx)^{1+n}}{b^3}-\frac{3ad(a+bx)^{2+n}}{b^3}+\frac{d(a+bx)^{3+n}}{b^3}\right)dx\\&=\frac{\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2d(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(a+bx)^{4+n}}{b^4(4+n)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 1.00

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 +
 n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)
^(4 + n))/(b^4*(4 + n))
```

fricas [B] time = 0.68, size = 222, normalized size = 2.36

$$\frac{\left(ab^3cn^3+9\,ab^3cn^2+26\,ab^3cn+24\,ab^3c-6\,a^4d+\left(b^4dn^3+6\,b^4dn^2+11\,b^4dn+6\,b^4d\right)x^4+\left(ab^3dn^3+3\,ab^3dn^2\right.}{b^4n^4+10\,b^4n^3+35\,b^4n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] (a*b^3*c*n^3 + 9*a*b^3*c*n^2 + 26*a*b^3*c*n + 24*a*b^3*c - 6*a^4*d + (b^4*d
*n^3 + 6*b^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 + (a*b^3*d*n^3 + 3*a*b^3*d*n
^2 + 2*a*b^3*d*n)*x^3 - 3*(a^2*b^2*d*n^2 + a^2*b^2*d*n)*x^2 + (b^4*c*n^3 +
9*b^4*c*n^2 + 24*b^4*c + 2*(13*b^4*c + 3*a^3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^
4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

giac [B] time = 0.33, size = 361, normalized size = 3.84

$$\frac{(bx+a)^n b^4 dn^3 x^4 + (bx+a)^n ab^3 dn^3 x^3 + 6\,(bx+a)^n b^4 dn^2 x^4 + 3\,(bx+a)^n ab^3 dn^2 x^3 + 11\,(bx+a)^n b^4 dnx^4 + (b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b^
4*d*n^2*x^4 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 11*(b*x + a)^n*b^4*d*n*x^4 +
(b*x + a)^n*b^4*c*n^3*x - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a
*b^3*d*n*x^3 + 6*(b*x + a)^n*b^4*d*x^4 + (b*x + a)^n*a*b^3*c*n^3 + 9*(b*x +
 a)^n*b^4*c*n^2*x - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 + 9*(b*x + a)^n*a*b^3*c*n
^2 + 26*(b*x + a)^n*b^4*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 26*(b*x + a)^n*
a*b^3*c*n + 24*(b*x + a)^n*b^4*c*x + 24*(b*x + a)^n*a*b^3*c - 6*(b*x + a)^n
*a^4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

maple [A] time = 0.00, size = 167, normalized size = 1.78

$$-\frac{\left(-b^3 d\,n^3 x^3 - 6b^3 d\,n^2 x^3 + 3a\,b^2 d\,n^2 x^2 - 11b^3 dn\,x^3 + 9a\,b^2 dn\,x^2 - b^3 c\,n^3 - 6d\,x^3 b^3 - 6a^2 bdnx + 6ad\,x^2 b^2 - 9b}{\left(n^4 + 10n^3 + 35n^2 + 50n + 24\right) b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c),x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^3*d*n^3*x^3-6*b^3*d*n^2*x^3+3*a*b^2*d*n^2*x^2-11*b^3*d*n
*x^3+9*a*b^2*d*n*x^2-b^3*c*n^3-6*b^3*d*x^3-6*a^2*b*d*n*x+6*a*b^2*d*x^2-9*b^
3*c*n^2-6*a^2*b*d*x-26*b^3*c*n+6*a^3*d-24*b^3*c)/b^4/(n^4+10*n^3+35*n^2+50*
n+24)
```

maxima [A] time = 0.96, size = 122, normalized size = 1.30

$$\frac{(bx+a)^{n+1}c}{b(n+1)} + \frac{\left(\left(n^3 + 6\,n^2 + 11\,n + 6\right)b^4 x^4 + \left(n^3 + 3\,n^2 + 2\,n\right)ab^3 x^3 - 3\left(n^2 + n\right)a^2 b^2 x^2 + 6\,a^3 bnx - 6\,a^4\right)(bx +}{\left(n^4 + 10\,n^3 + 35\,n^2 + 50\,n + 24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3
+ 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(
b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

mupad [B] time = 2.95, size = 247, normalized size = 2.63

$$(a+b\,x)^n \left(\frac{x\,\left(6\,d\,a^3\,b\,n + c\,b^4\,n^3 + 9\,c\,b^4\,n^2 + 26\,c\,b^4\,n + 24\,c\,b^4\right)}{b^4\,\left(n^4 + 10\,n^3 + 35\,n^2 + 50\,n + 24\right)} + \frac{a\,\left(-6\,d\,a^3 + c\,b^3\,n^3 + 9\,c\,b^3\,n^2 + 26\,c\,b^3\,n}{b^4\,\left(n^4 + 10\,n^3 + 35\,n^2 + 50\,n + 2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((x*(24*b^4*c + 9*b^4*c*n^2 + b^4*c*n^3 + 26*b^4*c*n + 6*a^3*b*
d*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*(24*b^3*c - 6*a^3*d +
9*b^3*c*n^2 + b^3*c*n^3 + 26*b^3*c*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 +
 24)) + (d*x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 + 24
) - (3*a^2*d*n*x^2*(n + 1))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*
d*n*x^3*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

sympy [A] time = 2.79, size = 1906, normalized size = 20.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6*a**3
*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*a**3*
b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*log(a
/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*
a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**
2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 2*b**3*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(a/b + x)/(
2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*b**5*
x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2
*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*
b**2*d*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c/
(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 + 4*a
*b**5*x + 2*b**6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 + 2*b*
*5*x) + 6*a**3*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*a*b**
4 + 2*b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c/(2*a*b**4
+ 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**3*d*log(a
/b + x)/b**4 + a**2*d*x/b**3 - a*d*x**2/(2*b**2) + c*log(a/b + x)/b + d*x**
3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35
*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a + b*x)**n/(b**4*n**4
+ 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d*n**2*x
**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b*
*4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*c*n**3*(a + b*x)**n/(b**4*n**4 + 10
*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*a*b**3*c*n**2*(a + b*x
)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*a
*b**3*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n
 + 24*b**4) + 24*a*b**3*c*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 +
10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(
a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
 + 2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2
+ 50*b**4*n + 24*b**4) + b**4*c*n**3*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**4*c*n**2*x*(a + b*x)**n/(b*
*4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*b**4*c*n*
x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b*
*4) + 24*b**4*c*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 5
0*b**4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**4*(a + b*x)**n/
(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*d
*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
 24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4), True))
```

3.177 $\int \frac{(a+bx)^n\left(c+dx^3\right)}{x}\,dx$

Optimal. Leaf size=99

$$\frac{a^2d(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}-\frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[Out] a^2*d*(b*x+a)^(1+n)/b^3/(1+n)-2*a*d*(b*x+a)^(2+n)/b^3/(2+n)+d*(b*x+a)^(3+n)
/b^3/(3+n)-c*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1620, 65}

$$\frac{a^2d(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}-\frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^3))/x,x]
```

```
[Out] (a^2*d*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n))/(b^3*(2
 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n)) - (c*(a + b*x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n\left(c+dx^3\right)}{x}\,dx &= \int \left(\frac{a^2d(a+bx)^n}{b^2}+\frac{c(a+bx)^n}{x}-\frac{2ad(a+bx)^{1+n}}{b^2}+\frac{d(a+bx)^{2+n}}{b^2}\right)dx\\ &= \frac{a^2d(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}+c\int \frac{(a+bx)^n}{x}\,dx\\ &= \frac{a^2d(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}-\frac{c(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+\right.}{a(1+n)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 94, normalized size = 0.95

$$\frac{(a+bx)^{n+1}\left(ad\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)-b^3c\left(n^2+5n+6\right)\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)\right)}{ab^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^3))/x,x]
```

```
[Out] ((a + b*x)^(1 + n)*(a*d*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2)
 - b^3*c*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a]))/
(a*b^3*(1 + n)*(2 + n)*(3 + n))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^3 + c\right)(bx + a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)*(b*x + a)^n/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)(bx + a)^n}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3 + c\right)(bx + a)^n}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^3+c)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)(bx + a)^n}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3 + c\right)\,(a + b\,x)^n}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^3)*(a + b*x)^n)/x,x)
```

```
[Out] int(((c + d*x^3)*(a + b*x)^n)/x, x)
```

<u>**sympy**</u> **[B]** time = 6.43, size = 741, normalized size = 7.48

$$-\frac{b^n cn\left(\frac{a}{b}+x\right)^n\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{\Gamma(n+2)}-\frac{b^n c\left(\frac{a}{b}+x\right)^n\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{\Gamma(n+2)}+d\left(\begin{cases}\frac{a^n x^3}{3}\\ \frac{2a^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}\\ -\frac{2a^2\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\\ \frac{a^2\log\left(\frac{a}{b}+x\right)}{b^3}-\frac{ax}{b^2}+\\ \frac{2a^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)/x,x)
```

```
[Out] -b**n*c*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(n +
 2) - b**n*c*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) + d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*
b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**
5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4
*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/
(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)
/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**
3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b
**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 +
6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 +
6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**
3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n*
*3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b**3*
n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(b**3
*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*
n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) - b*b**n*c*n*x*(a/b + x)**
n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b*b**n*c*x*
(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

3.178 $\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=294

$$-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^9(n+2)}-\frac{10ad\left(b^3c-7a^3d\right)(a+bx)^{n+5}}{b^9(n+5)}+\frac{2d\left(b^3c-28a^3d\right)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^2(\cdots}{b^9(\cdots}$$

```
[Out] a^2*(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^9/(1+n)-2*a*(-4*a^3*d+b^3*c)*(-a^3*d+b
^3*c)*(b*x+a)^(2+n)/b^9/(2+n)+(28*a^6*d^2-20*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(
3+n)/b^9/(3+n)+4*a^2*d*(-14*a^3*d+5*b^3*c)*(b*x+a)^(4+n)/b^9/(4+n)-10*a*d*(
-7*a^3*d+b^3*c)*(b*x+a)^(5+n)/b^9/(5+n)+2*d*(-28*a^3*d+b^3*c)*(b*x+a)^(6+n)
/b^9/(6+n)+28*a^2*d^2*(b*x+a)^(7+n)/b^9/(7+n)-8*a*d^2*(b*x+a)^(8+n)/b^9/(8+
n)+d^2*(b*x+a)^(9+n)/b^9/(9+n)
```

Rubi [A] time = 0.20, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1620}

$$\frac{\left(-20a^3b^3cd+28a^6d^2+b^6c^2\right)(a+bx)^{n+3}}{b^9(n+3)}+\frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^9(n+1)}-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^9(n+2)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] (a^2*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^3*c - 4*a
^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^6*c^2 - 20*a^3
*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) + (4*a^2*d*(5*b^3*c
 - 14*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4 + n)) - (10*a*d*(b^3*c - 7*a^3*d)*(
a + b*x)^(5 + n))/(b^9*(5 + n)) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^(6 + n)
)/(b^9*(6 + n)) + (28*a^2*d^2*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^2*(
a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^2*(a + b*x)^(9 + n))/(b^9*(9 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx &= \int\left(\frac{\left(ab^3c-a^4d\right)^2(a+bx)^n}{b^8}-\frac{2\left(ab^6c^2-5a^4b^3cd+4a^7d^2\right)(a+bx)^{1+n}}{b^8}+\frac{\left(b^6c^2-\cdots\right.}{\cdots}\right.\\ &= \frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^9(1+n)}-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^9(2+n)}+\frac{\left(b^6c^2-20\cdots\right.}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.27, size = 252, normalized size = 0.86

$$\frac{(a+bx)^{n+1}\left(\frac{\left(ab^3c-a^4d\right)^2}{n+1}+\frac{2d(a+bx)^5\left(b^3c-28a^3d\right)}{n+6}+\frac{10ad(a+bx)^4\left(7a^3d-b^3c\right)}{n+5}-\frac{2a(a+bx)\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)}{n+2}+\frac{28a^2d^2(a+bx)^6}{n+7}+\frac{(a+bx)\cdots}{\cdots}\right.}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*b^3*c - a^4*d)^2/(1 + n) - (2*a*(b^3*c - 4*a^3*d)*(b
^3*c - a^3*d)*(a + b*x))/(2 + n) + ((b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)
*(a + b*x)^2)/(3 + n) + (4*a^2*d*(5*b^3*c - 14*a^3*d)*(a + b*x)^3)/(4 + n)
+ (10*a*d*(-(b^3*c) + 7*a^3*d)*(a + b*x)^4)/(5 + n) + (2*d*(b^3*c - 28*a^3*
d)*(a + b*x)^5)/(6 + n) + (28*a^2*d^2*(a + b*x)^6)/(7 + n) - (8*a*d^2*(a +
b*x)^7)/(8 + n) + (d^2*(a + b*x)^8)/(9 + n)))/b^9
```

fricas [B] time = 0.70, size = 1565, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^2*n^6 + 78*a^3*b^6*c^2*n^5 + 1250*a^3*b^6*c^2*n^4 + 120960*a^3
*b^6*c^2 - 120960*a^6*b^3*c*d + 40320*a^9*d^2 + (b^9*d^2*n^8 + 36*b^9*d^2*n
^7 + 546*b^9*d^2*n^6 + 4536*b^9*d^2*n^5 + 22449*b^9*d^2*n^4 + 67284*b^9*d^2
*n^3 + 118124*b^9*d^2*n^2 + 109584*b^9*d^2*n + 40320*b^9*d^2)*x^9 + (a*b^8*
d^2*n^8 + 28*a*b^8*d^2*n^7 + 322*a*b^8*d^2*n^6 + 1960*a*b^8*d^2*n^5 + 6769*
a*b^8*d^2*n^4 + 13132*a*b^8*d^2*n^3 + 13068*a*b^8*d^2*n^2 + 5040*a*b^8*d^2*
n)*x^8 - 8*(a^2*b^7*d^2*n^7 + 21*a^2*b^7*d^2*n^6 + 175*a^2*b^7*d^2*n^5 + 73
5*a^2*b^7*d^2*n^4 + 1624*a^2*b^7*d^2*n^3 + 1764*a^2*b^7*d^2*n^2 + 720*a^2*b
^7*d^2*n)*x^7 + 2*(b^9*c*d*n^8 + 39*b^9*c*d*n^7 + 60480*b^9*c*d + 4*(159*b^
9*c*d + 7*a^3*b^6*d^2)*n^6 + 6*(939*b^9*c*d + 70*a^3*b^6*d^2)*n^5 + (29469*
b^9*c*d + 2380*a^3*b^6*d^2)*n^4 + 9*(10279*b^9*c*d + 700*a^3*b^6*d^2)*n^3 +
 2*(84307*b^9*c*d + 3836*a^3*b^6*d^2)*n^2 + 24*(6709*b^9*c*d + 140*a^3*b^6*
d^2)*n)*x^6 + 2*(a*b^8*c*d*n^8 + 34*a*b^8*c*d*n^7 + 466*a*b^8*c*d*n^6 + 56*
(59*a*b^8*c*d - 3*a^4*b^5*d^2)*n^5 + (12949*a*b^8*c*d - 1680*a^4*b^5*d^2)*n
^4 + 2*(13883*a*b^8*c*d - 2940*a^4*b^5*d^2)*n^3 + 24*(1241*a*b^8*c*d - 350*
a^4*b^5*d^2)*n^2 + 4032*(3*a*b^8*c*d - a^4*b^5*d^2)*n)*x^5 - 10*(a^2*b^7*c*
d*n^7 + 30*a^2*b^7*c*d*n^6 + 346*a^2*b^7*c*d*n^5 + 24*(80*a^2*b^7*c*d - 7*a
^5*b^4*d^2)*n^4 + (5269*a^2*b^7*c*d - 1008*a^5*b^4*d^2)*n^3 + 6*(1115*a^2*b
^7*c*d - 308*a^5*b^4*d^2)*n^2 + 1008*(3*a^2*b^7*c*d - a^5*b^4*d^2)*n)*x^4 +
 30*(351*a^3*b^6*c^2 - 8*a^6*b^3*c*d)*n^3 + (b^9*c^2*n^8 + 42*b^9*c^2*n^7 +
 120960*b^9*c^2 + 8*(93*b^9*c^2 + 5*a^3*b^6*c*d)*n^6 + 18*(401*b^9*c^2 + 60
*a^3*b^6*c*d)*n^5 + (41619*b^9*c^2 + 10600*a^3*b^6*c*d)*n^4 + 12*(12039*b^9
*c^2 + 3750*a^3*b^6*c*d - 560*a^6*b^3*d^2)*n^3 + 4*(72569*b^9*c^2 + 18940*a
^3*b^6*c*d - 5040*a^6*b^3*d^2)*n^2 + 48*(6289*b^9*c^2 + 840*a^3*b^6*c*d - 2
80*a^6*b^3*d^2)*n)*x^3 + 4*(12287*a^3*b^6*c^2 - 1440*a^6*b^3*c*d)*n^2 + (a*
b^8*c^2*n^8 + 40*a*b^8*c^2*n^7 + 664*a*b^8*c^2*n^6 + 10*(589*a*b^8*c^2 - 12
*a^4*b^5*c*d)*n^5 + (29839*a*b^8*c^2 - 3000*a^4*b^5*c*d)*n^4 + 10*(8479*a*b
^8*c^2 - 2580*a^4*b^5*c*d)*n^3 + 24*(5029*a*b^8*c^2 - 3475*a^4*b^5*c*d + 84
0*a^7*b^2*d^2)*n^2 + 20160*(3*a*b^8*c^2 - 3*a^4*b^5*c*d + a^7*b^2*d^2)*n)*x
^2 + 48*(2509*a^3*b^6*c^2 - 955*a^6*b^3*c*d)*n - 2*(a^2*b^7*c^2*n^7 + 39*a^
2*b^7*c^2*n^6 + 625*a^2*b^7*c^2*n^5 + 15*(351*a^2*b^7*c^2 - 8*a^5*b^4*c*d)*
n^4 + 2*(12287*a^2*b^7*c^2 - 1440*a^5*b^4*c*d)*n^3 + 24*(2509*a^2*b^7*c^2 -
 955*a^5*b^4*c*d)*n^2 + 20160*(3*a^2*b^7*c^2 - 3*a^5*b^4*c*d + a^8*b*d^2)*n
)*x)*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*b^9*n^6 + 63273
*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 1026576*b^9*
n + 362880*b^9)
```

giac [B] time = 0.61, size = 2660, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^9*d^2*n^8*x^9 + (b*x + a)^n*a*b^8*d^2*n^8*x^8 + 36*(b*x + a)
^n*b^9*d^2*n^7*x^9 + 28*(b*x + a)^n*a*b^8*d^2*n^7*x^8 + 546*(b*x + a)^n*b^9
*d^2*n^6*x^9 + 2*(b*x + a)^n*b^9*c*d*n^8*x^6 - 8*(b*x + a)^n*a^2*b^7*d^2*n^
7*x^7 + 322*(b*x + a)^n*a*b^8*d^2*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^2*n^5*x^
9 + 2*(b*x + a)^n*a*b^8*c*d*n^8*x^5 + 78*(b*x + a)^n*b^9*c*d*n^7*x^6 - 168*
(b*x + a)^n*a^2*b^7*d^2*n^6*x^7 + 1960*(b*x + a)^n*a*b^8*d^2*n^5*x^8 + 2244
9*(b*x + a)^n*b^9*d^2*n^4*x^9 + 68*(b*x + a)^n*a*b^8*c*d*n^7*x^5 + 1272*(b*
x + a)^n*b^9*c*d*n^6*x^6 + 56*(b*x + a)^n*a^3*b^6*d^2*n^6*x^6 - 1400*(b*x +
 a)^n*a^2*b^7*d^2*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^2*n^4*x^8 + 67284*(b*x
 + a)^n*b^9*d^2*n^3*x^9 + (b*x + a)^n*b^9*c^2*n^8*x^3 - 10*(b*x + a)^n*a^2*
b^7*c*d*n^7*x^4 + 932*(b*x + a)^n*a*b^8*c*d*n^6*x^5 + 11268*(b*x + a)^n*b^9
*c*d*n^5*x^6 + 840*(b*x + a)^n*a^3*b^6*d^2*n^5*x^6 - 5880*(b*x + a)^n*a^2*b
^7*d^2*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^2*n^3*x^8 + 118124*(b*x + a)^n*b
^9*d^2*n^2*x^9 + (b*x + a)^n*a*b^8*c^2*n^8*x^2 + 42*(b*x + a)^n*b^9*c^2*n^7
*x^3 - 300*(b*x + a)^n*a^2*b^7*c*d*n^6*x^4 + 6608*(b*x + a)^n*a*b^8*c*d*n^5
*x^5 - 336*(b*x + a)^n*a^4*b^5*d^2*n^5*x^5 + 58938*(b*x + a)^n*b^9*c*d*n^4*
x^6 + 4760*(b*x + a)^n*a^3*b^6*d^2*n^4*x^6 - 12992*(b*x + a)^n*a^2*b^7*d^2*
n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^2*n^2*x^8 + 109584*(b*x + a)^n*b^9*d^2*
n*x^9 + 40*(b*x + a)^n*a*b^8*c^2*n^7*x^2 + 744*(b*x + a)^n*b^9*c^2*n^6*x^3
+ 40*(b*x + a)^n*a^3*b^6*c*d*n^6*x^3 - 3460*(b*x + a)^n*a^2*b^7*c*d*n^5*x^4
 + 25898*(b*x + a)^n*a*b^8*c*d*n^4*x^5 - 3360*(b*x + a)^n*a^4*b^5*d^2*n^4*x
^5 + 185022*(b*x + a)^n*b^9*c*d*n^3*x^6 + 12600*(b*x + a)^n*a^3*b^6*d^2*n^3
*x^6 - 14112*(b*x + a)^n*a^2*b^7*d^2*n^2*x^7 + 5040*(b*x + a)^n*a*b^8*d^2*n
*x^8 + 40320*(b*x + a)^n*b^9*d^2*x^9 - 2*(b*x + a)^n*a^2*b^7*c^2*n^7*x + 66
4*(b*x + a)^n*a*b^8*c^2*n^6*x^2 + 7218*(b*x + a)^n*b^9*c^2*n^5*x^3 + 1080*(
b*x + a)^n*a^3*b^6*c*d*n^5*x^3 - 19200*(b*x + a)^n*a^2*b^7*c*d*n^4*x^4 + 16
80*(b*x + a)^n*a^5*b^4*d^2*n^4*x^4 + 55532*(b*x + a)^n*a*b^8*c*d*n^3*x^5 -
11760*(b*x + a)^n*a^4*b^5*d^2*n^3*x^5 + 337228*(b*x + a)^n*b^9*c*d*n^2*x^6
+ 15344*(b*x + a)^n*a^3*b^6*d^2*n^2*x^6 - 5760*(b*x + a)^n*a^2*b^7*d^2*n*x^
7 - 78*(b*x + a)^n*a^2*b^7*c^2*n^6*x + 5890*(b*x + a)^n*a*b^8*c^2*n^5*x^2 -
 120*(b*x + a)^n*a^4*b^5*c*d*n^5*x^2 + 41619*(b*x + a)^n*b^9*c^2*n^4*x^3 +
10600*(b*x + a)^n*a^3*b^6*c*d*n^4*x^3 - 52690*(b*x + a)^n*a^2*b^7*c*d*n^3*x
^4 + 10080*(b*x + a)^n*a^5*b^4*d^2*n^3*x^4 + 59568*(b*x + a)^n*a*b^8*c*d*n^
2*x^5 - 16800*(b*x + a)^n*a^4*b^5*d^2*n^2*x^5 + 322032*(b*x + a)^n*b^9*c*d*
n*x^6 + 6720*(b*x + a)^n*a^3*b^6*d^2*n*x^6 + 2*(b*x + a)^n*a^3*b^6*c^2*n^6
- 1250*(b*x + a)^n*a^2*b^7*c^2*n^5*x + 29839*(b*x + a)^n*a*b^8*c^2*n^4*x^2
- 3000*(b*x + a)^n*a^4*b^5*c*d*n^4*x^2 + 144468*(b*x + a)^n*b^9*c^2*n^3*x^3
 + 45000*(b*x + a)^n*a^3*b^6*c*d*n^3*x^3 - 6720*(b*x + a)^n*a^6*b^3*d^2*n^3
*x^3 - 66900*(b*x + a)^n*a^2*b^7*c*d*n^2*x^4 + 18480*(b*x + a)^n*a^5*b^4*d^
2*n^2*x^4 + 24192*(b*x + a)^n*a*b^8*c*d*n*x^5 - 8064*(b*x + a)^n*a^4*b^5*d^
2*n*x^5 + 120960*(b*x + a)^n*b^9*c*d*x^6 + 78*(b*x + a)^n*a^3*b^6*c^2*n^5 -
 10530*(b*x + a)^n*a^2*b^7*c^2*n^4*x + 240*(b*x + a)^n*a^5*b^4*c*d*n^4*x +
84790*(b*x + a)^n*a*b^8*c^2*n^3*x^2 - 25800*(b*x + a)^n*a^4*b^5*c*d*n^3*x^2
 + 290276*(b*x + a)^n*b^9*c^2*n^2*x^3 + 75760*(b*x + a)^n*a^3*b^6*c*d*n^2*x
^3 - 20160*(b*x + a)^n*a^6*b^3*d^2*n^2*x^3 - 30240*(b*x + a)^n*a^2*b^7*c*d*
n*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^2*n*x^4 + 1250*(b*x + a)^n*a^3*b^6*c^2*
n^4 - 49148*(b*x + a)^n*a^2*b^7*c^2*n^3*x + 5760*(b*x + a)^n*a^5*b^4*c*d*n^
3*x + 120696*(b*x + a)^n*a*b^8*c^2*n^2*x^2 - 83400*(b*x + a)^n*a^4*b^5*c*d*
n^2*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^2*n^2*x^2 + 301872*(b*x + a)^n*b^9*c^
2*n*x^3 + 40320*(b*x + a)^n*a^3*b^6*c*d*n*x^3 - 13440*(b*x + a)^n*a^6*b^3*d
^2*n*x^3 + 10530*(b*x + a)^n*a^3*b^6*c^2*n^3 - 240*(b*x + a)^n*a^6*b^3*c*d*
n^3 - 120432*(b*x + a)^n*a^2*b^7*c^2*n^2*x + 45840*(b*x + a)^n*a^5*b^4*c*d*
n^2*x + 60480*(b*x + a)^n*a*b^8*c^2*n*x^2 - 60480*(b*x + a)^n*a^4*b^5*c*d*n
*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^2*n*x^2 + 120960*(b*x + a)^n*b^9*c^2*x^3
 + 49148*(b*x + a)^n*a^3*b^6*c^2*n^2 - 5760*(b*x + a)^n*a^6*b^3*c*d*n^2 - 1
20960*(b*x + a)^n*a^2*b^7*c^2*n*x + 120960*(b*x + a)^n*a^5*b^4*c*d*n*x - 40
320*(b*x + a)^n*a^8*b*d^2*n*x + 120432*(b*x + a)^n*a^3*b^6*c^2*n - 45840*(b
*x + a)^n*a^6*b^3*c*d*n + 120960*(b*x + a)^n*a^3*b^6*c^2 - 120960*(b*x + a)
^n*a^6*b^3*c*d + 40320*(b*x + a)^n*a^9*d^2)/(b^9*n^9 + 45*b^9*n^8 + 870*b^9
```

```
*n^7 + 9450*b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 117
2700*b^9*n^2 + 1026576*b^9*n + 362880*b^9)
```

maple [B] time = 0.02, size = 1565, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] (b*x+a)^(n+1)*(b^8*d^2*n^8*x^8+36*b^8*d^2*n^7*x^8-8*a*b^7*d^2*n^7*x^7+546*b
^8*d^2*n^6*x^8-224*a*b^7*d^2*n^6*x^7+2*b^8*c*d*n^8*x^5+4536*b^8*d^2*n^5*x^8
+56*a^2*b^6*d^2*n^6*x^6-2576*a*b^7*d^2*n^5*x^7+78*b^8*c*d*n^7*x^5+22449*b^8
*d^2*n^4*x^8+1176*a^2*b^6*d^2*n^5*x^6-10*a*b^7*c*d*n^7*x^4-15680*a*b^7*d^2*
n^4*x^7+1272*b^8*c*d*n^6*x^5+67284*b^8*d^2*n^3*x^8-336*a^3*b^5*d^2*n^5*x^5+
9800*a^2*b^6*d^2*n^4*x^6-340*a*b^7*c*d*n^6*x^4-54152*a*b^7*d^2*n^3*x^7+b^8*
c^2*n^8*x^2+11268*b^8*c*d*n^5*x^5+118124*b^8*d^2*n^2*x^8-5040*a^3*b^5*d^2*n
^4*x^5+40*a^2*b^6*c*d*n^6*x^3+41160*a^2*b^6*d^2*n^3*x^6-4660*a*b^7*c*d*n^5*
x^4-105056*a*b^7*d^2*n^2*x^7+42*b^8*c^2*n^7*x^2+58938*b^8*c*d*n^4*x^5+10958
4*b^8*d^2*n*x^8+1680*a^4*b^4*d^2*n^4*x^4-28560*a^3*b^5*d^2*n^3*x^5+1200*a^2
*b^6*c*d*n^5*x^3+90944*a^2*b^6*d^2*n^2*x^6-2*a*b^7*c^2*n^7*x-33040*a*b^7*c*
d*n^4*x^4-104544*a*b^7*d^2*n*x^7+744*b^8*c^2*n^6*x^2+185022*b^8*c*d*n^3*x^5
+40320*b^8*d^2*x^8+16800*a^4*b^4*d^2*n^3*x^4-120*a^3*b^5*c*d*n^5*x^2-75600*
a^3*b^5*d^2*n^2*x^5+13840*a^2*b^6*c*d*n^4*x^3+98784*a^2*b^6*d^2*n*x^6-80*a*
b^7*c^2*n^6*x-129490*a*b^7*c*d*n^3*x^4-40320*a*b^7*d^2*x^7+7218*b^8*c^2*n^5
*x^2+337228*b^8*c*d*n^2*x^5-6720*a^5*b^3*d^2*n^3*x^3+58800*a^4*b^4*d^2*n^2*
x^4-3240*a^3*b^5*c*d*n^4*x^2-92064*a^3*b^5*d^2*n*x^5+2*a^2*b^6*c^2*n^6+7680
0*a^2*b^6*c*d*n^3*x^3+40320*a^2*b^6*d^2*x^6-1328*a*b^7*c^2*n^5*x-277660*a*b
^7*c*d*n^2*x^4+41619*b^8*c^2*n^4*x^2+322032*b^8*c*d*n*x^5-40320*a^5*b^3*d^2
*n^2*x^3+240*a^4*b^4*c*d*n^4*x+84000*a^4*b^4*d^2*n*x^4-31800*a^3*b^5*c*d*n^
3*x^2-40320*a^3*b^5*d^2*x^5+78*a^2*b^6*c^2*n^5+210760*a^2*b^6*c*d*n^2*x^3-1
1780*a*b^7*c^2*n^4*x-297840*a*b^7*c*d*n*x^4+144468*b^8*c^2*n^3*x^2+120960*b
^8*c*d*x^5+20160*a^6*b^2*d^2*n^2*x^2-73920*a^5*b^3*d^2*n*x^3+6000*a^4*b^4*c
*d*n^3*x+40320*a^4*b^4*d^2*x^4-135000*a^3*b^5*c*d*n^2*x^2+1250*a^2*b^6*c^2*
n^4+267600*a^2*b^6*c*d*n*x^3-59678*a*b^7*c^2*n^3*x-120960*a*b^7*c*d*x^4+290
276*b^8*c^2*n^2*x^2+60480*a^6*b^2*d^2*n*x^2-240*a^5*b^3*c*d*n^3-40320*a^5*b
^3*d^2*x^3+51600*a^4*b^4*c*d*n^2*x-227280*a^3*b^5*c*d*n*x^2+10530*a^2*b^6*c
^2*n^3+120960*a^2*b^6*c*d*x^3-169580*a*b^7*c^2*n^2*x+301872*b^8*c^2*n*x^2-4
0320*a^7*b*d^2*n*x+40320*a^6*b^2*d^2*x^2-5760*a^5*b^3*c*d*n^2+166800*a^4*b^
4*c*d*n*x-120960*a^3*b^5*c*d*x^2+49148*a^2*b^6*c^2*n^2-241392*a*b^7*c^2*n*x
+120960*b^8*c^2*x^2-40320*a^7*b*d^2*x-45840*a^5*b^3*c*d*n+120960*a^4*b^4*c*
d*x+120432*a^2*b^6*c^2*n-120960*a*b^7*c^2*x+40320*a^8*d^2-120960*a^5*b^3*c*
d+120960*a^2*b^6*c^2)/b^9/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4
+723680*n^3+1172700*n^2+1026576*n+362880)
```

maxima [B] time = 0.81, size = 601, normalized size = 2.04

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^nc^2}{\left(n^3+6\,n^2+11\,n+6\right)b^3}+\frac{2\left(\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^5 + 15*n^4 + 85*n^3 + 225
*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5
*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)
^n*c*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) +
```

```
((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 +
 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4
+ 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 7
35*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n
^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 +
50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4
- 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 4032
0*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*d^2/((n^9 + 45*n^8 + 870*n^7 + 9450*n^
6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 362880)
*b^9)
```

mupad [B] time = 3.73, size = 1410, normalized size = 4.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (d^2*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*
n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(1026576*n + 1172700*n^2 + 723680*n^
3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880) +
(2*a^3*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 60216*b^6*c^2*n + 24574
*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b^6*c^
2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 2880*a^3*b^3*c*d*n^2 - 12
0*a^3*b^3*c*d*n^3))/(b^9*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4
 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (x^3*(a + b*x
)^n*(3*n + n^2 + 2)*(60480*b^6*c^2 - 6720*a^6*d^2*n + 60216*b^6*c^2*n + 245
74*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b^6*
c^2*n^6 + 20160*a^3*b^3*c*d*n + 7640*a^3*b^3*c*d*n^2 + 960*a^3*b^3*c*d*n^3
+ 40*a^3*b^3*c*d*n^4))/(b^6*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*
n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (2*d*x^6*(
a + b*x)^n*(504*b^3*c + 24*b^3*c*n^2 + b^3*c*n^3 + 28*a^3*d*n + 191*b^3*c*n
)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^3*(1026576*n + 117270
0*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
 n^9 + 362880)) - (2*a^2*n*x*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 6
0216*b^6*c^2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 3
9*b^6*c^2*n^5 + b^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 288
0*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^8*(1026576*n + 1172700*n^2 + 7
23680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 36
2880)) + (a*n*x^2*(n + 1)*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 6021
6*b^6*c^2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b
^6*c^2*n^5 + b^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 2880*a
^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^7*(1026576*n + 1172700*n^2 + 7236
80*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 36288
0)) + (a*d^2*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 +
 322*n^5 + 28*n^6 + n^7 + 5040))/(b*(1026576*n + 1172700*n^2 + 723680*n^3 +
 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) - (8
*a^2*d^2*n*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5
+ n^6 + 720))/(b^2*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 632
73*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) - (10*a^2*d*n*x^4*(a
+ b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(504*b^3*c - 168*a^3*d + 24*b^3*c*n^2 + b
^3*c*n^3 + 191*b^3*c*n))/(b^5*(1026576*n + 1172700*n^2 + 723680*n^3 + 26932
5*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (2*a*d*n
*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(504*b^3*c - 168*a^3*d
 + 24*b^3*c*n^2 + b^3*c*n^3 + 191*b^3*c*n))/(b^4*(1026576*n + 1172700*n^2 +
 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 +
362880))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

3.179 $\int x(a+bx)^n \left(c+dx^3\right)^2 dx$

Optimal. Leaf size=248

$$-\frac{a\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^8(n+2)}-\frac{ad\left(8b^3c-35a^3d\right)(a+bx)^{n+4}}{b^8(n+4)}+\frac{d\left(2b^3c-35a^3\right.}{b^8(n}$$

```
[Out] -a*(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^8/(1+n)+(-7*a^3*d+b^3*c)*(-a^3*d+b^3*c)
*(b*x+a)^(2+n)/b^8/(2+n)+3*a^2*d*(-7*a^3*d+4*b^3*c)*(b*x+a)^(3+n)/b^8/(3+n)
-a*d*(-35*a^3*d+8*b^3*c)*(b*x+a)^(4+n)/b^8/(4+n)+d*(-35*a^3*d+2*b^3*c)*(b*x
+a)^(5+n)/b^8/(5+n)+21*a^2*d^2*(b*x+a)^(6+n)/b^8/(6+n)-7*a*d^2*(b*x+a)^(7+n
)/b^8/(7+n)+d^2*(b*x+a)^(8+n)/b^8/(8+n)
```

Rubi [A] time = 0.15, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1620}

$$-\frac{a\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^8(n+2)}+\frac{3a^2d\left(4b^3c-7a^3d\right)(a+bx)^{n+3}}{b^8(n+3)}-\frac{ad\left(8b^3c-35\right.}{b^8(}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] -((a*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^3*c - 7*a^3*
d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) + (3*a^2*d*(4*b^3*c - 7
*a^3*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) - (a*d*(8*b^3*c - 35*a^3*d)*(a + b
*x)^(4 + n))/(b^8*(4 + n)) + (d*(2*b^3*c - 35*a^3*d)*(a + b*x)^(5 + n))/(b^
8*(5 + n)) + (21*a^2*d^2*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^2*(a + b
*x)^(7 + n))/(b^8*(7 + n)) + (d^2*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n\left(c+dx^3\right)^2 dx &= \int\left(-\frac{a\left(-b^3c+a^3d\right)^2(a+bx)^n}{b^7}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^7}-\frac{3a^2d\left(-4b\right.}{}\right.\\ &= -\frac{a\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^8(1+n)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^8(2+n)}+\frac{3a^2d\left(4b^3c-\right.}{b^8}\end{aligned}$$

Mathematica [A] time = 0.21, size = 211, normalized size = 0.85

$$\frac{(a+bx)^{n+1}\left(\frac{d(a+bx)^4\left(2b^3c-35a^3d\right)}{n+5}+\frac{ad(a+bx)^3\left(35a^3d-8b^3c\right)}{n+4}+\frac{(a+bx)\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)}{n+2}-\frac{a\left(b^3c-a^3d\right)^2}{n+1}+\frac{21a^2d^2(a+bx)^5}{n+6}+\frac{3a^2d(a+b}{}\right.}{b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-((a*(b^3*c - a^3*d)^2)/(1 + n)) + ((b^3*c - 7*a^3*d)*(
b^3*c - a^3*d)*(a + b*x))/(2 + n) + (3*a^2*d*(4*b^3*c - 7*a^3*d)*(a + b*x)^
2)/(3 + n) + (a*d*(-8*b^3*c + 35*a^3*d)*(a + b*x)^3)/(4 + n) + (d*(2*b^3*c
- 35*a^3*d)*(a + b*x)^4)/(5 + n) + (21*a^2*d^2*(a + b*x)^5)/(6 + n) - (7*a*
d^2*(a + b*x)^6)/(7 + n) + (d^2*(a + b*x)^7)/(8 + n)))/b^8
```

fricas [B] time = 0.53, size = 1216, normalized size = 4.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^2*n^6 + 33*a^2*b^6*c^2*n^5 + 445*a^2*b^6*c^2*n^4 + 20160*a^2*b^
6*c^2 - 16128*a^5*b^3*c*d + 5040*a^8*d^2 - (b^8*d^2*n^7 + 28*b^8*d^2*n^6 +
322*b^8*d^2*n^5 + 1960*b^8*d^2*n^4 + 6769*b^8*d^2*n^3 + 13132*b^8*d^2*n^2 +
 13068*b^8*d^2*n + 5040*b^8*d^2)*x^8 - (a*b^7*d^2*n^7 + 21*a*b^7*d^2*n^6 +
175*a*b^7*d^2*n^5 + 735*a*b^7*d^2*n^4 + 1624*a*b^7*d^2*n^3 + 1764*a*b^7*d^2
*n^2 + 720*a*b^7*d^2*n)*x^7 + 7*(a^2*b^6*d^2*n^6 + 15*a^2*b^6*d^2*n^5 + 85*
a^2*b^6*d^2*n^4 + 225*a^2*b^6*d^2*n^3 + 274*a^2*b^6*d^2*n^2 + 120*a^2*b^6*d
^2*n)*x^6 - 2*(b^8*c*d*n^7 + 31*b^8*c*d*n^6 + 8064*b^8*c*d + (391*b^8*c*d +
 21*a^3*b^5*d^2)*n^5 + (2581*b^8*c*d + 210*a^3*b^5*d^2)*n^4 + (9544*b^8*c*d
 + 735*a^3*b^5*d^2)*n^3 + 2*(9782*b^8*c*d + 525*a^3*b^5*d^2)*n^2 + 72*(282*
b^8*c*d + 7*a^3*b^5*d^2)*n)*x^5 - 2*(a*b^7*c*d*n^7 + 27*a*b^7*c*d*n^6 + 283
*a*b^7*c*d*n^5 + 21*(69*a*b^7*c*d - 5*a^4*b^4*d^2)*n^4 + 2*(1874*a*b^7*c*d
- 315*a^4*b^4*d^2)*n^3 + 3*(1524*a*b^7*c*d - 385*a^4*b^4*d^2)*n^2 + 126*(16
*a*b^7*c*d - 5*a^4*b^4*d^2)*n)*x^4 + 3*(1045*a^2*b^6*c^2 - 16*a^5*b^3*c*d)*
n^3 + 8*(a^2*b^6*c*d*n^6 + 24*a^2*b^6*c*d*n^5 + 211*a^2*b^6*c*d*n^4 + 3*(27
2*a^2*b^6*c*d - 35*a^5*b^3*d^2)*n^3 + 5*(260*a^2*b^6*c*d - 63*a^5*b^3*d^2)*
n^2 + 42*(16*a^2*b^6*c*d - 5*a^5*b^3*d^2)*n)*x^3 + 2*(6077*a^2*b^6*c^2 - 50
4*a^5*b^3*c*d)*n^2 - (b^8*c^2*n^7 + 34*b^8*c^2*n^6 + 20160*b^8*c^2 + 2*(239
*b^8*c^2 + 12*a^3*b^5*c*d)*n^5 + 4*(895*b^8*c^2 + 132*a^3*b^5*c*d)*n^4 + (1
5289*b^8*c^2 + 4008*a^3*b^5*c*d)*n^3 + 2*(18353*b^8*c^2 + 5784*a^3*b^5*c*d
- 1260*a^6*b^2*d^2)*n^2 + 72*(621*b^8*c^2 + 112*a^3*b^5*c*d - 35*a^6*b^2*d^
2)*n)*x^2 + 24*(1023*a^2*b^6*c^2 - 292*a^5*b^3*c*d)*n - (a*b^7*c^2*n^7 + 33
*a*b^7*c^2*n^6 + 445*a*b^7*c^2*n^5 + 3*(1045*a*b^7*c^2 - 16*a^4*b^4*c*d)*n^
4 + 2*(6077*a*b^7*c^2 - 504*a^4*b^4*c*d)*n^3 + 24*(1023*a*b^7*c^2 - 292*a^4
*b^4*c*d)*n^2 + 1008*(20*a*b^7*c^2 - 16*a^4*b^4*c*d + 5*a^7*b*d^2)*n)*x)*(b
*x + a)^n/(b^8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^
4 + 67284*b^8*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

giac [B] time = 0.61, size = 2034, normalized size = 8.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^8*d^2*n^7*x^8 + (b*x + a)^n*a*b^7*d^2*n^7*x^7 + 28*(b*x + a)
^n*b^8*d^2*n^6*x^8 + 21*(b*x + a)^n*a*b^7*d^2*n^6*x^7 + 322*(b*x + a)^n*b^8
*d^2*n^5*x^8 + 2*(b*x + a)^n*b^8*c*d*n^7*x^5 - 7*(b*x + a)^n*a^2*b^6*d^2*n^
6*x^6 + 175*(b*x + a)^n*a*b^7*d^2*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^2*n^4*x^
8 + 2*(b*x + a)^n*a*b^7*c*d*n^7*x^4 + 62*(b*x + a)^n*b^8*c*d*n^6*x^5 - 105*
(b*x + a)^n*a^2*b^6*d^2*n^5*x^6 + 735*(b*x + a)^n*a*b^7*d^2*n^4*x^7 + 6769*
(b*x + a)^n*b^8*d^2*n^3*x^8 + 54*(b*x + a)^n*a*b^7*c*d*n^6*x^4 + 782*(b*x +
 a)^n*b^8*c*d*n^5*x^5 + 42*(b*x + a)^n*a^3*b^5*d^2*n^5*x^5 - 595*(b*x + a)^
n*a^2*b^6*d^2*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^2*n^3*x^7 + 13132*(b*x + a
)^n*b^8*d^2*n^2*x^8 + (b*x + a)^n*b^8*c^2*n^7*x^2 - 8*(b*x + a)^n*a^2*b^6*c
*d*n^6*x^3 + 566*(b*x + a)^n*a*b^7*c*d*n^5*x^4 + 5162*(b*x + a)^n*b^8*c*d*n
^4*x^5 + 420*(b*x + a)^n*a^3*b^5*d^2*n^4*x^5 - 1575*(b*x + a)^n*a^2*b^6*d^2
```

```
*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^2*n^2*x^7 + 13068*(b*x + a)^n*b^8*d^2*n
*x^8 + (b*x + a)^n*a*b^7*c^2*n^7*x + 34*(b*x + a)^n*b^8*c^2*n^6*x^2 - 192*(
b*x + a)^n*a^2*b^6*c*d*n^5*x^3 + 2898*(b*x + a)^n*a*b^7*c*d*n^4*x^4 - 210*(
b*x + a)^n*a^4*b^4*d^2*n^4*x^4 + 19088*(b*x + a)^n*b^8*c*d*n^3*x^5 + 1470*(
b*x + a)^n*a^3*b^5*d^2*n^3*x^5 - 1918*(b*x + a)^n*a^2*b^6*d^2*n^2*x^6 + 720
*(b*x + a)^n*a*b^7*d^2*n*x^7 + 5040*(b*x + a)^n*b^8*d^2*x^8 + 33*(b*x + a)^
n*a*b^7*c^2*n^6*x + 478*(b*x + a)^n*b^8*c^2*n^5*x^2 + 24*(b*x + a)^n*a^3*b^
5*c*d*n^5*x^2 - 1688*(b*x + a)^n*a^2*b^6*c*d*n^4*x^3 + 7496*(b*x + a)^n*a*b
^7*c*d*n^3*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^2*n^3*x^4 + 39128*(b*x + a)^n*b
^8*c*d*n^2*x^5 + 2100*(b*x + a)^n*a^3*b^5*d^2*n^2*x^5 - 840*(b*x + a)^n*a^2
*b^6*d^2*n*x^6 - (b*x + a)^n*a^2*b^6*c^2*n^6 + 445*(b*x + a)^n*a*b^7*c^2*n^
5*x + 3580*(b*x + a)^n*b^8*c^2*n^4*x^2 + 528*(b*x + a)^n*a^3*b^5*c*d*n^4*x^
2 - 6528*(b*x + a)^n*a^2*b^6*c*d*n^3*x^3 + 840*(b*x + a)^n*a^5*b^3*d^2*n^3*
x^3 + 9144*(b*x + a)^n*a*b^7*c*d*n^2*x^4 - 2310*(b*x + a)^n*a^4*b^4*d^2*n^2
*x^4 + 40608*(b*x + a)^n*b^8*c*d*n*x^5 + 1008*(b*x + a)^n*a^3*b^5*d^2*n*x^5
 - 33*(b*x + a)^n*a^2*b^6*c^2*n^5 + 3135*(b*x + a)^n*a*b^7*c^2*n^4*x - 48*(
b*x + a)^n*a^4*b^4*c*d*n^4*x + 15289*(b*x + a)^n*b^8*c^2*n^3*x^2 + 4008*(b*
x + a)^n*a^3*b^5*c*d*n^3*x^2 - 10400*(b*x + a)^n*a^2*b^6*c*d*n^2*x^3 + 2520
*(b*x + a)^n*a^5*b^3*d^2*n^2*x^3 + 4032*(b*x + a)^n*a*b^7*c*d*n*x^4 - 1260*
(b*x + a)^n*a^4*b^4*d^2*n*x^4 + 16128*(b*x + a)^n*b^8*c*d*x^5 - 445*(b*x +
a)^n*a^2*b^6*c^2*n^4 + 12154*(b*x + a)^n*a*b^7*c^2*n^3*x - 1008*(b*x + a)^n
*a^4*b^4*c*d*n^3*x + 36706*(b*x + a)^n*b^8*c^2*n^2*x^2 + 11568*(b*x + a)^n*
a^3*b^5*c*d*n^2*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^2*n^2*x^2 - 5376*(b*x + a)
^n*a^2*b^6*c*d*n*x^3 + 1680*(b*x + a)^n*a^5*b^3*d^2*n*x^3 - 3135*(b*x + a)^
n*a^2*b^6*c^2*n^3 + 48*(b*x + a)^n*a^5*b^3*c*d*n^3 + 24552*(b*x + a)^n*a*b^
7*c^2*n^2*x - 7008*(b*x + a)^n*a^4*b^4*c*d*n^2*x + 44712*(b*x + a)^n*b^8*c^
2*n*x^2 + 8064*(b*x + a)^n*a^3*b^5*c*d*n*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^2
*n*x^2 - 12154*(b*x + a)^n*a^2*b^6*c^2*n^2 + 1008*(b*x + a)^n*a^5*b^3*c*d*n
^2 + 20160*(b*x + a)^n*a*b^7*c^2*n*x - 16128*(b*x + a)^n*a^4*b^4*c*d*n*x +
5040*(b*x + a)^n*a^7*b*d^2*n*x + 20160*(b*x + a)^n*b^8*c^2*x^2 - 24552*(b*x
 + a)^n*a^2*b^6*c^2*n + 7008*(b*x + a)^n*a^5*b^3*c*d*n - 20160*(b*x + a)^n*
a^2*b^6*c^2 + 16128*(b*x + a)^n*a^5*b^3*c*d - 5040*(b*x + a)^n*a^8*d^2)/(b^
8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8
*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

maple [B] time = 0.02, size = 1142, normalized size = 4.60

$$-\frac{\left(-b^7d^2n^7x^7 - 28b^7d^2n^6x^7 + 7a\,b^6d^2n^6x^6 - 322b^7d^2n^5x^7 + 147a\,b^6d^2n^5x^6 - 2b^7cd\,n^7x^4 - 1960b^7d^2n^4x^7 - 42a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^7*d^2*n^7*x^7-28*b^7*d^2*n^6*x^7+7*a*b^6*d^2*n^6*x^6-322
*b^7*d^2*n^5*x^7+147*a*b^6*d^2*n^5*x^6-2*b^7*c*d*n^7*x^4-1960*b^7*d^2*n^4*x
^7-42*a^2*b^5*d^2*n^5*x^5+1225*a*b^6*d^2*n^4*x^6-62*b^7*c*d*n^6*x^4-6769*b^
7*d^2*n^3*x^7-630*a^2*b^5*d^2*n^4*x^5+8*a*b^6*c*d*n^6*x^3+5145*a*b^6*d^2*n^
3*x^6-782*b^7*c*d*n^5*x^4-13132*b^7*d^2*n^2*x^7+210*a^3*b^4*d^2*n^4*x^4-357
0*a^2*b^5*d^2*n^3*x^5+216*a*b^6*c*d*n^5*x^3+11368*a*b^6*d^2*n^2*x^6-b^7*c^2
*n^7*x-5162*b^7*c*d*n^4*x^4-13068*b^7*d^2*n*x^7+2100*a^3*b^4*d^2*n^3*x^4-24
*a^2*b^5*c*d*n^5*x^2-9450*a^2*b^5*d^2*n^2*x^5+2264*a*b^6*c*d*n^4*x^3+12348*
a*b^6*d^2*n*x^6-34*b^7*c^2*n^6*x-19088*b^7*c*d*n^3*x^4-5040*b^7*d^2*x^7-840
*a^4*b^3*d^2*n^3*x^3+7350*a^3*b^4*d^2*n^2*x^4-576*a^2*b^5*c*d*n^4*x^2-11508
*a^2*b^5*d^2*n*x^5+a*b^6*c^2*n^6+11592*a*b^6*c*d*n^3*x^3+5040*a*b^6*d^2*x^6
-478*b^7*c^2*n^5*x-39128*b^7*c*d*n^2*x^4-5040*a^4*b^3*d^2*n^2*x^3+48*a^3*b^
4*c*d*n^4*x+10500*a^3*b^4*d^2*n*x^4-5064*a^2*b^5*c*d*n^3*x^2-5040*a^2*b^5*d
^2*x^5+33*a*b^6*c^2*n^5+29984*a*b^6*c*d*n^2*x^3-3580*b^7*c^2*n^4*x-40608*b^
7*c*d*n*x^4+2520*a^5*b^2*d^2*n^2*x^2-9240*a^4*b^3*d^2*n*x^3+1056*a^3*b^4*c*
d*n^3*x+5040*a^3*b^4*d^2*x^4-19584*a^2*b^5*c*d*n^2*x^2+445*a*b^6*c^2*n^4+36
```

```
576*a*b^6*c*d*n*x^3-15289*b^7*c^2*n^3*x-16128*b^7*c*d*x^4+7560*a^5*b^2*d^2*
n*x^2-48*a^4*b^3*c*d*n^3-5040*a^4*b^3*d^2*x^3+8016*a^3*b^4*c*d*n^2*x-31200*
a^2*b^5*c*d*n*x^2+3135*a*b^6*c^2*n^3+16128*a*b^6*c*d*x^3-36706*b^7*c^2*n^2*
x-5040*a^6*b*d^2*n*x+5040*a^5*b^2*d^2*x^2-1008*a^4*b^3*c*d*n^2+23136*a^3*b^
4*c*d*n*x-16128*a^2*b^5*c*d*x^2+12154*a*b^6*c^2*n^2-44712*b^7*c^2*n*x-5040*
a^6*b*d^2*x-7008*a^4*b^3*c*d*n+16128*a^3*b^4*c*d*x+24552*a*b^6*c^2*n-20160*
b^7*c^2*x+5040*a^7*d^2-16128*a^4*b^3*c*d+20160*a*b^6*c^2)/b^8/(n^8+36*n^7+5
46*n^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2+109584*n+40320)
```

maxima [A] time = 0.98, size = 474, normalized size = 1.91

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n c^2}{\left(n^2+3n+2\right)b^2}+\frac{2\left(\left(n^4+10n^3+35n^2+50n+24\right)b^5x^5+\left(n^4+6n^3+11n^2+6n\right)ab^4\right.}{\left(n^5+15n^4+85\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2
*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
- 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2
+ 274*n + 120)*b^5) + ((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 1313
2*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*
n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 27
4*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3
*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2
+ 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8
)*(b*x + a)^n*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n
^3 + 118124*n^2 + 109584*n + 40320)*b^8)
```

mupad [B] time = 3.39, size = 1136, normalized size = 4.58

$$\frac{d^2\,x^8\,{\left(a+b\,x\right)}^n\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}{n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+67284\,n^3+118124\,n^2+109584\,n+40320}-\frac{a^2\,{\left(a+b\,x\right)}^n\,\left(5040\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (d^2*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 453
6*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^2 +
20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 44
5*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3*b^3*c*d - 7008*a^3
*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))/(b^8*(109584*n + 1
18124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 403
20)) + (x^2*(n + 1)*(a + b*x)^n*(20160*b^6*c^2 - 2520*a^6*d^2*n + 24552*b^6
*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^
2*n^5 + b^6*c^2*n^6 + 8064*a^3*b^3*c*d*n + 3504*a^3*b^3*c*d*n^2 + 504*a^3*b
^3*c*d*n^3 + 24*a^3*b^3*c*d*n^4))/(b^6*(109584*n + 118124*n^2 + 67284*n^3 +
 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*n*x*(a + b*x)
^n*(5040*a^6*d^2 + 20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 31
35*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3
*b^3*c*d - 7008*a^3*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))
/(b^7*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 +
 36*n^7 + n^8 + 40320)) + (2*d*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^
4 + 24)*(336*b^3*c + 21*b^3*c*n^2 + b^3*c*n^3 + 21*a^3*d*n + 146*b^3*c*n))/
(b^3*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 +
36*n^7 + n^8 + 40320)) + (a*d^2*n*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*
```

```
n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b*(109584*n + 118124*n^2 + 67284*n^3
+ 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (7*a^2*d^2*n*x^
6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^2*(109584
*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8
 + 40320)) + (2*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(336*b^3*c -
 105*a^3*d + 21*b^3*c*n^2 + b^3*c*n^3 + 146*b^3*c*n))/(b^4*(109584*n + 1181
24*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)
) - (8*a^2*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(336*b^3*c - 105*a^3*d + 21*
b^3*c*n^2 + b^3*c*n^3 + 146*b^3*c*n))/(b^5*(109584*n + 118124*n^2 + 67284*n
^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

3.180 $\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=203

$$\frac{\left(b^3c-a^3d\right)^2 (a+bx)^{n+1}}{b^7(n+1)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{15a^2d^2(a+bx)^{n+5}}{b^7(n+5)}+\frac{6}{}$$

```
[Out] (-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^7/(1+n)+6*a^2*d*(-a^3*d+b^3*c)*(b*x+a)^(2+
n)/b^7/(2+n)-3*a*d*(-5*a^3*d+2*b^3*c)*(b*x+a)^(3+n)/b^7/(3+n)+2*d*(-10*a^3*
d+b^3*c)*(b*x+a)^(4+n)/b^7/(4+n)+15*a^2*d^2*(b*x+a)^(5+n)/b^7/(5+n)-6*a*d^2
*(b*x+a)^(6+n)/b^7/(6+n)+d^2*(b*x+a)^(7+n)/b^7/(7+n)
```

Rubi [A] time = 0.11, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1850}

$$\frac{\left(b^3c-a^3d\right)^2 (a+bx)^{n+1}}{b^7(n+1)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{2d\left(b^3c-10a^3d\right)(a+}{b^7(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) + (6*a^2*d*(b^3*c - a^3
*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) - (3*a*d*(2*b^3*c - 5*a^3*d)*(a + b*x)
^(3 + n))/(b^7*(3 + n)) + (2*d*(b^3*c - 10*a^3*d)*(a + b*x)^(4 + n))/(b^7*(
4 + n)) + (15*a^2*d^2*(a + b*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)
^(6 + n))/(b^7*(6 + n)) + (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n\left(c+dx^3\right)^2 dx &= \int \left(\frac{\left(b^3c-a^3d\right)^2(a+bx)^n}{b^6}-\frac{6a^2d\left(-b^3c+a^3d\right)(a+bx)^{1+n}}{b^6}+\frac{3ad\left(-2b^3c+5a^3d\right)}{b^6}\right.\\ &= \frac{\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^7(1+n)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^7(2+n)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+}{b^7(3+n)}\end{aligned}$$

Mathematica [A] time = 0.17, size = 172, normalized size = 0.85

$$\frac{(a+bx)^{n+1}\left(\frac{2d(a+bx)^3\left(b^3c-10a^3d\right)}{n+4}+\frac{3ad(a+bx)^2\left(5a^3d-2b^3c\right)}{n+3}+\frac{\left(b^3c-a^3d\right)^2}{n+1}+\frac{15a^2d^2(a+bx)^4}{n+5}+\frac{6a^2d(a+bx)\left(b^3c-a^3d\right)}{n+2}+\frac{d^2(a+bx)^6}{n+7}-\right.}{b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b^3*c - a^3*d)^2/(1 + n) + (6*a^2*d*(b^3*c - a^3*d)*(a
 + b*x))/(2 + n) + (3*a*d*(-2*b^3*c + 5*a^3*d)*(a + b*x)^2)/(3 + n) + (2*d*
```

```
(b^3*c - 10*a^3*d)*(a + b*x)^3)/(4 + n) + (15*a^2*d^2*(a + b*x)^4)/(5 + n)
- (6*a*d^2*(a + b*x)^5)/(6 + n) + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

fricas [B] time = 0.44, size = 893, normalized size = 4.40

$$\frac{\left(ab^6c^2n^6 + 27\,ab^6c^2n^5 + 295\,ab^6c^2n^4 + 5040\,ab^6c^2 - 2520\,a^4b^3cd + 720\,a^7d^2 + \left(b^7d^2n^6 + 21\,b^7d^2n^5 + 175\,b^7d^2n^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^2*n^6 + 27*a*b^6*c^2*n^5 + 295*a*b^6*c^2*n^4 + 5040*a*b^6*c^2 - 25
20*a^4*b^3*c*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*
n^4 + 735*b^7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^
7 + (a*b^6*d^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^
3 + 274*a*b^6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 - 6*(a^2*b^5*d^2*n^5 + 10*a^2*
b^5*d^2*n^4 + 35*a^2*b^5*d^2*n^3 + 50*a^2*b^5*d^2*n^2 + 24*a^2*b^5*d^2*n)*x
^5 + 2*(b^7*c*d*n^6 + 24*b^7*c*d*n^5 + 1260*b^7*c*d + (226*b^7*c*d + 15*a^3
*b^4*d^2)*n^4 + 6*(176*b^7*c*d + 15*a^3*b^4*d^2)*n^3 + 5*(509*b^7*c*d + 33*
a^3*b^4*d^2)*n^2 + 18*(164*b^7*c*d + 5*a^3*b^4*d^2)*n)*x^4 + 3*(555*a*b^6*c
^2 - 4*a^4*b^3*c*d)*n^3 + 2*(a*b^6*c*d*n^6 + 21*a*b^6*c*d*n^5 + 163*a*b^6*c
*d*n^4 + 3*(189*a*b^6*c*d - 20*a^4*b^3*d^2)*n^3 + 4*(211*a*b^6*c*d - 45*a^4
*b^3*d^2)*n^2 + 60*(7*a*b^6*c*d - 2*a^4*b^3*d^2)*n)*x^3 + 8*(638*a*b^6*c^2
- 27*a^4*b^3*c*d)*n^2 - 6*(a^2*b^5*c*d*n^5 + 19*a^2*b^5*c*d*n^4 + 125*a^2*b
^5*c*d*n^3 + (317*a^2*b^5*c*d - 60*a^5*b^2*d^2)*n^2 + 30*(7*a^2*b^5*c*d - 2
*a^5*b^2*d^2)*n)*x^2 + 12*(669*a*b^6*c^2 - 107*a^4*b^3*c*d)*n + (b^7*c^2*n^
6 + 27*b^7*c^2*n^5 + 5040*b^7*c^2 + (295*b^7*c^2 + 12*a^3*b^4*c*d)*n^4 + 9*
(185*b^7*c^2 + 24*a^3*b^4*c*d)*n^3 + 4*(1276*b^7*c^2 + 321*a^3*b^4*c*d)*n^2
 + 36*(223*b^7*c^2 + 70*a^3*b^4*c*d - 20*a^6*b*d^2)*n)*x)*(b*x + a)^n/(b^7*
n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^
2 + 13068*b^7*n + 5040*b^7)
```

giac [B] time = 0.50, size = 1477, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^2*n^5*x^7 + 15*(b*x + a)^n*a*b^6*d^2*n^5*x^6 + 175*(b*x + a)^n*b^7
*d^2*n^4*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^4 - 6*(b*x + a)^n*a^2*b^5*d^2*n^
5*x^5 + 85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7
+ 2*(b*x + a)^n*a*b^6*c*d*n^6*x^3 + 48*(b*x + a)^n*b^7*c*d*n^5*x^4 - 60*(b*
x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^6*d^2*n^3*x^6 + 1624*(b*
x + a)^n*b^7*d^2*n^2*x^7 + 42*(b*x + a)^n*a*b^6*c*d*n^5*x^3 + 452*(b*x + a)
^n*b^7*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 - 210*(b*x + a)^n*a
^2*b^5*d^2*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b
^7*d^2*n*x^7 + (b*x + a)^n*b^7*c^2*n^6*x - 6*(b*x + a)^n*a^2*b^5*c*d*n^5*x^
2 + 326*(b*x + a)^n*a*b^6*c*d*n^4*x^3 + 2112*(b*x + a)^n*b^7*c*d*n^3*x^4 +
180*(b*x + a)^n*a^3*b^4*d^2*n^3*x^4 - 300*(b*x + a)^n*a^2*b^5*d^2*n^2*x^5 +
 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*d^2*x^7 + (b*x + a)^
n*a*b^6*c^2*n^6 + 27*(b*x + a)^n*b^7*c^2*n^5*x - 114*(b*x + a)^n*a^2*b^5*c*
d*n^4*x^2 + 1134*(b*x + a)^n*a*b^6*c*d*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^
2*n^3*x^3 + 5090*(b*x + a)^n*b^7*c*d*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^2*
n^2*x^4 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 + 27*(b*x + a)^n*a*b^6*c^2*n^5
+ 295*(b*x + a)^n*b^7*c^2*n^4*x + 12*(b*x + a)^n*a^3*b^4*c*d*n^4*x - 750*(b
*x + a)^n*a^2*b^5*c*d*n^3*x^2 + 1688*(b*x + a)^n*a*b^6*c*d*n^2*x^3 - 360*(b
*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 5904*(b*x + a)^n*b^7*c*d*n*x^4 + 180*(b*x +
```

```
a)^n*a^3*b^4*d^2*n*x^4 + 295*(b*x + a)^n*a*b^6*c^2*n^4 + 1665*(b*x + a)^n*
b^7*c^2*n^3*x + 216*(b*x + a)^n*a^3*b^4*c*d*n^3*x - 1902*(b*x + a)^n*a^2*b^
5*c*d*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2*n^2*x^2 + 840*(b*x + a)^n*a*b^6
*c*d*n*x^3 - 240*(b*x + a)^n*a^4*b^3*d^2*n*x^3 + 2520*(b*x + a)^n*b^7*c*d*x
^4 + 1665*(b*x + a)^n*a*b^6*c^2*n^3 - 12*(b*x + a)^n*a^4*b^3*c*d*n^3 + 5104
*(b*x + a)^n*b^7*c^2*n^2*x + 1284*(b*x + a)^n*a^3*b^4*c*d*n^2*x - 1260*(b*x
 + a)^n*a^2*b^5*c*d*n*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2*n*x^2 + 5104*(b*x +
 a)^n*a*b^6*c^2*n^2 - 216*(b*x + a)^n*a^4*b^3*c*d*n^2 + 8028*(b*x + a)^n*b^
7*c^2*n*x + 2520*(b*x + a)^n*a^3*b^4*c*d*n*x - 720*(b*x + a)^n*a^6*b*d^2*n*
x + 8028*(b*x + a)^n*a*b^6*c^2*n - 1284*(b*x + a)^n*a^4*b^3*c*d*n + 5040*(b
*x + a)^n*b^7*c^2*x + 5040*(b*x + a)^n*a*b^6*c^2 - 2520*(b*x + a)^n*a^4*b^3
*c*d + 720*(b*x + a)^n*a^7*d^2)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*
b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

maple [B] time = 0.01, size = 793, normalized size = 3.91

$$\frac{\left(b^6d^2n^6x^6 + 21b^6d^2n^5x^6 - 6a\,b^5d^2n^5x^5 + 175b^6d^2n^4x^6 - 90a\,b^5d^2n^4x^5 + 2b^6cd\,n^6x^3 + 735b^6d^2n^3x^6 + 30a^2b^4c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] (b*x+a)^(n+1)*(b^6*d^2*n^6*x^6+21*b^6*d^2*n^5*x^6-6*a*b^5*d^2*n^5*x^5+175*b
^6*d^2*n^4*x^6-90*a*b^5*d^2*n^4*x^5+2*b^6*c*d*n^6*x^3+735*b^6*d^2*n^3*x^6+3
0*a^2*b^4*d^2*n^4*x^4-510*a*b^5*d^2*n^3*x^5+48*b^6*c*d*n^5*x^3+1624*b^6*d^2
*n^2*x^6+300*a^2*b^4*d^2*n^3*x^4-6*a*b^5*c*d*n^5*x^2-1350*a*b^5*d^2*n^2*x^5
+452*b^6*c*d*n^4*x^3+1764*b^6*d^2*n*x^6-120*a^3*b^3*d^2*n^3*x^3+1050*a^2*b^
4*d^2*n^2*x^4-126*a*b^5*c*d*n^4*x^2-1644*a*b^5*d^2*n*x^5+b^6*c^2*n^6+2112*b
^6*c*d*n^3*x^3+720*b^6*d^2*x^6-720*a^3*b^3*d^2*n^2*x^3+12*a^2*b^4*c*d*n^4*x
+1500*a^2*b^4*d^2*n*x^4-978*a*b^5*c*d*n^3*x^2-720*a*b^5*d^2*x^5+27*b^6*c^2*
n^5+5090*b^6*c*d*n^2*x^3+360*a^4*b^2*d^2*n^2*x^2-1320*a^3*b^3*d^2*n*x^3+228
*a^2*b^4*c*d*n^3*x+720*a^2*b^4*d^2*x^4-3402*a*b^5*c*d*n^2*x^2+295*b^6*c^2*n
^4+5904*b^6*c*d*n*x^3+1080*a^4*b^2*d^2*n*x^2-12*a^3*b^3*c*d*n^3-720*a^3*b^3
*d^2*x^3+1500*a^2*b^4*c*d*n^2*x-5064*a*b^5*c*d*n*x^2+1665*b^6*c^2*n^3+2520*
b^6*c*d*x^3-720*a^5*b*d^2*n*x+720*a^4*b^2*d^2*x^2-216*a^3*b^3*c*d*n^2+3804*
a^2*b^4*c*d*n*x-2520*a*b^5*c*d*x^2+5104*b^6*c^2*n^2-720*a^5*b*d^2*x-1284*a^
3*b^3*c*d*n+2520*a^2*b^4*c*d*x+8028*b^6*c^2*n+720*a^6*d^2-2520*a^3*b^3*c*d+
5040*b^6*c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5
040)
```

maxima [A] time = 0.66, size = 359, normalized size = 1.77

$$\frac{(bx+a)^{n+1}c^2}{b(n+1)} + \frac{2\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4 + \left(n^3+3\,n^2+2\,n\right)ab^3x^3 - 3\left(n^2+n\right)a^2b^2x^2 + 6\,a^3bnx - 6\,a^4\right)(b}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (
n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^
4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + ((n^6 + 21*n
^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n^5 +
 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n^3 +
 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x^4 -
 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*a^6*
b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769
*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)
```

mupad [B] time = 3.19, size = 878, normalized size = 4.33

$$\frac{a\,(a+b\,x)^n\,\left(720\,a^6\,d^2-12\,a^3\,b^3\,c\,d\,n^3-216\,a^3\,b^3\,c\,d\,n^2-1284\,a^3\,b^3\,c\,d\,n-2520\,a^3\,b^3\,c\,d+b^6\,c^2\,n^6+27\,b^6\,c^2\,n^5}{b^7\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (a*(a + b*x)^n*(720*a^6*d^2 + 5040*b^6*c^2 + 8028*b^6*c^2*n + 5104*b^6*c^2*
n^2 + 1665*b^6*c^2*n^3 + 295*b^6*c^2*n^4 + 27*b^6*c^2*n^5 + b^6*c^2*n^6 - 2
520*a^3*b^3*c*d - 1284*a^3*b^3*c*d*n - 216*a^3*b^3*c*d*n^2 - 12*a^3*b^3*c*d
*n^3))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 +
 n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4
 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^
5 + 28*n^6 + n^7 + 5040) + (x*(a + b*x)^n*(5040*b^7*c^2 + 8028*b^7*c^2*n +
5104*b^7*c^2*n^2 + 1665*b^7*c^2*n^3 + 295*b^7*c^2*n^4 + 27*b^7*c^2*n^5 + b^
7*c^2*n^6 - 720*a^6*b*d^2*n + 2520*a^3*b^4*c*d*n + 1284*a^3*b^4*c*d*n^2 + 2
16*a^3*b^4*c*d*n^3 + 12*a^3*b^4*c*d*n^4))/(b^7*(13068*n + 13132*n^2 + 6769*
n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^4*(a + b*x)^n*(11
*n + 6*n^2 + n^3 + 6)*(210*b^3*c + 18*b^3*c*n^2 + b^3*c*n^3 + 15*a^3*d*n +
107*b^3*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 2
8*n^6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 +
 15*n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n
^5 + 28*n^6 + n^7 + 5040)) - (6*a^2*d^2*n*x^5*(a + b*x)^n*(50*n + 35*n^2 +
10*n^3 + n^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n
^5 + 28*n^6 + n^7 + 5040)) + (2*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(210*
b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n^3 + 107*b^3*c*n))/(b^4*(13068*n +
 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (6*a^2
*d*n*x^2*(n + 1)*(a + b*x)^n*(210*b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n
^3 + 107*b^3*c*n))/(b^5*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^
5 + 28*n^6 + n^7 + 5040))
```

sympy [A] time = 14.44, size = 11851, normalized size = 58.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x + c*d*x**4/2 + d**2*x**7/7), Eq(b, 0)), (60*a**6*d*
*2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200
*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6)
+ 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200
*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6)
+ 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4
*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5
+ 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*
a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x
**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b + x)/(60*a**6*b**7 +
 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**
11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a**4*b**2*d**2*x**2/(60*
a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 9
00*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 2*a**3*b**3*c*d/(6
0*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 +
 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1200*a**3*b**3*d
**2*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
+ 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*
x**6) + 2200*a**3*b**3*d**2*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4
```

```
*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5
+ 60*b**13*x**6) - 12*a**2*b**4*c*d*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900
*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*
x**5 + 60*b**13*x**6) + 900*a**2*b**4*d**2*x**4*log(a/b + x)/(60*a**6*b**7
+ 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b*
*11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1350*a**2*b**4*d**2*x**4/(60
*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 +
900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 30*a*b**5*c*d*x**
2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x*
*3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 360*a*b**5*d
**2*x**5*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
+ 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*
x**6) + 360*a*b**5*d**2*x**5/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**
9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60
*b**13*x**6) - 10*b**6*c**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9
*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*
b**13*x**6) - 40*b**6*c*d*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b
**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 +
60*b**13*x**6) + 60*b**6*d**2*x**6*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b*
*8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 36
0*a*b**12*x**5 + 60*b**13*x**6), Eq(n, -7)), (-60*a**6*d**2*log(a/b + x)/(1
0*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 5
0*a*b**11*x**4 + 10*b**12*x**5) - 137*a**6*d**2/(10*a**5*b**7 + 50*a**4*b**
8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12
*x**5) - 300*a**5*b*d**2*x*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 10
0*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) -
 625*a**5*b*d**2*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 10
0*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 600*a**4*b**2*d**2*x
**2*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*
a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 1100*a**4*b**2*d**2*x*
*2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**
3 + 50*a*b**11*x**4 + 10*b**12*x**5) - a**3*b**3*c*d/(10*a**5*b**7 + 50*a**
4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*
b**12*x**5) - 600*a**3*b**3*d**2*x**3*log(a/b + x)/(10*a**5*b**7 + 50*a**4*
b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b*
*12*x**5) - 900*a**3*b**3*d**2*x**3/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a*
*3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 5*a
**2*b**4*c*d*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a*
*2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**2*b**4*d**2*x**4*
log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2
*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**2*b**4*d**2*x**4/(1
0*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 5
0*a*b**11*x**4 + 10*b**12*x**5) - 10*a*b**5*c*d*x**2/(10*a**5*b**7 + 50*a**
4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*
b**12*x**5) - 60*a*b**5*d**2*x**5*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8
*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*
x**5) - 2*b**6*c**2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 1
00*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 10*b**6*c*d*x**3/(1
0*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 5
0*a*b**11*x**4 + 10*b**12*x**5) + 10*b**6*d**2*x**6/(10*a**5*b**7 + 50*a**4
*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b
**12*x**5), Eq(n, -6)), (60*a**6*d**2*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b
**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 125*a**6*d**2
/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b*
*11*x**4) + 240*a**5*b*d**2*x*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x +
24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 440*a**5*b*d**2*x/(4*
a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*
x**4) + 360*a**4*b**2*d**2*x**2*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x
+ 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 540*a**4*b**2*d**2*
```

```
x**2/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 +
4*b**11*x**4) - 2*a**3*b**3*c*d/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**
9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 240*a**3*b**3*d**2*x**3*log(a/b
+ x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 +
4*b**11*x**4) + 240*a**3*b**3*d**2*x**3/(4*a**4*b**7 + 16*a**3*b**8*x + 24*
a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 8*a**2*b**4*c*d*x/(4*a**
4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**
4) + 60*a**2*b**4*d**2*x**4*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24
*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 12*a*b**5*c*d*x**2/(4*a
**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x
**4) - 12*a*b**5*d**2*x**5/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**
2 + 16*a*b**10*x**3 + 4*b**11*x**4) - b**6*c**2/(4*a**4*b**7 + 16*a**3*b**8
*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 8*b**6*c*d*x**3/
(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**
11*x**4) + 2*b**6*d**2*x**6/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x*
*2 + 16*a*b**10*x**3 + 4*b**11*x**4), Eq(n, -5)), (-60*a**6*d**2*log(a/b +
x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 110*a**6*
d**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**
5*b*d**2*x*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b*
*10*x**3) - 270*a**5*b*d**2*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2
+ 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a**
2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2/(3*a**3*
b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 6*a**3*b**3*c*d*log(
a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 11*
a**3*b**3*c*d/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3)
- 60*a**3*b**3*d**2*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b*
*9*x**2 + 3*b**10*x**3) + 18*a**2*b**4*c*d*x*log(a/b + x)/(3*a**3*b**7 + 9*
a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 27*a**2*b**4*c*d*x/(3*a**3*b*
*7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 15*a**2*b**4*d**2*x**4
/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 18*a*b**5*c
*d*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10
*x**3) + 18*a*b**5*c*d*x**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 +
3*b**10*x**3) - 3*a*b**5*d**2*x**5/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*
x**2 + 3*b**10*x**3) - b**6*c**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x*
*2 + 3*b**10*x**3) + 6*b**6*c*d*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**
8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + b**6*d**2*x**6/(3*a**3*b**7 + 9*a**2*
b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3), Eq(n, -4)), (60*a**6*d**2*log(a/b +
 x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 90*a**6*d**2/(4*a**2*b**7 +
8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x*log(a/b + x)/(4*a**2*b**7 + 8
*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x/(4*a**2*b**7 + 8*a*b**8*x + 4*
b**9*x**2) + 60*a**4*b**2*d**2*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x
+ 4*b**9*x**2) - 24*a**3*b**3*c*d*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x +
4*b**9*x**2) - 36*a**3*b**3*c*d/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) -
20*a**3*b**3*d**2*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 48*a**2*b
**4*c*d*x*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 48*a**2*b
**4*c*d*x/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 5*a**2*b**4*d**2*x**4/
(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 24*a*b**5*c*d*x**2*log(a/b + x)/
(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 2*a*b**5*d**2*x**5/(4*a**2*b**7
+ 8*a*b**8*x + 4*b**9*x**2) - 2*b**6*c**2/(4*a**2*b**7 + 8*a*b**8*x + 4*b**
9*x**2) + 8*b**6*c*d*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + b**6*d
**2*x**6/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2), Eq(n, -3)), (-60*a**6*d*
*2*log(a/b + x)/(10*a*b**7 + 10*b**8*x) - 60*a**6*d**2/(10*a*b**7 + 10*b**8
*x) - 60*a**5*b*d**2*x*log(a/b + x)/(10*a*b**7 + 10*b**8*x) + 30*a**4*b**2*
d**2*x**2/(10*a*b**7 + 10*b**8*x) + 60*a**3*b**3*c*d*log(a/b + x)/(10*a*b**
7 + 10*b**8*x) + 60*a**3*b**3*c*d/(10*a*b**7 + 10*b**8*x) - 10*a**3*b**3*d*
*2*x**3/(10*a*b**7 + 10*b**8*x) + 60*a**2*b**4*c*d*x*log(a/b + x)/(10*a*b**
7 + 10*b**8*x) + 5*a**2*b**4*d**2*x**4/(10*a*b**7 + 10*b**8*x) - 30*a*b**5*
c*d*x**2/(10*a*b**7 + 10*b**8*x) - 3*a*b**5*d**2*x**5/(10*a*b**7 + 10*b**8*
x) - 10*b**6*c**2/(10*a*b**7 + 10*b**8*x) + 10*b**6*c*d*x**3/(10*a*b**7 + 1
```

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0*b**8*x) + 2*b**6*d**2*x**6/(10*a*b**7 + 10*b**8*x), Eq(n, -2)), (a**6*d**
2*log(a/b + x)/b**7 - a**5*d**2*x/b**6 + a**4*d**2*x**2/(2*b**5) - 2*a**3*c
*d*log(a/b + x)/b**4 - a**3*d**2*x**3/(3*b**4) + 2*a**2*c*d*x/b**3 + a**2*d
**2*x**4/(4*b**3) - a*c*d*x**2/b**2 - a*d**2*x**5/(5*b**2) + c**2*log(a/b +
 x)/b + 2*c*d*x**3/(3*b) + d**2*x**6/(6*b), Eq(n, -1)), (720*a**7*d**2*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 720*a**6*b*d**2*n
*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
+ 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a**5*b
**2*d**2*n**2*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b*
*7) + 360*a**5*b**2*d**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**
7*n + 5040*b**7) - 12*a**4*b**3*c*d*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*
n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
13068*b**7*n + 5040*b**7) - 216*a**4*b**3*c*d*n**2*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b*
*7*n**2 + 13068*b**7*n + 5040*b**7) - 1284*a**4*b**3*c*d*n*(a + b*x)**n/(b*
*7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 2520*a**4*b**3*c*d*(a + b*x)*
*n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n
**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 120*a**4*b**3*d**2*n**3
*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 360*a**
4*b**3*d**2*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**
5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040
*b**7) - 240*a**4*b**3*d**2*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) + 12*a**3*b**4*c*d*n**4*x*(a + b*x)**n/(b**7*n**7 + 28*
b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n*
*2 + 13068*b**7*n + 5040*b**7) + 216*a**3*b**4*c*d*n**3*x*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1
3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1284*a**3*b**4*c*d*n**2*x*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2520*a**3*b**4*c*
d*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 30*a**3
*b**4*d**2*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) + 180*a**3*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 330*a**3*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**7*n**
7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*
b**7*n**2 + 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n*x**4*(a + b*x)
**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*
n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*c*d*n**5*x
**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 114*a**2*
b**5*c*d*n**4*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b*
*7) - 750*a**2*b**5*c*d*n**3*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b
**7*n + 5040*b**7) - 1902*a**2*b**5*c*d*n**2*x**2*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) - 1260*a**2*b**5*c*d*n*x**2*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*d**2*n**5*x**
5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 60*a**2*b**
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5*d**2*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1
960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7
) - 210*a**2*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 3
22*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b*
*7*n + 5040*b**7) - 300*a**2*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) - 144*a**2*b**5*d**2*n*x**5*(a + b*x)**n/
(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + a*b**6*c**2*n**6*(a + b*x)
**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*
n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 27*a*b**6*c**2*n**5*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 295*a*b**6*c**2
*n**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1665*a*
b**6*c**2*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 196
0*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
 + 5104*a*b**6*c**2*n**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n
**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 50
40*b**7) + 8028*a*b**6*c**2*n*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) + 5040*a*b**6*c**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b
**7*n + 5040*b**7) + 2*a*b**6*c*d*n**6*x**3*(a + b*x)**n/(b**7*n**7 + 28*b*
*7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2
 + 13068*b**7*n + 5040*b**7) + 42*a*b**6*c*d*n**5*x**3*(a + b*x)**n/(b**7*n
**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1313
2*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 326*a*b**6*c*d*n**4*x**3*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1134*a*b**6*c*d*n**3*
x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1688*a*b
**6*c*d*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 840*a*b**6*c*d*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**
7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
 5040*b**7) + a*b**6*d**2*n**6*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
+ 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068
*b**7*n + 5040*b**7) + 15*a*b**6*d**2*n**5*x**6*(a + b*x)**n/(b**7*n**7 + 2
8*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*
n**2 + 13068*b**7*n + 5040*b**7) + 85*a*b**6*d**2*n**4*x**6*(a + b*x)**n/(b
**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 225*a*b**6*d**2*n**3*x**6*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 274*a*b**6*d**2
*n**2*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b*
*7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 12
0*a*b**6*d**2*n*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) + b**7*c**2*n**6*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) + 27*b**7*c**2*n**5*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**
7*n + 5040*b**7) + 295*b**7*c**2*n**4*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n
**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1
3068*b**7*n + 5040*b**7) + 1665*b**7*c**2*n**3*x*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) + 5104*b**7*c**2*n**2*x*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1
```

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3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 8028*b**7*c**2*n*x*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 5040*b**7*c**2*x*(a + b*
x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**
7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2*b**7*c*d*n**6*x**4
*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 48*b**7*c*d*
n**5*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**
7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 452
*b**7*c*d*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b
**7) + 2112*b**7*c*d*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322
*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7
*n + 5040*b**7) + 5090*b**7*c*d*n**2*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7
*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
 13068*b**7*n + 5040*b**7) + 5904*b**7*c*d*n*x**4*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 2520*b**7*c*d*x**4*(a + b*x)**n/(b**7*
n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 131
32*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*d**2*n**6*x**7*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 21*b**7*d**2*n**5*x**7*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 175*b**7*d**2*n
**4*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7
*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 735*
b**7*d**2*n**3*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b
**7) + 1624*b**7*d**2*n**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**
7*n + 5040*b**7) + 1764*b**7*d**2*n*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*
n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
13068*b**7*n + 5040*b**7) + 720*b**7*d**2*x**7*(a + b*x)**n/(b**7*n**7 + 28
*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n
**2 + 13068*b**7*n + 5040*b**7), True))
```

3.181 $\int \frac{(a+bx)^n\left(c+dx^3\right)^2}{x}\,dx$

Optimal. Leaf size=209

$$-\frac{ad\left(4b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{2d\left(b^3c-5a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d^2(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2d\left(2b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-5$$

```
[Out] a^2*d*(-a^3*d+2*b^3*c)*(b*x+a)^(1+n)/b^6/(1+n)-a*d*(-5*a^3*d+4*b^3*c)*(b*x+
a)^(2+n)/b^6/(2+n)+2*d*(-5*a^3*d+b^3*c)*(b*x+a)^(3+n)/b^6/(3+n)+10*a^2*d^2*
(b*x+a)^(4+n)/b^6/(4+n)-5*a*d^2*(b*x+a)^(5+n)/b^6/(5+n)+d^2*(b*x+a)^(6+n)/b
^6/(6+n)-c^2*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] time = 0.13, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1620, 65}

$$\frac{a^2d\left(2b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{ad\left(4b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{2d\left(b^3c-5a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d^2(a+bx)^{n+4}}{b^6(n+4)}-5a$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```

```
[Out] (a^2*d*(2*b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*d*(4*b^3*c -
 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a + b*
x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^(4 + n))/(b^6*(4 + n)) -
(5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a + b*x)^(6 + n))/(b^6*(6
 + n)) - (c^2*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x
)/a])/(a*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(a+bx)^n\left(c+dx^3\right)^2}{x}\,dx=\int \left(-\frac{a^2d\left(-2b^3c+a^3d\right)(a+bx)^n}{b^5}+\frac{c^2(a+bx)^n}{x}+\frac{ad\left(-4b^3c+5a^3d\right)(a+bx)^{1+n}}{b^5}+\right.$$

$$=\frac{a^2d\left(2b^3c-a^3d\right)(a+bx)^{1+n}}{b^6(1+n)}-\frac{ad\left(4b^3c-5a^3d\right)(a+bx)^{2+n}}{b^6(2+n)}+\frac{2d\left(b^3c-5a^3d\right)(a+}{b^6(3+n)}$$

$$=\frac{a^2d\left(2b^3c-a^3d\right)(a+bx)^{1+n}}{b^6(1+n)}-\frac{ad\left(4b^3c-5a^3d\right)(a+bx)^{2+n}}{b^6(2+n)}+\frac{2d\left(b^3c-5a^3d\right)(a+}{b^6(3+n)}$$

Mathematica [A] time = 0.17, size = 188, normalized size = 0.90

$$(a+bx)^{n+1}\left(\frac{2d(a+bx)^2\left(b^3c-5a^3d\right)}{b^6(n+3)}+\frac{ad(a+bx)\left(5a^3d-4b^3c\right)}{b^6(n+2)}+\frac{10a^2d^2(a+bx)^3}{b^6(n+4)}+\frac{a^2d\left(2b^3c-a^3d\right)}{b^6(n+1)}+\frac{d^2(a}{b^6(}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^3)^2)/x,x]
[Out] (a + b*x)^(1 + n)*((a^2*d*(2*b^3*c - a^3*d))/(b^6*(1 + n)) + (a*d*(-4*b^3*c
 + 5*a^3*d)*(a + b*x))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a + b*x)^2)/
(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^3)/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)
^4)/(b^6*(5 + n)) + (d^2*(a + b*x)^5)/(b^6*(6 + n)) - (c^2*Hypergeometric2F
1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^6+2\,cdx^3+c^2\right)(bx+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="fricas")
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x + a)^n/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(dx^3+c\right)^2(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{\left(d\,x^3+c\right)^2(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^2/x,x)
[Out] int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(dx^3+c\right)^2(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^3+c\right)^2\,{\left(a+b\,x\right)}^n}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^3)^2*(a + b*x)^n)/x,x)
```

```
[Out] int(((c + d*x^3)^2*(a + b*x)^n)/x, x)
```

sympy [B] time = 12.99, size = 4760, normalized size = 22.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**2/x,x)
```

```
[Out] -b**n*c**2*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) - b**n*c**2*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/
gamma(n + 2) + 2*c*d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x
)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4
*x + 2*b**5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5
*x**2) + 4*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log
(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*lo
g(a/b + x)/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b +
 x)/(a*b**3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(
a/b + x)/b**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n
/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**
n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)
**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x
)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)
**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) + d**2*Piecewise
((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b*
*7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**1
1*x**5) + 137*a**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 6
00*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*x*log(a/
b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9
*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 625*a**4*b*x/(60*a**5*b**6 + 30
0*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4
+ 60*b**11*x**5) + 600*a**3*b**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4
*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b
**11*x**5) + 1100*a**3*b**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3
*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a
**2*b**3*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*
x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 900*a**2*b*
*3*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**
9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4*log(a/b + x)/(
60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4/(60*a**5*b**6 + 300*a**
4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*
b**11*x**5) + 60*b**5*x**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 6
00*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5),
 Eq(n, -6)), (-60*a**5*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**
2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 125*a**5/(12*a**4*b**6 + 48
*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a*
*4*b*x*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48
```

```
*a*b**9*x**3 + 12*b**10*x**4) - 440*a**4*b*x/(12*a**4*b**6 + 48*a**3*b**7*x
 + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*x**2
*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**
9*x**3 + 12*b**10*x**4) - 540*a**3*b**2*x**2/(12*a**4*b**6 + 48*a**3*b**7*x
 + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*x**3
*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**
9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*x**3/(12*a**4*b**6 + 48*a**3*b**7*x
 + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*x**4*log
(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x*
*3 + 12*b**10*x**4) + 12*b**5*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2
*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*log(a/b
+ x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a*
*5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4
*b*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x
**3) + 270*a**4*b*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9
*x**3) + 180*a**3*b**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18
*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*x**2/(6*a**3*b**6 + 18*a**2*b**
7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*x**3*log(a/b + x)/(6*a**
3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*x**4/(6
*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*b**5*x**5/(
6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3), Eq(n, -4)), (
-60*a**5*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 90*a**5/(
6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x*log(a/b + x)/(6*a**
2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x/(6*a**2*b**6 + 12*a*b**7
*x + 6*b**8*x**2) - 60*a**3*b**2*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7
*x + 6*b**8*x**2) + 20*a**2*b**3*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x
**2) - 5*a*b**4*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*x**
5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*log(a/b +
 x)/(12*a*b**6 + 12*b**7*x) + 60*a**5/(12*a*b**6 + 12*b**7*x) + 60*a**4*b*x
*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 30*a**3*b**2*x**2/(12*a*b**6 + 12*b
**7*x) + 10*a**2*b**3*x**3/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*x**4/(12*a*b*
*6 + 12*b**7*x) + 3*b**5*x**5/(12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*l
og(a/b + x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x**3/(3*b**3) -
a*x**4/(4*b**2) + x**5/(5*b), Eq(n, -1)), (-120*a**6*(a + b*x)**n/(b**6*n**
6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b*
*6*n + 720*b**6) + 120*a**5*b*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) -
60*a**4*b**2*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
*4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**
2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*
n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 20*a**3*b**3*n**3*x**3*(a
 + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624
*b**6*n**2 + 1764*b**6*n + 720*b**6) + 60*a**3*b**3*n**2*x**3*(a + b*x)**n/
(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
+ 1764*b**6*n + 720*b**6) + 40*a**3*b**3*n*x**3*(a + b*x)**n/(b**6*n**6 + 2
1*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n
+ 720*b**6) - 5*a**2*b**4*n**4*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5
+ 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6)
- 30*a**2*b**4*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*
n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 55*a**2*b
**4*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*
b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*n*x**4*
(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 16
24*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*n**5*x**5*(a + b*x)**n/(b**
6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 17
64*b**6*n + 720*b**6) + 10*a*b**5*n**4*x**5*(a + b*x)**n/(b**6*n**6 + 21*b*
*6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 72
0*b**6) + 35*a*b**5*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*
b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 50*a
```

```
*b**5*n**2*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 24*a*b**5*n*x**5*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*n**5*x**6*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 15*b**6*n**4*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n*
*5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**
6) + 85*b**6*n**3*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
*4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 225*b**6*n*
*2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 274*b**6*n*x**6*(a + b*x)*
*n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n*
*2 + 1764*b**6*n + 720*b**6) + 120*b**6*x**6*(a + b*x)**n/(b**6*n**6 + 21*b
**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 7
20*b**6), True)) - b*b**n*c**2*n*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n +
1)*gamma(n + 1)/(a*gamma(n + 2)) - b*b**n*c**2*x*(a/b + x)**n*lerchphi(1 +
b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

3.182 $\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx$

Optimal. Leaf size=459

$$-\frac{6ad^2\left(4b^3c-55a^3d\right)(a+bx)^{n+8}}{b^{12}(n+8)}+\frac{3d^2\left(b^3c-55a^3d\right)(a+bx)^{n+9}}{b^{12}(n+9)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{12}(n+2)}+$$

```
[Out] a^2*(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^12/(1+n)-a*(-11*a^3*d+2*b^3*c)*(-a^3*d
+b^3*c)^2*(b*x+a)^(2+n)/b^12/(2+n)+(-a^3*d+b^3*c)*(55*a^6*d^2-29*a^3*b^3*c*
d+b^6*c^2)*(b*x+a)^(3+n)/b^12/(3+n)+3*a^2*d*(55*a^6*d^2-56*a^3*b^3*c*d+10*b
^6*c^2)*(b*x+a)^(4+n)/b^12/(4+n)-15*a*d*(22*a^6*d^2-14*a^3*b^3*c*d+b^6*c^2)
*(b*x+a)^(5+n)/b^12/(5+n)+3*d*(154*a^6*d^2-56*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^
(6+n)/b^12/(6+n)+42*a^2*d^2*(-11*a^3*d+2*b^3*c)*(b*x+a)^(7+n)/b^12/(7+n)-6*
a*d^2*(-55*a^3*d+4*b^3*c)*(b*x+a)^(8+n)/b^12/(8+n)+3*d^2*(-55*a^3*d+b^3*c)*
(b*x+a)^(9+n)/b^12/(9+n)+55*a^2*d^3*(b*x+a)^(10+n)/b^12/(10+n)-11*a*d^3*(b*
x+a)^(11+n)/b^12/(11+n)+d^3*(b*x+a)^(12+n)/b^12/(12+n)
```

Rubi [A] time = 0.32, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1620}

$$\frac{\left(b^3c-a^3d\right)\left(-29a^3b^3cd+55a^6d^2+b^6c^2\right)(a+bx)^{n+3}}{b^{12}(n+3)}+\frac{3a^2d\left(-56a^3b^3cd+55a^6d^2+10b^6c^2\right)(a+bx)^{n+4}}{b^{12}(n+4)}-\frac{15ad}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] (a^2*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^12*(1 + n)) - (a*(2*b^3*c - 11
*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^12*(2 + n)) + ((b^3*c - a^3
*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(3 + n))/(b^12*(3 + n
)) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(4 + n))
/(b^12*(4 + n)) - (15*a*d*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)
^(5 + n))/(b^12*(5 + n)) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d^2)*(a
 + b*x)^(6 + n))/(b^12*(6 + n)) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a + b*x
)^(7 + n))/(b^12*(7 + n)) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a + b*x)^(8 + n)
)/(b^12*(8 + n)) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^(9 + n))/(b^12*(9 +
n)) + (55*a^2*d^3*(a + b*x)^(10 + n))/(b^12*(10 + n)) - (11*a*d^3*(a + b*x)
^(11 + n))/(b^12*(11 + n)) + (d^3*(a + b*x)^(12 + n))/(b^12*(12 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx=\int\left(-\frac{a^2\left(-b^3c+a^3d\right)^3(a+bx)^n}{b^{11}}+\frac{a\left(-b^3c+a^3d\right)^2\left(-2b^3c+11a^3d\right)(a+bx)^{1+n}}{b^{11}}\right.$$

$$=\frac{a^2\left(b^3c-a^3d\right)^3(a+bx)^{1+n}}{b^{12}(1+n)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{2+n}}{b^{12}(2+n)}+\frac{\left(b^3c\right.}{}$$

Mathematica [A] time = 0.47, size = 402, normalized size = 0.88

$$(a+bx)^{n+1}\left(\frac{3d^2(a+bx)^8\left(b^3c-55a^3d\right)}{n+9}+\frac{6ad^2(a+bx)^7\left(55a^3d-4b^3c\right)}{n+8}+\frac{a(a+bx)\left(b^3c-a^3d\right)^2\left(11a^3d-2b^3c\right)}{n+2}+\frac{55a^2d^3(a+bx)^9}{n+10}+\frac{3d(a+bx)^5\left(154a^6\right.}{n}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*(b^3*c - a^3*d)^3)/(1 + n) + (a*(b^3*c - a^3*d)^2*
(-2*b^3*c + 11*a^3*d)*(a + b*x))/(2 + n) + ((b^3*c - a^3*d)*(b^6*c^2 - 29*a
^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^2)/(3 + n) + (3*a^2*d*(10*b^6*c^2 - 56*a
^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^3)/(4 + n) - (15*a*d*(b^6*c^2 - 14*a^3*b
^3*c*d + 22*a^6*d^2)*(a + b*x)^4)/(5 + n) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d
+ 154*a^6*d^2)*(a + b*x)^5)/(6 + n) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a +
 b*x)^6)/(7 + n) + (6*a*d^2*(-4*b^3*c + 55*a^3*d)*(a + b*x)^7)/(8 + n) + (3
*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^8)/(9 + n) + (55*a^2*d^3*(a + b*x)^9)/(10
 + n) - (11*a*d^3*(a + b*x)^10)/(11 + n) + (d^3*(a + b*x)^11)/(12 + n)))/b^
12
```

fricas [B] time = 0.53, size = 3564, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^9*c^3*n^9 + 144*a^3*b^9*c^3*n^8 + 4548*a^3*b^9*c^3*n^7 + 159667200
*a^3*b^9*c^3 - 239500800*a^6*b^6*c^2*d + 159667200*a^9*b^3*c*d^2 - 39916800
*a^12*d^3 + (b^12*d^3*n^11 + 66*b^12*d^3*n^10 + 1925*b^12*d^3*n^9 + 32670*b
^12*d^3*n^8 + 357423*b^12*d^3*n^7 + 2637558*b^12*d^3*n^6 + 13339535*b^12*d^
3*n^5 + 45995730*b^12*d^3*n^4 + 105258076*b^12*d^3*n^3 + 150917976*b^12*d^3
*n^2 + 120543840*b^12*d^3*n + 39916800*b^12*d^3)*x^12 + (a*b^11*d^3*n^11 +
55*a*b^11*d^3*n^10 + 1320*a*b^11*d^3*n^9 + 18150*a*b^11*d^3*n^8 + 157773*a*
b^11*d^3*n^7 + 902055*a*b^11*d^3*n^6 + 3416930*a*b^11*d^3*n^5 + 8409500*a*b
^11*d^3*n^4 + 12753576*a*b^11*d^3*n^3 + 10628640*a*b^11*d^3*n^2 + 3628800*a
*b^11*d^3*n)*x^11 - 11*(a^2*b^10*d^3*n^10 + 45*a^2*b^10*d^3*n^9 + 870*a^2*b
^10*d^3*n^8 + 9450*a^2*b^10*d^3*n^7 + 63273*a^2*b^10*d^3*n^6 + 269325*a^2*b
^10*d^3*n^5 + 723680*a^2*b^10*d^3*n^4 + 1172700*a^2*b^10*d^3*n^3 + 1026576*
a^2*b^10*d^3*n^2 + 362880*a^2*b^10*d^3*n)*x^10 + (3*b^12*c*d^2*n^11 + 207*b
^12*c*d^2*n^10 + 159667200*b^12*c*d^2 + 2*(3144*b^12*c*d^2 + 55*a^3*b^9*d^3
)*n^9 + 18*(6151*b^12*c*d^2 + 220*a^3*b^9*d^3)*n^8 + 3*(417309*b^12*c*d^2 +
 20020*a^3*b^9*d^3)*n^7 + 567*(16777*b^12*c*d^2 + 880*a^3*b^9*d^3)*n^6 + 6*
(8226277*b^12*c*d^2 + 411565*a^3*b^9*d^3)*n^5 + 36*(4833097*b^12*c*d^2 + 20
5590*a^3*b^9*d^3)*n^4 + 40*(10142427*b^12*c*d^2 + 324841*a^3*b^9*d^3)*n^3 +
 288*(2051288*b^12*c*d^2 + 41855*a^3*b^9*d^3)*n^2 + 5760*(82941*b^12*c*d^2
+ 770*a^3*b^9*d^3)*n)*x^9 + 3*(a*b^11*c*d^2*n^11 + 61*a*b^11*c*d^2*n^10 + 1
608*a*b^11*c*d^2*n^9 + 6*(4007*a*b^11*c*d^2 - 55*a^4*b^8*d^3)*n^8 + 21*(107
13*a*b^11*c*d^2 - 440*a^4*b^8*d^3)*n^7 + 21*(65289*a*b^11*c*d^2 - 5060*a^4*
b^8*d^3)*n^6 + 2*(2742001*a*b^11*c*d^2 - 323400*a^4*b^8*d^3)*n^5 + 2*(70625
74*a*b^11*c*d^2 - 1116885*a^4*b^8*d^3)*n^4 + 264*(84209*a*b^11*c*d^2 - 1641
5*a^4*b^8*d^3)*n^3 + 360*(52984*a*b^11*c*d^2 - 11979*a^4*b^8*d^3)*n^2 + 166
3200*(4*a*b^11*c*d^2 - a^4*b^8*d^3)*n)*x^8 - 24*(a^2*b^10*c*d^2*n^10 + 54*a
^2*b^10*c*d^2*n^9 + 1230*a^2*b^10*c*d^2*n^8 + 6*(2572*a^2*b^10*c*d^2 - 55*a
^5*b^7*d^3)*n^7 + 21*(5569*a^2*b^10*c*d^2 - 330*a^5*b^7*d^3)*n^6 + 42*(1315
3*a^2*b^10*c*d^2 - 1375*a^5*b^7*d^3)*n^5 + 10*(161702*a^2*b^10*c*d^2 - 2425
5*a^5*b^7*d^3)*n^4 + 24*(116917*a^2*b^10*c*d^2 - 22330*a^5*b^7*d^3)*n^3 + 3
60*(7192*a^2*b^10*c*d^2 - 1617*a^5*b^7*d^3)*n^2 + 237600*(4*a^2*b^10*c*d^2
- a^5*b^7*d^3)*n)*x^7 + 72*(1148*a^3*b^9*c^3 - 5*a^6*b^6*c^2*d)*n^6 + 3*(b^
```

```
12*c^2*d*n^11 + 72*b^12*c^2*d*n^10 + 79833600*b^12*c^2*d + (2285*b^12*c^2*d
 + 56*a^3*b^9*c*d^2)*n^9 + 12*(3505*b^12*c^2*d + 224*a^3*b^9*c*d^2)*n^8 + 3
*(165701*b^12*c^2*d + 17584*a^3*b^9*c*d^2)*n^7 + 48*(82167*b^12*c^2*d + 114
10*a^3*b^9*c*d^2 - 385*a^6*b^6*d^3)*n^6 + (21326135*b^12*c^2*d + 3263064*a^
3*b^9*c*d^2 - 277200*a^6*b^6*d^3)*n^5 + 12*(6509445*b^12*c^2*d + 946456*a^3
*b^9*c*d^2 - 130900*a^6*b^6*d^3)*n^4 + 4*(47131699*b^12*c^2*d + 5602072*a^3
*b^9*c*d^2 - 1039500*a^6*b^6*d^3)*n^3 + 96*(2946397*b^12*c^2*d + 236320*a^3
*b^9*c*d^2 - 52745*a^6*b^6*d^3)*n^2 + 2880*(81401*b^12*c^2*d + 3080*a^3*b^9
*c*d^2 - 770*a^6*b^6*d^3)*n)*x^6 + 6*(158683*a^3*b^9*c^3 - 3420*a^6*b^6*c^2
*d)*n^5 + 3*(a*b^11*c^2*d*n^11 + 67*a*b^11*c^2*d*n^10 + 1950*a*b^11*c^2*d*n
^9 + 6*(5385*a*b^11*c^2*d - 56*a^4*b^8*c*d^2)*n^8 + 3*(111851*a*b^11*c^2*d
- 4816*a^4*b^8*c*d^2)*n^7 + 3*(755417*a*b^11*c^2*d - 81424*a^4*b^8*c*d^2)*n
^6 + 560*(17848*a*b^11*c^2*d - 3687*a^4*b^8*c*d^2 + 198*a^7*b^5*d^3)*n^5 +
4*(7034735*a*b^11*c^2*d - 2313696*a^4*b^8*c*d^2 + 277200*a^7*b^5*d^3)*n^4 +
 96*(498251*a*b^11*c^2*d - 227822*a^4*b^8*c*d^2 + 40425*a^7*b^5*d^3)*n^3 +
576*(75857*a*b^11*c^2*d - 43568*a^4*b^8*c*d^2 + 9625*a^7*b^5*d^3)*n^2 + 266
1120*(6*a*b^11*c^2*d - 4*a^4*b^8*c*d^2 + a^7*b^5*d^3)*n)*x^5 + 72*(100058*a
^3*b^9*c^3 - 6725*a^6*b^6*c^2*d)*n^4 - 15*(a^2*b^10*c^2*d*n^10 + 63*a^2*b^1
0*c^2*d*n^9 + 1698*a^2*b^10*c^2*d*n^8 + 6*(4253*a^2*b^10*c^2*d - 56*a^5*b^7
*c*d^2)*n^7 + 3*(77827*a^2*b^10*c^2*d - 4368*a^5*b^7*c*d^2)*n^6 + 3*(444109
*a^2*b^10*c^2*d - 63952*a^5*b^7*c*d^2)*n^5 + 4*(1166393*a^2*b^10*c^2*d - 32
4324*a^5*b^7*c*d^2 + 27720*a^8*b^4*d^3)*n^4 + 12*(789721*a^2*b^10*c^2*d - 3
38800*a^5*b^7*c*d^2 + 55440*a^8*b^4*d^3)*n^3 + 144*(68927*a^2*b^10*c^2*d -
38948*a^5*b^7*c*d^2 + 8470*a^8*b^4*d^3)*n^2 + 665280*(6*a^2*b^10*c^2*d - 4*
a^5*b^7*c*d^2 + a^8*b^4*d^3)*n)*x^4 + 8*(4473299*a^3*b^9*c^3 - 756675*a^6*b
^6*c^2*d + 15120*a^9*b^3*c*d^2)*n^3 + (b^12*c^3*n^11 + 75*b^12*c^3*n^10 + 1
59667200*b^12*c^3 + 4*(623*b^12*c^3 + 15*a^3*b^9*c^2*d)*n^9 + 18*(2683*b^12
*c^3 + 200*a^3*b^9*c^2*d)*n^8 + 3*(201527*b^12*c^3 + 30360*a^3*b^9*c^2*d)*n
^7 + 9*(568099*b^12*c^3 + 139760*a^3*b^9*c^2*d - 2240*a^6*b^6*c*d^2)*n^6 +
2*(14825779*b^12*c^3 + 5117670*a^3*b^9*c^2*d - 362880*a^6*b^6*c*d^2)*n^5 +
12*(9759623*b^12*c^3 + 4102800*a^3*b^9*c^2*d - 777840*a^6*b^6*c*d^2)*n^4 +
8*(38232551*b^12*c^3 + 16529190*a^3*b^9*c^2*d - 6229440*a^6*b^6*c*d^2 + 831
600*a^9*b^3*d^3)*n^3 + 576*(861864*b^12*c^3 + 298435*a^3*b^9*c^2*d - 163940
*a^6*b^6*c*d^2 + 34650*a^9*b^3*d^3)*n^2 + 5760*(76781*b^12*c^3 + 13860*a^3*
b^9*c^2*d - 9240*a^6*b^6*c*d^2 + 2310*a^9*b^3*d^3)*n)*x^3 + 144*(780996*a^3
*b^9*c^3 - 293635*a^6*b^6*c^2*d + 27720*a^9*b^3*c*d^2)*n^2 + (a*b^11*c^3*n^
11 + 73*a*b^11*c^3*n^10 + 2346*a*b^11*c^3*n^9 + 6*(7267*a*b^11*c^3 - 30*a^4
*b^8*c^2*d)*n^8 + 3*(172459*a*b^11*c^3 - 3480*a^4*b^8*c^2*d)*n^7 + 3*(13593
79*a*b^11*c^3 - 84120*a^4*b^8*c^2*d)*n^6 + 4*(5373821*a*b^11*c^3 - 817200*a
^4*b^8*c^2*d + 15120*a^7*b^5*c*d^2)*n^5 + 4*(18531227*a*b^11*c^3 - 6042105*
a^4*b^8*c^2*d + 514080*a^7*b^5*c*d^2)*n^4 + 72*(2189036*a*b^11*c^3 - 138005
5*a^4*b^8*c^2*d + 331800*a^7*b^5*c*d^2)*n^3 + 1440*(125842*a*b^11*c^3 - 137
481*a^4*b^8*c^2*d + 70644*a^7*b^5*c*d^2 - 13860*a^10*b^2*d^3)*n^2 + 1995840
0*(4*a*b^11*c^3 - 6*a^4*b^8*c^2*d + 4*a^7*b^5*c*d^2 - a^10*b^2*d^3)*n)*x^2
+ 2880*(70402*a^3*b^9*c^3 - 54321*a^6*b^6*c^2*d + 15204*a^9*b^3*c*d^2)*n -
2*(a^2*b^10*c^3*n^10 + 72*a^2*b^10*c^3*n^9 + 2274*a^2*b^10*c^3*n^8 + 36*(11
48*a^2*b^10*c^3 - 5*a^5*b^7*c^2*d)*n^7 + 3*(158683*a^2*b^10*c^3 - 3420*a^5*
b^7*c^2*d)*n^6 + 36*(100058*a^2*b^10*c^3 - 6725*a^5*b^7*c^2*d)*n^5 + 4*(447
3299*a^2*b^10*c^3 - 756675*a^5*b^7*c^2*d + 15120*a^8*b^4*c*d^2)*n^4 + 72*(7
80996*a^2*b^10*c^3 - 293635*a^5*b^7*c^2*d + 27720*a^8*b^4*c*d^2)*n^3 + 1440
*(70402*a^2*b^10*c^3 - 54321*a^5*b^7*c^2*d + 15204*a^8*b^4*c*d^2)*n^2 + 199
58400*(4*a^2*b^10*c^3 - 6*a^5*b^7*c^2*d + 4*a^8*b^4*c*d^2 - a^11*b*d^3)*n)*
x)*(b*x + a)^n/(b^12*n^12 + 78*b^12*n^11 + 2717*b^12*n^10 + 55770*b^12*n^9
+ 749463*b^12*n^8 + 6926634*b^12*n^7 + 44990231*b^12*n^6 + 206070150*b^12*n
^5 + 657206836*b^12*n^4 + 1414014888*b^12*n^3 + 1931559552*b^12*n^2 + 14864
42880*b^12*n + 479001600*b^12)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Polynomial exponent overflow. Error: Bad Arg
ument Value
```

maple [B] time = 0.06, size = 3780, normalized size = 8.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^11*d^3*n^11*x^11-66*b^11*d^3*n^10*x^11+11*a*b^10*d^3*n^1
0*x^10-1925*b^11*d^3*n^9*x^11+605*a*b^10*d^3*n^9*x^10-3*b^11*c*d^2*n^11*x^8
-32670*b^11*d^3*n^8*x^11-110*a^2*b^9*d^3*n^9*x^9+14520*a*b^10*d^3*n^8*x^10-
207*b^11*c*d^2*n^10*x^8-357423*b^11*d^3*n^7*x^11-4950*a^2*b^9*d^3*n^8*x^9+2
4*a*b^10*c*d^2*n^10*x^7+199650*a*b^10*d^3*n^7*x^10-6288*b^11*c*d^2*n^9*x^8-
2637558*b^11*d^3*n^6*x^11+990*a^3*b^8*d^3*n^8*x^8-95700*a^2*b^9*d^3*n^7*x^9
+1464*a*b^10*c*d^2*n^9*x^7+1735503*a*b^10*d^3*n^6*x^10-3*b^11*c^2*d*n^11*x^
5-110718*b^11*c*d^2*n^8*x^8-13339535*b^11*d^3*n^5*x^11+35640*a^3*b^8*d^3*n^
7*x^8-168*a^2*b^9*c*d^2*n^9*x^6-1039500*a^2*b^9*d^3*n^6*x^9+38592*a*b^10*c*
d^2*n^8*x^7+9922605*a*b^10*d^3*n^5*x^10-216*b^11*c^2*d*n^10*x^5-1251927*b^1
1*c*d^2*n^7*x^8-45995730*b^11*d^3*n^4*x^11-7920*a^4*b^7*d^3*n^7*x^7+540540*
a^3*b^8*d^3*n^6*x^8-9072*a^2*b^9*c*d^2*n^8*x^6-6960030*a^2*b^9*d^3*n^5*x^9+
15*a*b^10*c^2*d*n^10*x^4+577008*a*b^10*c*d^2*n^7*x^7+37586230*a*b^10*d^3*n^
4*x^10-6855*b^11*c^2*d*n^9*x^5-9512559*b^11*c*d^2*n^6*x^8-105258076*b^11*d^
3*n^3*x^11-221760*a^4*b^7*d^3*n^6*x^7+1008*a^3*b^8*c*d^2*n^8*x^5+4490640*a^
3*b^8*d^3*n^5*x^8-206640*a^2*b^9*c*d^2*n^7*x^6-29625750*a^2*b^9*d^3*n^4*x^9
+1005*a*b^10*c^2*d*n^9*x^4+5399352*a*b^10*c*d^2*n^6*x^7+92504500*a*b^10*d^3
*n^3*x^10-b^11*c^3*n^11*x^2-126180*b^11*c^2*d*n^8*x^5-49357662*b^11*c*d^2*n
^5*x^8-150917976*b^11*d^3*n^2*x^11+55440*a^5*b^6*d^3*n^6*x^6-2550240*a^4*b^
7*d^3*n^5*x^7+48384*a^3*b^8*c*d^2*n^7*x^5+22224510*a^3*b^8*d^3*n^4*x^8-60*a
^2*b^9*c^2*d*n^9*x^3-2592576*a^2*b^9*c*d^2*n^6*x^6-79604800*a^2*b^9*d^3*n^3
*x^9+29250*a*b^10*c^2*d*n^8*x^4+32905656*a*b^10*c*d^2*n^5*x^7+140289336*a*b
^10*d^3*n^2*x^10-75*b^11*c^3*n^10*x^2-1491309*b^11*c^2*d*n^7*x^5-173991492*
b^11*c*d^2*n^4*x^8-120543840*b^11*d^3*n*x^11+1164240*a^5*b^6*d^3*n^5*x^6-50
40*a^4*b^7*c*d^2*n^7*x^4-15523200*a^4*b^7*d^3*n^4*x^7+949536*a^3*b^8*c*d^2*
n^6*x^5+66611160*a^3*b^8*d^3*n^3*x^8-3780*a^2*b^9*c^2*d*n^8*x^3-19647432*a^
2*b^9*c*d^2*n^5*x^6-128997000*a^2*b^9*d^3*n^2*x^9+2*a*b^10*c^3*n^10*x+48465
0*a*b^10*c^2*d*n^7*x^4+131616048*a*b^10*c*d^2*n^4*x^7+116915040*a*b^10*d^3*
n*x^10-2492*b^11*c^3*n^9*x^2-11832048*b^11*c^2*d*n^6*x^5-405697080*b^11*c*d
^2*n^3*x^8-39916800*b^11*d^3*x^11-332640*a^6*b^5*d^3*n^5*x^5+9702000*a^5*b^
6*d^3*n^4*x^6-216720*a^4*b^7*c*d^2*n^6*x^4-53610480*a^4*b^7*d^3*n^3*x^7+180
*a^3*b^8*c^2*d*n^8*x^2+9858240*a^3*b^8*c*d^2*n^5*x^5+116942760*a^3*b^8*d^3*
n^2*x^8-101880*a^2*b^9*c^2*d*n^7*x^3-92807568*a^2*b^9*c*d^2*n^4*x^6-1129233
60*a^2*b^9*d^3*n*x^9+146*a*b^10*c^3*n^9*x+5033295*a*b^10*c^2*d*n^6*x^4+3390
03552*a*b^10*c*d^2*n^3*x^7+39916800*a*b^10*d^3*x^10-48294*b^11*c^3*n^8*x^2-
63978405*b^11*c^2*d*n^5*x^5-590770944*b^11*c*d^2*n^2*x^8-4989600*a^6*b^5*d^
3*n^4*x^5+20160*a^5*b^6*c*d^2*n^6*x^3+40748400*a^5*b^6*d^3*n^3*x^6-3664080*
a^4*b^7*c*d^2*n^5*x^4-104005440*a^4*b^7*d^3*n^2*x^7+10800*a^3*b^8*c^2*d*n^7
*x^2+58735152*a^3*b^8*c*d^2*n^4*x^5+108488160*a^3*b^8*d^3*n*x^8-2*a^2*b^9*c
^3*n^9-1531080*a^2*b^9*c^2*d*n^6*x^3-271659360*a^2*b^9*c*d^2*n^3*x^6-399168
00*a^2*b^9*d^3*x^9+4692*a*b^10*c^3*n^8*x+33993765*a*b^10*c^2*d*n^5*x^4+5335
48224*a*b^10*c*d^2*n^2*x^7-604581*b^11*c^3*n^7*x^2-234340020*b^11*c^2*d*n^4
*x^5-477740160*b^11*c*d^2*n*x^8+1663200*a^7*b^4*d^3*n^4*x^4-28274400*a^6*b^
5*d^3*n^3*x^5+786240*a^5*b^6*c*d^2*n^5*x^3+90034560*a^5*b^6*d^3*n^2*x^6-360
*a^4*b^7*c^2*d*n^7*x-30970800*a^4*b^7*c*d^2*n^4*x^4-103498560*a^4*b^7*d^3*n
```

```
*x^7+273240*a^3*b^8*c^2*d*n^6*x^2+204434496*a^3*b^8*c*d^2*n^3*x^5+39916800*
a^3*b^8*d^3*x^8-144*a^2*b^9*c^3*n^8-14008860*a^2*b^9*c^2*d*n^5*x^3-47140934
4*a^2*b^9*c*d^2*n^2*x^6+87204*a*b^10*c^3*n^7*x+149923200*a*b^10*c^2*d*n^4*x
^4+457781760*a*b^10*c*d^2*n*x^7-5112891*b^11*c^3*n^6*x^2-565580388*b^11*c^2
*d*n^3*x^5-159667200*b^11*c*d^2*x^8+16632000*a^7*b^4*d^3*n^3*x^4-60480*a^6*
b^5*c*d^2*n^5*x^2-74844000*a^6*b^5*d^3*n^2*x^5+11511360*a^5*b^6*c*d^2*n^4*x
^3+97796160*a^5*b^6*d^3*n*x^6-20880*a^4*b^7*c^2*d*n^6*x-138821760*a^4*b^7*c
*d^2*n^3*x^4-39916800*a^4*b^7*d^3*x^7+3773520*a^3*b^8*c^2*d*n^5*x^2+4033491
84*a^3*b^8*c*d^2*n^2*x^5-4548*a^2*b^9*c^3*n^7-79939620*a^2*b^9*c^2*d*n^4*x^
3-434972160*a^2*b^9*c*d^2*n*x^6+1034754*a*b^10*c^3*n^6*x+422084100*a*b^10*c
^2*d*n^3*x^4+159667200*a*b^10*c*d^2*x^7-29651558*b^11*c^3*n^5*x^2-848562336
*b^11*c^2*d*n^2*x^5-6652800*a^8*b^3*d^3*n^3*x^3+58212000*a^7*b^4*d^3*n^2*x^
4-2177280*a^6*b^5*c*d^2*n^4*x^2-91143360*a^6*b^5*d^3*n*x^5+360*a^5*b^6*c^2*
d*n^6+77837760*a^5*b^6*c*d^2*n^3*x^3+39916800*a^5*b^6*d^3*x^6-504720*a^4*b^
7*c^2*d*n^5*x-328063680*a^4*b^7*c*d^2*n^2*x^4+30706020*a^3*b^8*c^2*d*n^4*x^
2+408360960*a^3*b^8*c*d^2*n*x^5-82656*a^2*b^9*c^3*n^6-279934320*a^2*b^9*c^2
*d*n^3*x^3-159667200*a^2*b^9*c*d^2*x^6+8156274*a*b^10*c^3*n^5*x+717481440*a
*b^10*c^2*d*n^2*x^4-117115476*b^11*c^3*n^4*x^2-703304640*b^11*c^2*d*n*x^5-3
9916800*a^8*b^3*d^3*n^2*x^3+120960*a^7*b^4*c*d^2*n^4*x+83160000*a^7*b^4*d^3
*n*x^4-28002240*a^6*b^5*c*d^2*n^3*x^2-39916800*a^6*b^5*d^3*x^5+20520*a^5*b^
6*c^2*d*n^5+243936000*a^5*b^6*c*d^2*n^2*x^3-6537600*a^4*b^7*c^2*d*n^4*x-376
427520*a^4*b^7*c*d^2*n*x^4+147700800*a^3*b^8*c^2*d*n^3*x^2+159667200*a^3*b^
8*c*d^2*x^5-952098*a^2*b^9*c^3*n^5-568599120*a^2*b^9*c^2*d*n^2*x^3+42990568
*a*b^10*c^3*n^4*x+655404480*a*b^10*c^2*d*n*x^4-305860408*b^11*c^3*n^3*x^2-2
39500800*b^11*c^2*d*x^5+19958400*a^9*b^2*d^3*n^2*x^2-73180800*a^8*b^3*d^3*n
*x^3+4112640*a^7*b^4*c*d^2*n^3*x+39916800*a^7*b^4*d^3*x^4-149506560*a^6*b^5
*c*d^2*n^2*x^2+484200*a^5*b^6*c^2*d*n^4+336510720*a^5*b^6*c*d^2*n*x^3-48336
840*a^4*b^7*c^2*d*n^3*x-159667200*a^4*b^7*c*d^2*x^4+396700560*a^3*b^8*c^2*d
*n^2*x^2-7204176*a^2*b^9*c^3*n^4-595529280*a^2*b^9*c^2*d*n*x^3+148249816*a*
b^10*c^3*n^3*x+239500800*a*b^10*c^2*d*x^4-496433664*b^11*c^3*n^2*x^2+598752
00*a^9*b^2*d^3*n*x^2-120960*a^8*b^3*c*d^2*n^3-39916800*a^8*b^3*d^3*x^3+4777
9200*a^7*b^4*c*d^2*n^2*x-283288320*a^6*b^5*c*d^2*n*x^2+6053400*a^5*b^6*c^2*
d*n^3+159667200*a^5*b^6*c*d^2*x^3-198727920*a^4*b^7*c^2*d*n^2*x+515695680*a
^3*b^8*c^2*d*n*x^2-35786392*a^2*b^9*c^3*n^3-239500800*a^2*b^9*c^2*d*x^3+315
221184*a*b^10*c^3*n^2*x-442258560*b^11*c^3*n*x^2-39916800*a^10*b*d^3*n*x+39
916800*a^9*b^2*d^3*x^2-3991680*a^8*b^3*c*d^2*n^2+203454720*a^7*b^4*c*d^2*n*
x-159667200*a^6*b^5*c*d^2*x^2+42283440*a^5*b^6*c^2*d*n^2-395945280*a^4*b^7*
c^2*d*n*x+239500800*a^3*b^8*c^2*d*x^2-112463424*a^2*b^9*c^3*n^2+362424960*a
*b^10*c^3*n*x-159667200*b^11*c^3*x^2-39916800*a^10*b*d^3*x-43787520*a^8*b^3
*c*d^2*n+159667200*a^7*b^4*c*d^2*x+156444480*a^5*b^6*c^2*d*n-239500800*a^4*
b^7*c^2*d*x-202757760*a^2*b^9*c^3*n+159667200*a*b^10*c^3*x+39916800*a^11*d^
3-159667200*a^8*b^3*c*d^2+239500800*a^5*b^6*c^2*d-159667200*a^2*b^9*c^3)/b^
12/(n^12+78*n^11+2717*n^10+55770*n^9+749463*n^8+6926634*n^7+44990231*n^6+20
6070150*n^5+657206836*n^4+1414014888*n^3+1931559552*n^2+1486442880*n+479001
600)
```

maxima [B] time = 0.90, size = 1153, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^5 + 15*n^4 + 85*n^3 + 225
*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5
*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)
^n*c^2*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
 + 3*((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n
```

```
^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*
n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5
 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 +
85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^
3 + 50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*
x^4 - 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 -
40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*c*d^2/((n^9 + 45*n^8 + 870*n^7 + 9
450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 3
62880)*b^9) + ((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 263755
8*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 + 12054
3840*n + 39916800)*b^12*x^12 + (n^11 + 55*n^10 + 1320*n^9 + 18150*n^8 + 157
773*n^7 + 902055*n^6 + 3416930*n^5 + 8409500*n^4 + 12753576*n^3 + 10628640*
n^2 + 3628800*n)*a*b^11*x^11 - 11*(n^10 + 45*n^9 + 870*n^8 + 9450*n^7 + 632
73*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362880*n)*a^
2*b^10*x^10 + 110*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67284*n^
4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^3*b^9*x^9 - 990*(n^8 + 28*n^7 + 32
2*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^4*b^8*x^8 +
 7920*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^5*
b^7*x^7 - 55440*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^6*b^6
*x^6 + 332640*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^7*b^5*x^5 - 1663200
*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^8*b^4*x^4 + 6652800*(n^3 + 3*n^2 + 2*n)*a^9
*b^3*x^3 - 19958400*(n^2 + n)*a^10*b^2*x^2 + 39916800*a^11*b*n*x - 39916800
*a^12)*(b*x + a)^n*d^3/((n^12 + 78*n^11 + 2717*n^10 + 55770*n^9 + 749463*n^
8 + 6926634*n^7 + 44990231*n^6 + 206070150*n^5 + 657206836*n^4 + 1414014888
*n^3 + 1931559552*n^2 + 1486442880*n + 479001600)*b^12)
```

mupad [B] time = 7.14, size = 2896, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] (2*a^3*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3
*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 47
6049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 +
b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a
^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 19
95840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n
^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d
*n^6))/(b^12*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^
4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2
717*n^10 + 78*n^11 + n^12 + 479001600)) + (d^3*x^12*(a + b*x)^n*(120543840*
n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n
^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800))/(14864
42880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 +
 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11
+ n^12 + 479001600) + (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(79833600*b^9*c^3 +
6652800*a^9*d^3*n + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 + 17893196*b
^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 +
 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 + 39916800*a^3*b^6*c^2*d*n
 - 26611200*a^6*b^3*c*d^2*n + 26074080*a^3*b^6*c^2*d*n^2 - 7297920*a^6*b^3*
c*d^2*n^2 + 7047240*a^3*b^6*c^2*d*n^3 - 665280*a^6*b^3*c*d^2*n^3 + 1008900*
a^3*b^6*c^2*d*n^4 - 20160*a^6*b^3*c*d^2*n^4 + 80700*a^3*b^6*c^2*d*n^5 + 342
0*a^3*b^6*c^2*d*n^6 + 60*a^3*b^6*c^2*d*n^7))/(b^9*(1486442880*n + 193155955
2*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 692
6634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)
) + (3*d*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)*(6
65280*b^6*c^2 - 18480*a^6*d^2*n + 434568*b^6*c^2*n + 117454*b^6*c^2*n^2 + 1
6815*b^6*c^2*n^3 + 1345*b^6*c^2*n^4 + 57*b^6*c^2*n^5 + b^6*c^2*n^6 + 73920*
```

a^3*b^3*c*d*n + 20272*a^3*b^3*c*d*n^2 + 1848*a^3*b^3*c*d*n^3 + 56*a^3*b^3*c
*d*n^4))/(b^6*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n
^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 +
2717*n^10 + 78*n^11 + n^12 + 479001600)) - (2*a^2*n*x*(a + b*x)^n*(79833600
*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 +
17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9
*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 - 119750400*a^3*
b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a^3*b^6*c^2*d*n + 21893760*a^
6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 1995840*a^6*b^3*c*d^2*n^2 - 30
26700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^
4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d*n^6))/(b^11*(1486442880*n +
 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231
*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 +
479001600)) + (d^2*x^9*(a + b*x)^n*(3960*b^3*c + 99*b^3*c*n^2 + 3*b^3*c*n^3
 + 110*a^3*d*n + 1086*b^3*c*n)*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n
^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(b^3*(1486442880*n + 19315
59552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 +
 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001
600)) + (a*d^3*n*x^11*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3
+ 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n
^10 + 3628800))/(b*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206
836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n
^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) - (11*a^2*d^3*n*x^10*(a + b*x
)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n
^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(b^2*(1486442880*n + 1931559552*n^2
+ 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n
^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (a
*n*x^2*(n + 1)*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880
*b^9*c^3*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*
n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^
3*n^8 + b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78
222240*a^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*
n^2 + 1995840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3
*c*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b
^6*c^2*d*n^6))/(b^10*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 6572
06836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770
*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (3*a*d*n*x^5*(a + b*x)^n*
(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(110880*a^6*d^2 + 665280*b^6*c^2 + 4345
68*b^6*c^2*n + 117454*b^6*c^2*n^2 + 16815*b^6*c^2*n^3 + 1345*b^6*c^2*n^4 +
57*b^6*c^2*n^5 + b^6*c^2*n^6 - 443520*a^3*b^3*c*d - 121632*a^3*b^3*c*d*n -
11088*a^3*b^3*c*d*n^2 - 336*a^3*b^3*c*d*n^3))/(b^7*(1486442880*n + 19315595
52*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 69
26634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600
)) - (15*a^2*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(110880*a^6*d^2 +
 665280*b^6*c^2 + 434568*b^6*c^2*n + 117454*b^6*c^2*n^2 + 16815*b^6*c^2*n^3
 + 1345*b^6*c^2*n^4 + 57*b^6*c^2*n^5 + b^6*c^2*n^6 - 443520*a^3*b^3*c*d - 1
21632*a^3*b^3*c*d*n - 11088*a^3*b^3*c*d*n^2 - 336*a^3*b^3*c*d*n^3))/(b^8*(1
486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n
^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n
^11 + n^12 + 479001600)) + (3*a*d^2*n*x^8*(a + b*x)^n*(1320*b^3*c - 330*a^3
*d + 33*b^3*c*n^2 + b^3*c*n^3 + 362*b^3*c*n)*(13068*n + 13132*n^2 + 6769*n^
3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b^4*(1486442880*n + 1931559
552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6
926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 47900160
0)) - (24*a^2*d^2*n*x^7*(a + b*x)^n*(1320*b^3*c - 330*a^3*d + 33*b^3*c*n^2
+ b^3*c*n^3 + 362*b^3*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5
+ n^6 + 720))/(b^5*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206
836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n
^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.183 $\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=396

$$-\frac{21ad^2\left(b^3c-10a^3d\right)(a+bx)^{n+7}}{b^{11}(n+7)}+\frac{3d^2\left(b^3c-40a^3d\right)(a+bx)^{n+8}}{b^{11}(n+8)}-\frac{a\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{11}(n+1)}+\frac{\left(b^3c-10a^3d\right)(\cdots}{b\cdots}$$

```
[Out] -a*(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^11/(1+n)+(-10*a^3*d+b^3*c)*(-a^3*d+b^3*
c)^2*(b*x+a)^(2+n)/b^11/(2+n)+9*a^2*d*(-5*a^3*d+2*b^3*c)*(-a^3*d+b^3*c)*(b*
x+a)^(3+n)/b^11/(3+n)-3*a*d*(40*a^6*d^2-35*a^3*b^3*c*d+4*b^6*c^2)*(b*x+a)^(
4+n)/b^11/(4+n)+3*d*(70*a^6*d^2-35*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(5+n)/b^11/
(5+n)+63*a^2*d^2*(-4*a^3*d+b^3*c)*(b*x+a)^(6+n)/b^11/(6+n)-21*a*d^2*(-10*a^
3*d+b^3*c)*(b*x+a)^(7+n)/b^11/(7+n)+3*d^2*(-40*a^3*d+b^3*c)*(b*x+a)^(8+n)/b
^11/(8+n)+45*a^2*d^3*(b*x+a)^(9+n)/b^11/(9+n)-10*a*d^3*(b*x+a)^(10+n)/b^11/
(10+n)+d^3*(b*x+a)^(11+n)/b^11/(11+n)
```

Rubi [A] time = 0.26, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1620}

$$-\frac{3ad\left(-35a^3b^3cd+40a^6d^2+4b^6c^2\right)(a+bx)^{n+4}}{b^{11}(n+4)}+\frac{3d\left(-35a^3b^3cd+70a^6d^2+b^6c^2\right)(a+bx)^{n+5}}{b^{11}(n+5)}+\frac{63a^2d^2\left(b^3c-\cdots\right.}{b^{11}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] -((a*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^11*(1 + n))) + ((b^3*c - 10*a^
3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^11*(2 + n)) + (9*a^2*d*(2*b^3*
c - 5*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^11*(3 + n)) - (3*a*d*(4*
b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d^2)*(a + b*x)^(4 + n))/(b^11*(4 + n)) +
(3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^2)*(a + b*x)^(5 + n))/(b^11*(5 +
n)) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^(6 + n))/(b^11*(6 + n)) - (21
*a*d^2*(b^3*c - 10*a^3*d)*(a + b*x)^(7 + n))/(b^11*(7 + n)) + (3*d^2*(b^3*c
 - 40*a^3*d)*(a + b*x)^(8 + n))/(b^11*(8 + n)) + (45*a^2*d^3*(a + b*x)^(9 +
 n))/(b^11*(9 + n)) - (10*a*d^3*(a + b*x)^(10 + n))/(b^11*(10 + n)) + (d^3*
(a + b*x)^(11 + n))/(b^11*(11 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n\left(c+dx^3\right)^3\,dx &= \int\left(\frac{a\left(-b^3c+a^3d\right)^3(a+bx)^n}{b^{10}}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^{10}}+\frac{9a^2d\,(\cdots}{\cdots}\right.\\ &= -\frac{a\left(b^3c-a^3d\right)^3(a+bx)^{1+n}}{b^{11}(1+n)}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{2+n}}{b^{11}(2+n)}+\frac{9a^2d\,(2\cdots}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.38, size = 345, normalized size = 0.87

$$(a+bx)^{n+1}\left(\frac{3d^2(a+bx)^7\left(b^3c-40a^3d\right)}{n+8}+\frac{21ad^2(a+bx)^6\left(10a^3d-b^3c\right)}{n+7}+\frac{(a+bx)\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2}{n+2}+\frac{a\left(a^3d-b^3c\right)^3}{n+1}+\frac{45a^2d^3(a+bx)^8}{n+9}+\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d)^3)/(1 + n) + ((b^3*c - 10*a^3*d)*
(b^3*c - a^3*d)^2*(a + b*x))/(2 + n) + (9*a^2*d*(-(b^3*c) + a^3*d)*(-2*b^3*
c + 5*a^3*d)*(a + b*x)^2)/(3 + n) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d + 40
*a^6*d^2)*(a + b*x)^3)/(4 + n) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^
2)*(a + b*x)^4)/(5 + n) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^5)/(6 + n
) + (21*a*d^2*(-(b^3*c) + 10*a^3*d)*(a + b*x)^6)/(7 + n) + (3*d^2*(b^3*c -
40*a^3*d)*(a + b*x)^7)/(8 + n) + (45*a^2*d^3*(a + b*x)^8)/(9 + n) - (10*a*d
^3*(a + b*x)^9)/(10 + n) + (d^3*(a + b*x)^10)/(11 + n)))/b^11
```

fricas [B] time = 0.51, size = 2919, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*b^9*c^3*n^9 + 63*a^2*b^9*c^3*n^8 + 1734*a^2*b^9*c^3*n^7 + 19958400*a^
2*b^9*c^3 - 23950080*a^5*b^6*c^2*d + 14968800*a^8*b^3*c*d^2 - 3628800*a^11*
d^3 - (b^11*d^3*n^10 + 55*b^11*d^3*n^9 + 1320*b^11*d^3*n^8 + 18150*b^11*d^3
*n^7 + 157773*b^11*d^3*n^6 + 902055*b^11*d^3*n^5 + 3416930*b^11*d^3*n^4 + 8
409500*b^11*d^3*n^3 + 12753576*b^11*d^3*n^2 + 10628640*b^11*d^3*n + 3628800
*b^11*d^3)*x^11 - (a*b^10*d^3*n^10 + 45*a*b^10*d^3*n^9 + 870*a*b^10*d^3*n^8
 + 9450*a*b^10*d^3*n^7 + 63273*a*b^10*d^3*n^6 + 269325*a*b^10*d^3*n^5 + 723
680*a*b^10*d^3*n^4 + 1172700*a*b^10*d^3*n^3 + 1026576*a*b^10*d^3*n^2 + 3628
80*a*b^10*d^3*n)*x^10 + 10*(a^2*b^9*d^3*n^9 + 36*a^2*b^9*d^3*n^8 + 546*a^2*
b^9*d^3*n^7 + 4536*a^2*b^9*d^3*n^6 + 22449*a^2*b^9*d^3*n^5 + 67284*a^2*b^9*
d^3*n^4 + 118124*a^2*b^9*d^3*n^3 + 109584*a^2*b^9*d^3*n^2 + 40320*a^2*b^9*d
^3*n)*x^9 - 3*(b^11*c*d^2*n^10 + 58*b^11*c*d^2*n^9 + 4989600*b^11*c*d^2 + 3
*(487*b^11*c*d^2 + 10*a^3*b^8*d^3)*n^8 + 6*(3497*b^11*c*d^2 + 140*a^3*b^8*d
^3)*n^7 + 21*(9027*b^11*c*d^2 + 460*a^3*b^8*d^3)*n^6 + 294*(3813*b^11*c*d^2
 + 200*a^3*b^8*d^3)*n^5 + (4371359*b^11*c*d^2 + 203070*a^3*b^8*d^3)*n^4 + 2
*(5512429*b^11*c*d^2 + 196980*a^3*b^8*d^3)*n^3 + 36*(473867*b^11*c*d^2 + 10
890*a^3*b^8*d^3)*n^2 + 360*(40123*b^11*c*d^2 + 420*a^3*b^8*d^3)*n)*x^8 - 3*
(a*b^10*c*d^2*n^10 + 51*a*b^10*c*d^2*n^9 + 1104*a*b^10*c*d^2*n^8 + 6*(2209*
a*b^10*c*d^2 - 40*a^4*b^7*d^3)*n^7 + 21*(4609*a*b^10*c*d^2 - 240*a^4*b^7*d^
3)*n^6 + 21*(21119*a*b^10*c*d^2 - 2000*a^4*b^7*d^3)*n^5 + 2*(633433*a*b^10*
c*d^2 - 88200*a^4*b^7*d^3)*n^4 + 12*(179733*a*b^10*c*d^2 - 32480*a^4*b^7*d^
3)*n^3 + 360*(5449*a*b^10*c*d^2 - 1176*a^4*b^7*d^3)*n^2 + 21600*(33*a*b^10*
c*d^2 - 8*a^4*b^7*d^3)*n)*x^7 + 18*(1519*a^2*b^9*c^3 - 4*a^5*b^6*c^2*d)*n^6
 + 21*(a^2*b^9*c*d^2*n^9 + 45*a^2*b^9*c*d^2*n^8 + 834*a^2*b^9*c*d^2*n^7 + 3
0*(275*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n^6 + 3*(15763*a^2*b^9*c*d^2 - 1200*a
^5*b^6*d^3)*n^5 + 15*(10651*a^2*b^9*c*d^2 - 1360*a^5*b^6*d^3)*n^4 + 4*(7706
9*a^2*b^9*c*d^2 - 13500*a^5*b^6*d^3)*n^3 + 60*(5119*a^2*b^9*c*d^2 - 1096*a^
5*b^6*d^3)*n^2 + 3600*(33*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n)*x^6 + 3*(90643*
a^2*b^9*c^3 - 1224*a^5*b^6*c^2*d)*n^5 - 3*(b^11*c^2*d*n^10 + 61*b^11*c^2*d*
n^9 + 7983360*b^11*c^2*d + 6*(270*b^11*c^2*d + 7*a^3*b^8*c*d^2)*n^8 + 210*(
117*b^11*c^2*d + 8*a^3*b^8*c*d^2)*n^7 + 3*(78191*b^11*c^2*d + 8876*a^3*b^8*
c*d^2)*n^6 + 3*(488231*b^11*c^2*d + 71120*a^3*b^8*c*d^2 - 3360*a^6*b^5*d^3)
*n^5 + 2*(3008035*b^11*c^2*d + 459669*a^3*b^8*c*d^2 - 50400*a^6*b^5*d^3)*n^
4 + 20*(795769*b^11*c^2*d + 105672*a^3*b^8*c*d^2 - 17640*a^6*b^5*d^3)*n^3 +
 72*(356683*b^11*c^2*d + 33061*a^3*b^8*c*d^2 - 7000*a^6*b^5*d^3)*n^2 + 288*
(78167*b^11*c^2*d + 3465*a^3*b^8*c*d^2 - 840*a^6*b^5*d^3)*n)*x^5 + 9*(19634
3*a^2*b^9*c^3 - 8600*a^5*b^6*c^2*d)*n^4 - 3*(a*b^10*c^2*d*n^10 + 57*a*b^10*
c^2*d*n^9 + 1392*a*b^10*c^2*d*n^8 + 6*(3167*a*b^10*c^2*d - 35*a^4*b^7*c*d^2
)*n^7 + 15*(10571*a*b^10*c^2*d - 504*a^4*b^7*c*d^2)*n^6 + 3*(276811*a*b^10*
c^2*d - 34300*a^4*b^7*c*d^2)*n^5 + 2*(1347169*a*b^10*c^2*d - 327600*a^4*b^7
```

```
*c*d^2 + 25200*a^7*b^4*d^3)*n^4 + 42*(122334*a*b^10*c^2*d - 47045*a^4*b^7*c
*d^2 + 7200*a^7*b^4*d^3)*n^3 + 72*(71237*a*b^10*c^2*d - 36995*a^4*b^7*c*d^2
 + 7700*a^7*b^4*d^3)*n^2 + 7560*(264*a*b^10*c^2*d - 165*a^4*b^7*c*d^2 + 40*
a^7*b^4*d^3)*n)*x^4 + 8*(936802*a^2*b^9*c^3 - 107865*a^5*b^6*c^2*d + 1890*a
^8*b^3*c*d^2)*n^3 + 12*(a^2*b^9*c^2*d*n^9 + 54*a^2*b^9*c^2*d*n^8 + 1230*a^2
*b^9*c^2*d*n^7 + 6*(2552*a^2*b^9*c^2*d - 35*a^5*b^6*c*d^2)*n^6 + 33*(3413*a
^2*b^9*c^2*d - 210*a^5*b^6*c*d^2)*n^5 + 6*(82091*a^2*b^9*c^2*d - 13685*a^5*
b^6*c*d^2)*n^4 + 10*(121670*a^2*b^9*c^2*d - 40887*a^5*b^6*c*d^2 + 5040*a^8*
b^3*d^3)*n^3 + 24*(61997*a^2*b^9*c^2*d - 31220*a^5*b^6*c*d^2 + 6300*a^8*b^3
*d^3)*n^2 + 2520*(264*a^2*b^9*c^2*d - 165*a^5*b^6*c*d^2 + 40*a^8*b^3*d^3)*n
)*x^3 + 36*(554953*a^2*b^9*c^3 - 149048*a^5*b^6*c^2*d + 12600*a^8*b^3*c*d^2
)*n^2 - (b^11*c^3*n^10 + 64*b^11*c^3*n^9 + 19958400*b^11*c^3 + 3*(599*b^11*
c^3 + 12*a^3*b^8*c^2*d)*n^8 + 12*(2423*b^11*c^3 + 156*a^3*b^8*c^2*d)*n^7 +
3*(99757*b^11*c^3 + 13512*a^3*b^8*c^2*d)*n^6 + 24*(84959*b^11*c^3 + 19590*a
^3*b^8*c^2*d - 315*a^6*b^5*c*d^2)*n^5 + (9261503*b^11*c^3 + 3114324*a^3*b^8
*c^2*d - 234360*a^6*b^5*c*d^2)*n^4 + 4*(6868181*b^11*c^3 + 2875752*a^3*b^8*
c^2*d - 621810*a^6*b^5*c*d^2)*n^3 + 36*(1397573*b^11*c^3 + 577644*a^3*b^8*c
^2*d - 270690*a^6*b^5*c*d^2 + 50400*a^9*b^2*d^3)*n^2 + 720*(69851*b^11*c^3
+ 16632*a^3*b^8*c^2*d - 10395*a^6*b^5*c*d^2 + 2520*a^9*b^2*d^3)*n)*x^2 + 14
4*(210655*a^2*b^9*c^3 - 122502*a^5*b^6*c^2*d + 31395*a^8*b^3*c*d^2)*n - (a*
b^10*c^3*n^10 + 63*a*b^10*c^3*n^9 + 1734*a*b^10*c^3*n^8 + 18*(1519*a*b^10*c
^3 - 4*a^4*b^7*c^2*d)*n^7 + 3*(90643*a*b^10*c^3 - 1224*a^4*b^7*c^2*d)*n^6 +
 9*(196343*a*b^10*c^3 - 8600*a^4*b^7*c^2*d)*n^5 + 8*(936802*a*b^10*c^3 - 10
7865*a^4*b^7*c^2*d + 1890*a^7*b^4*c*d^2)*n^4 + 36*(554953*a*b^10*c^3 - 1490
48*a^4*b^7*c^2*d + 12600*a^7*b^4*c*d^2)*n^3 + 144*(210655*a*b^10*c^3 - 1225
02*a^4*b^7*c^2*d + 31395*a^7*b^4*c*d^2)*n^2 + 90720*(220*a*b^10*c^3 - 264*a
^4*b^7*c^2*d + 165*a^7*b^4*c*d^2 - 40*a^10*b*d^3)*n)*x)*(b*x + a)^n/(b^11*n
^11 + 66*b^11*n^10 + 1925*b^11*n^9 + 32670*b^11*n^8 + 357423*b^11*n^7 + 263
7558*b^11*n^6 + 13339535*b^11*n^5 + 45995730*b^11*n^4 + 105258076*b^11*n^3
+ 150917976*b^11*n^2 + 120543840*b^11*n + 39916800*b^11)
```

giac [B] time = 0.72, size = 4934, normalized size = 12.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^11*d^3*n^10*x^11 + (b*x + a)^n*a*b^10*d^3*n^10*x^10 + 55*(b*
x + a)^n*b^11*d^3*n^9*x^11 + 45*(b*x + a)^n*a*b^10*d^3*n^9*x^10 + 1320*(b*x
 + a)^n*b^11*d^3*n^8*x^11 + 3*(b*x + a)^n*b^11*c*d^2*n^10*x^8 - 10*(b*x + a
)^n*a^2*b^9*d^3*n^9*x^9 + 870*(b*x + a)^n*a*b^10*d^3*n^8*x^10 + 18150*(b*x
+ a)^n*b^11*d^3*n^7*x^11 + 3*(b*x + a)^n*a*b^10*c*d^2*n^10*x^7 + 174*(b*x +
 a)^n*b^11*c*d^2*n^9*x^8 - 360*(b*x + a)^n*a^2*b^9*d^3*n^8*x^9 + 9450*(b*x
+ a)^n*a*b^10*d^3*n^7*x^10 + 157773*(b*x + a)^n*b^11*d^3*n^6*x^11 + 153*(b*
x + a)^n*a*b^10*c*d^2*n^9*x^7 + 4383*(b*x + a)^n*b^11*c*d^2*n^8*x^8 + 90*(b
*x + a)^n*a^3*b^8*d^3*n^8*x^8 - 5460*(b*x + a)^n*a^2*b^9*d^3*n^7*x^9 + 6327
3*(b*x + a)^n*a*b^10*d^3*n^6*x^10 + 902055*(b*x + a)^n*b^11*d^3*n^5*x^11 +
3*(b*x + a)^n*b^11*c^2*d*n^10*x^5 - 21*(b*x + a)^n*a^2*b^9*c*d^2*n^9*x^6 +
3312*(b*x + a)^n*a*b^10*c*d^2*n^8*x^7 + 62946*(b*x + a)^n*b^11*c*d^2*n^7*x^
8 + 2520*(b*x + a)^n*a^3*b^8*d^3*n^7*x^8 - 45360*(b*x + a)^n*a^2*b^9*d^3*n^
6*x^9 + 269325*(b*x + a)^n*a*b^10*d^3*n^5*x^10 + 3416930*(b*x + a)^n*b^11*d
^3*n^4*x^11 + 3*(b*x + a)^n*a*b^10*c^2*d*n^10*x^4 + 183*(b*x + a)^n*b^11*c^
2*d*n^9*x^5 - 945*(b*x + a)^n*a^2*b^9*c*d^2*n^8*x^6 + 39762*(b*x + a)^n*a*b
^10*c*d^2*n^7*x^7 - 720*(b*x + a)^n*a^4*b^7*d^3*n^7*x^7 + 568701*(b*x + a)^
n*b^11*c*d^2*n^6*x^8 + 28980*(b*x + a)^n*a^3*b^8*d^3*n^6*x^8 - 224490*(b*x
+ a)^n*a^2*b^9*d^3*n^5*x^9 + 723680*(b*x + a)^n*a*b^10*d^3*n^4*x^10 + 84095
00*(b*x + a)^n*b^11*d^3*n^3*x^11 + 171*(b*x + a)^n*a*b^10*c^2*d*n^9*x^4 + 4
860*(b*x + a)^n*b^11*c^2*d*n^8*x^5 + 126*(b*x + a)^n*a^3*b^8*c*d^2*n^8*x^5
- 17514*(b*x + a)^n*a^2*b^9*c*d^2*n^7*x^6 + 290367*(b*x + a)^n*a*b^10*c*d^2
```

*n^6*x^7 - 15120*(b*x + a)^n*a^4*b^7*d^3*n^6*x^7 + 3363066*(b*x + a)^n*b^11
*c*d^2*n^5*x^8 + 176400*(b*x + a)^n*a^3*b^8*d^3*n^5*x^8 - 672840*(b*x + a)^
n*a^2*b^9*d^3*n^4*x^9 + 1172700*(b*x + a)^n*a*b^10*d^3*n^3*x^10 + 12753576*
(b*x + a)^n*b^11*d^3*n^2*x^11 + (b*x + a)^n*b^11*c^3*n^10*x^2 - 12*(b*x + a
)^n*a^2*b^9*c^2*d*n^9*x^3 + 4176*(b*x + a)^n*a*b^10*c^2*d*n^8*x^4 + 73710*(
b*x + a)^n*b^11*c^2*d*n^7*x^5 + 5040*(b*x + a)^n*a^3*b^8*c*d^2*n^7*x^5 - 17
3250*(b*x + a)^n*a^2*b^9*c*d^2*n^6*x^6 + 5040*(b*x + a)^n*a^5*b^6*d^3*n^6*x
^6 + 1330497*(b*x + a)^n*a*b^10*c*d^2*n^5*x^7 - 126000*(b*x + a)^n*a^4*b^7*
d^3*n^5*x^7 + 13114077*(b*x + a)^n*b^11*c*d^2*n^4*x^8 + 609210*(b*x + a)^n*
a^3*b^8*d^3*n^4*x^8 - 1181240*(b*x + a)^n*a^2*b^9*d^3*n^3*x^9 + 1026576*(b*
x + a)^n*a*b^10*d^3*n^2*x^10 + 10628640*(b*x + a)^n*b^11*d^3*n*x^11 + (b*x
+ a)^n*a*b^10*c^3*n^10*x + 64*(b*x + a)^n*b^11*c^3*n^9*x^2 - 648*(b*x + a)^
n*a^2*b^9*c^2*d*n^8*x^3 + 57006*(b*x + a)^n*a*b^10*c^2*d*n^7*x^4 - 630*(b*x
 + a)^n*a^4*b^7*c*d^2*n^7*x^4 + 703719*(b*x + a)^n*b^11*c^2*d*n^6*x^5 + 798
84*(b*x + a)^n*a^3*b^8*c*d^2*n^6*x^5 - 993069*(b*x + a)^n*a^2*b^9*c*d^2*n^5
x^6 + 75600(b*x + a)^n*a^5*b^6*d^3*n^5*x^6 + 3800598*(b*x + a)^n*a*b^10*c
*d^2*n^4*x^7 - 529200*(b*x + a)^n*a^4*b^7*d^3*n^4*x^7 + 33074574*(b*x + a)^
n*b^11*c*d^2*n^3*x^8 + 1181880*(b*x + a)^n*a^3*b^8*d^3*n^3*x^8 - 1095840*(b
*x + a)^n*a^2*b^9*d^3*n^2*x^9 + 362880*(b*x + a)^n*a*b^10*d^3*n*x^10 + 3628
800*(b*x + a)^n*b^11*d^3*x^11 + 63*(b*x + a)^n*a*b^10*c^3*n^9*x + 1797*(b*x
 + a)^n*b^11*c^3*n^8*x^2 + 36*(b*x + a)^n*a^3*b^8*c^2*d*n^8*x^2 - 14760*(b*
x + a)^n*a^2*b^9*c^2*d*n^7*x^3 + 475695*(b*x + a)^n*a*b^10*c^2*d*n^6*x^4 -
22680*(b*x + a)^n*a^4*b^7*c*d^2*n^6*x^4 + 4394079*(b*x + a)^n*b^11*c^2*d*n^
5*x^5 + 640080*(b*x + a)^n*a^3*b^8*c*d^2*n^5*x^5 - 30240*(b*x + a)^n*a^6*b^
5*d^3*n^5*x^5 - 3355065*(b*x + a)^n*a^2*b^9*c*d^2*n^4*x^6 + 428400*(b*x + a
)^n*a^5*b^6*d^3*n^4*x^6 + 6470388*(b*x + a)^n*a*b^10*c*d^2*n^3*x^7 - 116928
0*(b*x + a)^n*a^4*b^7*d^3*n^3*x^7 + 51177636*(b*x + a)^n*b^11*c*d^2*n^2*x^8
 + 1176120*(b*x + a)^n*a^3*b^8*d^3*n^2*x^8 - 403200*(b*x + a)^n*a^2*b^9*d^3
*n*x^9 - (b*x + a)^n*a^2*b^9*c^3*n^9 + 1734*(b*x + a)^n*a*b^10*c^3*n^8*x +
29076*(b*x + a)^n*b^11*c^3*n^7*x^2 + 1872*(b*x + a)^n*a^3*b^8*c^2*d*n^7*x^2
 - 183744*(b*x + a)^n*a^2*b^9*c^2*d*n^6*x^3 + 2520*(b*x + a)^n*a^5*b^6*c*d^
2*n^6*x^3 + 2491299*(b*x + a)^n*a*b^10*c^2*d*n^5*x^4 - 308700*(b*x + a)^n*a
^4*b^7*c*d^2*n^5*x^4 + 18048210*(b*x + a)^n*b^11*c^2*d*n^4*x^5 + 2758014*(b
*x + a)^n*a^3*b^8*c*d^2*n^4*x^5 - 302400*(b*x + a)^n*a^6*b^5*d^3*n^4*x^5 -
6473796*(b*x + a)^n*a^2*b^9*c*d^2*n^3*x^6 + 1134000*(b*x + a)^n*a^5*b^6*d^3
*n^3*x^6 + 5884920*(b*x + a)^n*a*b^10*c*d^2*n^2*x^7 - 1270080*(b*x + a)^n*a
^4*b^7*d^3*n^2*x^7 + 43332840*(b*x + a)^n*b^11*c*d^2*n*x^8 + 453600*(b*x +
a)^n*a^3*b^8*d^3*n*x^8 - 63*(b*x + a)^n*a^2*b^9*c^3*n^8 + 27342*(b*x + a)^n
*a*b^10*c^3*n^7*x - 72*(b*x + a)^n*a^4*b^7*c^2*d*n^7*x + 299271*(b*x + a)^n
*b^11*c^3*n^6*x^2 + 40536*(b*x + a)^n*a^3*b^8*c^2*d*n^6*x^2 - 1351548*(b*x
+ a)^n*a^2*b^9*c^2*d*n^5*x^3 + 83160*(b*x + a)^n*a^5*b^6*c*d^2*n^5*x^3 + 80
83014*(b*x + a)^n*a*b^10*c^2*d*n^4*x^4 - 1965600*(b*x + a)^n*a^4*b^7*c*d^2*
n^4*x^4 + 151200*(b*x + a)^n*a^7*b^4*d^3*n^4*x^4 + 47746140*(b*x + a)^n*b^1
1*c^2*d*n^3*x^5 + 6340320*(b*x + a)^n*a^3*b^8*c*d^2*n^3*x^5 - 1058400*(b*x
+ a)^n*a^6*b^5*d^3*n^3*x^5 - 6449940*(b*x + a)^n*a^2*b^9*c*d^2*n^2*x^6 + 13
80960*(b*x + a)^n*a^5*b^6*d^3*n^2*x^6 + 2138400*(b*x + a)^n*a*b^10*c*d^2*n*
x^7 - 518400*(b*x + a)^n*a^4*b^7*d^3*n*x^7 + 14968800*(b*x + a)^n*b^11*c*d^
2*x^8 - 1734*(b*x + a)^n*a^2*b^9*c^3*n^7 + 271929*(b*x + a)^n*a*b^10*c^3*n^
6*x - 3672*(b*x + a)^n*a^4*b^7*c^2*d*n^6*x + 2039016*(b*x + a)^n*b^11*c^3*n
^5*x^2 + 470160*(b*x + a)^n*a^3*b^8*c^2*d*n^5*x^2 - 7560*(b*x + a)^n*a^6*b^
5*c*d^2*n^5*x^2 - 5910552*(b*x + a)^n*a^2*b^9*c^2*d*n^4*x^3 + 985320*(b*x +
 a)^n*a^5*b^6*c*d^2*n^4*x^3 + 15414084*(b*x + a)^n*a*b^10*c^2*d*n^3*x^4 - 5
927670*(b*x + a)^n*a^4*b^7*c*d^2*n^3*x^4 + 907200*(b*x + a)^n*a^7*b^4*d^3*n
^3*x^4 + 77043528*(b*x + a)^n*b^11*c^2*d*n^2*x^5 + 7141176*(b*x + a)^n*a^3*
b^8*c*d^2*n^2*x^5 - 1512000*(b*x + a)^n*a^6*b^5*d^3*n^2*x^5 - 2494800*(b*x
+ a)^n*a^2*b^9*c*d^2*n*x^6 + 604800*(b*x + a)^n*a^5*b^6*d^3*n*x^6 - 27342*(
b*x + a)^n*a^2*b^9*c^3*n^6 + 72*(b*x + a)^n*a^5*b^6*c^2*d*n^6 + 1767087*(b*
x + a)^n*a*b^10*c^3*n^5*x - 77400*(b*x + a)^n*a^4*b^7*c^2*d*n^5*x + 9261503
*(b*x + a)^n*b^11*c^3*n^4*x^2 + 3114324*(b*x + a)^n*a^3*b^8*c^2*d*n^4*x^2 -

```
234360*(b*x + a)^n*a^6*b^5*c*d^2*n^4*x^2 - 14600400*(b*x + a)^n*a^2*b^9*c^
2*d*n^3*x^3 + 4906440*(b*x + a)^n*a^5*b^6*c*d^2*n^3*x^3 - 604800*(b*x + a)^
n*a^8*b^3*d^3*n^3*x^3 + 15387192*(b*x + a)^n*a*b^10*c^2*d*n^2*x^4 - 7990920
*(b*x + a)^n*a^4*b^7*c*d^2*n^2*x^4 + 1663200*(b*x + a)^n*a^7*b^4*d^3*n^2*x^
4 + 67536288*(b*x + a)^n*b^11*c^2*d*n*x^5 + 2993760*(b*x + a)^n*a^3*b^8*c*d
^2*n*x^5 - 725760*(b*x + a)^n*a^6*b^5*d^3*n*x^5 - 271929*(b*x + a)^n*a^2*b^
9*c^3*n^5 + 3672*(b*x + a)^n*a^5*b^6*c^2*d*n^5 + 7494416*(b*x + a)^n*a*b^10
*c^3*n^4*x - 862920*(b*x + a)^n*a^4*b^7*c^2*d*n^4*x + 15120*(b*x + a)^n*a^7
*b^4*c*d^2*n^4*x + 27472724*(b*x + a)^n*b^11*c^3*n^3*x^2 + 11503008*(b*x +
a)^n*a^3*b^8*c^2*d*n^3*x^2 - 2487240*(b*x + a)^n*a^6*b^5*c*d^2*n^3*x^2 - 17
855136*(b*x + a)^n*a^2*b^9*c^2*d*n^2*x^3 + 8991360*(b*x + a)^n*a^5*b^6*c*d^
2*n^2*x^3 - 1814400*(b*x + a)^n*a^8*b^3*d^3*n^2*x^3 + 5987520*(b*x + a)^n*a
*b^10*c^2*d*n*x^4 - 3742200*(b*x + a)^n*a^4*b^7*c*d^2*n*x^4 + 907200*(b*x +
 a)^n*a^7*b^4*d^3*n*x^4 + 23950080*(b*x + a)^n*b^11*c^2*d*x^5 - 1767087*(b*
x + a)^n*a^2*b^9*c^3*n^4 + 77400*(b*x + a)^n*a^5*b^6*c^2*d*n^4 + 19978308*(
b*x + a)^n*a*b^10*c^3*n^3*x - 5365728*(b*x + a)^n*a^4*b^7*c^2*d*n^3*x + 453
600*(b*x + a)^n*a^7*b^4*c*d^2*n^3*x + 50312628*(b*x + a)^n*b^11*c^3*n^2*x^2
 + 20795184*(b*x + a)^n*a^3*b^8*c^2*d*n^2*x^2 - 9744840*(b*x + a)^n*a^6*b^5
*c*d^2*n^2*x^2 + 1814400*(b*x + a)^n*a^9*b^2*d^3*n^2*x^2 - 7983360*(b*x + a
)^n*a^2*b^9*c^2*d*n*x^3 + 4989600*(b*x + a)^n*a^5*b^6*c*d^2*n*x^3 - 1209600
*(b*x + a)^n*a^8*b^3*d^3*n*x^3 - 7494416*(b*x + a)^n*a^2*b^9*c^3*n^3 + 8629
20*(b*x + a)^n*a^5*b^6*c^2*d*n^3 - 15120*(b*x + a)^n*a^8*b^3*c*d^2*n^3 + 30
334320*(b*x + a)^n*a*b^10*c^3*n^2*x - 17640288*(b*x + a)^n*a^4*b^7*c^2*d*n^
2*x + 4520880*(b*x + a)^n*a^7*b^4*c*d^2*n^2*x + 50292720*(b*x + a)^n*b^11*c
^3*n*x^2 + 11975040*(b*x + a)^n*a^3*b^8*c^2*d*n*x^2 - 7484400*(b*x + a)^n*a
^6*b^5*c*d^2*n*x^2 + 1814400*(b*x + a)^n*a^9*b^2*d^3*n*x^2 - 19978308*(b*x
+ a)^n*a^2*b^9*c^3*n^2 + 5365728*(b*x + a)^n*a^5*b^6*c^2*d*n^2 - 453600*(b*
x + a)^n*a^8*b^3*c*d^2*n^2 + 19958400*(b*x + a)^n*a*b^10*c^3*n*x - 23950080
*(b*x + a)^n*a^4*b^7*c^2*d*n*x + 14968800*(b*x + a)^n*a^7*b^4*c*d^2*n*x - 3
628800*(b*x + a)^n*a^10*b*d^3*n*x + 19958400*(b*x + a)^n*b^11*c^3*x^2 - 303
34320*(b*x + a)^n*a^2*b^9*c^3*n + 17640288*(b*x + a)^n*a^5*b^6*c^2*d*n - 45
20880*(b*x + a)^n*a^8*b^3*c*d^2*n - 19958400*(b*x + a)^n*a^2*b^9*c^3 + 2395
0080*(b*x + a)^n*a^5*b^6*c^2*d - 14968800*(b*x + a)^n*a^8*b^3*c*d^2 + 36288
00*(b*x + a)^n*a^11*d^3)/(b^11*n^11 + 66*b^11*n^10 + 1925*b^11*n^9 + 32670*
b^11*n^8 + 357423*b^11*n^7 + 2637558*b^11*n^6 + 13339535*b^11*n^5 + 4599573
0*b^11*n^4 + 105258076*b^11*n^3 + 150917976*b^11*n^2 + 120543840*b^11*n + 3
9916800*b^11)
```

maple [B] time = 0.04, size = 2972, normalized size = 7.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] (b*x+a)^(n+1)*(b^10*d^3*n^10*x^10+55*b^10*d^3*n^9*x^10-10*a*b^9*d^3*n^9*x^9
+1320*b^10*d^3*n^8*x^10-450*a*b^9*d^3*n^8*x^9+3*b^10*c*d^2*n^10*x^7+18150*b
^10*d^3*n^7*x^10+90*a^2*b^8*d^3*n^8*x^8-8700*a*b^9*d^3*n^7*x^9+174*b^10*c*d
^2*n^9*x^7+157773*b^10*d^3*n^6*x^10+3240*a^2*b^8*d^3*n^7*x^8-21*a*b^9*c*d^2
*n^9*x^6-94500*a*b^9*d^3*n^6*x^9+4383*b^10*c*d^2*n^8*x^7+902055*b^10*d^3*n^
5*x^10-720*a^3*b^7*d^3*n^7*x^7+49140*a^2*b^8*d^3*n^6*x^8-1071*a*b^9*c*d^2*n
^8*x^6-632730*a*b^9*d^3*n^5*x^9+3*b^10*c^2*d*n^10*x^4+62946*b^10*c*d^2*n^7*
x^7+3416930*b^10*d^3*n^4*x^10-20160*a^3*b^7*d^3*n^6*x^7+126*a^2*b^8*c*d^2*n
^8*x^5+408240*a^2*b^8*d^3*n^5*x^8-23184*a*b^9*c*d^2*n^7*x^6-2693250*a*b^9*d
^3*n^4*x^9+183*b^10*c^2*d*n^9*x^4+568701*b^10*c*d^2*n^6*x^7+8409500*b^10*d^
3*n^3*x^10+5040*a^4*b^6*d^3*n^6*x^6-231840*a^3*b^7*d^3*n^5*x^7+5670*a^2*b^8
*c*d^2*n^7*x^5+2020410*a^2*b^8*d^3*n^4*x^8-12*a*b^9*c^2*d*n^9*x^3-278334*a*
b^9*c*d^2*n^6*x^6-7236800*a*b^9*d^3*n^3*x^9+4860*b^10*c^2*d*n^8*x^4+3363066
*b^10*c*d^2*n^5*x^7+12753576*b^10*d^3*n^2*x^10+105840*a^4*b^6*d^3*n^5*x^6-6
30*a^3*b^7*c*d^2*n^7*x^4-1411200*a^3*b^7*d^3*n^4*x^7+105084*a^2*b^8*c*d^2*n
```

```
^6*x^5+6055560*a^2*b^8*d^3*n^3*x^8-684*a*b^9*c^2*d*n^8*x^3-2032569*a*b^9*c*
d^2*n^5*x^6-11727000*a*b^9*d^3*n^2*x^9+b^10*c^3*n^10*x+73710*b^10*c^2*d*n^7
*x^4+13114077*b^10*c*d^2*n^4*x^7+10628640*b^10*d^3*n*x^10-30240*a^5*b^5*d^3
*n^5*x^5+882000*a^4*b^6*d^3*n^4*x^6-25200*a^3*b^7*c*d^2*n^6*x^4-4873680*a^3
*b^7*d^3*n^3*x^7+36*a^2*b^8*c^2*d*n^8*x^2+1039500*a^2*b^8*c*d^2*n^5*x^5+106
31160*a^2*b^8*d^3*n^2*x^8-16704*a*b^9*c^2*d*n^7*x^3-9313479*a*b^9*c*d^2*n^4
*x^6-10265760*a*b^9*d^3*n*x^9+64*b^10*c^3*n^9*x+703719*b^10*c^2*d*n^6*x^4+3
3074574*b^10*c*d^2*n^3*x^7+3628800*b^10*d^3*x^10-453600*a^5*b^5*d^3*n^4*x^5
+2520*a^4*b^6*c*d^2*n^6*x^3+3704400*a^4*b^6*d^3*n^3*x^6-399420*a^3*b^7*c*d^
2*n^5*x^4-9455040*a^3*b^7*d^3*n^2*x^7+1944*a^2*b^8*c^2*d*n^7*x^2+5958414*a^
2*b^8*c*d^2*n^4*x^5+9862560*a^2*b^8*d^3*n*x^8-a*b^9*c^3*n^9-228024*a*b^9*c^
2*d*n^6*x^3-26604186*a*b^9*c*d^2*n^3*x^6-3628800*a*b^9*d^3*x^9+1797*b^10*c^
3*n^8*x+4394079*b^10*c^2*d*n^5*x^4+51177636*b^10*c*d^2*n^2*x^7+151200*a^6*b
^4*d^3*n^4*x^4-2570400*a^5*b^5*d^3*n^3*x^5+90720*a^4*b^6*c*d^2*n^5*x^3+8184
960*a^4*b^6*d^3*n^2*x^6-72*a^3*b^7*c^2*d*n^7*x-3200400*a^3*b^7*c*d^2*n^4*x^
4-9408960*a^3*b^7*d^3*n*x^7+44280*a^2*b^8*c^2*d*n^6*x^2+20130390*a^2*b^8*c*
d^2*n^3*x^5+3628800*a^2*b^8*d^3*x^8-63*a*b^9*c^3*n^8-1902780*a*b^9*c^2*d*n^
5*x^3-45292716*a*b^9*c*d^2*n^2*x^6+29076*b^10*c^3*n^7*x+18048210*b^10*c^2*d
*n^4*x^4+43332840*b^10*c*d^2*n*x^7+1512000*a^6*b^4*d^3*n^3*x^4-7560*a^5*b^5
*c*d^2*n^5*x^2-6804000*a^5*b^5*d^3*n^2*x^5+1234800*a^4*b^6*c*d^2*n^4*x^3+88
90560*a^4*b^6*d^3*n*x^6-3744*a^3*b^7*c^2*d*n^6*x-13790070*a^3*b^7*c*d^2*n^3
*x^4-3628800*a^3*b^7*d^3*x^7+551232*a^2*b^8*c^2*d*n^5*x^2+38842776*a^2*b^8*
c*d^2*n^2*x^5-1734*a*b^9*c^3*n^7-9965196*a*b^9*c^2*d*n^4*x^3-41194440*a*b^9
*c*d^2*n*x^6+299271*b^10*c^3*n^6*x+47746140*b^10*c^2*d*n^3*x^4+14968800*b^1
0*c*d^2*x^7-604800*a^7*b^3*d^3*n^3*x^3+5292000*a^6*b^4*d^3*n^2*x^4-249480*a
^5*b^5*c*d^2*n^4*x^2-8285760*a^5*b^5*d^3*n*x^5+72*a^4*b^6*c^2*d*n^6+7862400
*a^4*b^6*c*d^2*n^3*x^3+3628800*a^4*b^6*d^3*x^6-81072*a^3*b^7*c^2*d*n^5*x-31
701600*a^3*b^7*c*d^2*n^2*x^4+4054644*a^2*b^8*c^2*d*n^4*x^2+38699640*a^2*b^8
*c*d^2*n*x^5-27342*a*b^9*c^3*n^6-32332056*a*b^9*c^2*d*n^3*x^3-14968800*a*b^
9*c*d^2*x^6+2039016*b^10*c^3*n^5*x+77043528*b^10*c^2*d*n^2*x^4-3628800*a^7*
b^3*d^3*n^2*x^3+15120*a^6*b^4*c*d^2*n^4*x+7560000*a^6*b^4*d^3*n*x^4-2955960
*a^5*b^5*c*d^2*n^3*x^2-3628800*a^5*b^5*d^3*x^5+3672*a^4*b^6*c^2*d*n^5+23710
680*a^4*b^6*c*d^2*n^2*x^3-940320*a^3*b^7*c^2*d*n^4*x-35705880*a^3*b^7*c*d^2
*n*x^4+17731656*a^2*b^8*c^2*d*n^3*x^2+14968800*a^2*b^8*c*d^2*x^5-271929*a*b
^9*c^3*n^5-61656336*a*b^9*c^2*d*n^2*x^3+9261503*b^10*c^3*n^4*x+67536288*b^1
0*c^2*d*n*x^4+1814400*a^8*b^2*d^3*n^2*x^2-6652800*a^7*b^3*d^3*n*x^3+468720*
a^6*b^4*c*d^2*n^3*x+3628800*a^6*b^4*d^3*x^4-14719320*a^5*b^5*c*d^2*n^2*x^2+
77400*a^4*b^6*c^2*d*n^4+31963680*a^4*b^6*c*d^2*n*x^3-6228648*a^3*b^7*c^2*d*
n^3*x-14968800*a^3*b^7*c*d^2*x^4+43801200*a^2*b^8*c^2*d*n^2*x^2-1767087*a*b
^9*c^3*n^4-61548768*a*b^9*c^2*d*n*x^3+27472724*b^10*c^3*n^3*x+23950080*b^10
*c^2*d*x^4+5443200*a^8*b^2*d^3*n*x^2-15120*a^7*b^3*c*d^2*n^3-3628800*a^7*b^
3*d^3*x^3+4974480*a^6*b^4*c*d^2*n^2*x-26974080*a^5*b^5*c*d^2*n*x^2+862920*a
^4*b^6*c^2*d*n^3+14968800*a^4*b^6*c*d^2*x^3-23006016*a^3*b^7*c^2*d*n^2*x+53
565408*a^2*b^8*c^2*d*n*x^2-7494416*a*b^9*c^3*n^3-23950080*a*b^9*c^2*d*x^3+5
0312628*b^10*c^3*n^2*x-3628800*a^9*b*d^3*n*x+3628800*a^8*b^2*d^3*x^2-453600
*a^7*b^3*c*d^2*n^2+19489680*a^6*b^4*c*d^2*n*x-14968800*a^5*b^5*c*d^2*x^2+53
65728*a^4*b^6*c^2*d*n^2-41590368*a^3*b^7*c^2*d*n*x+23950080*a^2*b^8*c^2*d*x
^2-19978308*a*b^9*c^3*n^2+50292720*b^10*c^3*n*x-3628800*a^9*b*d^3*x-4520880
*a^7*b^3*c*d^2*n+14968800*a^6*b^4*c*d^2*x+17640288*a^4*b^6*c^2*d*n-23950080
*a^3*b^7*c^2*d*x-30334320*a*b^9*c^3*n+19958400*b^10*c^3*x+3628800*a^10*d^3-
14968800*a^7*b^3*c*d^2+23950080*a^4*b^6*c^2*d-19958400*a*b^9*c^3)/b^11/(n^1
1+66*n^10+1925*n^9+32670*n^8+357423*n^7+2637558*n^6+13339535*n^5+45995730*n
^4+105258076*n^3+150917976*n^2+120543840*n+39916800)
```

maxima [B] time = 0.85, size = 953, normalized size = 2.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) + 3
*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
- 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^
2 + 274*n + 120)*b^5) + 3*((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 +
13132*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1
624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3
+ 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)
*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*
n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040
*a^8)*(b*x + a)^n*c*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 6
7284*n^3 + 118124*n^2 + 109584*n + 40320)*b^8) + ((n^10 + 55*n^9 + 1320*n^8
 + 18150*n^7 + 157773*n^6 + 902055*n^5 + 3416930*n^4 + 8409500*n^3 + 127535
76*n^2 + 10628640*n + 3628800)*b^11*x^11 + (n^10 + 45*n^9 + 870*n^8 + 9450*
n^7 + 63273*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362
880*n)*a*b^10*x^10 - 10*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67
284*n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^2*b^9*x^9 + 90*(n^8 + 28*n^7
 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^3*b^8*
x^8 - 720*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*
a^4*b^7*x^7 + 5040*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^5*
b^6*x^6 - 30240*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^6*b^5*x^5 + 15120
0*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^7*b^4*x^4 - 604800*(n^3 + 3*n^2 + 2*n)*a^8
*b^3*x^3 + 1814400*(n^2 + n)*a^9*b^2*x^2 - 3628800*a^10*b*n*x + 3628800*a^1
1)*(b*x + a)^n*d^3/((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 2
637558*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 +
120543840*n + 39916800)*b^11)
```

mupad [B]   time = 5.60, size = 2436, normalized size = 6.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^11*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^
4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362880
0))/(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*
n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39
916800) - (a^2*(a + b*x)^n*(19958400*b^9*c^3 - 3628800*a^9*d^3 + 30334320*b
^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 1767087*b^9*c^3*n^4
 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n^7 + 63*b^9*c^3*n
^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 14968800*a^6*b^3*c*d^2 - 176402
88*a^3*b^6*c^2*d*n + 4520880*a^6*b^3*c*d^2*n - 5365728*a^3*b^6*c^2*d*n^2 +
453600*a^6*b^3*c*d^2*n^2 - 862920*a^3*b^6*c^2*d*n^3 + 15120*a^6*b^3*c*d^2*n
^3 - 77400*a^3*b^6*c^2*d*n^4 - 3672*a^3*b^6*c^2*d*n^5 - 72*a^3*b^6*c^2*d*n^
6))/(b^11*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 133
39535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^1
1 + 39916800)) + (x^2*(n + 1)*(a + b*x)^n*(19958400*b^9*c^3 + 1814400*a^9*d
^3*n + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 17
67087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n
^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 + 11975040*a^3*b^6*c^2*d*n - 7484400*a^6*
b^3*c*d^2*n + 8820144*a^3*b^6*c^2*d*n^2 - 2260440*a^6*b^3*c*d^2*n^2 + 26828
64*a^3*b^6*c^2*d*n^3 - 226800*a^6*b^3*c*d^2*n^3 + 431460*a^3*b^6*c^2*d*n^4
- 7560*a^6*b^3*c*d^2*n^4 + 38700*a^3*b^6*c^2*d*n^5 + 1836*a^3*b^6*c^2*d*n^6
 + 36*a^3*b^6*c^2*d*n^7))/(b^9*(120543840*n + 150917976*n^2 + 105258076*n^3
 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 192
5*n^9 + 66*n^10 + n^11 + 39916800)) + (a*n*x*(a + b*x)^n*(19958400*b^9*c^3
- 3628800*a^9*d^3 + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9
*c^3*n^3 + 1767087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1
734*b^9*c^3*n^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 1
```

```
4968800*a^6*b^3*c*d^2 - 17640288*a^3*b^6*c^2*d*n + 4520880*a^6*b^3*c*d^2*n
- 5365728*a^3*b^6*c^2*d*n^2 + 453600*a^6*b^3*c*d^2*n^2 - 862920*a^3*b^6*c^2
*d*n^3 + 15120*a^6*b^3*c*d^2*n^3 - 77400*a^3*b^6*c^2*d*n^4 - 3672*a^3*b^6*c
^2*d*n^5 - 72*a^3*b^6*c^2*d*n^6))/(b^10*(120543840*n + 150917976*n^2 + 1052
58076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*
n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*d*x^5*(a + b*x)^n*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)*(332640*b^6*c^2 - 10080*a^6*d^2*n + 245004*b^6
*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 + 1075*b^6*c^2*n^4 + 51*b^6*
c^2*n^5 + b^6*c^2*n^6 + 41580*a^3*b^3*c*d*n + 12558*a^3*b^3*c*d*n^2 + 1260*
a^3*b^3*c*d*n^3 + 42*a^3*b^3*c*d*n^4))/(b^6*(120543840*n + 150917976*n^2 +
105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32
670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*d^2*x^8*(a + b*x)^n*(
990*b^3*c + 30*b^3*c*n^2 + b^3*c*n^3 + 30*a^3*d*n + 299*b^3*c*n)*(13068*n +
 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b^3*(12
0543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2
637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)
) + (a*d^3*n*x^10*(a + b*x)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 26932
5*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(b*(120543
840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 26375
58*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) -
(10*a^2*d^3*n*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^
4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(b^2*(120543840*n + 1509179
76*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423
*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*a*d^2*n*x^7*
(a + b*x)^n*(990*b^3*c - 240*a^3*d + 30*b^3*c*n^2 + b^3*c*n^3 + 299*b^3*c*n
)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b^4*(12054
3840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637
558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) -
 (21*a^2*d^2*n*x^6*(a + b*x)^n*(990*b^3*c - 240*a^3*d + 30*b^3*c*n^2 + b^3*
c*n^3 + 299*b^3*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^5*
(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5
+ 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 399168
00)) + (3*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(50400*a^6*d^2 + 3
32640*b^6*c^2 + 245004*b^6*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 +
1075*b^6*c^2*n^4 + 51*b^6*c^2*n^5 + b^6*c^2*n^6 - 207900*a^3*b^3*c*d - 6279
0*a^3*b^3*c*d*n - 6300*a^3*b^3*c*d*n^2 - 210*a^3*b^3*c*d*n^3))/(b^7*(120543
840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 26375
58*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) -
(12*a^2*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(50400*a^6*d^2 + 332640*b^6*c^2
 + 245004*b^6*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 + 1075*b^6*c^2*
n^4 + 51*b^6*c^2*n^5 + b^6*c^2*n^6 - 207900*a^3*b^3*c*d - 62790*a^3*b^3*c*d
*n - 6300*a^3*b^3*c*d*n^2 - 210*a^3*b^3*c*d*n^3))/(b^8*(120543840*n + 15091
7976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 3574
23*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.184 $\int (a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=337

$$-\frac{18ad^2\left(b^3c-7a^3d\right)(a+bx)^{n+6}}{b^{10}(n+6)}+\frac{3d^2\left(b^3c-28a^3d\right)(a+bx)^{n+7}}{b^{10}(n+7)}+\frac{\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{10}(n+1)}-\frac{9ad\left(b^3c-4a^3d\right)(}{b^{10}}$$

```
[Out] (-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^10/(1+n)+9*a^2*d*(-a^3*d+b^3*c)^2*(b*x+a)^
(2+n)/b^10/(2+n)-9*a*d*(-4*a^3*d+b^3*c)*(-a^3*d+b^3*c)*(b*x+a)^(3+n)/b^10/(
3+n)+3*d*(28*a^6*d^2-20*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(4+n)/b^10/(4+n)+9*a^2
*d^2*(-14*a^3*d+5*b^3*c)*(b*x+a)^(5+n)/b^10/(5+n)-18*a*d^2*(-7*a^3*d+b^3*c)
*(b*x+a)^(6+n)/b^10/(6+n)+3*d^2*(-28*a^3*d+b^3*c)*(b*x+a)^(7+n)/b^10/(7+n)+
36*a^2*d^3*(b*x+a)^(8+n)/b^10/(8+n)-9*a*d^3*(b*x+a)^(9+n)/b^10/(9+n)+d^3*(b
*x+a)^(10+n)/b^10/(10+n)
```

Rubi [A] time = 0.21, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1850}

$$\frac{3d\left(-20a^3b^3cd+28a^6d^2+b^6c^2\right)(a+bx)^{n+4}}{b^{10}(n+4)}+\frac{9a^2d^2\left(5b^3c-14a^3d\right)(a+bx)^{n+5}}{b^{10}(n+5)}-\frac{18ad^2\left(b^3c-7a^3d\right)(a+bx)^{n+}}{b^{10}(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^10*(1 + n)) + (9*a^2*d*(b^3*c - a^
3*d)^2*(a + b*x)^(2 + n))/(b^10*(2 + n)) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c
- a^3*d)*(a + b*x)^(3 + n))/(b^10*(3 + n)) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d
 + 28*a^6*d^2)*(a + b*x)^(4 + n))/(b^10*(4 + n)) + (9*a^2*d^2*(5*b^3*c - 14
*a^3*d)*(a + b*x)^(5 + n))/(b^10*(5 + n)) - (18*a*d^2*(b^3*c - 7*a^3*d)*(a
+ b*x)^(6 + n))/(b^10*(6 + n)) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^(7 + n
))/(b^10*(7 + n)) + (36*a^2*d^3*(a + b*x)^(8 + n))/(b^10*(8 + n)) - (9*a*d^
3*(a + b*x)^(9 + n))/(b^10*(9 + n)) + (d^3*(a + b*x)^(10 + n))/(b^10*(10 +
n))
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \int \left(\frac{\left(b^3c-a^3d\right)^3(a+bx)^n}{b^9}+\frac{9d\left(ab^3c-a^4d\right)^2(a+bx)^{1+n}}{b^9}+\frac{9ad\left(b^3c-4a^3d\right)\left(-b\right.}{b}\right.$$

$$=\frac{\left(b^3c-a^3d\right)^3(a+bx)^{1+n}}{b^{10}(1+n)}+\frac{9a^2d\left(b^3c-a^3d\right)^2(a+bx)^{2+n}}{b^{10}(2+n)}-\frac{9ad\left(b^3c-4a^3d\right)\left(b^3c\right.}{b^{10}(3+}$$

Mathematica [A] time = 0.36, size = 290, normalized size = 0.86

$$\frac{(a+bx)^{n+1}\left(\frac{9d(a+bx)\left(ab^3c-a^4d\right)^2}{n+2}+\frac{3d^2(a+bx)^6\left(b^3c-28a^3d\right)}{n+7}+\frac{18ad^2(a+bx)^5\left(7a^3d-b^3c\right)}{n+6}-\frac{9ad(a+bx)^2\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)}{n+3}+\frac{\left(b^3c-a^3\right.}{n+1}\right.}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b^3*c - a^3*d)^3/(1 + n) + (9*d*(a*b^3*c - a^4*d)^2*(a
 + b*x))/(2 + n) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^2)/(3
 + n) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^3)/(4 + n) +
 (9*a^2*d^2*(5*b^3*c - 14*a^3*d)*(a + b*x)^4)/(5 + n) + (18*a*d^2*(-(b^3*c)
 + 7*a^3*d)*(a + b*x)^5)/(6 + n) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^6)/(
7 + n) + (36*a^2*d^3*(a + b*x)^7)/(8 + n) - (9*a*d^3*(a + b*x)^8)/(9 + n) +
 (d^3*(a + b*x)^9)/(10 + n)))/b^10
```

fricas [B] time = 0.49, size = 2313, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^9*c^3*n^9 + 54*a*b^9*c^3*n^8 + 1266*a*b^9*c^3*n^7 + 3628800*a*b^9*c^3
- 2721600*a^4*b^6*c^2*d + 1555200*a^7*b^3*c*d^2 - 362880*a^10*d^3 + (b^10*d
^3*n^9 + 45*b^10*d^3*n^8 + 870*b^10*d^3*n^7 + 9450*b^10*d^3*n^6 + 63273*b^1
0*d^3*n^5 + 269325*b^10*d^3*n^4 + 723680*b^10*d^3*n^3 + 1172700*b^10*d^3*n^
2 + 1026576*b^10*d^3*n + 362880*b^10*d^3)*x^10 + (a*b^9*d^3*n^9 + 36*a*b^9*
d^3*n^8 + 546*a*b^9*d^3*n^7 + 4536*a*b^9*d^3*n^6 + 22449*a*b^9*d^3*n^5 + 67
284*a*b^9*d^3*n^4 + 118124*a*b^9*d^3*n^3 + 109584*a*b^9*d^3*n^2 + 40320*a*b
^9*d^3*n)*x^9 - 9*(a^2*b^8*d^3*n^8 + 28*a^2*b^8*d^3*n^7 + 322*a^2*b^8*d^3*n
^6 + 1960*a^2*b^8*d^3*n^5 + 6769*a^2*b^8*d^3*n^4 + 13132*a^2*b^8*d^3*n^3 +
13068*a^2*b^8*d^3*n^2 + 5040*a^2*b^8*d^3*n)*x^8 + 3*(b^10*c*d^2*n^9 + 48*b^
10*c*d^2*n^8 + 518400*b^10*c*d^2 + 24*(41*b^10*c*d^2 + a^3*b^7*d^3)*n^7 + 6
*(1877*b^10*c*d^2 + 84*a^3*b^7*d^3)*n^6 + 21*(3759*b^10*c*d^2 + 200*a^3*b^7
*d^3)*n^5 + 42*(8321*b^10*c*d^2 + 420*a^3*b^7*d^3)*n^4 + 4*(242639*b^10*c*d
^2 + 9744*a^3*b^7*d^3)*n^3 + 72*(22439*b^10*c*d^2 + 588*a^3*b^7*d^3)*n^2 +
1440*(1003*b^10*c*d^2 + 12*a^3*b^7*d^3)*n)*x^7 + 18*(938*a*b^9*c^3 - a^4*b^
6*c^2*d)*n^6 + 3*(a*b^9*c*d^2*n^9 + 42*a*b^9*c*d^2*n^8 + 732*a*b^9*c*d^2*n^
7 + 6*(1145*a*b^9*c*d^2 - 28*a^4*b^6*d^3)*n^6 + 9*(4191*a*b^9*c*d^2 - 280*a
^4*b^6*d^3)*n^5 + 24*(5132*a*b^9*c*d^2 - 595*a^4*b^6*d^3)*n^4 + 4*(57887*a*
b^9*c*d^2 - 9450*a^4*b^6*d^3)*n^3 + 48*(4715*a*b^9*c*d^2 - 959*a^4*b^6*d^3)
*n^2 + 2880*(30*a*b^9*c*d^2 - 7*a^4*b^6*d^3)*n)*x^6 + 3*(46963*a*b^9*c^3 -
270*a^4*b^6*c^2*d)*n^5 - 18*(a^2*b^8*c*d^2*n^8 + 37*a^2*b^8*c*d^2*n^7 + 547
*a^2*b^8*c*d^2*n^6 + (4135*a^2*b^8*c*d^2 - 168*a^5*b^5*d^3)*n^5 + 4*(4261*a
^2*b^8*c*d^2 - 420*a^5*b^5*d^3)*n^4 + 4*(9487*a^2*b^8*c*d^2 - 1470*a^5*b^5*
d^3)*n^3 + 48*(871*a^2*b^8*c*d^2 - 175*a^5*b^5*d^3)*n^2 + 576*(30*a^2*b^8*c
*d^2 - 7*a^5*b^5*d^3)*n)*x^5 + 18*(42287*a*b^9*c^3 - 835*a^4*b^6*c^2*d)*n^4
 + 3*(b^10*c^2*d*n^9 + 51*b^10*c^2*d*n^8 + 907200*b^10*c^2*d + 6*(186*b^10*
c^2*d + 5*a^3*b^7*c*d^2)*n^7 + 6*(2281*b^10*c^2*d + 165*a^3*b^7*c*d^2)*n^6
+ 3*(34343*b^10*c^2*d + 4150*a^3*b^7*c*d^2)*n^5 + 3*(163313*b^10*c^2*d + 24
750*a^3*b^7*c*d^2 - 1680*a^6*b^4*d^3)*n^4 + 2*(728587*b^10*c^2*d + 107160*a
^3*b^7*c*d^2 - 15120*a^6*b^4*d^3)*n^3 + 36*(71689*b^10*c^2*d + 7810*a^3*b^7
*c*d^2 - 1540*a^6*b^4*d^3)*n^2 + 360*(6751*b^10*c^2*d + 360*a^3*b^7*c*d^2 -
 84*a^6*b^4*d^3)*n)*x^4 + 2*(1327882*a*b^9*c^3 - 73575*a^4*b^6*c^2*d + 1080
*a^7*b^3*c*d^2)*n^3 + 3*(a*b^9*c^2*d*n^9 + 48*a*b^9*c^2*d*n^8 + 972*a*b^9*c
^2*d*n^7 + 30*(359*a*b^9*c^2*d - 4*a^4*b^6*c*d^2)*n^6 + 3*(23573*a*b^9*c^2*
d - 1200*a^4*b^6*c*d^2)*n^5 + 6*(46297*a*b^9*c^2*d - 6500*a^4*b^6*c*d^2)*n^
4 + 4*(155957*a*b^9*c^2*d - 45000*a^4*b^6*c*d^2 + 5040*a^7*b^3*d^3)*n^3 + 1
20*(5911*a*b^9*c^2*d - 2644*a^4*b^6*c*d^2 + 504*a^7*b^3*d^3)*n^2 + 2880*(10
5*a*b^9*c^2*d - 60*a^4*b^6*c*d^2 + 14*a^7*b^3*d^3)*n)*x^3 + 72*(79913*a*b^9
*c^3 - 11131*a^4*b^6*c^2*d + 810*a^7*b^3*c*d^2)*n^2 - 9*(a^2*b^8*c^2*d*n^8
+ 46*a^2*b^8*c^2*d*n^7 + 880*a^2*b^8*c^2*d*n^6 + 10*(901*a^2*b^8*c^2*d - 12
*a^5*b^5*c*d^2)*n^5 + (52699*a^2*b^8*c^2*d - 3360*a^5*b^5*c*d^2)*n^4 + 8*(2
```

```
1548*a^2*b^8*c^2*d - 4035*a^5*b^5*c*d^2)*n^3 + 60*(4651*a^2*b^8*c^2*d - 192
4*a^5*b^5*c*d^2 + 336*a^8*b^2*d^3)*n^2 + 1440*(105*a^2*b^8*c^2*d - 60*a^5*b
^5*c*d^2 + 14*a^8*b^2*d^3)*n)*x^2 + 360*(19444*a*b^9*c^3 - 6393*a^4*b^6*c^2
*d + 1452*a^7*b^3*c*d^2)*n + (b^10*c^3*n^9 + 54*b^10*c^3*n^8 + 3628800*b^10
*c^3 + 6*(211*b^10*c^3 + 3*a^3*b^7*c^2*d)*n^7 + 18*(938*b^10*c^3 + 45*a^3*b
^7*c^2*d)*n^6 + 3*(46963*b^10*c^3 + 5010*a^3*b^7*c^2*d)*n^5 + 18*(42287*b^1
0*c^3 + 8175*a^3*b^7*c^2*d - 120*a^6*b^4*c*d^2)*n^4 + 4*(663941*b^10*c^3 +
200358*a^3*b^7*c^2*d - 14580*a^6*b^4*c*d^2)*n^3 + 72*(79913*b^10*c^3 + 3196
5*a^3*b^7*c^2*d - 7260*a^6*b^4*c*d^2)*n^2 + 1440*(4861*b^10*c^3 + 1890*a^3*
b^7*c^2*d - 1080*a^6*b^4*c*d^2 + 252*a^9*b*d^3)*n)*x)*(b*x + a)^n/(b^10*n^1
0 + 55*b^10*n^9 + 1320*b^10*n^8 + 18150*b^10*n^7 + 157773*b^10*n^6 + 902055
*b^10*n^5 + 3416930*b^10*n^4 + 8409500*b^10*n^3 + 12753576*b^10*n^2 + 10628
640*b^10*n + 3628800*b^10)
```

giac [B] time = 0.67, size = 3874, normalized size = 11.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^10*d^3*n^9*x^10 + (b*x + a)^n*a*b^9*d^3*n^9*x^9 + 45*(b*x +
a)^n*b^10*d^3*n^8*x^10 + 36*(b*x + a)^n*a*b^9*d^3*n^8*x^9 + 870*(b*x + a)^n
*b^10*d^3*n^7*x^10 + 3*(b*x + a)^n*b^10*c*d^2*n^9*x^7 - 9*(b*x + a)^n*a^2*b
^8*d^3*n^8*x^8 + 546*(b*x + a)^n*a*b^9*d^3*n^7*x^9 + 9450*(b*x + a)^n*b^10*
d^3*n^6*x^10 + 3*(b*x + a)^n*a*b^9*c*d^2*n^9*x^6 + 144*(b*x + a)^n*b^10*c*d
^2*n^8*x^7 - 252*(b*x + a)^n*a^2*b^8*d^3*n^7*x^8 + 4536*(b*x + a)^n*a*b^9*d
^3*n^6*x^9 + 63273*(b*x + a)^n*b^10*d^3*n^5*x^10 + 126*(b*x + a)^n*a*b^9*c*
d^2*n^8*x^6 + 2952*(b*x + a)^n*b^10*c*d^2*n^7*x^7 + 72*(b*x + a)^n*a^3*b^7*
d^3*n^7*x^7 - 2898*(b*x + a)^n*a^2*b^8*d^3*n^6*x^8 + 22449*(b*x + a)^n*a*b^
9*d^3*n^5*x^9 + 269325*(b*x + a)^n*b^10*d^3*n^4*x^10 + 3*(b*x + a)^n*b^10*c
^2*d*n^9*x^4 - 18*(b*x + a)^n*a^2*b^8*c*d^2*n^8*x^5 + 2196*(b*x + a)^n*a*b^
9*c*d^2*n^7*x^6 + 33786*(b*x + a)^n*b^10*c*d^2*n^6*x^7 + 1512*(b*x + a)^n*a
^3*b^7*d^3*n^6*x^7 - 17640*(b*x + a)^n*a^2*b^8*d^3*n^5*x^8 + 67284*(b*x + a
)^n*a*b^9*d^3*n^4*x^9 + 723680*(b*x + a)^n*b^10*d^3*n^3*x^10 + 3*(b*x + a)^
n*a*b^9*c^2*d*n^9*x^3 + 153*(b*x + a)^n*b^10*c^2*d*n^8*x^4 - 666*(b*x + a)^
n*a^2*b^8*c*d^2*n^7*x^5 + 20610*(b*x + a)^n*a*b^9*c*d^2*n^6*x^6 - 504*(b*x
+ a)^n*a^4*b^6*d^3*n^6*x^6 + 236817*(b*x + a)^n*b^10*c*d^2*n^5*x^7 + 12600*
(b*x + a)^n*a^3*b^7*d^3*n^5*x^7 - 60921*(b*x + a)^n*a^2*b^8*d^3*n^4*x^8 + 1
18124*(b*x + a)^n*a*b^9*d^3*n^3*x^9 + 1172700*(b*x + a)^n*b^10*d^3*n^2*x^10
 + 144*(b*x + a)^n*a*b^9*c^2*d*n^8*x^3 + 3348*(b*x + a)^n*b^10*c^2*d*n^7*x^
4 + 90*(b*x + a)^n*a^3*b^7*c*d^2*n^7*x^4 - 9846*(b*x + a)^n*a^2*b^8*c*d^2*n
^6*x^5 + 113157*(b*x + a)^n*a*b^9*c*d^2*n^5*x^6 - 7560*(b*x + a)^n*a^4*b^6*
d^3*n^5*x^6 + 1048446*(b*x + a)^n*b^10*c*d^2*n^4*x^7 + 52920*(b*x + a)^n*a^
3*b^7*d^3*n^4*x^7 - 118188*(b*x + a)^n*a^2*b^8*d^3*n^3*x^8 + 109584*(b*x +
a)^n*a*b^9*d^3*n^2*x^9 + 1026576*(b*x + a)^n*b^10*d^3*n*x^10 + (b*x + a)^n*
b^10*c^3*n^9*x - 9*(b*x + a)^n*a^2*b^8*c^2*d*n^8*x^2 + 2916*(b*x + a)^n*a*b
^9*c^2*d*n^7*x^3 + 41058*(b*x + a)^n*b^10*c^2*d*n^6*x^4 + 2970*(b*x + a)^n*
a^3*b^7*c*d^2*n^6*x^4 - 74430*(b*x + a)^n*a^2*b^8*c*d^2*n^5*x^5 + 3024*(b*x
 + a)^n*a^5*b^5*d^3*n^5*x^5 + 369504*(b*x + a)^n*a*b^9*c*d^2*n^4*x^6 - 4284
0*(b*x + a)^n*a^4*b^6*d^3*n^4*x^6 + 2911668*(b*x + a)^n*b^10*c*d^2*n^3*x^7
+ 116928*(b*x + a)^n*a^3*b^7*d^3*n^3*x^7 - 117612*(b*x + a)^n*a^2*b^8*d^3*n
^2*x^8 + 40320*(b*x + a)^n*a*b^9*d^3*n*x^9 + 362880*(b*x + a)^n*b^10*d^3*x^
10 + (b*x + a)^n*a*b^9*c^3*n^9 + 54*(b*x + a)^n*b^10*c^3*n^8*x - 414*(b*x +
 a)^n*a^2*b^8*c^2*d*n^7*x^2 + 32310*(b*x + a)^n*a*b^9*c^2*d*n^6*x^3 - 360*(
b*x + a)^n*a^4*b^6*c*d^2*n^6*x^3 + 309087*(b*x + a)^n*b^10*c^2*d*n^5*x^4 +
37350*(b*x + a)^n*a^3*b^7*c*d^2*n^5*x^4 - 306792*(b*x + a)^n*a^2*b^8*c*d^2*
n^4*x^5 + 30240*(b*x + a)^n*a^5*b^5*d^3*n^4*x^5 + 694644*(b*x + a)^n*a*b^9*
c*d^2*n^3*x^6 - 113400*(b*x + a)^n*a^4*b^6*d^3*n^3*x^6 + 4846824*(b*x + a)^
n*b^10*c*d^2*n^2*x^7 + 127008*(b*x + a)^n*a^3*b^7*d^3*n^2*x^7 - 45360*(b*x
```

```
+ a)^n*a^2*b^8*d^3*n*x^8 + 54*(b*x + a)^n*a*b^9*c^3*n^8 + 1266*(b*x + a)^n*
b^10*c^3*n^7*x + 18*(b*x + a)^n*a^3*b^7*c^2*d*n^7*x - 7920*(b*x + a)^n*a^2*
b^8*c^2*d*n^6*x^2 + 212157*(b*x + a)^n*a*b^9*c^2*d*n^5*x^3 - 10800*(b*x + a
)^n*a^4*b^6*c*d^2*n^5*x^3 + 1469817*(b*x + a)^n*b^10*c^2*d*n^4*x^4 + 222750
*(b*x + a)^n*a^3*b^7*c*d^2*n^4*x^4 - 15120*(b*x + a)^n*a^6*b^4*d^3*n^4*x^4
- 683064*(b*x + a)^n*a^2*b^8*c*d^2*n^3*x^5 + 105840*(b*x + a)^n*a^5*b^5*d^3
*n^3*x^5 + 678960*(b*x + a)^n*a*b^9*c*d^2*n^2*x^6 - 138096*(b*x + a)^n*a^4*
b^6*d^3*n^2*x^6 + 4332960*(b*x + a)^n*b^10*c*d^2*n*x^7 + 51840*(b*x + a)^n*
a^3*b^7*d^3*n*x^7 + 1266*(b*x + a)^n*a*b^9*c^3*n^7 + 16884*(b*x + a)^n*b^10
*c^3*n^6*x + 810*(b*x + a)^n*a^3*b^7*c^2*d*n^6*x - 81090*(b*x + a)^n*a^2*b^
8*c^2*d*n^5*x^2 + 1080*(b*x + a)^n*a^5*b^5*c*d^2*n^5*x^2 + 833346*(b*x + a)
^n*a*b^9*c^2*d*n^4*x^3 - 117000*(b*x + a)^n*a^4*b^6*c*d^2*n^4*x^3 + 4371522
*(b*x + a)^n*b^10*c^2*d*n^3*x^4 + 642960*(b*x + a)^n*a^3*b^7*c*d^2*n^3*x^4
- 90720*(b*x + a)^n*a^6*b^4*d^3*n^3*x^4 - 752544*(b*x + a)^n*a^2*b^8*c*d^2*
n^2*x^5 + 151200*(b*x + a)^n*a^5*b^5*d^3*n^2*x^5 + 259200*(b*x + a)^n*a*b^9
*c*d^2*n*x^6 - 60480*(b*x + a)^n*a^4*b^6*d^3*n*x^6 + 1555200*(b*x + a)^n*b^
10*c*d^2*x^7 + 16884*(b*x + a)^n*a*b^9*c^3*n^6 - 18*(b*x + a)^n*a^4*b^6*c^2
*d*n^6 + 140889*(b*x + a)^n*b^10*c^3*n^5*x + 15030*(b*x + a)^n*a^3*b^7*c^2*
d*n^5*x - 474291*(b*x + a)^n*a^2*b^8*c^2*d*n^4*x^2 + 30240*(b*x + a)^n*a^5*
b^5*c*d^2*n^4*x^2 + 1871484*(b*x + a)^n*a*b^9*c^2*d*n^3*x^3 - 540000*(b*x +
 a)^n*a^4*b^6*c*d^2*n^3*x^3 + 60480*(b*x + a)^n*a^7*b^3*d^3*n^3*x^3 + 77424
12*(b*x + a)^n*b^10*c^2*d*n^2*x^4 + 843480*(b*x + a)^n*a^3*b^7*c*d^2*n^2*x^
4 - 166320*(b*x + a)^n*a^6*b^4*d^3*n^2*x^4 - 311040*(b*x + a)^n*a^2*b^8*c*d
^2*n*x^5 + 72576*(b*x + a)^n*a^5*b^5*d^3*n*x^5 + 140889*(b*x + a)^n*a*b^9*c
^3*n^5 - 810*(b*x + a)^n*a^4*b^6*c^2*d*n^5 + 761166*(b*x + a)^n*b^10*c^3*n^
4*x + 147150*(b*x + a)^n*a^3*b^7*c^2*d*n^4*x - 2160*(b*x + a)^n*a^6*b^4*c*d
^2*n^4*x - 1551456*(b*x + a)^n*a^2*b^8*c^2*d*n^3*x^2 + 290520*(b*x + a)^n*a
^5*b^5*c*d^2*n^3*x^2 + 2127960*(b*x + a)^n*a*b^9*c^2*d*n^2*x^3 - 951840*(b*
x + a)^n*a^4*b^6*c*d^2*n^2*x^3 + 181440*(b*x + a)^n*a^7*b^3*d^3*n^2*x^3 + 7
291080*(b*x + a)^n*b^10*c^2*d*n*x^4 + 388800*(b*x + a)^n*a^3*b^7*c*d^2*n*x^
4 - 90720*(b*x + a)^n*a^6*b^4*d^3*n*x^4 + 761166*(b*x + a)^n*a*b^9*c^3*n^4
- 15030*(b*x + a)^n*a^4*b^6*c^2*d*n^4 + 2655764*(b*x + a)^n*b^10*c^3*n^3*x
+ 801432*(b*x + a)^n*a^3*b^7*c^2*d*n^3*x - 58320*(b*x + a)^n*a^6*b^4*c*d^2*
n^3*x - 2511540*(b*x + a)^n*a^2*b^8*c^2*d*n^2*x^2 + 1038960*(b*x + a)^n*a^5
*b^5*c*d^2*n^2*x^2 - 181440*(b*x + a)^n*a^8*b^2*d^3*n^2*x^2 + 907200*(b*x +
 a)^n*a*b^9*c^2*d*n*x^3 - 518400*(b*x + a)^n*a^4*b^6*c*d^2*n*x^3 + 120960*(
b*x + a)^n*a^7*b^3*d^3*n*x^3 + 2721600*(b*x + a)^n*b^10*c^2*d*x^4 + 2655764
*(b*x + a)^n*a*b^9*c^3*n^3 - 147150*(b*x + a)^n*a^4*b^6*c^2*d*n^3 + 2160*(b
*x + a)^n*a^7*b^3*c*d^2*n^3 + 5753736*(b*x + a)^n*b^10*c^3*n^2*x + 2301480*
(b*x + a)^n*a^3*b^7*c^2*d*n^2*x - 522720*(b*x + a)^n*a^6*b^4*c*d^2*n^2*x -
1360800*(b*x + a)^n*a^2*b^8*c^2*d*n*x^2 + 777600*(b*x + a)^n*a^5*b^5*c*d^2*
n*x^2 - 181440*(b*x + a)^n*a^8*b^2*d^3*n*x^2 + 5753736*(b*x + a)^n*a*b^9*c^
3*n^2 - 801432*(b*x + a)^n*a^4*b^6*c^2*d*n^2 + 58320*(b*x + a)^n*a^7*b^3*c*
d^2*n^2 + 6999840*(b*x + a)^n*b^10*c^3*n*x + 2721600*(b*x + a)^n*a^3*b^7*c^
2*d*n*x - 1555200*(b*x + a)^n*a^6*b^4*c*d^2*n*x + 362880*(b*x + a)^n*a^9*b*
d^3*n*x + 6999840*(b*x + a)^n*a*b^9*c^3*n - 2301480*(b*x + a)^n*a^4*b^6*c^2
*d*n + 522720*(b*x + a)^n*a^7*b^3*c*d^2*n + 3628800*(b*x + a)^n*b^10*c^3*x
+ 3628800*(b*x + a)^n*a*b^9*c^3 - 2721600*(b*x + a)^n*a^4*b^6*c^2*d + 15552
00*(b*x + a)^n*a^7*b^3*c*d^2 - 362880*(b*x + a)^n*a^10*d^3)/(b^10*n^10 + 55
*b^10*n^9 + 1320*b^10*n^8 + 18150*b^10*n^7 + 157773*b^10*n^6 + 902055*b^10*
n^5 + 3416930*b^10*n^4 + 8409500*b^10*n^3 + 12753576*b^10*n^2 + 10628640*b^
10*n + 3628800*b^10)
```

maple [B] time = 0.03, size = 2280, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^9*d^3*n^9*x^9-45*b^9*d^3*n^8*x^9+9*a*b^8*d^3*n^8*x^8-870
*b^9*d^3*n^7*x^9+324*a*b^8*d^3*n^7*x^8-3*b^9*c*d^2*n^9*x^6-9450*b^9*d^3*n^6
*x^9-72*a^2*b^7*d^3*n^7*x^7+4914*a*b^8*d^3*n^6*x^8-144*b^9*c*d^2*n^8*x^6-63
273*b^9*d^3*n^5*x^9-2016*a^2*b^7*d^3*n^6*x^7+18*a*b^8*c*d^2*n^8*x^5+40824*a
*b^8*d^3*n^5*x^8-2952*b^9*c*d^2*n^7*x^6-269325*b^9*d^3*n^4*x^9+504*a^3*b^6*
d^3*n^6*x^6-23184*a^2*b^7*d^3*n^5*x^7+756*a*b^8*c*d^2*n^7*x^5+202041*a*b^8*
d^3*n^4*x^8-3*b^9*c^2*d*n^9*x^3-33786*b^9*c*d^2*n^6*x^6-723680*b^9*d^3*n^3*
x^9+10584*a^3*b^6*d^3*n^5*x^6-90*a^2*b^7*c*d^2*n^7*x^4-141120*a^2*b^7*d^3*n
^4*x^7+13176*a*b^8*c*d^2*n^6*x^5+605556*a*b^8*d^3*n^3*x^8-153*b^9*c^2*d*n^8
*x^3-236817*b^9*c*d^2*n^5*x^6-1172700*b^9*d^3*n^2*x^9-3024*a^4*b^5*d^3*n^5*
x^5+88200*a^3*b^6*d^3*n^4*x^6-3330*a^2*b^7*c*d^2*n^6*x^4-487368*a^2*b^7*d^3
*n^3*x^7+9*a*b^8*c^2*d*n^8*x^2+123660*a*b^8*c*d^2*n^5*x^5+1063116*a*b^8*d^3
*n^2*x^8-3348*b^9*c^2*d*n^7*x^3-1048446*b^9*c*d^2*n^4*x^6-1026576*b^9*d^3*n
*x^9-45360*a^4*b^5*d^3*n^4*x^5+360*a^3*b^6*c*d^2*n^6*x^3+370440*a^3*b^6*d^3
*n^3*x^6-49230*a^2*b^7*c*d^2*n^5*x^4-945504*a^2*b^7*d^3*n^2*x^7+432*a*b^8*c
^2*d*n^7*x^2+678942*a*b^8*c*d^2*n^4*x^5+986256*a*b^8*d^3*n*x^8-b^9*c^3*n^9-
41058*b^9*c^2*d*n^6*x^3-2911668*b^9*c*d^2*n^3*x^6-362880*b^9*d^3*x^9+15120*
a^5*b^4*d^3*n^4*x^4-257040*a^4*b^5*d^3*n^3*x^5+11880*a^3*b^6*c*d^2*n^5*x^3+
818496*a^3*b^6*d^3*n^2*x^6-18*a^2*b^7*c^2*d*n^7*x-372150*a^2*b^7*c*d^2*n^4*
x^4-940896*a^2*b^7*d^3*n*x^7+8748*a*b^8*c^2*d*n^6*x^2+2217024*a*b^8*c*d^2*n
^3*x^5+362880*a*b^8*d^3*x^8-54*b^9*c^3*n^8-309087*b^9*c^2*d*n^5*x^3-4846824
*b^9*c*d^2*n^2*x^6+151200*a^5*b^4*d^3*n^3*x^4-1080*a^4*b^5*c*d^2*n^5*x^2-68
0400*a^4*b^5*d^3*n^2*x^5+149400*a^3*b^6*c*d^2*n^4*x^3+889056*a^3*b^6*d^3*n*
x^6-828*a^2*b^7*c^2*d*n^6*x-1533960*a^2*b^7*c*d^2*n^3*x^4-362880*a^2*b^7*d^
3*x^7+96930*a*b^8*c^2*d*n^5*x^2+4167864*a*b^8*c*d^2*n^2*x^5-1266*b^9*c^3*n^
7-1469817*b^9*c^2*d*n^4*x^3-4332960*b^9*c*d^2*n*x^6-60480*a^6*b^3*d^3*n^3*x
^3+529200*a^5*b^4*d^3*n^2*x^4-32400*a^4*b^5*c*d^2*n^4*x^2-828576*a^4*b^5*d^
3*n*x^5+18*a^3*b^6*c^2*d*n^6+891000*a^3*b^6*c*d^2*n^3*x^3+362880*a^3*b^6*d^
3*x^6-15840*a^2*b^7*c^2*d*n^5*x-3415320*a^2*b^7*c*d^2*n^2*x^4+636471*a*b^8*
c^2*d*n^4*x^2+4073760*a*b^8*c*d^2*n*x^5-16884*b^9*c^3*n^6-4371522*b^9*c^2*d
*n^3*x^3-1555200*b^9*c*d^2*x^6-362880*a^6*b^3*d^3*n^2*x^3+2160*a^5*b^4*c*d^
2*n^4*x+756000*a^5*b^4*d^3*n*x^4-351000*a^4*b^5*c*d^2*n^3*x^2-362880*a^4*b^
5*d^3*x^5+810*a^3*b^6*c^2*d*n^5+2571840*a^3*b^6*c*d^2*n^2*x^3-162180*a^2*b^
7*c^2*d*n^4*x-3762720*a^2*b^7*c*d^2*n*x^4+2500038*a*b^8*c^2*d*n^3*x^2+15552
00*a*b^8*c*d^2*x^5-140889*b^9*c^3*n^5-7742412*b^9*c^2*d*n^2*x^3+181440*a^7*
b^2*d^3*n^2*x^2-665280*a^6*b^3*d^3*n*x^3+60480*a^5*b^4*c*d^2*n^3*x+362880*a
^5*b^4*d^3*x^4-1620000*a^4*b^5*c*d^2*n^2*x^2+15030*a^3*b^6*c^2*d*n^4+337392
0*a^3*b^6*c*d^2*n*x^3-948582*a^2*b^7*c^2*d*n^3*x-1555200*a^2*b^7*c*d^2*x^4+
5614452*a*b^8*c^2*d*n^2*x^2-761166*b^9*c^3*n^4-7291080*b^9*c^2*d*n*x^3+5443
20*a^7*b^2*d^3*n*x^2-2160*a^6*b^3*c*d^2*n^3-362880*a^6*b^3*d^3*x^3+581040*a
^5*b^4*c*d^2*n^2*x-2855520*a^4*b^5*c*d^2*n*x^2+147150*a^3*b^6*c^2*d*n^3+155
5200*a^3*b^6*c*d^2*x^3-3102912*a^2*b^7*c^2*d*n^2*x+6383880*a*b^8*c^2*d*n*x^
2-2655764*b^9*c^3*n^3-2721600*b^9*c^2*d*x^3-362880*a^8*b*d^3*n*x+362880*a^7
*b^2*d^3*x^2-58320*a^6*b^3*c*d^2*n^2+2077920*a^5*b^4*c*d^2*n*x-1555200*a^4*
b^5*c*d^2*x^2+801432*a^3*b^6*c^2*d*n^2-5023080*a^2*b^7*c^2*d*n*x+2721600*a*
b^8*c^2*d*x^2-5753736*b^9*c^3*n^2-362880*a^8*b*d^3*x-522720*a^6*b^3*c*d^2*n
+1555200*a^5*b^4*c*d^2*x+2301480*a^3*b^6*c^2*d*n-2721600*a^2*b^7*c^2*d*x-69
99840*b^9*c^3*n+362880*a^9*d^3-1555200*a^6*b^3*c*d^2+2721600*a^3*b^6*c^2*d-
3628800*b^9*c^3)/b^10/(n^10+55*n^9+1320*n^8+18150*n^7+157773*n^6+902055*n^5
+3416930*n^4+8409500*n^3+12753576*n^2+10628640*n+3628800)
```

maxima [B] time = 0.78, size = 770, normalized size = 2.28

$$\frac{(bx+a)^{n+1}c^3}{b(n+1)}+\frac{3\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(b}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (
n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^
4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + 3*((n^6 +
21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n
^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x
^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*
a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4
+ 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^9 + 45*n^8 + 870*n^7 +
9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n +
362880)*b^10*x^10 + (n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67284*
n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a*b^9*x^9 - 9*(n^8 + 28*n^7 + 322*
n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^2*b^8*x^8 + 7
2*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^3*b^7*
x^7 - 504*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^4*b^6*x^6 +
 3024*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^5*b^5*x^5 - 15120*(n^4 + 6*
n^3 + 11*n^2 + 6*n)*a^6*b^4*x^4 + 60480*(n^3 + 3*n^2 + 2*n)*a^7*b^3*x^3 - 1
81440*(n^2 + n)*a^8*b^2*x^2 + 362880*a^9*b*n*x - 362880*a^10)*(b*x + a)^n*d
^3/((n^10 + 55*n^9 + 1320*n^8 + 18150*n^7 + 157773*n^6 + 902055*n^5 + 34169
30*n^4 + 8409500*n^3 + 12753576*n^2 + 10628640*n + 3628800)*b^10)
```

mupad [B] time = 4.34, size = 2001, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*(3628800*a*b^9*c^3 - 362880*a^10*d^3 - 2721600*a^4*b^6*c^2*d +
 1555200*a^7*b^3*c*d^2 + 5753736*a*b^9*c^3*n^2 + 2655764*a*b^9*c^3*n^3 + 76
1166*a*b^9*c^3*n^4 + 140889*a*b^9*c^3*n^5 + 16884*a*b^9*c^3*n^6 + 1266*a*b^
9*c^3*n^7 + 54*a*b^9*c^3*n^8 + a*b^9*c^3*n^9 + 6999840*a*b^9*c^3*n - 230148
0*a^4*b^6*c^2*d*n + 522720*a^7*b^3*c*d^2*n - 801432*a^4*b^6*c^2*d*n^2 + 583
20*a^7*b^3*c*d^2*n^2 - 147150*a^4*b^6*c^2*d*n^3 + 2160*a^7*b^3*c*d^2*n^3 -
15030*a^4*b^6*c^2*d*n^4 - 810*a^4*b^6*c^2*d*n^5 - 18*a^4*b^6*c^2*d*n^6))/(b
^10*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 1
57773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (x*(a + b*x)
^n*(3628800*b^10*c^3 + 6999840*b^10*c^3*n + 5753736*b^10*c^3*n^2 + 2655764*
b^10*c^3*n^3 + 761166*b^10*c^3*n^4 + 140889*b^10*c^3*n^5 + 16884*b^10*c^3*n
^6 + 1266*b^10*c^3*n^7 + 54*b^10*c^3*n^8 + b^10*c^3*n^9 + 362880*a^9*b*d^3*
n + 2721600*a^3*b^7*c^2*d*n - 1555200*a^6*b^4*c*d^2*n + 2301480*a^3*b^7*c^2
*d*n^2 - 522720*a^6*b^4*c*d^2*n^2 + 801432*a^3*b^7*c^2*d*n^3 - 58320*a^6*b^
4*c*d^2*n^3 + 147150*a^3*b^7*c^2*d*n^4 - 2160*a^6*b^4*c*d^2*n^4 + 15030*a^3
*b^7*c^2*d*n^5 + 810*a^3*b^7*c^2*d*n^6 + 18*a^3*b^7*c^2*d*n^7))/(b^10*(1062
8640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6
 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (d^3*x^10*(a + b*x)^n
*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6
+ 870*n^7 + 45*n^8 + n^9 + 362880))/(10628640*n + 12753576*n^2 + 8409500*n^
3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 +
 n^10 + 3628800) + (3*d^2*x^7*(a + b*x)^n*(720*b^3*c + 27*b^3*c*n^2 + b^3*c
*n^3 + 24*a^3*d*n + 242*b^3*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 2
1*n^5 + n^6 + 720))/(b^3*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930
*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362
8800)) + (3*d*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(151200*b^6*c^2 - 50
40*a^6*d^2*n + 127860*b^6*c^2*n + 44524*b^6*c^2*n^2 + 8175*b^6*c^2*n^3 + 83
5*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^2*n^6 + 21600*a^3*b^3*c*d*n + 7260*a
^3*b^3*c*d*n^2 + 810*a^3*b^3*c*d*n^3 + 30*a^3*b^3*c*d*n^4))/(b^6*(10628640*
n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18
150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (a*d^3*n*x^9*(a + b*x)^n*(
109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7
```

```
 + n^8 + 40320))/(b*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4
+ 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)
) - (9*a^2*d^3*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4
 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b^2*(10628640*n + 12753576*n^2 + 840950
0*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n
^9 + n^10 + 3628800)) + (3*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(20160*a^6
*d^2 + 151200*b^6*c^2 + 127860*b^6*c^2*n + 44524*b^6*c^2*n^2 + 8175*b^6*c^2
*n^3 + 835*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^2*n^6 - 86400*a^3*b^3*c*d -
 29040*a^3*b^3*c*d*n - 3240*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^7*(1
0628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*
n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) - (9*a^2*d*n*x^2*(n
+ 1)*(a + b*x)^n*(20160*a^6*d^2 + 151200*b^6*c^2 + 127860*b^6*c^2*n + 44524
*b^6*c^2*n^2 + 8175*b^6*c^2*n^3 + 835*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^
2*n^6 - 86400*a^3*b^3*c*d - 29040*a^3*b^3*c*d*n - 3240*a^3*b^3*c*d*n^2 - 12
0*a^3*b^3*c*d*n^3))/(b^8*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930
*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362
8800)) + (3*a*d^2*n*x^6*(a + b*x)^n*(720*b^3*c - 168*a^3*d + 27*b^3*c*n^2 +
 b^3*c*n^3 + 242*b^3*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/
(b^4*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 +
157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) - (18*a^2*d^2
*n*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(720*b^3*c - 168*a^3
*d + 27*b^3*c*n^2 + b^3*c*n^3 + 242*b^3*c*n))/(b^5*(10628640*n + 12753576*n
^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320
*n^8 + 55*n^9 + n^10 + 3628800))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.185 $\int \frac{(a+bx)^n\left(c+dx^3\right)^3}{x}\,dx$

Optimal. Leaf size=358

$$-\frac{5ad^2\left(3b^3c-14a^3d\right)(a+bx)^{n+5}}{b^9(n+5)}+\frac{d^2\left(3b^3c-56a^3d\right)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^3(a+bx)^{n+7}}{b^9(n+7)}-\frac{ad\left(8a^6d^2-15a^3b^3cd+6\right.}{b^9(n+2)}$$

```
[Out] a^2*d*(a^6*d^2-3*a^3*b^3*c*d+3*b^6*c^2)*(b*x+a)^(1+n)/b^9/(1+n)-a*d*(8*a^6*
d^2-15*a^3*b^3*c*d+6*b^6*c^2)*(b*x+a)^(2+n)/b^9/(2+n)+d*(28*a^6*d^2-30*a^3*
b^3*c*d+3*b^6*c^2)*(b*x+a)^(3+n)/b^9/(3+n)+2*a^2*d^2*(-28*a^3*d+15*b^3*c)*(
b*x+a)^(4+n)/b^9/(4+n)-5*a*d^2*(-14*a^3*d+3*b^3*c)*(b*x+a)^(5+n)/b^9/(5+n)+
d^2*(-56*a^3*d+3*b^3*c)*(b*x+a)^(6+n)/b^9/(6+n)+28*a^2*d^3*(b*x+a)^(7+n)/b^
9/(7+n)-8*a*d^3*(b*x+a)^(8+n)/b^9/(8+n)+d^3*(b*x+a)^(9+n)/b^9/(9+n)-c^3*(b*
x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] time = 0.22, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1620, 65}

$$\frac{a^2d\left(-3a^3b^3cd+a^6d^2+3b^6c^2\right)(a+bx)^{n+1}}{b^9(n+1)}-\frac{ad\left(-15a^3b^3cd+8a^6d^2+6b^6c^2\right)(a+bx)^{n+2}}{b^9(n+2)}+\frac{d\left(-30a^3b^3cd+28a^6d\right.}{b^9(n}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

```
[Out] (a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2)*(a + b*x)^(1 + n))/(b^9*(1 + n
)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x)^(2 + n))/(b^9*
(2 + n)) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/
(b^9*(3 + n)) + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4
 + n)) - (5*a*d^2*(3*b^3*c - 14*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5 + n)) + (
d^2*(3*b^3*c - 56*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n)) + (28*a^2*d^3*(a
+ b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*(a + b*x)^(8 + n))/(b^9*(8 + n)) +
 (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n)) - (c^3*(a + b*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(a+bx)^n \left(c+dx^3\right)^3}{x}\,dx = \int \left(\frac{a^2d\left(3b^6c^2-3a^3b^3cd+a^6d^2\right)(a+bx)^n}{b^8} + \frac{c^3(a+bx)^n}{x} - \frac{ad\left(6b^6c^2-15a^3b^3c\right.}{}\right.$$
$$= \frac{a^2d\left(3b^6c^2-3a^3b^3cd+a^6d^2\right)(a+bx)^{1+n}}{b^9(1+n)} - \frac{ad\left(6b^6c^2-15a^3b^3cd+8a^6d^2\right)(a+b}{b^9(2+n)}$$
$$= \frac{a^2d\left(3b^6c^2-3a^3b^3cd+a^6d^2\right)(a+bx)^{1+n}}{b^9(1+n)} - \frac{ad\left(6b^6c^2-15a^3b^3cd+8a^6d^2\right)(a+b}{b^9(2+n)}$$

Mathematica [A] time = 0.35, size = 332, normalized size = 0.93

$$(a+bx)^{n+1}\left(\frac{d^2(a+bx)^5\left(3b^3c-56a^3d\right)}{b^9(n+6)} + \frac{5ad^2(a+bx)^4\left(14a^3d-3b^3c\right)}{b^9(n+5)} + \frac{28a^2d^3(a+bx)^6}{b^9(n+7)} + \frac{d(a+bx)^2\left(28a^6\right.}{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*((a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2))/(b^9*(1 +
n)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x))/(b^9*(2 + n)
) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^2)/(b^9*(3 + n))
 + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^3)/(b^9*(4 + n)) + (5*a*d^2*(
-3*b^3*c + 14*a^3*d)*(a + b*x)^4)/(b^9*(5 + n)) + (d^2*(3*b^3*c - 56*a^3*d)
*(a + b*x)^5)/(b^9*(6 + n)) + (28*a^2*d^3*(a + b*x)^6)/(b^9*(7 + n)) - (8*a
*d^3*(a + b*x)^7)/(b^9*(8 + n)) + (d^3*(a + b*x)^8)/(b^9*(9 + n)) - (c^3*Hy
pergeometric2F1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^3x^9+3\,cd^2x^6+3\,c^2dx^3+c^3\right)(bx+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x + a)^n/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^3(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3+c\right)^3(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^3/x,x)
[Out] int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^3(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^3+c\right)^3\,{\left(a+b\,x\right)}^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^3)^3*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^3)^3*(a + b*x)^n)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**3/x,x)
[Out] Timed out
```

3.186 $\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=324

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1}{3b^{5/3}(n}$$

```
[Out] e^2*(f*x+e)^(1+n)/b/f^3/(1+n)-2*e*(f*x+e)^(2+n)/b/f^3/(2+n)+(f*x+e)^(3+n)/b
/f^3/(3+n)+1/3*a*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^
(1/3)*e-a^(1/3)*f))/b^(5/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*a*(f*x+e)^(1+n)
*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))
/b^(5/3)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)+1/3*a*(f*x+e)^(1+n)*hyperge
om([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/b^(5/3)
/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f)/(1+n)
```

Rubi [A] time = 0.86, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6725, 68}

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1}{3b^{5/3}(n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (e^2*(e + f*x)^(1 + n))/(b*f^3*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*f^3*(2
 + n)) + (e + f*x)^(3 + n)/(b*f^3*(3 + n)) + (a*(e + f*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*
b^(5/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a*(e + f*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)
*f)])/(3*b^(5/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (a*(e + f*x)
^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e
- (-1)^(2/3)*a^(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)*(1
+ n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \int \left(\frac{e^2(e+fx)^n}{bf^2} - \frac{2e(e+fx)^{1+n}}{bf^2} + \frac{(e+fx)^{2+n}}{bf^2} - \frac{ax^2(e+fx)^n}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx}{b}$$

$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \cdots\right.}{b}$$

$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{5/3}} - \frac{a\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{5/3}} - \frac{a\int \cdots}{\cdots}$$

$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} + \frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \cdots$$

Mathematica [A] time = 0.61, size = 284, normalized size = 0.88

$$\frac{(e+fx)^{n+1}\left(\frac{a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)} + \frac{a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)} + \frac{a\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)} + \frac{3b^{2/3}e^2}{f^3(n+1)} - \frac{6b^{2/3}e(e+f\cdots}{f^3(n+2)\cdots}\right.}{3b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*((3*b^(2/3)*e^2)/(f^3*(1 + n)) - (6*b^(2/3)*e*(e + f*x))
/(f^3*(2 + n)) + (3*b^(2/3)*(e + f*x)^2)/(f^3*(3 + n)) + (a*Hypergeometric2
F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1/3)
*e - a^(1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(
e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/((b^(1/3)*e + (-1)^(1/3)*a^(
1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x)
)/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/((b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)*
(1 + n))))/(3*b^(5/3))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n x^5}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^5/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x^5}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(f x + e\right)^{n}}{b \,x^{3} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x+e)^n/(b*x^3+a),x)
[Out] int(x^5*(f*x+e)^n/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(f x + e\right)}^{n} x^{5}}{b x^{3} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(e+f\,x\right)}^n}{b\,x^3+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(e + f*x)^n)/(a + b*x^3),x)
[Out] int((x^5*(e + f*x)^n)/(a + b*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

3.187 $\int \frac{x^4(e+fx)^n}{a+bx^3} \, dx$

Optimal. Leaf size=332

$$-\frac{a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}\,a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}\cdots}{\cdots}$$

```
[Out] -e*(f*x+e)^(1+n)/b/f^2/(1+n)+(f*x+e)^(2+n)/b/f^2/(2+n)-1/3*a^(2/3)*(f*x+e)^
(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/3)*f))/b^(4/
3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(1/3)*a^(2/3)*(f*x+e)^(1+n)*hyperge
om([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*
f))/b^(4/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(2/3)*a^(2/3)*(
f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/((-1)^(1/3
)*b^(1/3)*e+a^(1/3)*f))/b^(4/3)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+n)
```

Rubi [A] time = 0.86, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6725, 68}

$$-\frac{a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}\,a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] -((e*(e + f*x)^(1 + n))/(b*f^2*(1 + n))) + (e + f*x)^(2 + n)/(b*f^2*(2 + n)
) - (a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*
(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1
+ n)) + ((-1)^(1/3)*a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(
3*b^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*a^(2/3)
*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(
e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b^(4/3)*((-1)^(1/3)*b^(1/
3)*e + a^(1/3)*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{e(e+fx)^n}{bf} + \frac{(e+fx)^{1+n}}{bf} - \frac{ax(e+fx)^n}{b\left(a+bx^3\right)}\right) dx$$
$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} - \frac{a\int \frac{x(e+fx)^n}{a+bx^3}\,dx}{b}$$
$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} - \frac{a\int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{}\right.}\right.}{b}$$
$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} + \frac{a^{2/3}\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}\,a^{2/3}\right)\int \frac{(e+fx)^n}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b^{4/3}} + \frac{((-}{}$$
$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} - \frac{a^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{-1}\,a^{2/3}}{}$$

Mathematica [A] time = 0.68, size = 292, normalized size = 0.88

$$\frac{(e+fx)^{n+1}\left(-\frac{a^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)} + \frac{\sqrt[3]{-1}\,a^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)} + \frac{(-1)^{2/3}a^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}\right.}{3b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*((-3*b^(1/3)*e)/(f^2*(1 + n)) + (3*b^(1/3)*(e + f*x))/(f
^2*(2 + n)) - (a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x
))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(1/3
)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))
/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(
1 + n)) + ((-1)^(2/3)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3
)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(((-1)^(1/3)*b^(1
/3)*e + a^(1/3)*f)*(1 + n))))/(3*b^(4/3))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n x^4}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^4/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x^4}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(f x + e\right)^n}{b x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x+e)^n/(b*x^3+a),x)
[Out] int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f x + e\right)^n x^4}{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(e + f x\right)^n}{b x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(e + f*x)^n)/(a + b*x^3),x)
[Out] int((x^4*(e + f*x)^n)/(a + b*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

3.188 $\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=293

$$\frac{\sqrt[3]{a}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b(n+1)\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}+\frac{\sqrt[3]{a}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e+fx)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b(n+1)\left((-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}-\frac{\sqrt[3]{a}\,(e+f}{}$$

```
[Out] (f*x+e)^(1+n)/b/f/(1+n)+1/3*a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],
b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/3)*f))/b/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*a^
(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-
1)^(2/3)*b^(1/3)*e-a^(1/3)*f))/b/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3
*a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/
((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/b/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+n)
```

Rubi [A] time = 0.48, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6725, 68}

$$\frac{\sqrt[3]{a}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b(n+1)\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}+\frac{\sqrt[3]{a}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e+fx)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b(n+1)\left((-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}-\frac{\sqrt[3]{a}\,(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (e + f*x)^(1 + n)/(b*f*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b*(b^
(1/3)*e - a^(1/3)*f)*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a
^(1/3)*f)])/(3*b*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (a^(1/3)*(e
+ f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e +
f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b*((-1)^(1/3)*b^(1/3)*e + a^(
1/3)*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \left(\frac{(e+fx)^n}{b} - \frac{a(e+fx)^n}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{(e+fx)^{1+n}}{bf(1+n)} - \frac{a\int \frac{(e+fx)^n}{a+bx^3}\,dx}{b}$$

$$= \frac{(e+fx)^{1+n}}{bf(1+n)} - \frac{a\int \left(-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{b}$$

$$= \frac{(e+fx)^{1+n}}{bf(1+n)} + \frac{\sqrt[3]{a}\int \frac{(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{(e+fx)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b}$$

$$= \frac{(e+fx)^{1+n}}{bf(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;\right.}{3b\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{\right.}$$

Mathematica [A] time = 0.24, size = 239, normalized size = 0.82

$$\frac{(e+fx)^{n+1}\left(\frac{\sqrt[3]{a}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} + \frac{\sqrt[3]{a}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{\sqrt[3]{a}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e} + \frac{3}{f}\right)}{3b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(3/f + (a^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1
/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (a^(1/3)
*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2
/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - (a^(1/3)*
Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/
3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*b*(1 +
n))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^n x^3}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^3/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x^3}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(f x +e \right)^{n}}{b \,x^{3}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x+e)^n/(b*x^3+a),x)
[Out] int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(fx + e\right)}^{n} x^{3}}{bx^{3} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(e+f\,x\right)}^n}{b\,x^3+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(e + f*x)^n)/(a + b*x^3),x)
[Out] int((x^3*(e + f*x)^n)/(a + b*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

3.189 $\int \frac{x^2(e+fx)^n}{a+bx^3} \, dx$

Optimal. Leaf size=253

$$-\frac{(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}-\frac{(e+fx)^{n+1} {}_2F_1\left(1,n\right.}{3b^{2/3}(n+1)\left(\right.}$$

```
[Out] -1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1
/3)*f))/b^(2/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([1,
 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))/b^(2/3)/(b^(1
/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n
],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/b^(2/3)/(b^(1/3)*e-(-1)
^(2/3)*a^(1/3)*f)/(1+n)
```

Rubi [A] time = 0.28, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6725, 68}

$$-\frac{(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}-\frac{(e+fx)^{n+1} {}_2F_1\left(1,n\right.}{3b^{2/3}(n+1)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] -((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(
1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)
*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/
3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e -
(-1)^(2/3)*a^(1/3)*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)} \right) dx$$

$$= \frac{\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{(e+fx)^n}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}\ldots}\right.}{3b^{2/3}\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)}$$

Mathematica [A] time = 0.16, size = 213, normalized size = 0.84

$$\frac{(e+fx)^{n+1}\left(-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e} - \frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x)
)/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1,
1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)]/(b^(1
/3)*e + (-1)^(1/3)*a^(1/3)*f) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)
*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)]/(b^(1/3)*e - (-1)^(2/3)*a^(
1/3)*f)))/(3*b^(2/3)*(1 + n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n x^2}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^2/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x^2}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(fx+e\right)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(e + f*x)^n)/(a + b*x^3),x)
```

```
[Out] int((x^2*(e + f*x)^n)/(a + b*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.190 $\int \frac{x(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=288

$$\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{-1}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(-1)^{2/3}(e+f}{3\sqrt[3]{}}$$

```
[Out] 1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/
3)*f))/a^(1/3)/b^(1/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(-1)^(1/3)*(f*x+e)^(
1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3
)*e-a^(1/3)*f))/a^(1/3)/b^(1/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*
(-1)^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e
)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a^(1/3)/b^(1/3)/((-1)^(1/3)*b^(1/3)*e+a
^(1/3)*f)/(1+n)
```

Rubi [A] time = 0.28, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6725, 68}

$$\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{-1}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(-1)^{2/3}(e+f}{3\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(
b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)
) - ((-1)^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^
(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^
(1/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((-1)^(2/3)*(e + f*x)^(
1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((
-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*((-1)^(1/3)*b^(1/3)*e
 + a^(1/3)*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)} \right) dx$$

$$= -\frac{\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{(e+fx)^n}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{(e+fx)^n}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$= \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{\sqrt[3]{-1}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}$$

Mathematica [A] time = 0.19, size = 237, normalized size = 0.82

$$\frac{(e+fx)^{n+1}\left(\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{\sqrt[3]{-1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{(-1)^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f) - ((-1)^(1/3)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e
 - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - ((-1)^(2/3)*Hypergeome
tric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)
*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(1/3)*b^(1/3)*(
1 + n))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n x}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x\left(fx+e\right)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x+e)^n/(b*x^3+a),x)
[Out] int(x*(f*x+e)^n/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{n} x}{bx^{3}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(e+f\,x\right)}^{n}}{b\,x^{3}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(e + f*x)^n)/(a + b*x^3),x)
[Out] int((x*(e + f*x)^n)/(a + b*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

3.191 $\int \frac{(e+fx)^n}{a+bx^3} dx$

Optimal. Leaf size=263

$$-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,\right.}{3a^{2/3}(n+1)}$$

```
[Out] -1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1
/3)*f))/a^(2/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([1,
 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f))/a^
(2/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(f*x+e)^(1+n)*hypergeom([1
, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a
^(2/3)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+n)
```

Rubi [A] time = 0.16, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6725, 68}

$$-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,\right.}{3a^{2/3}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(a + b*x^3),x]
```

```
[Out] -((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e +
 f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*((-1)^(2/3)*b^(1/3)*
e - a^(1/3)*f)*(1 + n)) + ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3
*a^(2/3)*((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$
$$= -\frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$
$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}$$

Mathematica [A] time = 0.12, size = 222, normalized size = 0.84

$$\frac{(e+fx)^{n+1}\left(-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f} + \frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{3a^{2/3}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x)
)/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1,
1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3
)*f)]/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) + Hypergeometric2F1[1, 1 + n, 2 +
n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)]/((-1)
^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(2/3)*(1 + n))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/(b*x^3 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(a + b*x^3),x)
```

```
[Out] int((e + f*x)^n/(a + b*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.192 $\int \frac{(e+fx)^n}{x\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=300

$$\frac{\sqrt[3]{b}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3a(n+1)\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}+\frac{\sqrt[3]{b}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f}\right)}{3a(n+1)\left(\sqrt[3]{-1}\,\sqrt[3]{a}\,f+\sqrt[3]{b}\,e\right)}+\frac{\sqrt[3]{b}\,(e+fx}{3}$$

```
[Out] 1/3*b^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)
*e-a^(1/3)*f))/a/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*b^(1/3)*(f*x+e)^(1+n)*hype
rgeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))/a/(b
^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)+1/3*b^(1/3)*(f*x+e)^(1+n)*hypergeom([1
, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/a/(b^(1/3)*e
-(-1)^(2/3)*a^(1/3)*f)/(1+n)-(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e
)/a/e/(1+n)
```

Rubi [A] time = 0.56, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6725, 65, 68}

$$\frac{\sqrt[3]{b}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3a(n+1)\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}+\frac{\sqrt[3]{b}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f}\right)}{3a(n+1)\left(\sqrt[3]{-1}\,\sqrt[3]{a}\,f+\sqrt[3]{b}\,e\right)}+\frac{\sqrt[3]{b}\,(e+fx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x*(a + b*x^3)),x]
```

```
[Out] (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e +
 f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (b
^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f
*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*a*(b^(1/3)*e + (-1)^(1/3)*a^(1
/3)*f)*(1 + n)) + (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*a*(b^(1/3)
*e - (-1)^(2/3)*a^(1/3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)^n}{x\left(a+bx^3\right)}\,dx = \int \left(\frac{(e+fx)^n}{ax} - \frac{bx^2(e+fx)^n}{a\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int \frac{(e+fx)^n}{x}\,dx}{a} - \frac{b\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx}{a}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)} - \frac{b\int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{}{3b^{2/3}\left(-\right.}\right.}{a}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)} - \frac{\sqrt[3]{b}\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a} - \frac{\sqrt[3]{b}}{}$$

$$= \frac{\sqrt[3]{b}\,(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{b}\,(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+f}{\sqrt[3]{b}e+\sqrt[3]{-1}}\right.}{3a\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)}$$

Mathematica [A] time = 0.24, size = 244, normalized size = 0.81

$$\frac{(e+fx)^{n+1}\left(\frac{\sqrt[3]{b}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} + \frac{\sqrt[3]{b}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e} + \frac{\sqrt[3]{b}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f} - \frac{3\,{}_2F_1\left(1,n+1;n+\right.}{e}\right.}{3a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x^3)),x]
```

```
[Out] ((e + f*x)^(1 + n)*((b^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
 + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (b^(1/3)*Hyper
geometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a
^(1/3)*f)])/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f) + (b^(1/3)*Hypergeometric2F1
[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/
(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f) - (3*Hypergeometric2F1[1, 1 + n, 2 + n,
1 + (f*x)/e])/e))/(3*a*(1 + n))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n}{bx^4+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^4 + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n}{\left(bx^3+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{n}}{\left(b\,x^{3}+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x/(b*x^3+a),x)
[Out] int((f*x+e)^n/x/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{n}}{\left(bx^{3}+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e+f\,x\right)^{n}}{x\,\left(b\,x^{3}+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x*(a + b*x^3)),x)
[Out] int((e + f*x)^n/(x*(a + b*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e+fx\right)^{n}}{x\left(a+bx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x/(b*x**3+a),x)
[Out] Integral((e + f*x)**n/(x*(a + b*x**3)), x)
```

3.193 $\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=326

$$-\frac{b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}\,b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}\cdots}{\cdots}$$

```
[Out] -1/3*b^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3
)*e-a^(1/3)*f))/a^(4/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(1/3)*b^(2/3)*
(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/
3)*b^(1/3)*e-a^(1/3)*f))/a^(4/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3
*(-1)^(2/3)*b^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/
3)*(f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a^(4/3)/((-1)^(1/3)*b^(1/3)*e+
a^(1/3)*f)/(1+n)+f*(f*x+e)^(1+n)*hypergeom([2, 1+n],[2+n],1+f*x/e)/a/e^2/(1
+n)
```

Rubi [A] time = 0.61, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6725, 65, 68}

$$-\frac{b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}\,b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

```
[Out] -(b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
+ f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n
)) + ((-1)^(1/3)*b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a
^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*b^(2/3)*(e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e +
 f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(4/3)*((-1)^(1/3)*b^(1/3)*
e + a^(1/3)*f)*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometric2F1[2, 1 + n,
2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)^n}{x^2\left(a+bx^3\right)}\,dx = \int\left(\frac{(e+fx)^n}{ax^2}-\frac{bx(e+fx)^n}{a\left(a+bx^3\right)}\right)dx$$
$$=\frac{\int \frac{(e+fx)^n}{x^2}\,dx}{a}-\frac{b\int \frac{x(e+fx)^n}{a+bx^3}\,dx}{a}$$
$$=\frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)}-\frac{b\int\left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}\right.}{a}$$
$$=\frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)}+\frac{b^{2/3}\int\frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}}-\frac{\left(\sqrt[3]{-1}\,b^{2/3}\right)\int\frac{(e+fx)^n}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{}}}{3a^{4/3}}$$
$$=-\frac{b^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}+\frac{\sqrt[3]{-1}\,b^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+\right.}{3a^{4/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(}$$

Mathematica [A] time = 0.27, size = 273, normalized size = 0.84

$$\frac{(e+fx)^{n+1}\left(-\frac{b^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}+\frac{\sqrt[3]{-1}\,b^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}+\frac{(-1)^{2/3}b^{2/3}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e}\right.}{3a^{4/3}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*
(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f)) + ((-1)^(1/3)
*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/
((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) + (
(-1)^(2/3)*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(
e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1
/3)*f) + (3*a^(1/3)*f*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/e^2)
)/(3*a^(4/3)*(1 + n))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(fx+e\right)^n}{bx^5+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^5 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n}{\left(bx^3+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(f x+e\right)^{n}}{\left(b \,x^{3}+a\right) x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x^2/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/x^2/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(f x + e\right)}^{n}}{{\left(b x^{3} + a\right)} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e+f\,x\right)}^n}{x^2\,\left(b\,x^3+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x^2*(a + b*x^3)),x)
```

```
[Out] int((e + f*x)^n/(x^2*(a + b*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.194 $\int \frac{x^2(c+dx)^{1+n}}{a+bx^3} dx$

Optimal. Leaf size=253

$$-\frac{(c+dx)^{n+2}\,_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)}-\frac{(c+dx)^{n+2}\,_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c\right)}-\frac{(c+dx)^{n+2}\,_2F_1\left(1\right.}{3b^{2/3}(n+}$$

```
[Out] -1/3*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1
/3)*d))/b^(2/3)/(b^(1/3)*c-a^(1/3)*d)/(2+n)-1/3*(d*x+c)^(2+n)*hypergeom([1,
 2+n],[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))/b^(2/3)/(b^(1
/3)*c+(-1)^(1/3)*a^(1/3)*d)/(2+n)-1/3*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n
],b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))/b^(2/3)/(b^(1/3)*c-(-1)
^(2/3)*a^(1/3)*d)/(2+n)
```

Rubi [A] time = 0.59, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6725, 68}

$$-\frac{(c+dx)^{n+2}\,_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)}-\frac{(c+dx)^{n+2}\,_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c\right)}-\frac{(c+dx)^{n+2}\,_2F_1\left(1\right.}{3b^{2/3}(n+}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

```
[Out] -((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/
(b^(1/3)*c - a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c - a^(1/3)*d)*(2 + n)) - ((c
 + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/(b^(
1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c + (-1)^(1/3)*a^(1/3)
*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/
3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c -
(-1)^(2/3)*a^(1/3)*d)*(2 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \left(\frac{(c+dx)^{1+n}}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{(c+dx)^{1+n}}{3b^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{(c+dx)^{1+n}}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\right)dx$$

$$= \frac{\int \frac{(c+dx)^{1+n}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}+\frac{\int \frac{(c+dx)^{1+n}}{-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}+\frac{\int \frac{(c+dx)^{1+n}}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}$$

$$= -\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)(2+n)}-\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\,\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}\,c+\sqrt[3]{-1}\,\sqrt[3]{a}\,d\right)(2+n)}$$

Mathematica [A] time = 0.39, size = 213, normalized size = 0.84

$$\frac{(c+dx)^{n+2}\left(-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c}-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

```
[Out] ((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x)
)/(b^(1/3)*c - a^(1/3)*d)]/(b^(1/3)*c - a^(1/3)*d)) - Hypergeometric2F1[1,
2 + n, 3 + n, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(b^(1
/3)*c + (-1)^(1/3)*a^(1/3)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)
*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(b^(1/3)*c - (-1)^(2/3)*a^(
1/3)*d)))/(3*b^(2/3)*(2 + n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx+c)^{n+1}x^2}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^2\,(dx+c)^{n+1}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x+c)^(n+1)/(b*x^3+a),x)
```

```
[Out] int(x^2*(d*x+c)^(n+1)/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c+d\,x\right)}^{n+1}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x)^(n + 1))/(a + b*x^3),x)
```

```
[Out] int((x^2*(c + d*x)^(n + 1))/(a + b*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x+c)**(1+n)/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.195 $\int \frac{x^m(e+fx)^n}{a+bx^3} dx$

Optimal. Leaf size=211

$$\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;\right.}{3a(m+1)}$$

```
[Out] 1/3*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-b^(1/3)*x/a^(1/3),-f*x/e)/a/(1
+m)/((1+f*x/e)^n)+1/3*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,(-1)^(1/3)*b^
(1/3)*x/a^(1/3),-f*x/e)/a/(1+m)/((1+f*x/e)^n)+1/3*x^(1+m)*(f*x+e)^n*AppellF
1(1+m,-n,1,2+m,-f*x/e,-(-1)^(2/3)*b^(1/3)*x/a^(1/3))/a/(1+m)/((1+f*x/e)^n)
```

Rubi [A] time = 0.46, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6725, 135, 133}

$$\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;\right.}{3a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b^(1/3)
*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*Appel
lF1[1 + m, -n, 1, 2 + m, -((f*x)/e), ((-1)^(1/3)*b^(1/3)*x)/a^(1/3)])/(3*a*
(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2
+ m, -((f*x)/e), -(((-1)^(2/3)*b^(1/3)*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x
)/e)^n)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} - \frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x\right)}\right) dx$$
$$= -\frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{2/3}} - \frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3a^{2/3}} - \frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}$$
$$= -\frac{\left((e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{2/3}} - \frac{\left((e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3a^{2/3}} - \frac{(}{}$$
$$= \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n} F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{3a(1+m)} + \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-}}{}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(e + f*x)^n)/(a + b*x^3),x]
[Out] Integrate[(x^m*(e + f*x)^n)/(a + b*x^3), x]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n x^m}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^m/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x^m}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(fx+e\right)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(f*x+e)^n/(b*x^3+a),x)
[Out] int(x^m*(f*x+e)^n/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{n}x^{m}}{bx^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{m}\left(e+f\,x\right)^{n}}{b\,x^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e + f*x)^n)/(a + b*x^3),x)
[Out] int((x^m*(e + f*x)^n)/(a + b*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

3.196 $\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$

Optimal. Leaf size=1480

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)a}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}+\frac{2\sqrt{2+\sqrt{3}}\;(}{}$$

```
[Out] 2/3*(d*x^3+c)^(1/2)/b-2*a*d^(1/3)*(d*x^3+c)^(1/2)/b^2/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))+3^(1/4)*a*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)
*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^2/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*a*d^(1/3)*(c^(1/3)+d^(1/3)*
x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(a*d^(1/3)+b*c^(1/3)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2)
)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/b^3/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/3*(-a^3*d+b^3*c)*(c^(1/3)+d^(1/3)*x)*Elli
pticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^3/(-a*d^(1/3)+b*c^(1/3
)*(1+3^(1/2)))/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))^2)^(1/2)-c^(1/6)*(c^(1/3)+d^(1/3)*x)*arctanh(1/3*(b^2*c^(2
/3)+a*b*c^(1/3)*d^(1/3)+a^2*d^(2/3))^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-(d^
(1/3)*x+c^(1/3)*(1-3^(1/2)))^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(
3/4)/c^(1/6)/b^(1/2)/(b*c^(1/3)-a*d^(1/3))^(1/2)/(7-4*3^(1/2)+(d^(1/3)*x+c^
(1/3)*(1-3^(1/2)))^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2))*(b*c^(1/3)-a
*d^(1/3))^(1/2)*(b^2*c^(2/3)+a*b*c^(1/3)*d^(1/3)+a^2*d^(2/3))^(1/2)*(c^(2/3
)*(1-d^(1/3)*x/c^(1/3)+d^(2/3)*x^2/c^(2/3))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)/b^(5/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))^2)^(1/2)-4*3^(1/4)*c^(1/3)*(-a^3*d+b^3*c)*(c^(1/3)+d^(1
/3)*x)*EllipticPi((-d^(1/3)*x-c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2))),(-a*d^(1/3)+b*c^(1/3)*(1+3^(1/2)))^2/(-a*d^(1/3)+b*c^(1/3)*(1-3^(1/
2)))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(c^(2/3)*(1-d^(1/3)*x/c^(1/
3)+d^(2/3)*x^2/c^(2/3))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^2/(2*b^2
*c^(2/3)+2*a*b*c^(1/3)*d^(1/3)-a^2*d^(2/3))/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 2.81, antiderivative size = 1482, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {2147, 261, 1878, 218, 1877, 2136, 2142, 2113, 537, 571, 93, 208}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*a*d^(1/3)*Sqrt[c + d*x^3])/(b^2*((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)) - (c^(1/6)*Sqrt[b*c^(1/3) - a*d^(1/3)]*Sqrt[b^2*c^
(2/3) + a*b*c^(1/3)*d^(1/3) + a^2*d^(2/3)]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2
/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/
```

```
3) + d^(1/3)*x)^2]*ArcTanh[(Sqrt[2 - Sqrt[3]]*Sqrt[b^2*c^(2/3) + a*b*c^(1/3
)*d^(1/3) + a^2*d^(2/3)]*Sqrt[1 - ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])/(3^(1/4)*Sqrt[b]*c^(1/6)*Sqrt[b*c^(1/3
) - a*d^(1/3)]*Sqrt[7 - 4*Sqrt[3] + ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])])/(b^(5/2)*Sqrt[(c^(1/3)*(c^(1/3) +
d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a*c^(1/3)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^2*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2 + Sqrt[
3]]*a*((1 - Sqrt[3])*b*c^(1/3) + a*d^(1/3))*d^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (2*Sqrt[2 + Sqrt[3]]*(b^3*c - a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*((1 + Sqrt[3])*b*c^(1/3) - a*
d^(1/3))*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(1/3)*(b^3*c -
a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2
/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticPi[((1 +
Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2/((1 - Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2,
-ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(b^2*(2*b^2*c^(2/3) + 2*a*b*c^(1/3)*d^(1/3) - a^2*d
^(2/3))*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2147

```
Int[Sqrt[(a_) + (b_.)*(x_)^3]/((c_) + (d_.)*(x_)), x_Symbol] :> Dist[b/d, I
nt[x^2/Sqrt[a + b*x^3], x], x] + (Dist[(b*c)/d^3, Int[(c - d*x)/Sqrt[a + b*
x^3], x], x] - Dist[(b*c^3 - a*d^3)/d^3, Int[1/((c + d*x)*Sqrt[a + b*x^3]),
 x], x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \frac{(ad)\int \frac{a-bx}{\sqrt{c+dx^3}}\,dx}{b^3} + \frac{d\int \frac{x^2}{\sqrt{c+dx^3}}\,dx}{b} - \left(-c+\frac{a^3d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{c+dx^3}}\,dx$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{\left(ad^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{b^2} + \frac{\left(a\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)d\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{b^3} + \frac{\left(b\left(c-\frac{a^3}{b^3}\right.\right.\cdots}{b\cdots}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\cdots\right.}}}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right.}}}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\cdots\right.}}}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right.}}}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\cdots\right.}}}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right.}}}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\cdots\right.}}}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right.}}}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt[6]{c}\sqrt{b\sqrt[3]{c}-a\sqrt[3]{d}}\sqrt{b^2c^{2/3}+ab\sqrt[3]{c}\sqrt[3]{d}+a^2d^{2/3}}\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 2.15, size = 820, normalized size = 0.55

$$2\left(\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}\,d\sqrt{\frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}}}\sqrt{\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1}\,\Pi\left(\frac{i\sqrt{3}b\sqrt[3]{c}}{\sqrt[3]{d}a+\sqrt[3]{-1}b\sqrt[3]{c}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{d}x+\sqrt[3]{c}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}}}\right)\middle|\sqrt[3]{-1}\right)a^3}{b^2\left(\sqrt[3]{d}a+\sqrt[3]{-1}b\sqrt[3]{c}\right)}-\frac{3^{3/4}d^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{\cdots}}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x),x]
```

```
[Out] (2*(c + d*x^3 - (3^(3/4)*a^2*d^(2/3)*((-1)^(1/3)*c^(1/3) - d^(1/3)*x)*Sqrt[
(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]*Sqrt[(-1)^(1/6) - (I*d^(1
/3)*x)/c^(1/3)]*EllipticF[ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1
+ (-1)^(1/3))*c^(1/3))]], (-1)^(1/3)])/(b^2*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1
/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]) + (3^(3/4)*a*c^(1/3)*d^(1/3)*((-1)^(1/3
)*c^(1/3) - d^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I)*d^(1/3)*x)/c^(1/3)]*Sqrt[(
I*(1 + (d^(1/3)*x)/c^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[
ArcSin[Sqrt[(-1)^(1/6) - (I*d^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 +
(-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*d^(1/3)*x)/c^(1/3)]/3^
(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(b*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/
3)*x)/((1 + (-1)^(1/3))*c^(1/3))]) - ((3*I)*b*c^(4/3)*Sqrt[(c^(1/3) + d^(1/
3)*x)/((1 + (-1)^(1/3))*c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x
^2)/c^(2/3)]*EllipticPi[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(
1/3)), ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/
3))]], (-1)^(1/3)])/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)) + ((-1)^(1/3)*Sqrt[3
]*(1 + (-1)^(1/3))*a^3*c^(1/3)*d*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3
))*c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)]*Elliptic
Pi[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)), ArcSin[Sqrt[(c
^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]], (-1)^(1/3)])/(
b^2*((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)))))/(3*b*Sqrt[c + d*x^3])
```

fricas [F] time = 22.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{dx^3+c}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

maple [A] time = 0.22, size = 1126, normalized size = 0.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(b*x+a),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b-2/3*I*a^2/b^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+2/3*I*a/b^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*(
(x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
)+2/3*I*(a^3*d-b^3*c)/b^4*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*
(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)/(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)+a/b)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)+a/b),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{bx + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(a + b*x),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{a + bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(a + b*x), x)
```

3.197 $\int \frac{(d^3+e^3x^3)^p}{d+ex} dx$

Optimal. Leaf size=135

$$\frac{(d^3+e^3x^3)^p \left(1+\frac{2(d+ex)}{(-3+i\sqrt{3})d}\right)^{-p} \left(1-\frac{2(d+ex)}{(3+i\sqrt{3})d}\right)^{-p} F_1\left(p;-p,-p;p+1;-\frac{2(d+ex)}{(-3+i\sqrt{3})d},\frac{2(d+ex)}{(3+i\sqrt{3})d}\right)}{ep}$$

```
[Out] (e^3*x^3+d^3)^p*AppellF1(p,-p,-p,1+p,-2*(e*x+d)/d/(-3+I*3^(1/2)),2*(e*x+d)/
d/(3+I*3^(1/2)))/e/p/((1+2*(e*x+d)/d/(-3+I*3^(1/2)))^p)/((1-2*(e*x+d)/d/(3+
I*3^(1/2)))^p)
```

Rubi [F] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d^3+e^3x^3)^p}{d+ex} dx$$

Verification is Not applicable to the result.

```
[In] Int[(d^3 + e^3*x^3)^p/(d + e*x),x]
[Out] Defer[Int][(d^3 + e^3*x^3)^p/(d + e*x), x]
```

Rubi steps

$$\int \frac{(d^3+e^3x^3)^p}{d+ex} dx = \int \frac{(d^3+e^3x^3)^p}{d+ex} dx$$

Mathematica [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(d^3+e^3x^3)^p}{d+ex} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d^3 + e^3*x^3)^p/(d + e*x),x]
[Out] Integrate[(d^3 + e^3*x^3)^p/(d + e*x), x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e^3x^3+d^3)^p}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((e^3*x^3 + d^3)^p/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e^3x^3+d^3)^p}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(e^{3} x^{3}+d^{3}\right)^{p}}{e x+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e^3*x^3+d^3)^p/(e*x+d),x)
[Out] int((e^3*x^3+d^3)^p/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e^{3} x^{3}+d^{3}\right)^{p}}{e x+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^3+e^3\,x^3\right)^p}{d+e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^3 + e^3*x^3)^p/(d + e*x),x)
[Out] int((d^3 + e^3*x^3)^p/(d + e*x), x)
```

sympy [B] time = 30.80, size = 636, normalized size = 4.71

$$\frac{0^{p} \log\left(1+\frac{e^{3} x^{3}}{d^{3}}\right)\Gamma\left(-\frac{2}{3}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{5}{3}\right)}{4\pi^{2} e}+\frac{0^{p} e^{\frac{i\pi}{3}} \log\left(1-\frac{exe^{\frac{i\pi}{3}}}{d}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma^{2}\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}{6\pi^{2} e\Gamma\left(\frac{5}{3}\right)}+\frac{0^{p} e^{\frac{2i\pi}{3}} \log\left(1-\frac{exe}{d}\right.}{12\pi^{2} e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e**3*x**3+d**3)**p/(e*x+d),x)
[Out] 0**p*log(1 + e**3*x**3/d**3)*gamma(-2/3)*gamma(-1/3)*gamma(4/3)*gamma(5/3)/
(4*pi**2*e) + 0**p*exp(I*pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(-1/3)
*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*exp(2*I*
pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**
2*e*gamma(4/3)) - 0**p*log(1 - e*x*exp_polar(I*pi)/d)*gamma(-1/3)*gamma(1/3
)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*log(1 - e*x*exp_po
lar(I*pi)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**2*e*gamma(4/3)) + 0**p*exp
(-2*I*pi/3)*log(1 - e*x*exp_polar(5*I*pi/3)/d)*gamma(1/3)**3*gamma(2/3)**2/
(12*pi**2*e*gamma(4/3)) + 0**p*exp(-I*pi/3)*log(1 - e*x*exp_polar(5*I*pi/3)
/d)*gamma(-1/3)*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3))
- d**2*e**(3*p)*p*x**(3*p)*gamma(-2/3)*gamma(-1/3)*gamma(4/3)*gamma(5/3)*ga
mma(p)*gamma(2/3 - p)*hyper((1 - p, 2/3 - p), (5/3 - p,), d**3*exp_polar(I*
pi)/(e**3*x**3))/(4*pi**2*e**3*x**2*gamma(5/3 - p)*gamma(p + 1)) - d*e**(3*
```

```
p)*p*x**(3*p)*gamma(-1/3)*gamma(1/3)*gamma(2/3)*gamma(4/3)*gamma(p)*gamma(1
/3 - p)*hyper((1 - p, 1/3 - p), (4/3 - p,), d**3*exp_polar(I*pi)/(e**3*x**3
))/(4*pi**2*e**2*x*gamma(4/3 - p)*gamma(p + 1)) - d**(3*p)*e**2*x**3*gamma(
1/3)**2*gamma(2/3)**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e*
*3*x**3*exp_polar(I*pi)/d**3)/(4*pi**2*d**3*gamma(-p)*gamma(p + 1))
```

3.198 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=16

$$2\tan^{-1}\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

```
[Out] 2*arctan((1+x)/(x^3+1)^(1/2))
```

Rubi [A] time = 0.08, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2146, 203}

$$2\tan^{-1}\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] 2*ArcTan[(1 + x)/Sqrt[1 + x^3]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\\ &= 2\tan^{-1}\left(\frac{1+x}{\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.87, size = 296, normalized size = 18.50

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\sqrt{2}-i\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{5/6}+\sqrt{2}}+\right.}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
```

```
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*(-I + Sqrt[2])*EllipticPi[(
2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) + Sqrt[2]) + (3*(5 + I*Sqrt[2] + I*
Sqrt[3] + Sqrt[6])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSi
n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(5*I + 2*Sqrt[2]
 + Sqrt[3] + (2*I)*Sqrt[6])))/(3*Sqrt[1 + x^3])
```

fricas [A] time = 0.44, size = 19, normalized size = 1.19

$$-\arctan\left(\frac{x^2-2x}{2\sqrt{x^3+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(x^2 - 2*x)/sqrt(x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2+2x-2}{\sqrt{x^3+1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

maple [C] time = 0.07, size = 1640, normalized size = 102.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*I*2^(1/2)*(1/(3/2-1/2*I*3^(1/2
))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I
*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))
*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+
1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+
1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(
1/2))-2^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/
2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((x+1)
/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+
1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1
/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/
2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1
/2)/(-1-I*2^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I
*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(
1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I
```

```
*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3*I*2^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1
/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/
2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2
/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/
2)/(I*2^(1/2)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3
^(1/2))/(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2^
(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*
3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(I*2^(1/2)-1)*EllipticPi(((x+1)/(3/2-1/2
*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2
*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I
/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/
2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(I*2^(
1/2)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(
I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(3/2-1
/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/
(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2
*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^
(1/2)/(x^3+1)^(1/2)/(I*2^(1/2)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1
/2),(-3/2+1/2*I*3^(1/2))/(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2+2\,x-2}{\sqrt{x^3+1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

mupad [B] time = 0.20, size = 273, normalized size = 17.06

$$\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{1+\sqrt{2}\,1i};\mathrm{asin}\left(\sqrt{\frac{x+\cdots}{\frac{3}{2}+\cdots}}\right.\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((x^2 + 2)*(x^3 + 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i + 1),
 asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(1/2)*
1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))
, -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(x^3 - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \frac{x^2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \left(-\frac{2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(x**2+2)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2*x/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(x**2/
(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(x**2*sqrt(x**3
+ 1) + 2*sqrt(x**3 + 1)), x)
```

3.199 $\int \frac{2+2x-x^2}{\left(2+x^2\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=20

$$-2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

```
[Out] -2*arctan((1-x)/(-x^3+1)^(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2146, 203}

$$-2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] -2*ArcTan[(1 - x)/Sqrt[1 - x^3]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{\left(2+x^2\right)\sqrt{1-x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\right)\\ &= -2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.73, size = 280, normalized size = 14.00

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\,\sqrt{x^2+x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{6\left(1+i\sqrt{2}\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{i+2\sqrt{2}-\sqrt{3}} \cdots\right.}{3\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
```

```
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + (6*(1 + I*Sqrt[2])*EllipticPi[(2*
Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(I + 2*Sqrt[2] - Sqrt[3]) + (3*(1 - I*Sqrt[2])*El
lipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) - Sqrt[2])))/(3*Sqrt[1 -
x^3])
```

fricas [A] time = 0.49, size = 28, normalized size = 1.40

$$-\arctan\left(\frac{\sqrt{-x^3+1}\left(x^2+2x\right)}{2\left(x^3-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*sqrt(-x^3 + 1)*(x^2 + 2*x)/(x^3 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-2x-2}{\sqrt{-x^3+1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

maple [C] time = 0.06, size = 732, normalized size = 36.60

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2i\sqrt{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(
1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*
3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*
3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*2^(1/2)*3^(1
/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2
+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2
)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(-
3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1
/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1
/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(I*2^(1/2)-1/2+1/2*I*3^(1/2))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(I*2^(1/
2)-1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*2^(1/2)*3
^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(
```

```
1/2)/(I*2^(1/2)-1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(I*2^(1/2)-1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(
-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-2x-2}{\sqrt{-x^3+1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

mupad [B] time = 2.83, size = 292, normalized size = 14.60

$$-\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}}{2}}{1+\sqrt{2}}\right.\right.}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((x^2 + 2)*(1 - x^3)^(1/2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2
)/(2^(1/2)*1i + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + e
llipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(
(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2x}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx-\int\frac{x^2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int\left(-\frac{2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(x**2+2)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(x**2
/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(x**2*sqrt(1 -
x**3) + 2*sqrt(1 - x**3)), x)
```

3.200 $\int \frac{2+2x-x^2}{\left(2+x^2\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=18

$$-2\tanh^{-1}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

```
[Out] -2*arctanh((1-x)/(x^3-1)^(1/2))
```

Rubi [A] time = 0.08, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2146, 206}

$$-2\tanh^{-1}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{\left(2+x^2\right)\sqrt{-1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\right)\\ &= -2\tanh^{-1}\left(\frac{1-x}{\sqrt{-1+x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.23, size = 278, normalized size = 15.44

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{6\left(1+i\sqrt{2}\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{i+2\sqrt{2}-\sqrt{3}}+ \cdots\right.}{3\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
```

```
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + (6*(1 + I*Sqrt[2])*EllipticPi[(2*
Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(I + 2*Sqrt[2] - Sqrt[3]) + (3*(1 - I*Sqrt[2])*El
lipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) - Sqrt[2])))/(3*Sqrt[-1 +
 x^3])
```

fricas [A] time = 0.47, size = 25, normalized size = 1.39

$$\log\left(\frac{x^2 + 2x + 2\sqrt{x^3 - 1}}{x^2 + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] log((x^2 + 2*x + 2*sqrt(x^3 - 1))/(x^2 + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 - 2x - 2}{\sqrt{x^3 - 1}\left(x^2 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

maple [C] time = 0.06, size = 1656, normalized size = 92.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3
/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1
/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1
/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(-I*2^
(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/
(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(1/(-3/2-
1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2
/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I
*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(x^3-1)^(1/2)/(-I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2
),(3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2
)))^(1/2))*3^(1/2)+3*I*2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1
/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))
+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(-I*2^(1/2)+1)*Elli
pticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1
),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2^(1/2)*(1/(-3/2-1/2*I*3
^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1
/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2
))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-
1)^(1/2)/(-I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+
```

```
1/2*I*3^(1/2))/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/
2))*3^(1/2)-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3
/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/
2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I
*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3
^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/
2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*El
lipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+
1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*I*2^(1/2)*(1/
(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))
*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2
+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))
^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^
(1/2)))^(1/2))+2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1
/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(
3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-2\,x-2}{\sqrt{x^3-1}\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

mupad [B] time = 2.77, size = 276, normalized size = 15.33

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{-\ldots}\right.\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\ldots}{2}\right.\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((x^2 + 2)*(x^3 - 1)^(1/2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i +
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(
1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2x}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx-\int\frac{x^2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int\left(-\frac{2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(x**2+2)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(x**2
/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(x**2*sqrt(x**3
 - 1) + 2*sqrt(x**3 - 1)), x)
```

3.201 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=18

$$2\tanh^{-1}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

```
[Out] 2*arctanh((1+x)/(-x^3-1)^(1/2))
```

Rubi [A] time = 0.08, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2146, 206}

$$2\tanh^{-1}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] 2*ArcTanh[(1 + x)/Sqrt[-1 - x^3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{\left(2+x^2\right)\sqrt{-1-x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\\ &= 2\tanh^{-1}\left(\frac{1+x}{\sqrt{-1-x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.56, size = 298, normalized size = 16.56

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\sqrt{2}-i\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{5/6}+\sqrt{2}}+\frac{3(}{}\right.}{3\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
```

```
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*(-I + Sqrt[2])*EllipticPi[(
2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) + Sqrt[2]) + (3*(5 + I*Sqrt[2] + I*
Sqrt[3] + Sqrt[6])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSi
n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(5*I + 2*Sqrt[2]
 + Sqrt[3] + (2*I)*Sqrt[6])))/(3*Sqrt[-1 - x^3])
```

fricas [A] time = 0.47, size = 28, normalized size = 1.56

$$\log\left(-\frac{x^2-2x-2\sqrt{-x^3-1}}{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] log(-(x^2 - 2*x - 2*sqrt(-x^3 - 1))/(x^2 + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2+2x-2}{\sqrt{-x^3-1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

maple [C] time = 0.06, size = 724, normalized size = 40.22

$$\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2\sqrt{2}\,\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x)
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))-2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/
2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*
I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticP
i(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*
3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*
3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/
2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^
(1/2)))^(1/2))+2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1
/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(
1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(I*2^(1/2)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(I*2^(1/2)+1/2+1
/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*3^(1/
2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)
))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(I*2^(1/2)+1
```

```
/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),I*3^(1/2)/(I*2^(1/2)+1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)
))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2+2x-2}{\sqrt{-x^3-1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

mupad [B] time = 0.11, size = 289, normalized size = 16.06

$$\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{1+\sqrt{2}\,1i};\mathrm{asin}\left(\right.\right.\right.}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((x^2 + 2)*(- x^3 - 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(
2^(1/2)*1i + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipt
icPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/((- x^3
 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx-\int \frac{x^2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx-\int \left(-\frac{2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(x**2+2)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(2*x/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**
2/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(x**2*sqrt(-
x**3 - 1) + 2*sqrt(-x**3 - 1)), x)
```

3.202 $\int \frac{2-2x-x^2}{\left(2+d+dx+x^2\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=30

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d+1}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

```
[Out] 2*arctan((1+x)*(1+d)^(1/2)/(x^3+1)^(1/2))/(1+d)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2145, 204}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d+1}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 + d]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[1 + d]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{\left(2+d+dx+x^2\right)\sqrt{1+x^3}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-2-(2+2d)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{1+d}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{1+d}}\end{aligned}$$

Mathematica [C] time = 1.36, size = 424, normalized size = 14.13

$$\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\,\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\left(-\left(\left(1+\sqrt[3]{-1}\right)d^2\right)+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2-4d-8}+4\right)d-2\sqrt[3]{-\cdots}\right.\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*((8 + 8*(-1)^(1/3) - (1 + (
-1)^(1/3))*d^2 + 4*Sqrt[-8 - 4*d + d^2] - 2*(-1)^(1/3)*Sqrt[-8 - 4*d + d^2]
 + (1 + (-1)^(1/3))*d*(4 + Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3]
)/(2*(-1)^(1/3) + d - Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)] + ((1 + (-1)^(1/3))*d^2 + (1 + (-1)^(1/3))
*d*(-4 + Sqrt[-8 - 4*d + d^2]) - 2*(4 + 4*(-1)^(1/3) - 2*Sqrt[-8 - 4*d + d^
2] + (-1)^(1/3)*Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(
1/3) + d + Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^
(1/3))]], (-1)^(1/3)]))/((2 + (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 - 4*d
+ d^2])))/(3*Sqrt[1 + x^3])
```

fricas [A] time = 0.48, size = 181, normalized size = 6.03

$$\left[-\frac{\sqrt{-d-1}\,\log\left(-\frac{2\,(3\,d+4)x^3-x^4-\left(d^2+2\,d+4\right)x^2-d^2+4\,\sqrt{x^3+1}\,\left((d+2)x-x^2+d\right)\sqrt{-d-1}-2\left(d^2+2\,d\right)x+4\,d+4}{2\,dx^3+x^4+\left(d^2+2\,d+4\right)x^2+d^2+2\left(d^2+2\,d\right)x+4\,d+4}\right)}{2\,(d+1)},-\frac{\arctan\left(-\frac{\sqrt{x^3+1}\,\left((d+2)x-x\right.}{2\left((d+1)x^3+\right.}\right.}{\sqrt{d+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-d - 1)*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - d^2
+ 4*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(-d - 1) - 2*(d^2 + 2*d)*x + 4*
d + 4)/(2*d*x^3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d +
 4))/(d + 1), -arctan(-1/2*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(d + 1)/
((d + 1)*x^3 + d + 1))/sqrt(d + 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2+2\,x-2}{\sqrt{x^3+1}\left(dx+x^2+d+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)), x)
```

maple [C] time = 0.07, size = 4397, normalized size = 146.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3/2/(d^2-4*d-8)^(1/2)*(1/(3/2-1/
2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(
-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*
I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-
1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))
,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2-1/2*I*(1/(3/2-1/2*I
*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/
2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3
```

```
^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2
*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)+3/2*(1/(3/2-1/2*
I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3
/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*
3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/
2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(
(-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d-4*I/(d^2-4*d-8)^(1/2)*(
1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2)
)*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(
-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3
^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x
+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-
8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+6/(d^2
-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/
2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))
*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*
d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
*d+2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1
/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/
2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*
d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2
))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2))*d*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/
2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2
*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d
-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2)
)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))+4*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1
/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(
1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2
*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*
I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2))*3^(1/2)+12/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1
/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/
2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2
/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/
2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^
(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+1/2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3
^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-
1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(
1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I
*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)+3/2/(d^2-4*d-8)^
(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*
3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*Ellipti
cPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d
^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2+I*(
```

```
1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2)
)*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(
-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3
^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x
+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-
8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3/2*(1
/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))
*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-
3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+
1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8
)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2*I*(1/(3/2
-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/
2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1
/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3
/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/
2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-6/(d^2-4*d
-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/
2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))
^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I
*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*Ell
ipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/
2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d-2
*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*
(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/
(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)
^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(
-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))*d*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(
1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2)
)*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(
-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^
(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-
1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2
*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*Elli
pticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2
*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1
/2)-12/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1
/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/
2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*
d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2
))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2))-1/2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3
^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*
3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+
1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1
/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-
1/2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` for
more details)Is d^2-4*(d+2) positive, negative or zero?
```

mupad [B] time = 2.82, size = 632, normalized size = 21.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((x^3 + 1)^(1/2)*(d + d*x + x^2 + 2)),x)
```

```
[Out] - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)
/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((
3^(1/2)*1i)/2 + 3/2)/((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
)*(d - (d - 2)*(d/2 - (d^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/2)/2 - d/2 +
1)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(d/
2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2)*(d/2 + (d
^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(d^2
 - 4*d - 8)^(1/2)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \frac{x^2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1)
+ 2*sqrt(x**3 + 1)), x) - Integral(x**2/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 +
 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(d*x*sqrt(x
**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x)
```

3.203 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

```
[Out] -2*arctan((1-x)*(1-d)^(1/2)/(-x^3+1)^(1/2))/(1-d)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2145, 204}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[1 - d]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[1 - d]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{\left(2-d+dx+x^2\right)\sqrt{1-x^3}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{-2-(2-2d)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}\end{aligned}$$

Mathematica [C] time = 1.49, size = 427, normalized size = 11.24

$$\frac{\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{3i\left(\left(-\left(\left(1+\sqrt[3]{-1}\right)d^2\right)+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2+4d-8}-4\right)d+2\sqrt[3]{-1}\sqrt{\ldots}\right.\right.}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + ((3*I)*((8 + 8*(-1)^(1/3) - (1 +
(-1)^(1/3))*d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2
] + (1 + (-1)^(1/3))*d*(-4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[
3])/(2*(-1)^(1/3) - d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)] + (-8 - 8*(-1)^(1/3) + (1 + (-1)^(1/3))*
d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2] + (1 + (-1
)^(1/3))*d*(4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((-2*I)*Sqrt[3])/(-2*(-1)
^(1/3) + d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1
)^(1/3))]], (-1)^(1/3)]))/((-2 - (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 + 4
*d + d^2])))/(3*Sqrt[1 - x^3])
```

fricas [A] time = 0.48, size = 191, normalized size = 5.03

$$\left[\frac{\log\left(-\frac{2\,(3\,d-4)x^3-x^4-\left(d^2-2\,d+4\right)x^2-4\,\sqrt{-x^3+1}\,\left((d-2)x-x^2-d\right)\sqrt{d-1}-d^2+2\left(d^2-2\,d\right)x-4\,d+4}{2\,dx^3+x^4+\left(d^2-2\,d+4\right)x^2+d^2-2\left(d^2-2\,d\right)x-4\,d+4}\right)}{2\,\sqrt{d-1}}, -\frac{\sqrt{-d+1}\,\arctan\left(-\frac{\sqrt{-x^3+1}\,\left((d-2)x\right.}{2\left((d-1)x\right.}\right.}{d-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - 4*sqrt(-x^3 + 1)*(
(d - 2)*x - x^2 - d)*sqrt(d - 1) - d^2 + 2*(d^2 - 2*d)*x - 4*d + 4)/(2*d*x^
3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d + 4))/sqrt(d -
1), -sqrt(-d + 1)*arctan(-1/2*sqrt(-x^3 + 1)*((d - 2)*x - x^2 - d)*sqrt(-d
+ 1)/((d - 1)*x^3 - d + 1))/(d - 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 - 2\,x - 2}{\sqrt{-x^3 + 1}\,\left(dx + x^2 - d + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)), x)
```

maple [C] time = 0.07, size = 1908, normalized size = 50.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^
(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-
I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2
*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I
*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*
3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*
(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1
/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
```

```
*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),
(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(
I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1
/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/2)*(
I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1
/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-8/3*I/(d^2+4*d-8)
^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2)
)*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-
x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(
1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-1
/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3
/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/
2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)
/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^
(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(
-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(
1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1
/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/
2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*
3^(1/2)))^(1/2))*d-4/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/
2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3
^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+
1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^
(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/
2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3
^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+
1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^
(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+8/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/
2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/
2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*
I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d
-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` for
more details)Is d^2-4*(2-d) positive, negative or zero?
```

mupad [B] time = 0.14, size = 677, normalized size = 17.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((1 - x^3)^(1/2)*(d*x - d + x^2 + 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) + (2*((3^(1
/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*
(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(
d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d + 2)*(d/2 -
 (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1
) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)^(1/2)) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/
2 + 3/2)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d +
 2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) + 1) + x^3)^(1/2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2x}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx-\int\frac{x^2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3)
 + 2*sqrt(1 - x**3)), x) - Integral(x**2/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x
**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(d*x*sqrt(
1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

3.204 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=36

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

```
[Out] -2*arctanh((1-x)*(1-d)^(1/2)/(x^3-1)^(1/2))/(1-d)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2145, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[1 - d]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[1 - d]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{\left(2-d+dx+x^2\right)\sqrt{-1+x^3}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{-2-(-2+2d)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{1-d}}\end{aligned}$$

Mathematica [C] time = 0.46, size = 425, normalized size = 11.81

$$\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{3i\left(\left(-\left(\left(1+\sqrt[3]{-1}\right)d^2\right)+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2+4d-8}-4\right)d+2\sqrt[3]{-1}\sqrt{\cdots}\right.\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + ((3*I)*((8 + 8*(-1)^(1/3) - (1 +
(-1)^(1/3))*d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2
] + (1 + (-1)^(1/3))*d*(-4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[
3])/(2*(-1)^(1/3) - d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)] + (-8 - 8*(-1)^(1/3) + (1 + (-1)^(1/3))*
d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2] + (1 + (-1
)^(1/3))*d*(4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((-2*I)*Sqrt[3])/(-2*(-1)
^(1/3) + d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1
)^(1/3))]], (-1)^(1/3)]))/((-2 - (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 + 4
*d + d^2])))/(3*Sqrt[-1 + x^3])
```

fricas [A] time = 0.49, size = 187, normalized size = 5.19

$$\left[-\frac{\sqrt{-d+1}\log\left(-\frac{2\,(3\,d-4)x^3-x^4-\left(d^2-2\,d+4\right)x^2-d^2+4\,\sqrt{x^3-1}\left((d-2)x-x^2-d\right)\sqrt{-d+1}+2\left(d^2-2\,d\right)x-4\,d+4}{2\,dx^3+x^4+\left(d^2-2\,d+4\right)x^2+d^2-2\left(d^2-2\,d\right)x-4\,d+4}\right)}{2\,(d-1)}, -\frac{\arctan\left(-\frac{\sqrt{x^3-1}\left((d-2)\right.}{2\left((d-1)\right.}\right.}{\sqrt{d-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-d + 1)*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - d^2
+ 4*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(-d + 1) + 2*(d^2 - 2*d)*x - 4*
d + 4)/(2*d*x^3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d +
 4))/(d - 1), -arctan(-1/2*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(d - 1)/
((d - 1)*x^3 - d + 1))/sqrt(d - 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-2\,x-2}{\sqrt{x^3-1}\left(dx+x^2-d+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)), x)
```

maple [C] time = 0.05, size = 4437, normalized size = 123.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+3/2/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/
2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(
3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3
^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I
*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2+2*I/(d^2+4*d-8)^(1/2)*(1/(-3
/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+
1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/
```

```
2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-
1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(
(3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-3/2*(1/(-3/2-1/2*
I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/
2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(
1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x
^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3
^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-
1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/
2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(
3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(
1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2
),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)+6/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^
(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/
2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2)
)*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1
)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/
2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-
3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-
1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+
1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/
2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3
/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2
-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I
*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(
1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(
d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*
3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^
(1/2)))^(1/2))-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1
/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/
2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*E
llipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1
/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1
/2)-12/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^
(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I
/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^
(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1
+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/
2))+4*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))
^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*
I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)
^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(
1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1
/2))*3^(1/2)-3/2/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*
3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1
/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d
^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))*d^2+1/2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2
-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2
```

```
*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2
*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d
-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+
1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/
2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)-3/2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*
I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^
(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*
(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2))*d-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)
*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2
+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)
)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*
d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3
^(1/2)-6/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2))
)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2
*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8
)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/
(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(
1/2))*d-4*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/
2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+
1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*
d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2
))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2))*3^(1/2)-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*
(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+
1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))
*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d
+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/
2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/
2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3
/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/
2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/
2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
*d^2*3^(1/2)+12/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3
^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/
2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^
2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^
(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1
/2)))^(1/2))-2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*
3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1
/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d
^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))*d*3^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` for
more details)Is d^2-4*(2-d) positive, negative or zero?
```

mupad [B] time = 2.80, size = 629, normalized size = 17.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((x^3 - 1)^(1/2)*(d*x - d + x^2 + 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2) + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1
i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi
(((3^(1/2)*1i)/2 + 3/2)/(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2))*(d + (d + 2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)
^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2
)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d + 2)*(d/
2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2
)*(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2x}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx-\int\frac{x^2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1)
 + 2*sqrt(x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3
- 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(d*x*sqrt(
x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

3.205 $\int \frac{2-2x-x^2}{\left(2+d+dx+x^2\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d+1}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

```
[Out] 2*arctanh((1+x)*(1+d)^(1/2)/(-x^3-1)^(1/2))/(1+d)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2145, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d+1}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[1 + d]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[1 + d]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{\left(2+d+dx+x^2\right)\sqrt{-1-x^3}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-2-(-2-2d)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{1+d}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{1+d}}\end{aligned}$$

Mathematica [C] time = 0.55, size = 426, normalized size = 13.31

$$\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(-\left(\left(1+\sqrt[3]{-1}\right)d^2\right)+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2-4d-8}+4\right)d-2\sqrt[3]{-\cdots}\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/
3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*((8 + 8*(-1)^(1/3) - (1 + (
-1)^(1/3))*d^2 + 4*Sqrt[-8 - 4*d + d^2] - 2*(-1)^(1/3)*Sqrt[-8 - 4*d + d^2]
 + (1 + (-1)^(1/3))*d*(4 + Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3]
)/(2*(-1)^(1/3) + d - Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)] + ((1 + (-1)^(1/3))*d^2 + (1 + (-1)^(1/3))
*d*(-4 + Sqrt[-8 - 4*d + d^2]) - 2*(4 + 4*(-1)^(1/3) - 2*Sqrt[-8 - 4*d + d^
2] + (-1)^(1/3)*Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(
1/3) + d + Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^
(1/3))]], (-1)^(1/3)]))/((2 + (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 - 4*d
+ d^2])))/(3*Sqrt[-1 - x^3])
```

fricas [B] time = 0.48, size = 185, normalized size = 5.78

$$\left[\frac{\log\left(-\frac{2\,(3\,d+4)x^3-x^4-\left(d^2+2\,d+4\right)x^2-4\,\sqrt{-x^3-1}\,\left((d+2)x-x^2+d\right)\sqrt{d+1}-d^2-2\,\left(d^2+2\,d\right)x+4\,d+4}{2\,dx^3+x^4+\left(d^2+2\,d+4\right)x^2+d^2+2\,\left(d^2+2\,d\right)x+4\,d+4}\right)}{2\,\sqrt{d+1}}, -\frac{\sqrt{-d-1}\,\arctan\left(-\frac{\sqrt{-x^3-1}\,\left((d+2)x-x^2\right.}{2\,\left((d+1)x^3+\right.}\right.}{d+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - 4*sqrt(-x^3 - 1)*(
(d + 2)*x - x^2 + d)*sqrt(d + 1) - d^2 - 2*(d^2 + 2*d)*x + 4*d + 4)/(2*d*x^
3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d + 4))/sqrt(d +
1), -sqrt(-d - 1)*arctan(-1/2*sqrt(-x^3 - 1)*((d + 2)*x - x^2 + d)*sqrt(-d
- 1)/((d + 1)*x^3 + d + 1))/(d + 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2+2\,x-2}{\sqrt{-x^3-1}\,\left(dx+x^2+d+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)), x)
```

maple [C] time = 0.06, size = 1888, normalized size = 59.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+1/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1
/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^
(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/
2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/
2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)
+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/
2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
```

```
(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))*d-4/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-
1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1
/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2
)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I
*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(
-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2
*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*
3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-8/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1
/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2
)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*
3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)
^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-1/3*I/(d^2-4*d-8)^(1/2)*3^(1
/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1
/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/
(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*
(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)
*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/
2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^
2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2-4*d-8)^
(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*
x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3
-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^
(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+
1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I
*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(
1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)
*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d
+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+8/3*I/(d^2-4
*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(
1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)
/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(
1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^
(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` for
more details)Is d^2-4*(d+2) positive, negative or zero?
```

mupad [B] time = 0.12, size = 680, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((- x^3 - 1)^(1/2)*(d + d*x + x^2 + 2)),x)
```

```
[Out] - (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3^(
1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/((
d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2)*(d/2 - (
d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1)) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)
*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2
 + 3/2)/(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2
)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*(d/2 + (d^2 - 4*d - 8)^(1/2)
/2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx-\int \frac{x^2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 -
1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(-x**3 - 1) + d*sqrt(-
x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(d*
x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3
 - 1)), x)
```

3.206 $\int (d+ex)^3\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=355

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(9\sqrt{a}\,e^2+5\sqrt{c}\,d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)}}}{5c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 1/6*e^3*(c*x^4+a)^(3/2)/c+3/4*a*d^2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/
c^(1/2)+3/4*d^2*e*x^2*(c*x^4+a)^(1/2)+1/15*d*x*(9*e^2*x^2+5*d^2)*(c*x^4+a)^
(1/2)+6/5*a*d*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-6/5*a^(5/
4)*d*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1
/2)+1/15*a^(3/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))
*(9*e^2*a^(1/2)+5*d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^
2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1885, 1177, 1198, 220, 1196, 1248, 641, 195, 217, 206}

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(9\sqrt{a}\,e^2+5\sqrt{c}\,d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)}}}{5c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

```
[Out] (3*d^2*e*x^2*Sqrt[a + c*x^4])/4 + (6*a*d*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*
(Sqrt[a] + Sqrt[c]*x^2)) + (d*x*(5*d^2 + 9*e^2*x^2)*Sqrt[a + c*x^4])/15 + (
e^3*(a + c*x^4)^(3/2))/(6*c) + (3*a*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*
x^4]])/(4*Sqrt[c]) - (6*a^(5/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*d*(5*Sqrt[c]*d^2 + 9*Sqrt[a]*e^2)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \int \left(\left(d^3+3de^2x^2\right)\sqrt{a+cx^4} + x\left(3d^2e+e^3x^2\right)\sqrt{a+cx^4}\right)dx$$
$$= \int \left(d^3+3de^2x^2\right)\sqrt{a+cx^4}\,dx + \int x\left(3d^2e+e^3x^2\right)\sqrt{a+cx^4}\,dx$$
$$= \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{1}{15}\int \frac{10ad^3+18ade^2x^2}{\sqrt{a+cx^4}}\,dx + \frac{1}{2}\,\mathrm{Subst}\left(\int \left(3d^2e+\right.\right.$$
$$= \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3\left(a+cx^4\right)^{3/2}}{6c} + \frac{1}{2}\left(3d^2e\right)\mathrm{Subst}\left(\int \sqrt{a+cx^2}\,dx,\right.$$
$$= \frac{3}{4}d^2ex^2\sqrt{a+cx^4} + \frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3\left(a+c\right.}{6c}$$
$$= \frac{3}{4}d^2ex^2\sqrt{a+cx^4} + \frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3\left(a+c\right.}{6c}$$
$$= \frac{3}{4}d^2ex^2\sqrt{a+cx^4} + \frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3\left(a+c\right.}{6c}$$

Mathematica [C] time = 0.14, size = 186, normalized size = 0.52

$$\frac{\sqrt{a+cx^4}\left(12cd^3x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^4}{a}\right)+9\sqrt{a}\sqrt{c}\,d^2e\sinh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+9cd^2ex^2\sqrt{\frac{cx^4}{a}+1}+12cde^2x^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\right.\right.}{12c\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[a + c*x^4]*(2*a*e^3*Sqrt[1 + (c*x^4)/a] + 9*c*d^2*e*x^2*Sqrt[1 + (c*x
^4)/a] + 2*c*e^3*x^4*Sqrt[1 + (c*x^4)/a] + 9*Sqrt[a]*Sqrt[c]*d^2*e*ArcSinh[
(Sqrt[c]*x^2)/Sqrt[a]] + 12*c*d^3*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^4)/a)] + 12*c*d*e^2*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^4)/a)]))
/(12*c*Sqrt[1 + (c*x^4)/a])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+a}\,(ex+d)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

maple [C] time = 0.06, size = 334, normalized size = 0.94

$$\frac{3\sqrt{c\,x^4+a}\,d\,e^2x^3}{5}-\frac{6i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,a^{\frac{3}{2}}d\,e^2\,\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}+\frac{6i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,a^{\frac{3}{2}}d}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/6*e^3*(c*x^4+a)^(3/2)/c+3/5*d*e^2*x^3*(c*x^4+a)^(1/2)+6/5*I*d*e^2*a^(3/2)
/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/
2),I)-6/5*I*d*e^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*EllipticE(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/4*d^2*e*x^2*(c*x^4+a)^(1/2)+3/4*e*d^2*a/c^
(1/2)*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))+1/3*d^3*x*(c*x^4+a)^(1/2)+2/3*d^3*a/(
I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+a}\,(ex+d)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x^4+a}\,\left(d+e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)*(d + e*x)^3,x)
```

```
[Out] int((a + c*x^4)^(1/2)*(d + e*x)^3, x)
```

sympy [A] time = 4.65, size = 175, normalized size = 0.49

$$\frac{\sqrt{a}\,d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{3\sqrt{a}\,d^2ex^2\sqrt{1+\frac{cx^4}{a}}}{4}+\frac{3\sqrt{a}\,de^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{3ad^2e\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{c}}+e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)
/a)/(4*gamma(5/4)) + 3*sqrt(a)*d**2*e*x**2*sqrt(1 + c*x**4/a)/4 + 3*sqrt(a)
*d*e**2*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a
)/(4*gamma(7/4)) + 3*a*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(4*sqrt(c)) + e**
3*Piecewise((sqrt(a)*x**4/4, Eq(c, 0)), ((a + c*x**4)**(3/2)/(6*c), True))
```

3.207 $\int (d+ex)^2\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=326

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(3\sqrt{a}\,e^2+5\sqrt{c}\,d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E}{5c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 1/2*a*d*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*d*e*x^2*(c*x^4+a
)^(1/2)+1/15*x*(3*e^2*x^2+5*d^2)*(c*x^4+a)^(1/2)+2/5*a*e^2*x*(c*x^4+a)^(1/2
)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-2/5*a^(5/4)*e^2*(cos(2*arctan(c^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2
*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/15*a^(3/4)*(cos(2*arctan(c^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*ar
ctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e^2*a^(1/2)+5*d^2*c^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^
(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1885, 275, 195, 217, 206, 1177, 1198, 220, 1196}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(3\sqrt{a}\,e^2+5\sqrt{c}\,d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E}{5c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

```
[Out] (d*e*x^2*Sqrt[a + c*x^4])/2 + (2*a*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*(Sqrt[
a] + Sqrt[c]*x^2)) + (x*(5*d^2 + 3*e^2*x^2)*Sqrt[a + c*x^4])/15 + (a*d*e*Ar
cTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) - (2*a^(5/4)*e^2*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*(5*S
qrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(
3/4)*Sqrt[a + c*x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \int \left(2dex\sqrt{a+cx^4} + \left(d^2+e^2x^2\right)\sqrt{a+cx^4}\right)dx$$
$$= (2de)\int x\sqrt{a+cx^4}\,dx + \int \left(d^2+e^2x^2\right)\sqrt{a+cx^4}\,dx$$
$$= \frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4} + \frac{1}{15}\int \frac{10ad^2+6ae^2x^2}{\sqrt{a+cx^4}}\,dx + (de)\,\text{Subst}\left(\int \sqrt{a+cx^2}\right.$$
$$= \frac{1}{2}dex^2\sqrt{a+cx^4} + \frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4} + \frac{1}{2}(ade)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,\right.$$
$$= \frac{1}{2}dex^2\sqrt{a+cx^4} + \frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4} - \frac{2a^{5/4}e^2\left(\sqrt{a}\right.}{}$$
$$= \frac{1}{2}dex^2\sqrt{a+cx^4} + \frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4} + \frac{ade\tanh^{-1}}{2\sqrt{}}$$

Mathematica [C] time = 0.14, size = 146, normalized size = 0.45

$$\frac{\sqrt{a+cx^4}\left(6\sqrt{c}d^2x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^4}{a}\right)+e\left(3d\left(\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+\sqrt{c}x^2\sqrt{\frac{cx^4}{a}+1}\right)+2\sqrt{c}ex^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};\right.\right.}{6\sqrt{c}\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[a + c*x^4]*(6*Sqrt[c]*d^2*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^
4)/a)] + e*(3*d*(Sqrt[c]*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]
*x^2)/Sqrt[a]]) + 2*Sqrt[c]*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^
4)/a)])))/(6*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+a}\left(e^2x^2+2\,dex+d^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+a}\,(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

maple [C] time = 0.02, size = 310, normalized size = 0.95

$$\frac{\sqrt{c\,x^4+a}\;e^2x^3}{5}-\frac{2i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;a^{\frac{3}{2}}e^2\,\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;x,i\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;\sqrt{c\,x^4+a}\;\sqrt{c}}+\frac{2i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;a^{\frac{3}{2}}e^2\,\mathrm{Elli}}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;\sqrt{c\,x^4+a}\;\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/5*e^2*x^3*(c*x^4+a)^(1/2)+2/5*I*e^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I
/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/
2)/c^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-2/5*I*e^2*a^(3/2)/(I/a^
(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^
2+1)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
+1/2*d*e*x^2*(c*x^4+a)^(1/2)+1/2*d*e*a/c^(1/2)*ln(c^(1/2)*x^2+(c*x^4+a)^(1/
2))+1/3*d^2*x*(c*x^4+a)^(1/2)+2/3*d^2*a/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/
2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*Ell
ipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+a}\,(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x^4+a}\;(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)*(d + e*x)^2,x)
```

```
[Out] int((a + c*x^4)^(1/2)*(d + e*x)^2, x)
```

sympy [C] time = 4.09, size = 138, normalized size = 0.42

$$\frac{\sqrt{a}\,d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}\,dex^2\sqrt{1+\frac{cx^4}{a}}}{2}+\frac{\sqrt{a}\,e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ade\,\mathrm{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)
/a)/(4*gamma(5/4)) + sqrt(a)*d*e*x**2*sqrt(1 + c*x**4/a)/2 + sqrt(a)*e**2*x
**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*gamm
a(7/4)) + a*d*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))
```

3.208 $\int (d+ex)\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=158

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*a*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/3*d*x*(c*x^4+a)^(1/2
)+1/4*e*x^2*(c*x^4+a)^(1/2)+1/3*a^(3/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1885, 195, 220, 275, 217, 206}

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[a + c*x^4],x]
```

```
[Out] (d*x*Sqrt[a + c*x^4])/3 + (e*x^2*Sqrt[a + c*x^4])/4 + (a*e*ArcTanh[(Sqrt[c]
*x^2)/Sqrt[a + c*x^4]])/(4*Sqrt[c]) + (a^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\sqrt{a+cx^4}\,dx &= \int \left(d\sqrt{a+cx^4}+ex\sqrt{a+cx^4}\right)dx\\
&= d\int \sqrt{a+cx^4}\,dx+e\int x\sqrt{a+cx^4}\,dx\\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{3}(2ad)\int \frac{1}{\sqrt{a+cx^4}}\,dx+\frac{1}{2}e\,\text{Subst}\left(\int \sqrt{a+cx^2}\,dx,x,x^2\right)\\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}\\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}\\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}}{3\sqrt[4]{c}\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.07, size = 109, normalized size = 0.69

$$\frac{\sqrt{a+cx^4}\left(4\sqrt{c}\,dx\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^4}{a}\right)+\sqrt{a}\,e\sinh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+\sqrt{c}\,ex^2\sqrt{\frac{cx^4}{a}+1}\right)}{4\sqrt{c}\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[a + c*x^4]*(Sqrt[c]*e*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*e*ArcSinh[(Sq
rt[c]*x^2)/Sqrt[a]] + 4*Sqrt[c]*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^4)/a)]))/(4*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+a}\,(ex+d),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+a}\,(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

maple [C] time = 0.01, size = 127, normalized size = 0.80

$$\frac{2\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,ad\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{\sqrt{c\,x^4+a}\,e\,x^2}{4}+\frac{ae\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{4\sqrt{c}}+\frac{\sqrt{c\,x^4+a}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/4*e*x^2*(c*x^4+a)^(1/2)+1/4*e*a/c^(1/2)*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))+1
/3*d*x*(c*x^4+a)^(1/2)+2/3*d*a/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2
)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I
/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+a}\,(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x^4+a}\,\,(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)*(d + e*x),x)
```

```
[Out] int((a + c*x^4)^(1/2)*(d + e*x), x)
```

sympy [C] time = 3.45, size = 88, normalized size = 0.56

$$\frac{\sqrt{a}\,dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}\,ex^2\sqrt{1+\frac{cx^4}{a}}}{4}+\frac{ae\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4)) + sqrt(a)*e*x**2*sqrt(1 + c*x**4/a)/4 + a*e*asinh(sqrt(c)*x
**2/sqrt(a))/(4*sqrt(c))
```

3.209 $\int \sqrt{a+cx^4}\, dx$

Optimal. Leaf size=105

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

```
[Out] 1/3*x*(c*x^4+a)^(1/2)+1/3*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
1/2)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{195, 220\}$

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3
*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+cx^4}\, dx &= \frac{1}{3}x\sqrt{a+cx^4}+\frac{1}{3}(2a)\int \frac{1}{\sqrt{a+cx^4}}\, dx\\ &= \frac{1}{3}x\sqrt{a+cx^4}+\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.11, size = 89, normalized size = 0.85

$$\frac{x\left(a+cx^4\right)-\frac{2ia\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4],x]
[Out] (x*(a + c*x^4) - ((2*I)*a*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*S
qrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(3*Sqrt[a + c*x^4])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + a), x)
```

maple [C] time = 0.00, size = 85, normalized size = 0.81

$$\frac{2\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,a\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{\sqrt{c\,x^4+a}\,x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2),x)
[Out] 1/3*x*(c*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x
^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^
(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + a), x)
```

mupad [B] time = 2.64, size = 37, normalized size = 0.35

$$\frac{x\sqrt{c\,x^4+a}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\;\frac{5}{4};\;-\frac{c\,x^4}{a}\right)}{\sqrt{\frac{c\,x^4}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2),x)
```

```
[Out] (x*(a + c*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -(c*x^4)/a))/((c*x^4)/a +
1)^(1/2)
```

sympy [C] time = 0.79, size = 37, normalized size = 0.35

$$\frac{\sqrt{a}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4))
```

3.210 $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

Optimal. Leaf size=730

$$-\frac{\sqrt{-ae^4-cd^4}\,\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2e^3}+\frac{\sqrt{c}\,d^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2e^3}-\frac{\sqrt[4]{a}\,\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)}{2e^4\sqrt{a+cx^4}}$$

```
[Out] 1/2*d^2*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))*c^(1/2)/e^3-1/2*arctan(x*(-a*e
^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))*(-a*e^4-c*d^4)^(1/2)/e^3-1/2*arctanh((
c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))*(a*e^4+c*d^4)^(1/2)/e
^3+1/2*(c*x^4+a)^(1/2)/e-d*x*c^(1/2)*(c*x^4+a)^(1/2)/e^2/(a^(1/2)+x^2*c^(1/
2))+a^(1/4)*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/e^2/(c*
x^4+a)^(1/2)+1/2*c^(1/4)*d*(a*e^4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2
))^2)^(1/2)/a^(1/4)/e^4/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/4*(a*e^
4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*
c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*
(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(
1/4)/d/e^4/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/2*a^(1/4)*c^(1/4)*d*
(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))
*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/
2))*(e^2+d^2*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/e^4
/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1729, 1209, 1198, 220, 1196, 1217, 1707, 1248, 735, 844, 217, 206, 725}

$$-\frac{\sqrt{-ae^4-cd^4}\,\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2e^3}+\frac{\sqrt{c}\,d^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2e^3}-\frac{\sqrt{ae^4+cd^4}\,\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\,\sqrt{ae^4+cd^4}}\right)}{2e^3}-\frac{\sqrt[4]{a}\,\sqrt[4]{c}\,d\left(\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/(d + e*x),x]
```

```
[Out] Sqrt[a + c*x^4]/(2*e) - (Sqrt[c]*d*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[
c]*x^2)) - (Sqrt[-(c*d^4) - a*e^4]*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*S
qrt[a + c*x^4])])/(2*e^3) + (Sqrt[c]*d^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x
^4]])/(2*e^3) - (Sqrt[c*d^4 + a*e^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^
4 + a*e^4]*Sqrt[a + c*x^4])])/(2*e^3) + (a^(1/4)*c^(1/4)*d*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)*d*((Sqrt[c
]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*e^4*Sqrt[a
 + c*x^4]) + (c^(1/4)*d*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c
]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*S
```

```
qrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^
(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = d\int \frac{\sqrt{a+cx^4}}{d^2-e^2x^2}\,dx - e\int \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2}\,dx$$

$$= \left(d\left(a+\frac{cd^4}{e^4}\right)\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx - \frac{d\int \frac{cd^2+ce^2x^2}{\sqrt{a+cx^4}}\,dx}{e^4} - \frac{1}{2}e\,\text{Subst}\left(\int \frac{\sqrt{a+cx^2}}{d^2-e^2x}\,dx, x, x^2\right.$$

$$= \frac{\sqrt{a+cx^4}}{2e} + \frac{\left(\sqrt{a}\sqrt{c}\,d\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{e^2} + \frac{\text{Subst}\left(\int \frac{-ae^2-cd^2x}{\left(d^2-e^2x\right)\sqrt{a+cx^2}}\,dx, x, x^2\right)}{2e} + \frac{\left(\sqrt{c}\,d\left(a+\frac{cd^4}{e^4}\right)\right)\int}{\sqrt{c}\,d^2+\sqrt{a}}$$

$$= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}\,dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\,\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2e^3} + \frac{\sqrt[4]{a}\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{}}{e^2\sqrt{}}$$

$$= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}\,dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\,\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2e^3} + \frac{\sqrt[4]{a}\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{}}{e^2\sqrt{}}$$

$$= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}\,dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\,\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2e^3} + \frac{\sqrt{c}\,d^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2e^3} -$$

Mathematica [C] time = 0.84, size = 405, normalized size = 0.55

$$2c^{3/4}d^2\sqrt{\frac{cx^4}{a}+1}\left(\sqrt{a}\,e^2+i\sqrt{c}\,d^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right) - 2\sqrt{a}\,c^{3/4}d^2e^2\sqrt{\frac{cx^4}{a}+1}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/(d + e*x),x]
```

```
[Out] (-2*Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[a]]*x], -1] + 2*c^(3/4)*d^2*(I*Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sq
rt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + S
qrt[(I*Sqrt[c])/Sqrt[a]]*(c^(1/4)*d*e*(e^2*(a + c*x^4) + Sqrt[c]*d^2*Sqrt[a
 + c*x^4]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]] - Sqrt[c*d^4 + a*e^4]*Sqrt
[a + c*x^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4
])]) - 2*(-1)^(1/4)*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[1 + (c*x^4)/a]*EllipticPi[
(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1])
)/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/4)*d*e^4*Sqrt[a + c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

maple [C] time = 0.02, size = 565, normalized size = 0.77

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,a\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,d}-\frac{a\,\mathrm{arctanh}\left(\frac{\frac{2c\,d^2x^2}{e^2}+2a}{2\sqrt{a+\frac{c\,d^4}{e^4}}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{a+\frac{c\,d^4}{e^4}}\,e}-\frac{c\,d^4\,\mathrm{arctanh}}{2\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2*(c*x^4+a)^(1/2)/e-c*d^3/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/
2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((
I/a^(1/2)*c^(1/2))^(1/2)*x,I)+1/2*c^(1/2)*d^2/e^3*ln(2*c^(1/2)*x^2+2*(c*x^4
+a)^(1/2))-I*c^(1/2)*d/e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^
(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*(Ellipti
cF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))
-1/2/e/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^
(1/2)/(c*x^4+a)^(1/2))*a-1/2/e^5/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d
^2/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))*c*d^4+1/(I/a^(1/2)*c^(1/2)
)^(1/2)/d*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/
(c*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)*c^(1/2))^(1/2)*x,-I*a^(1/2)/c^(1/2)/d
^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a+1/e^4/(I/a^(
1/2)*c^(1/2))^(1/2)*d^3*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)
*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)*c^(1/2))^(1/2)*x,-I*a^(
1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*
c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + c*x^4)^(1/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**4)/(d + e*x), x)
```

3.211 $\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$

Optimal. Leaf size=1221

$$\frac{c\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\,\sqrt{cx^4+a}}\right)d^3}{e^3\sqrt{cd^4+ae^4}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{cx^4+a}}\right)d}{e^3}-\frac{\sqrt{cx^4+a}\,d}{e\left(d^2-e^2x^2\right)}-\frac{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{c}\,x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}\,x^2+\sqrt{a}\right)^2}}\,E\left(2\right.}{e^2\sqrt{cx^4+a}}$$

```
[Out] -d*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))*c^(1/2)/e^3-1/2*(-a*e^4+c*d^4)*arct
an(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/d/e^3/(-a*e^4-c*d^4)^(1/2)+1
/2*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))*(-a*e^4-c*d^4)^(1/2)/
d/e^3+c*d^3*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/
e^3/(a*e^4+c*d^4)^(1/2)-d*(c*x^4+a)^(1/2)/e/(-e^2*x^2+d^2)+x*(c*x^4+a)^(1/2
)/(-e^2*x^2+d^2)+2*x*c^(1/2)*(c*x^4+a)^(1/2)/e^2/(a^(1/2)+x^2*c^(1/2))-2*a^
(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/
4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/e^2/(c*x^4+a)^(
1/2)-1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(
-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(
1/2))^2)^(1/2)/a^(1/4)/e^4/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),1/
2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))^2*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a
^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^2/e^4/(c*x^4+a)^(1/2)-1/2*c^
(1/4)*(a*e^4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/e
^4/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)+1/4*(a*e^4+c*d^4)*(cos(2*arcta
n(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^
(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^2/e^4/(e^2*a^(
1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)+1/4*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/
4)*x/a^(1/4))),1/2*2^(1/2))*(e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/e^4/(c*x^4+a)^(1/2)+3/4*
a^(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*c^(1/2))*(e^2+d^2*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/e^4/(c*x^4+a)^(1/2)
```

Rubi [A] time = 1.80, antiderivative size = 1221, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.790, Rules used = {2153, 1227, 1198, 220, 1196, 1217, 1707, 1248, 733, 844, 217, 206, 725, 1336, 1209}

$$\frac{c\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\,\sqrt{cx^4+a}}\right)d^3}{e^3\sqrt{cd^4+ae^4}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{cx^4+a}}\right)d}{e^3}-\frac{\sqrt{cx^4+a}\,d}{e\left(d^2-e^2x^2\right)}-\frac{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{c}\,x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}\,x^2+\sqrt{a}\right)^2}}\,E\left(2\right.}{e^2\sqrt{cx^4+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[c]*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[c]*x^2)) - (d*Sqrt[a + c
*x^4])/(e*(d^2 - e^2*x^2)) + (x*Sqrt[a + c*x^4])/(d^2 - e^2*x^2) + (Sqrt[-(
```

```
c*d^4) - a*e^4]*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(
2*d*e^3) - ((c*d^4 - a*e^4)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a +
 c*x^4])])/(2*d*e^3*Sqrt[-(c*d^4) - a*e^4]) - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x
^2)/Sqrt[a + c*x^4]])/e^3 + (c*d^3*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4
+ a*e^4]*Sqrt[a + c*x^4])])/(e^3*Sqrt[c*d^4 + a*e^4]) - (2*a^(1/4)*c^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) + (3*a^(1/4)*
c^(1/4)*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(4*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*e^4*Sqrt[a + c*x^4]) + (c^(1/4)*(
Sqrt[c]*d^2 + Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/
4)*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
 + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt
[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/
4)*d^2*e^4*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d^2*e^4*(Sqrt[c]*d^2 + Sqrt[a]*
e^2)*Sqrt[a + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1227

```
Int[Sqrt[(a_) + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Symbol] :> Simp[(x
*Sqrt[a + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2*d*e^2), Int[(d - e*x^2
)/Sqrt[a + c*x^4], x], x] - Dist[(c*d^2 - a*e^2)/(2*d*e^2), Int[1/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]) /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^
2, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
```

```
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2153

```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^nn)^p, (c/(c^2 - d^2*x^(2*n)) - (d*x^n)/
(c^2 - d^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !
IntegerQ[p] && ILtQ[q, 0] && IGtQ[Log[2, nn/n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx &= \int \left(\frac{d^2\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2}-\frac{2dex\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2}+\frac{e^2x^2\sqrt{a+cx^4}}{\left(-d^2+e^2x^2\right)^2}\right)dx\\
&= d^2\int \frac{\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2}\,dx-(2de)\int \frac{x\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2}\,dx+e^2\int \frac{x^2\sqrt{a+cx^4}}{\left(-d^2+e^2x^2\right)^2}\,dx\\
&= \frac{x\sqrt{a+cx^4}}{2\left(d^2-e^2x^2\right)}+\frac{1}{2}\left(a-\frac{cd^4}{e^4}\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx+\frac{c\int \frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx}{2e^4}-(de)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}}{(d^2}\right.\\
&= -\frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)}+\frac{x\sqrt{a+cx^4}}{2\left(d^2-e^2x^2\right)}+d^2\int \frac{\sqrt{a+cx^4}}{\left(-d^2+e^2x^2\right)^2}\,dx+\frac{1}{2}\left(\sqrt{a}\left(\sqrt{a}-\frac{\sqrt{c}\,d^2}{e^2}\right)\right)\int \frac{1+}{(d^2-e^2x}\\
&= \frac{\sqrt{c}\,x\sqrt{a+cx^4}}{2e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)}+\frac{x\sqrt{a+cx^4}}{d^2-e^2x^2}-\frac{\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{4de^3\sqrt{-cd^4-ae^4}}-\frac{\sqrt[4]{a}\,\sqrt[4]{c}\,(\sqrt{}}{}\\
&= \frac{\sqrt{c}\,x\sqrt{a+cx^4}}{2e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)}+\frac{x\sqrt{a+cx^4}}{d^2-e^2x^2}-\frac{\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{4de^3\sqrt{-cd^4-ae^4}}-\frac{\sqrt[4]{a}\,\sqrt[4]{c}\,(\sqrt{}}{}\\
&= \frac{2\sqrt{c}\,x\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)}+\frac{x\sqrt{a+cx^4}}{d^2-e^2x^2}+\frac{\sqrt{-cd^4-ae^4}\,\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2de^3}-\frac{\left(cd^4-ae^{}\right.}{2de}\end{aligned}$$

Mathematica [C] time = 1.93, size = 382, normalized size = 0.31

$$\frac{2\sqrt[4]{-1}\,\sqrt[4]{a}\,c^{3/4}d^2\sqrt{\frac{cx^4}{a}+1}\,\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)-\frac{2\sqrt{c}\,\sqrt{\frac{cx^4}{a}+1}\left(\sqrt{a}\,e^2+i\sqrt{c}\,d^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}-\frac{cd^3e\sqrt{a+c}}{}}{e^4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

```
[Out] (-((e^3*(a + c*x^4))/(d + e*x)) - Sqrt[c]*d*e*Sqrt[a + c*x^4]*ArcTanh[(Sqrt
[c]*x^2)/Sqrt[a + c*x^4]] - (c*d^3*e*Sqrt[a + c*x^4]*ArcTanh[(-(a*e^2) - c*
d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/Sqrt[c*d^4 + a*e^4] - (2*I
)*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[S
qrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (2*Sqrt[c]*(I*Sqrt[c]*d^2 + Sqrt[a]*e^2)
*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])
/Sqrt[(I*Sqrt[c])/Sqrt[a]] + 2*(-1)^(1/4)*a^(1/4)*c^(3/4)*d^2*Sqrt[1 + (c*x
^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)
*x)/a^(1/4)], -1])/(e^4*Sqrt[a + c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d)^2, x)
```

maple [C] time = 0.03, size = 402, normalized size = 0.33

$$\frac{c\,d^3 \operatorname{arctanh}\left(\frac{\frac{2 c\,d^2 x^2}{e^2}+2a}{2\sqrt{a+\frac{c\,d^4}{e^4}}\,\sqrt{c\,x^4+a}}\right)}{\sqrt{a+\frac{c\,d^4}{e^4}}\;e^5}+\frac{2\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;c\,d^2\operatorname{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;x, i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;\sqrt{c\,x^4+a}\;e^4}-\frac{2\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{c}}{\cdots}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/(e*x+d)^2,x)
```

```
[Out] -1/e*(c*x^4+a)^(1/2)/(e*x+d)+2*c*d^2/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1
/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*El
lipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-c^(1/2)*d/e^3*ln(2*c^(1/2)*x^2+2*(c*
x^4+a)^(1/2))+2*I*c^(1/2)/e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)
*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*(Elli
pticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,
I))+c*d^3/e^5/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*d^2/e^2*x^2+2*a)/(a+c*d^
4/e^4)^(1/2)/(c*x^4+a)^(1/2))-2*c*d^2/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(
1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*E
llipticPi((I/a^(1/2)*c^(1/2))^(1/2)*x,-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2
)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+a}}{{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/(d + e*x)^2,x)
```

```
[Out] int((a + c*x^4)^(1/2)/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^4}}{\left(d + ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + c*x**4)/(d + e*x)**2, x)
```

3.212 $\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=295

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(3\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,c^{3/4}\sqrt{a+cx^4}}-\frac{3\sqrt[4]{a}\,de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(\ldots\right.}{c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 3/2*d^2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*e^3*(c*x^4+a)^(1
/2)/c+3*d*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-3*a^(1/4)*d*e
^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4
)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^
(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/
2*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e^2*a^(1/2)
+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/
2)/a^(1/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1885, 1198, 220, 1196, 1248, 641, 217, 206}

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(3\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,c^{3/4}\sqrt{a+cx^4}}-\frac{3\sqrt[4]{a}\,de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(\ldots\right.}{c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (e^3*Sqrt[a + c*x^4])/(2*c) + (3*d*e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a]
 + Sqrt[c]*x^2)) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt
[c]) - (3*a^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sq
rt[a + c*x^4]) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], 1/2])/(2*a^(1/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \left(\frac{d^3+3de^2x^2}{\sqrt{a+cx^4}}+\frac{x\left(3d^2e+e^3x^2\right)}{\sqrt{a+cx^4}}\right)dx$$

$$= \int \frac{d^3+3de^2x^2}{\sqrt{a+cx^4}}\,dx+\int \frac{x\left(3d^2e+e^3x^2\right)}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{3d^2e+e^3x}{\sqrt{a+cx^2}}\,dx,x,x^2\right)-\frac{\left(3\sqrt{a}\,de^2\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}+\left(d\left(d^2+\frac{3\sqrt{a}\,e^2}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,d$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{3\sqrt[4]{a}\,de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}-$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{3\sqrt[4]{a}\,de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}-$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{3d^2e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}-\frac{3\sqrt[4]{a}\,de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x\right.}}}{c^{3/4}\sqrt{a+cx}}$$

Mathematica [C] time = 0.14, size = 157, normalized size = 0.53

$$\frac{d^3x\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}+\frac{3d^2e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{de^2x^3\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}+\frac{e^3\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (e^3*Sqrt[a + c*x^4])/(2*c) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4
]])/(2*Sqrt[c]) + (d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/
4, -((c*x^4)/a)])/Sqrt[a + c*x^4] + (d*e^2*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^3}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

maple [C] time = 0.02, size = 218, normalized size = 0.74

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,d^3\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{3d^2e\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}+\frac{3i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*e^3*(c*x^4+a)^(1/2)/c+3*I*d*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a
^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)
/c^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(
1/2))^(1/2)*x,I))+3/2*e*d^2*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))/c^(1/2)+d^3/(I/
a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*
x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^3}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x)^3/(a + c*x^4)^(1/2), x)
```

sympy [A] time = 4.19, size = 141, normalized size = 0.48

$$e^3\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for}\ c=0\\ \frac{\sqrt{a+cx^4}}{2c} & \text{otherwise}\end{cases}\right)+\frac{3d^2e\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}}+\frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{3de^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] e**3*Piecewise((x**4/(4*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**4)/(2*c), True)
) + 3*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d**3*x*gamma(1/4)*hy
per((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) +
3*d*e**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a
)/(4*sqrt(a)*gamma(7/4))
```

3.213 $\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=263

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\,e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\right.}{c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] d*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+e^2*x*(c*x^4+a)^(1/2)/c^(1
/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^
2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e^2+d^2*c^(1/2)/a^(1/2))*
((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1885, 275, 217, 206, 1198, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\,e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\right.}{c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (d*e*ArcTanh[(S
qrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] - (a^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sqrt[c]*d^2)/Sqrt[
a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4
])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx &= \int \left(\frac{2dex}{\sqrt{a+cx^4}}+\frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\right)dx\\
&= (2de)\int \frac{x}{\sqrt{a+cx^4}}\,dx+\int \frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx\\
&= (de)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right)-\frac{\left(\sqrt{a}\,e^2\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}+\left(d^2+\frac{\sqrt{a}\,e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx\\
&= \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right.}{}\\
&= \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{de\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}}-\frac{\sqrt[4]{a}\,e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.15, size = 133, normalized size = 0.51

$$\frac{d^2x\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}+\frac{de\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}}+\frac{e^2x^3\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (d*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] + (d^2*x*Sqrt[1 + (c*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4] + (e
```

```
^2*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/
(3*Sqrt[a + c*x^4])
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^2x^2+2\,dex+d^2}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

maple [C] time = 0.02, size = 197, normalized size = 0.75

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,d^2\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{de\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{\sqrt{c}}+\frac{i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] I*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I
/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF((I/a^(1/2)
*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))+d*e*ln(c^(1/
2)*x^2+(c*x^4+a)^(1/2))/c^(1/2)+d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c
^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*Ellipti
cF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x)^2/(a + c*x^4)^(1/2), x)
```

sympy [C] time = 3.44, size = 105, normalized size = 0.40

$$\frac{de\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}}+\frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] d*e*asinh(sqrt(c)*x**2/sqrt(a))/sqrt(c) + d**2*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e**2*x**3*gam
ma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamm
a(7/4))
```

3.214 $\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] 1/2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*d*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^
(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1885, 220, 275, 217, 206}

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
```

```
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx &= \int \left(\frac{d}{\sqrt{a+cx^4}}+\frac{ex}{\sqrt{a+cx^4}}\right)dx \\
&= d\int \frac{1}{\sqrt{a+cx^4}}\,dx + e\int \frac{x}{\sqrt{a+cx^4}}\,dx \\
&= \frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}} + \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, x^2\right) \\
&= \frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}} + \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{a+cx^4}}\right) \\
&= \frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.65

$$\frac{dx\sqrt{\frac{cx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}} + \frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*x*Sqrt[1 + (c*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ex+d}{\sqrt{cx^4+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex+d}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

maple [C] time = 0.01, size = 96, normalized size = 0.79

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}+1}\,d\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{e\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*e*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))/c^(1/2)+d/(I/a^(1/2)*c^(1/2))^(1/2)*(
-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(
1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ex+d}{\sqrt{cx^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{d+e\,x}{\sqrt{c\,x^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x)/(a + c*x^4)^(1/2), x)
```

sympy [C] time = 2.32, size = 61, normalized size = 0.50

$$\frac{e\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}\right)}{2\sqrt{c}}+\frac{dx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^{4}e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d*x*gamma(1/4)*hyper((1/4, 1/2)
, (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

3.215 $\int \frac{1}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

```
[Out] 1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^4+a)
^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {220}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + c*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]
)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx=\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.03, size = 74, normalized size = 0.84

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + c*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
 -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

maple [C] time = 0.00, size = 70, normalized size = 0.80

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^(1/2),x)
[Out] 1/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(
1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

mupad [B] time = 2.63, size = 37, normalized size = 0.42

$$\frac{x\,\sqrt{\frac{c\,x^4}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{c\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^(1/2),x)
[Out] (x*((c*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(c*x^4)/a))/(a + c*x^4
)^(1/2)
```

sympy [C] time = 0.73, size = 36, normalized size = 0.41

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*gamma(5/4))
```

3.216 $\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=405

$$\frac{e\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}}+\frac{\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\cdots\right.}{4\sqrt[4]{a}\sqrt[4]{c}\,d\cdots}$$

```
[Out] 1/2*e*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)^(1/
2)-1/2*e*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(a*
e^4+c*d^4)^(1/2)+1/2*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2
)/a^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/4*(cos(2*arctan(c^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c
^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^
4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/(e^2*a^(1/2)+d^2*c^(1
/2))/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1725, 1217, 220, 1707, 1248, 725, 206}

$$\frac{e\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}}-\frac{e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}+\frac{\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}-\cdots\left(\sqrt{a}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (e*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*Sqrt[-(c*d^
4) - a*e^4]) - (e*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a +
 c*x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + (c^(1/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[
c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*(Sqr
t[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = d\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx - e\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx$$

$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{\left(d^2-e^2x\right)\sqrt{a+cx^2}}\,dx, x, x^2\right)\right) + \frac{\left(\sqrt{c}\,d\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}\,d^2+\sqrt{a}\,e^2} + \frac{\left(\sqrt{a}\,de^2\right)\int \frac{}{(d}}{\sqrt{c}\,d^2}$$

$$= \frac{e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} + \frac{\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+cx^4}} - \frac{\left(\sqrt{c}\,d^2\right.}{}$$

$$= \frac{e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}} + \frac{\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,I}{2\sqrt[4]{a}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)}$$

Mathematica [C] time = 0.26, size = 200, normalized size = 0.49

$$\frac{\sqrt{\frac{cx^4}{a}+1}\left(\sqrt[4]{c}\,d\log\left(\frac{e^2x^2-d^2}{ae^2\left(\sqrt{\frac{cx^4}{a}+1}\sqrt{\frac{cd^4}{ae^4}+1}+1\right)+cd^2x^2}\right)-2\sqrt[4]{-1}\sqrt[4]{a}\,e\sqrt{\frac{cd^4}{ae^4}+1}\,\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)\right)}{2\sqrt[4]{c}\,de\sqrt{a+cx^4}\sqrt{\frac{cd^4}{ae^4}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(-2*(-1)^(1/4)*a^(1/4)*Sqrt[1 + (c*d^4)/(a*e^4)]*e*Ell
ipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4
)], -1] + c^(1/4)*d*Log[(-d^2 + e^2*x^2)/(c*d^2*x^2 + a*e^2*(1 + Sqrt[1 + (
c*d^4)/(a*e^4)]*Sqrt[1 + (c*x^4)/a]))]))/(2*c^(1/4)*d*Sqrt[1 + (c*d^4)/(a*e
^4)]*e*Sqrt[a + c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

maple [C] time = 0.02, size = 169, normalized size = 0.42

$$\frac{\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,e\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}\,d}-\frac{\mathrm{arctanh}\left(\frac{\frac{2c\,d^2x^2}{e^2}+2a}{2\sqrt{a+\frac{c\,d^4}{e^4}}\sqrt{c\,x^4+a}}\right)}{2\sqrt{a+\frac{c\,d^4}{e^4}}}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/e*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*d^2/e^2*x^2+2*a)/(a+c*d^4/e^
4)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(-I/a^(1/2)*c^(1/
2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(
(I/a^(1/2)*c^(1/2))^(1/2)*x,-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))
^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^4+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)), x)
```

3.217 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=610

$$-\frac{c^{3/4}d^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\Pi\left(\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\sqrt{a}\,\sqrt{c}\,d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt{a+cx^4}\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)\left(ae^4+cd^4\right)}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}+$$

```
[Out] -c*d^3*e*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)^
(3/2)-c*d^3*e*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2)
)/(a*e^4+c*d^4)^(3/2)-e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)+e^2*x*c^(1/
2)*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*c^(1/4)*e^2*
(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))
*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/
2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)
+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1
/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(e^2*a^(
1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/2*c^(3/4)*d^2*(cos(2*arctan(c^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2)
,1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^4)/(e^2*a^(1/2)+d^2*c^(1/2
))/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1727, 1742, 12, 1248, 725, 206, 1715, 1196, 1709, 220, 1707}

$$-\frac{c^{3/4}d^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\Pi\left(\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\sqrt{a}\,\sqrt{c}\,d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt{a+cx^4}\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)\left(ae^4+cd^4\right)}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -((e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x))) + (Sqrt[c]*e^2*x*Sqrt[
a + c*x^4])/((c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) - (c*d^3*e*ArcTan[(Sq
rt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(-(c*d^4) - a*e^4)^(3/2) -
(c*d^3*e*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
)/(c*d^4 + a*e^4)^(3/2) - (a^(1/4)*c^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], 1/2])/((c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/
4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4
]) - (c^(3/4)*d^2*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^
2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a
^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1727

```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[(
e^3*(d + e*x)^(q + 1)*Sqrt[a + c*x^4])/((q + 1)*(c*d^4 + a*e^4)), x] + Dist
[c/((q + 1)*(c*d^4 + a*e^4)), Int[((d + e*x)^(q + 1)*Simp[d^3*(q + 1) - d^2
*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x])/Sqrt[a + c*x^4], x]
, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

Rule 1742

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} - \frac{c\int \frac{-d^3+d^2ex-de^2x^2-e^3x^3}{(d+ex)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} - \frac{c\int \frac{2d^3ex}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4} - \frac{c\int \frac{-d^4-2d^2e^2x^2+e^4x^4}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} + \frac{\int \frac{cd^4e^2+\sqrt{a}\sqrt{c}\,d^2e^4+\left(2cd^2e^4-e^4\left(cd^2+\sqrt{a}\sqrt{c}\,e^2\right)\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx}{e^2\left(cd^4+ae^4\right)} - \frac{\left(2cd^3e\right)\int \frac{}{}}{cd}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} + \frac{\sqrt{c}\,e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\sqrt[4]{c}\,e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{}{(\sqrt{}}}}{\left(cd^4+ae^4\right)}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} + \frac{\sqrt{c}\,e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{cd^3e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{\left(-cd^4-ae^4\right)^{3/2}} - \frac{\sqrt[4]{}}{}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)(d+ex)} + \frac{\sqrt{c}\,e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{cd^3e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{\left(-cd^4-ae^4\right)^{3/2}} - \frac{cd}{}$$

Mathematica [C] time = 1.11, size = 425, normalized size = 0.70

$$-\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(2\sqrt[4]{-1}\sqrt[4]{a}\,c^{3/4}d^2\sqrt{\frac{cx^4}{a}+1}\,(d+ex)\sqrt{ae^4+cd^4}\,\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)+e^3\left(a+cx^4\right)\sqrt{ae^4+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*e^2*Sqrt[c*d^4 + a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]*Elli
pticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*Sqrt[c]*(Sqrt[c]*d^2
```

```
+ I*Sqrt[a]*e^2)*Sqrt[c*d^4 + a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]*Elliptic
F[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - Sqrt[(I*Sqrt[c])/Sqrt[a]]*(
e^3*Sqrt[c*d^4 + a*e^4]*(a + c*x^4) + c*d^3*e*(d + e*x)*Sqrt[a + c*x^4]*Arc
Tanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])] + 2*(-1)^(1
/4)*a^(1/4)*c^(3/4)*d^2*Sqrt[c*d^4 + a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]*E
llipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1
/4)], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*(c*d^4 + a*e^4)^(3/2)*(d + e*x)*Sqrt
[a + c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + a}\,(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

maple [C] time = 0.03, size = 421, normalized size = 0.69

$$-\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;c\,d^2\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x, i\right)}{\left(a\,e^4+c\,d^4\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x, i\right)+\right.}{\left(a\,e^4+c\,d^4\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] -e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)-c*d^2/(a*e^4+c*d^4)/(I/a^(1/2)*c
^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1
/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)+I*c^(1/2)*e^2/
(a*e^4+c*d^4)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^
(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF((I/a^(1/2)
*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))+2*c*d^3/(a*e
^4+c*d^4)/e*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*d^2/e^2*x^2+2*a)/(a+
c*d^4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(-I/a^(1/
2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*Ell
ipticPi((I/a^(1/2)*c^(1/2))^(1/2)*x,-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*
c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + a}\,(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^4+a}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + c x^{4}} \left(d + e x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)**2), x)
```

3.218 $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=659

$$\frac{c^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(ae^4+cd^4\right)}+\frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(ae^4+cd^4\right)^2}-\frac{3\sqrt[4]{a}\,c^{5/4}d^3e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt{a+cx^4}}$$

```
[Out] 3/2*c*d^2*e*(-a*e^4+c*d^4)*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2
))/(-a*e^4-c*d^4)^(5/2)-3/2*c*d^2*e*(-a*e^4+c*d^4)*arctanh((c*d^2*x^2+a*e^2
)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(5/2)-1/2*e^3*(c*x^4+a
)^(1/2)/(a*e^4+c*d^4)/(e*x+d)^2-3*c*d^3*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)^2
/(e*x+d)+3*c^(3/2)*d^3*e^2*x*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)^2/(a^(1/2)+x^2*c
^(1/2))-3*a^(1/4)*c^(5/4)*d^3*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
1/2)/(a*e^4+c*d^4)^2/(c*x^4+a)^(1/2)+1/2*c^(3/4)*d*(cos(2*arctan(c^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+
x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)-3/4*c^(3/4)*d*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2
/d^2/e^2/a^(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))^2*(a^(1/2)
+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^
4)^2/(c*x^4+a)^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1727, 1739, 1742, 12, 1248, 725, 206, 1715, 1196, 1709, 220, 1707}

$$\frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(ae^4+cd^4\right)^2}+\frac{c^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(ae^4+cd^4\right)}-\frac{3\sqrt[4]{a}\,c^{5/4}d^3e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] -(e^3*Sqrt[a + c*x^4])/(2*(c*d^4 + a*e^4)*(d + e*x)^2) - (3*c*d^3*e^3*Sqrt[
a + c*x^4])/((c*d^4 + a*e^4)^2*(d + e*x)) + (3*c^(3/2)*d^3*e^2*x*Sqrt[a + c
*x^4])/((c*d^4 + a*e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (3*c*d^2*e*(c*d^4 - a*
e^4)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*(-(c*d^4)
 - a*e^4)^(5/2)) - (3*c*d^2*e*(c*d^4 - a*e^4)*ArcTanh[(a*e^2 + c*d^2*x^2)/(
Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*(c*d^4 + a*e^4)^(5/2)) - (3*a^(1/
4)*c^(5/4)*d^3*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/((c*d^4 + a*e^4)
^2*Sqrt[a + c*x^4]) + (c^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2
*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (3*c^(3/4)*d*(Sqrt[c]*d^2 - Sqr
t[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2),
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(c*d^4 + a*e^4)^2*Sqrt[a +
c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
```

```
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1727

```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[(
e^3*(d + e*x)^(q + 1)*Sqrt[a + c*x^4])/((q + 1)*(c*d^4 + a*e^4)), x] + Dist
[c/((q + 1)*(c*d^4 + a*e^4)), Int[((d + e*x)^(q + 1)*Simp[d^3*(q + 1) - d^2
*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x])/Sqrt[a + c*x^4], x]
, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

Rule 1739

```
Int[((Px_)*((d_) + (e_.)*(x_))^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :
> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D =
Coeff[Px, x, 3]}, -Simp[((d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*(d + e*x)^(q +
 1)*Sqrt[a + c*x^4])/((q + 1)*(c*d^4 + a*e^4)), x] + Dist[1/((q + 1)*(c*d^4
 + a*e^4)), Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp[(q + 1)*(a*e*(d^2*
D - C*d*e + B*e^2) + A*d*(c*d^2)) - (e*(q + 1)*(A*c*d^2 + a*e*(d*D - C*e))
- B*d*(c*d^2*(q + 1)))*x + (q + 1)*(D*e*(a*e^2) + c*d*(C*d^2 - e*(B*d - A*e
)))*x^2 + c*(q + 3)*(d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*x^3, x], x], x]] /;
 FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^
4 + a*e^4, 0] && LtQ[q, -1]
```

Rule 1742

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{c\int \frac{-2d^3+2d^2ex-2de^2x^2}{(d+ex)^2\sqrt{a+cx^4}}\,dx}{2\left(cd^4+ae^4\right)}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\int \frac{2d^2\left(cd^4-2ae^4\right)-2de\left(2cd^4-ae^4\right)x+6cd}{(d+ex)\sqrt{a+cx^4}}}{2\left(cd^4+ae^4\right)^2}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\int \frac{\left(-2d^2e\left(cd^4-2ae^4\right)-2d^2e\left(2cd^4-ae^4\right)\right)}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}}{2\left(cd^4+ae^4\right)^2}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} - \frac{\int \frac{-6\sqrt{a}c^{3/2}d^5e^4-2cd^3e^2\left(cd^4-2ae^4\right)+(6c}{(a}}{2e}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{3}{}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{3}{}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{3}{}$$

Mathematica [C] time = 2.52, size = 513, normalized size = 0.78

$$-\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(e^3\sqrt{ae^4+cd^4}\left(a^2e^4+ac\left(7d^4+6d^3ex+e^4x^4\right)+c^2d^3x^4(7d+6ex)\right)+6\sqrt[4]{-1}\sqrt[4]{a}c^{3/4}d\sqrt{\frac{cx^4}{a}+1}(d+ex\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (6*Sqrt[a]*c^(3/2)*d^3*e^2*Sqrt[c*d^4 + a*e^4]*(d + e*x)^2*Sqrt[1 + (c*x^4)
/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + (2*I)*c*d*(2*c*
d^4 + (3*I)*Sqrt[a]*Sqrt[c]*d^2*e^2 - a*e^4)*Sqrt[c*d^4 + a*e^4]*(d + e*x)^
2*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]
 - Sqrt[(I*Sqrt[c])/Sqrt[a]]*(e^3*Sqrt[c*d^4 + a*e^4]*(a^2*e^4 + c^2*d^3*x^
4*(7*d + 6*e*x) + a*c*(7*d^4 + 6*d^3*e*x + e^4*x^4)) + 3*c*d^2*e*(c*d^4 - a
*e^4)*(d + e*x)^2*Sqrt[a + c*x^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 +
 a*e^4]*Sqrt[a + c*x^4])] + 6*(-1)^(1/4)*a^(1/4)*c^(3/4)*d*(c*d^4 - a*e^4)*
Sqrt[c*d^4 + a*e^4]*(d + e*x)^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e
^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1]))/(2*Sqrt[(I
*Sqrt[c])/Sqrt[a]]*(c*d^4 + a*e^4)^(5/2)*(d + e*x)^2*Sqrt[a + c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}\,(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

maple [C] time = 0.03, size = 483, normalized size = 0.73

$$\frac{3i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)\right)\sqrt{a}\,c^{\frac{3}{2}}d^3e^2}{\left(a\,e^4+c\,d^4\right)^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{3\sqrt{c\,x^4+a}\,c\,d^3e^3}{\left(a\,e^4+c\,d^4\right)^2(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] -1/2*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)^2-3*c*d^3*e^3*(c*x^4+a)^(1/2
)/(a*e^4+c*d^4)^2/(e*x+d)+c*d*(a*e^4-2*c*d^4)/(a*e^4+c*d^4)^2/(I/a^(1/2)*c^
(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/
2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)+3*I*c^(3/2)*e^2
*d^3/(a*e^4+c*d^4)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*
x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF((I/
a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))-3*c*d
^2*(a*e^4-c*d^4)/(a*e^4+c*d^4)^2/e*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2
*c*d^2/e^2*x^2+2*a)/(a+c*d^4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/
2))^(1/2)/d*e*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1
/2)/(c*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)*c^(1/2))^(1/2)*x,-I*a^(1/2)/c^(1/
2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+a}\,(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^4+a}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x)^3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^4}\,(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)**3), x)
```

3.219 $\int \frac{(d+ex)^3}{\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=298

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{3de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}E\left(2\tan^{-1}\right.}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 1/2*(-a*e^3+c*x*(3*d*e^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)^(1/2)-3/2*d*e^2*
x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1/2)+x^2*c^(1/2))+3/2*d*e^2*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/4*d*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellip
ticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-3*e^2*a^(1/2)+d^2*c^(1
/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(5/4
)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1854, 1198, 220, 1196}

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{3de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}E\left(2\tan^{-1}\right.}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

```
[Out] (-3*d*e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a*e^3
 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(2*a*c*Sqrt[a + c*x^4]) + (3*d*e^2*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4])
 + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])
/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+cx^4\right)^{3/2}}\,dx = -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}-\frac{\int \frac{-d^3+3de^2x^2}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}+\frac{\left(3de^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\sqrt{c}}+\frac{\left(d\left(d^2-\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= -\frac{3de^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}+\frac{3de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x\right.}}}{2a^{3/4}c^{3/4}\sqrt{a}}$$

Mathematica [C] time = 0.07, size = 126, normalized size = 0.42

$$\frac{cd^3x\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+2cde^2x^3\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^4}{a}\right)-ae^3+cd^3x+3cd^2ex^2}{2ac\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

```
[Out] (-(a*e^3) + c*d^3*x + 3*c*d^2*e*x^2 + c*d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + 2*c*d*e^2*x^3*Sqrt[1 + (c*x^4)/a]*H
ypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(2*a*c*Sqrt[a + c*x^4])
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3\right)\sqrt{cx^4+a}}{c^2x^8+2\,acx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^4 + a)/(c^2*x^8
 + 2*a*c*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

maple [C] time = 0.03, size = 261, normalized size = 0.88

$$\frac{3d^2 e\,x^2}{2\sqrt{c\,x^4+a}\;a}+\left(\frac{x}{2\sqrt{\left(x^4+\frac{a}{c}\right)c}\;a}+\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;x,i\right)}{2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;\sqrt{c\,x^4+a}\;a}\right)d^3+3\left(\frac{x^3}{2\sqrt{\left(x^4+\frac{a}{c}\right)c}\;a}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^(3/2),x)
[Out] -1/2*e^3/c/(c*x^4+a)^(1/2)+3*d*e^2*(1/2*x^3/a/((x^4+a/c)*c)^(1/2)-1/2*I/a^(
1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*
c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))
^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I)))+3/2*e*d^2/(c*x^4+a)^
(1/2)/a*x^2+d^3*(1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2
)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a
)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^3}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(d+e\,x)^3}{\left(c\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4)^(3/2),x)
[Out] int((d + e*x)^3/(a + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(d+ex)^3}{\left(a+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**(3/2),x)
[Out] Integral((d + e*x)**3/(a + c*x**4)**(3/2), x)
```

3.220 $\int \frac{(d+ex)^2}{\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=270

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\cdots\right.\right.}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 1/2*x*(e*x+d)^2/a/(c*x^4+a)^(1/2)-1/2*e^2*x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1
/2)+x^2*c^(1/2))+1/2*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3
/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4)))
,1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(5/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1855, 1198, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\cdots\right.\right.}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(d + e*x)^2)/(2*a*Sqrt[a + c*x^4]) - (e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c
]*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rubi steps

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{\int \frac{-d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} + \frac{e^2\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\,\sqrt{c}} + \frac{\left(d^2-\frac{\sqrt{a}\,e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} +$$

Mathematica [C] time = 0.06, size = 108, normalized size = 0.40

$$\frac{x\left(3d^2\sqrt{\frac{cx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+2e^2x^2\sqrt{\frac{cx^4}{a}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^4}{a}\right)+3d(d+2ex)\right)}{6a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(3*d*(d + 2*e*x) + 3*d^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2,
 5/4, -((c*x^4)/a)] + 2*e^2*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[3/4,
3/2, 7/4, -((c*x^4)/a)]))/(6*a*Sqrt[a + c*x^4])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+a}\left(e^2x^2+2\,dex+d^2\right)}{c^2x^8+2\,acx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*(e^2*x^2 + 2*d*e*x + d^2)/(c^2*x^8 + 2*a*c*x^4 + a
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

maple [C] time = 0.02, size = 239, normalized size = 0.89

$$\frac{d e x^2}{\sqrt{c x^4+a}\, a}+\left(\frac{x}{2\sqrt{\left(x^4+\frac{a}{c}\right) c}\, a}+\frac{\sqrt{-\frac{i\sqrt{c}\, x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\, x^2}{\sqrt{a}}+1}\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x, i\right)}{2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}\, a}\right) d^2+\left(\frac{x^3}{2\sqrt{\left(x^4+\frac{a}{c}\right) c}\, a}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^(3/2),x)
```

```
[Out] e^2*(1/2/((x^4+a/c)*c)^(1/2)/a*x^3-1/2*I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*
(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^
(1/2)/c^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2
)*c^(1/2))^(1/2)*x,I)))+d*e/(c*x^4+a)^(1/2)/a*x^2+d^2*(1/2/((x^4+a/c)*c)^(1
/2)/a*x+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I
/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))
^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{\left(cx^4+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{\left(c\,x^4+a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4)^(3/2),x)
```

```
[Out] int((d + e*x)^2/(a + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{\left(a+cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2/(a + c*x**4)**(3/2), x)
```

3.221 $\int \frac{d+ex}{\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

```
[Out] 1/2*x*(e*x+d)/a/(c*x^4+a)^(1/2)+1/4*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^
2)^(1/2)/a^(5/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1855, 12, 220}

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(d + e*x))/(2*a*Sqrt[a + c*x^4]) + (d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
1/2])/(4*a^(5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+cx^4\right)^{3/2}}\,dx = \frac{x(d+ex)}{2a\sqrt{a+cx^4}} + \frac{\int \frac{d}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= \frac{x(d+ex)}{2a\sqrt{a+cx^4}} + \frac{d\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= \frac{x(d+ex)}{2a\sqrt{a+cx^4}} + \frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.52

$$\frac{x\left(d\sqrt{\frac{cx^4}{a}+1}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+d+ex\right)}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4)^(3/2),x]
[Out] (x*(d + e*x + d*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x
^4)/a)]))/(2*a*Sqrt[a + c*x^4])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+a}\,(ex+d)}{c^2x^8+2\,acx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + a)*(e*x + d)/(c^2*x^8 + 2*a*c*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex+d}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

maple [C] time = 0.01, size = 115, normalized size = 1.01

$$\frac{e\,x^2}{2\sqrt{c\,x^4+a}\,a} + \left(\frac{x}{2\sqrt{\left(x^4+\frac{a}{c}\right)c}\,a} + \frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,a}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^(3/2),x)
```

```
[Out] 1/2*e/(c*x^4+a)^(1/2)/a*x^2+d*(1/2/((x^4+a/c)*c)^(1/2)/a*x+1/2/a/(I/a^(1/2)
*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^
(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex+d}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

mupad [B] time = 2.88, size = 57, normalized size = 0.50

$$\frac{e\,x^2}{2\,a\,\sqrt{c\,x^4+a}}+\frac{d\,x\,{\left(\frac{c\,x^4}{a}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{{\left(c\,x^4+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4)^(3/2),x)
```

```
[Out] (e*x^2)/(2*a*(a + c*x^4)^(1/2)) + (d*x*((c*x^4)/a + 1)^(3/2)*hypergeom([1/4
, 3/2], 5/4, -(c*x^4)/a))/(a + c*x^4)^(3/2)
```

sympy [C] time = 8.09, size = 61, normalized size = 0.54

$$\frac{dx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{cx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3
/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + c*x**4/a))
```

3.222 $\int \frac{1}{(a+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{x}{2a\sqrt{a+cx^4}}$$

```
[Out] 1/2*x/a/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a
^(5/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {199, 220}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{x}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-3/2),x]
```

```
[Out] x/(2*a*Sqrt[a + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/
4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+cx^4\right)^{3/2}}\,dx &= \frac{x}{2a\sqrt{a+cx^4}}+\frac{\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}\\ &= \frac{x}{2a\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.51

$$\frac{x\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+x}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-3/2),x]
[Out] (x + x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/
(2*a*Sqrt[a + c*x^4])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+a}}{c^2x^8+2\,acx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + a)/(c^2*x^8 + 2*a*c*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + a)^(-3/2), x)
```

maple [C] time = 0.00, size = 94, normalized size = 0.87

$$\frac{x}{2\sqrt{\left(x^4+\frac{a}{c}\right)c}\;a}+\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;\operatorname{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;x,i\right)}{2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;\sqrt{c\,x^4+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^(3/2),x)
[Out] 1/2/((x^4+a/c)*c)^(1/2)/a*x+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(
1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF
((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + a)^(-3/2), x)
```

mupad [B] time = 2.66, size = 37, normalized size = 0.34

$$\frac{x\left(\frac{c\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\left(c\,x^4+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^(3/2),x)
```

```
[Out] (x*((c*x^4)/a + 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(c*x^4)/a))/(a + c*x^4
)^(3/2)
```

sympy [C] time = 0.81, size = 36, normalized size = 0.33

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**(3/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3/2
)*gamma(5/4))
```

3.223 $\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=818

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{cx^4+a}}\right)e^5}{2\left(-cd^4-ae^4\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\,\sqrt{cx^4+a}}\right)e^5}{2\left(cd^4+ae^4\right)^{3/2}}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}\,x^2+\sqrt{a}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^4\ (\cdots}{2\sqrt[4]{a}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}-\cdots$$

```
[Out] -1/2*e^5*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)^
(3/2)-1/2*e^5*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2)
)/(a*e^4+c*d^4)^(3/2)+1/2*e*(-c*d^2*x^2+a*e^2)/a/(a*e^4+c*d^4)/(c*x^4+a)^(1
/2)+1/2*c*d*x*(e^2*x^2+d^2)/a/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)-1/2*d*e^2*x*c^(
1/2)*(c*x^4+a)^(1/2)/a/(a*e^4+c*d^4)/(a^(1/2)+x^2*c^(1/2))+1/2*c^(1/4)*d*e^
2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)
))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(
1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(a*e^4+c*d^4)/(c*x^
4+a)^(1/2)+1/4*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)
+x^2*c^(1/2))^2)^(1/2)/a^(5/4)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)+1/2*c^(1/4)*d*
e^4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^4)/(e^
2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/4*e^4*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),1/
2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/(a*e^4+c*d^4)/(e^2*a^(1/2)+d^2
*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1729, 1222, 1179, 1198, 220, 1196, 1217, 1707, 1248, 741, 12, 725, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{cx^4+a}}\right)e^5}{2\left(-cd^4-ae^4\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\,\sqrt{cx^4+a}}\right)e^5}{2\left(cd^4+ae^4\right)^{3/2}}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}\,x^2+\sqrt{a}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^4\ (\cdots}{2\sqrt[4]{a}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

```
[Out] (e*(a*e^2 - c*d^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c*d*x*(d^2
 + e^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (Sqrt[c]*d*e^2*x*Sqrt[
a + c*x^4])/(2*a*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) - (e^5*ArcTan[(Sq
rt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*(-(c*d^4) - a*e^4)^(3/2)
) - (e^5*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
)/(2*(c*d^4 + a*e^4)^(3/2)) + (c^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(2*a^(3/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*(Sqrt[c]
*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c*d
^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*e^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a +
```

```
c*x^4]) - (e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^
2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1222

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/(
c*d^2 + a*e^2), Int[(c*d - c*e*x^2)*(a + c*x^4)^p, x], x] + Dist[e^2/(c*d^2
 + a*e^2), Int[(a + c*x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = d\int \frac{1}{\left(d^2-e^2x^2\right)\left(a+cx^4\right)^{3/2}}\,dx - e\int \frac{x}{\left(d^2-e^2x^2\right)\left(a+cx^4\right)^{3/2}}\,dx$$

$$= -\left(\frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{\left(d^2-e^2x\right)\left(a+cx^2\right)^{3/2}}\,dx, x, x^2\right)\right) + \frac{d\int \frac{cd^2+ce^2x^2}{\left(a+cx^4\right)^{3/2}}\,dx}{cd^4+ae^4} + \frac{\left(de^4\right)\int \frac{}{\left(\right.}}{c}$$

$$= \frac{e\left(ae^2-cd^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{d\int \frac{-cd^2+ce^2x^2}{\sqrt{a+cx^4}}\,dx}{2a\left(cd^4+ae^4\right)} - \frac{e\,\text{Subs}}{}$$

$$= \frac{e\left(ae^2-cd^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{e^5\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\left(-cd^4-ae^4\right)^{3/2}} + -\frac{\sqrt[4]{}}{}$$

$$= \frac{e\left(ae^2-cd^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{\sqrt{c}\,de^2x\sqrt{a+cx^4}}{2a\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}\,x\right.}$$

$$= \frac{e\left(ae^2-cd^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{\sqrt{c}\,de^2x\sqrt{a+cx^4}}{2a\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}\,x\right.}$$

Mathematica [C] time = 0.88, size = 434, normalized size = 0.53

$$\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(\sqrt[4]{c}\,d\left(\sqrt{ae^4+cd^4}\left(ae^3+cdx\left(d^2-dex+e^2x^2\right)\right)-ae^5\sqrt{a+cx^4}\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)\right)-2\sqrt[4]{-1}\,a^{5/4}e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[c*d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a]*Elliptic
E[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]) + c^(3/4)*d^2*((-I)*Sqrt[c]*
d^2 + Sqrt[a]*e^2)*Sqrt[c*d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcS
inh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[(I*Sqrt[c])/Sqrt[a]]*(c^(1/4)*
d*(Sqrt[c*d^4 + a*e^4]*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)) - a*e^5*Sqrt
[a + c*x^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4
])]) - 2*(-1)^(1/4)*a^(5/4)*e^4*Sqrt[c*d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a]*Ell
ipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4
)], -1]))/(2*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/4)*d*(c*d^4 + a*e^4)^(3/2)*Sq
rt[a + c*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+a\right)^{\frac{3}{2}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

maple [C] time = 0.02, size = 496, normalized size = 0.61

$$\frac{\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,c\,d^3\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{2\left(a\,e^4+c\,d^4\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,a}-\frac{i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\mathrm{El}\right.}{2\left(a\,e^4+c\,d^4\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^(3/2),x)
```

```
[Out] -2*c*(-1/4/a*e^2*d/(a*e^4+c*d^4)*x^3+1/4/a*e*d^2/(a*e^4+c*d^4)*x^2-1/4/a*d^
3/(a*e^4+c*d^4)*x-1/4*e^3/(a*e^4+c*d^4)/c)/((x^4+a/c)*c)^(1/2)+1/2/a*c*d^3/
(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I
/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))
^(1/2)*x,I)-1/2*I/a^(1/2)*c^(1/2)*e^2*d/(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2))^(
1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^
4+a)^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c
^(1/2))^(1/2)*x,I))+e^3/(a*e^4+c*d^4)*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2
*(2*c*d^2/e^2*x^2+2*a)/(a+c*d^4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^
(1/2))^(1/2)/d*e*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)
^(1/2)/(c*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)*c^(1/2))^(1/2)*x,-I*a^(1/2)/c^
(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+a\right)^{\frac{3}{2}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c\,x^4+a\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/((a + c*x^4)^(3/2)*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^4\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**(3/2),x)
[Out] Integral(1/((a + c*x**4)**(3/2)*(d + e*x)), x)
```

3.224 $\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx$

Optimal. Leaf size=349

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+1)\left(\sqrt{-\sqrt{-a}}\,d+\sqrt[4]{b}\,c\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\Big(}{4b^{3/4}(n+}$$

```
[Out] -1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)
^(1/4)*d))/b^(3/4)/(b^(1/4)*c-(-a)^(1/4)*d)/(1+n)-1/4*(d*x+c)^(1+n)*hyperge
om([1, 1+n],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(3/4)/(b^(1/4
)*c+(-a)^(1/4)*d)/(1+n)-1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*
(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(1+n)/(b^(1/4)*c-d*(-(-a
)^(1/2))^(1/2))-1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*(d*x+c)/
(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(1+n)/(b^(1/4)*c+d*(-(-a)^(1/2))
^(1/2))
```

Rubi [A] time = 0.73, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6725, 831, 68}

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+1)\left(\sqrt{-\sqrt{-a}}\,d+\sqrt[4]{b}\,c\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\Big(}{4b^{3/4}(n+}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

```
[Out] -((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/
(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d
)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)
*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqrt[
-Sqrt[-a]]*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
 n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*c
- (-a)^(1/4)*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*
c + (-a)^(1/4)*d)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rubi steps

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \left(\frac{x(c+dx)^n}{2\left(-\sqrt{-a}\sqrt{b}+bx^2\right)}+\frac{x(c+dx)^n}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx$$
$$= \frac{1}{2}\int \frac{x(c+dx)^n}{-\sqrt{-a}\sqrt{b}+bx^2}\,dx+\frac{1}{2}\int \frac{x(c+dx)^n}{\sqrt{-a}\sqrt{b}+bx^2}\,dx$$
$$= \frac{1}{2}\int \left(-\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}\,x\right)}+\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}\,x\right)}\right)dx+\frac{1}{2}\int \left(-\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt[4]{-a}-\right.}\right.$$
$$= -\frac{\int \frac{(c+dx)^n}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}-\frac{\int \frac{(c+dx)^n}{\sqrt[4]{-a}-\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^n}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^n}{\sqrt[4]{-a}+\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}$$
$$= -\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}\,d\right)(1+n)}-\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}(c+}{\sqrt[4]{b}c+\sqrt{-}}\right.}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}\,d\right)(1+n)}$$

Mathematica [C] time = 0.43, size = 274, normalized size = 0.79

$$\frac{(c+dx)^{n+1}\left(-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}\right.}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right.}{4b^{3/4}(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

```
[Out] ((c + d*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x)
)/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometric2
F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b^(1
/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c +
d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hypergeo
metric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/
(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(1 + n))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(dx+c)^n x^3}{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n*x^3/(b*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(d x +c \right)^{n}}{b \,x^{4}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

```
[Out] int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(d x + c\right)}^{n} x^{3}}{b x^{4} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c+d\,x\right)}^n}{b\,x^4+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x)^n)/(a + b*x^4),x)
```

```
[Out] int((x^3*(c + d*x)^n)/(a + b*x^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x+c)**n/(b*x**4+a),x)
```

```
[Out] Timed out
```

3.225 $\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx$

Optimal. Leaf size=349

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}\,d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+2)\left(\sqrt{-\sqrt{-a}}\,d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+2}\,{}_2}{4b^{3/4}(}$$

```
[Out] -1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)
^(1/4)*d))/b^(3/4)/(b^(1/4)*c-(-a)^(1/4)*d)/(2+n)-1/4*(d*x+c)^(2+n)*hyperge
om([1, 2+n],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(3/4)/(b^(1/4
)*c+(-a)^(1/4)*d)/(2+n)-1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*
(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(2+n)/(b^(1/4)*c-d*(-(-a
)^(1/2))^(1/2))-1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*(d*x+c)/
(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(2+n)/(b^(1/4)*c+d*(-(-a)^(1/2))
^(1/2))
```

Rubi [A] time = 0.57, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6725, 831, 68}

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}\,d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+2)\left(\sqrt{-\sqrt{-a}}\,d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+2}\,{}_2}{4b^{3/4}(}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

```
[Out] -((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/
(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d
)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)
*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqrt[
-Sqrt[-a]]*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 +
 n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*c
- (-a)^(1/4)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3
 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*
c + (-a)^(1/4)*d)*(2 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \left(\frac{x(c+dx)^{1+n}}{2\left(-\sqrt{-a}\sqrt{b}+bx^2\right)}+\frac{x(c+dx)^{1+n}}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx$$
$$= \frac{1}{2}\int \frac{x(c+dx)^{1+n}}{-\sqrt{-a}\sqrt{b}+bx^2}\,dx+\frac{1}{2}\int \frac{x(c+dx)^{1+n}}{\sqrt{-a}\sqrt{b}+bx^2}\,dx$$
$$= \frac{1}{2}\int \left(-\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}\,x\right)}+\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}\,x\right)}\right)dx+\frac{1}{2}\int \left(-\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt[4]{-a}-\sqrt[4]{\right.}}\right.$$
$$= -\frac{\int \frac{(c+dx)^{1+n}}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}-\frac{\int \frac{(c+dx)^{1+n}}{\sqrt[4]{-a}-\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^{1+n}}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^{1+n}}{\sqrt[4]{-a}+\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}$$
$$= -\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}\,d\right)(2+n)}-\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}(c+\ldots}{\sqrt[4]{b}c+\sqrt{-\ldots}}\right.}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}\,d\right)(2+n)}$$

Mathematica [C] time = 0.25, size = 274, normalized size = 0.79

$$\frac{(c+dx)^{n+2}\left(-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c+i\sqrt[4]{-a}d}-\frac{{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{\sqrt[4]{-a}d+\sqrt[4]{b}c}\right)}{4b^{3/4}(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

```
[Out] ((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x)
)/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometric2
F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b^(1
/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c +
d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hypergeo
metric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/
(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(2 + n))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx+c)^{n+1}x^3}{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(dx+c\right)^{n+1}}{b\,x^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x+c)^(n+1)/(b*x^4+a),x)
```

```
[Out] int(x^3*(d*x+c)^(n+1)/(b*x^4+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx+c\right)^{n+1} x^{3}}{bx^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3} \left(c+d\,x\right)^{n+1}}{b\,x^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x)^(n + 1))/(a + b*x^4),x)
```

```
[Out] int((x^3*(c + d*x)^(n + 1))/(a + b*x^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x+c)**(1+n)/(b*x**4+a),x)
```

```
[Out] Timed out
```

3.226 $\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=1605

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bced^2-b\sqrt{d^2-4ce}\,(d^2-2ce)d-2ae^4-2bc^2e^2}\,x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4+\sqrt{d^2-4ce}\,d^3-4ced^2-2ce\sqrt{d^2-4ce}\,d+2c^2e^2\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-bd^4 \cdots}}{\cdots}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d \cdots\right.}}$$

```
[Out] 1/2*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x^2*b^(1/2))*(d-(-4*c*e+d^2)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2
)^(1/2)/a^(1/4)/(-4*c*e+d^2)^(1/2)/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*(-4*
c*e+d^2)^(1/2)))/(b*x^4+a)^(1/2)+1/2*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/4*(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2)))^2/e^2
/a^(1/2)/b^(1/2)/(d-(-4*c*e+d^2)^(1/2))^2,1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2)
)*(2*e^2*a^(1/2)-b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2)))*((b*x^4+a)/(a^(1
/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(d-(-4*c*e+d^2)^(1/2))/(-4*c*e+d^
2)^(1/2)/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2)))/(b*x^4+a)
^(1/2)-1/2*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x^2*b^(1/2))*(d+(-4*c*e+d^2)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(
1/2))^2)^(1/2)/a^(1/4)/(-4*c*e+d^2)^(1/2)/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e
+d*(-4*c*e+d^2)^(1/2)))/(b*x^4+a)^(1/2)-1/2*e*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1
/4)*x/a^(1/4))),1/4*(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e+d*(-4*c*e+d^2)^(1/2))
)^2/e^2/a^(1/2)/b^(1/2)/(d+(-4*c*e+d^2)^(1/2))^2,1/2*2^(1/2))*(a^(1/2)+x^2*
b^(1/2))*(2*e^2*a^(1/2)-b^(1/2)*(d^2-2*c*e+d*(-4*c*e+d^2)^(1/2)))*((b*x^4+a
)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(-4*c*e+d^2)^(1/2)/(d+(-4*
c*e+d^2)^(1/2))/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e+d*(-4*c*e+d^2)^(1/2)))/(b
*x^4+a)^(1/2)-1/2*e^2*arctanh(1/4*(4*a*e^2+b*x^2*(d-(-4*c*e+d^2)^(1/2))^2)*
2^(1/2)/(b*x^4+a)^(1/2)/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4-b*d*(-2*c*e+
d^2)*(-4*c*e+d^2)^(1/2))^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(b*d^4-4*b*c*d^2
*e+2*b*c^2*e^2+2*a*e^4-b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)+1/2*e^2*a
rctanh(1/4*(4*a*e^2+b*x^2*(d+(-4*c*e+d^2)^(1/2))^2)*2^(1/2)/(b*x^4+a)^(1/2)
/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4+b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2)
)^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4+
b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)-1/2*e^2*arctan(x*2^(1/2)*(-b*d^4
+4*b*c*d^2*e-2*b*c^2*e^2-2*a*e^4-b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)
/e/(d+(-4*c*e+d^2)^(1/2))/(b*x^4+a)^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(-2*a
*e^4-(-2*(-4*c*e+d^2)^(1/2)*c*d*e+d^3*(-4*c*e+d^2)^(1/2)+2*c^2*e^2-4*c*d^2*
e+d^4)*b)^(1/2)+1/2*e^2*arctan(x*2^(1/2)*(-b*d^4+4*b*c*d^2*e-2*b*c^2*e^2-2*
a*e^4+b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)/e/(d-(-4*c*e+d^2)^(1/2))/(
b*x^4+a)^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(-2*a*e^4-(2*(-4*c*e+d^2)^(1/2)*
c*d*e-d^3*(-4*c*e+d^2)^(1/2)+2*c^2*e^2-4*c*d^2*e+d^4)*b)^(1/2)
```

Rubi [A] time = 9.68, antiderivative size = 1605, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 1725, 1217, 220, 1707, 1248, 725, 206}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

```
[Out] -((e^2*ArcTan[(Sqrt[2]*Sqrt[-(b*d^4) + 4*b*c*d^2*e - 2*b*c^2*e^2 - 2*a*e^4
- b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d + Sqrt[d^2 - 4*c*e])*Sqrt[a
 + b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[-2*a*e^4 - b*(d^4 - 4*c*d^2*e
+ 2*c^2*e^2 + d^3*Sqrt[d^2 - 4*c*e] - 2*c*d*e*Sqrt[d^2 - 4*c*e])])) + (e^2*
ArcTan[(Sqrt[2]*Sqrt[-(b*d^4) + 4*b*c*d^2*e - 2*b*c^2*e^2 - 2*a*e^4 + b*d*S
qrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d - Sqrt[d^2 - 4*c*e])*Sqrt[a + b*x^
4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[-2*a*e^4 - b*(d^4 - 4*c*d^2*e + 2*c^2
*e^2 - d^3*Sqrt[d^2 - 4*c*e] + 2*c*d*e*Sqrt[d^2 - 4*c*e])]) - (e^2*ArcTanh[
(4*a*e^2 + b*(d - Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d
^2*e + 2*b*c^2*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a
+ b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e
^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (e^2*ArcTanh[(4*a*e^
2 + b*(d + Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*e +
2*b*c^2*e^2 + 2*a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a + b*x^4
])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*
a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (b^(1/4)*e*(d - Sqrt[d^2 -
4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[d^2 - 4*c*e
]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*Sqrt[a + b*
x^4]) - (b^(1/4)*e*(d + Sqrt[d^2 - 4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(2*a^(1/4)*Sqrt[d^2 - 4*c*e]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e
+ d*Sqrt[d^2 - 4*c*e]))*Sqrt[a + b*x^4]) + (e*(2*Sqrt[a]*e^2 - Sqrt[b]*(d^2
 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e
- d*Sqrt[d^2 - 4*c*e]))^2/(4*Sqrt[a]*Sqrt[b]*e^2*(d - Sqrt[d^2 - 4*c*e])^2)
, 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]
*(d - Sqrt[d^2 - 4*c*e])*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2
 - 4*c*e]))*Sqrt[a + b*x^4]) - (e*(2*Sqrt[a]*e^2 - Sqrt[b]*(d^2 - 2*c*e + d
*Sqrt[d^2 - 4*c*e]))*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
rt[b]*x^2)^2]*EllipticPi[(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e + d*Sqrt[d^2
 - 4*c*e]))^2/(4*Sqrt[a]*Sqrt[b]*e^2*(d + Sqrt[d^2 - 4*c*e])^2), 2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]*(d + Sqrt[d
^2 - 4*c*e])*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e + d*Sqrt[d^2 - 4*c*e]))*
Sqrt[a + b*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
```

```
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(c+dx+ex^2\right)\sqrt{a+bx^4}}\,dx = \int\left(\frac{2e}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}} - \frac{2e}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4c\right.}\right.$$

$$= \frac{(2e)\int \frac{1}{\left(d-\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{1}{\left(d+\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}}$$

$$= -\frac{\left(4e^2\right)\int \frac{x}{\left(\left(d-\sqrt{d^2-4ce}\right)^2-4e^2x^2\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}} + \frac{\left(4e^2\right)\int \frac{x}{\left(\left(d+\sqrt{d^2-4ce}\right)^2-4e^2x^2\right)\sqrt{a+bx^4}}\,d}{\sqrt{d^2-4ce}}$$

$$= -\frac{\left(2e^2\right)\operatorname{Subst}\left(\int \frac{1}{\left(\left(d-\sqrt{d^2-4ce}\right)^2-4e^2x\right)\sqrt{a+bx^2}}\,dx, x, x^2\right)}{\sqrt{d^2-4ce}} + \frac{\left(2e^2\right)\operatorname{Subst}\left(\int \frac{}{\left(\left(d+\sqrt{d\right.\right.}}\right.}{\sqrt{}}$$

$$= -\frac{e^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bcd^2e-2bc^2e^2-2ae^4-bd\sqrt{d^2-4ce}\left(d^2-2ce\right)}x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{a+bx^4}}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-4cd^2e+2c^2e^2+d^3\sqrt{d^2-4ce}-2cde\sqrt{d^2-}\right.}}$$

$$= -\frac{e^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bcd^2e-2bc^2e^2-2ae^4-bd\sqrt{d^2-4ce}\left(d^2-2ce\right)}x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{a+bx^4}}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-4cd^2e+2c^2e^2+d^3\sqrt{d^2-4ce}-2cde\sqrt{d^2-}\right.}}$$

Mathematica [C] time = 7.37, size = 1416, normalized size = 0.88

$$-\frac{i\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{\frac{i\sqrt{b}x^2}{\sqrt{a}}+1}\,\Pi\left(-\frac{2i\sqrt{a}e^2}{\sqrt{b}\left(-d^2+2ce-\sqrt{d^4-4cd^2e}\right)};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right)\middle|-1\right)d^2}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}e\left(-d^2+2ce-\sqrt{d^4-4cd^2e}\right)\left(\frac{d^2-2ce+\sqrt{d^4-4cd^2e}}{2e^2}-\frac{d^2-2ce-\sqrt{d^4-4cd^2e}}{2e^2}\right)\sqrt{bx^4+a}} - \frac{i\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{\frac{i\sqrt{b}x^2}{\sqrt{a}}+}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}e\left(-d^2+2ce+\sqrt{d}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

```
[Out] -((Sqrt[2]*e^2*(ArcTanh[(2*a*e^2 + b*(d^2 - 2*c*e - d*Sqrt[d^2 - 4*c*e])*x^
2)/(Sqrt[4*a*e^4 + b*(2*d^4 - 8*c*d^2*e + 4*c^2*e^2 - 2*d^3*Sqrt[d^2 - 4*c*
e] + 4*c*d*e*Sqrt[d^2 - 4*c*e])]*Sqrt[a + b*x^4])]/(2*Sqrt[2*a*e^4 + b*(d^4
 - 4*c*d^2*e + 2*c^2*e^2 - d^3*Sqrt[d^2 - 4*c*e] + 2*c*d*e*Sqrt[d^2 - 4*c*e
])]) - ArcTanh[(2*a*e^2 + b*(d^2 - 2*c*e + d*Sqrt[d^2 - 4*c*e])*x^2)/(Sqrt[
4*a*e^4 + 2*b*(d^4 - 4*c*d^2*e + 2*c^2*e^2 + d^3*Sqrt[d^2 - 4*c*e] - 2*c*d*
e*Sqrt[d^2 - 4*c*e])]*Sqrt[a + b*x^4])]/(2*Sqrt[2*a*e^4 + b*(d^4 - 4*c*d^2*
e + 2*c^2*e^2 + d^3*Sqrt[d^2 - 4*c*e] - 2*c*d*e*Sqrt[d^2 - 4*c*e])])))/Sqrt
[d^2 - 4*c*e]) - (I*d^2*Sqrt[1 - (I*Sqrt[b]*x^2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[
b]*x^2)/Sqrt[a]]*EllipticPi[((-2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(-d^2 + 2*c*e - S
qrt[d^4 - 4*c*d^2*e])), I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(Sqrt[
(I*Sqrt[b])/Sqrt[a]]*e*(-d^2 + 2*c*e - Sqrt[d^4 - 4*c*d^2*e])*(-1/2*(d^2 -
```

```
2*c*e - Sqrt[d^4 - 4*c*d^2*e])/e^2 + (d^2 - 2*c*e + Sqrt[d^4 - 4*c*d^2*e])/
(2*e^2))*Sqrt[a + b*x^4]) - (I*Sqrt[d^4 - 4*c*d^2*e]*Sqrt[1 - (I*Sqrt[b]*x^
2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[b]*x^2)/Sqrt[a]]*EllipticPi[((-2*I)*Sqrt[a]*e^
2)/(Sqrt[b]*(-d^2 + 2*c*e - Sqrt[d^4 - 4*c*d^2*e])), I*ArcSinh[Sqrt[(I*Sqrt
[b])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*e*(-d^2 + 2*c*e - Sqrt[d^
4 - 4*c*d^2*e])*(-1/2*(d^2 - 2*c*e - Sqrt[d^4 - 4*c*d^2*e])/e^2 + (d^2 - 2*
c*e + Sqrt[d^4 - 4*c*d^2*e])/(2*e^2))*Sqrt[a + b*x^4]) - (I*d^2*Sqrt[1 - (I
*Sqrt[b]*x^2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[b]*x^2)/Sqrt[a]]*EllipticPi[((-2*I)
*Sqrt[a]*e^2)/(Sqrt[b]*(-d^2 + 2*c*e + Sqrt[d^4 - 4*c*d^2*e])), I*ArcSinh[S
qrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*e*(-d^2 + 2*c*
e + Sqrt[d^4 - 4*c*d^2*e])*((d^2 - 2*c*e - Sqrt[d^4 - 4*c*d^2*e])/(2*e^2) -
 (d^2 - 2*c*e + Sqrt[d^4 - 4*c*d^2*e])/(2*e^2))*Sqrt[a + b*x^4]) + (I*Sqrt[
d^4 - 4*c*d^2*e]*Sqrt[1 - (I*Sqrt[b]*x^2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[b]*x^2)
/Sqrt[a]]*EllipticPi[((-2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(-d^2 + 2*c*e + Sqrt[d^4
 - 4*c*d^2*e])), I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt
[b])/Sqrt[a]]*e*(-d^2 + 2*c*e + Sqrt[d^4 - 4*c*d^2*e])*((d^2 - 2*c*e - Sqrt
[d^4 - 4*c*d^2*e])/(2*e^2) - (d^2 - 2*c*e + Sqrt[d^4 - 4*c*d^2*e])/(2*e^2))
*Sqrt[a + b*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}\left(ex^2+dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

maple [C] time = 0.10, size = 1153, normalized size = 0.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/2/(-4*c*e+d^2)^(1/2)/(1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b
/e^3*c*d^2+b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1/2
*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*c*e
+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d^2-1/2/(1/2*b/e^4
*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*c*e+d^2)^
(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(
1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*
c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e*c+1/(1/2*b/e^4*d^
4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*c*e+d^2)^(1/
2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*a)-2/(-4*c*e+d^2)^(1/2)/(I/a^(1/2)*b^
(1/2))^(1/2)*e/(-d+(-4*c*e+d^2)^(1/2))*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2)
)^(1/2),-4*I*a^(1/2)/b^(1/2)*e^2/(-d+(-4*c*e+d^2)^(1/2))^2,(-I/a^(1/2)*b^(1
/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))+1/2/(-4*c*e+d^2)^(1/2)/(1/2*b/e^4*d^4
```

```
+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2
)+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^
(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+
a)^(1/2)*b*x^2/e^2*d^2+1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-
2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/
2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^
2)^(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x
^4+a)^(1/2)*b*x^2/e*c+1/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b
/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*
a)-2/(-4*c*e+d^2)^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)/(d+(-4*c*e+d^2)^(1/2))*e*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(
1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),-4*I*a^(1/2)/b^(1/2)/(d+(-4*c*e
+d^2)^(1/2))^2*e^2,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4 + a}\left(ex^2 + dx + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\,x^4 + a}\,\left(e\,x^2 + d\,x + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/2)*(c + d*x + e*x^2)),x)
[Out] int(1/((a + b*x^4)^(1/2)*(c + d*x + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^4}\left(c + dx + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**4)*(c + d*x + e*x**2)), x)
```

3.227 $\int x^m \left(c \left(a + bx^2\right)^2\right)^{3/2} dx$

Optimal. Leaf size=161

$$\frac{a^3 c x^{m+1} \sqrt{c \left(a + bx^2\right)^2}}{(m+1)\left(a + bx^2\right)} + \frac{3a^2 b c x^{m+3} \sqrt{c \left(a + bx^2\right)^2}}{(m+3)\left(a + bx^2\right)} + \frac{b^3 c x^{m+7} \sqrt{c \left(a + bx^2\right)^2}}{(m+7)\left(a + bx^2\right)} + \frac{3ab^2 c x^{m+5} \sqrt{c \left(a + bx^2\right)^2}}{(m+5)\left(a + bx^2\right)}$$

```
[Out] a^3*c*x^(1+m)*(c*(b*x^2+a)^2)^(1/2)/(1+m)/(b*x^2+a)+3*a^2*b*c*x^(3+m)*(c*(b
*x^2+a)^2)^(1/2)/(3+m)/(b*x^2+a)+3*a*b^2*c*x^(5+m)*(c*(b*x^2+a)^2)^(1/2)/(5
+m)/(b*x^2+a)+b^3*c*x^(7+m)*(c*(b*x^2+a)^2)^(1/2)/(7+m)/(b*x^2+a)
```

Rubi [A] time = 0.13, antiderivative size = 205, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{a^3 c x^{m+1} \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{(m+1)\left(a + bx^2\right)} + \frac{3a^2 b c x^{m+3} \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{(m+3)\left(a + bx^2\right)} + \frac{3ab^2 c x^{m+5} \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{(m+5)\left(a + bx^2\right)} + \frac{b^3 c x^{m+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^(1 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/((1 + m)*(a + b*x^2
)) + (3*a^2*b*c*x^(3 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/((3 + m)*(
a + b*x^2)) + (3*a*b^2*c*x^(5 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(
(5 + m)*(a + b*x^2)) + (b^3*c*x^(7 + m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^
4])/((7 + m)*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int x^m \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \int x^m \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} dx$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int x^m \left(abc+b^2cx^2\right)^3 dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int \left(a^3b^3c^3x^m+3a^2b^4c^3x^{2+m}+3ab^5c^3x^{4+m}+b^6c^3x^{6+m}\right) d}{b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{a^3cx^{1+m}\sqrt{a^2c+2abcx^2+b^2cx^4}}{(1+m)\left(a+bx^2\right)} + \frac{3a^2bcx^{3+m}\sqrt{a^2c+2abcx^2+b^2cx^4}}{(3+m)\left(a+bx^2\right)} + \frac{3ab^2cx^{5+}}{(}$$

Mathematica [A] time = 0.10, size = 132, normalized size = 0.82

$$\frac{x^{m+1}\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(a^3\left(m^3+15m^2+71m+105\right)+3a^2b\left(m^3+13m^2+47m+35\right)x^2+3ab^2\left(m^3+11m^2+\right.\right.}{(m+1)(m+3)(m+5)(m+7)\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(c*(a + b*x^2)^2)^(3/2)*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^2
*b*(35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^4
+ b^3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(a +
 b*x^2)^3)
```

fricas [A] time = 0.91, size = 233, normalized size = 1.45

$$\frac{\left(\left(b^3cm^3+9\,b^3cm^2+23\,b^3cm+15\,b^3c\right)x^7+3\left(ab^2cm^3+11\,ab^2cm^2+31\,ab^2cm+21\,ab^2c\right)x^5+3\left(a^2bcm^3+13\right.\right.}{am^4+16\,am^3+86\,am^2+\left(bm^4+16\,bm^3+86\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*c*m^3 + 9*b^3*c*m^2 + 23*b^3*c*m + 15*b^3*c)*x^7 + 3*(a*b^2*c*m^3 + 1
1*a*b^2*c*m^2 + 31*a*b^2*c*m + 21*a*b^2*c)*x^5 + 3*(a^2*b*c*m^3 + 13*a^2*b*
c*m^2 + 47*a^2*b*c*m + 35*a^2*b*c)*x^3 + (a^3*c*m^3 + 15*a^3*c*m^2 + 71*a^3
*c*m + 105*a^3*c)*x)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*x^m/(a*m^4 + 16*
a*m^3 + 86*a*m^2 + (b*m^4 + 16*b*m^3 + 86*b*m^2 + 176*b*m + 105*b)*x^2 + 17
6*a*m + 105*a)
```

giac [B] time = 0.38, size = 355, normalized size = 2.20

$$\frac{\left(b^3m^3x^7x^m\mathrm{sgn}\left(bx^2+a\right)+9\,b^3m^2x^7x^m\mathrm{sgn}\left(bx^2+a\right)+3\,ab^2m^3x^5x^m\mathrm{sgn}\left(bx^2+a\right)+23\,b^3mx^7x^m\mathrm{sgn}\left(bx^2+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^7*x^m*sgn(b*x^2 + a) + 9*b^3*m^2*x^7*x^m*sgn(b*x^2 + a) + 3*a*b^
2*m^3*x^5*x^m*sgn(b*x^2 + a) + 23*b^3*m*x^7*x^m*sgn(b*x^2 + a) + 33*a*b^2*m
^2*x^5*x^m*sgn(b*x^2 + a) + 15*b^3*x^7*x^m*sgn(b*x^2 + a) + 3*a^2*b*m^3*x^3
*x^m*sgn(b*x^2 + a) + 93*a*b^2*m*x^5*x^m*sgn(b*x^2 + a) + 39*a^2*b*m^2*x^3*
x^m*sgn(b*x^2 + a) + 63*a*b^2*x^5*x^m*sgn(b*x^2 + a) + a^3*m^3*x*x^m*sgn(b*
```

```
x^2 + a) + 141*a^2*b*m*x^3*x^m*sgn(b*x^2 + a) + 15*a^3*m^2*x*x^m*sgn(b*x^2
+ a) + 105*a^2*b*x^3*x^m*sgn(b*x^2 + a) + 71*a^3*m*x*x^m*sgn(b*x^2 + a) + 1
05*a^3*x*x^m*sgn(b*x^2 + a))*c^(3/2)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

maple [A] time = 0.01, size = 200, normalized size = 1.24

$$\frac{\left(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23b^3m\,x^6+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93a\,b^2m\,x^4+39a^2b\,m^2x^2+\cdots\right.}{(m+7)(m+5)(m+3)(m+1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*(b*x^2+a)^2)^(3/2),x)
```

```
[Out] x^(m+1)*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^
2*x^4+15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x
^4+a^3*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)*(c*(b
*x^2+a)^2)^(3/2)/(7+m)/(5+m)/(3+m)/(m+1)/(b*x^2+a)^3
```

maxima [A] time = 0.99, size = 119, normalized size = 0.74

$$\frac{\left(\left(m^3+9\,m^2+23\,m+15\right)b^3c^{\frac{3}{2}}x^7+3\left(m^3+11\,m^2+31\,m+21\right)ab^2c^{\frac{3}{2}}x^5+3\left(m^3+13\,m^2+47\,m+35\right)a^2bc^{\frac{3}{2}}x^3+\cdots\right.}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*c^(3/2)*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a
*b^2*c^(3/2)*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*c^(3/2)*x^3 + (m^3 +
15*m^2 + 71*m + 105)*a^3*c^(3/2)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 10
5)
```

mupad [B] time = 2.97, size = 234, normalized size = 1.45

$$\frac{x^m\left(\frac{3\,a^2\,c\,x^3\,\sqrt{c\,{\left(b\,x^2+a\right)}^2}\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b^2\,c\,x^7\,\sqrt{c\,{\left(b\,x^2+a\right)}^2}\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{3\,a\,b\,c\,x^5\,\sqrt{c\,{\left(b\,x^2+a\right)}^2}\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}\cdots\right.}{\frac{a}{b}+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] (x^m*((3*a^2*c*x^3*(c*(a + b*x^2)^2)^(1/2)*(47*m + 13*m^2 + m^3 + 35))/(176
*m + 86*m^2 + 16*m^3 + m^4 + 105) + (b^2*c*x^7*(c*(a + b*x^2)^2)^(1/2)*(23*
m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (3*a*b*c*x^5
*(c*(a + b*x^2)^2)^(1/2)*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86*m^2 + 16*m
^3 + m^4 + 105) + (a^3*c*x*(c*(a + b*x^2)^2)^(1/2)*(71*m + 15*m^2 + m^3 + 1
05))/(b*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))))/(a/b + x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.228 $\int x^5 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx$

Optimal. Leaf size=143

$$\frac{a^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)}+\frac{3a^2bcx^8\sqrt{c\left(a+bx^2\right)^2}}{8\left(a+bx^2\right)}+\frac{b^3cx^{12}\sqrt{c\left(a+bx^2\right)^2}}{12\left(a+bx^2\right)}+\frac{3ab^2cx^{10}\sqrt{c\left(a+bx^2\right)^2}}{10\left(a+bx^2\right)}$$

```
[Out] 1/6*a^3*c*x^6*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/8*a^2*b*c*x^8*(c*(b*x^2+a)^
2)^(1/2)/(b*x^2+a)+3/10*a*b^2*c*x^10*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/12*b
^3*c*x^12*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.16, antiderivative size = 134, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1111, 645}

$$\frac{c\left(a+bx^2\right)^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{12b^3}-\frac{ac\left(a+bx^2\right)^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{5b^3}+\frac{a^2c\left(a+bx^2\right)^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^2*c*(a + b*x^2)^3*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(8*b^3) - (a*c*
(a + b*x^2)^4*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*b^3) + (c*(a + b*x^
2)^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(12*b^3)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int x^5\left(c\left(a+bx^2\right)^2\right)^{3/2}dx=\int x^5\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}dx$$
$$=\frac{1}{2}\text{Subst}\left(\int x^2\left(a^2c+2abcx+b^2cx^2\right)^{3/2}dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int\left(\frac{a^2(abc+b^2cx)^3}{b^2}-\frac{2a(abc+b^2cx)^4}{b^3c}+\frac{(abc+b^2cx)^5}{b^4c^2}\right)dx,x,x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}$$
$$=\frac{a^2c\left(a+bx^2\right)^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{8b^3}-\frac{ac\left(a+bx^2\right)^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{5b^3}+\frac{c\left(a\right.}{}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.44

$$\frac{x^6\left(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{120\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^6*(c*(a + b*x^2)^2)^(3/2)*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3
*x^6))/(120*(a + b*x^2)^3)
```

fricas [A] time = 0.77, size = 74, normalized size = 0.52

$$\frac{\left(10\,b^3cx^{12}+36\,ab^2cx^{10}+45\,a^2bcx^8+20\,a^3cx^6\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{120\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/120*(10*b^3*c*x^12 + 36*a*b^2*c*x^10 + 45*a^2*b*c*x^8 + 20*a^3*c*x^6)*sqr
t(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

giac [A] time = 0.30, size = 72, normalized size = 0.50

$$\frac{1}{120}\left(10\,b^3x^{12}\text{sgn}\left(bx^2+a\right)+36\,ab^2x^{10}\text{sgn}\left(bx^2+a\right)+45\,a^2bx^8\text{sgn}\left(bx^2+a\right)+20\,a^3x^6\text{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/120*(10*b^3*x^12*sgn(b*x^2 + a) + 36*a*b^2*x^10*sgn(b*x^2 + a) + 45*a^2*b
*x^8*sgn(b*x^2 + a) + 20*a^3*x^6*sgn(b*x^2 + a))*c^(3/2)
```

maple [A] time = 0.01, size = 60, normalized size = 0.42

$$\frac{\left(10b^3x^6+36a\,b^2x^4+45a^2b\,x^2+20a^3\right)\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}x^6}{120\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*((b*x^2+a)^2*c)^(3/
2)/(b*x^2+a)^3
```

maxima [A] time = 1.03, size = 136, normalized size = 0.95

$$\frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}a^2x^2}{8\,b^2}+\frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}a^3}{8\,b^3}+\frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{5}{2}}x^2}{12\,b^2c}-\frac{7\left(b^2cx^4+2\,abcx^2+\right.}{60\,b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a^2*x^2/b^2 + 1/8*(b^2*c*x^4 +
2*a*b*c*x^2 + a^2*c)^(3/2)*a^3/b^3 + 1/12*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)
^(5/2)*x^2/(b^2*c) - 7/60*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(5/2)*a/(b^3*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(c\left(b\,x^2+a\right)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] int(x^5*(c*(a + b*x^2)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.229 $\int x^4 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx$

Optimal. Leaf size=143

$$\frac{a^3 c x^5 \sqrt{c \left(a+b x^2\right)^2}}{5 \left(a+b x^2\right)}+\frac{3 a^2 b c x^7 \sqrt{c \left(a+b x^2\right)^2}}{7 \left(a+b x^2\right)}+\frac{b^3 c x^{11} \sqrt{c \left(a+b x^2\right)^2}}{11 \left(a+b x^2\right)}+\frac{a b^2 c x^9 \sqrt{c \left(a+b x^2\right)^2}}{3 \left(a+b x^2\right)}$$

```
[Out] 1/5*a^3*c*x^5*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/7*a^2*b*c*x^7*(c*(b*x^2+a)^
2)^(1/2)/(b*x^2+a)+1/3*a*b^2*c*x^9*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/11*b^3
*c*x^11*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.11, antiderivative size = 187, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{b^3 c x^{11} \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{11 \left(a+b x^2\right)}+\frac{a b^2 c x^9 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{3 \left(a+b x^2\right)}+\frac{3 a^2 b c x^7 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{7 \left(a+b x^2\right)}+\frac{a^3 c x^5 \sqrt{a^2 c+2 a}}{5 \left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2)) + (3*a^2*
b*c*x^7*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(7*(a + b*x^2)) + (a*b^2*c*x
^9*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(3*(a + b*x^2)) + (b^3*c*x^11*Sqr
t[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(11*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int x^4\left(c\left(a+bx^2\right)^2\right)^{3/2}dx=\int x^4\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}dx$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int x^4\left(abc+b^2cx^2\right)^3dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int\left(a^3b^3c^3x^4+3a^2b^4c^3x^6+3ab^5c^3x^8+b^6c^3x^{10}\right)dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$=\frac{a^3cx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)}+\frac{3a^2bcx^7\sqrt{a^2c+2abcx^2+b^2cx^4}}{7\left(a+bx^2\right)}+\frac{ab^2cx^9\sqrt{a^2c+}}{3\left(a\right.}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.44

$$\frac{x^5\left(231a^3+495a^2bx^2+385ab^2x^4+105b^3x^6\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{1155\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^5*(c*(a + b*x^2)^2)^(3/2)*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105
*b^3*x^6))/(1155*(a + b*x^2)^3)
```

fricas [A] time = 0.72, size = 74, normalized size = 0.52

$$\frac{\left(105\,b^3cx^{11}+385\,ab^2cx^9+495\,a^2bcx^7+231\,a^3cx^5\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{1155\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*b^3*c*x^11 + 385*a*b^2*c*x^9 + 495*a^2*b*c*x^7 + 231*a^3*c*x^5)
*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

giac [A] time = 0.29, size = 72, normalized size = 0.50

$$\frac{1}{1155}\left(105\,b^3x^{11}\mathrm{sgn}\left(bx^2+a\right)+385\,ab^2x^9\mathrm{sgn}\left(bx^2+a\right)+495\,a^2bx^7\mathrm{sgn}\left(bx^2+a\right)+231\,a^3x^5\mathrm{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1155*(105*b^3*x^11*sgn(b*x^2 + a) + 385*a*b^2*x^9*sgn(b*x^2 + a) + 495*a^
2*b*x^7*sgn(b*x^2 + a) + 231*a^3*x^5*sgn(b*x^2 + a))*c^(3/2)
```

maple [A] time = 0.01, size = 60, normalized size = 0.42

$$\frac{\left(105b^3x^6+385a\,b^2x^4+495a^2b\,x^2+231a^3\right)\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}x^5}{1155\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*((b*x^2+a)^2*c
)^(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.22, size = 47, normalized size = 0.33

$$\frac{1}{11}\,b^3c^{\frac{3}{2}}x^{11}+\frac{1}{3}\,ab^2c^{\frac{3}{2}}x^9+\frac{3}{7}\,a^2bc^{\frac{3}{2}}x^7+\frac{1}{5}\,a^3c^{\frac{3}{2}}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*c^(3/2)*x^11 + 1/3*a*b^2*c^(3/2)*x^9 + 3/7*a^2*b*c^(3/2)*x^7 + 1/5
*a^3*c^(3/2)*x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(c\left(b\,x^2+a\right)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] int(x^4*(c*(a + b*x^2)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.230 $\int x^3 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx$

Optimal. Leaf size=66

$$\frac{c\left(a+bx^2\right)^4\sqrt{c\left(a+bx^2\right)^2}}{10b^2}-\frac{ac\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^2}}{8b^2}$$

```
[Out] -1/8*a*c*(b*x^2+a)^3*(c*(b*x^2+a)^2)^(1/2)/b^2+1/10*c*(b*x^2+a)^4*(c*(b*x^2
+a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1989, 1111, 640, 609}

$$\frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{5/2}}{10b^2c}-\frac{a\left(a+bx^2\right)\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] -(a*(a + b*x^2)*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(8*b^2) + (a^2*c +
 2*a*b*c*x^2 + b^2*c*x^4)^(5/2)/(10*b^2*c)
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int x^3\left(c\left(a+bx^2\right)^2\right)^{3/2}dx=\int x^3\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}dx$$
$$=\frac{1}{2}\text{Subst}\left(\int x\left(a^2c+2abcx+b^2cx^2\right)^{3/2}dx,x,x^2\right)$$
$$=\frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{5/2}}{10b^2c}-\frac{a\,\text{Subst}\left(\int\left(a^2c+2abcx+b^2cx^2\right)^{3/2}dx,x,x^2\right)}{2b}$$
$$=-\frac{a\left(a+bx^2\right)\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{8b^2}+\frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{5/2}}{10b^2c}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.95

$$\frac{x^4\left(10a^3+20a^2bx^2+15ab^2x^4+4b^3x^6\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{40\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^4*(c*(a + b*x^2)^2)^(3/2)*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*
x^6))/(40*(a + b*x^2)^3)
```

fricas [A] time = 0.64, size = 74, normalized size = 1.12

$$\frac{\left(4\,b^3cx^{10}+15\,ab^2cx^8+20\,a^2bcx^6+10\,a^3cx^4\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{40\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(4*b^3*c*x^10 + 15*a*b^2*c*x^8 + 20*a^2*b*c*x^6 + 10*a^3*c*x^4)*sqrt(b
^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

giac [A] time = 0.30, size = 48, normalized size = 0.73

$$\frac{1}{40}\left(4\,b^3x^{10}+15\,ab^2x^8+20\,a^2bx^6+10\,a^3x^4\right)c^{\frac{3}{2}}\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*c^(3/2)*sgn(b*
x^2 + a)
```

maple [A] time = 0.01, size = 60, normalized size = 0.91

$$\frac{\left(4b^3x^6+15a\,b^2x^4+20a^2b\,x^2+10a^3\right)\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}x^4}{40\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*((b*x^2+a)^2*c)^(3/2)
/(b*x^2+a)^3
```

maxima [A] time = 1.04, size = 98, normalized size = 1.48

$$-\frac{\left(b^2cx^4 + 2\,abcx^2 + a^2c\right)^{\frac{3}{2}}ax^2}{8\,b} - \frac{\left(b^2cx^4 + 2\,abcx^2 + a^2c\right)^{\frac{3}{2}}a^2}{8\,b^2} + \frac{\left(b^2cx^4 + 2\,abcx^2 + a^2c\right)^{\frac{5}{2}}}{10\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a*x^2/b - 1/8*(b^2*c*x^4 + 2*a
*b*c*x^2 + a^2*c)^(3/2)*a^2/b^2 + 1/10*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(5
/2)/(b^2*c)
```

mupad [B] time = 2.83, size = 50, normalized size = 0.76

$$\frac{\left(-a^2 + 3\,a\,b\,x^2 + 4\,b^2\,x^4\right)\,\left(c\,a^2 + 2\,c\,a\,b\,x^2 + c\,b^2\,x^4\right)^{3/2}}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] ((4*b^2*x^4 - a^2 + 3*a*b*x^2)*(a^2*c + b^2*c*x^4 + 2*a*b*c*x^2)^(3/2))/(40
*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.231 $\int x^2 \left(c \left(a+bx^2\right)^2\right)^{3/2} dx$

Optimal. Leaf size=143

$$\frac{a^3cx^3\sqrt{c\left(a+bx^2\right)^2}}{3\left(a+bx^2\right)}+\frac{3a^2bcx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{b^3cx^9\sqrt{c\left(a+bx^2\right)^2}}{9\left(a+bx^2\right)}+\frac{3ab^2cx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}$$

```
[Out] 1/3*a^3*c*x^3*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/5*a^2*b*c*x^5*(c*(b*x^2+a)^
2)^(1/2)/(b*x^2+a)+3/7*a*b^2*c*x^7*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/9*b^3*
c*x^9*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.10, antiderivative size = 187, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{b^3cx^9\sqrt{a^2c+2abcx^2+b^2cx^4}}{9\left(a+bx^2\right)}+\frac{3ab^2cx^7\sqrt{a^2c+2abcx^2+b^2cx^4}}{7\left(a+bx^2\right)}+\frac{3a^2bcx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)}+\frac{a^3cx^3\sqrt{a^2c+2}}{3\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^3*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(3*(a + b*x^2)) + (3*a^2*
b*c*x^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2)) + (3*a*b^2*c
*x^7*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(7*(a + b*x^2)) + (b^3*c*x^9*Sq
rt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(9*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int x^2\left(c\left(a+bx^2\right)^2\right)^{3/2}dx=\int x^2\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}dx$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int x^2\left(abc+b^2cx^2\right)^3dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int\left(a^3b^3c^3x^2+3a^2b^4c^3x^4+3ab^5c^3x^6+b^6c^3x^8\right)dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$=\frac{a^3cx^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{3\left(a+bx^2\right)}+\frac{3a^2bcx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)}+\frac{3ab^2cx^7\sqrt{a^2c+}}{7\left(a\right.}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.44

$$\frac{\left(105a^3x^3+189a^2bx^5+135ab^2x^7+35b^3x^9\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{315\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7 + 35*
b^3*x^9))/(315*(a + b*x^2)^3)
```

fricas [A] time = 0.71, size = 74, normalized size = 0.52

$$\frac{\left(35\,b^3cx^9+135\,ab^2cx^7+189\,a^2bcx^5+105\,a^3cx^3\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{315\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^3*c*x^9 + 135*a*b^2*c*x^7 + 189*a^2*b*c*x^5 + 105*a^3*c*x^3)*sq
rt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

giac [A] time = 0.27, size = 72, normalized size = 0.50

$$\frac{1}{315}\left(35\,b^3x^9\mathrm{sgn}\left(bx^2+a\right)+135\,ab^2x^7\mathrm{sgn}\left(bx^2+a\right)+189\,a^2bx^5\mathrm{sgn}\left(bx^2+a\right)+105\,a^3x^3\mathrm{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*b^3*x^9*sgn(b*x^2 + a) + 135*a*b^2*x^7*sgn(b*x^2 + a) + 189*a^2*b
*x^5*sgn(b*x^2 + a) + 105*a^3*x^3*sgn(b*x^2 + a))*c^(3/2)
```

maple [A] time = 0.01, size = 60, normalized size = 0.42

$$\frac{\left(35b^3x^6+135a\,b^2x^4+189a^2b\,x^2+105a^3\right)\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}x^3}{315\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*((b*x^2+a)^2*c)^
(3/2)/(b*x^2+a)^3
```

maxima [A] time = 1.20, size = 47, normalized size = 0.33

$$\frac{1}{9}\,b^{3}c^{\frac{3}{2}}x^{9}+\frac{3}{7}\,ab^{2}c^{\frac{3}{2}}x^{7}+\frac{3}{5}\,a^{2}bc^{\frac{3}{2}}x^{5}+\frac{1}{3}\,a^{3}c^{\frac{3}{2}}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*c^(3/2)*x^9 + 3/7*a*b^2*c^(3/2)*x^7 + 3/5*a^2*b*c^(3/2)*x^5 + 1/3*a
^3*c^(3/2)*x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(c\left(b\,x^2+a\right)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] int(x^2*(c*(a + b*x^2)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(c\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral(x**2*(c*(a + b*x**2)**2)**(3/2), x)
```

3.232 $\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx$

Optimal. Leaf size=32

$$\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^2}}{8b}$$

```
[Out] 1/8*c*(b*x^2+a)^3*(c*(b*x^2+a)^2)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1591, 15, 30}

$$\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^2])/(8*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx &= \frac{\text{Subst}\left(\int \left(cx^2\right)^{3/2}\,dx,x,a+bx^2\right)}{2b}\\ &= \frac{\left(c\sqrt{c\left(a+bx^2\right)^2}\right)\text{Subst}\left(\int x^3\,dx,x,a+bx^2\right)}{2b\left(a+bx^2\right)}\\ &= \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^2}}{8b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.91

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(c*(a + b*x^2)^2)^(3/2))/(8*b)
```

fricas [B] time = 0.63, size = 73, normalized size = 2.28

$$\frac{\left(b^3cx^8 + 4\,ab^2cx^6 + 6\,a^2bcx^4 + 4\,a^3cx^2\right)\sqrt{b^2cx^4 + 2\,abcx^2 + a^2c}}{8\left(bx^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(b^3*c*x^8 + 4*a*b^2*c*x^6 + 6*a^2*b*c*x^4 + 4*a^3*c*x^2)*sqrt(b^2*c*x^
4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

giac [A] time = 0.25, size = 25, normalized size = 0.78

$$\frac{\left(bx^2 + a\right)^4 c^{\frac{3}{2}} \operatorname{sgn}\left(bx^2 + a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(b*x^2 + a)^4*c^(3/2)*sgn(b*x^2 + a)/b
```

maple [B] time = 0.01, size = 59, normalized size = 1.84

$$\frac{\left(b^3x^6 + 4a\,b^2x^4 + 6a^2b\,x^2 + 4a^3\right)\left(\left(b\,x^2 + a\right)^2 c\right)^{\frac{3}{2}} x^2}{8\left(b\,x^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/8*x^2*(b^3*x^6+4*a*b^2*x^4+6*a^2*b*x^2+4*a^3)*((b*x^2+a)^2*c)^(3/2)/(b*x^
2+a)^3
```

maxima [B] time = 0.93, size = 60, normalized size = 1.88

$$\frac{1}{8}\left(b^2cx^4 + 2\,abcx^2 + a^2c\right)^{\frac{3}{2}} x^2 + \frac{\left(b^2cx^4 + 2\,abcx^2 + a^2c\right)^{\frac{3}{2}} a}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*x^2 + 1/8*(b^2*c*x^4 + 2*a*b*c*
x^2 + a^2*c)^(3/2)*a/b
```

mupad [B] time = 2.84, size = 40, normalized size = 1.25

$$\frac{\left(b^2\,x^2 + a\,b\right)\,\left(c\,a^2 + 2\,c\,a\,b\,x^2 + c\,b^2\,x^4\right)^{3/2}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] ((a*b + b^2*x^2)*(a^2*c + b^2*c*x^4 + 2*a*b*c*x^2)^(3/2))/(8*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral(x*(c*(a + b*x**2)**2)**(3/2), x)
```

3.233 $\int \left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx$

Optimal. Leaf size=135

$$\frac{a^3cx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{a^2bcx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}+\frac{3ab^2cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}$$

```
[Out] a^3*c*x*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^2*b*c*x^3*(c*(b*x^2+a)^2)^(1/2)/(
b*x^2+a)+3/5*a*b^2*c*x^5*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/7*b^3*c*x^7*(c*(
b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.05, antiderivative size = 175, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1988, 1088, 194}

$$\frac{b^3x^7\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{7\left(a+bx^2\right)^3}+\frac{3ab^2x^5\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{5\left(a+bx^2\right)^3}+\frac{a^2bx^3\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{a^3x\left(a^2c+2\right.}{\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*x*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(a + b*x^2)^3 + (a^2*b*x^3*
(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(a + b*x^2)^3 + (3*a*b^2*x^5*(a^2*
c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(5*(a + b*x^2)^3) + (b^3*x^7*(a^2*c + 2
*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(7*(a + b*x^2)^3)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1988

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && T
rinomialQ[u, x] && !TrinomialMatchQ[u, x]
```

Rubi steps

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \int \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} dx$$
$$= \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} \int \left(2abc+2b^2cx^2\right)^3 dx}{\left(2abc+2b^2cx^2\right)^3}$$
$$= \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} \int \left(8a^3b^3c^3+24a^2b^4c^3x^2+24ab^5c^3x^4+8b^6c^3x^6\right) dx}{\left(2abc+2b^2cx^2\right)^3}$$
$$= \frac{a^3x\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{a^2bx^3\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{3ab^2x^5\left(a^2c+2a\right.}{5\left(a+\right.}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.45

$$\frac{\left(35a^3x+35a^2bx^3+21ab^2x^5+5b^3x^7\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{35\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*b^3*x^
7))/(35*(a + b*x^2)^3)
```

fricas [A] time = 0.41, size = 72, normalized size = 0.53

$$\frac{\left(5\,b^3cx^7+21\,ab^2cx^5+35\,a^2bcx^3+35\,a^3cx\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{35\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^3*c*x^7 + 21*a*b^2*c*x^5 + 35*a^2*b*c*x^3 + 35*a^3*c*x)*sqrt(b^2*
c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

giac [A] time = 0.33, size = 46, normalized size = 0.34

$$\frac{1}{35}\left(5\,b^3x^7+21\,ab^2x^5+35\,a^2bx^3+35\,a^3x\right)c^{\frac{3}{2}}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35*(5*b^3*x^7 + 21*a*b^2*x^5 + 35*a^2*b*x^3 + 35*a^3*x)*c^(3/2)*sgn(b*x^2
 + a)
```

maple [A] time = 0.00, size = 58, normalized size = 0.43

$$\frac{\left(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}x}{35\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2*c)^(3/2),x)
```

```
[Out] 1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*((b*x^2+a)^2*c)^(3/2)/(
b*x^2+a)^3
```

maxima [A] time = 0.96, size = 43, normalized size = 0.32

$$\frac{1}{7}b^3c^{\frac{3}{2}}x^7+\frac{3}{5}ab^2c^{\frac{3}{2}}x^5+a^2bc^{\frac{3}{2}}x^3+a^3c^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*b^3*c^(3/2)*x^7 + 3/5*a*b^2*c^(3/2)*x^5 + a^2*b*c^(3/2)*x^3 + a^3*c^(3/
2)*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\left(b\,x^2+a\right)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2), x)
```

3.234 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=139

$$\frac{a^3c\log(x)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{3a^2bcx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}+\frac{b^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)}+\frac{3ab^2cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}$$

```
[Out] 3/2*a^2*b*c*x^2*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/4*a*b^2*c*x^4*(c*(b*x^2+a
)^2)^(1/2)/(b*x^2+a)+1/6*b^3*c*x^6*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^3*c*ln
(x)*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1989, 1112, 266, 43}

$$\frac{b^3cx^6\sqrt{a^2c+2abcx^2+b^2cx^4}}{6\left(a+bx^2\right)}+\frac{3ab^2cx^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{4\left(a+bx^2\right)}+\frac{3a^2bcx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)}+\frac{a^3c\log(x)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

```
[Out] (3*a^2*b*c*x^2*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(2*(a + b*x^2)) + (3*
a*b^2*c*x^4*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(4*(a + b*x^2)) + (b^3*c
*x^6*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(6*(a + b*x^2)) + (a^3*c*Sqrt[a
^2*c + 2*a*b*c*x^2 + b^2*c*x^4]*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x}\,dx = \int \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{x}\,dx$$

$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{\left(abc+b^2cx^2\right)^3}{x}\,dx}{b^2c\left(abc+b^2cx^2\right)}$$

$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\ \text{Subst}\left(\int \frac{\left(abc+b^2cx\right)^3}{x}\,dx,x,x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}$$

$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\ \text{Subst}\left(\int \left(3a^2b^4c^3+\frac{a^3b^3c^3}{x}+3ab^5c^3x+b^6c^3x^2\right)dx,x,x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}$$

$$= \frac{3a^2bcx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)}+\frac{3ab^2cx^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{4\left(a+bx^2\right)}+\frac{b^3cx^6\sqrt{a^2c+2abc}}{6\left(a+bx\right.}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.45

$$\frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(12a^3\log(x)+bx^2\left(18a^2+9abx^2+2b^2x^4\right)\right)}{12\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(b*x^2*(18*a^2 + 9*a*b*x^2 + 2*b^2*x^4) + 12*a^3*L
og[x]))/(12*(a + b*x^2)^3)
```

fricas [A] time = 0.44, size = 73, normalized size = 0.53

$$\frac{\left(2\,b^3cx^6+9\,ab^2cx^4+18\,a^2bcx^2+12\,a^3c\log(x)\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{12\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*c*x^6 + 9*a*b^2*c*x^4 + 18*a^2*b*c*x^2 + 12*a^3*c*log(x))*sqrt(
b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

giac [A] time = 0.35, size = 73, normalized size = 0.53

$$\frac{1}{12}\left(2\,b^3x^6\text{sgn}\left(bx^2+a\right)+9\,ab^2x^4\text{sgn}\left(bx^2+a\right)+18\,a^2bx^2\text{sgn}\left(bx^2+a\right)+6\,a^3\log\left(x^2\right)\text{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/12*(2*b^3*x^6*sgn(b*x^2 + a) + 9*a*b^2*x^4*sgn(b*x^2 + a) + 18*a^2*b*x^2*
sgn(b*x^2 + a) + 6*a^3*log(x^2)*sgn(b*x^2 + a))*c^(3/2)
```

maple [A] time = 0.02, size = 59, normalized size = 0.42

$$\frac{\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}\left(2b^3x^6+9a\,b^2x^4+18a^2b\,x^2+12a^3\ln(x)\right)}{12\left(b\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2*c)^(3/2)/x,x)
```

```
[Out] 1/12*((b*x^2+a)^2*c)^(3/2)*(2*b^3*x^6+9*a*b^2*x^4+18*a^2*b*x^2+12*a^3*ln(x)
)/(b*x^2+a)^3
```

maxima [A] time = 0.98, size = 171, normalized size = 1.23

$$\frac{1}{2}\,(-1)^{2\,b^2cx^2+2\,abc}\,a^3c^{\frac{3}{2}}\log\left(2\,b^2cx^2+2\,abc\right)-\frac{1}{2}\,(-1)^{2\,abcx^2+2\,a^2c}\,a^3c^{\frac{3}{2}}\log\left(2\,abc+\frac{2\,a^2c}{x^2}\right)+\frac{1}{4}\,\sqrt{b^2cx^4+2\,abcx^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^3*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c) - 1
/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^3*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2) + 1/
4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*a*b*c*x^2 + 3/4*sqrt(b^2*c*x^4 + 2*
a*b*c*x^2 + a^2*c)*a^2*c + 1/6*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\left(b\,x^2+a\right)^2\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x,x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x, x)
```

3.235 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=134

$$-\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}+\frac{3a^2bcx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{ab^2cx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}$$

```
[Out] -a^3*c*(c*(b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+3*a^2*b*c*x*(c*(b*x^2+a)^2)^(1/2)/
(b*x^2+a)+a*b^2*c*x^3*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/5*b^3*c*x^5*(c*(b*x
^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.09, antiderivative size = 178, normalized size of antiderivative = 1.33, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1989, 1112, 270}

$$\frac{b^3cx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)}+\frac{ab^2cx^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}+\frac{3a^2bcx\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}-\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

```
[Out] -((a^3*c*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(x*(a + b*x^2))) + (3*a^2*b
*c*x*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(a + b*x^2) + (a*b^2*c*x^3*Sqrt
[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(a + b*x^2) + (b^3*c*x^5*Sqrt[a^2*c + 2*
a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2}\,dx = \int \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{x^2}\,dx$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{\left(abc+b^2cx^2\right)^3}{x^2}\,dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \left(3a^2b^4c^3+\frac{a^3b^3c^3}{x^2}+3ab^5c^3x^2+b^6c^3x^4\right)dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= -\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{x\left(a+bx^2\right)}+\frac{3a^2bcx\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}+\frac{ab^2cx^3\sqrt{a^2c+2abc}}{a+bx^2}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.46

$$\frac{\left(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{5x\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(
5*x*(a + b*x^2)^3)
```

fricas [A] time = 0.43, size = 72, normalized size = 0.54

$$\frac{\left(b^3cx^6+5\,ab^2cx^4+15\,a^2bcx^2-5\,a^3c\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{5\left(bx^3+ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*c*x^6 + 5*a*b^2*c*x^4 + 15*a^2*b*c*x^2 - 5*a^3*c)*sqrt(b^2*c*x^4 +
 2*a*b*c*x^2 + a^2*c)/(b*x^3 + a*x)
```

giac [A] time = 0.24, size = 69, normalized size = 0.51

$$\frac{1}{5}\left(b^3x^5\mathrm{sgn}\left(bx^2+a\right)+5\,ab^2x^3\mathrm{sgn}\left(bx^2+a\right)+15\,a^2bx\mathrm{sgn}\left(bx^2+a\right)-\frac{5\,a^3\mathrm{sgn}\left(bx^2+a\right)}{x}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*(b^3*x^5*sgn(b*x^2 + a) + 5*a*b^2*x^3*sgn(b*x^2 + a) + 15*a^2*b*x*sgn(b
*x^2 + a) - 5*a^3*sgn(b*x^2 + a)/x)*c^(3/2)
```

maple [A] time = 0.01, size = 60, normalized size = 0.45

$$-\frac{\left(-b^3x^6-5a\,b^2x^4-15a^2b\,x^2+5a^3\right)\left(\left(b\,x^2+a\right)^2c\right)^{\frac{3}{2}}}{5\left(b\,x^2+a\right)^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2*c)^(3/2)/x^2,x)
```

```
[Out] -1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*((b*x^2+a)^2*c)^(3/2)/x/(b*x
^2+a)^3
```

maxima [A] time = 0.97, size = 48, normalized size = 0.36

$$\frac{b^3c^{\frac{3}{2}}x^6 + 5\,ab^2c^{\frac{3}{2}}x^4 + 15\,a^2bc^{\frac{3}{2}}x^2 - 5\,a^3c^{\frac{3}{2}}}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(b^3*c^(3/2)*x^6 + 5*a*b^2*c^(3/2)*x^4 + 15*a^2*b*c^(3/2)*x^2 - 5*a^3*c
^(3/2))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\left(b\,x^2+a\right)^2\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x^2,x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x**2,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x**2, x)
```

3.236 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=140

$$-\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{2x^2\left(a+bx^2\right)}+\frac{3a^2bc\log(x)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}$$

```
[Out] -1/2*a^3*c*(c*(b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+3/2*a*b^2*c*x^2*(c*(b*x^2+a)
^2)^(1/2)/(b*x^2+a)+1/4*b^3*c*x^4*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3*a^2*b*c
*ln(x)*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] time = 0.10, antiderivative size = 184, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1989, 1112, 266, 43}

$$\frac{b^3cx^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{4\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)}-\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{2x^2\left(a+bx^2\right)}+\frac{3a^2bc\log(x)\sqrt{a^2 \ldots}}{a \ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

```
[Out] -(a^3*c*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(2*x^2*(a + b*x^2)) + (3*a*b
^2*c*x^2*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(2*(a + b*x^2)) + (b^3*c*x^
4*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(4*(a + b*x^2)) + (3*a^2*b*c*Sqrt[
a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]*Log[x])/(a + b*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx = \int \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{x^3}\,dx$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{\left(abc+b^2cx^2\right)^3}{x^3}\,dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int \frac{\left(abc+b^2cx\right)^3}{x^2}\,dx,x,x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int \left(3ab^5c^3+\frac{a^3b^3c^3}{x^2}+\frac{3a^2b^4c^3}{x}+b^6c^3x\right)dx,x,x^2\right)}{2b^2c\left(abc+b^2cx^2\right)}$$
$$= -\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{2x^2\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\left(a+bx^2\right)}+\frac{b^3cx^4\sqrt{a^2c+2abcx^2}}{4\left(a+bx^2\right)}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.46

$$-\frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(2a^3-12a^2bx^2\log(x)-6ab^2x^4-b^3x^6\right)}{4x^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

```
[Out] -1/4*((c*(a + b*x^2)^2)^(3/2)*(2*a^3 - 6*a*b^2*x^4 - b^3*x^6 - 12*a^2*b*x^2
*Log[x]))/(x^2*(a + b*x^2)^3)
```

fricas [A] time = 0.44, size = 76, normalized size = 0.54

$$\frac{\left(b^3cx^6+6\,ab^2cx^4+12\,a^2bcx^2\log(x)-2\,a^3c\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{4\left(bx^4+ax^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*c*x^6 + 6*a*b^2*c*x^4 + 12*a^2*b*c*x^2*log(x) - 2*a^3*c)*sqrt(b^2*
c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^4 + a*x^2)
```

giac [A] time = 0.29, size = 91, normalized size = 0.65

$$\frac{1}{4}\left(b^3x^4\text{sgn}\left(bx^2+a\right)+6\,ab^2x^2\text{sgn}\left(bx^2+a\right)+6\,a^2b\log\left(x^2\right)\text{sgn}\left(bx^2+a\right)-\frac{2\left(3\,a^2bx^2\text{sgn}\left(bx^2+a\right)+a^3\text{sgn}\left(b\right.\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*x^4*sgn(b*x^2 + a) + 6*a*b^2*x^2*sgn(b*x^2 + a) + 6*a^2*b*log(x^2)
*sgn(b*x^2 + a) - 2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^2)*
c^(3/2)
```

maple [A] time = 0.02, size = 61, normalized size = 0.44

$$\frac{\left(\left(b\,x^2+a\right)^2 c\right)^{\frac{3}{2}} \left(b^3x^6+6a\,b^2x^4+12a^2b\,x^2\ln(x)-2a^3\right)}{4\left(b\,x^2+a\right)^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2*c)^(3/2)/x^3,x)
```

```
[Out] 1/4*((b*x^2+a)^2*c)^(3/2)*(b^3*x^6+6*a*b^2*x^4+12*b*a^2*ln(x)*x^2-2*a^3)/(b
*x^2+a)^3/x^2
```

maxima [A] time = 0.99, size = 176, normalized size = 1.26

$$\frac{3}{2}\,(-1)^{2\,b^2cx^2+2\,abc}\,a^2bc^{\frac{3}{2}}\log\left(2\,b^2cx^2+2\,abc\right)-\frac{3}{2}\,(-1)^{2\,abcx^2+2\,a^2c}\,a^2bc^{\frac{3}{2}}\log\left(2\,abc+\frac{2\,a^2c}{x^2}\right)+\frac{3}{4}\,\sqrt{b^2cx^4+2\,abcx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^2*b*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c) -
 3/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^2*b*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2)
+ 3/4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*b^2*c*x^2 + 9/4*sqrt(b^2*c*x^4
+ 2*a*b*c*x^2 + a^2*c)*a*b*c - 1/2*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)/
x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\left(b\,x^2+a\right)^2\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x^3,x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x**3,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x**3, x)
```

3.237 $\int x^2 \left(c \left(a+b x^2\right)^3\right)^{3/2} \, dx$

Optimal. Leaf size=253

$$-\frac{21 a^{9/2} c \sqrt{c \left(a+b x^2\right)^3} \sinh^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{1024 b^{3/2} \left(\frac{b x^2}{a}+1\right)^{3/2}}+\frac{21 a^5 c x \sqrt{c \left(a+b x^2\right)^3}}{1024 b \left(a+b x^2\right)}+\frac{21 a^4 c x^3 \sqrt{c \left(a+b x^2\right)^3}}{512 \left(a+b x^2\right)}+\frac{7}{128} a^3 c x^3 \sqrt{c \left(a+b x^2\right)^3}+$$

```
[Out] 7/128*a^3*c*x^3*(c*(b*x^2+a)^3)^(1/2)+21/1024*a^5*c*x*(c*(b*x^2+a)^3)^(1/2)
/b/(b*x^2+a)+21/512*a^4*c*x^3*(c*(b*x^2+a)^3)^(1/2)/(b*x^2+a)+21/320*a^2*c*
x^3*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+3/40*a*c*x^3*(b*x^2+a)^2*(c*(b*x^2+a)^3
)^(1/2)+1/12*c*x^3*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)-21/1024*a^(9/2)*c*arcs
inh(x*b^(1/2)/a^(1/2))*(c*(b*x^2+a)^3)^(1/2)/b^(3/2)/(1+b*x^2/a)^(3/2)
```

Rubi [A] time = 0.25, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 279, 321, 217, 206}

$$-\frac{21 a^6 c \sqrt{c \left(a+b x^2\right)^3} \tanh^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a+b x^2}}\right)}{1024 b^{3/2} \left(a+b x^2\right)^{3/2}}+\frac{21 a^5 c x \sqrt{c \left(a+b x^2\right)^3}}{1024 b \left(a+b x^2\right)}+\frac{21 a^4 c x^3 \sqrt{c \left(a+b x^2\right)^3}}{512 \left(a+b x^2\right)}+\frac{7}{128} a^3 c x^3 \sqrt{c \left(a+b x^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (7*a^3*c*x^3*Sqrt[c*(a + b*x^2)^3])/128 + (21*a^5*c*x*Sqrt[c*(a + b*x^2)^3]
)/(1024*b*(a + b*x^2)) + (21*a^4*c*x^3*Sqrt[c*(a + b*x^2)^3])/(512*(a + b*x
^2)) + (21*a^2*c*x^3*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/320 + (3*a*c*x^3*(a
 + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/40 + (c*x^3*(a + b*x^2)^3*Sqrt[c*(a + b*
x^2)^3])/12 - (21*a^6*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(1024*b^(3/2)*(a + b*x^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2\left(c\left(a+bx^2\right)^3\right)^{3/2}dx=\frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2\left(a+bx^2\right)^{9/2}dx}{\left(a+bx^2\right)^{3/2}}$$

$$=\frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(3ac\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2\left(a+bx^2\right)^{7/2}dx}{4\left(a+bx^2\right)^{3/2}}$$

$$=\frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(21a^2c\sqrt{c\cdots}\right.}{\cdots}$$

$$=\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{12}cx^3\left(\cdots\right.$$

$$=\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{3}{40}acx^3\left(a+bx^2\cdots\right.$$

$$=\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+\cdots\right.}$$

$$=\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}\cdots$$

$$=\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}\cdots$$

$$=\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}\cdots$$

Mathematica [A] time = 0.15, size = 143, normalized size = 0.57

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{b}\,x\sqrt{\frac{bx^2}{a}+1}\left(315a^5+4910a^4bx^2+11432a^3b^2x^4+12144a^2b^3x^6+6272ab^4x^8+1280b^5x^{10}\cdots\right.\right.}{15360b^{3/2}\left(a+bx^2\right)^4\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*(315*a^5 + 4910*a^4
*b*x^2 + 11432*a^3*b^2*x^4 + 12144*a^2*b^3*x^6 + 6272*a*b^4*x^8 + 1280*b^5*
x^10) - 315*a^(11/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/(15360*b^(3/2)*(a + b*x
^2)^4*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 0.59, size = 433, normalized size = 1.71

$$\left[\frac{315\left(a^6bcx^2+a^7c\right)\sqrt{\frac{c}{b}}\log\left(-\frac{2b^2cx^4+3abcx^2+a^2c-2\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\,bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\left(1280b^5cx^{11}+6272ab^4cx^9+\right.}{30720\left(b^2x^2+ab\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(315*(a^6*b*c*x^2 + a^7*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x
^2 + a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*
sqrt(c/b))/(b*x^2 + a)) + 2*(1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2
*b^3*c*x^7 + 11432*a^3*b^2*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3
*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b), 1/15360*(
315*(a^6*b*c*x^2 + a^7*c)*sqrt(-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4
+ 3*a^2*b*c*x^2 + a^3*c)*b*x*sqrt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c))
+ (1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2*b^3*c*x^7 + 11432*a^3*b^2
*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3
*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b)]
```

giac [A] time = 0.51, size = 177, normalized size = 0.70

$$\frac{1}{15360}\left(\frac{315a^6c\log\left(\left|-\sqrt{bc}\,x+\sqrt{bcx^2+ac}\right|\right)\operatorname{sgn}\left(bx^2+a\right)}{\sqrt{bc}\,b}+\left(\frac{315a^5\operatorname{sgn}\left(bx^2+a\right)}{b}+2\left(2455a^4\operatorname{sgn}\left(bx^2+a\right)+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15360*(315*a^6*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 +
 a)/(sqrt(b*c)*b) + (315*a^5*sgn(b*x^2 + a)/b + 2*(2455*a^4*sgn(b*x^2 + a)
+ 4*(1429*a^3*b*sgn(b*x^2 + a) + 2*(759*a^2*b^2*sgn(b*x^2 + a) + 8*(10*b^4*
x^2*sgn(b*x^2 + a) + 49*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*x^2)*x^2)*sqrt(b*c*
x^2 + a*c)*x)*c
```

maple [A] time = 0.04, size = 236, normalized size = 0.93

$$-\frac{\left(\left(bx^2+a\right)^3c\right)^{\frac{3}{2}}\left(315a^6c^3\ln\left(\frac{bcx+\sqrt{bcx^2+ac}\,\sqrt{bc}}{\sqrt{bc}}\right)-1280\left(bcx^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,b^3x^7+315\sqrt{bc}\,\sqrt{bcx^2+ac}\,a^5c^2x-37\right.}{15360\left(bx^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(b*x^2+a)^3)^(3/2),x)
```

```
[Out] -1/15360*(c*(b*x^2+a)^3)^(3/2)/b*(-1280*x^7*(b*c*x^2+a*c)^(5/2)*b^3*(b*c)^(
1/2)-3712*(b*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^5*a*b^2-3440*(b*c)^(1/2)*(b*c*x
^2+a*c)^(5/2)*x^3*a^2*b+315*ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c
)^(1/2))*a^6*c^3-840*(b*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x*a^3+210*(b*c)^(1/2)*
```

```
(b*c*x^2+a*c)^(3/2)*x*a^4*c+315*(b*c)^(1/2)*(b*c*x^2+a*c)^(1/2)*x*a^5*c^2)/
(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c/(b*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(bx^2 + a\right)^3 c\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(c \left(b\,x^2 + a\right)^3\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(a + b*x^2)^3)^(3/2),x)
[Out] int(x^2*(c*(a + b*x^2)^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*(b*x**2+a)**3)**(3/2),x)
[Out] Timed out
```

3.238 $\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx$

Optimal. Leaf size=32

$$\frac{c\left(a+bx^2\right)^4\sqrt{c\left(a+bx^2\right)^3}}{11b}$$

```
[Out] 1/11*c*(b*x^2+a)^4*(c*(b*x^2+a)^3)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1591, 15, 30}

$$\frac{c\left(a+bx^2\right)^4\sqrt{c\left(a+bx^2\right)^3}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (c*(a + b*x^2)^4*Sqrt[c*(a + b*x^2)^3])/(11*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}
\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx &= \frac{\text{Subst}\left(\int\left(cx^3\right)^{3/2}\,dx,x,a+bx^2\right)}{2b}\\
&= \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int x^{9/2}\,dx,x,a+bx^2\right)}{2b\left(a+bx^2\right)^{3/2}}\\
&= \frac{c\left(a+bx^2\right)^4\sqrt{c\left(a+bx^2\right)^3}}{11b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.91

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^3\right)^{3/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(c*(a + b*x^2)^3)^(3/2))/(11*b)
```

fricas [B] time = 0.48, size = 87, normalized size = 2.72

$$\frac{\left(b^4cx^8 + 4\,ab^3cx^6 + 6\,a^2b^2cx^4 + 4\,a^3bcx^2 + a^4c\right)\sqrt{b^3cx^6 + 3\,ab^2cx^4 + 3\,a^2bcx^2 + a^3c}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11*(b^4*c*x^8 + 4*a*b^3*c*x^6 + 6*a^2*b^2*c*x^4 + 4*a^3*b*c*x^2 + a^4*c)*
sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)/b
```

giac [A] time = 0.31, size = 28, normalized size = 0.88

$$\frac{\left(bcx^2 + ac\right)^{\frac{11}{2}} \operatorname{sgn}\left(bx^2 + a\right)}{11\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/11*(b*c*x^2 + a*c)^(11/2)*sgn(b*x^2 + a)/(b*c^4)
```

maple [A] time = 0.01, size = 26, normalized size = 0.81

$$\frac{\left(b\,x^2 + a\right)\left(\left(b\,x^2 + a\right)^3 c\right)^{\frac{3}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)^3*c)^(3/2),x)
```

```
[Out] 1/11*(b*x^2+a)/b*((b*x^2+a)^3*c)^(3/2)
```

maxima [B] time = 1.08, size = 70, normalized size = 2.19

$$\frac{\left(b^4c^{\frac{3}{2}}x^8 + 4\,ab^3c^{\frac{3}{2}}x^6 + 6\,a^2b^2c^{\frac{3}{2}}x^4 + 4\,a^3bc^{\frac{3}{2}}x^2 + a^4c^{\frac{3}{2}}\right)\left(bx^2 + a\right)^{\frac{3}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b^4*c^(3/2)*x^8 + 4*a*b^3*c^(3/2)*x^6 + 6*a^2*b^2*c^(3/2)*x^4 + 4*a^3
*b*c^(3/2)*x^2 + a^4*c^(3/2))*(b*x^2 + a)^(3/2)/b
```

mupad [B] time = 2.71, size = 62, normalized size = 1.94

$$\sqrt{c\left(b\,x^2 + a\right)^3}\,\left(\frac{a^4\,c}{11\,b} + \frac{4\,a^3\,c\,x^2}{11} + \frac{b^3\,c\,x^8}{11} + \frac{6\,a^2\,b\,c\,x^4}{11} + \frac{4\,a\,b^2\,c\,x^6}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(a + b*x^2)^3)^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^3)^(1/2)*((a^4*c)/(11*b) + (4*a^3*c*x^2)/11 + (b^3*c*x^8)/11
 + (6*a^2*b*c*x^4)/11 + (4*a*b^2*c*x^6)/11)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Timed out
```

3.239 $\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx$

Optimal. Leaf size=207

$$\frac{63a^{7/2}c\sqrt{c\left(a+bx^2\right)^3}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)}+\frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(\right.}$$

```
[Out] 21/128*a^3*c*x*(c*(b*x^2+a)^3)^(1/2)+63/256*a^4*c*x*(c*(b*x^2+a)^3)^(1/2)/(
b*x^2+a)+21/160*a^2*c*x*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/80*a*c*x*(b*x^2+a
)^2*(c*(b*x^2+a)^3)^(1/2)+1/10*c*x*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)+63/256
*a^(7/2)*c*arcsinh(x*b^(1/2)/a^(1/2))*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)^(3/
2)/b^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6720, 195, 217, 206}

$$\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)}+\frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{63a^5c\sqrt{c\left(a+bx^2\right)^3}\,\tanh}{256\sqrt{b}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (21*a^3*c*x*Sqrt[c*(a + b*x^2)^3])/128 + (63*a^4*c*x*Sqrt[c*(a + b*x^2)^3])
/(256*(a + b*x^2)) + (21*a^2*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/160 + (
9*a*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/80 + (c*x*(a + b*x^2)^3*Sqrt[c
*(a + b*x^2)^3])/10 + (63*a^5*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(256*Sqrt[b]*(a + b*x^2)^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2} dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{9/2} dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(9ac\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{7/2} dx}{10\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(63a^2c\sqrt{c\left(a+bx\right.}\right.}{80\left(a\right.}$$

$$= \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{10}cx\left(a+bx^2\right)^3$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a\right.}$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} +$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} +$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} +$$

Mathematica [A] time = 0.11, size = 132, normalized size = 0.64

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(315a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x\sqrt{\frac{bx^2}{a}+1}\left(965a^4+1490a^3bx^2+1368a^2b^2x^4+656ab^3x^6+128b^4x\right.\right.}{1280\sqrt{b}\left(a+bx^2\right)^4\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*(965*a^4 + 1490*a^3
*b*x^2 + 1368*a^2*b^2*x^4 + 656*a*b^3*x^6 + 128*b^4*x^8) + 315*a^(9/2)*ArcS
inh[(Sqrt[b]*x)/Sqrt[a]]))/(1280*Sqrt[b]*(a + b*x^2)^4*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 0.50, size = 402, normalized size = 1.94

$$\left[\frac{315\left(a^5bcx^2+a^6c\right)\sqrt{\frac{c}{b}}\log\left(-\frac{2b^2cx^4+3abcx^2+a^2c+2\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\,bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\left(128b^4cx^9+656ab^3cx^7+13\right.}{2560\left(bx^2+a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x^
2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*s
qrt(c/b))/(b*x^2 + a)) + 2*(128*b^4*c*x^9 + 656*a*b^3*c*x^7 + 1368*a^2*b^2*
c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*
a^2*b*c*x^2 + a^3*c))/(b*x^2 + a), -1/1280*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(
-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*sq
rt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (128*b^4*c*x^9 + 656*a*b^3*c*
x^7 + 1368*a^2*b^2*c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 +
 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

giac [A] time = 0.33, size = 153, normalized size = 0.74

$$-\frac{1}{1280}\left(\frac{315\,a^5c\log\left(\left|-\sqrt{bc}\,x+\sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bc}}-\left(965\,a^4\mathrm{sgn}\left(bx^2+a\right)+2\left(745\,a^3b\mathrm{sgn}\left(bx^2+a\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/1280*(315*a^5*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 +
 a)/sqrt(b*c) - (965*a^4*sgn(b*x^2 + a) + 2*(745*a^3*b*sgn(b*x^2 + a) + 4*(
171*a^2*b^2*sgn(b*x^2 + a) + 2*(8*b^4*x^2*sgn(b*x^2 + a) + 41*a*b^3*sgn(b*x
^2 + a))*x^2)*x^2)*x^2)*sqrt(b*c*x^2 + a*c)*x)*c
```

maple [A] time = 0.02, size = 205, normalized size = 0.99

$$\frac{\left(\left(b\,x^2+a\right)^3c\right)^{\frac{3}{2}}\left(315a^5c^3\ln\left(\frac{bcx+\sqrt{bc\,x^2+ac}\,\sqrt{bc}}{\sqrt{bc}}\right)+315\sqrt{bc\,x^2+ac}\,\sqrt{bc}\,a^4c^2x+128\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,b^2x^5+2\cdots\right.}{1280\left(b\,x^2+a\right)^3\left(\left(b\,x^2+a\right)c\right)^{\frac{3}{2}}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^3*c)^(3/2),x)
```

```
[Out] 1/1280*((b*x^2+a)^3*c)^(3/2)*(128*x^5*(b*c*x^2+a*c)^(5/2)*b^2*(b*c)^(1/2)+4
00*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^3*a*b+440*(b*c*x^2+a*c)^(5/2)*(b*c)^(1
/2)*x*a^2+210*(b*c*x^2+a*c)^(3/2)*(b*c)^(1/2)*x*a^3*c+315*(b*c*x^2+a*c)^(1/
2)*(b*c)^(1/2)*x*a^4*c^2+315*ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*
c)^(1/2))*a^5*c^3)/(b*x^2+a)^3/((b*x^2+a)*c)^(3/2)/c/(b*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(bx^2+a\right)^3c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\left(c\left(b\,x^2+a\right)^3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2),x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Timed out
```

3.240 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=192

$$\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}-\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}$$

```
[Out] 1/3*a^3*c*(c*(b*x^2+a)^3)^(1/2)+a^4*c*(c*(b*x^2+a)^3)^(1/2)/(b*x^2+a)+1/5*a
^2*c*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+1/7*a*c*(b*x^2+a)^2*(c*(b*x^2+a)^3)^(1
/2)+1/9*c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)-a^3*c*arctanh((1+b*x^2/a)^(1/2)
)*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)^(3/2)
```

Rubi [A] time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 266, 50, 63, 208}

$$\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{a^{9/2}c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] (a^3*c*Sqrt[c*(a + b*x^2)^3])/3 + (a^4*c*Sqrt[c*(a + b*x^2)^3])/(a + b*x^2)
 + (a^2*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/5 + (a*c*(a + b*x^2)^2*Sqrt[c*
(a + b*x^2)^3])/7 + (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/9 - (a^(9/2)*c*
Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(a + b*x^2)^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x}\,dx,x,x^2\right)}{2\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(ac\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx,x,x^2\right)}{2\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Su}}{2\left(a+\right.}$$

$$= \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+\right.}$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} +$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx\right.$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx\right.$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx\right.$$

Mathematica [A] time = 0.08, size = 111, normalized size = 0.58

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{a+bx^2}\left(563a^4+506a^3bx^2+408a^2b^2x^4+185ab^3x^6+35b^4x^8\right)-315a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{315\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^
2*b^2*x^4 + 185*a*b^3*x^6 + 35*b^4*x^8) - 315*a^(9/2)*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]]))/(315*(a + b*x^2)^(9/2))
```

fricas [A] time = 0.49, size = 391, normalized size = 2.04

$$\left[\frac{315\left(a^4bcx^2+a^5c\right)\sqrt{ac}\,\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,\sqrt{ac}}{bx^4+ax^2}\right)+2\left(35\,b^4cx^8+185\,ab^3cx^6+4\right.}{630\left(bx^2+a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b*c*x^2 +
 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(a
*c))/(b*x^4 + a*x^2)) + 2*(35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2*b^2*c*x
^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*
c*x^2 + a^3*c))/(b*x^2 + a), 1/315*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(-a*c)*ar
ctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-a*c)/(b^
2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2
*b^2*c*x^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 +
3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

giac [A] time = 0.36, size = 185, normalized size = 0.96

$$\frac{1}{315}\left(\frac{315\,a^5\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{-ac}}+\frac{315\,\sqrt{bcx^2+ac}\,a^4c^{44}\mathrm{sgn}\left(bx^2+a\right)+105\left(bcx^2+ac\right)^{\frac{3}{2}}a^3c^{43}\mathrm{sg}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/315*(315*a^5*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/sqrt(-
a*c) + (315*sqrt(b*c*x^2 + a*c)*a^4*c^44*sgn(b*x^2 + a) + 105*(b*c*x^2 + a*
c)^(3/2)*a^3*c^43*sgn(b*x^2 + a) + 63*(b*c*x^2 + a*c)^(5/2)*a^2*c^42*sgn(b*
x^2 + a) + 45*(b*c*x^2 + a*c)^(7/2)*a*c^41*sgn(b*x^2 + a) + 35*(b*c*x^2 + a
*c)^(9/2)*c^40*sgn(b*x^2 + a))/c^45)*c^2
```

maple [A] time = 0.02, size = 221, normalized size = 1.15

$$-\frac{\left(\left(b\,x^2+a\right)^3c\right)^{\frac{3}{2}}\left(315a^5c^3\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^2+ac}}{x}\right)-315\sqrt{ac}\,\sqrt{bc\,x^2+ac}\,a^4c^2-35\sqrt{ac}\,\left(bc\,x^2+ac\right)^{\frac{5}{2}}b^2x^4-1\right.}{315\left(b\,x^2+a\right)^3\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^3*c)^(3/2)/x,x)
```

```
[Out] -1/315*((b*x^2+a)^3*c)^(3/2)*(-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^4*b^2-1
15*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^2*a*b+315*ln(2*((a*c)^(1/2)*(b*c*x^2+a
*c)^(1/2)+a*c)/x)*a^5*c^3-189*a^2*((b*x^2+a)*c)^(5/2)*(a*c)^(1/2)+46*(a*c)^
(1/2)*(b*c*x^2+a*c)^(5/2)*a^2-105*(a*c)^(1/2)*(b*c*x^2+a*c)^(3/2)*a^3*c-315
*(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)*a^4*c^2)/(b*x^2+a)^3/((b*x^2+a)*c)^(3/2)/c
/(a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(bx^2+a\right)^3 c\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\left(b\,x^2+a\right)^3\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x,x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x, x)
```

3.241 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=208

$$\frac{315a^{5/2}\sqrt{b}\,c\sqrt{c\left(a+bx^2\right)^3}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)}+\frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{16}abcx\left(a+bx^2\right.$$

```
[Out] 105/64*a^2*b*c*x*(c*(b*x^2+a)^3)^(1/2)+315/128*a^3*b*c*x*(c*(b*x^2+a)^3)^(1
/2)/(b*x^2+a)+21/16*a*b*c*x*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/8*b*c*x*(b*x^
2+a)^2*(c*(b*x^2+a)^3)^(1/2)-c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)/x+315/128*
a^(5/2)*c*arcsinh(x*b^(1/2)/a^(1/2))*b^(1/2)*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2
/a)^(3/2)
```

Rubi [A] time = 0.19, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 277, 195, 217, 206}

$$\frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)}+\frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3}+\frac{315a^4\sqrt{b}\,c\sqrt{c\left(a+bx^2\right)^3}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128\left(a+bx^2\right)^{3/2}}+\frac{21}{16}abcx\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] (105*a^2*b*c*x*Sqrt[c*(a + b*x^2)^3])/64 + (315*a^3*b*c*x*Sqrt[c*(a + b*x^2
)^3])/(128*(a + b*x^2)) + (21*a*b*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/16
 + (9*b*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/8 - (c*(a + b*x^2)^3*Sqrt[
c*(a + b*x^2)^3])/x + (315*a^4*Sqrt[b]*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(128*(a + b*x^2)^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^2}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= -\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(9bc\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{7/2}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(63abc\sqrt{c\left(a+bx^2\right)^3}\right)}{8\left(a+bx^2\right.}$$

$$= \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c(a}}{x}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+\right.}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.31

$$-\frac{a^4\left(c\left(a+bx^2\right)^3\right)^{3/2}\,{}_2F_1\left(-\frac{9}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(a+bx^2\right)^4\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] -((a^4*(c*(a + b*x^2)^3)^(3/2)*Hypergeometric2F1[-9/2, -1/2, 1/2, -((b*x^2)
/a)])/(x*(a + b*x^2)^4*Sqrt[1 + (b*x^2)/a]))
```

fricas [A] time = 0.49, size = 396, normalized size = 1.90

$$\left[\frac{315\left(a^4bcx^3+a^5cx\right)\sqrt{bc}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c+2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,\sqrt{bc}\,x}{bx^2+a}\right)+2\left(16\,b^4cx^8+88\,ab^3cx^6+\cdots\right.}{256\left(bx^3+ax\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/256*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(b*c)*log(-(2*b^2*c*x^4 + 3*a*b*c*x
^2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt
(b*c)*x)/(b*x^2 + a)) + 2*(16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 210*a^2*b^2*c*x^
4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c
*x^2 + a^3*c))/(b*x^3 + a*x), -1/128*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(-b*c
)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-b*c)
*x/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 21
0*a^2*b^2*c*x^4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x
^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^3 + a*x)]
```

giac [A] time = 0.54, size = 185, normalized size = 0.89

$$\frac{1}{256}\left(\frac{512\,\sqrt{bc}\,a^5c\mathrm{sgn}\left(bx^2+a\right)}{\left(\sqrt{bc}\,x-\sqrt{bcx^2+ac}\right)^2-ac}-315\,\sqrt{bc}\,a^4\log\left(\left(\sqrt{bc}\,x-\sqrt{bcx^2+ac}\right)^2\right)\mathrm{sgn}\left(bx^2+a\right)+2\left(325\,a^3b\mathrm{sgn}\left(\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/256*(512*sqrt(b*c)*a^5*c*sgn(b*x^2 + a)/((sqrt(b*c)*x - sqrt(b*c*x^2 + a*
c))^2 - a*c) - 315*sqrt(b*c)*a^4*log((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2)
*sgn(b*x^2 + a) + 2*(325*a^3*b*sgn(b*x^2 + a) + 2*(105*a^2*b^2*sgn(b*x^2 +
a) + 4*(2*b^4*x^2*sgn(b*x^2 + a) + 11*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*sqrt(
b*c*x^2 + a*c)*x)*c
```

maple [A] time = 0.02, size = 215, normalized size = 1.03

$$\frac{\left(\left(b\,x^2+a\right)^3c\right)^{\frac{3}{2}}\left(315a^4b\,c^3x\ln\left(\frac{bcx+\sqrt{bc\,x^2+ac}\,\sqrt{bc}}{\sqrt{bc}}\right)+315\sqrt{bc\,x^2+ac}\,\sqrt{bc}\,a^3b\,c^2x^2+210\left(bc\,x^2+ac\right)^{\frac{3}{2}}\sqrt{bc}\,a^2b\cdots\right.}{128\left(b\,x^2+a\right)^3\left(\left(b\,x^2+a\right)c\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^3*c)^(3/2)/x^2,x)
```

```
[Out] 1/128*((b*x^2+a)^3*c)^(3/2)*(16*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^4*b^2+56*
(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^2*a*b+210*(b*c*x^2+a*c)^(3/2)*(b*c)^(1/2)
*x^2*a^2*b*c+315*(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2)*x^2*a^3*b*c^2+315*ln((b*c*
x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*x*a^4*b*c^3-128*(b*c*x^2+a*
c)^(5/2)*(b*c)^(1/2)*a^2)/(b*x^2+a)^3/((b*x^2+a)*c)^(3/2)/c/(b*c)^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(bx^2+a\right)^3c\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\left(b\,x^2+a\right)^3\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x^2,x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**2,x)
```

```
[Out] Timed out
```

3.242 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=202

$$\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}-\frac{9a^2bc\sqrt{c\left(a+bx^2\right)^3}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a \cdots\right.}$$

```
[Out] 3/2*a^2*b*c*(c*(b*x^2+a)^3)^(1/2)+9/2*a^3*b*c*(c*(b*x^2+a)^3)^(1/2)/(b*x^2+
a)+9/10*a*b*c*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/14*b*c*(b*x^2+a)^2*(c*(b*x^
2+a)^3)^(1/2)-1/2*c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)/x^2-9/2*a^2*b*c*arcta
nh((1+b*x^2/a)^(1/2))*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)^(3/2)
```

Rubi [A] time = 0.22, antiderivative size = 204, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6720, 266, 47, 50, 63, 208}

$$\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}-\frac{9a^{7/2}bc\sqrt{c\left(a+bx^2\right)^3}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\left(a+bx^2\right)^{3/2}}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+ \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] (3*a^2*b*c*Sqrt[c*(a + b*x^2)^3])/2 + (9*a^3*b*c*Sqrt[c*(a + b*x^2)^3])/(2*
(a + b*x^2)) + (9*a*b*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/10 + (9*b*c*(a +
 b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/14 - (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3
])/(2*x^2) - (9*a^(7/2)*b*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(2*(a + b*x^2)^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^3}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^2}\,dx,x,x^2\right)}{2\left(a+bx^2\right)^{3/2}}$$

$$= -\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2} + \frac{\left(9bc\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx,x,x^2\right)}{4\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2} + \frac{\left(9abc\sqrt{c\left(a+bx^2\right)^3}\right)\text{Su}}{4\left(a+\right.}$$

$$= \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a-\right.}}{2x^2}$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)}$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(\right.$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(\right.$$

$$= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3} + \frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{14}bc\left(\right.$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.24

$$\frac{b\left(a+bx^2\right)\left(c\left(a+bx^2\right)^3\right)^{3/2}\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] (b*(a + b*x^2)*(c*(a + b*x^2)^3)^(3/2)*Hypergeometric2F1[2, 11/2, 13/2, 1 +
 (b*x^2)/a])/(11*a^2)
```

fricas [A] time = 0.47, size = 411, normalized size = 2.03

$$\left[\frac{315\left(a^3b^2cx^4+a^4bcx^2\right)\sqrt{ac}\,\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,\sqrt{ac}}{bx^4+ax^2}\right)+2\left(10\,b^4cx^8+58\,ab^3cx^6\right.}{140\left(bx^4+ax^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a^3*b^2*c*x^4 + a^4*b*c*x^2)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b
*c*x^2 + 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c
)*sqrt(a*c))/(b*x^4 + a*x^2)) + 2*(10*b^4*c*x^8 + 58*a*b^3*c*x^6 + 156*a^2*
b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*
a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2), 1/70*(315*(a^3*b^2*c*x^4 + a^4*b*c*x
^2)*sqrt(-a*c)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*
c)*sqrt(-a*c)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (10*b^4*c*x^8 + 58*a*b^3
*c*x^6 + 156*a^2*b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3
*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2)]
```

giac [A] time = 0.39, size = 204, normalized size = 1.01

$$\frac{\left(\frac{315\,a^4b^2c\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{-ac}}-\frac{35\,\sqrt{bcx^2+ac}\,a^4b\,\mathrm{sgn}\left(bx^2+a\right)}{x^2}+\frac{2\left(140\,\sqrt{bcx^2+ac}\,a^3b^2c^{21}\mathrm{sgn}\left(bx^2+a\right)+35\left(bcx^2+ac\right)^{\frac{3}{2}}a^2b^2c^{20}\mathrm{sgn}\left(b\right.\right.}{}\right.}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/70*(315*a^4*b^2*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/s
qrt(-a*c) - 35*sqrt(b*c*x^2 + a*c)*a^4*b*sgn(b*x^2 + a)/x^2 + 2*(140*sqrt(b
*c*x^2 + a*c)*a^3*b^2*c^21*sgn(b*x^2 + a) + 35*(b*c*x^2 + a*c)^(3/2)*a^2*b^
2*c^20*sgn(b*x^2 + a) + 14*(b*c*x^2 + a*c)^(5/2)*a*b^2*c^19*sgn(b*x^2 + a)
+ 5*(b*c*x^2 + a*c)^(7/2)*b^2*c^18*sgn(b*x^2 + a))/c^21)*c/b
```

maple [A] time = 0.02, size = 238, normalized size = 1.18

$$\frac{\left(\left(b\,x^2+a\right)^3c\right)^{\frac{3}{2}}\left(-315a^4b\,c^3x^2\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^2+ac}}{x}\right)+315\sqrt{bc\,x^2+ac}\,\sqrt{ac}\,a^3b\,c^2x^2+105\left(bc\,x^2+ac\right)^{\frac{3}{2}}\sqrt{ac}\right.}{70\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^3*c)^(3/2)/x^3,x)
```

```
[Out] 1/70*((b*x^2+a)^3*c)^(3/2)*(10*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*b^2*x^4-315*
ln(2*(a*c+(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2))/x)*x^2*a^4*b*c^3-4*(a*c)^(1/2)*(
b*c*x^2+a*c)^(5/2)*a*b*x^2+105*(b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a^2*b*c+
315*(b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^2*a^3*b*c^2+42*a*b*((b*x^2+a)*c)^(5/2
)*x^2*(a*c)^(1/2)-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^2)/(b*x^2+a)^3/((b*x
^2+a)*c)^(3/2)/c/x^2/(a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(bx^2+a\right)^3 c\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\left(b\,x^2+a\right)^3\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x^3,x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**3,x)
```

```
[Out] Timed out
```

3.243 $\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=77

$$\frac{\sqrt{a}\, c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}$$

```
[Out] -c*x*(c/(b*x^2+a))^(1/2)/b+c*arcsinh(x*b^(1/2)/a^(1/2))*a^(1/2)*(c/(b*x^2+a
))^(1/2)*(1+b*x^2/a)^(1/2)/b^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6720, 288, 217, 206}

$$\frac{c\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] -((c*x*Sqrt[c/(a + b*x^2)])/b) + (c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}dx=\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int\frac{x^2}{\left(a+bx^2\right)^{3/2}}dx$$
$$=-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{a+bx^2}}dx}{b}$$
$$=-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{1-bx^2}dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}$$
$$=-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.07, size = 89, normalized size = 1.16

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\left(\frac{c}{a+bx^2}\right)^{3/2}\left(\left(a+bx^2\right)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{b}x\sqrt{\frac{bx^2}{a}+1}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] (Sqrt[a]*(c/(a + b*x^2))^(3/2)*Sqrt[1 + (b*x^2)/a]*(-(Sqrt[a]*Sqrt[b]*x*Sqr
t[1 + (b*x^2)/a]) + (a + b*x^2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/b^(3/2)
```

fricas [A] time = 0.46, size = 141, normalized size = 1.83

$$\left[-\frac{2\,cx\sqrt{\frac{c}{bx^2+a}}-c\sqrt{\frac{c}{b}}\log\left(-2\,bcx^2-ac-2\left(b^2x^3+abx\right)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{b}}\right)}{2\,b},-\frac{cx\sqrt{\frac{c}{bx^2+a}}+c\sqrt{-\frac{c}{b}}\arctan\left(\frac{bx\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{b}}}{c}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*c*x*sqrt(c/(b*x^2 + a)) - c*sqrt(c/b)*log(-2*b*c*x^2 - a*c - 2*(b^
2*x^3 + a*b*x)*sqrt(c/(b*x^2 + a))*sqrt(c/b)))/b, -(c*x*sqrt(c/(b*x^2 + a))
 + c*sqrt(-c/b)*arctan(b*x*sqrt(c/(b*x^2 + a))*sqrt(-c/b)/c))/b]
```

giac [A] time = 0.46, size = 71, normalized size = 0.92

$$-\left(\frac{cx\text{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}\,b}+\frac{c\log\left(\left|-\sqrt{bc}\,x+\sqrt{bcx^2+ac}\right|\right)\text{sgn}\left(bx^2+a\right)}{\sqrt{bc}\,b}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

```
[Out] -(c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b) + c*log(abs(-sqrt(b*c)*x + sqr
t(b*c*x^2 + a*c)))*sgn(b*x^2 + a)/(sqrt(b*c)*b))*c
```

maple [A] time = 0.01, size = 60, normalized size = 0.78

$$-\frac{\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}\left(b\,x^2+a\right)\left(b^{\frac{3}{2}}x-\sqrt{b\,x^2+a}\;b\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c/(b*x^2+a))^(3/2),x)
```

```
[Out] -(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(x*b^(3/2)-ln(b^(1/2)*x+(b*x^2+a)^(1/2))*b*(
b*x^2+a)^(1/2))/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*(c/(b*x^2 + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(\frac{c}{b\,x^2+a}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c/(a + b*x^2))^(3/2),x)
```

```
[Out] int(x^2*(c/(a + b*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(\frac{c}{a + bx^{2}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] Integral(x**2*(c/(a + b*x**2))**(3/2), x)
```

3.244 $\int x \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=21

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

[Out] -c*(c/(b*x^2+a))^(1/2)/b

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = $\{1591, 15, 30\}$

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

[In] Int[x*(c/(a + b*x^2))^(3/2),x]

[Out] -((c*Sqrt[c/(a + b*x^2)])/b)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int x\left(\frac{c}{a+bx^2}\right)^{3/2} dx &= \frac{\text{Subst}\left(\int \left(\frac{c}{x}\right)^{3/2} dx, x, a+bx^2\right)}{2b}\\ &= \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}} dx, x, a+bx^2\right)}{2b}\\ &= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c/(a + b*x^2))^(3/2),x]
[Out] -((c*Sqrt[c/(a + b*x^2)])/b)
```

fricas [A] time = 0.44, size = 19, normalized size = 0.90

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

giac [A] time = 0.39, size = 28, normalized size = 1.33

$$-\frac{c^2\text{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
[Out] -c^2*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b)
```

maple [A] time = 0.00, size = 26, normalized size = 1.24

$$-\frac{\left(b\,x^2+a\right)\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1/(b*x^2+a)*c)^(3/2),x)
[Out] -(b*x^2+a)/b*(1/(b*x^2+a)*c)^(3/2)
```

maxima [A] time = 1.06, size = 19, normalized size = 0.90

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

mupad [B] time = 2.64, size = 19, normalized size = 0.90

$$-\frac{c\sqrt{\frac{c}{b\,x^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c/(a + b*x^2))^(3/2),x)
[Out] -(c*(c/(a + b*x^2))^(1/2))/b
```

sympy [**A**] time = 1.51, size = 53, normalized size = 2.52

$$\begin{cases} -\frac{ac^{\frac{3}{2}}\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}}}{b} - c^{\frac{3}{2}}x^2\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}} & \text{for } b \neq 0 \\ \frac{x^2\left(\frac{c}{a}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] Piecewise((-a*c**(3/2)*(1/(a + b*x**2))**(3/2)/b - c**(3/2)*x**2*(1/(a + b*
x**2))**(3/2), Ne(b, 0)), (x**2*(c/a)**(3/2)/2, True))
```

3.245 $\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=21

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

[Out] c*x*(c/(b*x^2+a))^(1/2)/a

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6720, 191}

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

[In] Int[(c/(a + b*x^2))^(3/2),x]

[Out] (c*x*Sqrt[c/(a + b*x^2)])/a

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx\\ &= \frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(c/(a + b*x^2))^(3/2),x]

[Out] (c*x*Sqrt[c/(a + b*x^2)])/a

fricas [A] time = 0.46, size = 19, normalized size = 0.90

$$\frac{cx\sqrt{\frac{c}{bx^2+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

```
[Out] c*x*sqrt(c/(b*x^2 + a))/a
```

giac [A] time = 0.48, size = 28, normalized size = 1.33

$$\frac{c^2x\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

```
[Out] c^2*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a)
```

maple [A] time = 0.00, size = 26, normalized size = 1.24

$$\frac{\left(b\,x^2+a\right)\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x^2+a)*c)^(3/2),x)
```

```
[Out] (b*x^2+a)/a*x*(1/(b*x^2+a)*c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c/(b*x^2 + a))^(3/2), x)
```

mupad [B] time = 2.74, size = 19, normalized size = 0.90

$$\frac{c\,x\,\sqrt{\frac{c}{b\,x^2+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2),x)
```

```
[Out] (c*x*(c/(a + b*x^2))^(1/2))/a
```

sympy [A] time = 1.46, size = 66, normalized size = 3.14

$$\begin{cases} c^{\frac{3}{2}}x\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}}+\frac{bc^{\frac{3}{2}}x^3\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}}}{a} & \text{for } a\neq 0 \\ -\frac{c^{\frac{3}{2}}x\left(\frac{1}{b}\right)^{\frac{3}{2}}\left(\frac{1}{x^2}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2),x)
```

```
[Out] Piecewise((c**(3/2)*x*(1/(a + b*x**2))**(3/2) + b*c**(3/2)*x**3*(1/(a + b*x
**2))**(3/2)/a, Ne(a, 0)), (-c**(3/2)*x*(1/b)**(3/2)*(x**(-2))**(3/2)/2, Tr
ue))
```

3.246 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=71

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{\frac{bx^2}{a}+1}\,\sqrt{\frac{c}{a+bx^2}}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{a}$$

```
[Out] c*(c/(b*x^2+a))^(1/2)/a-c*arctanh((1+b*x^2/a)^(1/2))*(c/(b*x^2+a))^(1/2)*(1
+b*x^2/a)^(1/2)/a
```

Rubi [A] time = 0.14, antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 266, 51, 63, 208}

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{a+bx^2}\,\sqrt{\frac{c}{a+bx^2}}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)])/a - (c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
```

```
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx$$
$$= \frac{1}{2}\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a} + \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}$$
$$= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a} + \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}$$
$$= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a} - \frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.54

$$\frac{c\sqrt{\frac{c}{a+bx^2}}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)]*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^2)/a])/a
```

fricas [A] time = 0.52, size = 138, normalized size = 1.94

$$\left[\frac{c\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac-2\left(abx^2+a^2\right)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)+2\,c\sqrt{\frac{c}{bx^2+a}}}{2\,a}, \frac{c\sqrt{-\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}{c}\right)+c\sqrt{\frac{c}{bx^2+a}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(c*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c - 2*(a*b*x^2 + a^2)*sqrt(c/(b*x^2 +
 a))*sqrt(c/a))/x^2) + 2*c*sqrt(c/(b*x^2 + a)))/a, (c*sqrt(-c/a)*arctan(a*s
qrt(c/(b*x^2 + a))*sqrt(-c/a)/c) + c*sqrt(c/(b*x^2 + a)))/a]
```

giac [A] time = 0.25, size = 59, normalized size = 0.83

$$c\left(\frac{c\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,a} + \frac{c}{\sqrt{bcx^2+ac}\,a}\right)\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="giac")
```

```
[Out] c*(c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a) + c/(sqrt(b*c*x^
2 + a*c)*a))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 64, normalized size = 0.90

$$-\frac{\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}} \left(b\,x^{2}+a\right) \left(\sqrt{b\,x^{2}+a}\;a \ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)-a^{\frac{3}{2}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x^2+a)*c)^(3/2)/x,x)
```

```
[Out] -(1/(b*x^2+a)*c)^(3/2)*(b*x^2+a)*(ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a*(b*
x^2+a)^(1/2)-a^(3/2))/a^(5/2)
```

maxima [A] time = 1.95, size = 80, normalized size = 1.13

$$\frac{1}{2}\,c\left(\frac{c\log\left(\frac{a\sqrt{\frac{c}{bx^2+a}}-\sqrt{ac}}{a\sqrt{\frac{c}{bx^2+a}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a}+\frac{2\sqrt{\frac{c}{bx^2+a}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*c*(c*log((a*sqrt(c/(b*x^2 + a)) - sqrt(a*c))/(a*sqrt(c/(b*x^2 + a)) + s
qrt(a*c)))/(sqrt(a*c)*a) + 2*sqrt(c/(b*x^2 + a))/a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{c}{b\,x^2+a}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2)/x,x)
```

```
[Out] int((c/(a + b*x^2))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{c}{a+bx^{2}}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x, x)
```

3.247 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} dx$

Optimal. Leaf size=48

$$-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

```
[Out] -c*(c/(b*x^2+a))^(1/2)/a/x-2*b*c*x*(c/(b*x^2+a))^(1/2)/a^2
```

Rubi [A] time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6720, 271, 191}

$$-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x^2,x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/(a*x)) - (2*b*c*x*Sqrt[c/(a + b*x^2)])/a^2
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}} dx \\
&= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}-\frac{\left(2bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}} dx}{a} \\
&= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.67

$$-\frac{c\left(a+2bx^2\right)\sqrt{\frac{c}{a+bx^2}}}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x^2,x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)]*(a + 2*b*x^2))/(a^2*x))
```

fricas [A] time = 0.45, size = 32, normalized size = 0.67

$$-\frac{\left(2\,bcx^2+ac\right)\sqrt{\frac{c}{bx^2+a}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a))/(a^2*x)
```

giac [A] time = 0.34, size = 81, normalized size = 1.69

$$-\left(\frac{bcx\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}\,a^2}-\frac{2\,\sqrt{bc}\,c\mathrm{sgn}\left(bx^2+a\right)}{\left(\left(\sqrt{bc}\,x-\sqrt{bcx^2+ac}\right)^2-ac\right)a}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -(b*c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a^2) - 2*sqrt(b*c)*c*sgn(b*x^2
+ a)/(((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2 - a*c)*a))*c
```

maple [A] time = 0.01, size = 37, normalized size = 0.77

$$-\frac{\left(b\,x^2+a\right)\left(2b\,x^2+a\right)\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x^2+a)*c)^(3/2)/x^2,x)
```

```
[Out] -(b*x^2+a)*(2*b*x^2+a)*(1/(b*x^2+a)*c)^(3/2)/a^2/x
```

maxima [A] time = 0.96, size = 46, normalized size = 0.96

$$-\frac{2\,b^2c^{\frac{3}{2}}x^4+3\,abc^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -(2*b^2*c^(3/2)*x^4 + 3*a*b*c^(3/2)*x^2 + a^2*c^(3/2))/((b*x^2 + a)^(3/2)*a
^2*x)
```

mupad [B] time = 2.83, size = 54, normalized size = 1.12

$$-\frac{\left(\frac{b\,c}{a}+\frac{2\,b^2\,c\,x^2}{a^2}\right)\sqrt{\frac{c}{b\,x^2+a}}\,\left(\frac{a}{b}+x^2\right)}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2)/x^2,x)
```

```
[Out] -((((b*c)/a + (2*b^2*c*x^2)/a^2)*(c/(a + b*x^2))^(1/2)*(a/b + x^2))/(a*x + b
*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x**2,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**2, x)
```

3.248 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=104

$$-\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2}+\frac{3bc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2}$$

```
[Out] -3/2*b*c*(c/(b*x^2+a))^(1/2)/a^2-1/2*c*(c/(b*x^2+a))^(1/2)/a/x^2+3/2*b*c*ar
ctanh((1+b*x^2/a)^(1/2))*(c/(b*x^2+a))^(1/2)*(1+b*x^2/a)^(1/2)/a^2
```

Rubi [A] time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6720, 266, 51, 63, 208}

$$-\frac{3c\left(a+bx^2\right)\sqrt{\frac{c}{a+bx^2}}}{2a^2x^2}+\frac{3bc\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)])/(a*x^2) - (3*c*Sqrt[c/(a + b*x^2)]*(a + b*x^2))/(2*
a^2*x^2) + (3*b*c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/(2*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{x^3\left(a+bx^2\right)^{3/2}}\,dx\\
&= \frac{1}{2}\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}+\frac{\left(3c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}-\frac{3c\sqrt{\frac{c}{a+bx^2}}\left(a+bx^2\right)}{2a^2x^2}-\frac{\left(3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a^2}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}-\frac{3c\sqrt{\frac{c}{a+bx^2}}\left(a+bx^2\right)}{2a^2x^2}-\frac{\left(3c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a^2}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}-\frac{3c\sqrt{\frac{c}{a+bx^2}}\left(a+bx^2\right)}{2a^2x^2}+\frac{3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.38

$$-\frac{bc\sqrt{\frac{c}{a+bx^2}}\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] -((b*c*Sqrt[c/(a + b*x^2)]*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^2)/a])/
a^2)
```

fricas [A] time = 0.48, size = 175, normalized size = 1.68

$$\left[\frac{3\,bcx^2\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac+2\left(abx^2+a^2\right)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)-2\left(3\,bcx^2+ac\right)\sqrt{\frac{c}{bx^2+a}}}{4\,a^2x^2},-\frac{3\,bcx^2\sqrt{-\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}{c}\right)+(3\right.}{2\,a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*b*c*x^2*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c + 2*(a*b*x^2 + a^2)*sqrt(c/
(b*x^2 + a))*sqrt(c/a))/x^2) - 2*(3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(a^
2*x^2), -1/2*(3*b*c*x^2*sqrt(-c/a)*arctan(a*sqrt(c/(b*x^2 + a))*sqrt(-c/a)/
c) + (3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(a^2*x^2)]
```

giac [A] time = 0.33, size = 103, normalized size = 0.99

$$-\frac{1}{2}\,c\left(\frac{3\,bc\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,a^2}+\frac{2\,abc^2-3\left(bcx^2+ac\right)bc}{\left(\sqrt{bcx^2+ac}\,ac-\left(bcx^2+ac\right)^{\frac{3}{2}}\right)a^2}\right)\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*c*(3*b*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^2) + (2*
a*b*c^2 - 3*(b*c*x^2 + a*c)*b*c)/((sqrt(b*c*x^2 + a*c)*a*c - (b*c*x^2 + a*c
)^(3/2))*a^2))*sgn(b*x^2 + a)
```

maple [A] time = 0.01, size = 81, normalized size = 0.78

$$\frac{\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}\left(b\,x^{2}+a\right)\left(3\sqrt{b\,x^{2}+a}\;a b\,x^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)-3a^{\frac{3}{2}}b\,x^{2}-a^{\frac{5}{2}}\right)}{2a^{\frac{7}{2}}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x^2+a)*c)^(3/2)/x^3,x)
```

```
[Out] 1/2*(1/(b*x^2+a)*c)^(3/2)*(b*x^2+a)*(3*(b*x^2+a)^(1/2)*ln(2*(a+(b*x^2+a)^(1
/2)*a^(1/2))/x)*x^2*a*b-3*a^(3/2)*x^2*b-a^(5/2))/a^(7/2)/x^2
```

maxima [A] time = 1.99, size = 121, normalized size = 1.16

$$-\frac{1}{4}\,bc\left(\frac{2\,c\sqrt{\frac{c}{bx^{2}+a}}}{a^{2}c-\frac{a^{3}c}{bx^{2}+a}}+\frac{3\,c\log\left(\frac{a\sqrt{\frac{c}{bx^{2}+a}}-\sqrt{ac}}{a\sqrt{\frac{c}{bx^{2}+a}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a^{2}}+\frac{4\sqrt{\frac{c}{bx^{2}+a}}}{a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*c*(2*c*sqrt(c/(b*x^2 + a))/(a^2*c - a^3*c/(b*x^2 + a)) + 3*c*log((a*
sqrt(c/(b*x^2 + a)) - sqrt(a*c))/(a*sqrt(c/(b*x^2 + a)) + sqrt(a*c)))/(sqrt
(a*c)*a^2) + 4*sqrt(c/(b*x^2 + a))/a^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{c}{b\,x^{2}+a}\right)^{3/2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2)/x^3,x)
```

```
[Out] int((c/(a + b*x^2))^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{c}{a+bx^{2}}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x**3,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**3, x)
```

3.249 $\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=138

$$-\frac{2a^3\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^4}+\frac{6a^2\left(a+bx^2\right)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^4}+\frac{2\left(a+bx^2\right)^4\left(c\sqrt{a+bx^2}\right)^{3/2}}{19b^4}-\frac{2a\left(a+bx^2\right)^3\left(c\sqrt{a+bx^2}\right)^{3/2}}{5b^4}$$

```
[Out] -2/7*a^3*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^4+6/11*a^2*(b*x^2+a)^2*(c*(b
*x^2+a)^(1/2))^(3/2)/b^4-2/5*a*(b*x^2+a)^3*(c*(b*x^2+a)^(1/2))^(3/2)/b^4+2/
19*(b*x^2+a)^4*(c*(b*x^2+a)^(1/2))^(3/2)/b^4
```

Rubi [A] time = 0.19, antiderivative size = 152, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6720, 266, 43}

$$\frac{6a^2c\left(a+bx^2\right)^{5/2}\sqrt{c\sqrt{a+bx^2}}}{11b^4}-\frac{2a^3c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b^4}+\frac{2c\left(a+bx^2\right)^{9/2}\sqrt{c\sqrt{a+bx^2}}}{19b^4}-\frac{2ac\left(a+bx^2\right)^{7/2}\sqrt{c\sqrt{a+bx^2}}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-2*a^3*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^4) + (6*a^2*c*Sqr
t[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^4) - (2*a*c*Sqrt[c*Sqrt[a + b
*x^2]]*(a + b*x^2)^(7/2))/(5*b^4) + (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2
)^(9/2))/(19*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^7\left(a+bx^2\right)^{3/4}dx}{\sqrt[4]{a+bx^2}}$$
$$=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x^3(a+bx)^{3/4}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{3/4}}{b^3}+\frac{3a^2(a+bx)^{7/4}}{b^3}-\frac{3a(a+bx)^{11/4}}{b^3}+\frac{(a+bx)^{15/4}}{b^3}\right)dx,x,$$
$$2\sqrt[4]{a+bx^2}$$
$$=-\frac{2a^3c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b^4}+\frac{6a^2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{5/2}}{11b^4}-\frac{2ac\sqrt{c\sqrt{a+bx}}}{5}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.46

$$\frac{2\left(a+bx^2\right)\left(-128a^3+224a^2bx^2-308ab^2x^4+385b^3x^6\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-128*a^3 + 224*a^2*b*x^2 - 308*a*
b^2*x^4 + 385*b^3*x^6))/(7315*b^4)
```

fricas [A] time = 0.50, size = 75, normalized size = 0.54

$$\frac{2\left(385\,b^4cx^8+77\,ab^3cx^6-84\,a^2b^2cx^4+96\,a^3bcx^2-128\,a^4c\right)\sqrt{bx^2+a}\,\sqrt{\sqrt{bx^2+a}\,c}}{7315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^4*c*x^8 + 77*a*b^3*c*x^6 - 84*a^2*b^2*c*x^4 + 96*a^3*b*c*x^2
- 128*a^4*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^4
```

giac [A] time = 0.34, size = 137, normalized size = 0.99

$$\frac{2\,c^{\frac{3}{2}}\left(\frac{19\left(77\left(bx^2+a\right)^{\frac{15}{4}}-315\left(bx^2+a\right)^{\frac{11}{4}}a+495\left(bx^2+a\right)^{\frac{7}{4}}a^2-385\left(bx^2+a\right)^{\frac{3}{4}}a^3\right)a}{b^3}+\frac{1155\left(bx^2+a\right)^{\frac{19}{4}}-5852\left(bx^2+a\right)^{\frac{15}{4}}a+11970\left(bx^2+a\right)^{\frac{11}{4}}a^2-125}{b^3}\right.}{21945\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/21945*c^(3/2)*(19*(77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4)*a + 495
*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)*a/b^3 + (1155*(b*x^2 +
a)^(19/4) - 5852*(b*x^2 + a)^(15/4)*a + 11970*(b*x^2 + a)^(11/4)*a^2 - 1254
0*(b*x^2 + a)^(7/4)*a^3 + 7315*(b*x^2 + a)^(3/4)*a^4)/b^3)/b
```

maple [A] time = 0.01, size = 58, normalized size = 0.42

$$-\frac{2\left(b\,x^2+a\right)\left(-385b^3x^6+308a\,b^2x^4-224a^2b\,x^2+128a^3\right)\left(\sqrt{b\,x^2+a}\;c\right)^{\frac{3}{2}}}{7315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

```
[Out] -2/7315*(b*x^2+a)*(-385*b^3*x^6+308*a*b^2*x^4-224*a^2*b*x^2+128*a^3)*(c*(b*
x^2+a)^(1/2))^(3/2)/b^4
```

maxima [A] time = 0.98, size = 85, normalized size = 0.62

$$-\frac{2\left(1045\left(\sqrt{bx^2+a}\,c\right)^{\frac{7}{2}}a^3c^6-1995\left(\sqrt{bx^2+a}\,c\right)^{\frac{11}{2}}a^2c^4+1463\left(\sqrt{bx^2+a}\,c\right)^{\frac{15}{2}}ac^2-385\left(\sqrt{bx^2+a}\,c\right)^{\frac{19}{2}}\right)}{7315\,b^4c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/7315*(1045*(sqrt(b*x^2 + a)*c)^(7/2)*a^3*c^6 - 1995*(sqrt(b*x^2 + a)*c)^
(11/2)*a^2*c^4 + 1463*(sqrt(b*x^2 + a)*c)^(15/2)*a*c^2 - 385*(sqrt(b*x^2 +
a)*c)^(19/2))/(b^4*c^8)
```

mupad [B] time = 2.96, size = 109, normalized size = 0.79

$$\sqrt{c\,\sqrt{b\,x^2+a}}\,\left(\frac{2\,c\,x^8\,\sqrt{b\,x^2+a}}{19}-\frac{256\,a^4\,c\,\sqrt{b\,x^2+a}}{7315\,b^4}+\frac{2\,a\,c\,x^6\,\sqrt{b\,x^2+a}}{95\,b}-\frac{24\,a^2\,c\,x^4\,\sqrt{b\,x^2+a}}{1045\,b^2}+\frac{192\,a^3\,c\,x^2\,\sqrt{}}{7315\,}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^8*(a + b*x^2)^(1/2))/19 - (256*a^4*c*(a
 + b*x^2)^(1/2))/(7315*b^4) + (2*a*c*x^6*(a + b*x^2)^(1/2))/(95*b) - (24*a^
2*c*x^4*(a + b*x^2)^(1/2))/(1045*b^2) + (192*a^3*c*x^2*(a + b*x^2)^(1/2))/(
7315*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Timed out
```

3.250 $\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=102

$$\frac{2a^2\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^3}+\frac{2\left(a+bx^2\right)^3\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b^3}-\frac{4a\left(a+bx^2\right)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^3}$$

```
[Out] 2/7*a^2*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^3-4/11*a*(b*x^2+a)^2*(c*(b*x^
2+a)^(1/2))^(3/2)/b^3+2/15*(b*x^2+a)^3*(c*(b*x^2+a)^(1/2))^(3/2)/b^3
```

Rubi [A] time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6720, 266, 43}

$$\frac{2a^2c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b^3}+\frac{2c\left(a+bx^2\right)^{7/2}\sqrt{c\sqrt{a+bx^2}}}{15b^3}-\frac{4ac\left(a+bx^2\right)^{5/2}\sqrt{c\sqrt{a+bx^2}}}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*a^2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^3) - (4*a*c*Sqrt[c
*Sqrt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^3) + (2*c*Sqrt[c*Sqrt[a + b*x^2]
]*(a + b*x^2)^(7/2))/(15*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^5\left(c\sqrt{a+bx^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^5\left(a+bx^2\right)^{3/4}dx}{\sqrt[4]{a+bx^2}}$$
$$=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x^2(a+bx)^{3/4}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{3/4}}{b^2}-\frac{2a(a+bx)^{7/4}}{b^2}+\frac{(a+bx)^{11/4}}{b^2}\right)dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$=\frac{2a^2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b^3}-\frac{4ac\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{5/2}}{11b^3}+\frac{2c\sqrt{c\sqrt{a+bx^2}}\left(a+\right.}{15b^3}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.51

$$\frac{2\left(a+bx^2\right)\left(32a^2-56abx^2+77b^2x^4\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{1155b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(32*a^2 - 56*a*b*x^2 + 77*b^2*x^4)
)/(1155*b^3)
```

fricas [A] time = 0.49, size = 63, normalized size = 0.62

$$\frac{2\left(77\,b^3cx^6+21\,ab^2cx^4-24\,a^2bcx^2+32\,a^3c\right)\sqrt{bx^2+a}\,\sqrt{\sqrt{bx^2+a}\,c}}{1155\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*c*x^6 + 21*a*b^2*c*x^4 - 24*a^2*b*c*x^2 + 32*a^3*c)*sqrt(b*x
^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^3
```

giac [A] time = 0.24, size = 109, normalized size = 1.07

$$\frac{2\,c^{\frac{3}{2}}\left(\frac{5\left(21\left(bx^2+a\right)^{\frac{11}{4}}-66\left(bx^2+a\right)^{\frac{7}{4}}a+77\left(bx^2+a\right)^{\frac{3}{4}}a^2\right)a}{b^2}+\frac{77\left(bx^2+a\right)^{\frac{15}{4}}-315\left(bx^2+a\right)^{\frac{11}{4}}a+495\left(bx^2+a\right)^{\frac{7}{4}}a^2-385\left(bx^2+a\right)^{\frac{3}{4}}a^3}{b^2}\right)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/1155*c^(3/2)*(5*(21*(b*x^2 + a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*x
^2 + a)^(3/4)*a^2)*a/b^2 + (77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4)*
a + 495*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)/b^2)/b
```

maple [A] time = 0.01, size = 47, normalized size = 0.46

$$\frac{2\left(b\,x^2+a\right)\left(77x^4b^2-56ab\,x^2+32a^2\right)\left(\sqrt{b\,x^2+a}\;c\right)^{\frac{3}{2}}}{1155b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^2+a)^(1/2)*c)^(3/2),x)
```

```
[Out] 2/1155*(b*x^2+a)*(77*b^2*x^4-56*a*b*x^2+32*a^2)*((b*x^2+a)^(1/2)*c)^(3/2)/b
^3
```

maxima [A] time = 0.93, size = 64, normalized size = 0.63

$$\frac{2\left(165\left(\sqrt{bx^2+a}\,c\right)^{\frac{7}{2}}a^2c^4-210\left(\sqrt{bx^2+a}\,c\right)^{\frac{11}{2}}ac^2+77\left(\sqrt{bx^2+a}\,c\right)^{\frac{15}{2}}\right)}{1155\,b^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(165*(sqrt(b*x^2 + a)*c)^(7/2)*a^2*c^4 - 210*(sqrt(b*x^2 + a)*c)^(11
/2)*a*c^2 + 77*(sqrt(b*x^2 + a)*c)^(15/2))/(b^3*c^6)
```

mupad [B] time = 2.90, size = 88, normalized size = 0.86

$$\sqrt{c\,\sqrt{b\,x^2+a}}\,\left(\frac{2\,c\,x^6\,\sqrt{b\,x^2+a}}{15}+\frac{64\,a^3\,c\,\sqrt{b\,x^2+a}}{1155\,b^3}+\frac{2\,a\,c\,x^4\,\sqrt{b\,x^2+a}}{55\,b}-\frac{16\,a^2\,c\,x^2\,\sqrt{b\,x^2+a}}{385\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^6*(a + b*x^2)^(1/2))/15 + (64*a^3*c*(a
+ b*x^2)^(1/2))/(1155*b^3) + (2*a*c*x^4*(a + b*x^2)^(1/2))/(55*b) - (16*a^2
*c*x^2*(a + b*x^2)^(1/2))/(385*b^2))
```

sympy [A] time = 88.61, size = 116, normalized size = 1.14

$$\begin{cases} \frac{64a^3c^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{4}}}{1155b^3}-\frac{16a^2c^{\frac{3}{2}}x^2\left(a+bx^2\right)^{\frac{3}{4}}}{385b^2}+\frac{2ac^{\frac{3}{2}}x^4\left(a+bx^2\right)^{\frac{3}{4}}}{55b}+\frac{2c^{\frac{3}{2}}x^6\left(a+bx^2\right)^{\frac{3}{4}}}{15} & \text{for } b\neq 0 \\ \frac{x^6\left(\sqrt{a}\,c\right)^{\frac{3}{2}}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((64*a**3*c**(3/2)*(a + b*x**2)**(3/4)/(1155*b**3) - 16*a**2*c**(3
/2)*x**2*(a + b*x**2)**(3/4)/(385*b**2) + 2*a*c**(3/2)*x**4*(a + b*x**2)**(
3/4)/(55*b) + 2*c**(3/2)*x**6*(a + b*x**2)**(3/4)/15, Ne(b, 0)), (x**6*(sqr
t(a)*c)**(3/2)/6, True))
```

3.251 $\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=66

$$\frac{2\left(a+bx^2\right)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^2}-\frac{2a\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^2}$$

```
[Out] -2/7*a*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^2+2/11*(b*x^2+a)^2*(c*(b*x^2+a
)^(1/2))^(3/2)/b^2
```

Rubi [A] time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6720, 266, 43}

$$\frac{2c\left(a+bx^2\right)^{5/2}\sqrt{c\sqrt{a+bx^2}}}{11b^2}-\frac{2ac\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-2*a*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^2) + (2*c*Sqrt[c*Sq
rt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^3\left(c\sqrt{a+bx^2}\right)^{3/2}dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^3\left(a+bx^2\right)^{3/4}dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x(a+bx)^{3/4}dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int\left(-\frac{a(a+bx)^{3/4}}{b}+\frac{(a+bx)^{7/4}}{b}\right)dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= -\frac{2ac\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b^2}+\frac{2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{5/2}}{11b^2}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.62

$$\frac{2\left(a+bx^2\right)\left(7bx^2-4a\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{77b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-4*a + 7*b*x^2))/(77*b^2)
```

fricas [A] time = 0.49, size = 51, normalized size = 0.77

$$\frac{2\left(7\,b^2cx^4+3\,abcx^2-4\,a^2c\right)\sqrt{bx^2+a}\,\sqrt{\sqrt{bx^2+a}\,c}}{77\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
[Out] 2/77*(7*b^2*c*x^4 + 3*a*b*c*x^2 - 4*a^2*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2
+ a)*c)/b^2
```

giac [A] time = 0.30, size = 81, normalized size = 1.23

$$\frac{2\left(\frac{11\left(3\left(bx^2+a\right)^{\frac{7}{4}}-7\left(bx^2+a\right)^{\frac{3}{4}}a\right)a}{b}+\frac{21\left(bx^2+a\right)^{\frac{11}{4}}-66\left(bx^2+a\right)^{\frac{7}{4}}a+77\left(bx^2+a\right)^{\frac{3}{4}}a^2}{b}\right)c^{\frac{3}{2}}}{231\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
[Out] 2/231*(11*(3*(b*x^2 + a)^(7/4) - 7*(b*x^2 + a)^(3/4)*a)*a/b + (21*(b*x^2 +
a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*x^2 + a)^(3/4)*a^2)/b)*c^(3/2)/b
```

maple [A] time = 0.01, size = 36, normalized size = 0.55

$$-\frac{2\left(b\,x^2+a\right)\left(-7b\,x^2+4a\right)\left(\sqrt{b\,x^2+a}\;c\right)^{\frac{3}{2}}}{77b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)^(1/2)*c)^(3/2),x)
```

```
[Out] -2/77*(b*x^2+a)*(-7*b*x^2+4*a)*((b*x^2+a)^(1/2)*c)^(3/2)/b^2
```

maxima [A] time = 0.93, size = 43, normalized size = 0.65

$$-\frac{2\left(11\left(\sqrt{bx^2+a}\,c\right)^{\frac{7}{2}}ac^2-7\left(\sqrt{bx^2+a}\,c\right)^{\frac{11}{2}}\right)}{77\,b^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/77*(11*(sqrt(b*x^2 + a)*c)^(7/2)*a*c^2 - 7*(sqrt(b*x^2 + a)*c)^(11/2))/(
b^2*c^4)
```

mupad [B] time = 2.89, size = 67, normalized size = 1.02

$$\sqrt{c\,\sqrt{b\,x^2+a}}\,\left(\frac{2\,c\,x^4\,\sqrt{b\,x^2+a}}{11}-\frac{8\,a^2\,c\,\sqrt{b\,x^2+a}}{77\,b^2}+\frac{6\,a\,c\,x^2\,\sqrt{b\,x^2+a}}{77\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^4*(a + b*x^2)^(1/2))/11 - (8*a^2*c*(a +
 b*x^2)^(1/2))/(77*b^2) + (6*a*c*x^2*(a + b*x^2)^(1/2))/(77*b))
```

sympy [A] time = 38.30, size = 87, normalized size = 1.32

$$\begin{cases} -\frac{8a^2c^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{4}}}{77b^2}+\frac{6ac^{\frac{3}{2}}x^2\left(a+bx^2\right)^{\frac{3}{4}}}{77b}+\frac{2c^{\frac{3}{2}}x^4\left(a+bx^2\right)^{\frac{3}{4}}}{11} & \text{for } b \neq 0 \\ \frac{x^4\left(\sqrt{a}\,c\right)^{\frac{3}{2}}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((-8*a**2*c**(3/2)*(a + b*x**2)**(3/4)/(77*b**2) + 6*a*c**(3/2)*x*
*2*(a + b*x**2)**(3/4)/(77*b) + 2*c**(3/2)*x**4*(a + b*x**2)**(3/4)/11, Ne(
b, 0)), (x**4*(sqrt(a)*c)**(3/2)/4, True))
```

3.252 $\int x \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=36

$$\frac{2c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b}$$

```
[Out] 2/7*c*(b*x^2+a)^(3/2)*(c*(b*x^2+a)^(1/2))^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1591, 15, 30}

$$\frac{2c\left(a+bx^2\right)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int x\left(c\sqrt{a+bx^2}\right)^{3/2} dx &= \frac{\text{Subst}\left(\int \left(c\sqrt{x}\right)^{3/2} dx, x, a+bx^2\right)}{2b}\\ &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x^{3/4}\,dx, x, a+bx^2\right)}{2b\sqrt[4]{a+bx^2}}\\ &= \frac{2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.86

$$\frac{2\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b)
```

fricas [A] time = 0.47, size = 37, normalized size = 1.03

$$\frac{2\left(bcx^2+ac\right)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}\,c}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b*c*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b
```

giac [A] time = 0.27, size = 17, normalized size = 0.47

$$\frac{2\left(bx^2+a\right)^{\frac{7}{4}}c^{\frac{3}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*(b*x^2 + a)^(7/4)*c^(3/2)/b
```

maple [A] time = 0.00, size = 26, normalized size = 0.72

$$\frac{2\left(b\,x^2+a\right)\left(\sqrt{b\,x^2+a}\;c\right)^{\frac{3}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)^(1/2)*c)^(3/2),x)
```

```
[Out] 2/7*(b*x^2+a)*((b*x^2+a)^(1/2)*c)^(3/2)/b
```

maxima [A] time = 0.91, size = 25, normalized size = 0.69

$$\frac{2\left(bx^2+a\right)\left(\sqrt{bx^2+a}\,c\right)^{\frac{3}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x^2 + a)*(sqrt(b*x^2 + a)*c)^(3/2)/b
```

mupad [B] time = 2.77, size = 28, normalized size = 0.78

$$\frac{2\,c\left(b\,x^2+a\right)^{3/2}\sqrt{c\,\sqrt{b\,x^2+a}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (2*c*(a + b*x^2)^(3/2)*(c*(a + b*x^2)^(1/2))^(1/2))/(7*b)
```

sympy [A] time = 14.89, size = 58, normalized size = 1.61

$$\begin{cases} \frac{2ac^{\frac{3}{2}}(a+bx^2)^{\frac{3}{4}}}{7b} + \frac{2c^{\frac{3}{2}}x^2(a+bx^2)^{\frac{3}{4}}}{7} & \text{for } b \neq 0 \\ \frac{x^2(\sqrt{a}\,c)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((2*a*c**(3/2)*(a + b*x**2)**(3/4)/(7*b) + 2*c**(3/2)*x**2*(a + b*
x**2)**(3/4)/7, Ne(b, 0)), (x**2*(sqrt(a)*c)**(3/2)/2, True))
```

3.253 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=117

$$\frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\tan^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\tanh^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

```
[Out] 2/3*(c*(b*x^2+a)^(1/2))^(3/2)+arctan((1+b*x^2/a)^(1/4))*(c*(b*x^2+a)^(1/2))
^(3/2)/(1+b*x^2/a)^(3/4)-arctanh((1+b*x^2/a)^(1/4))*(c*(b*x^2+a)^(1/2))^(3/
2)/(1+b*x^2/a)^(3/4)
```

Rubi [A] time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 266, 50, 63, 298, 203, 206}

$$\frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}}-\frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}}+\frac{2}{3}c\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

```
[Out] (2*c*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2])/3 + (a^(3/4)*c*Sqrt[c*Sqrt[a
+ b*x^2]]*ArcTan[(a + b*x^2)^(1/4)/a^(1/4)])/(a + b*x^2)^(1/4) - (a^(3/4)*c
*Sqrt[c*Sqrt[a + b*x^2]]*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)])/(a + b*x^2)^(1
/4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{\left(a+bx^2\right)^{3/4}}{x}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{(a+bx)^{3/4}}{x}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$

$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} + \frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$

$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} + \frac{\left(2ac\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx,x,\sqrt[4]{a+bx^2}\right)}{b\sqrt[4]{a+bx^2}}$$

$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} - \frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx,x,\sqrt[4]{a+bx^2}\right)}{\sqrt[4]{a+bx^2}} + \frac{(a}{}$$

$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} + \frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\,\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}} - \frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\,\text{t}}{\sqrt[4]{a+b}}$$

Mathematica [A] time = 0.04, size = 96, normalized size = 0.82

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(3a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)-3a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)+2\left(a+bx^2\right)^{3/4}\right)}{3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

```
[Out] ((c*Sqrt[a + b*x^2])^(3/2)*(2*(a + b*x^2)^(3/4) + 3*a^(3/4)*ArcTan[(a + b*x
^2)^(1/4)/a^(1/4)] - 3*a^(3/4)*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)]))/(3*(a +
 b*x^2)^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.41, size = 190, normalized size = 1.62

$$-\frac{1}{12}\left(6\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+6\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x
^2 + a)^(1/4))/(-a)^(1/4)) + 6*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4)) - 3*sqrt(2)*(-a)^(3/4)*lo
g(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(-a)) + 3*sq
rt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a
) + sqrt(-a)) - 8*(b*x^2 + a)^(3/4))*c^(3/2)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{b\,x^2+a}\;c\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^(1/2)*c)^(3/2)/x,x)
```

```
[Out] int(((b*x^2+a)^(1/2)*c)^(3/2)/x,x)
```

maxima [A] time = 1.97, size = 118, normalized size = 1.01

$$\frac{3\,ac^4\left(\frac{2\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}\,c}}{\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}+\frac{\log\left(\frac{\sqrt{\sqrt{bx^2+a}\,c}-\left(ac^2\right)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^2+a}\,c}+\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}\right)+4\left(\sqrt{bx^2+a}\,c\right)^{\frac{3}{2}}c^2}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a)*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4)
 + log((sqrt(sqrt(b*x^2 + a)*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a)*c) +
 (a*c^2)^(1/4)))/(a*c^2)^(1/4)) + 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^2)/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x,x)
```

```
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x,x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x, x)
```

3.254 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=133

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2}+\frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\tan^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\tanh^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

```
[Out] -1/2*(c*(b*x^2+a)^(1/2))^(3/2)/x^2+3/4*b*arctan((1+b*x^2/a)^(1/4))*(c*(b*x^
2+a)^(1/2))^(3/2)/a/(1+b*x^2/a)^(3/4)-3/4*b*arctanh((1+b*x^2/a)^(1/4))*(c*(
b*x^2+a)^(1/2))^(3/2)/a/(1+b*x^2/a)^(3/4)
```

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 266, 47, 63, 298, 203, 206}

$$-\frac{c\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}}{2x^2}+\frac{3bc\sqrt{c\sqrt{a+bx^2}}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{a+bx^2}}-\frac{3bc\sqrt{c\sqrt{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

```
[Out] -(c*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2])/(2*x^2) + (3*b*c*Sqrt[c*Sqrt[a
 + b*x^2]]*ArcTan[(a + b*x^2)^(1/4)/a^(1/4)])/(4*a^(1/4)*(a + b*x^2)^(1/4))
 - (3*b*c*Sqrt[c*Sqrt[a + b*x^2]]*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)])/(4*a^
(1/4)*(a + b*x^2)^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{\left(a+bx^2\right)^{3/4}}{x^3}\,dx}{\sqrt[4]{a+bx^2}}\\
&= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{(a+bx)^{3/4}}{x^2}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}\\
&= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2}+\frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx,x,x^2\right)}{8\sqrt[4]{a+bx^2}}\\
&= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2}+\frac{\left(3c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx,x,\sqrt[4]{a+bx^2}\right)}{2\sqrt[4]{a+bx^2}}\\
&= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2}-\frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx,x,\sqrt[4]{a+bx^2}\right)}{4\sqrt[4]{a+bx^2}}+\cdots\\
&= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2}+\frac{3bc\sqrt{c\sqrt{a+bx^2}}\,\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\,\sqrt[4]{a+bx^2}}-\frac{3bc\sqrt{c\sqrt{a+bx^2}}\,\tan\cdots}{4\sqrt[4]{a}\,\sqrt[4]{a+\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.38

$$
\frac{2b\left(a+bx^2\right)\left(c\sqrt{a+bx^2}\right)^{3/2}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};\frac{bx^2}{a}+1\right)}{7a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

```
[Out] (2*b*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*Hypergeometric2F1[7/4, 2, 11/4,
1 + (b*x^2)/a])/(7*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.40, size = 212, normalized size = 1.59

$$\frac{\frac{6\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}}+\frac{6\sqrt{2}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}}+\frac{3\sqrt{2}(-a)^{\frac{3}{4}}b^2\log\left(\sqrt{2}\left(bx^2+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^2+a}+\sqrt{-a}\right.}{a}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(6*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^2 + a)^
(1/4))/(-a)^(1/4))/(-a)^(1/4) + 6*sqrt(2)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 3*sqrt(2)*(-a)^(
3/4)*b^2*log(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(
-a))/a + 3*sqrt(2)*b^2*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x
^2 + a) + sqrt(-a))/(-a)^(1/4) - 8*(b*x^2 + a)^(3/4)*b/x^2)*c^(3/2)/b
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{b\,x^2+a}\;c\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^(1/2)*c)^(3/2)/x^3,x)
```

```
[Out] int(((b*x^2+a)^(1/2)*c)^(3/2)/x^3,x)
```

maxima [A] time = 2.40, size = 138, normalized size = 1.04

$$\frac{\left(3c^4\left(\frac{2\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}\,c}}{\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}+\frac{\log\left(\frac{\sqrt{\sqrt{bx^2+a}\,c}-\left(ac^2\right)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^2+a}\,c}+\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}\right)-\frac{4\left(\sqrt{bx^2+a}\,c\right)^{\frac{3}{2}}c^4}{\left(bx^2+a\right)c^2-ac^2}\right)b}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a)*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4) +
 log((sqrt(sqrt(b*x^2 + a)*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a)*c) + (
a*c^2)^(1/4)))/(a*c^2)^(1/4)) - 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^4/((b*x^2 + a
)*c^2 - a*c^2))*b/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x^3,x)
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**3,x)
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**3, x)
```

3.255 $\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=152

$$\frac{4a^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2} E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{4a^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b\left(a+bx^2\right)}+\frac{2ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b}+\frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2}$$

```
[Out] 2/15*a*x*(c*(b*x^2+a)^(1/2))^(3/2)/b+2/9*x^3*(c*(b*x^2+a)^(1/2))^(3/2)-4/15
*a^2*x*(c*(b*x^2+a)^(1/2))^(3/2)/b/(b*x^2+a)+4/15*a^(3/2)*(cos(1/2*arctan(x
*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(si
n(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*(b*x^2+a)^(1/2))^(3/2)/b^(3/2)
/(1+b*x^2/a)^(3/4)
```

Rubi [A] time = 0.17, antiderivative size = 191, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6720, 279, 321, 229, 227, 196}

$$\frac{4a^{5/2}c\sqrt[4]{\frac{bx^2}{a}+1}\sqrt{c\sqrt{a+bx^2}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt{a+bx^2}}-\frac{4a^2cx\sqrt{c\sqrt{a+bx^2}}}{15b\sqrt{a+bx^2}}+\frac{2}{9}cx^3\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}+\frac{2acx\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-4*a^2*c*x*Sqrt[c*Sqrt[a + b*x^2]])/(15*b*Sqrt[a + b*x^2]) + (2*a*c*x*Sqrt
[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2])/(15*b) + (2*c*x^3*Sqrt[c*Sqrt[a + b*x^
2]]*Sqrt[a + b*x^2])/9 + (4*a^(5/2)*c*Sqrt[c*Sqrt[a + b*x^2]]*(1 + (b*x^2)/
a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^(3/2)*Sqrt[a +
b*x^2])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2\left(c\sqrt{a+bx^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^2\left(a+bx^2\right)^{3/4}dx}{\sqrt[4]{a+bx^2}}$$

$$=\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}+\frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right)\int\frac{x^2}{\sqrt[4]{a+bx^2}}dx}{3\sqrt[4]{a+bx^2}}$$

$$=\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}-\frac{\left(2a^2c\sqrt{c\sqrt{a+bx^2}}\right)\cdot}{15b\sqrt[4]{a+b}}$$

$$=\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}-\frac{\left(2a^2c\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{}\right.}{15b\sqrt{}}$$

$$=-\frac{4a^2cx\sqrt{c\sqrt{a+bx^2}}}{15b\sqrt{a+bx^2}}+\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}$$

$$=-\frac{4a^2cx\sqrt{c\sqrt{a+bx^2}}}{15b\sqrt{a+bx^2}}+\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}$$

Mathematica [C] time = 0.06, size = 68, normalized size = 0.45

$$\frac{2x\left(c\sqrt{a+bx^2}\right)^{3/2}\left(-\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}+a+bx^2\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*x*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2 - (a*Hypergeometric2F1[-3/4, 1/2,
 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(3/4)))/(9*b)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}\,c}\,cx^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sqrt{bx^2 + a}\, c\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)*x^2, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(\sqrt{b\,x^2 + a}\; c\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^2+a)^(1/2)*c)^(3/2),x)
[Out] int(x^2*((b*x^2+a)^(1/2)*c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sqrt{bx^2 + a}\, c\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(c\,\sqrt{b\,x^2 + a}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(a + b*x^2)^(1/2))^(3/2),x)
[Out] int(x^2*(c*(a + b*x^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(c\sqrt{a + bx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*(b*x**2+a)**(1/2))**(3/2),x)
[Out] Integral(x**2*(c*sqrt(a + b*x**2))**(3/2), x)
```

3.256 $\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=119

$$\frac{2}{5}x\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{6ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{5\left(a+bx^2\right)}-\frac{6\sqrt{a}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

```
[Out] 2/5*x*(c*(b*x^2+a)^(1/2))^(3/2)+6/5*a*x*(c*(b*x^2+a)^(1/2))^(3/2)/(b*x^2+a)
-6/5*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/
a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(c*
(b*x^2+a)^(1/2))^(3/2)/(1+b*x^2/a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 146, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6720, 195, 229, 227, 196}

$$-\frac{6a^{3/2}c\sqrt[4]{\frac{bx^2}{a}+1}\sqrt{c\sqrt{a+bx^2}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt{a+bx^2}}+\frac{6acx\sqrt{c\sqrt{a+bx^2}}}{5\sqrt{a+bx^2}}+\frac{2}{5}cx\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (6*a*c*x*Sqrt[c*Sqrt[a + b*x^2]])/(5*Sqrt[a + b*x^2]) + (2*c*x*Sqrt[c*Sqrt[
a + b*x^2]]*Sqrt[a + b*x^2])/5 - (6*a^(3/2)*c*Sqrt[c*Sqrt[a + b*x^2]]*(1 +
(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*Sq
rt[a + b*x^2])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \left(a+bx^2\right)^{3/4} dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2} + \frac{\left(3ac\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{1}{\sqrt[4]{a+bx^2}} dx}{5\sqrt[4]{a+bx^2}}$$
$$= \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2} + \frac{\left(3ac\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}} dx}{5\sqrt{a+bx^2}}$$
$$= \frac{6acx\sqrt{c\sqrt{a+bx^2}}}{5\sqrt{a+bx^2}} + \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2} - \frac{\left(3ac\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right.}}{5\sqrt{a+bx^2}}$$
$$= \frac{6acx\sqrt{c\sqrt{a+bx^2}}}{5\sqrt{a+bx^2}} + \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2} - \frac{6a^{3/2}c\sqrt{c\sqrt{a+bx^2}}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan\right.}{5\sqrt{b}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.44

$$\frac{x\left(c\sqrt{a+bx^2}\right)^{3/2} {}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (x*(c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)
])/(1 + (b*x^2)/a)^(3/4)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}\,c}\,c, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sqrt{bx^2+a}\,c\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(\sqrt{b\,x^{2}+a}\;c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^(1/2)*c)^(3/2),x)
```

```
[Out] int(((b*x^2+a)^(1/2)*c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sqrt{bx^{2}+a}\,c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,\sqrt{b\,x^{2}+a}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] int((c*(a + b*x^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sqrt{a+bx^{2}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2), x)
```

3.257 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=115

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}+\frac{3bx\left(c\sqrt{a+bx^2}\right)^{3/2}}{a+bx^2}-\frac{3\sqrt{b}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

```
[Out] -(c*(b*x^2+a)^(1/2))^(3/2)/x+3*b*x*(c*(b*x^2+a)^(1/2))^(3/2)/(b*x^2+a)-3*(c
os(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)
))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)*(c*(b*x^2+
a)^(1/2))^(3/2)/(1+b*x^2/a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6720, 277, 229, 227, 196}

$$-\frac{c\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}}{x}+\frac{3bcx\sqrt{c\sqrt{a+bx^2}}}{\sqrt{a+bx^2}}-\frac{3\sqrt{a}\sqrt{b}\,c\sqrt[4]{\frac{bx^2}{a}+1}\sqrt{c\sqrt{a+bx^2}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^2,x]
```

```
[Out] (3*b*c*x*Sqrt[c*Sqrt[a + b*x^2]])/Sqrt[a + b*x^2] - (c*Sqrt[c*Sqrt[a + b*x^
2]]*Sqrt[a + b*x^2])/x - (3*Sqrt[a]*Sqrt[b]*c*Sqrt[c*Sqrt[a + b*x^2]]*(1 +
(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[a + b*x^
2]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{\left(a+bx^2\right)^{3/4}}{x^2}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{x} + \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{2\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{x} + \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{2\sqrt{a+bx^2}}$$

$$= \frac{3bcx\sqrt{c\sqrt{a+bx^2}}}{\sqrt{a+bx^2}} - \frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{x} - \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right.}}{2\sqrt{a+bx^2}}$$

$$= \frac{3bcx\sqrt{c\sqrt{a+bx^2}}}{\sqrt{a+bx^2}} - \frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{x} - \frac{3\sqrt{a}\,\sqrt{b}\,c\sqrt{c\sqrt{a+bx^2}}\,\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\right.}{\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.48

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^2,x]
```

```
[Out] -(((c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x^2)/a
)])/(x*(1 + (b*x^2)/a)^(3/4)))
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}\,c}\,c}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{bx^2+a}\,c\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^2, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{b\,x^{2}+a}\;c\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^(1/2)*c)^(3/2)/x^2,x)
[Out] int(((b*x^2+a)^(1/2)*c)^(3/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{bx^{2}+a}\,c\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,\sqrt{b\,x^{2}+a}\right)^{3/2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x^2,x)
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{a+bx^{2}}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**2,x)
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**2, x)
```

3.258 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=154

$$-\frac{b^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}}+\frac{b^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{2a\left(a+bx^2\right)}-\frac{b\left(c\sqrt{a+bx^2}\right)^{3/2}}{2ax}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{3x^3}$$

```
[Out] -1/3*(c*(b*x^2+a)^(1/2))^(3/2)/x^3-1/2*b*(c*(b*x^2+a)^(1/2))^(3/2)/a/x+1/2*
b^2*x*(c*(b*x^2+a)^(1/2))^(3/2)/a/(b*x^2+a)-1/2*b^(3/2)*(cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(
1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*(b*x^2+a)^(1/2))^(3/2)/a^(3/2)/(
1+b*x^2/a)^(3/4)
```

Rubi [A] time = 0.16, antiderivative size = 193, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6720, 277, 325, 229, 227, 196}

$$\frac{b^2cx\sqrt{c\sqrt{a+bx^2}}}{2a\sqrt{a+bx^2}}-\frac{b^{3/2}c\sqrt[4]{\frac{bx^2}{a}+1}\,\sqrt{c\sqrt{a+bx^2}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt{a+bx^2}}-\frac{bc\sqrt{a+bx^2}\,\sqrt{c\sqrt{a+bx^2}}}{2ax}-\frac{c\sqrt{a+bx^2}\,\sqrt{c\sqrt{a+bx^2}}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^4,x]
```

```
[Out] (b^2*c*x*Sqrt[c*Sqrt[a + b*x^2]])/(2*a*Sqrt[a + b*x^2]) - (c*Sqrt[c*Sqrt[a
+ b*x^2]]*Sqrt[a + b*x^2])/(3*x^3) - (b*c*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a +
b*x^2])/(2*a*x) - (b^(3/2)*c*Sqrt[c*Sqrt[a + b*x^2]]*(1 + (b*x^2)/a)^(1/4)*
EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*Sqrt[a + b*x^2])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{\left(a+bx^2\right)^{3/4}}{x^4}\,dx}{\sqrt[4]{a+bx^2}}\\
&= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{3x^3}+\frac{\left(bc\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{2\sqrt[4]{a+bx^2}}\\
&= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{3x^3}-\frac{bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2ax}+\frac{\left(b^2c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4a\sqrt[4]{a+bx^2}}\\
&= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{3x^3}-\frac{bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2ax}+\frac{\left(b^2c\sqrt{c\sqrt{a+bx^2}}\,\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \cdots}{4a\sqrt{a+bx^2}}\\
&= \frac{b^2cx\sqrt{c\sqrt{a+bx^2}}}{2a\sqrt{a+bx^2}}-\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{3x^3}-\frac{bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2ax}-\frac{\left(b^2c\sqrt{c\sqrt{a \cdots}}\right.}{\cdots}\\
&= \frac{b^2cx\sqrt{c\sqrt{a+bx^2}}}{2a\sqrt{a+bx^2}}-\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{3x^3}-\frac{bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2ax}-\frac{b^{3/2}c\sqrt{c\sqrt{a \cdots}}}{\cdots}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.37

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^4,x]
```

```
[Out] -1/3*((c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x^
2)/a)])/(x^3*(1 + (b*x^2)/a)^(3/4))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Shoul
dn't happen
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{bx^2+a}\,c\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^4, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{b\,x^2+a}\;c\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^(1/2)*c)^(3/2)/x^4,x)
```

```
[Out] int(((b*x^2+a)^(1/2)*c)^(3/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{bx^2+a}\,c\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**4,x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**4, x)
```

3.259 $\int \sqrt{(b-x)(-a+x)}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -1/8*(a-b)^2*arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))-1/4*(a+b-2*x)*(
-a*b+(a+b)*x-x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1981, 612, 621, 204}

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*ArcTan[(a +
b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{(b-x)(-a+x)}\,dx &= \int \sqrt{-ab+(a+b)x-x^2}\,dx\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}+\frac{1}{8}(a-b)^2\int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}+\frac{1}{4}(a-b)^2\,\mathrm{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+b-}{\sqrt{-ab+(a-}}\right.\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.16, size = 106, normalized size = 1.49

$$\frac{(a-x)\left((a-b)^{5/2}\sqrt{b-x}\sqrt{\frac{a-x}{a-b}}\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)-(a-x)(b-x)(a+b-2x)\right)}{4(x-a)\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] ((a - x)*(-((a + b - 2*x)*(a - x)*(b - x)) + (a - b)^(5/2)*Sqrt[(a - x)/(a
- b)]*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[a - b]]))/(4*(-a + x)*Sqrt[(a -
x)*(-b + x)])
```

fricas [A] time = 0.91, size = 80, normalized size = 1.13

$$-\frac{1}{8}\left(a^{2}-2\,ab+b^{2}\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^{2}}\,(a+b-2\,x)}{2\left(ab-(a+b)x+x^{2}\right)}\right)-\frac{1}{4}\sqrt{-ab+(a+b)x-x^{2}}\,(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2
*x)
```

giac [A] time = 0.29, size = 61, normalized size = 0.86

$$\frac{1}{8}\left(a^{2}-2\,ab+b^{2}\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}\left(-a+b\right)-\frac{1}{4}\sqrt{-ab+ax+bx-x^{2}}\,(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

maple [A] time = 0.02, size = 122, normalized size = 1.72

$$\frac{a^{2}\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^{2}+(a+b)x}}\right)}{8}-\frac{ab\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^{2}+(a+b)x}}\right)}{4}+\frac{b^{2}\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^{2}+(a+b)x}}\right)}{8}-\frac{(a+b-2x)\sqrt{-ab-x^{2}\cdots}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b-x)*(-a+x))^(1/2),x)
```

```
[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/4*arctan((x-1/2*b-1/2*a)/(-a*b+(a
+b)*x-x^2)^(1/2))*a*b+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*
a^2+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{-(a-x)(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int((-(a - x)*(b - x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(-a+x)(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)*(b - x)), x)
```

3.260 $\int \sqrt{(1-x^2)(3+x^2)}\,dx$

Optimal. Leaf size=48

$$\frac{1}{3}\sqrt{-x^4-2x^2+3}\,x+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -2/3*EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(x,1/3*I*3^(1/2))*3^(1
/2)+1/3*x*(-x^4-2*x^2+3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1988, 1091, 1180, 524, 424, 419}

$$\frac{1}{3}\sqrt{-x^4-2x^2+3}\,x+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt[3] + (4*E
llipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
```

```
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1988

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && T
rinomialQ[u, x] && !TrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\left(1-x^2\right)\left(3+x^2\right)}\,dx &= \int \sqrt{3-2x^2-x^4}\,dx\\ &= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{1}{3}\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{2}{3}\int \frac{6-2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx\\ &= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2}{3}\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx+8\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx\\ &= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2E\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{4F\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.06, size = 59, normalized size = 1.23

$$\frac{1}{3}\left(\sqrt{-x^4-2x^2+3}\,x-4iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-2iE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (4*I
)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-x^4-2\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - 2*x^2 + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(x^2+3\right)\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

maple [B] time = 0.02, size = 114, normalized size = 2.38

$$\frac{\sqrt{-x^4-2x^2+3}\;x}{3}+\frac{2\sqrt{-x^2+1}\;\sqrt{3x^2+9}\;\;\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\;\sqrt{3x^2+9}\;\;\left(-\mathrm{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)+\mathrm{Ellip}\right.}{3\sqrt{-x^4-2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x^2+1)*(x^2+3))^(1/2),x)
```

```
[Out] 1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3
)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4
-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(x^2+3\right)\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-\left(x^2-1\right)\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x^2 - 1)*(x^2 + 3))^(1/2),x)
```

```
[Out] int((-(x^2 - 1)*(x^2 + 3))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(1-x^2\right)\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x**2+1)*(x**2+3))**(1/2),x)
```

```
[Out] Integral(sqrt((1 - x**2)*(x**2 + 3)), x)
```

3.261 $\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx$

Optimal. Leaf size=32

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1981, 621, 204}

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx &= \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+b-2x}{\sqrt{-ab+(a+b)x-x^2}}\right)\\
&= -\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)
\end{aligned}$$

Mathematica [B] time = 0.03, size = 72, normalized size = 2.25

$$-\frac{2\sqrt{a-b}\,\sqrt{b-x}\,\sqrt{\frac{a-x}{a-b}}\,\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)}{\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] (-2*Sqrt[a - b]*Sqrt[(a - x)/(a - b)]*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[
a - b]])/Sqrt[(a - x)*(-b + x)]
```

fricas [A] time = 0.76, size = 43, normalized size = 1.34

$$-\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}\,(a+b-2\,x)}{2\left(ab-(a+b)x+x^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
x^2))
```

giac [A] time = 0.43, size = 22, normalized size = 0.69

$$\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}\left(-a+b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] arcsin((a + b - 2*x)/(a - b))*sgn(-a + b)
```

maple [A] time = 0.01, size = 28, normalized size = 0.88

$$\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)\,x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b-x)*(-a+x))^(1/2),x)
```

```
[Out] arctan((-1/2*a-1/2*b+x)/(-a*b-x^2+(a+b)*x)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-\left(a-x\right)\,\left(b-x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int(1/(-(a - x)*(b - x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(-a+x)(b-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-a + x)*(b - x)), x)
```

3.262 $\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}} \, dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1988, 1095, 419}

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1988

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && T
rinomialQ[u, x] &&  !TrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\left(1-x^2\right)\left(3+x^2\right)}} \, dx &= \int \frac{1}{\sqrt{3-2x^2-x^4}} \, dx \\
&= 2\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}} \, dx \\
&= \frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 18, normalized size = 1.50

$$-iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4-2\,x^2+3}}{x^4+2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 - 2*x^2 + 3)/(x^4 + 2*x^2 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x^2+3\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

maple [B] time = 0.01, size = 43, normalized size = 3.58

$$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\operatorname{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-x^2+1)*(x^2+3))^(1/2),x)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x^2+3\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{-\left(x^2-1\right)\,\left(x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x^2 - 1)*(x^2 + 3))^(1/2),x)
```

```
[Out] int(1/(-(x^2 - 1)*(x^2 + 3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(1-x^{2}\right)\left(x^{2}+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x**2+1)*(x**2+3))**(1/2),x)
```

```
[Out] Integral(1/sqrt((1 - x**2)*(x**2 + 3)), x)
```

3.263 $\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=244

$$\frac{(c+dx^2)(-a^2d^2-2abcd+11b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^2d^3}-\frac{\sqrt{e}\,(bc-ad)(a^2d^2+2abcd+5b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}-\frac{(c+}{}$$

```
[Out] 1/6*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)^3/b/d^2/e-1/16*(-a*d+b*c)*(a^2*
d^2+2*a*b*c*d+5*b^2*c^2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1
/2)/e^(1/2))*e^(1/2)/b^(5/2)/d^(7/2)+1/16*(-a^2*d^2-2*a*b*c*d+11*b^2*c^2)*(
d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^3-1/8*(a*d+3*b*c)*(d*x^2+c)^2*
(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^3
```

Rubi [A] time = 0.33, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 463, 455, 385, 208}

$$\frac{(c+dx^2)(-a^2d^2-2abcd+11b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^2d^3}-\frac{\sqrt{e}\,(bc-ad)(a^2d^2+2abcd+5b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}+\frac{(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((11*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c +
d*x^2))/(16*b^2*d^3) - ((3*b*c + a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c
+ d*x^2)^2)/(8*b*d^3) + (((e*(a + b*x^2))/(c + d*x^2))^(3/2)*(c + d*x^2)^3)
/(6*b*d^2*e) - ((b*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[e]*ArcTa
nh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(5
/2)*d^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2\left(-ae+cx^2\right)^2}{\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)^3}{6bd^2e} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^2\left(-3\left(2a^2d^2e^2-(bce-ade)^2\right)+6bc^2dex^2\right)}{\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e}{}}\right.}{6bd^2}$$

$$= -\frac{(3bc+ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{8bd^3} + \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)^3}{6bd^2e} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{3d(b}{}\right.}{}$$

$$= \frac{\left(11b^2c^2-2abcd-a^2d^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{8bd^3} + \frac{\left(\frac{e(}{}\right.}{}$$

$$= \frac{\left(11b^2c^2-2abcd-a^2d^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{8bd^3} + \frac{\left(\frac{e(}{}\right.}{}$$

Mathematica [A] time = 0.52, size = 198, normalized size = 0.81

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)(bc-ad)^{3/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{a+bx^2}} - b\sqrt{d}\left(c+dx^2\right)\left(3a^2d^2-2abd\left(dx^2-2c\right)+b^2\right.\right.}{48b^3d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(b*Sqrt[d]*(c + d*x^2)*(3*a^2*d^2 - 2*
a*b*d*(-2*c + d*x^2) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - (3*(b*c -
 a*d)^(3/2)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a
*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/
(48*b^3*d^(7/2))
```

fricas [A] time = 0.74, size = 541, normalized size = 2.22

$$\left[-\frac{3\left(5\,b^{3}c^{3}-3\,ab^{2}c^{2}d-a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{\frac{e}{bd}}\,\log\left(8\,b^{2}d^{2}ex^{4}+8\left(b^{2}cd+abd^{2}\right)ex^{2}+\left(b^{2}c^{2}+6\,abcd+a^{2}d^{2}\right)e+4\left(2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(e/(b*d)
)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d
+ a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*
b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(8*b^2*d^3
*x^6 + 15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4
 + (5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2
 + c)))/(b^2*d^3), 1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d
^3)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(8*b^2*d^3*x^6 + 15*b^2*c^
3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4 + (5*b^2*c^2*d
- 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d^3
)]
```

giac [A] time = 0.73, size = 243, normalized size = 1.00

$$\frac{1}{96}\left(2\,\sqrt{bdx^{4}e+bcx^{2}e+adx^{2}e+ace}\left(2\,x^{2}\left(\frac{4\,x^{2}}{d}-\frac{5\,b^{2}cd-abd^{2}}{b^{2}d^{3}}\right)+\frac{15\,b^{2}c^{2}-4\,abcd-3\,a^{2}d^{2}}{b^{2}d^{3}}\right)+\frac{3\left(5\,b^{3}c^{3}e-3\,ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)*(2*x^2*(4*x^2/d - (
5*b^2*c*d - a*b*d^2)/(b^2*d^3)) + (15*b^2*c^2 - 4*a*b*c*d - 3*a^2*d^2)/(b^2
*d^3)) + 3*(5*b^3*c^3*e - 3*a*b^2*c^2*d*e - a^2*b*c*d^2*e - a^3*d^3*e)*e^(-
1/2)*log(abs(-b*c*e - a*d*e - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b
*c*x^2*e + a*d*x^2*e + a*c*e))*sqrt(b*d)*e^(1/2)))/(sqrt(b*d)*b^2*d^3))*sgn
(d*x^2 + c)
```

maple [B] time = 0.07, size = 527, normalized size = 2.16

$$\frac{\sqrt{\frac{\left(b\,x^{2}+a\right)e}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(3a^{3}d^{3}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3a^{2}bc\,d^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}}}{2\sqrt{bd}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^3*(-12*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*x^2*a*b*d^2*(b*d)^(1/2)-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*x^2*c*b^2*d*(b*d)^(1/2)+3*d^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3+3*ln(1/2*(2*b*d*x^2+2*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*
```

```
b*d^2+9*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*a*c^2*b^2*d-15*b^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3+16*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(3/2)*b*d*(b*d)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*a^2*d^2*(b*d)^(1/2)-24*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d
)^(1/2)+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/((d*x^2
+c)*(b*x^2+a))^(1/2)/b^2/(b*d)^(1/2)
```

maxima [A] time = 2.28, size = 414, normalized size = 1.70

$$\frac{1}{96}e\left(\frac{2\left(3\left(11\,b^3c^3d^2-13\,ab^2c^2d^3+a^2bcd^4+a^3d^5\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{5}{2}}-8\left(5\,b^4c^3d-3\,ab^3c^2d^2-3\,a^2b^2cd^3+a^3bd^4\right)\left(\right.\right.}{b^5d^3e^3-\frac{3\left(bx^2+a\right)b^4d^4e^3}{dx^2+c}+\frac{3\left(bx^2+a\right)^2b^3d^5e^3}{\left(dx^2+c\right)^2}-\frac{\left(bx^2+\right.}{\left(d\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*e*(2*(3*(11*b^3*c^3*d^2 - 13*a*b^2*c^2*d^3 + a^2*b*c*d^4 + a^3*d^5)*((
b*x^2 + a)*e/(d*x^2 + c))^(5/2) - 8*(5*b^4*c^3*d - 3*a*b^3*c^2*d^2 - 3*a^2*
b^2*c*d^3 + a^3*b*d^4)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e + 3*(5*b^5*c^3 -
 3*a*b^4*c^2*d - a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 + c
))*e^2)/(b^5*d^3*e^3 - 3*(b*x^2 + a)*b^4*d^4*e^3/(d*x^2 + c) + 3*(b*x^2 + a
)^2*b^3*d^5*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*b^2*d^6*e^3/(d*x^2 + c)^3) +
3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*log((d*sqrt((b*x^2 +
a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(
b*d*e)))/(sqrt(b*d*e)*b^2*d^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.264 $\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{e}\,(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^2}-\frac{\left(c+dx^2\right)(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^2}$$

```
[Out] 1/8*(-a*d+b*c)*(a*d+3*b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^
(1/2)/e^(1/2))*e^(1/2)/b^(3/2)/d^(5/2)-1/8*(-a*d+5*b*c)*(d*x^2+c)*(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/b/d^2+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^
2
```

Rubi [A] time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 455, 385, 208}

$$\frac{\sqrt{e}\,(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^2}-\frac{\left(c+dx^2\right)(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -((5*b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b*d^2) +
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^2) + ((b*c - a*d)*(3
*b*c + a*d)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sq
rt[b]*Sqrt[e])])/(8*b^(3/2)*d^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
```

```
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2\left(-ae+cx^2\right)}{\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^2} - \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{(bc-ad)e+4cdx^2}{\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{4d^2}$$

$$= -\frac{(5bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8bd^2} + \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^2} + \frac{((bc-ad)(3bc+ad)e)\,\text{Sul}}{}$$

$$= -\frac{(5bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8bd^2} + \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^2} + \frac{(bc-ad)(3bc+ad)\sqrt{e}\ \text{ta}}{8b^{3/2}d^{5/}}$$

Mathematica [A] time = 0.38, size = 149, normalized size = 0.93

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(b\sqrt{d}\left(c+dx^2\right)\left(ad-3bc+2bdx^2\right)+\frac{(ad+3bc)(bc-ad)^{3/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{a+bx^2}}\right)}{8b^2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*(c + d*x^2)*(-3*b*c + a*d + 2
*b*d*x^2) + ((b*c - a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*
d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/(
8*b^2*d^(5/2))
```

fricas [A] time = 0.49, size = 407, normalized size = 2.53

$$\left[-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e-4\left(2\,b^2d^3x^4+\right.\right.}{32\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4
 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b
^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e
*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d
```

```
- (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^2), -1/16*((
3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c
 + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) -
 2*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d - (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/(b*d^2)]
```

giac [A] time = 0.82, size = 185, normalized size = 1.15

$$\frac{1}{16}\left(2\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\left(\frac{2x^2}{d}-\frac{3bc-ad}{bd^2}\right)-\frac{\left(3b^2c^2e-2abcde-a^2d^2e\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-bce-ade-2\left(\cdots\right.\right.\right.}{\sqrt{bd}\,b\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)*(2*x^2/d - (3*b*c -
 a*d)/(b*d^2)) - (3*b^2*c^2*e - 2*a*b*c*d*e - a^2*d^2*e)*e^(-1/2)*log(abs(-
b*c*e - a*d*e - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d
*x^2*e + a*c*e))*sqrt(b*d)*e^(1/2)))/(sqrt(b*d)*b*d^2))*sgn(d*x^2 + c)
```

maple [B] time = 0.04, size = 342, normalized size = 2.12

$$\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,\left(dx^2+c\right)\left(-a^2d^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-2abcd\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\cdots}{2\sqrt{bd}}\right.\right.\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/16*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)*x^2*b*d-d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^2-2*ln(1/2*(2*b*d*x^2+a*d
+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a*c*b*
d+3*b^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2))/(b*d)^(1/2))*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2
)*a*d-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*c)/((d*x^2+c)*(b*
x^2+a))^(1/2)/d^2/b/(b*d)^(1/2)
```

maxima [A] time = 2.25, size = 269, normalized size = 1.67

$$\frac{1}{16}e\left(\frac{2\left(\left(5b^2c^2d-6abcd^2+a^2d^3\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}-\left(3b^3c^2-2ab^2cd-a^2bd^2\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,e\right)}{b^3d^2e^2-\frac{2(bx^2+a)b^2d^3e^2}{dx^2+c}+\frac{(bx^2+a)^2bd^4e^2}{(dx^2+c)^2}}-\frac{\left(3b^2c^2-2abcd-a^2d^2\cdots\right.}{\sqrt{bde}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*((5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*((b*x^2 + a)*e/(d*x^2 + c)
)^(3/2) - (3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 +
 c))*e)/(b^3*d^2*e^2 - 2*(b*x^2 + a)*b^2*d^3*e^2/(d*x^2 + c) + (b*x^2 + a)^
2*b*d^4*e^2/(d*x^2 + c)^2) - (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*log((d*sqrt(
(b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)
) + sqrt(b*d*e)))/(sqrt(b*d*e)*b*d^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

3.265 $\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d}-\frac{\sqrt{e}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}\,d^{3/2}}$$

```
[Out] -1/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/
2))*e^(1/2)/d^(3/2)/b^(1/2)+1/2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1960, 288, 208}

$$\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d}-\frac{\sqrt{e}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*d) - ((b*c - a*d)*Sqrt[e
]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(
2*Sqrt[b]*d^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=((bc-ad)e)\,\text{Subst}\left(\int\frac{x^2}{\left(be-dx^2\right)^2}\,dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{2d}-\frac{((bc-ad)e)\,\text{Subst}\left(\int\frac{1}{be-dx^2}\,dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2d}$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{2d}-\frac{(bc-ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}\,d^{3/2}}$$

Mathematica [A] time = 0.28, size = 143, normalized size = 1.39

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(b\sqrt{d}\left(a+bx^2\right)\left(c+dx^2\right)-\sqrt{a+bx^2}\,(bc-ad)^{3/2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)\right)}{2bd^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*(a + b*x^2)*(c + d*x^2) - (b*
c - a*d)^(3/2)*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(S
qrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(2*b*d^(3/2)*(a + b*x^2))
```

fricas [A] time = 0.50, size = 313, normalized size = 3.04

$$\left[-\frac{(bc-ad)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e+4\left(2\,b^2d^3x^4+b^2c^2d+abcd^2\right.\right.}{8\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2
)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d
+ a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
)*sqrt(e/(b*d))) - 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d, 1/4*
((b*c - a*d)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(d*x^2 + c)*sqrt(
(b*e*x^2 + a*e)/(d*x^2 + c)))/d]
```

giac [A] time = 0.82, size = 149, normalized size = 1.45

$$\frac{1}{4}\left(\frac{(bce-ade)\sqrt{bd}\,e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bd}\,bce^{\frac{1}{2}}-\sqrt{bd}\,ade^{\frac{1}{2}}-2\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)bd\right|\right)}{bd^2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*((b*c*e - a*d*e)*sqrt(b*d)*e^(-1/2)*log(abs(-sqrt(b*d)*b*c*e^(1/2) - sq
rt(b*d)*a*d*e^(1/2) - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e
 + a*d*x^2*e + a*c*e))*b*d))/(b*d^2) + 2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x
^2*e + a*c*e)/d)*sgn(d*x^2 + c)
```

maple [B] time = 0.02, size = 200, normalized size = 1.94

$$\frac{\sqrt{\frac{(b\,x^2+a)e}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(ad\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-bc\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+\right.}{4\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{bd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/4*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(a*ln(1/2*(2*b*d*x^2+a*d+b*c+2*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*d-b*ln(1/2*(2
*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(
1/2))*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/((d*x^2+c)*(b*x^
2+a))^(1/2)/d/(b*d)^(1/2)
```

maxima [A] time = 2.25, size = 145, normalized size = 1.41

$$\frac{1}{4}e\left(\frac{2\,(bc-ad)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{bde-\frac{(bx^2+a)d^2e}{dx^2+c}}+\frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(b*d*e - (b*x^2 + a)*d
^2*e/(d*x^2 + c)) + (b*c - a*d)*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sq
rt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*
d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.266 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x} dx$

Optimal. Leaf size=112

$$\frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}}$$

```
[Out] -arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*a^(1/2)*e^(
1/2)/c^(1/2)+arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))
*b^(1/2)*e^(1/2)/d^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 481, 208}

$$\frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

```
[Out] -((Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])])/Sqrt[c]) + (Sqrt[b]*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b
*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/Sqrt[d]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = ((bc-ad)e)\,\mathrm{Subst}\left(\int \frac{x^2}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$
$$= (ae)\,\mathrm{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right) + (be)\,\mathrm{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$
$$= -\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}} + \frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}$$

Mathematica [A] time = 0.18, size = 173, normalized size = 1.54

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)-\sqrt{a}\sqrt{d}\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*
x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]] - Sqr
t[a]*Sqrt[d]*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqr
t[c + d*x^2])]))/(Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2])
```

fricas [A] time = 0.70, size = 865, normalized size = 7.72

$$\left[\frac{1}{4}\sqrt{\frac{be}{d}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e+4\left(2\,bd^3x^4+bc^2d+acd^2+\left(3\,bcd^2+ad^3\right)x^2\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*c*d
^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 1/4*sqrt(
a*e/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^
2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^
2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4), -1/2*sqrt(-b
*e/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 1/4*sqrt(a*e/c)*log(((b^2*c^2 + 6*a*b*c
*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2
*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*
e*x^2 + a*e)/(d*x^2 + c)))/x^4), 1/2*sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x
^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2
*e)) + 1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 +
(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3
*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))), 1/2*
sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - 1/2*sqrt(-b*e/d)*arctan(1/2*(2
*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e
*x^2 + a*b*e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Error: Bad Argument Type
```

maple [B] time = 0.04, size = 179, normalized size = 1.60

$$\frac{\sqrt{\frac{(b x^2+a)e}{d x^2+c}}\,\left(d x^2+c\right)\left(-\sqrt{bd}\,a\ln\left(\frac{ad x^2+bc x^2+2ac+2\sqrt{ac}\,\sqrt{bd x^4+ad x^2+bc x^2+ac}}{x^2}\right)+\sqrt{ac}\,b\ln\left(\frac{2bd x^2+ad+bc+2\sqrt{bd x^4+ad x^2+\cdots}}{2\sqrt{bd}}\right)\cdots\right.}{2\sqrt{\left(d x^2+c\right)\left(b x^2+a\right)}\,\sqrt{bd}\,\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x,x)
```

```
[Out] 1/2*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*b*(a*c)^(1/2)-a
*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*
c)/x^2)*(b*d)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

maxima [A] time = 2.24, size = 149, normalized size = 1.33

$$\frac{1}{2}\left(\frac{a\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}}-\frac{b\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2
 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/sqrt(a*c*e) - b*log((d*sqrt((b*x^2 + a
)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b
*d*e)))/sqrt(b*d*e))*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{e\left(b x^2+a\right)}{d x^2+c}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x,x)
```

```
[Out] Timed out
```

3.267 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$

Optimal. Leaf size=127

$$\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}\,c^{3/2}}$$

```
[Out] -1/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/
2))*e^(1/2)/c^(3/2)/a^(1/2)+1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/
(a-c*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 288, 208}

$$\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}\,c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c*(a - (c*(a + b*x^2))/(
c + d*x^2))) - ((b*c - a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*Sqrt[a]*c^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2c}$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}\,c^{3/2}}$$

Mathematica [A] time = 0.10, size = 133, normalized size = 1.05

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,c^{3/2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)}{2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-((Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/(c*x^2)) - ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqr
t[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*c^(3/2))))/(2*Sqrt[a + b*x^2])
```

fricas [A] time = 0.64, size = 333, normalized size = 2.62

$$\left[-\frac{(bc-ad)x^2\sqrt{\frac{e}{ac}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(2\,a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3\,a^2c^2d\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{e}{ac}}}{x^4}\right)}{8\,cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*x^2*sqrt(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e
*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*
d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c))*sqrt(e/(a*c)))/x^4) + 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(c*x^2), 1/4*((b*c - a*d)*x^2*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x
^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e
)) - 2*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*x^2)]
```

giac [B] time = 0.68, size = 269, normalized size = 2.12

$$\frac{1}{2}\left(\frac{(bce-ade)\arctan\left(-\frac{\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}\,c} - \frac{\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}\right)bce+\left(\sqrt{bd}\right.}{\left(ace-\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*((b*c*e - a*d*e)*arctan(-(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*
x^2*e + a*d*x^2*e + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*c) - ((sqrt(b*d)*x^
2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*b*c*e + (sqrt(
b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a*d*e +
 2*sqrt(b*d)*a*c*e^(3/2))/((a*c*e - (sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e
 + b*c*x^2*e + a*d*x^2*e + a*c*e))^2)*c))*sgn(d*x^2 + c)
```

maple [B] time = 0.05, size = 326, normalized size = 2.57

$$-\frac{\sqrt{\frac{(b x^2+a)e}{d x^2+c}}\,(d x^2+c)\left(-a^2cd x^2\ln\left(\frac{ad x^2+bc x^2+2ac+2\sqrt{ac}\,\sqrt{bd x^4+ad x^2+bc x^2+ac}}{x^2}\right)+ab c^2x^2\ln\left(\frac{ad x^2+bc x^2+2ac+2\sqrt{ac}\,\sqrt{b}}{x^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^3,x)
```

```
[Out] -1/4*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(-2*b*d*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*x^4*(a*c)^(1/2)-a^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d*c*x^2+c^2*ln((a*d*x^2+b*c*x^2+2*
a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*b*a*x^2-2*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^2*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*
c)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/c^2/a/x^2/(a*c)^(1/2)
```

maxima [A] time = 2.23, size = 145, normalized size = 1.14

$$\frac{1}{4}e\left(\frac{2(bc-ad)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{ace-\frac{(bx^2+a)c^2e}{dx^2+c}}+\frac{(bc-ad)\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(a*c*e - (b*x^2 + a)*c
^2*e/(d*x^2 + c)) + (b*c - a*d)*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sq
rt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*
c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

3.268 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$

Optimal. Leaf size=208

$$\frac{\sqrt{e}\,(3ad+bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}+\frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

```
[Out] 1/8*(-a*d+b*c)*(3*a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^
(1/2)/e^(1/2))*e^(1/2)/a^(3/2)/c^(5/2)-1/4*(-a*d+b*c)^2*(e*(b*x^2+a)/(d*x^2
+c))^(1/2)/c^2/(a-c*(b*x^2+a)/(d*x^2+c))^2+1/8*(-5*a*d+b*c)*(-a*d+b*c)*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2/(a-c*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] time = 0.17, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 455, 385, 208}

$$\frac{\sqrt{e}\,(3ad+bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}+\frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

```
[Out] -((b*c - a*d)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^2*(a - (c*(a + b*x^
2))/(c + d*x^2))^2) + ((b*c - 5*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(8*a*c^2*(a - (c*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b*c +
3*a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]
*Sqrt[e])])/(8*a^(3/2)*c^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2\left(be-dx^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{-(bc-ad)e+4cdx^2}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4c^2}$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{((bc-ad)(bc+3ad)e)\,\text{Subst}\left(\int \frac{1}{-ae+\dots}\right.}{8ac^2}$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)(bc+3ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\dots}{\dots}\right.}{8a^{3/2}c^{5/2}}$$

Mathematica [A] time = 0.11, size = 174, normalized size = 0.84

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(x^4\left(-3a^2d^2+2abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2ac+3adx^2\dots\right.\right.}{8a^{3/2}c^{5/2}x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2) + (b^2*c^2 + 2*a*b*
c*d - 3*a^2*d^2)*x^4*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*
x^2])]))/(8*a^(3/2)*c^(5/2)*x^4*Sqrt[a + b*x^2])
```

fricas [A] time = 1.19, size = 427, normalized size = 2.05

$$\left[-\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)x^4\sqrt{\frac{e}{ac}}\,\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2-4\left(2\,a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3\,a^2c^2d\right)x^2\dots\right.}{x^4}\right.}{32\,ac^2x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(e/(a*c))*log(((b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4
*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sq
rt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) + 4*((b*c*d - 3*a*d^2)*
x^4 + 2*a*c^2 + (b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*
c^2*x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(-e/(a*c))*arcta
n(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(
a*c))/(b*e*x^2 + a*e)) + 2*((b*c*d - 3*a*d^2)*x^4 + 2*a*c^2 + (b*c^2 - a*c*
d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^2*x^4)]
```

giac [B] time = 0.82, size = 598, normalized size = 2.88

$$-\frac{1}{8}\left(\frac{\left(b^2c^2e+2\,abcde-3\,a^2d^2e\right)\arctan\left(-\frac{\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+adx^2e+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}\,ac^2}-\frac{\left(\sqrt{bd}\,x^2e^{\frac{1}{2}}-\sqrt{bdx^4e+bcx^2e+ad}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*((b^2*c^2*e + 2*a*b*c*d*e - 3*a^2*d^2*e)*arctan(-(sqrt(b*d)*x^2*e^(1/2
) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))/sqrt(-a*c*e))/(sqrt(-a
*c*e)*a*c^2) - ((sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x
^2*e + a*c*e))*a*b^2*c^3*e^2 + 10*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e +
 b*c*x^2*e + a*d*x^2*e + a*c*e))*a^2*b*c^2*d*e^2 + 5*(sqrt(b*d)*x^2*e^(1/2)
 - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^3*c*d^2*e^2 + (sqrt(b
*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*b^2*c^
2*e + 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a
*c*e))^3*a*b*c*d*e - 3*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e
+ a*d*x^2*e + a*c*e))^3*a^2*d^2*e + 8*sqrt(b*d)*a^3*c^2*d*e^(5/2) + 8*(sqrt
(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^2*sqrt
(b*d)*a*b*c^2*e^(3/2))/((a*c*e - (sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e +
b*c*x^2*e + a*d*x^2*e + a*c*e))^2)^2*a*c^2))*sgn(d*x^2 + c)
```

maple [B] time = 0.07, size = 558, normalized size = 2.68

$$\frac{\sqrt{\frac{\left(b\,x^2+a\right)e}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-3a^3c\,d^2x^4\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+2a^2b\,c^2d\,x^4\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^5,x)
```

```
[Out] 1/16*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(-10*b*d^2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*x^6*a*(a*c)^(1/2)-2*b^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*x^6*c*(a*c)^(1/2)-3*a^3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d^2*c*x^4+2*ln((a*d*x^2+b*c*x^2+2*a*c+2*
(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d*b*a^2*c^2*x^4+c^3*l
n((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))
/x^2)*b^2*a*x^4-10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2*a^2*x^4*(a*c)^(1
/2)-8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b*c*d-2*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2*c^2*x^4*(a*c)^(1/2)+10*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(3/2)*d*a*x^2*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*
```

```
b*c*x^2*(a*c)^(1/2)-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*a*c*(a*c)^(1/2))/
((d*x^2+c)*(b*x^2+a))^(1/2)/c^3/a^2/x^4/(a*c)^(1/2)
```

maxima [A] time = 2.17, size = 265, normalized size = 1.27

$$-\frac{1}{16}e\left(\frac{2\left(\left(b^2c^3-6\,abc^2d+5\,a^2cd^2\right)\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}+\left(ab^2c^2+2\,a^2bcd-3\,a^3d^2\right)\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}\,e\right)}{a^3c^2e^2-\frac{2\left(bx^2+a\right)a^2c^3e^2}{dx^2+c}+\frac{\left(bx^2+a\right)^2ac^4e^2}{\left(dx^2+c\right)^2}}+\frac{\left(b^2c^2+2\,abcd-3\,a^2c\right.}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*e*(2*((b^2*c^3 - 6*a*b*c^2*d + 5*a^2*c*d^2)*((b*x^2 + a)*e/(d*x^2 + c
))^(3/2) + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt((b*x^2 + a)*e/(d*x^2
+ c))*e)/(a^3*c^2*e^2 - 2*(b*x^2 + a)*a^2*c^3*e^2/(d*x^2 + c) + (b*x^2 + a)
^2*a*c^4*e^2/(d*x^2 + c)^2) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log((c*sqrt
((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c
)) + sqrt(a*c*e)))/(sqrt(a*c*e)*a*c^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**5,x)
```

```
[Out] Timed out
```

3.269 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7} dx$

Optimal. Leaf size=318

$$-\frac{\left(-11a^2d^2+2abcd+b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}\left(5a^2d^2+2abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}+\cdots$$

```
[Out] 1/6*(-a*d+b*c)^3*e^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/a/c^2/(a*e-c*e*(b*x^2+a)
/(d*x^2+c))^3-1/16*(-a*d+b*c)*(5*a^2*d^2+2*a*b*c*d+b^2*c^2)*arctanh(c^(1/2)
*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*e^(1/2)/a^(5/2)/c^(7/2)+1/8
*(-a*d+b*c)^2*(3*a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^3/(a-c*(b*x^2+a
)/(d*x^2+c))^2-1/16*(-a*d+b*c)*(-11*a^2*d^2+2*a*b*c*d+b^2*c^2)*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/a^2/c^3/(a-c*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] time = 0.31, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 463, 455, 385, 208}

$$-\frac{\left(-11a^2d^2+2abcd+b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}\left(5a^2d^2+2abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

```
[Out] ((b*c - a*d)^2*(b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(8*a*c^3*(a
 - (c*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(b^2*c^2 + 2*a*b*c*d - 11
*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(16*a^2*c^3*(a - (c*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)^3*e^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2))/
(6*a*c^2*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^3) - ((b*c - a*d)*(b^2*c^2 +
 2*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(5/2)*c^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
```

```
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^7}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^2\left(be-dx^2\right)^2}{\left(-ae+cx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^3} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^2\left(-3\left(2b^2c^2e^2-(bce-ade)^2\right)+6acd^2ex^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right.}{6ac^2}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8ac^3\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)^2} + \frac{(bc-ad)^3e^2\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce\left(a+bx^2\right)}{c+dx^2}\right)^3} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{3c(bc-ad)(bc+3a\ldots}{(-ae+\ldots}\right.}{24a\ldots}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8ac^3\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(b^2c^2+2abcd-11a^2d^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)} + \frac{(bc-ad)^3e^2\ldots}{6ac^2\left(ae-\ldots\right.}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8ac^3\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(b^2c^2+2abcd-11a^2d^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c\left(a+bx^2\right)}{c+dx^2}\right)} + \frac{(bc-ad)^3e^2\ldots}{6ac^2\left(ae-\ldots\right.}$$

Mathematica [A] time = 0.18, size = 222, normalized size = 0.70

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2\left(-8c^2+10cdx^2-15d^2x^4\right)-2abcx^2\left(c-2dx^2\right)+3b^2c^2x^4\right)-3\ldots\right.}{48a^{5/2}c^{7/2}x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2]*(3*b^2*c^2*x^4 - 2*a*b*c*x^2*(c - 2*d*x^2) + a^2*(
-8*c^2 + 10*c*d*x^2 - 15*d^2*x^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d
^2 - 5*a^3*d^3)*x^6*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x
^2])]))/(48*a^(5/2)*c^(7/2)*x^6*Sqrt[a + b*x^2])
```

fricas [A] time = 3.20, size = 561, normalized size = 1.76

$$\left[-\frac{3\left(b^3c^3 + ab^2c^2d + 3a^2bcd^2 - 5a^3d^3\right)x^6\sqrt{\frac{e}{ac}}\log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)ex^4+8a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(2a^2c^3+\left(abc^2d+a^2cd^2\right)\right.}{x^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(e/(
a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2
 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 +
 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) -
4*((3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c^3 +
2*a*b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c)))/(a^2*c^3*x^6), 1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2
*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*
c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e)) + 2*((
3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c^3 + 2*a*
b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(a^2*c^3*x^6)]
```

giac [B] time = 0.98, size = 1076, normalized size = 3.38

$$\frac{1}{48}\left(\frac{3\left(b^3c^3e + ab^2c^2de + 3a^2bcd^2e - 5a^3d^3e\right)\arctan\left(-\frac{\sqrt{bd}\,x^2e^{\frac{1}{2}} - \sqrt{bdx^4e+bcx^2e+adx^2e+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}\,a^2c^3} - \frac{3\left(\sqrt{bd}\,x^2e^{\frac{1}{2}} - \sqrt{bdx^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(b^3*c^3*e + a*b^2*c^2*d*e + 3*a^2*b*c*d^2*e - 5*a^3*d^3*e)*arctan(
-(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))/
sqrt(-a*c*e))/(sqrt(-a*c*e)*a^2*c^3) - (3*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d
*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^2*b^3*c^5*e^3 + 51*(sqrt(b*d)*x^
2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^3*b^2*c^4*d*
e^3 + 105*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e +
 a*c*e))*a^4*b*c^3*d^2*e^3 + 33*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b
*c*x^2*e + a*d*x^2*e + a*c*e))*a^5*c^2*d^3*e^3 + 8*(sqrt(b*d)*x^2*e^(1/2) -
 sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a*b^3*c^4*e^2 + 72*(sqr
t(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a^2
*b^2*c^3*d*e^2 + 24*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a
*d*x^2*e + a*c*e))^3*a^3*b*c^2*d^2*e^2 - 40*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b
*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a^4*c*d^3*e^2 - 3*(sqrt(b*d)*x
```

```
^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^5*b^3*c^3*e -
 3*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)
)^5*a*b^2*c^2*d*e - 9*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e +
 a*d*x^2*e + a*c*e))^5*a^2*b*c*d^2*e + 15*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d
*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^5*a^3*d^3*e + 16*sqrt(b*d)*a^4*b*c
^4*d*e^(7/2) + 48*sqrt(b*d)*a^5*c^3*d^2*e^(7/2) + 48*(sqrt(b*d)*x^2*e^(1/2)
 - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^2*sqrt(b*d)*a^2*b^2*c^4
*e^(5/2) + 144*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^
2*e + a*c*e))^2*sqrt(b*d)*a^3*b*c^3*d*e^(5/2))/((a*c*e - (sqrt(b*d)*x^2*e^(
1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^2)^3*a^2*c^3))*sgn(
d*x^2 + c)
```

maple [B] time = 0.08, size = 849, normalized size = 2.67

$$-\frac{\sqrt{\frac{(b x^2+a)e}{d x^2+c}}\,\left(d x^2+c\right)\left(-15a^4 c\, d^3 x^6 \ln\left(\frac{ad\, x^2+bc\, x^2+2ac+2\sqrt{ac}\,\sqrt{bd\, x^4+ad\, x^2+bc\, x^2+ac}}{x^2}\right)+9a^3 b\, c^2 d^2 x^6 \ln\left(\frac{ad\, x^2+bc\, x^2+2ac+2\sqrt{}\cdots}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^7,x)
```

```
[Out] -1/96*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(-66*b*d^3*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*x^8*a^2*(a*c)^(1/2)-24*b^2*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*x^8*a*c*(a*c)^(1/2)-6*b^3*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x
^8*c^2*(a*c)^(1/2)-15*a^4*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d^3*c*x^6+9*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a
*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d^2*b*a^3*c^2*x^6+3*ln(
(a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x
^2)*d*b^2*a^2*c^3*x^6+3*c^4*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*b^3*a*x^6-66*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*d^3*a^3*x^6*(a*c)^(1/2)-54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2
*b*a^2*c*x^6*(a*c)^(1/2)-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2*d*a*c^2
*x^6*(a*c)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^3*c^3*x^6*(a*c)^(1
/2)+66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*d^2*a^2*x^4*(a*c)^(1/2)+24*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*d*b*a*c*x^4*(a*c)^(1/2)+6*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(3/2)*b^2*c^2*x^4*(a*c)^(1/2)-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(3/2)*d*a^2*c*x^2*(a*c)^(1/2)-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*a*c^
2*x^2*(a*c)^(1/2)+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*a^2*c^2*(a*c)^(1/2
))/((d*x^2+c)*(b*x^2+a))^(1/2)/c^4/a^3/x^6/(a*c)^(1/2)
```

maxima [A] time = 2.29, size = 410, normalized size = 1.29

$$-\frac{1}{96}e\left(\frac{2\left(3\left(b^3c^5+ab^2c^4d-13\,a^2bc^3d^2+11\,a^3c^2d^3\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{5}{2}}-8\left(ab^3c^4-3\,a^2b^2c^3d-3\,a^3bc^2d^2+5\,a^4cd^3\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right.\cdots\right.}{a^5c^3e^3-\frac{3\left(bx^2+a\right)a^4c^4e^3}{dx^2+c}+\frac{3\left(bx^2+a\right)^2a^3c^5e^3}{\left(dx^2+c\right)^2}-\frac{\left(bx^2+a\right)\cdots}{\left(dx^2+c\right)\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/96*e*(2*(3*(b^3*c^5 + a*b^2*c^4*d - 13*a^2*b*c^3*d^2 + 11*a^3*c^2*d^3)*(
(b*x^2 + a)*e/(d*x^2 + c))^(5/2) - 8*(a*b^3*c^4 - 3*a^2*b^2*c^3*d - 3*a^3*b
*c^2*d^2 + 5*a^4*c*d^3)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e - 3*(a^2*b^3*c^
3 + a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - 5*a^5*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 +
c))*e^2)/(a^5*c^3*e^3 - 3*(b*x^2 + a)*a^4*c^4*e^3/(d*x^2 + c) + 3*(b*x^2 +
```

```
a)^2*a^3*c^5*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*a^2*c^6*e^3/(d*x^2 + c)^3) -
 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log((c*sqrt((b*x^2 +
 a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt
(a*c*e)))/(sqrt(a*c*e)*a^2*c^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**7,x)
[Out] Timed out
```

3.270 $\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=357

$$\frac{x\left(-2a^2d^2-3abcd+8b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15b^2d^2}-\frac{\sqrt{c}\left(-2a^2d^2-3abcd+8b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}(4b$$

```
[Out] 1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d
^2-1/15*(-a*d+4*b*c)*x*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^2+1/5*x^
3*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d+1/15*c^(3/2)*(-a*d+4*b*c)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^(5/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)-1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^(5/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 478, 582, 531, 418, 492, 411}

$$\frac{x\left(-2a^2d^2-3abcd+8b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15b^2d^2}-\frac{\sqrt{c}\left(-2a^2d^2-3abcd+8b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}(4b$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
15*b^2*d^2) - ((4*b*c - a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2
))/(15*b*d^2) + (x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d) -
 (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(5/
2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*(4*b*c - a*d)*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\sqrt{c+dx^2}\right)\int\frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\left(c+dx^2\right)}{5d} - \frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\sqrt{c+dx^2}\right)\int\frac{x^2\left(3ac+(4bc-ad)x^2\right)}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{5d\sqrt{a+bx^2}}$$

$$= -\frac{(4bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\left(c+dx^2\right)}{15bd^2} + \frac{x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\left(c+dx^2\right)}{5d} + \frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\sqrt{c+dx^2}\right)\int \cdots}{15bd^2 \cdots}$$

$$= -\frac{(4bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\left(c+dx^2\right)}{15bd^2} + \frac{x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\left(c+dx^2\right)}{5d} + \frac{\left(ac(4bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\sqrt{\cdots}\right.}{15bd^2\sqrt{\cdots}}$$

$$= \frac{\left(8b^2c^2-3abcd-2a^2d^2\right)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{15b^2d^2} - \frac{(4bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\left(c+dx^2\right)}{15bd^2} + \frac{x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\left(\cdots\right.}{5d}$$

$$= \frac{\left(8b^2c^2-3abcd-2a^2d^2\right)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{15b^2d^2} - \frac{(4bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\left(c+dx^2\right)}{15bd^2} + \frac{x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\left(\cdots\right.}{5d}$$

Mathematica [C] time = 0.48, size = 255, normalized size = 0.71

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^2d^2+7abcd-8b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(2a^2d^2\cdots\right.\right.}{15bd^3\sqrt{\frac{b}{a}}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(
-4*b*c + a*d + 3*b*d*x^2) + I*c*(-8*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] - I*c*(-8*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*b*Sqrt[b/a
]*d^3*(a + b*x^2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4\sqrt{\frac{bex^2+ae}{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4, x)
```

maple [A] time = 0.05, size = 552, normalized size = 1.55

$$\frac{\sqrt{\frac{(b\,x^2+a)e}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(3\sqrt{-\frac{b}{a}}\,b^2d^3x^7+4\sqrt{-\frac{b}{a}}\,ab\,d^3x^5-\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5+\sqrt{-\frac{b}{a}}\,a^2d^3x^3-4\sqrt{-\frac{b}{a}}\,b^2c^2d\,x^3+\sqrt{-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
[Out] 1/15*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(3*(-1/a*b)^(1/2)*x^7*b^2*d^3+
4*(-1/a*b)^(1/2)*x^5*a*b*d^3-(-1/a*b)^(1/2)*x^5*b^2*c*d^2+(-1/a*b)^(1/2)*x^
3*a^2*d^3-4*(-1/a*b)^(1/2)*x^3*b^2*c^2*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticF(x*(-1/a*b)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+7*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-1/a*b)^(1/2),(a*d/b/c)^(1/2))*a*b*
c^2*d-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-1/a*b)^(1/2),
(a*d/b/c)^(1/2))*b^2*c^3-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E(x*(-1/a*b)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2-3*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-1/a*b)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+8*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-1/a*b)^(1/2),(a*d/b/c)^(1/2
))*b^2*c^3+(-1/a*b)^(1/2)*x*a^2*c*d^2-4*(-1/a*b)^(1/2)*x*a*b*c^2*d)/d^3/((d
*x^2+c)*(b*x^2+a))^(1/2)/b/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.271 $\int x^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$

Optimal. Leaf size=266

$$-\frac{c^{3/2}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{c}\,(2bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\left(c+dx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{3d}$$

```
[Out] -1/3*(-a*d+2*b*c)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d+1/3*x*(d*x^2+c)*(e*(b
*x^2+a)/(d*x^2+c))^(1/2)/d-1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(b*
x^2+a)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+1/3*(-a*d+2
*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b
/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6719, 478, 531, 418, 492, 411}

$$-\frac{c^{3/2}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{\sqrt{c}\,(2bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\left(c+dx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -((2*b*c - a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*b*d) + (x*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*d) + (Sqrt[c]*(2*b*c - a*d)*Sqrt[(
e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (c^(3/2)*Sq
rt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
```

```
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int x^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$=\frac{x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d}-\frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{ac+(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d\sqrt{a+bx^2}}$$

$$=\frac{x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d}-\frac{\left(ac\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d\sqrt{a+bx^2}}-\frac{\left((2bc-ad)\sqrt{\frac{e(a+}{c+}}\right.}{}$$

$$=-\frac{(2bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{3bd}+\frac{x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d}-\frac{c^{3/2}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}$$

$$=-\frac{(2bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{3bd}+\frac{x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d}+\frac{\sqrt{c}\,(2bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{c}}\right.\right.}{3bd^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}$$

Mathematica [C] time = 0.31, size = 208, normalized size = 0.78

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)+2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}}\right.}{3d^2\sqrt{\frac{b}{a}}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) -
 I*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*S
qrt[b/a]*d^2*(a + b*x^2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\sqrt{\frac{bex^2+ae}{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2, x)
```

maple [A] time = 0.02, size = 356, normalized size = 1.34

$$\frac{\sqrt{\frac{(b\,x^2+a)e}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3+\sqrt{-\frac{b}{a}}\,acdx+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,acd\,\text{Elliptic}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/3*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*((-1/a*b)^(1/2)*x^5*b*d^2+(-1/a
*b)^(1/2)*x^3*a*d^2+(-1/a*b)^(1/2)*x^3*b*c*d-2*a*c*((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*d+2*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b*
c^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b
/c*d)^(1/2))*a*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/
a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b*c^2+(-1/a*b)^(1/2)*x*a*c*d)/((d*x^2+c)*(b*x
^2+a))^(1/2)/d^2/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{\frac{e \left(b x^2 + a\right)}{d x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

3.272 $\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=194

$$x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}+\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ell
ipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b
*x^2+a)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+(1/(1+d*x^
2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2)
,(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(b*x^2
+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6719, 422, 418, 492, 411}

$$x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}+\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)] - (Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{\left(a\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}} + \frac{\left(b\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}} + \frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}} - \frac{\left(c\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx}}{\left(c+dx^2\right)}}{\sqrt{a+bx^2}}$$

$$= x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}} - \frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}} + \frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}$$

Mathematica [A] time = 0.06, size = 86, normalized size = 0.44

$$\frac{\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqr
t[-(d/c)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\frac{bex^2+ae}{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((b*e*x^2 + a*e)/(d*x^2 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

maple [A] time = 0.02, size = 184, normalized size = 0.95

$$\frac{\sqrt{\frac{(b\,x^2+a)e}{d\,x^2+c}}\,\left(d\,x^2+c\right)\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\left(ad\,\mathrm{EllipticF}\left(\sqrt{-\frac{b}{a}}\,x,\sqrt{\frac{ad}{bc}}\right)+bc\,\mathrm{EllipticE}\left(\sqrt{-\frac{b}{a}}\,x,\sqrt{\frac{ad}{bc}}\right)-bc\,\mathrm{Ellipti}}{\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
[Out] ((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*(a*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*d-b*c*EllipticF((-1/a*b
)^(1/2)*x,(a/b/c*d)^(1/2))+b*c*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2)))
/((d*x^2+c)*(b*x^2+a))^(1/2)/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

3.273 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2} dx$

Optimal. Leaf size=239

$$\frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c}-\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cx}+\frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] d*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c-(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)
/c/x+b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)
^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(
1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 475, 21, 422, 418, 492, 411}

$$\frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c}-\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cx}+\frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

```
[Out] (d*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c - (Sqrt[(e*(a + b*x^2))/(c + d*x^
2)]*(c + d*x^2))/(c*x) - (Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]) + (b*Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{cx} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{cx} + \frac{\left(b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{cx} + \frac{\left(b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}} + \frac{\left(bd\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right.}{c\sqrt{a+}}$$

$$= \frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{cx} + \frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\left(d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right.}{}$$

$$= \frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{cx} - \frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{a}$$

Mathematica [A] time = 0.25, size = 111, normalized size = 0.46

$$\frac{\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\sqrt{\frac{bx^2}{a}+1}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\left(a+bx^2\right)\sqrt{\frac{dx^2}{c}+1}}-\frac{1}{x}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)*(-x^(-1) + (b*Sqrt[1 + (b*x^
2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*(a + b*
x^2)*Sqrt[1 + (d*x^2)/c])))/c
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^2, x)
```

maple [A] time = 0.03, size = 192, normalized size = 0.80

$$-\frac{\sqrt{\frac{\left(b\,x^2+a\right)e}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(\sqrt{-\frac{b}{a}}\,bd\,x^4+\sqrt{-\frac{b}{a}}\,ad\,x^2+\sqrt{-\frac{b}{a}}\,bc\,x^2-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,bcx\,\mathrm{EllipticE}\left(\sqrt{-\frac{b}{a}}\,x,\sqrt{\frac{ad}{bc}}\right.\right.}{\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^2,x)
```

```
[Out] -((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*((-1/a*b)^(1/2)*x^4*b*d-b*c*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1
/2))+(-1/a*b)^(1/2)*x^2*a*d+(-1/a*b)^(1/2)*x^2*b*c+(-1/a*b)^(1/2)*a*c)/((d*
x^2+c)*(b*x^2+a))^(1/2)/c/x/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^2,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

3.274 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4} dx$

Optimal. Leaf size=321

$$-\frac{\sqrt{d}(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2}-\frac{(c+dx^2)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2x}-\frac{b\sqrt{d}}{}$$

```
[Out] 1/3*d*(-2*a*d+b*c)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2-1/3*(d*x^2+c)*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)/c/x^3-1/3*(-2*a*d+b*c)*(d*x^2+c)*(e*(b*x^2+a)/(d*
x^2+c))^(1/2)/a/c^2/x-1/3*(-2*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2
)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1
/3*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1
+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/
c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 475, 583, 531, 418, 492, 411}

$$\frac{dx(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2}-\frac{(c+dx^2)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2x}-\frac{\sqrt{d}(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]
```

```
[Out] (d*(b*c - 2*a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*a*c^2) - (Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c*x^3) - ((b*c - 2*a*d)*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*a*c^2*x) - (Sqrt[d]*(b*c - 2*a*d)*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (b*Sq
rt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
```

```
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3cx^3} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{bc-2ad-bdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3ac^2x} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{abcd-bd(bc-2}{\sqrt{a+bx^2}\sqrt{c+}}}{3ac^2\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3ac^2x} - \frac{\left(bd\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{}}}{3c\sqrt{a+bx^2}}$$

$$= \frac{d(bc-2ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3ac^2x} - \frac{b\sqrt{d}\sqrt{\frac{e(a+}{c+}}}{}$$

$$= \frac{d(bc-2ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{3ac^2x} - \frac{\sqrt{d}\,(bc-2}{}$$

Mathematica [C] time = 0.66, size = 238, normalized size = 0.74

$$-\frac{\sqrt{\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(a\left(c-2dx^2\right)+bcx^2\right)+ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\right.\right.}{3bc^2x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]
```

```
[Out] -1/3*(Sqrt[b/a]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(c
 + d*x^2)*(b*c*x^2 + a*(c - 2*d*x^2)) - I*b*c*(-(b*c) + 2*a*d)*x^3*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
c)] + I*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c^2*x^3*(a + b*x^2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^4, x)
```

maple [A] time = 0.04, size = 444, normalized size = 1.38

$$\frac{\sqrt{\frac{(b\,x^2+a)e}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(2\sqrt{-\frac{b}{a}}\,ab\,d^2x^6-\sqrt{-\frac{b}{a}}\,b^2cd\,x^6+2\sqrt{-\frac{b}{a}}\,a^2d^2x^4-2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,abcd\,x^3\,\mathrm{EllipticE}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^4,x)
```

```
[Out] 1/3*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(2*(-1/a*b)^(1/2)*x^6*a*b*d^2-(
-1/a*b)^(1/2)*x^6*b^2*c*d+b*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^3*a*c-((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^3*b^2*c^2-2*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/
2))*x^3*a*b*c*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^
(1/2)*x,(a/b/c*d)^(1/2))*x^3*b^2*c^2+2*(-1/a*b)^(1/2)*x^4*a^2*d^2-(-1/a*b)^
(1/2)*x^4*b^2*c^2+(-1/a*b)^(1/2)*x^2*a^2*c*d-2*(-1/a*b)^(1/2)*x^2*a*b*c^2-(
-1/a*b)^(1/2)*a^2*c^2)/((d*x^2+c)*(b*x^2+a))^(1/2)/c^2/x^3/(-1/a*b)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^4,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**4,x)
```

```
[Out] Timed out
```

3.275 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6} dx$

Optimal. Leaf size=424

$$\frac{\sqrt{d}\left(-8a^2d^2+3abcd+2b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\left(-8a^2d^2+3abcd+2b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3}+\frac{(c+$$

```
[Out] -1/15*d*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c^2)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^
2/c^3-1/5*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/x^5-1/15*(-4*a*d+b*c)*(
d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2/x^3+1/15*(-8*a^2*d^2+3*a*b*c*d
+2*b^2*c^2)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/c^3/x+1/15*(-8*a^2*
d^2+3*a*b*c*d+2*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(
x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/a^2/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/15*b*(-4*a*
d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
a^2/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 475, 583, 531, 418, 492, 411}

$$-\frac{dx\left(-8a^2d^2+3abcd+2b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3}+\frac{\left(c+dx^2\right)\left(-8a^2d^2+3abcd+2b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3x}+\frac{\sqrt{d}\left(-8a^2d^2+3abcd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```

```
[Out] -(d*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]
)/(15*a^2*c^3) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*c*x^5)
- ((b*c - 4*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(15*a*c^2*x
^3) + ((2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)
]*(c + d*x^2))/(15*a^2*c^3*x) + (Sqrt[d]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2
)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*a^2*c^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) -
(b*Sqrt[d]*(b*c - 4*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^2*c^(3/2)*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{bc-4ad-3bdx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15ac^2x^3} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{2b^2c^2+3abcd-}{x^2\sqrt{a-}}}{15ac^2\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15ac^2x^3} + \frac{\left(2b^2c^2+3abcd-8a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3x}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{15ac^2x^3} + \frac{\left(2b^2c^2+3abcd-8a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3x}$$

$$= -\frac{d\left(2b^2c^2+3abcd-8a^2d^2\right)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+d\right.}{15ac^2x^3}$$

$$= -\frac{d\left(2b^2c^2+3abcd-8a^2d^2\right)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+d\right.}{15ac^2x^3}$$

Mathematica [C] time = 0.57, size = 302, normalized size = 0.71

$$-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-2ibcx^5\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(2a^2d^2-abcd-b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ibcx^5\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```

```
[Out] -1/15*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)
*(-2*b^2*c^2*x^4 + a*b*c*x^2*(c - 3*d*x^2) + a^2*(3*c^2 - 4*c*d*x^2 + 8*d^2
*x^4)) + I*b*c*(-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)
*b*c*(-(b^2*c^2) - a*b*c*d + 2*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(a^2*Sqrt[b/a]*c^
3*x^5*(a + b*x^2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
[Out] integral(sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^6, x)
```

maple [A] time = 0.04, size = 708, normalized size = 1.67

$$-\frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,(dx^2+c)\left(8\sqrt{-\frac{b}{a}}\,a^2bd^3x^8-3\sqrt{-\frac{b}{a}}\,ab^2cd^2x^8-2\sqrt{-\frac{b}{a}}\,b^3c^2dx^8+8\sqrt{-\frac{b}{a}}\,a^3d^3x^6+\sqrt{-\frac{b}{a}}\,a^2bcd^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^6,x)
[Out] -1/15*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(8*(-1/a*b)^(1/2)*x^8*a^2*b*d
^3-3*(-1/a*b)^(1/2)*x^8*a*b^2*c*d^2-2*(-1/a*b)^(1/2)*x^8*b^3*c^2*d+4*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2
))*x^5*a^2*b*c*d^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/
a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^5*a*b^2*c^2*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^5*b^3*c^3-8*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1
/2))*x^5*a^2*b*c*d^2+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-
1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^5*a*b^2*c^2*d+2*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^5*b^3*c^3+8*(-
1/a*b)^(1/2)*x^6*a^3*d^3+(-1/a*b)^(1/2)*x^6*a^2*b*c*d^2-4*(-1/a*b)^(1/2)*x^
6*a*b^2*c^2*d-2*(-1/a*b)^(1/2)*x^6*b^3*c^3+4*(-1/a*b)^(1/2)*x^4*a^3*c*d^2-3
*(-1/a*b)^(1/2)*x^4*a^2*b*c^2*d-(-1/a*b)^(1/2)*x^4*a*b^2*c^3-(-1/a*b)^(1/2)
*x^2*a^3*c^2*d+4*(-1/a*b)^(1/2)*x^2*a^2*b*c^3+3*(-1/a*b)^(1/2)*a^3*c^3)/((d
*x^2+c)*(b*x^2+a))^(1/2)/c^3/x^5/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e\,(bx^2+a)}{dx^2+c}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^6,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**6,x)
```

```
[Out] Timed out
```

3.276 $\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=282

$$\frac{e\left(c+dx^2\right)\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48bd^4}-\frac{e^{3/2}(bc-ad)\left(-a^2d^2-10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}$$

```
[Out] 1/6*(e*(b*x^2+a)/(d*x^2+c))^(5/2)*(d*x^2+c)^3/b/d^2/e-1/16*(-a*d+b*c)*(-a^2
*d^2-10*a*b*c*d+35*b^2*c^2)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))
^(1/2)/b^(1/2)/e^(1/2))/b^(3/2)/d^(9/2)+c^2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/d^4+1/48*(-5*a^2*d^2-50*a*b*c*d+79*b^2*c^2)*e*(d*x^2+c)*(e*(b*x
^2+a)/(d*x^2+c))^(1/2)/b/d^4-1/24*(a*d+11*b*c)*e*(d*x^2+c)^2*(e*(b*x^2+a)/(
d*x^2+c))^(1/2)/d^4
```

Rubi [A] time = 0.38, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1960, 463, 455, 1157, 388, 208}

$$-\frac{e^{3/2}(bc-ad)\left(-a^2d^2-10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}+\frac{e\left(c+dx^2\right)\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (c^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^4 + ((79*b^2*c^2 -
50*a*b*c*d - 5*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(4
8*b*d^4) - ((11*b*c + a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^
2)/(24*d^4) + (((e*(a + b*x^2))/(c + d*x^2))^(5/2)*(c + d*x^2)^3)/(6*b*d^2*
e) - ((b*c - a*d)*(35*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt
[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(3/2)*d^(9
/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx=((bc-ad)e)\,\text{Subst}\left(\int\frac{x^4\left(-ae+cx^2\right)^2}{\left(be-dx^2\right)^4}dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$=\frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{5/2}\left(c+dx^2\right)^3}{6bd^2e}-\frac{(bc-ad)\,\text{Subst}\left(\int\frac{x^4\left(-6a^2d^2e^2+5(bce-ade)^2+6bc^2dex^2\right)}{\left(be-dx^2\right)^3}dx,x,\sqrt{\frac{e(a\cdots}{c\cdots}}\right.}{6bd^2}$$

$$=-\frac{(11bc+ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{24d^4}+\frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{5/2}\left(c+dx^2\right)^3}{6bd^2e}-\frac{(bc-ad)\,\text{Subst}\left(\int\frac{-b\cdots}{\cdots}\right.}{\cdots}$$

$$=\frac{\left(79b^2c^2-50abcd-5a^2d^2\right)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{48bd^4}-\frac{(11bc+ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{24d^4}\cdots$$

$$=\frac{c^2(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d^4}+\frac{\left(79b^2c^2-50abcd-5a^2d^2\right)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{48bd^4}-\frac{(11bc+\cdots}{\cdots}$$

$$=\frac{c^2(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d^4}+\frac{\left(79b^2c^2-50abcd-5a^2d^2\right)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{48bd^4}-\frac{(11bc+\cdots}{\cdots}$$

Mathematica [A] time = 0.54, size = 294, normalized size = 1.04

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(b\sqrt{d}\sqrt{bc-ad}\left(3a^3d^2\left(c+dx^2\right)+a^2bd\left(-100c^2-35cdx^2+17d^2x^4\right)+ab^2\left(105c^3-65c^2dx^2-52c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*Sqrt[b*c - a*d]*(3*a^3*d^2*
(c + d*x^2) + a^2*b*d*(-100*c^2 - 35*c*d*x^2 + 17*d^2*x^4) + b^3*x^2*(105*c
^3 + 35*c^2*d*x^2 - 14*c*d^2*x^4 + 8*d^3*x^6) + a*b^2*(105*c^3 - 65*c^2*d*x
^2 - 52*c*d^2*x^4 + 22*d^3*x^6)) - 3*(b*c - a*d)^2*(35*b^2*c^2 - 10*a*b*c*d
 - a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt
[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(48*b^2*d^(9/2)*Sqrt[b*c - a*d]*(a
+ b*x^2))
```

fricas [A] time = 1.85, size = 553, normalized size = 1.96

$$\left[\frac{3\left(35\,b^3c^3-45\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)e\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(e/
(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*e - 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2
 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(8*b^
2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*c*d^2
 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4), 1/96*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d
+ 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a
*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(
8*b^2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*c
*d^2 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*sq
rt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.84Unable to divide, perhaps due to rounding error%
%%{%%%{2,[0,5,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,4,0]%%%},0]:[1,0,%%%{-
1,[1,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{2,[1,4,1]%%%},[0,0,0,2]%%%} / %%%{%
%%{1,[0,2,2]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-2,[0,1,2]%%%},0]:[1,0,%%%{-1,[1
,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,0,0,2]%%%} Error: Bad
Argument Value
```

maple [B] time = 0.08, size = 1027, normalized size = 3.64

$$\frac{\left(-3a^3d^4x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-27a^2bc\,d^3x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+135\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/96*(12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*a*b*d^3-60*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b^2*c*d^2-3*ln(1/2*(2*b*d*
x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))
*x^2*a^3*d^4-27*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a^2*b*c*d^3+135*ln(1/2*(2*b*d*x^2+a*d+b*
c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a*b^2
*c^2*d^2-105*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*b^3*c^3*d+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(3/2)*(b*d)^(1/2)*x^2*b*d^2+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/
2)*x^2*a^2*d^3-108*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*a*b*
c*d^2+54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b^2*c^2*d-3*ln
(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/
(b*d)^(1/2))*a^3*c*d^3-27*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^2*b*c^2*d^2+135*ln(1/2*(2*b*d*x^
2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a
*b^2*c^3*d-105*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*(b*d)^(1/2))/(b*d)^(1/2))*b^3*c^4+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2
)*(b*d)^(1/2)*b*c*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*c
*d^2-120*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b*c^2*d+114*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c^3-96*(b*d)^(1/2)*((d*x^2+
c)*(b*x^2+a))^(1/2)*a*b*c^2*d+96*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b^
2*c^3)/d^4/b*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(b*d)^(1/2)/((d*x^2+c)
*(b*x^2+a))^(1/2)/(b*x^2+a)
```

maxima [A] time = 2.12, size = 454, normalized size = 1.61

$$\frac{1}{96}e\left(\frac{2\left(3\left(29\,b^3c^3d^2-51\,ab^2c^2d^3+23\,a^2bcd^4-a^3d^5\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{5}{2}}e-8\left(17\,b^4c^3d-27\,ab^3c^2d^2+9\,a^2b^2cd^3+a^3bd\ldots\right.\right.}{b^4d^4e^3-\frac{3\left(bx^2+a\right)b^3d^5e^3}{dx^2+c}+\frac{3\left(bx^2+a\right)^2b^2d^6e^3}{\left(dx^2+c\right)^2}-\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*e*(2*(3*(29*b^3*c^3*d^2 - 51*a*b^2*c^2*d^3 + 23*a^2*b*c*d^4 - a^3*d^5)
*((b*x^2 + a)*e/(d*x^2 + c))^(5/2)*e - 8*(17*b^4*c^3*d - 27*a*b^3*c^2*d^2 +
 9*a^2*b^2*c*d^3 + a^3*b*d^4)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^2 + 3*(19
*b^5*c^3 - 29*a*b^4*c^2*d + 9*a^2*b^3*c*d^2 + a^3*b^2*d^3)*sqrt((b*x^2 + a)
*e/(d*x^2 + c))*e^3)/(b^4*d^4*e^3 - 3*(b*x^2 + a)*b^3*d^5*e^3/(d*x^2 + c) +
 3*(b*x^2 + a)^2*b^2*d^6*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*b*d^7*e^3/(d*x^2
 + c)^3) + 96*(b*c^3 - a*c^2*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/d^4 + 3*(35
*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*log((d*sqrt((b*x^2 +
 a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt
(b*d*e)))/(sqrt(b*d*e)*b*d^4))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

3.277 $\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=199

$$\frac{3e^{3/2}(bc-ad)(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{be\left(c+dx^2\right)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3}-\frac{e\left(c+dx^2\right)(9bc-5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8d^3}-\frac{ce(bc}{}$$

```
[Out] 3/8*(-a*d+b*c)*(-a*d+5*b*c)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))
^(1/2)/b^(1/2)/e^(1/2))/d^(7/2)/b^(1/2)-c*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+
c))^(1/2)/d^3-1/8*(-5*a*d+9*b*c)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
d^3+1/4*b*e*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^3
```

Rubi [A] time = 0.22, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 455, 1157, 388, 208}

$$\frac{3e^{3/2}(bc-ad)(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{be\left(c+dx^2\right)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3}-\frac{e\left(c+dx^2\right)(9bc-5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8d^3}-\frac{ce(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -((c*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^3) - ((9*b*c - 5*a*
d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*d^3) + (b*e*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^3) + (3*(b*c - a*d)*(5*b*c -
a*d)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*S
qrt[e])])/(8*Sqrt[b]*d^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx=((bc-ad)e)\,\text{Subst}\left(\int\frac{x^4\left(-ae+cx^2\right)}{\left(be-dx^2\right)^3}\,dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$=\frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^3}+\frac{((bc-ad)e)\,\text{Subst}\left(\int\frac{-b(bc-ad)e^2-4d(bc-ad)ex^2-4cd^2x^4}{\left(be-dx^2\right)^2}\,dx,x,\right.}{4d^3}$$

$$=-\frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8d^3}+\frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^3}-\frac{(bc-ad)\,\text{Subst}\left(\int\right.}{}$$

$$=-\frac{c(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3}-\frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8d^3}+\frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right.}{4d^3}$$

$$=-\frac{c(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3}-\frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8d^3}+\frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right.}{4d^3}$$

Mathematica [A] time = 0.62, size = 191, normalized size = 0.96

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(3\sqrt{bc-ad}\left(a^2d^2-6abcd+5b^2c^2\right)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)+b\sqrt{d}\sqrt{a+bx^2}\left(ad\left(13c+5dx\right.\right.\right.}{8bd^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*Sqrt[a + b*x^2]*(a*d*(13*c
+ 5*d*x^2) + b*(-15*c^2 - 5*c*d*x^2 + 2*d^2*x^4)) + 3*Sqrt[b*c - a*d]*(5*b^
2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqr
t[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(8*b*d^(7/2)*Sqrt[a + b*x^2])
```

fricas [A] time = 0.98, size = 417, normalized size = 2.10

$$\left[\frac{3\left(5b^2c^2-6abcd+a^2d^2\right)e\sqrt{\frac{e}{bd}}\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e+4\left(2b^2d^3x^4+b\right.\right.}{32}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(e/(b*d))*log(8*b^2*d^2*e*
x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(
2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(2*b*d^2*e*x^4 - 5*(b*c*d -
a*d^2)*e*x^2 - (15*b*c^2 - 13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
d^3, -1/16*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(-e/(b*d))*arctan(1/2
*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(
b*e*x^2 + a*e)) - 2*(2*b*d^2*e*x^4 - 5*(b*c*d - a*d^2)*e*x^2 - (15*b*c^2 -
13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.76Unable to divide, perhaps due to rounding error%
%%{%%%{2,[0,4,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,3,0]%%%},0]:[1,0,%%%{-
1,[1,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{2,[1,3,1]%%%},[0,0,0,2]%%%} / %%%{%
%%{1,[0,2,2]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-2,[0,1,2]%%%},0]:[1,0,%%%{-1,[1
,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,0,0,2]%%%} Error: Bad
Argument Value
```

maple [B] time = 0.06, size = 679, normalized size = 3.41

$$\left(3a^2d^3x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-18abc\,d^2x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+15b^2c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/16*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b*d^2+3*ln(1/2*
(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)
^(1/2))*x^2*a^2*d^3-18*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a*b*c*d^2+15*ln(1/2*(2*b*d*x^2+a*
d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*b
^2*c^2*d+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*a*d^2-10*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b*c*d+3*ln(1/2*(2*b*d*x^2
+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^
```

```
2*c*d^2-18*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
(b*d)^(1/2))/(b*d)^(1/2))*a*b*c^2*d+15*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*b^2*c^3+16*((d*x^2+c)
*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a*c*d-16*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1
/2)*b*c^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*c*d-14*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*c^2)/d^3*(d*x^2+c)*((b*x^2+a)/
(d*x^2+c)*e)^(3/2)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*x^2+a)
```

maxima [A] time = 2.29, size = 303, normalized size = 1.52

$$\frac{1}{16}e\left(\frac{2\left(\left(9\,b^2c^2d-14\,abcd^2+5\,a^2d^3\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}e-\left(7\,b^3c^2-10\,ab^2cd+3\,a^2bd^2\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,e^2\right)}{b^2d^3e^2-\frac{2(bx^2+a)bd^4e^2}{dx^2+c}+\frac{(bx^2+a)^2d^5e^2}{(dx^2+c)^2}}-\frac{3\left(5\,b^2c^2-6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*((9*b^2*c^2*d - 14*a*b*c*d^2 + 5*a^2*d^3)*((b*x^2 + a)*e/(d*x^2 +
 c))^(3/2)*e - (7*b^3*c^2 - 10*a*b^2*c*d + 3*a^2*b*d^2)*sqrt((b*x^2 + a)*e/
(d*x^2 + c))*e^2)/(b^2*d^3*e^2 - 2*(b*x^2 + a)*b*d^4*e^2/(d*x^2 + c) + (b*x
^2 + a)^2*d^5*e^2/(d*x^2 + c)^2) - 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*lo
g((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(
d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*d^3) - 16*(b*c^2 - a*c*d)*sqrt((b*
x^2 + a)*e/(d*x^2 + c))/d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

3.278 $\int x \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=141

$$-\frac{3\sqrt{b}\,e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2}+\frac{(c+dx^2)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2d}$$

```
[Out] 1/2*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)/d-3/2*(-a*d+b*c)*e^(3/2)*arctan
h(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))*b^(1/2)/d^(5/2)+3/
2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2
```

Rubi [A] time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{1960, 288, 321, 208\}$

$$-\frac{3\sqrt{b}\,e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2}+\frac{(c+dx^2)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*d^2) + (((e*(a + b*x
^2))/(c + d*x^2))^(3/2)*(c + d*x^2))/(2*d) - (3*Sqrt[b]*(b*c - a*d)*e^(3/2)
*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2
*d^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4}{\left(be-dx^2\right)^2}dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)}{2d} - \frac{(3(bc-ad)e)\,\text{Subst}\left(\int \frac{x^2}{be-dx^2}dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2d}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2d^2} + \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)}{2d} - \frac{\left(3b(bc-ad)e^2\right)\text{Subst}\left(\int \frac{1}{be-dx^2}dx\right.}{2d^2}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2d^2} + \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}\left(c+dx^2\right)}{2d} - \frac{3\sqrt{b}\,(bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a}{c+}}}{\sqrt{b}\sqrt{}}\right.}{2d^{5/2}}$$

Mathematica [C] time = 0.06, size = 96, normalized size = 0.68

$$\frac{e\left(a+bx^2\right)^2\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{d\left(bx^2+a\right)}{ad-bc}\right)}{5bc-5ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*(a + b*x^2)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*
c - a*d)]*Hypergeometric2F1[3/2, 5/2, 7/2, (d*(a + b*x^2))/(-(b*c) + a*d)])
/(5*b*c - 5*a*d)
```

fricas [A] time = 0.73, size = 328, normalized size = 2.33

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{be}{d}}\,e\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e+4\left(2\,bd^3x^4+bc^2d+acd^2\right.\right.}{8\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b*c - a*d)*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d
^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d +
a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c))) - 4*(b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/d^2, 1/4*(3*(b*c - a*d)*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d
)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 2*(
b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.67Unable to divide, perhaps due to rounding error%
%%{%%%{2,[0,3,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,2,0]%%%},0]:[1,0,%%%{-
1,[1,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{2,[1,2,1]%%%},[0,0,0,2]%%%} / %%%{%
%%{1,[0,2,2]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-2,[0,1,2]%%%},0]:[1,0,%%%{-1,[1
,1,1]%%%}]%%},[1,0,0,1]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,0,0,2]%%%} Error: Bad
Argument Value
```

maple [B] time = 0.05, size = 432, normalized size = 3.06

$$-\frac{\left(-3ab\,d^{2}x^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3b^{2}cd\,x^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-3abcd\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -1/4*(-3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b
*d)^(1/2))/(b*d)^(1/2))*x^2*a*b*d^2+3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*b^2*c*d-2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*d*x^2-3*a*b*c*d*ln(1/2*(2*b*d*x^2
+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+3*
b^2*c^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2))/(b*d)^(1/2))+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a*d-4*((d*
x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*b*c-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(b*d)^(1/2)*b*c)/d^2*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(b*d)^(1/2)
/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*x^2+a)
```

maxima [A] time = 2.21, size = 189, normalized size = 1.34

$$\frac{1}{4}\left(\frac{2\left(b^{2}c-abd\right)\sqrt{\frac{(bx^{2}+a)e}{dx^{2}+c}}\,e}{bd^{2}e-\frac{(bx^{2}+a)d^{3}e}{dx^{2}+c}}+\frac{3\left(b^{2}c-abd\right)e\log\left(\frac{d\sqrt{\frac{(bx^{2}+a)e}{dx^{2}+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^{2}+a)e}{dx^{2}+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,d^{2}}+\frac{4\left(bc-ad\right)\sqrt{\frac{(bx^{2}+a)e}{dx^{2}+c}}}{d^{2}}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(b^2*c - a*b*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e/(b*d^2*e - (b*x^2
 + a)*d^3*e/(d*x^2 + c)) + 3*(b^2*c - a*b*d)*e*log((d*sqrt((b*x^2 + a)*e/(d*
x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))
/(sqrt(b*d*e)*d^2) + 4*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/d^2)*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

3.279 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x} dx$

Optimal. Leaf size=151

$$-\frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}}+\frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}-\frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

```
[Out] -a^(3/2)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1
/2))/c^(3/2)+b^(3/2)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
b^(1/2)/e^(1/2))/d^(3/2)-(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d
```

Rubi [A] time = 0.19, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 479, 522, 208}

$$-\frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}}+\frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}-\frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d)) - (a^(3/2)*e^(3/
2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/
c^(3/2) + (b^(3/2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(Sqrt[b]*Sqrt[e])])/d^(3/2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
```

```
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{-abe^2+(bc+ad)ex^2}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{cd}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(a^2e^2)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{c} + \frac{(b^2e^2)\,\text{Subst}\left(\int \frac{1}{be-\ldots}\right.}{\ldots}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}$$

Mathematica [A] time = 1.28, size = 193, normalized size = 1.28

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{d}\left(-\frac{a^{3/2}d\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{c^{3/2}\sqrt{a+bx^2}}+\frac{ad}{c}-b\right)+\frac{b\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{a+bx^2}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*((b*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2
))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a
+ b*x^2] + Sqrt[d]*(-b + (a*d)/c - (a^(3/2)*d*Sqrt[c + d*x^2]*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(c^(3/2)*Sqrt[a + b*x^2]))
))/d^(3/2)
```

fricas [A] time = 1.30, size = 1049, normalized size = 6.95

$$\left[\frac{bc\sqrt{\frac{be}{d}}\,e\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e+4\left(2bd^3x^4+bc^2d+acd^2+\left(3bcd^2+\ldots\right.\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 +
 (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (
3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + a*
```

```
d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e +
8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3
 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) - 4*
(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), -1/4*(2*b*c*sqrt(-b
*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e
)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) - a*d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b
*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt
((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/(c*d), 1/4*(2*a*d*sqrt(-a*e/c)*e*arctan(1/2*((b*c + a*d)*x^2
+ 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)
) + b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (
b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*
b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(b
*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), 1/2*(a*d*sqrt(-a*e/c)
*e*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - b*c*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2
 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a
*b*e)) - 2*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.52Error: Bad Argument Type
```

maple [**B**] time = 0.06, size = 401, normalized size = 2.66

$$\left(-\sqrt{bd}\, a^2 d^2 x^2 \ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+\sqrt{ac}\, b^2 cd\,x^2 \ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x,x)
```

```
[Out] 1/2*(ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2))/(b*d)^(1/2))*(a*c)^(1/2)*x^2*b^2*c*d-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2
+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a^2*d^2+
ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
)/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c^2-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*a*c+2*
(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*a^2*c*d+2*((d*x^2+c)*
(b*x^2+a))^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)*a*d-2*((d*x^2+c)*(b*x^2+a))^(1/2)*
(b*d)^(1/2)*(a*c)^(1/2)*b*c)/c/d*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(a
*c)^(1/2)/(b*d)^(1/2)/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

maxima [**A**] time = 2.27, size = 197, normalized size = 1.30

$$\frac{1}{2}\left(\frac{a^2 e \log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,c}-\frac{b^2 e \log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,d}-\frac{2\,(bc-ad)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{cd}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*e*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b
*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*c) - b^2*e*log((d*sqr
t((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 +
c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*d) - 2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x
^2 + c))/(c*d))*e
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x,x)
```

```
[Out] Timed out
```

3.280 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=165

$$-\frac{3\sqrt{a}\,e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2}+\frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

```
[Out] 1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/c/(a-c*(b*x^2+a)/(d*x^2+c))-3/
2*(-a*d+b*c)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/
e^(1/2))*a^(1/2)/c^(5/2)+3/2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^2
```

Rubi [A] time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 288, 321, 208}

$$-\frac{3\sqrt{a}\,e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2}+\frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c^2) + ((b*c - a*d)*
((e*(a + b*x^2))/(c + d*x^2))^(3/2))/(2*c*(a - (c*(a + b*x^2))/(c + d*x^2))
) - (3*Sqrt[a]*(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*c^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
```

```
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(3(bc-ad)e)\,\text{Subst}\left(\int \frac{x^2}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2c}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{\left(3a(bc-ad)e^2\right)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \cdots\right.}{2c^2}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{3\sqrt{a}\,(bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}$$

Mathematica [A] time = 0.09, size = 146, normalized size = 0.88

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{c}\sqrt{a+bx^2}\left(2bcx^2-a\left(c+3dx^2\right)\right)-3\sqrt{a}\,x^2\sqrt{c+dx^2}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{2c^{5/2}x^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[a + b*x^2]*(2*b*c*x^2 -
a*(c + 3*d*x^2)) - 3*Sqrt[a]*(b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(2*c^(5/2)*x^2*Sqrt[a + b*
x^2])
```

fricas [A] time = 1.36, size = 350, normalized size = 2.12

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{ae}{c}}\,ex^2\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(\left(bc^2d+acd^2\right)x^4+2\,ac^3+\left(bc^3+3\,ac^2d\right)x^2\right)\sqrt{\frac{ae}{c}}\sqrt{\frac{bex^2+ae}{dx^2+c}}\cdots}{x^4}\right.}{8\,c^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b*c - a*d)*sqrt(a*e/c)*e*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)
*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*((b*c^2*d + a*c*d^2)
*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/x^4) - 4*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c)))/(c^2*x^2), 1/4*(3*(b*c - a*d)*sqrt(-a*e/c)*e*x^2*arctan(1/2*
((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a
*b*e*x^2 + a^2*e)) + 2*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c)))/(c^2*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.55Unable to divide, perhaps due to rounding error%
%%{%%%{2,[0,1,0]%%%},[6,0,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[5,1,0]%%%}+%%%{%%%{2,[1,0,1]%%%},[4,2,0]%%%}+%%%{%%%{-4,[0,1,1]%%%},[4,1,1
]%%%}+%%%{%%{[%%%{8,[0,0,1]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[3,2,1]%%%}+
%%%{%%%{-4,[1,0,2]%%%},[2,3,1]%%%}+%%%{%%%{2,[0,1,2]%%%},[2,2,2]%%%}+%%%{%%
{[%%%{-4,[0,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,3,2]%%%}+%%%{%%%{2,[
1,0,3]%%%},[0,4,2]%%%} / %%%{%%%{1,[0,2,0]%%%},[6,0,0]%%%}+%%%{%%{[%%%{-2,[
0,1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[5,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%}
,[4,2,0]%%%}+%%%{%%%{-2,[0,2,1]%%%},[4,1,1]%%%}+%%%{%%{[%%%{4,[0,1,1]%%%},0
]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[3,2,1]%%%}+%%%{%%%{-2,[1,1,2]%%%},[2,3,1]%%%
}+%%%{%%%{1,[0,2,2]%%%},[2,2,2]%%%}+%%%{%%{[%%%{-2,[0,1,2]%%%},0]:[1,0,%%%{
-1,[1,1,1]%%%}]%%},[1,3,2]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,4,2]%%%} Error: Bad
 Argument Value
```

maple [B] time = 0.08, size = 641, normalized size = 3.88

$$-\frac{\left(-3a^2c\,d^2x^4\ln\left(\frac{a d\,x^2+b c\,x^2+2ac+2\sqrt{ac}\;\sqrt{b d\,x^4+a d\,x^2+b c\,x^2+ac}}{x^2}\right)+3ab\,c^2d\,x^4\ln\left(\frac{a d\,x^2+b c\,x^2+2ac+2\sqrt{ac}\;\sqrt{b d\,x^4+a d\,x^2+b c\,x^2+ac}}{x^2}\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^3,x)
```

```
[Out] -1/4*(-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b*d^2-3*ln((a*
d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)
*x^4*a^2*c*d^2+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2))/x^2)*x^4*a*b*c^2*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(a*c)^(1/2)*x^4*a*d^2-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^
4*b*c*d-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2))/x^2)*x^2*a^2*c^2*d+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a*b*c^3+4*((d*x^2+c)*(b*x^2+a))
^(1/2)*(a*c)^(1/2)*x^2*a*c*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^2*
b*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*d-2*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^2*a*c*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(a*c)^(1/2)*x^2*b*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)
^(1/2)*c)*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(a*c)^(1/2)/x^2/c^3/(b*x^
2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

maxima [A] time = 2.26, size = 189, normalized size = 1.15

$$\frac{1}{4}\left(\frac{2\left(abc-a^{2}d\right)\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}\,e}{ac^{2}e-\frac{\left(bx^{2}+a\right)c^{3}e}{dx^{2}+c}}+\frac{3\left(abc-a^{2}d\right)e\log\left(\frac{c\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}-\sqrt{ace}}{c\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,c^{2}}+\frac{4\,(bc-ad)\sqrt{\frac{\left(bx^{2}+a\right)e}{dx^{2}+c}}}{c^{2}}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a*b*c - a^2*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e/(a*c^2*e - (b*x^2
+ a)*c^3*e/(d*x^2 + c)) + 3*(a*b*c - a^2*d)*e*log((c*sqrt((b*x^2 + a)*e/(d*
x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))
/(sqrt(a*c*e)*c^2) + 4*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/c^2)*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{3/2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

3.281 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=256

$$-\frac{3e^{3/2}(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}\,c^{7/2}}-\frac{ae^3(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}+\frac{e^2(5bc-9ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-de(b$$

```
[Out] -3/8*(-5*a*d+b*c)*(-a*d+b*c)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)/a^(1/2)/e^(1/2))/c^(7/2)/a^(1/2)-d*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2
+c))^(1/2)/c^3-1/4*a*(-a*d+b*c)^2*e^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3/(a*
e-c*e*(b*x^2+a)/(d*x^2+c))^2+1/8*(-9*a*d+5*b*c)*(-a*d+b*c)*e^2*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/c^3/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] time = 0.22, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 455, 1157, 388, 208}

$$-\frac{ae^3(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}+\frac{e^2(5bc-9ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{3e^{3/2}(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}\,c^{7/2}}-de(b$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] -((d*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^3) - (a*(b*c - a*d)
^2*e^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^3*(a*e - (c*e*(a + b*x^2))/(
c + d*x^2))^2) + ((5*b*c - 9*a*d)*(b*c - a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c +
 d*x^2)])/(8*c^3*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - 5*a*d)*
(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sq
rt[a]*Sqrt[e])])/(8*Sqrt[a]*c^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4\left(be-dx^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{((bc-ad)e)\,\text{Subst}\left(\int \frac{-a(bc-ad)e^2-4c(bc-ad)ex^2+4c^2dx^4}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e(a\cdots}{c+\cdots}}\right.}{4c^3}$$

$$= -\frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-a(3bc\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{d(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \cdots$$

$$= -\frac{d(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \cdots$$

Mathematica [A] time = 0.13, size = 186, normalized size = 0.73

$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(3x^4\sqrt{c+dx^2}\left(5a^2d^2-6abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\left(a\left(2c^2-5cdx^2-1\cdots\right.\right.\right.}{8\sqrt{a}\,c^{7/2}x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] -1/8*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*
(b*c*x^2*(5*c + 13*d*x^2) + a*(2*c^2 - 5*c*d*x^2 - 15*d^2*x^4)) + 3*(b^2*c^
2 - 6*a*b*c*d + 5*a^2*d^2)*x^4*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*
x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(Sqrt[a]*c^(7/2)*x^4*Sqrt[a + b*x^2])
```

fricas [A] time = 3.45, size = 435, normalized size = 1.70

$$\Bigg[\frac{3\left(b^2c^2-6\,abcd+5\,a^2d^2\right)ex^4\sqrt{\frac{e}{ac}}\,\log\Bigg(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2-4\left(2\,a^2c^3+\left(abc^2d+a^2cd^2\right)x^4+\left(abc^3+3\,a^2c^2d\right)\cdots\right.}{x^4}\cdots}{32\,c^3x^4}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*sqrt(e/(a*c))*log(((b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2
- 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) - 4*((13*b*c*d - 15*
a*d^2)*e*x^4 + 2*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d
*x^2 + c)))/(c^3*x^4), 1/16*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*sqrt
(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2
 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e)) - 2*((13*b*c*d - 15*a*d^2)*e*x^4 + 2
*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c^3
*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.56Unable to divide, perhaps due to rounding error%
%%{%%%{2,[4,1,4]%%%},[2,7,0]%%%}+%%%{%%%{-8,[3,2,4]%%%},[2,6,1]%%%}+%%%{%%%
{12,[2,3,4]%%%},[2,5,2]%%%}+%%%{%%%{-8,[1,4,4]%%%},[2,4,3]%%%}+%%%{%%%{2,[0
,5,4]%%%},[2,3,4]%%%}+%%%{%%{[%%%{-4,[4,0,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}
]%%},[1,8,0]%%%}+%%%{%%{[%%%{16,[3,1,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[1,7,1]%%%}+%%%{%%{[%%%{-24,[2,2,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,6
,2]%%%}+%%%{%%{[%%%{16,[1,3,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,5,3]%%
%}+%%%{%%{[%%%{-4,[0,4,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,4,4]%%%}+%%
%{%%%{2,[5,0,5]%%%},[0,9,0]%%%}+%%%{%%%{-8,[4,1,5]%%%},[0,8,1]%%%}+%%%{%%%{
12,[3,2,5]%%%},[0,7,2]%%%}+%%%{%%%{-8,[2,3,5]%%%},[0,6,3]%%%}+%%%{%%%{2,[1,
4,5]%%%},[0,5,4]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[0,
1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%},[
0,2,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.07, size = 1042, normalized size = 4.07

$$\left(-15a^3c\,d^3x^6\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+18a^2b\,c^2d^2x^6\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+}}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^5,x)
```

```
[Out] 1/16*(-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a*b*d^3+6*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*b^2*c*d^2-15*ln((a*d*x^2+b
*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^
3*c*d^3+18*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2))/x^2)*x^6*a^2*b*c^2*d^2-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a*b^2*c^3*d-18*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*d^3-26*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a*b*c*d^2+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(a*c)^(1/2)*x^6*b^2*c^2*d-15*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^3*c^2*d^2+18*ln((a*d*x^2+b*c*x
^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^2*b*
c^3*d-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2))/x^2)*x^4*a*b^2*c^4+16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^4
*a^2*c*d^2-16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^4*a*b*c^2*d+18*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*a*d^2-6*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*b*c*d-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(a*c)^(1/2)*x^4*a^2*c*d^2-8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1
/2)*x^4*a*b*c^2*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*b^2
*c^3+14*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a*c*d-6*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*b*c^2-4*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(3/2)*(a*c)^(1/2)*a*c^2)/a*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/
(a*c)^(1/2)/x^4/c^4/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

maxima [A] time = 2.48, size = 303, normalized size = 1.18

$$-\frac{1}{16}e\left(\frac{2\left(\left(5\,b^2c^3-14\,abc^2d+9\,a^2cd^2\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}e-\left(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,e^2\right)}{a^2c^3e^2-\frac{2\,(bx^2+a)ac^4e^2}{dx^2+c}+\frac{(bx^2+a)^2c^5e^2}{(dx^2+c)^2}}-\frac{3\left(b^2c^2-6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*e*(2*((5*b^2*c^3 - 14*a*b*c^2*d + 9*a^2*c*d^2)*((b*x^2 + a)*e/(d*x^2
 + c))^(3/2)*e - (3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*sqrt((b*x^2 + a)*e
/(d*x^2 + c))*e^2)/(a^2*c^3*e^2 - 2*(b*x^2 + a)*a*c^4*e^2/(d*x^2 + c) + (b*
x^2 + a)^2*c^5*e^2/(d*x^2 + c)^2) - 3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*l
og((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/
(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*c^3) + 16*(b*c*d - a*d^2)*sqrt((b
*x^2 + a)*e/(d*x^2 + c))/c^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

3.282 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=366

$$-\frac{e^2\left(-79a^2d^2+50abcd+5b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}+\frac{e^{3/2}\left(-35a^2d^2+10abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(\ldots}{\ldots}}}{\sqrt{a}\ldots}\right)}{16a^{3/2}c^{9/2}}$$

```
[Out] 1/6*(-a*d+b*c)^3*e^2*(e*(b*x^2+a)/(d*x^2+c))^(5/2)/a/c^2/(a*e-c*e*(b*x^2+a)
/(d*x^2+c))^3+1/16*(-a*d+b*c)*(-35*a^2*d^2+10*a*b*c*d+b^2*c^2)*e^(3/2)*arct
anh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^(3/2)/c^(9/2)+
d^2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^4+1/24*(-a*d+b*c)^2*(11*a*
d+b*c)*e^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^4/(a*e-c*e*(b*x^2+a)/(d*x^2+c))^
2-1/48*(-a*d+b*c)*(-79*a^2*d^2+50*a*b*c*d+5*b^2*c^2)*e^2*(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/a/c^4/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] time = 0.37, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1960, 463, 455, 1157, 388, 208}

$$-\frac{e^2\left(-79a^2d^2+50abcd+5b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}+\frac{e^{3/2}\left(-35a^2d^2+10abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(\ldots}{\ldots}}}{\sqrt{a}\ldots}\right)}{16a^{3/2}c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (d^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^4 + ((b*c - a*d)^3*
e^2*((e*(a + b*x^2))/(c + d*x^2))^(5/2))/(6*a*c^2*(a*e - (c*e*(a + b*x^2))/
(c + d*x^2))^3) + ((b*c - a*d)^2*(b*c + 11*a*d)*e^3*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(24*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)
*(5*b^2*c^2 + 50*a*b*c*d - 79*a^2*d^2)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)
])/(48*a*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b^2*c^2
 + 10*a*b*c*d - 35*a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(3/2)*c^(9/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{x^4\left(be-dx^2\right)^2}{\left(-ae+cx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^4\left(-6b^2c^2e^2+5(bce-ade)^2+6acd^2ex^2\right)}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e(a+b}{c+d}}\right.}{6ac^2}$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{ac(bc-}{}\right.}{}$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(5b^2c^2+50abc\right.}{48ac^4\left(ae-\right.}$$

$$= \frac{d^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} -$$

$$= \frac{d^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} -$$

Mathematica [A] time = 0.19, size = 245, normalized size = 0.67

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(3x^6\sqrt{c+dx^2}\left(35a^3d^3-45a^2bcd^2+9ab^2c^2d+b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)-\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\left(a^2\left(8\right.\right.\right.}{48a^{3/2}c^{9/2}x^6\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(3*
b^2*c^2*x^4*(c + d*x^2) + 2*a*b*c*x^2*(7*c^2 - 19*c*d*x^2 - 50*d^2*x^4) + a
^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6))) + 3*(b^3*c^3 + 9*a
*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*x^6*Sqrt[c + d*x^2]*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(48*a^(3/2)*c^(9/2)*x^6*S
qrt[a + b*x^2])
```

fricas [A] time = 14.42, size = 573, normalized size = 1.57

$$\left[\frac{3\left(b^3c^3+9\,ab^2c^2d-45\,a^2bcd^2+35\,a^3d^3\right)ex^6\sqrt{\frac{e}{ac}}\,\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(2\,a^2c^3+\left(abc^2d+a\right.\right.}{x^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqr
t(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*
b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*
c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^
4) - 4*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 + 8*a^2*c^3*e + (
3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3 - a^2*c^2*d)*e
*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6), -1/96*(3*(b^3*c^3 + 9
*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqrt(-e/(a*c))*arctan(1/2
*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))
/(b*e*x^2 + a*e)) + 2*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 +
8*a^2*c^3*e + (3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3
 - a^2*c^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.57Unable to divide, perhaps due to rounding error%
%%{%%%{2,[5,1,5]%%%},[2,9,0]%%%}+%%%{%%%{-10,[4,2,5]%%%},[2,8,1]%%%}+%%%{%%
%{20,[3,3,5]%%%},[2,7,2]%%%}+%%%{%%%{-20,[2,4,5]%%%},[2,6,3]%%%}+%%%{%%%{10
,[1,5,5]%%%},[2,5,4]%%%}+%%%{%%%{-2,[0,6,5]%%%},[2,4,5]%%%}+%%%{%%{[%%%{-4,
[5,0,5]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,10,0]%%%}+%%%{%%{[%%%{20,[4,1
,5]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,9,1]%%%}+%%%{%%{[%%%{-40,[3,2,5]%
%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,8,2]%%%}+%%%{%%{[%%%{40,[2,3,5]%%%},0
]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,7,3]%%%}+%%%{%%{[%%%{-20,[1,4,5]%%%},0]:[1
,0,%%%{-1,[1,1,1]%%%}]%%},[1,6,4]%%%}+%%%{%%{[%%%{4,[0,5,5]%%%},0]:[1,0,%%%
{-1,[1,1,1]%%%}]%%},[1,5,5]%%%}+%%%{%%%{2,[6,0,6]%%%},[0,11,0]%%%}+%%%{%%%{
-10,[5,1,6]%%%},[0,10,1]%%%}+%%%{%%%{20,[4,2,6]%%%},[0,9,2]%%%}+%%%{%%%{-20
,[3,3,6]%%%},[0,8,3]%%%}+%%%{%%%{10,[2,4,6]%%%},[0,7,4]%%%}+%%%{%%%{-2,[1,5
,6]%%%},[0,6,5]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[0,1
,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%},[0
,2,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.10, size = 1498, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^7,x)
```

```
[Out] -1/96*(-174*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^10*a^2*b*d^4+
72*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^10*a*b^2*c*d^3-216*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a^2*b*c*d^3+138*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a*b^2*c^2*d^2-72*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^6*a*b*c*d^2-42*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*b*c^2*d^2+66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*(a*c)^(1/2)*x^6*a*b^2*c^3*d-96*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*
x^6*a^2*b*c^2*d^2-60*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*a*
b*c^2*d-105*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2))/x^2)*x^6*a^4*c^2*d^3-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2
```

```
)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a*b^3*c^5+174*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^6*a^2*d^3+6*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(a*c)^(1/2)*x^6*b^3*c^4-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*
c)^(1/2)*x^4*b^2*c^3-105*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^8*a^4*c*d^4-174*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(a*c)^(1/2)*x^8*a^3*d^4+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a
*c)^(1/2)*x^10*b^3*c^2*d^2+135*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^8*a^3*b*c^2*d^3-27*ln((a*d*x^2+b*c*
x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^8*a^2*b
^2*c^3*d^2-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2))/x^2)*x^8*a*b^3*c^4*d+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
(a*c)^(1/2)*x^8*b^3*c^3*d+135*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^3*b*c^3*d^2-27*ln((a*d*x^2+b*c*x
^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^2*b^
2*c^4*d-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^6*b^2*c^2*d-174
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^3*c*d^3+96*(a*c)^(1/
2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^6*a^3*c*d^3+114*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(3/2)*(a*c)^(1/2)*x^4*a^2*c*d^2-44*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*
(a*c)^(1/2)*x^2*a^2*c^2*d+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2
)*x^2*a*b*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c^3)/a
^2*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(a*c)^(1/2)/x^6/c^5/(b*x^2+a)/((
d*x^2+c)*(b*x^2+a))^(1/2)
```

maxima [A] time = 3.04, size = 453, normalized size = 1.24

$$\frac{1}{96}e\left(\frac{2\left(3\left(b^3c^5-23\,ab^2c^4d+51\,a^2bc^3d^2-29\,a^3c^2d^3\right)\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{5}{2}}e+8\left(ab^3c^4+9\,a^2b^2c^3d-27\,a^3bc^2d^2+17\,a^4\right.\right.}{a^4c^4e^3-\frac{3\left(bx^2+a\right)a^3c^5e^3}{dx^2+c}+\frac{3\left(bx^2+a\right)^2a^2c^6e^3}{\left(dx^2+c\right)^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/96*e*(2*(3*(b^3*c^5 - 23*a*b^2*c^4*d + 51*a^2*b*c^3*d^2 - 29*a^3*c^2*d^3)
*((b*x^2 + a)*e/(d*x^2 + c))^(5/2)*e + 8*(a*b^3*c^4 + 9*a^2*b^2*c^3*d - 27*
a^3*b*c^2*d^2 + 17*a^4*c*d^3)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^2 - 3*(a^
2*b^3*c^3 + 9*a^3*b^2*c^2*d - 29*a^4*b*c*d^2 + 19*a^5*d^3)*sqrt((b*x^2 + a)
*e/(d*x^2 + c))*e^3)/(a^4*c^4*e^3 - 3*(b*x^2 + a)*a^3*c^5*e^3/(d*x^2 + c) +
 3*(b*x^2 + a)^2*a^2*c^6*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*a*c^7*e^3/(d*x^2
 + c)^3) + 96*(b*c*d^2 - a*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/c^4 - 3*(b^
3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*log((c*sqrt((b*x^2 +
 a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt
(a*c*e)))/(sqrt(a*c*e)*a*c^4))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**7,x)
```

```
[Out] Timed out
```

3.283 $\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=391

$$-\frac{\sqrt{c}\,e\left(a^2d^2-16abcd+16b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ex\left(-\frac{a^2d}{b}+16ac-\frac{16bc^2}{d}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2}+\frac{c^{3/2}e(}{}$$

```
[Out] -1/5*(16*a*c-16*b*c^2/d-a^2*d/b)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2-e*x^
3*(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d-1/5*(-7*a*d+8*b*c)*e*x*(d*x^2+c
)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^3+6/5*b*e*x^3*(d*x^2+c)*(e*(b*x^2+a)/(d*x
^2+c))^(1/2)/d^2+1/5*c^(3/2)*(-7*a*d+8*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/5*
(a^2*d^2-16*a*b*c*d+16*b^2*c^2)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6719, 467, 581, 582, 531, 418, 492, 411}

$$-\frac{\sqrt{c}\,e\left(a^2d^2-16abcd+16b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ex\left(-\frac{a^2d}{b}+16ac-\frac{16bc^2}{d}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2}+\frac{c^{3/2}e(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -((16*a*c - (16*b*c^2)/d - (a^2*d)/b)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]
)/(5*d^2) - (e*x^3*(a + b*x^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d - ((8*b
*c - 7*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d^3) + (6
*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d^2) - (Sqrt[c]*
(16*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b*d^(7/2)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*(8*b*c - 7*a*d)*e*Sqrt[(e*(a + b*
x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(5*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int x^4\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx=\frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^4\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$=-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}+\frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2\sqrt{a+bx^2}\left(3a+6bx^2\right)}{\sqrt{c+dx^2}}dx}{d\sqrt{a+bx^2}}$$

$$=-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}+\frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5d^2}+\frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int}{5d^2}$$

$$=-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}-\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5d^3}+\frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2}$$

$$=-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}-\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{5d^3}+\frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2}$$

$$=\frac{\left(16b^2c^2-16abcd+a^2d^2\right)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5bd^3}-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}-(8bc-7ad)ex$$

$$=\frac{\left(16b^2c^2-16abcd+a^2d^2\right)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5bd^3}-\frac{ex^3\left(a+bx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}-(8bc-7ad)ex$$

Mathematica [C] time = 0.60, size = 290, normalized size = 0.74

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(8ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^2d^2-3abcd+2b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^2d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a^2*d*(7*c + 2*d*x^2)
+ b^2*x^2*(-8*c^2 - 2*c*d*x^2 + d^2*x^4) + a*b*(-8*c^2 + 5*c*d*x^2 + 3*d^2*
x^4)) - I*c*(16*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (8*I)*c*(2*b^
2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(5*Sqrt[b/a]*d^4*(a + b*x^2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bex^6+aex^4\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*e*x^6 + a*e*x^4)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^2 + c),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4, x)
```

maple [B] time = 0.05, size = 933, normalized size = 2.39

$$\frac{\left(\frac{(b\,x^2+a)e}{d\,x^2+c}\right)^{\frac{3}{2}} \left(d\,x^2+c\right) \left(\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,b^2 d^3 x^7 + 3\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,ab\,d^3 x^5 - 2\sqrt{\left(d\,x^2+c\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/5*((b*x^2+a)/(d*x^2+c)*e)^(3/2)*(d*x^2+c)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-
1/a*b)^(1/2)*x^7*b^2*d^3+3*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^5*a
*b*d^3-2*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^5*b^2*c*d^2+2*((d*x^2
+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^3*a^2*d^3-3*((d*x^2+c)*(b*x^2+a))^(1/
2)*(-1/a*b)^(1/2)*x^3*b^2*c^2*d+5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a
*b)^(1/2)*x^3*a*b*c*d^2-5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2
)*x^3*b^2*c^2*d-8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a^2*c*d^2+24*((d*x^2
+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/
a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*b*c^2*d-16*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1
/2))*b^2*c^3+((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a^2*c*d^2-16*((d*x^2+c)*(
b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^
(1/2)*x,(a/b/c*d)^(1/2))*a*b*c^2*d+16*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*
b^2*c^3+2*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x*a^2*c*d^2-3*((d*x^2+
c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x*a*b*c^2*d+5*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(-1/a*b)^(1/2)*x*a^2*c*d^2-5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
(-1/a*b)^(1/2)*x*a*b*c^2*d)/d^4/(b*x^2+a)^2/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(\frac{e\left(b x^2+a\right)}{d x^2+c}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

3.284 $\int x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=310

$$-\frac{\sqrt{c}\,e(4bc-3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\,e(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{4bex\left(c+\right.}{}$$

```
[Out] -1/3*(-7*a*d+8*b*c)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2-e*x*(b*x^2+a)*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)/d+4/3*b*e*x*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1
/2)/d^2+1/3*(-7*a*d+8*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4
*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)
/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {6719, 467, 528, 531, 418, 492, 411}

$$\frac{4bex\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2}-\frac{ex(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2}-\frac{\sqrt{c}\,e(4bc-3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\,e(8bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -((8*b*c - 7*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*d^2) - (e*x*(a
+ b*x^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d + (4*b*e*x*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)]*(c + d*x^2))/(3*d^2) + (Sqrt[c]*(8*b*c - 7*a*d)*e*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (Sqrt[c]*(4*b*c
- 3*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int x^2\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx = \frac{\left(e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$= -\frac{ex\left(a+bx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d} + \frac{\left(e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}\left(a+4bx^2\right)}{\sqrt{c+dx^2}}dx}{d\sqrt{a+bx^2}}$$

$$= -\frac{ex\left(a+bx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d^2} + \frac{\left(e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{-a(4b\cdots}{\cdots}}{3d^2\sqrt{a+bx\cdots}}$$

$$= -\frac{ex\left(a+bx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d^2} - \frac{\left(b(8bc-7ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+\cdots}\right.}{3d^2\sqrt{a+\cdots}}$$

$$= -\frac{(8bc-7ad)ex\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{3d^2} - \frac{ex\left(a+bx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d^2} - \frac{\sqrt{c\cdots}}{\cdots}$$

$$= -\frac{(8bc-7ad)ex\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{3d^2} - \frac{ex\left(a+bx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{3d^2} + \frac{\sqrt{c\cdots}}{\cdots}$$

Mathematica [C] time = 0.42, size = 235, normalized size = 0.76

$$-\frac{e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(i\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-11abcd+8b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(3ad-b\left(\cdots\right.\right.\right.}{3d^3\sqrt{\frac{b}{a}}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -1/3*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(3*a*d
 - b*(4*c + d*x^2)) + I*b*c*(-8*b*c + 7*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(8*b^2*c^2 - 1
1*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*d^3*(a + b*x^2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bex^4+aex^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*e*x^4 + a*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^2 + c),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2, x)
```

maple [B] time = 0.03, size = 734, normalized size = 2.37

$$\frac{\left(\frac{\left(b\,x^2+a\right)e}{d\,x^2+c}\right)^{\frac{3}{2}}\left(d\,x^2+c\right)\left(\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,b^2d^2x^5+\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^3-3\sqrt{bd\,x^4+\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/3*((b*x^2+a)/(d*x^2+c)*e)^(3/2)*(d*x^2+c)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-
1/a*b)^(1/2)*x^5*b^2*d^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^3*a*b
*d^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^3*b^2*c*d-3*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^3*a*b*d^2+3*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^3*b^2*c*d+3*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/
2))*a^2*d^2-11*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*b*c*d+8*((d*x^2+c)*(b
*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(
1/2)*x,(a/b/c*d)^(1/2))*b^2*c^2+7*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*b*
c*d-8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b^2*c^2+((d*x^2+c)*(b*x^2+a))^(1
/2)*(-1/a*b)^(1/2)*x*a*b*c*d-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)
^(1/2)*x*a^2*d^2+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x*a*b
*c*d)/(b*x^2+a)^2/d^3/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

3.285 $\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=262

$$\frac{b\sqrt{c}\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{e(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ex(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

```
[Out] -(-a*d+b*c)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d+(-a*d+2*b*c)*e*x*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/c/d-(-a*d+2*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)/d^(3/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+b
*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d
*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(3
/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6719, 413, 531, 418, 492, 411}

$$\frac{b\sqrt{c}\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{e(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ex(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -(((b*c - a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d)) + ((2*b*c - a*
d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d) - ((2*b*c - a*d)*e*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[c]*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (b*Sqrt[c
]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2} dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{abc+b(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{\left(abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{d\sqrt{a+bx^2}} + \frac{\left(b(2bc-ad)e\cdots\right.}{\cdots}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(2bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{b\sqrt{c}\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\cdots\right.}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(2bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{(2bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\cdots\right.\right.}{\sqrt{c}\,d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] time = 0.32, size = 206, normalized size = 0.79

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left((ad-bc)\left(dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)-2ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)+ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+\cdots}\right.}{cd^2\sqrt{\frac{b}{a}}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(I*b*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-
(b*c) + a*d)*(Sqrt[b/a]*d*x*(a + b*x^2) - (2*I)*b*c*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(Sqrt[b/
a]*c*d^2*(a + b*x^2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bex^2 + ae\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*e*x^2 + a*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{\left(bx^2 + a\right)e}{dx^2 + c}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

maple [A] time = 0.03, size = 527, normalized size = 2.01

$$\left(\frac{\left(b\,x^2+a\right)e}{d\,x^2+c}\right)^{\frac{3}{2}} \left(d\,x^2 + c\right) \left(\sqrt{bd\,x^4 + ad\,x^2 + bc\,x^2 + ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^3 - \sqrt{bd\,x^4 + ad\,x^2 + bc\,x^2 + ac}\,\sqrt{-\frac{b}{a}}\,b^2cd\,x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] ((b*x^2+a)/(d*x^2+c)*e)^(3/2)*(d*x^2+c)*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(-1/a*b)^(1/2)*a*b*d^2*x^3-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(
1/2)*b^2*c*d*x^3+2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*b*c*d-2*((d*x^2+c
)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*
b)^(1/2)*x,(a/b/c*d)^(1/2))*b^2*c^2-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*
b*c*d+2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b^2*c^2+(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(-1/a*b)^(1/2)*a^2*d^2*x-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
(-1/a*b)^(1/2)*a*b*c*d*x)/(b*x^2+a)^2/d^2/c/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{\left(bx^2 + a\right)e}{dx^2 + c}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

3.286 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2} dx$

Optimal. Leaf size=307

$$\frac{e(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{e\left(c+dx^2\right)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2dx}-\frac{ex(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2}-e(bc$$

```
[Out] -(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x-(-2*a*d+b*c)*e*x*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/c^2+(-2*a*d+b*c)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^
(1/2)/c^2/d/x+(-2*a*d+b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*
x^2+c))^(1/2)/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+b*e*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^(1/2)/d^(1/2)/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 468, 583, 531, 418, 492, 411}

$$\frac{e\left(c+dx^2\right)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2dx}-\frac{ex(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2}+\frac{e(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-e(bc$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x)) - ((b*c - 2*a*
d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^2 + ((b*c - 2*a*d)*e*Sqrt[(e*(a
 + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(c^2*d*x) + ((b*c - 2*a*d)*e*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(c^(3/2)*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (b*e*Sqrt[
(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
```

```
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{x^2(c+dx^2)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a(bc-2ad)-abdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{c^2dx} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a^2bc}{\sqrt{}}}{ac^2d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{c^2dx} + \frac{\left(abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int -}{c\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{(bc-2ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{c^2dx} + \frac{be}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{(bc-2ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{c^2dx} + \frac{(bc}{}$$

Mathematica [C] time = 0.36, size = 228, normalized size = 0.74

$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(d\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(ac+2adx^2-bcx^2\right)-ibcx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+\right.}{c^2dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] -((e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*(a + b*x^2)*(a*c - b*c*
x^2 + 2*a*d*x^2) + I*b*c*(-(b*c) + 2*a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a
*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)]))/(Sqrt[b/a]*c^2*d*x*(a + b*x^2)))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bex^2+ae\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^4+cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*e*x^2 + a*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^4 + c*x^2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^2, x)
```

maple [A] time = 0.03, size = 670, normalized size = 2.18

$$-\frac{\left(\frac{(b\,x^2+a)e}{d\,x^2+c}\right)^{\frac{3}{2}}\left(d\,x^2+c\right)\left(\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^4+\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^4-\sqrt{bd\,x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^2,x)
```

```
[Out] -((b*x^2+a)/(d*x^2+c)*e)^(3/2)*(d*x^2+c)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a
*b)^(1/2)*x^4*a*b*d^2+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^
4*a*b*d^2-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^4*b^2*c*d+((
d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF
((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x*a*b*c*d-((d*x^2+c)*(b*x^2+a))^(1/2)*((
b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^
(1/2))*x*b^2*c^2-2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x*a*b*c*d+((d*x^2+c
)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*
b)^(1/2)*x,(a/b/c*d)^(1/2))*x*b^2*c^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^
(1/2)*x^2*a^2*d^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^2*a*b*c*d+(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^2*a^2*d^2-(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^2*a*b*c*d+((d*x^2+c)*(b*x^2+a))^(1/2
)*(-1/a*b)^(1/2)*a^2*c*d)/(b*x^2+a)^2/c^2/x/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

3.287 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4} dx$

Optimal. Leaf size=383

$$\frac{be(3bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}e(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{e\left(c+dx^2\right)(\cdots}{\cdots}$$

```
[Out] -(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x^3+1/3*d*(-8*a*d+7*b*c)*e*
x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3+1/3*(-4*a*d+3*b*c)*e*(d*x^2+c)*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/c^2/d/x^3-1/3*(-8*a*d+7*b*c)*e*(d*x^2+c)*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/c^3/x+1/3*b*(-4*a*d+3*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x
^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2)
)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)-1/3*(-8*a*d+7*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {6719, 468, 583, 531, 418, 492, 411}

$$-\frac{e\left(c+dx^2\right)(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3x}+\frac{e\left(c+dx^2\right)(3bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^2dx^3}+\frac{dex(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3}+\frac{be(3bc-4ad)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x^3)) + (d*(7*b*c
- 8*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*c^3) + ((3*b*c - 4*a*d)*
e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c^2*d*x^3) - ((7*b*c -
8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c^3*x) - (Sqrt[d
]*(7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]) + (b*(3*b*c - 4*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellipti
cF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{x^4(c+dx^2)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a(3bc-4ad)+b(2bc-3ad)x^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a^2d(7b}{x}}{3ac^2d\sqrt{a+b}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2}{3c^3x}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2}{3c^3x}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{d(7bc-8ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3} - (7b$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{d(7bc-8ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3} - (7b$$

Mathematica [C] time = 0.47, size = 275, normalized size = 0.72

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}\left(a^2\left(c^2-4cdx^2-8d^2x^4\right)+abx^2\left(5c^2+3cdx^2-8d^2x^4\right)+b^2cx^4\left(4c+7dx^2\right)\right)-4ibcx^3\sqrt{\frac{bx^2}{a}+1}\right.}{3c^3x^3\sqrt{\frac{b}{a}}\,(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(b^2*c*x^4*(4*c + 7*d*x^2
) + a^2*(c^2 - 4*c*d*x^2 - 8*d^2*x^4) + a*b*x^2*(5*c^2 + 3*c*d*x^2 - 8*d^2*
x^4))) + I*b*c*(-7*b*c + 8*a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (4*I)*b*c*(-(b*c) + a*d)*
x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)]))/(3*Sqrt[b/a]*c^3*x^3*(a + b*x^2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bex^2+ae\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^6+cx^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*e*x^2 + a*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^6 + c*x^4),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^4, x)
```

maple [A] time = 0.04, size = 791, normalized size = 2.07

$$\frac{\left(\frac{(b\,x^2+a)e}{d\,x^2+c}\right)^{\frac{3}{2}}\left(d\,x^2+c\right)\left(5\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^6+3\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^6-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^4,x)
```

```
[Out] 1/3*((b*x^2+a)/(d*x^2+c)*e)^(3/2)*(d*x^2+c)*(5*((d*x^2+c)*(b*x^2+a))^(1/2)*
(-1/a*b)^(1/2)*x^6*a*b*d^2-4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^6
*b^2*c*d+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^6*a*b*d^2-3
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^6*b^2*c*d+4*((d*x^2+c
)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*
b)^(1/2)*x,(a/b/c*d)^(1/2))*x^3*a*b*c*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/
2))*x^3*b^2*c^2-8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^3*a*b*c*d+7*((d*x^
2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1
/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^3*b^2*c^2+5*((d*x^2+c)*(b*x^2+a))^(1/2)*(-
1/a*b)^(1/2)*x^4*a^2*d^2-4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^4*b
^2*c^2+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^4*a^2*d^2-3*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^4*a*b*c*d+4*((d*x^2+c)*
(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^2*a^2*c*d-5*((d*x^2+c)*(b*x^2+a))^(1/2)*(
-1/a*b)^(1/2)*x^2*a*b*c^2-((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*a^2*c^
2)/(b*x^2+a)^2/c^3/x^3/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

3.288 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=480

$$-\frac{\sqrt{d}\,e\left(16a^2d^2-16abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5ac^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{e\left(c+dx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)\sqrt{\frac{e(a}{c}}}{5ac^4x}$$

```
[Out] -(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x^5+1/5*d*(16*a^2*d^2-16*a*
b*c*d+b^2*c^2)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^4+1/5*(-6*a*d+5*b*c)*e
*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^2/d/x^5-1/5*(-8*a*d+7*b*c)*e*(d*
x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3/x^3-1/5*(16*a^2*d^2-16*a*b*c*d+b^2
*c^2)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^4/x-1/5*(16*a^2*d^2-16*
a*b*c*d+b^2*c^2)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/a/c^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/5*b*(-8*a*d+7*b*c)*
e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^(
5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 468, 583, 531, 418, 492, 411}

$$-\frac{e\left(c+dx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4x}+\frac{dex\left(16a^2d^2-16abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4}-\frac{\sqrt{d}\,e\left(16a^2d^2-16\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x^5)) + (d*(b^2*c^
2 - 16*a*b*c*d + 16*a^2*d^2)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(5*a*c^
4) + ((5*b*c - 6*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*c
^2*d*x^5) - ((7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2
))/(5*c^3*x^3) - ((b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*e*Sqrt[(e*(a + b*x^2)
)/(c + d*x^2)]*(c + d*x^2))/(5*a*c^4*x) - (Sqrt[d]*(b^2*c^2 - 16*a*b*c*d +
16*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*a*c^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]) - (b*Sqrt[d]*(7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*a*c^(5/2)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{x^6(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a(5bc-6ad)+b(4bc-5ad)x^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{3a^2}{}}{5ac^2d\sqrt{a}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+}{5c^3x^3}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+}{5c^3x^3}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+}{5c^3x^3}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{d\left(b^2c^2-16abcd+16a^2d^2\right)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx}{c+dx}}}{5c^2dx^5}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{d\left(b^2c^2-16abcd+16a^2d^2\right)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx}{c+dx}}}{5c^2dx^5}$$

Mathematica [C] time = 0.67, size = 357, normalized size = 0.74

$$-\frac{e\sqrt{\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-ibcx^5\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(8a^2d^2-9abcd+b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ibcx^5\sqrt{\frac{bx^2}{a}+1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] -1/5*(Sqrt[b/a]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(b^3*c^2*x^6
*(c + d*x^2) + a*b^2*c*x^4*(3*c^2 - 8*c*d*x^2 - 16*d^2*x^4) + a^2*b*x^2*(3*
c^3 - 11*c^2*d*x^2 - 8*c*d^2*x^4 + 16*d^3*x^6) + a^3*(c^3 - 2*c^2*d*x^2 + 8
*c*d^2*x^4 + 16*d^3*x^6)) + I*b*c*(b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*x^5*S
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - I*b*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c
^4*x^5*(a + b*x^2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bex^2+ae\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{dx^8+cx^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*e*x^2 + a*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(d*x^8 + c*x^6),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^6, x)
```

maple [B] time = 0.04, size = 1197, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^6,x)
```

```
[Out] -1/5*((b*x^2+a)/(d*x^2+c)*e)^(3/2)*(d*x^2+c)*(11*((d*x^2+c)*(b*x^2+a))^(1/2
)*(-1/a*b)^(1/2)*x^8*a^2*b*d^3-11*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2
)*x^8*a*b^2*c*d^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^8*b^3*c^2*d+
5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^8*a^2*b*d^3-5*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^8*a*b^2*c*d^2+8*((d*x^2+c)*(
b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^
(1/2)*x,(a/b/c*d)^(1/2))*x^5*a^2*b*c*d^2-9*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1
/2))*x^5*a*b^2*c^2*d+((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^5*b^3*c^3-16*((
d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE
((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^5*a^2*b*c*d^2+16*((d*x^2+c)*(b*x^2+a))
^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(
a/b/c*d)^(1/2))*x^5*a*b^2*c^2*d-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^5*b^
3*c^3+11*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^6*a^3*d^3-3*((d*x^2+c
)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^6*a^2*b*c*d^2-8*((d*x^2+c)*(b*x^2+a))^(
1/2)*(-1/a*b)^(1/2)*x^6*a*b^2*c^2*d+((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1
/2)*x^6*b^3*c^3+5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^6*a^
3*d^3-5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^6*a^2*b*c*d^2+
8*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^4*a^3*c*d^2-11*((d*x^2+c)*(b
*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^4*a^2*b*c^2*d+3*((d*x^2+c)*(b*x^2+a))^(1/2)
*(-1/a*b)^(1/2)*x^4*a*b^2*c^3-2*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*
x^2*a^3*c^2*d+3*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^2*a^2*b*c^3+((
d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*a^3*c^3)/a/(b*x^2+a)^2/c^4/x^5/(-1
/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**6,x)
[Out] Timed out
```

3.289 $\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{1}{2}\sqrt{\frac{1-x^2}{x^2+1}}\left(x^2+1\right)-\tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)$$

```
[Out] -arctan(((-x^2+1)/(x^2+1))^(1/2))+1/2*(x^2+1)*((-x^2+1)/(x^2+1))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1960, 288, 204}

$$\frac{1}{2}\sqrt{\frac{1-x^2}{x^2+1}}\left(x^2+1\right)-\tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(1 + x^2))/2 - ArcTan[Sqrt[(1 - x^2)/(1 + x^2)]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx,x,\sqrt{\frac{1-x^2}{1+x^2}}\right)\right)\\
&= \frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}\left(1+x^2\right)+\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{\frac{1-x^2}{1+x^2}}\right)\\
&= \frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}\left(1+x^2\right)-\tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 86, normalized size = 1.69

$$\frac{\sqrt{\frac{1-x^2}{x^2+1}}\sqrt{x^2+1}\left(\sqrt{x^2+1}\left(x^2-1\right)+2\sqrt{1-x^2}\sin^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)\right)}{2\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*Sqrt[1 + x^2]*((-1 + x^2)*Sqrt[1 + x^2] + 2*Sqrt
[1 - x^2]*ArcSin[Sqrt[1 - x^2]/Sqrt[2]]))/(2*(-1 + x^2))
```

fricas [A] time = 0.42, size = 55, normalized size = 1.08

$$\frac{1}{2}\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}-\arctan\left(\frac{\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*sqrt(-(x^2 - 1)/(x^2 + 1)) - arctan(((x^2 + 1)*sqrt(-(x^2 - 1
)/(x^2 + 1)) - 1)/x^2)
```

giac [A] time = 0.28, size = 18, normalized size = 0.35

$$\frac{1}{2}\sqrt{-x^4+1}+\frac{1}{2}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^4 + 1) + 1/2*arcsin(x^2)
```

maple [A] time = 0.02, size = 52, normalized size = 1.02

$$\frac{\sqrt{-\frac{x^2-1}{x^2+1}}\left(x^2+1\right)\left(\arcsin\left(x^2\right)+\sqrt{-x^4+1}\right)}{2\sqrt{-\left(x^2-1\right)\left(x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-x^2+1)/(x^2+1))^(1/2),x)
```

```
[Out] 1/2*(-(x^2-1)/(x^2+1))^(1/2)*(x^2+1)*((-x^4+1)^(1/2)+arcsin(x^2))/(-(x^2-1)
*(x^2+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-\frac{x^2-1}{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)), x)
```

mupad [B] time = 2.67, size = 55, normalized size = 1.08

$$-\mathrm{atan}\left(\sqrt{-\frac{x^2-1}{x^2+1}}\right)-\frac{\sqrt{-\frac{x^2-1}{x^2+1}}}{\frac{x^2-1}{x^2+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(x^2 - 1)/(x^2 + 1))^(1/2),x)
```

```
[Out] - atan((-(x^2 - 1)/(x^2 + 1))^(1/2)) - (-(x^2 - 1)/(x^2 + 1))^(1/2)/((x^2 -
 1)/(x^2 + 1) - 1)
```

sympy [A] time = 21.54, size = 39, normalized size = 0.76

$$\begin{cases} \frac{\sqrt{1-x^2}\sqrt{x^2+1}}{2}-\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{1-x^2}}{2}\right) & \text{for } x>-1 \wedge x<1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x**2+1)/(x**2+1))**(1/2),x)
```

```
[Out] Piecewise((sqrt(1 - x**2)*sqrt(x**2 + 1)/2 - asin(sqrt(2)*sqrt(1 - x**2)/2)
, (x > -1) & (x < 1)))
```

3.290 $\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{10}\sqrt{\frac{5-7x^2}{5x^2+7}}\left(5x^2+7\right)-\frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{5\sqrt{35}}$$

```
[Out] -37/175*arctan(1/7*35^(1/2)*((-7*x^2+5)/(5*x^2+7))^(1/2))*35^(1/2)+1/10*(5*
x^2+7)*((-7*x^2+5)/(5*x^2+7))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1960, 288, 204}

$$\frac{1}{10}\sqrt{\frac{5-7x^2}{5x^2+7}}\left(5x^2+7\right)-\frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{5\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(7 + 5*x^2))/10 - (37*ArcTan[Sqrt[5/7]*Sqrt[
(5 - 7*x^2)/(7 + 5*x^2)]])/(5*Sqrt[35])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = -\left(74\,\text{Subst}\left(\int \frac{x^2}{\left(-7-5x^2\right)^2}\,dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}}\right)\right)$$
$$= \frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\left(7+5x^2\right) + \frac{37}{5}\,\text{Subst}\left(\int \frac{1}{-7-5x^2}\,dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}}\right)$$
$$= \frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\left(7+5x^2\right) - \frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{7+5x^2}}\right)}{5\sqrt{35}}$$

Mathematica [A] time = 0.05, size = 104, normalized size = 1.44

$$\frac{\sqrt{\frac{5-7x^2}{5x^2+7}}\sqrt{5x^2+7}\left(35\sqrt{5x^2+7}\left(7x^2-5\right)-74\sqrt{35}\sqrt{7x^2-5}\sinh^{-1}\left(\sqrt{\frac{5}{74}}\sqrt{7x^2-5}\right)\right)}{350\left(7x^2-5\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*Sqrt[7 + 5*x^2]*(35*Sqrt[7 + 5*x^2]*(-5 + 7*
x^2) - 74*Sqrt[35]*Sqrt[-5 + 7*x^2]*ArcSinh[Sqrt[5/74]*Sqrt[-5 + 7*x^2]]))/
(350*(-5 + 7*x^2))
```

fricas [A] time = 0.43, size = 77, normalized size = 1.07

$$\frac{1}{10}\left(5x^2+7\right)\sqrt{-\frac{7x^2-5}{5x^2+7}} - \frac{37}{350}\sqrt{35}\arctan\left(\frac{\sqrt{35}\left(35x^2+12\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}}{35\left(7x^2-5\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(5*x^2 + 7)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7)) - 37/350*sqrt(35)*arctan(1/
35*sqrt(35)*(35*x^2 + 12)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(7*x^2 - 5))
```

giac [A] time = 0.35, size = 30, normalized size = 0.42

$$\frac{37}{350}\sqrt{35}\arcsin\left(\frac{35}{37}x^2+\frac{12}{37}\right)+\frac{1}{10}\sqrt{-35x^4-24x^2+35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="giac")
```

```
[Out] 37/350*sqrt(35)*arcsin(35/37*x^2 + 12/37) + 1/10*sqrt(-35*x^4 - 24*x^2 + 35
)
```

maple [A] time = 0.03, size = 78, normalized size = 1.08

$$\frac{\sqrt{-\frac{7x^2-5}{5x^2+7}}\left(5x^2+7\right)\left(37\sqrt{35}\arcsin\left(\frac{35x^2}{37}+\frac{12}{37}\right)+35\sqrt{-35x^4-24x^2+35}\right)}{350\sqrt{-\left(7x^2-5\right)\left(5x^2+7\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x)
```

```
[Out] 1/350*(-(7*x^2-5)/(5*x^2+7))^(1/2)*(5*x^2+7)*(37*35^(1/2)*arcsin(35/37*x^2+
12/37)+35*(-35*x^4-24*x^2+35)^(1/2))/(-(7*x^2-5)*(5*x^2+7))^(1/2)
```

maxima [A] time = 2.05, size = 76, normalized size = 1.06

$$-\frac{37}{175}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{-\frac{7x^2-5}{5x^2+7}}\right)-\frac{37\sqrt{-\frac{7x^2-5}{5x^2+7}}}{5\left(\frac{5(7x^2-5)}{5x^2+7}-7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="maxima")
```

```
[Out] -37/175*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))) - 37/5
*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(5*(7*x^2 - 5)/(5*x^2 + 7) - 7)
```

mupad [B] time = 0.21, size = 88, normalized size = 1.22

$$-\frac{37\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\sqrt{7}\sqrt{-\frac{7x^2-5}{5x^2+7}}}{7}\right)}{175}-\frac{37\sqrt{5}\sqrt{7}\sqrt{35}\sqrt{-\frac{7x^2-5}{5x^2+7}}}{1225\left(\frac{5x^2-\frac{25}{7}}{5x^2+7}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2),x)
```

```
[Out] - (37*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/7))/
175 - (37*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/(1225*
((5*x^2 - 25/7)/(5*x^2 + 7) - 1))
```

sympy [A] time = 66.86, size = 66, normalized size = 0.92

$$\begin{cases} \frac{5\sqrt{35}\left(\frac{\sqrt{25-35x^2}\sqrt{35x^2+49}}{125}-\frac{74\,\mathrm{asin}\left(\frac{\sqrt{74}\sqrt{25-35x^2}}{74}\right)}{125}\right)}{14} & \text{for}\ x > -\frac{\sqrt{35}}{7} \wedge x < \frac{\sqrt{35}}{7} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x**2+5)/(5*x**2+7))**(1/2),x)
```

```
[Out] Piecewise((5*sqrt(35)*(sqrt(25 - 35*x**2)*sqrt(35*x**2 + 49)/125 - 74*asin(
sqrt(74)*sqrt(25 - 35*x**2)/74)/125)/14, (x > -sqrt(35)/7) & (x < sqrt(35)/
7)))
```

3.291 $\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{3}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

```
[Out] -2/3*arctan(((-x^3+1)/(x^3+1))^(1/2))+1/3*(x^3+1)*((-x^3+1)/(x^3+1))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1960, 288, 204}

$$\frac{1}{3}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/3 - (2*ArcTan[Sqrt[(1 - x^3)/(1 + x^3
)]])/3
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx &= -\left(\frac{4}{3}\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\right)\\
&= \frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)+\frac{2}{3}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\\
&= \frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 86, normalized size = 1.62

$$\frac{\sqrt{\frac{1-x^3}{x^3+1}}\sqrt{x^3+1}\left(\sqrt{x^3+1}\left(x^3-1\right)+2\sqrt{1-x^3}\sin^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{2}}\right)\right)}{3\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*Sqrt[1 + x^3]*((-1 + x^3)*Sqrt[1 + x^3] + 2*Sqrt
[1 - x^3]*ArcSin[Sqrt[1 - x^3]/Sqrt[2]]))/(3*(-1 + x^3))
```

fricas [A] time = 0.43, size = 55, normalized size = 1.04

$$\frac{1}{3}\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-\frac{2}{3}\arctan\left(\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-1}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 2/3*arctan(((x^3 + 1)*sqrt(-(x^3
 - 1)/(x^3 + 1)) - 1)/x^3)
```

giac [A] time = 0.29, size = 22, normalized size = 0.42

$$\frac{1}{3}\left(\sqrt{-x^6+1}+\arcsin\left(x^3\right)\right)\mathrm{sgn}\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(-x^6 + 1) + arcsin(x^3))*sgn(x^3 + 1)
```

maple [A] time = 0.10, size = 68, normalized size = 1.28

$$-\frac{\sqrt{-\frac{x^3-1}{x^3+1}}\sqrt{-\left(x^3+1\right)\left(x^3-1\right)}\arcsin\left(x^3\right)}{3\left(x^3-1\right)}+\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((-x^3+1)/(x^3+1))^(1/2),x)
```

```
[Out] 1/3*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/3*arcsin(x^3)*(-(x^3-1)/(x^3+1))^(1/
2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

mupad [B] time = 2.67, size = 56, normalized size = 1.06

$$-\frac{2\,\mathrm{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}-\frac{2\,\sqrt{-\frac{x^3-1}{x^3+1}}}{\frac{3\,\left(x^3-1\right)}{x^3+1}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
```

```
[Out] - (2*atan((-(x^3 - 1)/(x^3 + 1))^(1/2)))/3 - (2*(-(x^3 - 1)/(x^3 + 1))^(1/2
))/((3*(x^3 - 1))/(x^3 + 1) - 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] Timed out
```

3.292 $\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx$

Optimal. Leaf size=113

$$-\frac{1}{9}\left(\frac{1-x^3}{x^3+1}\right)^{3/2}\left(x^3+1\right)^3-\frac{1}{6}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)^2+\frac{1}{2}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

```
[Out] -1/9*((-x^3+1)/(x^3+1))^(3/2)*(x^3+1)^3-1/3*arctan(((-x^3+1)/(x^3+1))^(1/2)
)+1/2*(x^3+1)*((-x^3+1)/(x^3+1))^(1/2)-1/6*(x^3+1)^2*((-x^3+1)/(x^3+1))^(1/
2)
```

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1960, 463, 455, 385, 204}

$$-\frac{1}{9}\left(\frac{1-x^3}{x^3+1}\right)^{3/2}\left(x^3+1\right)^3-\frac{1}{6}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)^2+\frac{1}{2}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/2 - (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x
^3)^2)/6 - (((1 - x^3)/(1 + x^3))^(3/2)*(1 + x^3)^3)/9 - ArcTan[Sqrt[(1 - x
^3)/(1 + x^3)]]/3
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx &= -\left(\frac{4}{3}\text{Subst}\left(\int\frac{x^2\left(-1+x^2\right)^2}{\left(-1-x^2\right)^4}\,dx,x,\sqrt{\frac{1-x^3}{1+x^3}}\right)\right)\\
&= -\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3-\frac{2}{9}\text{Subst}\left(\int\frac{x^2\left(6-6x^2\right)}{\left(-1-x^2\right)^3}\,dx,x,\sqrt{\frac{1-x^3}{1+x^3}}\right)\\
&= -\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)^2-\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3+\frac{1}{18}\text{Subst}\left(\int\frac{12-24x^2}{\left(-1-x^2\right)^2}\,dx,x,\sqrt{\frac{1-x^3}{1+x^3}}\right)\\
&= \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)-\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)^2-\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3+\frac{1}{3}\text{Subst}\left(\int\frac{1}{-1-x^2}\right.\\
&= \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)-\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)^2-\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}\left(1+x^3\right)^3-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 98, normalized size = 0.87

$$\frac{\sqrt{\frac{1-x^3}{x^3+1}}\sqrt{x^3+1}\left(6\sqrt{1-x^3}\sin^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{2}}\right)+\sqrt{x^3+1}\left(2x^9-5x^6+7x^3-4\right)\right)}{18\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*Sqrt[1 + x^3]*(Sqrt[1 + x^3]*(-4 + 7*x^3 - 5*x^6
 + 2*x^9) + 6*Sqrt[1 - x^3]*ArcSin[Sqrt[1 - x^3]/Sqrt[2]]))/(18*(-1 + x^3))
```

fricas [A] time = 0.44, size = 65, normalized size = 0.58

$$\frac{1}{18}\left(2\,x^9-x^6+x^3+4\right)\sqrt{-\frac{x^3-1}{x^3+1}}-\frac{1}{3}\arctan\left(\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-1}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(2*x^9 - x^6 + x^3 + 4)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1/3*arctan(((x^3
+ 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1)/x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

maple [A] time = 0.07, size = 80, normalized size = 0.71

$$-\frac{\sqrt{-\frac{x^3-1}{x^3+1}}\,\sqrt{-\left(x^3+1\right)\left(x^3-1\right)}\,\arcsin\left(x^3\right)}{6\left(x^3-1\right)}+\frac{\left(2x^6-3x^3+4\right)\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*((-x^3+1)/(x^3+1))^(1/2),x)
```

```
[Out] 1/18*(2*x^6-3*x^3+4)*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/6*(-(x^3-1)/(x^3+1)
)^(1/2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)*arcsin(x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

mupad [B] time = 2.66, size = 101, normalized size = 0.89

$$\frac{2\sqrt{-\frac{x^3-1}{x^3+1}}}{9}-\frac{\operatorname{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}+\frac{x^3\sqrt{-\frac{x^3-1}{x^3+1}}}{18}-\frac{x^6\sqrt{-\frac{x^3-1}{x^3+1}}}{18}+\frac{x^9\sqrt{-\frac{x^3-1}{x^3+1}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
```

```
[Out] (2*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9 - atan((-(x^3 - 1)/(x^3 + 1))^(1/2))/3 +
 (x^3*(-(x^3 - 1)/(x^3 + 1))^(1/2))/18 - (x^6*(-(x^3 - 1)/(x^3 + 1))^(1/2))
/18 + (x^9*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] Timed out
```

3.293 $\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx$

Optimal. Leaf size=106

$$\frac{1}{250}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)^2-\frac{27}{350}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}$$

```
[Out] 2257/30625*arctan(1/7*35^(1/2)*((-7*x^5+5)/(5*x^5+7))^(1/2))*35^(1/2)-27/35
0*(5*x^5+7)*((-7*x^5+5)/(5*x^5+7))^(1/2)+1/250*(5*x^5+7)^2*((-7*x^5+5)/(5*x
^5+7))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1960, 455, 385, 204}

$$\frac{1}{250}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)^2-\frac{27}{350}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (-27*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5))/350 + (Sqrt[(5 - 7*x^5)/(7
+ 5*x^5)]*(7 + 5*x^5)^2)/250 + (2257*ArcTan[Sqrt[5/7]*Sqrt[(5 - 7*x^5)/(7 +
 5*x^5)]])/(875*Sqrt[35])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
```

```
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = -\left(\frac{148}{5}\text{Subst}\left(\int\frac{x^2\left(-5+7x^2\right)}{\left(-7-5x^2\right)^3}\,dx,x,\sqrt{\frac{5-7x^5}{7+5x^5}}\right)\right)$$
$$=\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2+\frac{37}{125}\text{Subst}\left(\int\frac{-74+140x^2}{\left(-7-5x^2\right)^2}\,dx,x,\sqrt{\frac{5-7x^5}{7+5x^5}}\right)$$
$$=-\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)+\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2-\frac{2257}{875}\text{Subst}\left(\int\frac{1}{-7-5x^2}\,dx,\right.$$
$$=-\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)+\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{7+5x^5}}\right)}{875\sqrt{35}}$$

Mathematica [A] time = 0.05, size = 109, normalized size = 1.03

$$\frac{\sqrt{\frac{5-7x^5}{5x^5+7}}\sqrt{5x^5+7}\left(4514\sqrt{35}\sqrt{7x^5-5}\sinh^{-1}\left(\sqrt{\frac{5}{74}}\sqrt{7x^5-5}\right)+35\sqrt{5x^5+7}\left(245x^{10}-777x^5+430\right)\right)}{61250\left(7x^5-5\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*Sqrt[7 + 5*x^5]*(35*Sqrt[7 + 5*x^5]*(430 - 7
77*x^5 + 245*x^10) + 4514*Sqrt[35]*Sqrt[-5 + 7*x^5]*ArcSinh[Sqrt[5/74]*Sqrt
[-5 + 7*x^5]]))/(61250*(-5 + 7*x^5))
```

fricas [A] time = 0.46, size = 82, normalized size = 0.77

$$\frac{1}{1750}\left(175\,x^{10}-185\,x^5-602\right)\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}+\frac{2257}{61250}\sqrt{35}\arctan\left(\frac{\sqrt{35}\left(35\,x^5+12\right)\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{35\left(7\,x^5-5\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1750*(175*x^10 - 185*x^5 - 602)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)) + 2257/612
50*sqrt(35)*arctan(1/35*sqrt(35)*(35*x^5 + 12)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7
))/(7*x^5 - 5))
```

giac [A] time = 0.37, size = 47, normalized size = 0.44

$$\frac{1}{61250}\left(35\sqrt{-35\,x^{10}-24\,x^5+35}\left(35\,x^5-86\right)-2257\sqrt{35}\arcsin\left(\frac{35}{37}\,x^5+\frac{12}{37}\right)\right)\text{sgn}\left(5\,x^5+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="giac")
```

```
[Out] 1/61250*(35*sqrt(-35*x^10 - 24*x^5 + 35)*(35*x^5 - 86) - 2257*sqrt(35)*arcs
in(35/37*x^5 + 12/37))*sgn(5*x^5 + 7)
```

maple [C] time = 0.28, size = 130, normalized size = 1.23

$$-\frac{2257\sqrt{-\frac{7x^5-5}{5x^5+7}}\,\sqrt{-\left(5x^5+7\right)\left(7x^5-5\right)}\;\mathrm{RootOf}\left(\_Z^2+35\right)\ln\left(35x^5\,\mathrm{RootOf}\left(\_Z^2+35\right)+12\,\mathrm{RootOf}\left(\_Z^2+35\right.\right.}{61250\left(7x^5-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x)
```

```
[Out] 1/1750*(35*x^5-86)*(5*x^5+7)*(-(7*x^5-5)/(5*x^5+7))^(1/2)-2257/61250*RootOf
(_Z^2+35)*ln(35*RootOf(_Z^2+35)*x^5+12*RootOf(_Z^2+35)+35*(-35*x^10-24*x^5+
35)^(1/2))*(-(7*x^5-5)/(5*x^5+7))^(1/2)*(-(5*x^5+7)*(7*x^5-5))^(1/2)/(7*x^5
-5)
```

maxima [A] time = 1.99, size = 121, normalized size = 1.14

$$\frac{2257}{30625}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{-\frac{7x^5-5}{5x^5+7}}\right)-\frac{37\left(675\left(-\frac{7x^5-5}{5x^5+7}\right)^{\frac{3}{2}}+427\sqrt{-\frac{7x^5-5}{5x^5+7}}\right)}{875\left(\frac{25\left(7x^5-5\right)^2}{\left(5x^5+7\right)^2}-\frac{70\left(7x^5-5\right)}{5x^5+7}+49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="maxima")
```

```
[Out] 2257/30625*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7))) - 3
7/875*(675*(-(7*x^5 - 5)/(5*x^5 + 7))^(3/2) + 427*sqrt(-(7*x^5 - 5)/(5*x^5
+ 7)))/(25*(7*x^5 - 5)^2/(5*x^5 + 7)^2 - 70*(7*x^5 - 5)/(5*x^5 + 7) + 49)
```

mupad [B] time = 2.99, size = 134, normalized size = 1.26

$$\frac{2257\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{-\frac{7x^5-5}{5x^5+7}}}{7}\right)}{30625}-\frac{43\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{-\frac{7x^5-5}{5x^5+7}}}{4375}-\frac{37\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,x^5\sqrt{-\frac{7x^5-5}{5x^5+7}}}{12250}+\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,x^{10}\sqrt{-}}{350}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2),x)
```

```
[Out] (2257*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/7))/
30625 - (43*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/4375
 - (37*5^(1/2)*7^(1/2)*35^(1/2)*x^5*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/12250
 + (5^(1/2)*7^(1/2)*35^(1/2)*x^10*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/350
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*((-7*x**5+5)/(5*x**5+7))**(1/2),x)
```

```
[Out] Timed out
```

3.294 $\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} \, dx$

Optimal. Leaf size=52

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

```
[Out] 1/2*arctan(1/2*(x^2-1)^(1/2)*2^(1/2))*(-x^2/(-x^2+1))^(1/2)*(x^2-1)^(1/2)/x
*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6719, 444, 63, 203}

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-(x^2/(1 - x^2))]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqr
t[2]*x)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\int \frac{x}{\sqrt{-1+x^2}\,(1+x^2)}\,dx}{x}$$

$$= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx, x, x^2\right)}{2x}$$

$$= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{-1+x^2}\right)}{x}$$

$$= \frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{-1+x^2}\,\tan^{-1}\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.94

$$\frac{\sqrt{\frac{x^2}{x^2-1}}\sqrt{x^2-1}\,\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
[Out] (Sqrt[x^2/(-1 + x^2)]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqrt[
2]*x)
```

fricas [A] time = 0.44, size = 32, normalized size = 0.62

$$\frac{1}{2}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^2-1\right)\sqrt{\frac{x^2}{x^2-1}}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 - 1)*sqrt(x^2/(x^2 - 1))/x)
```

giac [C] time = 0.42, size = 40, normalized size = 0.77

$$\frac{1}{2}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-1}\right)\text{sgn}\left(x^2-1\right)\text{sgn}(x) + \frac{1}{2}\sqrt{2}\,\arctan\left(\frac{1}{2}i\sqrt{2}\right)\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 1))*sgn(x^2 - 1)*sgn(x) + 1/2*sqr
t(2)*arctan(1/2*I*sqrt(2))*sgn(x)
```

maple [A] time = 0.02, size = 42, normalized size = 0.81

$$\frac{\sqrt{\frac{x^2}{x^2-1}}\sqrt{x^2-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{x^2-1}\,\sqrt{2}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2-1))^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*(x^2/(x^2-1))^(1/2)/x*(x^2-1)^(1/2)*2^(1/2)*arctan(1/2*(x^2-1)^(1/2)*2^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1),x)
```

```
[Out] int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(x**2-1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(x**2/(x**2 - 1))/(x**2 + 1), x)
```

3.295 $\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2} \, dx$

Optimal. Leaf size=68

$$\frac{\sqrt{-\frac{x^2}{-\left((a+1)x^2\right)-a+1}}\sqrt{(a+1)x^2+a-1}\,\tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

```
[Out] 1/2*arctan(1/2*(-1+a+(1+a)*x^2)^(1/2)*2^(1/2))*(-x^2/(1-a-(1+a)*x^2))^(1/2)
*(-1+a+(1+a)*x^2)^(1/2)/x*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{6719, 444, 63, 205\}$

$$\frac{\sqrt{-\frac{x^2}{-(a+1)x^2-a+1}}\sqrt{(a+1)x^2+a-1}\,\tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-(x^2/(1 - a - (1 + a)*x^2))]*Sqrt[-1 + a + (1 + a)*x^2]*ArcTan[Sqrt[
-1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\int \frac{x}{(1+x^2)\sqrt{-1+a+(1+a)x^2}}\,dx}{x}$$

$$= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+a+(1+a)x}}\,dx,x,x^2\right)}{2x}$$

$$= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{-1+a}{1+a}+\frac{x^2}{1+a}}\,dx,x,\sqrt{-1+a+(1+a)x^2}\right)}{(1+a)x}$$

$$= \frac{\sqrt{-\frac{x^2}{1-a-(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\tan^{-1}\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 0.96

$$\frac{\sqrt{ax^2+a+x^2-1}\sqrt{\frac{x^2}{(a+1)x^2+a-1}}\tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-1 + a + x^2 + a*x^2]*Sqrt[x^2/(-1 + a + (1 + a)*x^2)]*ArcTan[Sqrt[-1
 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

fricas [A] time = 0.43, size = 42, normalized size = 0.62

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left((a+1)x^2+a-3\right)\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{4\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*((a + 1)*x^2 + a - 3)*sqrt(x^2/((a + 1)*x^2
+ a - 1))/x)
```

giac [A] time = 0.45, size = 61, normalized size = 0.90

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{ax^2+x^2+a-1}\right)\text{sgn}\left(ax^2+x^2+a-1\right)\text{sgn}(x)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a-1}\right)\text{sgn}\,(a-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + x^2 + a - 1))*sgn(a*x^2 + x^2 +
 a - 1)*sgn(x) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a - 1))*sgn(a - 1)*sgn
(x)
```

maple [A] time = 0.04, size = 60, normalized size = 0.88

$$\frac{\sqrt{\frac{x^2}{a\,x^2+x^2+a-1}}\sqrt{a\,x^2+x^2+a-1}\sqrt{2}\arctan\left(\frac{\sqrt{a\,x^2+x^2+a-1}\sqrt{2}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*(x^2/(a*x^2+x^2+a-1))^(1/2)/x*(a*x^2+x^2+a-1)^(1/2)*2^(1/2)*arctan(1/2*
(a*x^2+x^2+a-1)^(1/2)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2/((a + 1)*x^2 + a - 1))/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{x^2}{(a+1)\,x^2+a-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1),x)
```

```
[Out] int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^2}{ax^2+a+x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(-1+a+(1+a)*x**2))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(x**2/(a*x**2 + a + x**2 - 1))/(x**2 + 1), x)
```

3.296 $\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$

Optimal. Leaf size=281

$$\frac{(bc-ad)\left(5a^2d^2+2abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}+\frac{\left(c+dx^2\right)\left(5a^2d^2+2abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^3d^2e}-\frac{(c+dx^2 \cdots}{\cdots}$$

```
[Out] 1/16*(-a*d+b*c)*(5*a^2*d^2+2*a*b*c*d+b^2*c^2)*arctanh(d^(1/2)*(e*(b*x^2+a)/
(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(7/2)/d^(5/2)/e^(1/2)+1/16*(5*a^2*d^2+2
*a*b*c*d+b^2*c^2)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^3/d^2/e-1/24*(5
*a*d+3*b*c)*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^2/e-1/6*(d*x^2+
c)^3*(a-c*(b*x^2+a)/(d*x^2+c))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d/(-a*d+b*c)
/e
```

Rubi [A] time = 0.29, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 413, 385, 199, 208}

$$\frac{\left(c+dx^2\right)\left(5a^2d^2+2abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^3d^2e}+\frac{(bc-ad)\left(5a^2d^2+2abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}-\frac{(c+dx^2 \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d
*x^2))/(16*b^3*d^2*e) - ((3*b*c + 5*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2)^2)/(24*b^2*d^2*e) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x
^2)^3*(a - (c*(a + b*x^2))/(c + d*x^2)))/(6*b*d*(b*c - a*d)*e) + ((b*c - a*
d)*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(7/2)*d^(5/2)*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{\left(-ae+cx^2\right)^2}{\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-a(bc+5ad)e^2+3c(bc+ad)ex^2}{\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e(a+\cdots}{c+d\cdots}}\right.}{6bd}$$

$$= -\frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} + \frac{\left((bc-ad)\left(b^2c^2+\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(b^2c^2+2abcd+5a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\cdots}{\cdots}$$

$$= \frac{\left(b^2c^2+2abcd+5a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\cdots}{\cdots}$$

Mathematica [A] time = 0.39, size = 224, normalized size = 0.80

$$\frac{\sqrt{a+bx^2}\left(3\sqrt{bc-ad}\left(5a^2d^2+2abcd+b^2c^2\right)\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)+\sqrt{d}\sqrt{a+bx^2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(15a^2d^2-2abd\left(2c+\cdots\right.\right.\right.}{48b^3d^{5/2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]
*(15*a^2*d^2 - 2*a*b*d*(2*c + 5*d*x^2) + b^2*(-3*c^2 + 2*c*d*x^2 + 8*d^2*x^
4)) + 3*Sqrt[b*c - a*d]*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcSinh[(Sqrt[d]*
```

```
Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(48*b^3*d^(5/2)*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

fricas [A] time = 0.48, size = 545, normalized size = 1.94

$$\left[-\frac{3\left(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3\right)\sqrt{bde}\log\left(8b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6abcd+a^2d^2\right)e-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b*d*e)*
log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d +
a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b
*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(8*b^3*d^4*x^6 - 3*b^3*c^3*d -
 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4 - (b^3*c
^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(b^4*d^3*e), -1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d
^3)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*
x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(8*b^3*d^4*x^6 - 3*b^3
*c^3*d - 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4
- (b^3*c^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c)))/(b^4*d^3*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[0,1,0]
%%%},[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{
1,[1,0,1]%%%},[0,2]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[0
,1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{1,[1,1,1]%%%},[0
,2]%%%} Error: Bad Argument Value
```

maple [B] time = 0.05, size = 527, normalized size = 1.88

$$\frac{\left(bx^2+a\right)\left(-15a^3d^3\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+9a^2bcd^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/96*(b*x^2+a)/b^3*(-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b
*d^2*x^2-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c*d*x^2-15*
a^3*d^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2))/(b*d)^(1/2))+9*a^2*b*c*d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a
```

```
*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+3*a*b^2*c^2*d*ln(1/2*(2
*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(
1/2))+3*b^3*c^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*(b*d)^(1/2))/(b*d)^(1/2))+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(b*d)
^(1/2)*b*d+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*d^2-24*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b*c*d-6*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c^2)/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2
+c)*(b*x^2+a))^(1/2)/d^2/(b*d)^(1/2)
```

maxima [A] time = 2.27, size = 413, normalized size = 1.47

$$\frac{1}{96}\,e\left(\frac{2\left(3\left(b^3c^3d^2+ab^2c^2d^3+3\,a^2bcd^4-5\,a^3d^5\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{5}{2}}+8\left(b^4c^3d-3\,ab^3c^2d^2-3\,a^2b^2cd^3+5\,a^3bd^4\right)\left(\frac{(bx^2+a}{dx^2+c}\right.\right.}{b^6d^2e^4-\frac{3(bx^2+a)b^5d^3e^4}{dx^2+c}+\frac{3(bx^2+a)^2b^4d^4e^4}{(dx^2+c)^2}-\frac{(bx^2+a)^3}{(dx^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*e*(2*(3*(b^3*c^3*d^2 + a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - 5*a^3*d^5)*((b*
x^2 + a)*e/(d*x^2 + c))^(5/2) + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 - 3*a^2*b^2*
c*d^3 + 5*a^3*b*d^4)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e - 3*(b^5*c^3 + a*b
^4*c^2*d - 13*a^2*b^3*c*d^2 + 11*a^3*b^2*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 + c
))*e^2)/(b^6*d^2*e^4 - 3*(b*x^2 + a)*b^5*d^3*e^4/(d*x^2 + c) + 3*(b*x^2 + a
)^2*b^4*d^4*e^4/(d*x^2 + c)^2 - (b*x^2 + a)^3*b^3*d^5*e^4/(d*x^2 + c)^3) -
3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log((d*sqrt((b*x^2 +
a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(
b*d*e)))/(sqrt(b*d*e)*b^3*d^2*e))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.297 $\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=169

$$-\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}-\frac{(c+dx^2)(3ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^2de}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bde}$$

```
[Out] -1/8*(-a*d+b*c)*(3*a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b
^(1/2)/e^(1/2))/b^(5/2)/d^(3/2)/e^(1/2)-1/8*(3*a*d+b*c)*(d*x^2+c)*(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/b^2/d/e+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
b/d/e
```

Rubi [A] time = 0.13, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 385, 199, 208}

$$-\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}-\frac{(c+dx^2)(3ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^2de}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bde}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -((b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^2*d*e)
+ (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b*d*e) - ((b*c - a*d
)*(b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])])/(8*b^(5/2)*d^(3/2)*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
```

```
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{-ae+cx^2}{\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4bde} - \frac{((bc-ad)(bc+3ad)e)\,\text{Subst}\left(\int \frac{1}{\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4bd}$$

$$= -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4bde} - \frac{((bc-ad)(bc+3ad))\,\text{Subst}\left(\int \frac{1}{be-\ldots}\right.}{8b^2d}$$

$$= -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4bde} - \frac{(bc-ad)(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\ldots}}{\sqrt{b}\ldots}\right.}{8b^{5/2}d^{3/2}\sqrt{e}}$$

Mathematica [A] time = 0.34, size = 172, normalized size = 1.02

$$\frac{\sqrt{d}\left(a+bx^2\right)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(b\left(c+2dx^2\right)-3ad\right)-\sqrt{a+bx^2}\sqrt{bc-ad}\,(3ad+bc)\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{8b^2d^{3/2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[d]*(a + b*x^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(-3*a*d + b*(c + 2*d
*x^2)) - Sqrt[b*c - a*d]*(b*c + 3*a*d)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqr
t[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^2*d^(3/2)*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

fricas [A] time = 0.46, size = 413, normalized size = 2.44

$$\left[-\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\sqrt{bde}\,\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)e+4\left(2\,bd^2x^4+bc\ldots\right.\right.}{32\,b^3d\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 +
 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d
^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 +
```

```
 a*e)/(d*x^2 + c))) - 4*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c
*d^2 - a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e), 1/16*(
(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c
+ a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*
e)) + 2*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c*d^2 - a*b*d^3)*
x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[0,1,0]
%%%},[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{
1,[1,0,1]%%%},[0,2]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[0
,1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{1,[1,1,1]%%%},[0
,2]%%%} Error: Bad Argument Value
```

maple [B] time = 0.04, size = 342, normalized size = 2.02

$$\frac{\left(b\,x^2+a\right)\left(3a^2d^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-2abcd\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-b\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*d*x^2+3
*d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2))/(b*d)^(1/2))*a^2-2*a*b*c*d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))-ln(1/2*(2*b*d*x^2+a*d+b*c+
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*b^2*c^2-6*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*d+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)*b*c)/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*
x^2+a))^(1/2)/b^2/d/(b*d)^(1/2)
```

maxima [A] time = 2.23, size = 268, normalized size = 1.59

$$\frac{1}{16}e\left(\frac{2\left(\left(b^2c^2d+2\,abcd^2-3\,a^2d^3\right)\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}+\left(b^3c^2-6\,ab^2cd+5\,a^2bd^2\right)\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}\,e\right)}{b^4de^3-\frac{2\left(bx^2+a\right)b^3d^2e^3}{dx^2+c}+\frac{\left(bx^2+a\right)^2b^2d^3e^3}{\left(dx^2+c\right)^2}}+\frac{\left(b^2c^2+2\,abcd-3\,a\cdots\right.}{\sqrt{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*((b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*((b*x^2 + a)*e/(d*x^2 + c)
)^(3/2) + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 +
 c))*e)/(b^4*d*e^3 - 2*(b*x^2 + a)*b^3*d^2*e^3/(d*x^2 + c) + (b*x^2 + a)^2*
b^2*d^3*e^3/(d*x^2 + c)^2) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log((d*sqrt(
```

```
(b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)
) + sqrt(b*d*e)))/(sqrt(b*d*e)*b^2*d*e))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.298 $\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$

Optimal. Leaf size=106

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}+\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be}$$

```
[Out] 1/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2
))/b^(3/2)/d^(1/2)/e^(1/2)+1/2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/e
```

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1960, 199, 208}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}+\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*b*e) + ((b*c - a*d)*ArcT
anh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3
/2)*Sqrt[d]*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{2be} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2b}$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{2be} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}$$

Mathematica [A] time = 0.13, size = 152, normalized size = 1.43

$$\frac{\sqrt{a+bx^2}\left(\sqrt{d}\sqrt{a+bx^2}\sqrt{\frac{b(c+dx^2)}{bc-ad}} + \sqrt{bc-ad}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)\right)}{2b\sqrt{d}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]
 + Sqrt[b*c - a*d]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(2*
b*Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d
)])
```

fricas [A] time = 0.44, size = 313, normalized size = 2.95

$$\left[-\frac{\sqrt{bde}\,(bc-ad)\log\left(8\,b^2d^2ex^4 + 8\left(b^2cd + abd^2\right)ex^2 + \left(b^2c^2 + 6\,abcd + a^2d^2\right)e - 4\left(2\,bd^2x^4 + bc^2 + acd + \left(3\,bcd\right.\right.\right.}{8\,b^2de}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(b*d*e)*(b*c - a*d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*
e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d
+ (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4
*(b*d^2*x^2 + b*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e), -1/4*(sq
rt(-b*d*e)*(b*c - a*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt
((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(b*d^2*x^2 + b*c
*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[0,1,0]
%%%},[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{
1,[1,0,1]%%%},[0,2]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[0
,1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1]%%%}+%%%{%%%{1,[1,1,1]%%%},[0
,2]%%%} Error: Bad Argument Value
```

maple [B] time = 0.03, size = 200, normalized size = 1.89

$$\frac{\left(b\,x^2+a\right)\left(-ad\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+bc\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+2\sqrt{bd}\right.}{4\sqrt{\frac{(b\,x^2+a)e}{d\,x^2+c}}\,\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{bd}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)*(-d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a+b*c*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2))/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x
^2+a))^(1/2)/b/(b*d)^(1/2)
```

maxima [A] time = 2.18, size = 153, normalized size = 1.44

$$\frac{1}{4}\,e\left(\frac{2\,(bc-ad)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{b^2e^2-\frac{(bx^2+a)bde^2}{dx^2+c}}-\frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,be}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(b^2*e^2 - (b*x^2 + a)
*b*d*e^2/(d*x^2 + c)) - (b*c - a*d)*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c))
- sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d
*e)*b*e))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.299 $\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=112

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}$$

```
[Out] -arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*c^(1/2)/a^(
1/2)/e^(1/2)+arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))
*d^(1/2)/b^(1/2)/e^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 391, 208}

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqr
t[e])])/(Sqrt[a]*Sqrt[e])) + (Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*Sqrt[e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= c\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right) + d\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}$$

Mathematica [A] time = 0.24, size = 190, normalized size = 1.70

$$\frac{\sqrt{a+bx^2}\left(\sqrt{a}\sqrt{d}\sqrt{c+dx^2}\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right) - b\sqrt{c}\,(c+dx^2)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{\sqrt{a}\,b\,(c+dx^2)^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]*Sqrt[(b*(
c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]
 - b*Sqrt[c]*(c + d*x^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c
+ d*x^2])]))/(Sqrt[a]*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^(3/2)
)
```

fricas [A] time = 0.66, size = 881, normalized size = 7.87

$$\left[\frac{1}{4}\sqrt{\frac{d}{be}}\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 + 8\left(b^2cd + abd^2\right)x^2 + 4\left(2\,b^2d^2x^4 + b^2c^2 + abcd + \left(3\,b^2cd + abd^2\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b
^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d +
 a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 1/4*sqrt(
c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2
+ a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*
c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), -1/2*sqrt(
-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) + 1/4*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d
 + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), 1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c +
a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2
 + a*c)) + 1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*
d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3
*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))),
1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e
```

```
)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) - 1/2*sqrt(-d/(b*e))*arctan(
1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e)
)/(b*d*x^2 + a*d))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[2,1,2]
%%%},[2,3,0]%%%}+%%%{%%%{-2,[1,2,2]%%%},[2,2,1]%%%}+%%%{%%%{1,[0,3,2]%%%},[
2,1,2]%%%}+%%%{%%{[%%%{-2,[2,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,4,0
]%%%}+%%%{%%{[%%%{4,[1,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,3,1]%%%}+
%%%{%%{[%%%{-2,[0,2,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,2,2]%%%}+%%%{%
%%{1,[3,0,3]%%%},[0,5,0]%%%}+%%%{%%%{-2,[2,1,3]%%%},[0,4,1]%%%}+%%%{%%%{1,[
1,2,3]%%%},[0,3,2]%%%} / %%%{%%%{1,[0,2,0]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[
0,1,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%}
,[0,2,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.04, size = 179, normalized size = 1.60

$$\frac{\left(b\,x^{2}+a\right)\left(-\sqrt{b d}\,c\ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2 a c+2\sqrt{a c}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}}{x^{2}}\right)+\sqrt{a c}\,d\ln\left(\frac{2 b d\,x^{2}+a d+b c+2\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\,\sqrt{b d}}{2\sqrt{b d}}\right.\right.}{2\sqrt{\frac{\left(b\,x^{2}+a\right) e}{d\,x^{2}+c}}\,\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{b d}\,\sqrt{a c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*(d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*(a*c)^(1/2)-c*ln((a*d*x^2+b*c*x^2+2*a*c+2*(
a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*(b*d)^(1/2))/((b*x^2+a
)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

maxima [A] time = 1.96, size = 155, normalized size = 1.38

$$\frac{1}{2}e\left(\frac{c\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,e}-\frac{d\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*(c*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x
^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*e) - d*log((d*sqrt((b*x
^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) +
sqrt(b*d*e)))/(sqrt(b*d*e)*e))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

3.300 $\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=130

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}+\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

```
[Out] 1/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2
))/a^(3/2)/c^(1/2)/e^(1/2)+1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/(
a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1960, 199, 208}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}+\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*a*(a*e - (c*e*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt[c]*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2a}$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}$$

Mathematica [**A**] time = 0.12, size = 133, normalized size = 1.02

$$\frac{\sqrt{a+bx^2}\left(-\frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{c}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax^2}\right)}{2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x^2)) - ((-(b*c) +
 a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2
)*Sqrt[c])))/(2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

fricas [**A**] time = 0.66, size = 333, normalized size = 2.56

$$\left[-\frac{\sqrt{ace}\,(bc-ad)x^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-4\,((bcd+ad^2)x^4+2\,ac^2+(bc^2+3\,acd)x^2)\sqrt{ace}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{x^4}\right)+4\,(a}{8\,a^2cex^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a*c*e)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x
^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 + 2
*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c
)))/x^4) + 4*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*
e*x^2), -1/4*(sqrt(-a*c*e)*(b*c - a*d)*x^2*arctan(1/2*sqrt(-a*c*e)*((b*c +
a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)
) + 2*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*e*x^2)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[0,1,0]
%%%},[6,0,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[5,1,0]%%%}+%%%{
%%%{1,[1,0,1]%%%},[4,2,0]%%%}+%%%{%%%{-2,[0,1,1]%%%},[4,1,1]%%%}+%%%{%%{[%%
%{4,[0,0,1]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[3,2,1]%%%}+%%%{%%%{-2,[1,0,
2]%%%},[2,3,1]%%%}+%%%{%%%{1,[0,1,2]%%%},[2,2,2]%%%}+%%%{%%{[%%%{-2,[0,0,2]
%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,3,2]%%%}+%%%{%%%{1,[1,0,3]%%%},[0,4,
2]%%%} / %%%{%%%{1,[0,2,0]%%%},[6,0,0]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:[1
,0,%%%{-1,[1,1,1]%%%}]%%},[5,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%},[4,2,0]%%%}+%%%
{%%%{-2,[0,2,1]%%%},[4,1,1]%%%}+%%%{%%{[%%%{4,[0,1,1]%%%},0]:[1,0,%%%{-1,[1
,1,1]%%%}]%%},[3,2,1]%%%}+%%%{%%%{-2,[1,1,2]%%%},[2,3,1]%%%}+%%%{%%%{1,[0,2
,2]%%%},[2,2,2]%%%}+%%%{%%{[%%%{-2,[0,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%
%},[1,3,2]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,4,2]%%%} Error: Bad Argument Value
```

maple [B] time = 0.05, size = 326, normalized size = 2.51

$$-\frac{\left(b\,x^2+a\right)\left(a^2cd\,x^2\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)-ab\,c^2x^2\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2}}{x^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)*(-2*b*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^4*(a*c)^(1/2)+
a^2*c*d*x^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2))/x^2)-a*b*c^2*x^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*d*a*x^2*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b*c*
x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2))/((b*x^2+a)/(d*x^2+c)
*e)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a^2/c/x^2/(a*c)^(1/2)
```

maxima [A] time = 1.52, size = 153, normalized size = 1.18

$$\frac{1}{4}\,e\left(\frac{2\,(bc-ad)\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}}{a^2e^2-\frac{\left(bx^2+a\right)ace^2}{dx^2+c}}-\frac{(bc-ad)\log\left(\frac{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,ae}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(a^2*e^2 - (b*x^2 + a)
*a*c*e^2/(d*x^2 + c)) - (b*c - a*d)*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c))
- sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c
*e)*a*e))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.301 $\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=218

$$-\frac{(ad+3bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}-\frac{(ad+3bc)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{e(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

```
[Out] -1/8*(-a*d+b*c)*(a*d+3*b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a
^(1/2)/e^(1/2))/a^(5/2)/c^(3/2)/e^(1/2)-1/4*(-a*d+b*c)^2*e*(e*(b*x^2+a)/(d*
x^2+c))^(1/2)/a/c/(a*e-c*e*(b*x^2+a)/(d*x^2+c))^2-1/8*(-a*d+b*c)*(a*d+3*b*c
)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/c/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] time = 0.14, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 385, 199, 208}

$$-\frac{(ad+3bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}-\frac{(ad+3bc)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{e(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*a*c*(a*e - (c*e*(a
+ b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(3*b*c + a*d)*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)])/(8*a^2*c*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - ((b*c - a*
d)*(3*b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[
a]*Sqrt[e])])/(8*a^(5/2)*c^(3/2)*Sqrt[e])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{be-dx^2}{\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2 e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{((bc-ad)(3bc+ad)e)\,\text{Subst}\left(\int \frac{1}{\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4ac}$$

$$= -\frac{(bc-ad)^2 e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int \ldots\right.}{8a^2c}$$

$$= -\frac{(bc-ad)^2 e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{\ldots}}{\sqrt{\ldots}}\right.}{8a^{5/2}c^{3/2}\sqrt{e}}$$

Mathematica [A] time = 0.16, size = 173, normalized size = 0.79

$$\frac{\sqrt{a}\sqrt{c}\left(a+bx^2\right)\sqrt{c+dx^2}\left(3bcx^2-a\left(2c+dx^2\right)\right)-x^4\sqrt{a+bx^2}\left(-a^2d^2-2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{3/2}x^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*(a + b*x^2)*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(2*c + d*x^2))
- (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*x^4*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sq
rt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(3/2)*x^4*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

fricas [A] time = 1.32, size = 443, normalized size = 2.03

$$\left[-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{ace}\,x^4\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2cd\right)ex^2+4\left(\left(bcd+ad^2\right)x^4+2\,ac^2+\left(bc^2+3\,acd\right)x^2\right)\sqrt{ace}\ldots}{x^4}\right.}{32\,a^3c^2ex^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(a*c*e)*x^4*log(((b^2*c^2 + 6
*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(
(b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*
e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*
x^4 - 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*
c^2*e*x^4), 1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-a*c*e)*x^4*arctan
(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c
))/(a*b*c*e*x^2 + a^2*c*e)) - 2*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*x^4
- 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*c^2*
e*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[4,1,4]
%%%},[2,7,0]%%%}+%%%{%%%{-4,[3,2,4]%%%},[2,6,1]%%%}+%%%{%%%{6,[2,3,4]%%%},[
2,5,2]%%%}+%%%{%%%{-4,[1,4,4]%%%},[2,4,3]%%%}+%%%{%%%{1,[0,5,4]%%%},[2,3,4]
%%%}+%%%{%%{[%%%{-2,[4,0,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,8,0]%%%}+
%%%{%%{[%%%{8,[3,1,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,7,1]%%%}+%%%{%%
{[%%%{-12,[2,2,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,6,2]%%%}+%%%{%%{[%%
%{8,[1,3,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,5,3]%%%}+%%%{%%{[%%%{-2,[
0,4,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,4,4]%%%}+%%%{%%%{1,[5,0,5]%%%}
,[0,9,0]%%%}+%%%{%%%{-4,[4,1,5]%%%},[0,8,1]%%%}+%%%{%%%{6,[3,2,5]%%%},[0,7,
2]%%%}+%%%{%%%{-4,[2,3,5]%%%},[0,6,3]%%%}+%%%{%%%{1,[1,4,5]%%%},[0,5,4]%%%}
 / %%%{%%%{1,[0,2,0]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:[1,0,%%%
{-1,[1,1,1]%%%}]%%},[1,1,0]%%%}+%%%{%%%{1,[1,1,1]%%%},[0,2,0]%%%} Error: Ba
d Argument Value
```

maple [B] time = 0.06, size = 558, normalized size = 2.56

$$\frac{\left(b\,x^{2}+a\right)\left(a^{3}c\,d^{2}x^{4}\ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2 a c+2\sqrt{a c}\;\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}}{x^{2}}\right)+2 a^{2} b\,c^{2} d\,x^{4}\ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2 a c+2\sqrt{a c}\;\sqrt{b d\,x^{4}+a d\,x^{2}+}}{x^{2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)*(-2*b*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*a*(a*c)^(1
/2)-10*b^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*c*(a*c)^(1/2)+a^3*ln((
a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^
2)*d^2*c*x^4+2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2))/x^2)*d*b*a^2*c^2*x^4-3*c^3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*
c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*b^2*a*x^4-2*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*d^2*a^2*x^4*(a*c)^(1/2)-8*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b*c*d-10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b
^2*c^2*x^4*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a*
d*x^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*c*x^2*(a*c)^(1/2)-4*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a*c)/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/
((d*x^2+c)*(b*x^2+a))^(1/2)/a^3/c^2/x^4/(a*c)^(1/2)
```

maxima [A] time = 1.76, size = 271, normalized size = 1.24

$$\frac{1}{16}e\left(\frac{2\left(\left(3\,b^2c^3-2\,abc^2d-a^2cd^2\right)\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}-\left(5\,ab^2c^2-6\,a^2bcd+a^3d^2\right)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,e\right)}{a^4ce^3-\frac{2\,(bx^2+a)a^3c^2e^3}{dx^2+c}+\frac{(bx^2+a)^2a^2c^3e^3}{(dx^2+c)^2}}+\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right.}{\sqrt{ace}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*((3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2)*((b*x^2 + a)*e/(d*x^2 + c)
)^(3/2) - (5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 +
 c))*e)/(a^4*c*e^3 - 2*(b*x^2 + a)*a^3*c^2*e^3/(d*x^2 + c) + (b*x^2 + a)^2*
a^2*c^3*e^3/(d*x^2 + c)^2) + (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*log((c*sqrt(
(b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)
) + sqrt(a*c*e)))/(sqrt(a*c*e)*a^2*c*e))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.302 $\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=403

$$-\frac{x\left(a+bx^2\right)\left(-8a^2d^2+3abcd+2b^2c^2\right)}{15b^3d\left(c+dx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}+\frac{\sqrt{c}\left(a+bx^2\right)\left(-8a^2d^2+3abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{3/2}\left(c+dx^2\right)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-c^{3/2}$$

```
[Out] 1/15*(-4*a*d+b*c)*x*(b*x^2+a)/b^2/d/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/5*x^3*(
b*x^2+a)/b/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/15*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c
^2)*x*(b*x^2+a)/b^3/d/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/15*c^(3/2)*
(-4*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*
d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/b^2/d^(3/2)/(d*x^2+c)/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/15*(-8*a^2*
d^2+3*a*b*c*d+2*b^2*c^2)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)/b^
3/d^(3/2)/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))
^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 478, 582, 531, 418, 492, 411}

$$-\frac{x\left(a+bx^2\right)\left(-8a^2d^2+3abcd+2b^2c^2\right)}{15b^3d\left(c+dx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}+\frac{\sqrt{c}\left(a+bx^2\right)\left(-8a^2d^2+3abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{3/2}\left(c+dx^2\right)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-c^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((b*c - 4*a*d)*x*(a + b*x^2))/(15*b^2*d*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
+ (x^3*(a + b*x^2))/(5*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((2*b^2*c^2 +
 3*a*b*c*d - 8*a^2*d^2)*x*(a + b*x^2))/(15*b^3*d*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)]*(c + d*x^2)) + (Sqrt[c]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*(a + b*
x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^3*d^(3/
2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2)) - (c^(3/2)*(b*c - 4*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^4\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x^3\left(a+bx^2\right)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{x^2(3ac+(-bc+4ad)x^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-4ad)x\left(a+bx^2\right)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3\left(a+bx^2\right)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{-ac(bc-4ad)+\left(-2b^2c^2-3abcd+8a^2d^2\right)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-4ad)x\left(a+bx^2\right)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3\left(a+bx^2\right)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(ac(bc-4ad)\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{((-}{}$$

$$= \frac{(bc-4ad)x\left(a+bx^2\right)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3\left(a+bx^2\right)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(2b^2c^2+3abcd-8a^2d^2\right)x\left(a+bx^2\right)}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{c^{3/2}(bc-4a}{15b^2d^3}$$

$$= \frac{(bc-4ad)x\left(a+bx^2\right)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3\left(a+bx^2\right)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(2b^2c^2+3abcd-8a^2d^2\right)x\left(a+bx^2\right)}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\sqrt{c}\left(2b^2c^2\right.}{}$$

Mathematica [C] time = 0.48, size = 258, normalized size = 0.64

$$\frac{2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(2a^2d^2-abcd-b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(8a^2d^2-3abcd-\right.}{15a^2d^2\left(\frac{b}{a}\right)^{5/2}\left(c+dx^2\right)\sqrt{\frac{e(a+b}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a*d - b*(c + 3*d*x^2))) - I*c*(
-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b^2*c^2) - a*b
*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d^2*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)]*(c + d*x^2))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^6+cx^4\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex^2+ae}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

[Out] integral((d*x^6 + c*x^4)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b*e*x^2 + a*e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] integrate(x^4/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)

maple [A] time = 0.03, size = 553, normalized size = 1.37

$$\left(b\,x^2+a\right)\left(3\sqrt{-\frac{b}{a}}\,b^2d^3x^7-\sqrt{-\frac{b}{a}}\,ab\,d^3x^5+4\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5-4\sqrt{-\frac{b}{a}}\,a^2d^3x^3+\sqrt{-\frac{b}{a}}\,b^2c^2d\,x^3-4\sqrt{-\frac{b}{a}}\,a^2c\,d^2x+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)

[Out] 1/15*(b*x^2+a)*(3*(-1/a*b)^(1/2)*b^2*d^3*x^7-(-1/a*b)^(1/2)*a*b*d^3*x^5+4*(-1/a*b)^(1/2)*b^2*c*d^2*x^5-4*(-1/a*b)^(1/2)*a^2*d^3*x^3+(-1/a*b)^(1/2)*b^2*c^2*d*x^3-4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a^2*c*d^2+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*b*c^2*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b^2*c^3+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a^2*c*d^2-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*b*c^2*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b^2*c^3-4*(-1/a*b)^(1/2)*a^2*c*d^2*x+(-1/a*b)^(1/2)*a*b*c^2*d*x)/b^2/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/d^2/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] integrate(x^4/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)

```
[Out] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.303 $\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=312

$$\frac{x\left(a+bx^2\right)(bc-2ad)}{3b^2\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\left(a+bx^2\right)(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[Out] 1/3*x*(b*x^2+a)/b/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-2*a*d+b*c)*x*(b*x^2+a
)/b^2/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*c^(3/2)*(b*x^2+a)*(1/(1+d
*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1
/2),(1-b*c/a/d)^(1/2))/b/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-2*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1
/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/
a/d)^(1/2))*c^(1/2)/b^2/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6719, 478, 531, 418, 492, 411}

$$\frac{x\left(a+bx^2\right)(bc-2ad)}{3b^2\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\left(a+bx^2\right)(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (x*(a + b*x^2))/(3*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b*c - 2*a*d)*x*
(a + b*x^2))/(3*b^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[
c]*(b*c - 2*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(3*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (c^(3/2)*(a + b*x^2)*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^2\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x\left(a+bx^2\right)}{3b\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{ac+(-bc+2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x\left(a+bx^2\right)}{3b\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{\left(ac\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}} - \frac{\left((-bc+2ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx}}}{3b\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x\left(a+bx^2\right)}{3b\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{(bc-2ad)x\left(a+bx^2\right)}{3b^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)} - \frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\left(c(-bc\right.}{}$$

$$= \frac{x\left(a+bx^2\right)}{3b\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{(bc-2ad)x\left(a+bx^2\right)}{3b^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)} - \frac{\sqrt{c}\,(bc-2ad)\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}$$

Mathematica [C] time = 0.28, size = 212, normalized size = 0.68

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(2ad-b}{3bd\sqrt{\frac{b}{a}}\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + I*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d*Sqrt[(e*(a + b*x^2))/(c +
 d*x^2)]*(c + d*x^2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(dx^4+cx^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex^2+ae},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b*e*x^2 + a*e),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

maple [A] time = 0.02, size = 358, normalized size = 1.15

$$\frac{\left(bx^2+a\right)\left(\sqrt{-\frac{b}{a}}\,bd^2x^5+\sqrt{-\frac{b}{a}}\,ad^2x^3+\sqrt{-\frac{b}{a}}\,bcdx^3+\sqrt{-\frac{b}{a}}\,acdx-2\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\,acd\,\mathrm{EllipticE}\left(\sqrt{-\frac{b}{a}}\,x,\sqrt{\right.\right.}{3\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\sqrt{(d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*((-1/a*b)^(1/2)*x^5*b*d^2+(-1/a*b)^(1/2)*x^3*a*d^2+(-1/a*b)^(
1/2)*x^3*b*c*d+a*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*
b)^(1/2)*x,(a/b/c*d)^(1/2))*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b*c^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*c*d+((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b*c^
2+(-1/a*b)^(1/2)*a*c*d*x)/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x^2+a
))^(1/2)/b/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

3.304 $\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=252

$$\frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)}{b\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\,\sqrt{d}\,\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[Out] d*x*(b*x^2+a)/b/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*(
1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2
/c)^(1/2),(1-b*c/a/d)^(1/2))/a/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
*c^(1/2)*d^(1/2)/b/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(
d*x^2+c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6719, 422, 418, 492, 411}

$$\frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)}{b\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\,\sqrt{d}\,\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (d*x*(a + b*x^2))/(b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt
[c]*Sqrt[d]*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*(c + d*x^2)) + (c^(3/2)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{\left(c\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{dx\left(a+bx^2\right)}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{\left(cd\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{dx\left(a+bx^2\right)}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{c^{3/2}\left(a+bx^2\right)F\left(\tan^{-1}\right.}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+}{c+}}}$$

Mathematica [A] time = 0.06, size = 86, normalized size = 0.34

$$\frac{\sqrt{\frac{a+bx^2}{a}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[
-(b/a)]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^2+c\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex^2+ae}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b*e*x^2 + a*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

maple [A] time = 0.02, size = 127, normalized size = 0.50

$$\frac{\left(b\,x^2+a\right)\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,c\,\mathrm{EllipticE}\left(\sqrt{-\frac{b}{a}}\,x,\sqrt{\frac{ad}{bc}}\right)}{\sqrt{\frac{\left(b\,x^2+a\right)e}{d\,x^2+c}}\,\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] (b*x^2+a)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2
)*x,(a/b/c*d)^(1/2))/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1
/2)/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(1/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.305 $\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=289

$$-\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)}{a\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)E\left(\tan^{-1}\left(\right.\right.}{a\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (-b*x^2-a)/a/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+d*x*(b*x^2+a)/a/(d*x^2+c)/(e*(
b*x^2+a)/(d*x^2+c))^(1/2)-(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d
^(1/2)/a/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^
(1/2)+(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)
/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a/(d*x^2+c)/(
c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 475, 21, 422, 418, 492, 411}

$$-\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)}{a\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)E\left(\tan^{-1}\left(\right.\right.}{a\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((a + b*x^2)/(a*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])) + (d*x*(a + b*x^2))/
(a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[c]*Sqrt[d]*(a + b
*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
 + (Sqrt[c]*Sqrt[d]*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(a*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)]*(c + d*x^2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{x^2\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{ad+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(bd\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx\left(a+bx^2\right)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{(c\cdots}{\cdots}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx\left(a+bx^2\right)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\sqrt{c}\cdots}{\cdots}$$

Mathematica [A] time = 0.26, size = 111, normalized size = 0.38

$$\frac{\left(a+bx^2\right)\left(\frac{d\sqrt{\frac{dx^2}{c}+1}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{bx^2}{a}+1}\left(c+dx^2\right)}-\frac{1}{x}\right)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] ((a + b*x^2)*(-x^(-1) + (d*Sqrt[1 + (d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)
]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[1 + (b*x^2)/a]*(c + d*x^2))))/(a*Sqr
t[(e*(a + b*x^2))/(c + d*x^2)])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^2+c\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex^4+aex^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b*e*x^4 + a*e*x^2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2), x)
```

maple [A] time = 0.02, size = 297, normalized size = 1.03

$$-\frac{\left(b\,x^2+a\right)\left(\sqrt{-\frac{b}{a}}\,bd\,x^4+\sqrt{-\frac{b}{a}}\,ad\,x^2-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,adx\,\mathrm{EllipticF}\left(\sqrt{-\frac{b}{a}}\,x,\sqrt{\frac{ad}{bc}}\right)+\sqrt{-\frac{b}{a}}\,bc\,x^2-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{}\right.}{\sqrt{\frac{\left(b\,x^2+a\right)e}{d\,x^2+c}}\,\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] -(b*x^2+a)*((-1/a*b)^(1/2)*b*d*x^4-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x*a*d+b*c*((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*x*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))-b*c*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1
/2))+(-1/a*b)^(1/2)*x^2*a*d+(-1/a*b)^(1/2)*x^2*b*c+(-1/a*b)^(1/2)*a*c)/((b*
x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a/x/(-1/a*b)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.306 $\int \frac{1}{x^4 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \, dx$

Optimal. Leaf size=375

$$\frac{(a+bx^2)(2bc-ad)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{dx(a+bx^2)(2bc-ad)}{3a^2c(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{b\sqrt{c}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{d}(a+bx^2)(2b\cdots}{3a^2\sqrt{c}(c+\cdots}$$

```
[Out] 1/3*(-b*x^2-a)/a/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-a*d+2*b*c)*(b*x^2+
a)/a^2/c/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*d*(-a*d+2*b*c)*x*(b*x^2+a)/a^2
/c/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-a*d+2*b*c)*(b*x^2+a)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)/a^2/(d*x^2+c)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^
2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*b*(b*x^2+a)*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c
/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^2/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 372, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 475, 583, 531, 418, 492, 411}

$$\frac{(a+bx^2)(2bc-ad)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{dx(a+bx^2)(2bc-ad)}{3a^2c(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{b\sqrt{c}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{d}(a+bx^2)(2b\cdots}{3a^2\sqrt{c}(c+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -(a + b*x^2)/(3*a*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((2*b*c - a*d)*(
a + b*x^2))/(3*a^2*c*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(2*b*c - a*d
)*x*(a + b*x^2))/(3*a^2*c*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) +
(Sqrt[d]*(2*b*c - a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[
(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (b*Sqrt[c]*Sqrt[d]*(a + b*x^2)*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{x^4\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{-2bc+ad-bdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)\left(a+bx^2\right)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{abcd+bd(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)\left(a+bx^2\right)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(bd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} - \frac{\left(bd(2bc\right.}{}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)\left(a+bx^2\right)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(2bc-ad)x\left(a+bx^2\right)}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} - \frac{b\sqrt{c}\sqrt{d}\left(a+bx^2\right)}{3a^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)\left(a+bx^2\right)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(2bc-ad)x\left(a+bx^2\right)}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\sqrt{d}(2bc-ad)\left(a\right.}{3a^2\sqrt{c}\sqrt{\frac{c(a}{a(c}}}$$

Mathematica [C] time = 0.41, size = 238, normalized size = 0.63

$$\frac{-\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(a\left(c+dx^2\right)-2bcx^2\right)+2ibcx^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-}{3a^2cx^3\sqrt{\frac{b}{a}}\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*b*c*x^2 + a*(c + d*x^2))) - I*b*c*
(-2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(3*a^2*Sqrt[b/a]*c*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^2+c\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex^6+aex^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b*e*x^6 + a*e*x^4),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4), x)
```

maple [A] time = 0.03, size = 444, normalized size = 1.18

$$-\frac{\left(b\,x^2+a\right)\left(\sqrt{-\frac{b}{a}}\,ab\,d^2x^6-2\sqrt{-\frac{b}{a}}\,b^2cd\,x^6+\sqrt{-\frac{b}{a}}\,a^2d^2x^4-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,abcd\,x^3\,\mathrm{EllipticE}\left(\sqrt{-\frac{b}{a}}\,x,\sqrt{\frac{ad}{bc}}\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
[Out] -1/3*(b*x^2+a)*((-1/a*b)^(1/2)*a*b*d^2*x^6-2*(-1/a*b)^(1/2)*b^2*c*d*x^6+2*b
*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/
c*d)^(1/2))*x^3*a*c-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1
/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^3*b^2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^3*a*b*c*d+2*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*
x^3*b^2*c^2+(-1/a*b)^(1/2)*a^2*d^2*x^4-2*(-1/a*b)^(1/2)*b^2*c^2*x^4+2*(-1/a
*b)^(1/2)*a^2*c*d*x^2-(-1/a*b)^(1/2)*a*b*c^2*x^2+(-1/a*b)^(1/2)*a^2*c^2)/((
b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a^2/x^3/c/(-1/a*b)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

3.307 $\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$

Optimal. Leaf size=354

$$\frac{\left(c+dx^2\right)^3\left(7a^2d^2-2abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6b^2de^2(bc-ad)^2}-\frac{a^2\left(c+dx^2\right)^3}{be(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(bc-ad)\left(5ad(2bc-7ad)+b^2c^2\right)\tanh^{-1}\left(\right.}{16b^{9/2}d^{3/2}e^{3/2}}$$

```
[Out] -1/16*(-a*d+b*c)*(b^2*c^2+5*a*d*(-7*a*d+2*b*c))*arctanh(d^(1/2)*(e*(b*x^2+a
)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(9/2)/d^(3/2)/e^(3/2)-a^2*(d*x^2+c)^3
/b/(-a*d+b*c)^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16*(b^2*c^2+5*a*d*(-7*a*d
+2*b*c))*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^4/d/e^2-1/24*(b^2*c^2+5*
a*d*(-7*a*d+2*b*c))*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^3/d/(-a*d+b
*c)/e^2+1/6*(7*a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x^2+c)^3*(e*(b*x^2+a)/(d*x^2+c
))^(1/2)/b^2/d/(-a*d+b*c)^2/e^2
```

Rubi [A] time = 0.38, antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1960, 462, 385, 199, 208}

$$-\frac{a^2\left(c+dx^2\right)^3}{be(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(bc-ad)\left(5ad(2bc-7ad)+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}}+\frac{\left(c+dx^2\right)^3\left(\frac{c^2}{d}-\frac{a(2bc-7ad)}{b^2}\right)\sqrt{\quad}}{6e^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -((b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c +
d*x^2))/(16*b^4*d*e^2) - ((c^2/d + (5*a*(2*b*c - 7*a*d))/b^2)*Sqrt[(e*(a +
b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(24*b*(b*c - a*d)*e^2) - (a^2*(c + d*x^
2)^3)/(b*(b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((c^2/d - (a*
(2*b*c - 7*a*d))/b^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^3)/(6*(
b*c - a*d)^2*e^2) - ((b*c - a*d)*(b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(9/2)
*d^(3/2)*e^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{\left(-ae+cx^2\right)^2}{x^2\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-a(2bc-7ad)e^2+bc^2ex^2}{\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{b}$$

$$= -\frac{a^2\left(c+dx^2\right)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(b^2c^2-2abcd+7a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^3}{6b^2d(bc-ad)^2e^2} - \frac{\left((bc-ad)\left(b^2\right.\right.}{}$$

$$= -\frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{24b^3d(bc-ad)e^2} - \frac{a^2\left(c+dx^2\right)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(b^2c^2-2abc\right.}{}$$

$$= -\frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^4de^2} - \frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+\right.}{24b^3d(bc-ad)e^2}$$

$$= -\frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{16b^4de^2} - \frac{\left(b^2c^2+5ad(2bc-7ad)\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+\right.}{24b^3d(bc-ad)e^2}$$

Mathematica [A] time = 0.48, size = 247, normalized size = 0.70

$$\frac{\sqrt{d}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(105a^3d^2+5a^2bd\left(7dx^2-20c\right)+ab^2\left(3c^2-38cdx^2-14d^2x^4\right)+b^3x^2\left(3c^2+14cdx^2+8d^2x^4\right)\right)-3}{48b^4d^{3/2}e\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(105*a^3*d^2 + 5*a^2*b*d*(-20*c
+ 7*d*x^2) + a*b^2*(3*c^2 - 38*c*d*x^2 - 14*d^2*x^4) + b^3*x^2*(3*c^2 + 14*
c*d*x^2 + 8*d^2*x^4)) - 3*Sqrt[b*c - a*d]*(b^2*c^2 + 10*a*b*c*d - 35*a^2*d^
2)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(48*
b^4*d^(3/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c -
 a*d)])
```

fricas [A] time = 1.81, size = 781, normalized size = 2.21

$$\left[\frac{3\left(ab^3c^3+9\,a^2b^2c^2d-45\,a^3bcd^2+35\,a^4d^3+\left(b^4c^3+9\,ab^3c^2d-45\,a^2b^2cd^2+35\,a^3bd^3\right)x^2\right)\sqrt{bde}\,\log\left(8\,b^2d^2ex^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a*b^3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4
*c^3 + 9*a*b^3*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(b*d*e)*lo
g(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d
*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d -
 100*a^2*b^2*c^2*d^2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6
 + (17*b^4*c^2*d^2 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d -
35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x^2 + a*b^5*d^2*e^2), 1/96*(3*(a*b^3*c^3 + 9
*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4*c^3 + 9*a*b^3*c^2*d - 4
5*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b
*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b
*d*e)) + 2*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d - 100*a^2*b^2*c^2*d^2 + 105*a^3*b
*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6 + (17*b^4*c^2*d^2 - 52*a*b^3*c*
d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d - 35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*
d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x
^2 + a*b^5*d^2*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
```

```
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,2,2]
%%%},[2,1,2,0]%%%}+%%%{%%%{-4,[2,1,2]%%%},[2,1,1,1]%%%}+%%%{%%%{2,[3,0,2]%%
%},[2,1,0,2]%%%}+%%%{%%{[%%%{-4,[0,2,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[1,1,3,0]%%%}+%%%{%%{[%%%{8,[1,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1
,2,1]%%%}+%%%{%%{[%%%{-4,[2,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,1,
2]%%%}+%%%{%%%{2,[0,3,3]%%%},[0,1,4,0]%%%}+%%%{%%%{-4,[1,2,3]%%%},[0,1,3,1]
%%%}+%%%{%%%{2,[2,1,3]%%%},[0,1,2,2]%%%} / %%%{%%%{1,[2,0,2]%%%},[2,0,0,0]%
%%}+%%%{%%{[%%%{-2,[1,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,1,0]%%%}
+%%%{%%%{1,[1,1,3]%%%},[0,0,2,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.07, size = 1027, normalized size = 2.90

$$\frac{\left(-105a^3b\,d^3x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+135a^2b^2c\,d^2x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{b}}{2\sqrt{bd}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/96*(-60*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*a*b^2*d^2+12*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b^3*c*d-105*ln(1/2*(2*b
*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/
2))*x^2*a^3*b*d^3+135*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a^2*b^2*c*d^2-27*ln(1/2*(2*b*d*x^2
+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^
2*a*b^3*c^2*d-3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*b^4*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(3/2)*(b*d)^(1/2)*x^2*b^2*d+54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(
1/2)*x^2*a^2*b*d^2-108*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*
a*b^2*c*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b^3*c^2-105
*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2
))/(b*d)^(1/2))*a^4*d^3+135*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^3*b*c*d^2-27*ln(1/2*(2*b*d*x^2
+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^
2*b^2*c^2*d-3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(b*d)^(1/2))/(b*d)^(1/2))*a*b^3*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/
2)*(b*d)^(1/2)*a*b*d+114*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^
3*d^2-120*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*b*c*d+6*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b^2*c^2+96*(b*d)^(1/2)*((d*x^2
+c)*(b*x^2+a))^(1/2)*a^3*d^2-96*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2
*b*c*d)/d/b^4*(b*x^2+a)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/(
(b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

maxima [A] time = 1.83, size = 465, normalized size = 1.31

$$\frac{1}{96}e\left(\frac{2\left(48\left(a^2b^4cd-a^3b^3d^2\right)e^3+\frac{3\left(b^3c^3d^2+9\,ab^2c^2d^3-45\,a^2bcd^4+35\,a^3d^5\right)\left(bx^2+a\right)^3e^3}{\left(dx^2+c\right)^3}-\frac{8\left(b^4c^3d+9\,ab^3c^2d^2-45\,a^2b^2cd^3+35\,a^3bd^4\right)\left(b\right.}{\left(dx^2+c\right)^2}\right.}{b^4d^4\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{7}{2}}e^2-3\,b^5d^3\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{5}{2}}e^3+3\,b^6d^2\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}e^4-b^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*e*(2*(48*(a^2*b^4*c*d - a^3*b^3*d^2)*e^3 + 3*(b^3*c^3*d^2 + 9*a*b^2*c^
2*d^3 - 45*a^2*b*c*d^4 + 35*a^3*d^5)*(b*x^2 + a)^3*e^3/(d*x^2 + c)^3 - 8*(b
```

```
^4*c^3*d + 9*a*b^3*c^2*d^2 - 45*a^2*b^2*c*d^3 + 35*a^3*b*d^4)*(b*x^2 + a)^2
*e^3/(d*x^2 + c)^2 - 3*(b^5*c^3 - 23*a*b^4*c^2*d + 99*a^2*b^3*c*d^2 - 77*a^
3*b^2*d^3)*(b*x^2 + a)*e^3/(d*x^2 + c))/(b^4*d^4*((b*x^2 + a)*e/(d*x^2 + c)
)^(7/2)*e^2 - 3*b^5*d^3*((b*x^2 + a)*e/(d*x^2 + c))^(5/2)*e^3 + 3*b^6*d^2*(
(b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^4 - b^7*d*sqrt((b*x^2 + a)*e/(d*x^2 + c)
)*e^5) + 3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*log((d*s
qrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2
+ c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*b^4*d*e^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

3.308 $\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$\frac{3(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}\,e^{3/2}}+\frac{(c+dx^2)(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^3e^2}+\frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e^2}$$

```
[Out] 3/8*(-5*a*d+b*c)*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b
^(1/2)/e^(1/2))/b^(7/2)/e^(3/2)/d^(1/2)+a*(-a*d+b*c)/b^3/e/(e*(b*x^2+a)/(d*
x^2+c))^(1/2)+1/8*(-7*a*d+3*b*c)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^
3/e^2+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/e^2
```

Rubi [A] time = 0.24, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 456, 453, 208}

$$\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e^2}+\frac{(c+dx^2)(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^3e^2}+\frac{3(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}\,e^{3/2}}+\frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (a*(b*c - a*d))/(b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((3*b*c - 7*a*d
)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^3*e^2) + (Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b^2*e^2) + (3*(b*c - 5*a*d)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])
])/(8*b^(7/2)*Sqrt[d]*e^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
```

```
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{-ae+cx^2}{x^2\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} - \frac{1}{4}((bc-ad)e)\,\text{Subst}\left(\int \frac{\frac{4a}{b}-\frac{3(bc-ad)x^2}{b^2e}}{x^2\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{1}{8}((bc-ad)e)\,\text{Subst}\left(\int \frac{-\frac{8a}{b^2e}+ \ldots}{x^2\left(be \ldots\right.} \ldots\right.$$

$$= \frac{a(bc-ad)}{b^3e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{(3(bc-5ad)(bc \ldots}{\ldots}$$

$$= \frac{a(bc-ad)}{b^3e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{3(bc-5ad)(bc- \ldots}{8 \ldots}$$

Mathematica [A] time = 0.32, size = 190, normalized size = 0.94

$$\frac{\sqrt{d}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\left(-15a^2d+ab\left(13c-5dx^2\right)+b^2x^2\left(5c+2dx^2\right)\right)+3\sqrt{a+bx^2}\,(bc-5ad)\sqrt{bc-ad}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}} \ldots\right.}{8b^3\sqrt{d}\,e\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(-15*a^2*d + a*b*(13*c - 5*d*x^2
) + b^2*x^2*(5*c + 2*d*x^2)) + 3*(b*c - 5*a*d)*Sqrt[b*c - a*d]*Sqrt[a + b*x
^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^3*Sqrt[d]*e*Sq
rt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

fricas [A] time = 1.09, size = 585, normalized size = 2.90

$$\left[\frac{3\left(ab^2c^2-6\,a^2bcd+5\,a^3d^2+\left(b^3c^2-6\,ab^2cd+5\,a^2bd^2\right)x^2\right)\sqrt{bde}\,\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*
a^2*b*d^2)*x^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x
^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (
3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(2
*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3
)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 + a*b^4*d*e^2), -1/16*(3*(a*b^2*c^2 - 6*a^
2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqrt(-b*d*
e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(2*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15
*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d
^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 +
 a*b^4*d*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,2,2]
%%%},[2,1,2,0]%%%}+%%%{%%%{-4,[2,1,2]%%%},[2,1,1,1]%%%}+%%%{%%%{2,[3,0,2]%%
%},[2,1,0,2]%%%}+%%%{%%{[%%%{-4,[0,2,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[1,1,3,0]%%%}+%%%{%%{[%%%{8,[1,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1
,2,1]%%%}+%%%{%%{[%%%{-4,[2,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,1,
2]%%%}+%%%{%%%{2,[0,3,3]%%%},[0,1,4,0]%%%}+%%%{%%%{-4,[1,2,3]%%%},[0,1,3,1]
%%%}+%%%{%%%{2,[2,1,3]%%%},[0,1,2,2]%%%} / %%%{%%%{1,[2,0,2]%%%},[2,0,0,0]%
%%}+%%%{%%{[%%%{-2,[1,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,1,0]%%%}
+%%%{%%%{1,[1,1,3]%%%},[0,0,2,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.06, size = 679, normalized size = 3.36

$$-\frac{\left(-15a^2b\,d^2x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+18a\,b^2cd\,x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -1/16*(-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b^2*d-15*ln(1
/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b
```

```
*d)^(1/2))*x^2*a^2*b*d^2+18*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a*b^2*c*d-3*ln(1/2*(2*b*d*x^
2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x
^2*b^3*c^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*a*b*d-10*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b^2*c-15*ln(1/2*(2*b*d*
x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))
*a^3*d^2+18*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(b*d)^(1/2))/(b*d)^(1/2))*a^2*b*c*d-3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a*b^2*c^2+16*((d*x^2+
c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a^2*d-16*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^
(1/2)*a*b*c+14*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*d-10*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b*c)/b^3*(b*x^2+a)/(b*d)^(1/
2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

maxima [A] time = 1.81, size = 311, normalized size = 1.54

$$\frac{1}{16}e\left(\frac{2\left(8\left(ab^3c-a^2b^2d\right)e^2-\frac{3\left(b^2c^2d-6\,abcd^2+5\,a^2d^3\right)\left(bx^2+a\right)^2e^2}{\left(dx^2+c\right)^2}+\frac{5\left(b^3c^2-6\,ab^2cd+5\,a^2bd^2\right)\left(bx^2+a\right)e^2}{dx^2+c}\right)}{b^3d^2\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{5}{2}}e^2-2\,b^4d\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}e^3+b^5\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}\,e^4}-\frac{3\left(b^2c^2-6\,abcd+5\,a\right.}{\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*(8*(a*b^3*c - a^2*b^2*d)*e^2 - 3*(b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2
*d^3)*(b*x^2 + a)^2*e^2/(d*x^2 + c)^2 + 5*(b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*
d^2)*(b*x^2 + a)*e^2/(d*x^2 + c))/(b^3*d^2*((b*x^2 + a)*e/(d*x^2 + c))^(5/2
)*e^2 - 2*b^4*d*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^3 + b^5*sqrt((b*x^2 + a
)*e/(d*x^2 + c))*e^4) - 3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*log((d*sqrt((b*
x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) +
 sqrt(b*d*e)))/(sqrt(b*d*e)*b^3*e^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

3.309 $\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[Out] 3/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2
))*d^(1/2)/b^(5/2)/e^(3/2)-3/2*(-a*d+b*c)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/
2)+1/2*(d*x^2+c)/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1960, 290, 325, 208}

$$\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (-3*(b*c - a*d))/(2*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (c + d*x^2)/
(2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(5/2)*
e^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
```

```
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{x^2\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)\\
&= \frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{x^2(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2b}\\
&= -\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3d(bc-ad))\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2b^2e}\\
&= -\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 86, normalized size = 0.59

$$-\frac{\left(a+bx^2\right)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{d(bx^2+a)}{ad-bc}\right)}{b\left(\frac{b(c+dx^2)}{bc-ad}\right)^{3/2}\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -(((a + b*x^2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (d*(a + b*x^2))/(-(b*c) +
 a*d)])/(b*((e*(a + b*x^2))/(c + d*x^2))^(3/2)*((b*(c + d*x^2))/(b*c - a*d)
)^(3/2)))
```

fricas [A] time = 0.86, size = 443, normalized size = 3.03

$$\left[-\frac{3\left(\left(b^2c-abd\right)ex^2+\left(abc-a^2d\right)e\right)\sqrt{\frac{d}{be}}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2-4\left(2\,b^2d^2x^4\right.\right.}{8\left(b^3e^2x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(d/(b*e))*log(8*b^
2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(
2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) - 4*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (
b*c*d - 3*a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b
^2*e^2), -1/4*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(-d/(b*e))
*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(
-d/(b*e))/(b*d*x^2 + a*d)) - 2*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (b*c*d - 3*
a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b^2*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,2,2]
%%%},[2,1,2,0]%%%}+%%%{%%%{-4,[2,1,2]%%%},[2,1,1,1]%%%}+%%%{%%%{2,[3,0,2]%%
%},[2,1,0,2]%%%}+%%%{%%{[%%%{-4,[0,2,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[1,1,3,0]%%%}+%%%{%%{[%%%{8,[1,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1
,2,1]%%%}+%%%{%%{[%%%{-4,[2,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,1,
2]%%%}+%%%{%%%{2,[0,3,3]%%%},[0,1,4,0]%%%}+%%%{%%%{-4,[1,2,3]%%%},[0,1,3,1]
%%%}+%%%{%%%{2,[2,1,3]%%%},[0,1,2,2]%%%} / %%%{%%%{1,[2,0,2]%%%},[2,0,0,0]%
%%}+%%%{%%{[%%%{-2,[1,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,1,0]%%%}
+%%%{%%%{1,[1,1,3]%%%},[0,0,2,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.05, size = 432, normalized size = 2.96

$$\frac{\left(-3ab\,d^2x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3b^2cd\,x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-3a^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/4*(-3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2))/(b*d)^(1/2))*x^2*a*b*d^2+3*b^2*c*d*x^2*ln(1/2*(2*b*d*x^2+a*d+b*c+
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*d*x^2-3*a^2*d^2*ln(1/2*(2*b*d*x^2+
a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+3*a
*b*c*d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d
)^(1/2))/(b*d)^(1/2))+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*d
+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a*d-4*((d*x^2+c)*(b*x^2+a))^(1/2
)*(b*d)^(1/2)*b*c)/b^2*(b*x^2+a)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d
*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

maxima [A] time = 1.74, size = 199, normalized size = 1.36

$$\frac{1}{4}e\left(\frac{2\left(2\left(b^2c-abd\right)e-\frac{3\left(bcd-ad^2\right)\left(bx^2+a\right)e}{dx^2+c}\right)}{b^2d\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}e^2-b^3\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,e^3}-\frac{3\left(bcd-ad^2\right)\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,b^2e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(2*(b^2*c - a*b*d)*e - 3*(b*c*d - a*d^2)*(b*x^2 + a)*e/(d*x^2 + c)
)/(b^2*d*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^2 - b^3*sqrt((b*x^2 + a)*e/(d*
x^2 + c))*e^3) - 3*(b*c*d - a*d^2)*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) -
 sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*
e)*b^2*e^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

3.310 $\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}+\frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[Out] -c^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^(
3/2)/e^(3/2)+d^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/
e^(1/2))/b^(3/2)/e^(3/2)+(-a*d+b*c)/a/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 480, 522, 208}

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}+\frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (c^(3/2)*ArcTanh[(S
qrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(a^(3/2)*e^(3
/2)) + (d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])])/(b^(3/2)*e^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{x^2\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-(bc+ad)e+cdx^2}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{abe}$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{ae} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{be}$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}} + \frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}$$

Mathematica [A] time = 0.35, size = 253, normalized size = 1.66

$$\frac{-a^{3/2}d^{3/2}\sqrt{a+bx^2}\sqrt{c+dx^2}(ad-bc)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right) - b\left(c+dx^2\right)\sqrt{bc-ad}\left(bc^{3/2}\sqrt{a+bx^2}\tanh^{-1}\ldots\right)}{a^{3/2}b^2e\left(c+dx^2\right)^{3/2}\sqrt{bc-ad}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-(a^(3/2)*d^(3/2)*(-(b*c) + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[(b*(
c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]
) - b*Sqrt[b*c - a*d]*(c + d*x^2)*(Sqrt[a]*(-(b*c) + a*d)*Sqrt[c + d*x^2] +
 b*c^(3/2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[
c + d*x^2])]))/(a^(3/2)*b^2*Sqrt[b*c - a*d]*e*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)]*(c + d*x^2)^(3/2))
```

fricas [B] time = 1.60, size = 1293, normalized size = 8.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*d*e*x^2 + a^2*d*e)*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6
*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2
 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*s
qrt(d/(b*e))) + (b^2*c*e*x^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d +
 a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), -1/4*(2*(a
*b*d*e*x^2 + a^2*d*e)*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) - (b^2*c*e*x
^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^
2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2
+ (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)
))/x^4) - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), 1/4*(2*(b^2*c*e*x^2 + a*b*c*e)*sqrt(
-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + (a*b*d*e*x^2 + a^2*d*e)*sqrt(d/(b*e
))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2
)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*s
qrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 4*(b*c^2 - a*c*d + (b*c*d
 - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^
2), 1/2*((b^2*c*e*x^2 + a*b*c*e)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2
 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c))
 - (a*b*d*e*x^2 + a^2*d*e)*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d
)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) + 2*(b*
c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^
2*e^2*x^2 + a^2*b*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,2,2]
%%%},[2,1,3,0]%%%}+%%%{%%%{-4,[2,1,2]%%%},[2,1,2,1]%%%}+%%%{%%%{2,[3,0,2]%%
%},[2,1,1,2]%%%}+%%%{%%{[%%%{-4,[0,2,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},
[1,1,4,0]%%%}+%%%{%%{[%%%{8,[1,1,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1
,3,1]%%%}+%%%{%%{[%%%{-4,[2,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,2,
2]%%%}+%%%{%%%{2,[0,3,3]%%%},[0,1,5,0]%%%}+%%%{%%%{-4,[1,2,3]%%%},[0,1,4,1]
%%%}+%%%{%%%{2,[2,1,3]%%%},[0,1,3,2]%%%} / %%%{%%%{1,[2,0,0]%%%},[2,0,0,0]%
%%}+%%%{%%{[%%%{-2,[1,0,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,1,0]%%%}
+%%%{%%%{1,[1,1,1]%%%},[0,0,2,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.06, size = 401, normalized size = 2.64

$$-\frac{\left(-\sqrt{ac}\,ab\,d^{2}x^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+\sqrt{bd}\,b^{2}c^{2}x^{2}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+\cdots}}{x^{2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -1/2*(-ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d
)^(1/2))/(b*d)^(1/2))*(a*c)^(1/2)*x^2*a*b*d^2+(b*d)^(1/2)*ln((a*d*x^2+b*c*x
^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*b^2*c^
2-ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/
2))/(b*d)^(1/2))*(a*c)^(1/2)*a^2*d^2+(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*a*c+
2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*a*b*c^2+2*((d*x^2+c
)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)*a*d-2*((d*x^2+c)*(b*x^2+a))^(1/2
)*(b*d)^(1/2)*(a*c)^(1/2)*b*c)/a/b*(b*x^2+a)/(a*c)^(1/2)/(b*d)^(1/2)/((d*x^
2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

maxima [A] time = 1.92, size = 204, normalized size = 1.34

$$\frac{1}{2}\,e\left(\frac{c^2\log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,ae^2}-\frac{d^2\log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}-\sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}}+\sqrt{bde}}\right)}{\sqrt{bde}\,be^2}+\frac{2\,(bc-ad)}{ab\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*(c^2*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b
*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*a*e^2) - d^2*log((d*s
qrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2
+ c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*b*e^2) + 2*(b*c - a*d)/(a*b*sqrt((b*x^2
+ a)*e/(d*x^2 + c))*e^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

3.311 $\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{3\sqrt{c}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

```
[Out] 3/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2
))*c^(1/2)/a^(5/2)/e^(3/2)-3/2*(-a*d+b*c)/a^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/
2)+1/2*(-a*d+b*c)/a/(a*e-c*e*(b*x^2+a)/(d*x^2+c))/(e*(b*x^2+a)/(d*x^2+c))^(
1/2)
```

Rubi [A] time = 0.11, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1960, 290, 325, 208}

$$\frac{3\sqrt{c}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-3*(b*c - a*d))/(2*a^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (b*c - a*d)/
(2*a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(a*e - (c*e*(a + b*x^2))/(c + d*x^2)
)) + (3*Sqrt[c]*(b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^
2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(5/2)*e^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{1}{x^2\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{x^2(-ae+cx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2a}$$

$$= -\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(3c(bc-ad))\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\dots}\right)}{2a^2e}$$

$$= -\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{3\sqrt{c}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}$$

Mathematica [A] time = 0.09, size = 148, normalized size = 0.87

$$\frac{3\sqrt{c}\,x^2\sqrt{a+bx^2}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)-\sqrt{a}\sqrt{c+dx^2}\left(a\left(c-2dx^2\right)+3bcx^2\right)}{2a^{5/2}ex^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[c + d*x^2]*(3*b*c*x^2 + a*(c - 2*d*x^2))) + 3*Sqrt[c]*(b*c
- a*d)*x^2*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[
c + d*x^2])])/(2*a^(5/2)*e*x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d
*x^2])
```

fricas [A] time = 2.18, size = 469, normalized size = 2.76

$$\left[-\frac{3\left(\left(b^2c-abd\right)ex^4+\left(abc-a^2d\right)ex^2\right)\sqrt{\frac{c}{ae}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left(\left(abcd+a^2d^2\right)x^4+2\,a^2c^2+\left(abc^2+3\,a\dots\right.\right.}{x^4}\right.}{8\left(a^2be^2x^4+a^3e^2x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*d)*e*x^2)*sqrt(c/(a*e))*log(
((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^
2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*((3*b*c*d - 2*a*d^2)
*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a
^2*b*e^2*x^4 + a^3*e^2*x^2), -1/4*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*
d)*e*x^2)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + 2*((3*b*c*d - 2*a*d^
2)*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
(a^2*b*e^2*x^4 + a^3*e^2*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,0,0]
%%%},[6,1,0,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[5,1,1,0]%%%}+
%%%{%%%{2,[0,1,1]%%%},[4,1,2,0]%%%}+%%%{%%%{-4,[1,0,1]%%%},[4,1,1,1]%%%}+%%
%{%%{[%%%{8,[0,0,1]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[3,1,2,1]%%%}+%%%{%%
%{-4,[0,1,2]%%%},[2,1,3,1]%%%}+%%%{%%%{2,[1,0,2]%%%},[2,1,2,2]%%%}+%%%{%%{[
%%%{-4,[0,0,2]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,3,2]%%%}+%%%{%%%{2,[
0,1,3]%%%},[0,1,4,2]%%%} / %%%{%%%{1,[2,0,0]%%%},[6,0,0,0]%%%}+%%%{%%{[%%%{
-2,[1,0,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[5,0,1,0]%%%}+%%%{%%%{1,[1,1,
1]%%%},[4,0,2,0]%%%}+%%%{%%%{-2,[2,0,1]%%%},[4,0,1,1]%%%}+%%%{%%{[%%%{4,[1,
0,1]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[3,0,2,1]%%%}+%%%{%%%{-2,[1,1,2]%%%
},[2,0,3,1]%%%}+%%%{%%%{1,[2,0,2]%%%},[2,0,2,2]%%%}+%%%{%%{[%%%{-2,[1,0,2]%
%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,3,2]%%%}+%%%{%%%{1,[1,1,3]%%%},[0,0
,4,2]%%%} Error: Bad Argument Value
```

maple [B] time = 0.07, size = 641, normalized size = 3.77

$$\frac{\left(-3a^2bcd\,x^4\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+3a\,b^2c^2x^4\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/4*(2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b^2*d-3*ln((a*d*
x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x
^4*a^2*b*c*d+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2))/x^2)*x^4*a*b^2*c^2+4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(
a*c)^(1/2)*x^4*a*b*d+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*
b^2*c-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2))/x^2)*x^2*a^3*c*d+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a^2*b*c^2+4*((d*x^2+c)*(b*x^2+a))^(
1/2)*(a*c)^(1/2)*x^2*a^2*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^2*a*
b*c-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*b+2*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^2*a^2*d+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(a*c)^(1/2)*x^2*a*b*c-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(
```

```
1/2)*a)*(b*x^2+a)/(a*c)^(1/2)/x^2/a^3/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)
/((b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

maxima [A] time = 1.97, size = 197, normalized size = 1.16

$$\frac{1}{4}\,e\left(\frac{2\left(2\left(abc-a^2d\right)e-\frac{3\left(bc^2-acd\right)\left(bx^2+a\right)e}{dx^2+c}\right)}{a^2c\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}e^2-a^3\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}\,e^3}-\frac{3\,(bc-ad)c\log\left(\frac{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}-\sqrt{ace}}{c\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}+\sqrt{ace}}\right)}{\sqrt{ace}\,a^2e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(2*(a*b*c - a^2*d)*e - 3*(b*c^2 - a*c*d)*(b*x^2 + a)*e/(d*x^2 + c)
)/(a^2*c*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^2 - a^3*sqrt((b*x^2 + a)*e/(d*
x^2 + c))*e^3) - 3*(b*c - a*d)*c*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - s
qrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)
*a^2*e^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

3.312 $\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=255

$$-\frac{3(5bc-ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}\,e^{3/2}}-\frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)}+\frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+}{c+}}}{4a^2\left(ae-\frac{ce(a+bx}{c+dx^2}\right.}$$

```
[Out] -3/8*(-a*d+b*c)*(-a*d+5*b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
a^(1/2)/e^(1/2))/a^(7/2)/e^(3/2)/c^(1/2)+b*(-a*d+b*c)/a^3/e/(e*(b*x^2+a)/(d
*x^2+c))^(1/2)-1/4*(-a*d+b*c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/(a*e-c*e*
(b*x^2+a)/(d*x^2+c))^2-1/8*(-3*a*d+7*b*c)*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)/a^3/(a*e^2-c*e^2*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] time = 0.23, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1960, 456, 453, 208}

$$-\frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)}-\frac{3(5bc-ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}\,e^{3/2}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}+\frac{b(bc-a}{a^3e\sqrt{\frac{e(a+}{c+}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*(b*c - a*d))/(a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((b*c - a*d)^2*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*a^2*(a*e - (c*e*(a + b*x^2))/(c + d*x
^2))^2) - ((7*b*c - 3*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
8*a^3*(a*e^2 - (c*e^2*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - a*d)*(5*b*c -
a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]
)/(8*a^(7/2)*Sqrt[c]*e^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\,\text{Subst}\left(\int \frac{be-dx^2}{x^2\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{1}{4}((bc-ad)e)\,\text{Subst}\left(\int \frac{\frac{4b}{a}+\frac{3(bc-ad)x^2}{a^2e}}{x^2\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} + \frac{1}{8}((bc-ad)e)\,\text{Subst}\left(\int \frac{\frac{8b}{a^2e}+}{x^2\left(-\right.}\right.$$

$$= \frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} + \frac{(3(bc-ad)(5bc}{}$$

$$= \frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} - \frac{3(bc-ad)(5bc}{}$$

Mathematica [A] time = 0.12, size = 189, normalized size = 0.74

$$\frac{\sqrt{a}\sqrt{c}\sqrt{c+dx^2}\left(-a^2\left(2c+5dx^2\right)+abx^2\left(5c-13dx^2\right)+15b^2cx^4\right)-3x^4\sqrt{a+bx^2}\left(a^2d^2-6abcd+5b^2c^2\right)\tanh^{-1}}{8a^{7/2}\sqrt{c}\,ex^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(5*c - 13*d*x^2) -
 a^2*(2*c + 5*d*x^2)) - 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4*Sqrt[a + b*
x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(7/
2)*Sqrt[c]*e*x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

fricas [A] time = 7.60, size = 613, normalized size = 2.40

$$\left[\frac{3\left(\left(5\,b^3c^2-6\,ab^2cd+a^2bd^2\right)x^6+\left(5\,ab^2c^2-6\,a^2bcd+a^3d^2\right)x^4\right)\sqrt{ace}\,\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\left(abc^2+a^2c\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2*
b*c*d + a^3*d^2)*x^4)*sqrt(a*c*e)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^
4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 + 2*
a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/x^4) + 4*((15*a*b^2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a*b^2*
c^3 - 8*a^2*b*c^2*d - 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*x^2)*s
qrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4), 1/16*(
3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2*b*c*d +
 a^3*d^2)*x^4)*sqrt(-a*c*e)*arctan(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*
c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)) + 2*((15*a*b^
2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a*b^2*c^3 - 8*a^2*b*c^2*d -
 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%%{%%%{2,[1,4,4]
%%%},[2,1,7,0]%%%}+%%%{%%%{-8,[2,3,4]%%%},[2,1,6,1]%%%}+%%%{%%%{12,[3,2,4]%
%%},[2,1,5,2]%%%}+%%%{%%%{-8,[4,1,4]%%%},[2,1,4,3]%%%}+%%%{%%%{2,[5,0,4]%%%
},[2,1,3,4]%%%}+%%%{%%{[%%%{-4,[0,4,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[
1,1,8,0]%%%}+%%%{%%{[%%%{16,[1,3,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1
,7,1]%%%}+%%%{%%{[%%%{-24,[2,2,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,6
,2]%%%}+%%%{%%{[%%%{16,[3,1,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,5,3]
%%%}+%%%{%%{[%%%{-4,[4,0,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,1,4,4]%%%
}+%%%{%%%{2,[0,5,5]%%%},[0,1,9,0]%%%}+%%%{%%%{-8,[1,4,5]%%%},[0,1,8,1]%%%}+
%%%{%%%{12,[2,3,5]%%%},[0,1,7,2]%%%}+%%%{%%%{-8,[3,2,5]%%%},[0,1,6,3]%%%}+%
%%{%%%{2,[4,1,5]%%%},[0,1,5,4]%%%} / %%%{%%%{1,[2,0,0]%%%},[2,0,0,0]%%%}+%%
%{%%{[%%%{-2,[1,0,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[1,0,1,0]%%%}+%%%{%
%%{1,[1,1,1]%%%},[0,0,2,0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.08, size = 1042, normalized size = 4.09

$$-\frac{\left(3a^3bc\,d^2x^6\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{x^2}\right)-18a^2b^2c^2d\,x^6\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+}}{x^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -1/16*(-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a*b^2*d^2+18*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*b^3*c*d+3*ln((a*d*x^2+b
*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^
3*b*c*d^2-18*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2))/x^2)*x^6*a^2*b^2*c^2*d+15*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^
(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a*b^3*c^3-12*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*b*d^2+26*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a*b^2*c*d+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(a*c)^(1/2)*x^6*b^3*c^2+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^4*c*d^2-18*ln((a*d*x^2+b*c*x^2
+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^3*b*c^
2*d+15*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2))/x^2)*x^4*a^2*b^2*c^3+16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^
4*a^2*b*c*d-16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^4*a*b^2*c^2+6*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*a*b*d-18*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*b^2*c-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(a*c)^(1/2)*x^4*a^3*d^2+8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2
)*x^4*a^2*b*c*d+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b^
2*c^2+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a^2*d-14*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a*b*c+4*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c)/c*(b*x^2+a)/(a*c)^(1/2)/x^4/a^4/((d*x^2+c
)*(b*x^2+a))^(1/2)/(d*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

maxima [A] time = 1.96, size = 311, normalized size = 1.22

$$\frac{1}{16}e\left(\frac{2\left(8\left(a^2b^2c-a^3bd\right)e^2+\frac{3\left(5b^2c^3-6abc^2d+a^2cd^2\right)\left(bx^2+a\right)^2e^2}{\left(dx^2+c\right)^2}-\frac{5\left(5ab^2c^2-6a^2bcd+a^3d^2\right)\left(bx^2+a\right)e^2}{dx^2+c}\right)}{a^3c^2\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{5}{2}}e^2-2a^4c\left(\frac{\left(bx^2+a\right)e}{dx^2+c}\right)^{\frac{3}{2}}e^3+a^5\sqrt{\frac{\left(bx^2+a\right)e}{dx^2+c}}e^4}+\frac{3\left(5b^2c^2-6abcd+a\right.}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*(8*(a^2*b^2*c - a^3*b*d)*e^2 + 3*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c
*d^2)*(b*x^2 + a)^2*e^2/(d*x^2 + c)^2 - 5*(5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*
d^2)*(b*x^2 + a)*e^2/(d*x^2 + c))/(a^3*c^2*((b*x^2 + a)*e/(d*x^2 + c))^(5/2
)*e^2 - 2*a^4*c*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^3 + a^5*sqrt((b*x^2 + a
)*e/(d*x^2 + c))*e^4) + 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*log((c*sqrt((b*
x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) +
 sqrt(a*c*e)))/(sqrt(a*c*e)*a^3*e^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

3.313 $\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=453

$$\frac{x\left(a+bx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)}{5b^4e\left(c+dx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-\frac{\sqrt{c}\left(a+bx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^4\sqrt{d}\,e\left(c+dx^2\right)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-\frac{c^{3/2}\left(a+\right.}{5b^3}$$

```
[Out] 1/5*(-8*a*d+7*b*c)*x*(b*x^2+a)/b^3/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+6/5*d*x^
3*(b*x^2+a)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/5*(16*a^2*d^2-16*a*b*c*d+
b^2*c^2)*x*(b*x^2+a)/b^4/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-x^3*(d*x
^2+c)/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/5*c^(3/2)*(-8*a*d+7*b*c)*(b*x^2+a
)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/b^3/e/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/5*(16*a^2*d^2-16*a*b*c*d+b^2*c
^2)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)/b^4/e/(d*x^2+c)/d^(1/2)
/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6719, 467, 581, 582, 531, 418, 492, 411}

$$\frac{x\left(a+bx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)}{5b^4e\left(c+dx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-\frac{\sqrt{c}\left(a+bx^2\right)\left(16a^2d^2-16abcd+b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^4\sqrt{d}\,e\left(c+dx^2\right)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-\frac{c^{3/2}\left(a+\right.}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] ((7*b*c - 8*a*d)*x*(a + b*x^2))/(5*b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
 + (6*d*x^3*(a + b*x^2))/(5*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b^
2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*x*(a + b*x^2))/(5*b^4*e*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)]*(c + d*x^2)) - (x^3*(c + d*x^2))/(b*e*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)]) - (Sqrt[c]*(b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*(a + b*x^2)*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b^4*Sqrt[d]*e*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d
*x^2)) - (c^(3/2)*(7*b*c - 8*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b^3*Sqrt[d]*e*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] && !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{x^2\sqrt{c+dx^2}(3c+6dx^2)}{\sqrt{a+bx^2}}\,dx}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{x^2(3c(5bc-6ad)+3d(7bc-8ad)x^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{3acd(7bc-8ad)-3d(b^2c^2-1}{\sqrt{a+bx^2}\sqrt{c+}}}{15b^3de\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+}}$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(ac(7bc-8ad)\sqrt{a+bx^2}\right)\int \frac{}{\sqrt{a+b}}}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx}}$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-16abcd+16a^2d^2)x(a+bx^2)}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{x^3(c+}{be\sqrt{\frac{e(a-}{c+}}}$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-16abcd+16a^2d^2)x(a+bx^2)}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{x^3(c+}{be\sqrt{\frac{e(a-}{c+}}}$$

Mathematica [C] time = 0.50, size = 271, normalized size = 0.60

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(8a^2d^2-9abcd+b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(16a^2d^2-\right.\right.}{5b^3de^2\sqrt{\frac{b}{a}}\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(c + d*x^2)*(-8*a^2*d + a
*b*(7*c - 2*d*x^2) + b^2*x^2*(2*c + d*x^2)) - I*c*(b^2*c^2 - 16*a*b*c*d + 1
6*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] + I*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])
)/(5*b^3*Sqrt[b/a]*d*e^2*(a + b*x^2))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^8+2\,cdx^6+c^2x^4\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{b^2e^2x^4+2\,abe^2x^2+a^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^8 + 2*c*d*x^6 + c^2*x^4)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/
(b^2*e^2*x^4 + 2*a*b*e^2*x^2 + a^2*e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

maple [A] time = 0.06, size = 935, normalized size = 2.06

$$\left(b\,x^2+a\right)\left(\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,b^2 d^3 x^7-2\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,a b\,d^3 x^5+3\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/5*(b*x^2+a)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^7*b^2*d^3-2*((d
*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^5*a*b*d^3+3*((d*x^2+c)*(b*x^2+a))
^(1/2)*(-1/a*b)^(1/2)*x^5*b^2*c*d^2-3*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^
(1/2)*a^2*d^3*x^3+2*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^3*b^2*c^2*
d-5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^3*a^2*d^3+5*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^3*a*b*c*d^2-8*((d*x^2+c)*(b*
x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1
/2)*x,(a/b/c*d)^(1/2))*a^2*c*d^2+9*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*b
*c^2*d-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b^2*c^3+16*((d*x^2+c)*(b*x^2+a)
)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,
(a/b/c*d)^(1/2))*a^2*c*d^2-16*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*b*c^2*
d+((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b^2*c^3-3*((d*x^2+c)*(b*x^2+a))^(1/2
)*(-1/a*b)^(1/2)*x*a^2*c*d^2+2*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x
*a*b*c^2*d-5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*a^2*c*d^2*x
+5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*a*b*c^2*d*x)/b^3/d/((
b*x^2+a)/(d*x^2+c)*e)^(3/2)/(d*x^2+c)^2/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

3.314 $\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=378

$$\frac{dx\left(a+bx^2\right)(7bc-8ad)}{3b^3e\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)(7bc-8ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^3e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c^{3/2}\left(a+bx^2\right)(3bc-4ad)F\left(\text{t}\right.}{3ab^2\sqrt{d}\,e\left(c+dx^2\right)\sqrt{\frac{c(a}{a(c}}}$$

```
[Out] 4/3*d*x*(b*x^2+a)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*d*(-8*a*d+7*b*c)*
x*(b*x^2+a)/b^3/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-x*(d*x^2+c)/b/e/(
e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*c^(3/2)*(-4*a*d+3*b*c)*(b*x^2+a)*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(1-b*c/a/d)^(1/2))/a/b^2/e/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-8*a*d+7*b*c)*(b*x^2+a)*(1/(1+d*x^2/
c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/b^3/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 467, 528, 531, 418, 492, 411}

$$\frac{c^{3/2}\left(a+bx^2\right)(3bc-4ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab^2\sqrt{d}\,e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{4dx\left(a+bx^2\right)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)(7bc-8ad)}{3b^3e\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right.}{3b^3e\left(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (4*d*x*(a + b*x^2))/(3*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (d*(7*b*c
 - 8*a*d)*x*(a + b*x^2))/(3*b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*
x^2)) - (x*(c + d*x^2))/(b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (Sqrt[c]*
Sqrt[d]*(7*b*c - 8*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(3*b^3*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a
 + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*(3*b*c - 4*a*d)*(a + b*x^2)
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^2*Sqrt[d]*
e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(
c + d*x^2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{x^2}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^2\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx}{e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{x\left(c+dx^2\right)}{be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}\left(c+4dx^2\right)}{\sqrt{a+bx^2}}\,dx}{be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{4dx\left(a+bx^2\right)}{3b^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{x\left(c+dx^2\right)}{be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{c(3bc-4ad)+d(7bc-8ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{4dx\left(a+bx^2\right)}{3b^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{x\left(c+dx^2\right)}{be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{\left(d(7bc-8ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(c(3bc-\right.}{}$$

$$= \frac{4dx\left(a+bx^2\right)}{3b^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{d(7bc-8ad)x\left(a+bx^2\right)}{3b^3e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)} - \frac{x\left(c+dx^2\right)}{be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{c^{3/2}(3bc-4ad)\left(a+bx^2\right)F\left(\right.}{3ab^2\sqrt{d}\,e\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e}{}}}$$

$$= \frac{4dx\left(a+bx^2\right)}{3b^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{d(7bc-8ad)x\left(a+bx^2\right)}{3b^3e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)} - \frac{x\left(c+dx^2\right)}{be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{d}(7bc-8ad)\left(a+bx^2\right.}{3b^3e\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e(}{}}}$$

Mathematica [C] time = 0.41, size = 219, normalized size = 0.58

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(x\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(4ad-3bc+bdx^2\right)-4ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ic\sqrt{\right.}{3a^2e^2\left(\frac{b}{a}\right)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*x*(c + d*x^2)*(-3*b*c + 4*a*d
 + b*d*x^2) + I*c*(-7*b*c + 8*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (4*I)*c*(-(b*c) + a*d)*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)]))/(3*a^2*(b/a)^(5/2)*e^2*(a + b*x^2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^6+2\,cdx^4+c^2x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{b^2e^2x^4+2\,abe^2x^2+a^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

[Out] integral((d^2*x^6 + 2*c*d*x^4 + c^2*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e^2*x^4 + 2*a*b*e^2*x^2 + a^2*e^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")

[Out] integrate(x^2/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)

maple [A] time = 0.03, size = 643, normalized size = 1.70

$$\frac{(b\,x^2+a)\left(\sqrt{(d\,x^2+c)\,(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{(d\,x^2+c)\,(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,a\,d^2x^3+3\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)

[Out] 1/3*(b*x^2+a)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^5*b*d^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^3*a*d^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^3*b*c*d+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^3*a*d^2-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^3*b*c*d+4*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*c*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b*c^2-8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*c*d+7*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b*c^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x*a*c*d+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x*a*c*d-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x*b*c^2)/b^2/((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(d*x^2+c)^2/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] integrate(x^2/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

3.315 $\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=327

$$-\frac{dx\left(a+bx^2\right)(bc-2ad)}{ab^2e\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c^{3/2}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{abe\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a}{c}}}$$

```
[Out] (-a*d+b*c)*x/a/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-d*(-2*a*d+b*c)*x*(b*x^2+a)
/a/b^2/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*(1/(1+d*
x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/
2),(1-b*c/a/d)^(1/2))*d^(1/2)/a/b/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+(-2*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/
2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a
/d)^(1/2))*c^(1/2)*d^(1/2)/a/b^2/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6719, 413, 531, 418, 492, 411}

$$-\frac{dx\left(a+bx^2\right)(bc-2ad)}{ab^2e\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c^{3/2}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{abe\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(b*c - 2*a*d
)*x*(a + b*x^2))/(a*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) +
(Sqrt[c]*Sqrt[d]*(b*c - 2*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(a*b^2*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*Sqrt[d]*(a + b*x^2)*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*b*e*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx}{e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{acd-d(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{\left(cd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{be\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}} - \frac{\left(d(bc-2ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+}}}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{d(bc-2ad)x\left(a+bx^2\right)}{ab^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)} + \frac{c^{3/2}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{abe\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)} +$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{d(bc-2ad)x\left(a+bx^2\right)}{ab^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)} + \frac{\sqrt{c}\sqrt{d}(bc-2ad)\left(a+bx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{ab^2e\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}$$

Mathematica [C] time = 0.47, size = 203, normalized size = 0.62

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left((bc-ad)\left(x\sqrt{\frac{b}{a}}\left(c+dx^2\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(2ad}{a^2e^2\left(\frac{b}{a}\right)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*((-I)*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (
b*c - a*d)*(Sqrt[b/a]*x*(c + d*x^2) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(a^2*(b/a)^(3/2)*e
^2*(a + b*x^2))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^4+2\,cdx^2+c^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{b^2e^2x^4+2\,abe^2x^2+a^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2
*e^2*x^4 + 2*a*b*e^2*x^2 + a^2*e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(-3/2), x)
```

maple [A] time = 0.03, size = 514, normalized size = 1.57

$$-\frac{\left(b\,x^2+a\right)\left(\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,a\,d^2x^3-\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,bcd\,x^3+\sqrt{bd\,x^4+ad\,x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -(b*x^2+a)/b*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*a*d^2*x^3-
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*b*c*d*x^3+((d*x^2+c)*(b*
x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1
/2)*x,(a/b/c*d)^(1/2))*a*c*d-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*b*c^2-2*(
(d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*a*c*d+((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x
```

```
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/
2))*b*c^2+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*a*c*d*x-(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*b*c^2*x)/((b*x^2+a)/(d*x^2+c)*
e)^(3/2)/(d*x^2+c)^2/a/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(1/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

3.316 $\int \frac{1}{x^2\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=380

$$\frac{c^{3/2}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2e\left(c+dx^2\right)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-\frac{\left(a+bx^2\right)(2bc-ad)}{a^2bex\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)(2bc-ad)}{a^2be\left(c+dx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)(2bc}{a^2be\left(c+dx^2\right.}$$

```
[Out] (-a*d+b*c)/a/b/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(-a*d+2*b*c)*(b*x^2+a)/a^2
/b/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+d*(-a*d+2*b*c)*x*(b*x^2+a)/a^2/b/e/(d*
x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/
d)^(1/2))*d^(1/2)/a^2/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2
+a)/(d*x^2+c))^(1/2)-(-a*d+2*b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/
c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c
^(1/2)*d^(1/2)/a^2/b/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+
a)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 468, 583, 531, 418, 492, 411}

$$\frac{c^{3/2}\sqrt{d}\left(a+bx^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2e\left(c+dx^2\right)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-\frac{\left(a+bx^2\right)(2bc-ad)}{a^2bex\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}+\frac{dx\left(a+bx^2\right)(2bc-ad)}{a^2be\left(c+dx^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}\left(a+bx^2\right)(2bc}{a^2be\left(c+dx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((2*b*c - a*d)*(a
 + b*x^2))/(a^2*b*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (d*(2*b*c - a*d)
*x*(a + b*x^2))/(a^2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (
Sqrt[c]*Sqrt[d]*(2*b*c - a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(a^2*b*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*Sqrt[d]*(a + b*x^2)*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a^2*e*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{-c(2bc-ad)-bcdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{abc^2d+bcd(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a^2bce\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(cd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{ae\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{(d(2bc-ad}{a^2 \cdots}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(2bc-ad)x(a+bx^2)}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{c^{3/2}\sqrt{d}(a+bx^2)F(\cdots}{a^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\cdots}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(2bc-ad)x(a+bx^2)}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\sqrt{c}\sqrt{d}(2bc-ad)(a\cdots}{a^2be\sqrt{\frac{c(a+bx\cdots}{a(c+dx\cdots}}}$$

Mathematica [C] time = 0.37, size = 223, normalized size = 0.59

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}(c+dx^2)(ac-adx^2+2bcx^2) - 2icx\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) + icx\sqrt{\cdots}\right.}{a^2e^2x\sqrt{\frac{b}{a}}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(a*c + 2*b*c*x^
2 - a*d*x^2)) + I*c*(-2*b*c + a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d)*x
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)]))/(a^2*Sqrt[b/a]*e^2*x*(a + b*x^2))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(d^2x^4+2\,cdx^2+c^2)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{b^2e^2x^6+2\,abe^2x^4+a^2e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2
*e^2*x^6 + 2*a*b*e^2*x^4 + a^2*e^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2), x)
```

maple [A] time = 0.04, size = 650, normalized size = 1.71

$$-\frac{\left(b\,x^2+a\right)\left(-\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,a\,d^2x^4+\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,bcd\,x^4+\sqrt{bd\,x^4+ad\,x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -(b*x^2+a)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^4*b*c*d-(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^4*a*d^2+(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*(-1/a*b)^(1/2)*x^4*b*c*d-2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))
*x*a*c*d+2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x*b*c^2+((d*x^2+c)*(b*x^2+a
))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x
,(a/b/c*d)^(1/2))*x*a*c*d-2*b*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x*((d*x^2+c)*(b*x^2+a))^(1/2)+((
d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*x^2*a*c*d+((d*x^2+c)*(b*x^2+a))^(1
/2)*(-1/a*b)^(1/2)*x^2*b*c^2-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(
1/2)*x^2*a*c*d+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^2*b*c^2
+((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*a*c^2)/((b*x^2+a)/(d*x^2+c)*e)^
(3/2)/(d*x^2+c)^2/a^2/x/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

[Out] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)

[Out] Timed out

3.317 $\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=444

$$\frac{(a+bx^2)(8bc-7ad)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{dx(a+bx^2)(8bc-7ad)}{3a^3e(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}(a+bx^2)(4bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^3e(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+$$

```
[Out] (-a*d+b*c)/a/b/e/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4*b*c)*(b*x^
2+a)/a^2/b/e/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-7*a*d+8*b*c)*(b*x^2+a)
/a^3/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*d*(-7*a*d+8*b*c)*x*(b*x^2+a)/a^3
/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-7*a*d+8*b*c)*(b*x^2+a)*(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c
)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^3/e/(d*x^2+c)/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4*b*c)*(b*x^2+a)*
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^3/e/(d*x^2+c)/(c*(b*x^2+a)/
a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 468, 583, 531, 418, 492, 411}

$$\frac{(a+bx^2)(8bc-7ad)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(a+bx^2)(4bc-3ad)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{dx(a+bx^2)(8bc-7ad)}{3a^3e(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}(a+bx^2)(4bc-3ad)F\left(\tan^{-1}\right.}{3a^3e(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((4*b*c - 3*a*d
)*(a + b*x^2))/(3*a^2*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((8*b*c
- 7*a*d)*(a + b*x^2))/(3*a^3*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(8
*b*c - 7*a*d)*x*(a + b*x^2))/(3*a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c
+ d*x^2)) + (Sqrt[c]*Sqrt[d]*(8*b*c - 7*a*d)*(a + b*x^2)*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^3*e*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[c]*Sqrt[d
]*(4*b*c - 3*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*a^3*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x
^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(a+bx^2\right)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{-c(4bc-3ad)-d(3bc-2ad)x^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)\left(a+bx^2\right)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{-bc^2(8bc-7ad)-bcd(4bc-3ad)x^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a^2bce\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)\left(a+bx^2\right)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)\left(a+bx^2\right)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{abc^2d(4}{}}{3a^3bc^2e\sqrt{}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)\left(a+bx^2\right)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)\left(a+bx^2\right)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(bd(8bc-7ad)\sqrt{a}\right.}{3a^3e\sqrt{}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)\left(a+bx^2\right)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)\left(a+bx^2\right)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(8bc-7ad)x\left(a\right.}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c\right.}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)\left(a+bx^2\right)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)\left(a+bx^2\right)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(8bc-7ad)x\left(a\right.}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c\right.}$$

Mathematica [C] time = 0.48, size = 266, normalized size = 0.60

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-ix^3\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-11abcd+8b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(a^2\left(c+\right.\right.\right.}{3a^3e^2x^3\sqrt{\frac{b}{a}}\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(-8*b^2*c*x^4 +
 a^2*(c + 4*d*x^2) + a*b*(-4*c*x^2 + 7*d*x^4))) - I*b*c*(-8*b*c + 7*a*d)*x^
3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] - I*(8*b^2*c^2 - 11*a*b*c*d + 3*a^2*d^2)*x^3*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3
*a^3*Sqrt[b/a]*e^2*x^3*(a + b*x^2))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^4+2\,cdx^2+c^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{b^2e^2x^8+2\,abe^2x^6+a^2e^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2
*e^2*x^8 + 2*a*b*e^2*x^6 + a^2*e^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4), x)
```

maple [A] time = 0.04, size = 866, normalized size = 1.95

$$-\frac{\left(b\,x^2+a\right)\left(4\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^6+3\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^6-5\sqrt{\left(d\,x^2+c\right)\left(\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -1/3*(b*x^2+a)*(4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*a*b*d^2*x^6-5*
((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*b^2*c*d*x^6+3*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*a*b*d^2*x^6-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(-1/a*b)^(1/2)*b^2*c*d*x^6-3*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x
^3*a^2*d^2+11*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^3*a*b*c*d-8*((d*x^2+c)
*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/a*b
)^(1/2)*x,(a/b/c*d)^(1/2))*x^3*b^2*c^2-7*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2
))*x^3*a*b*c*d+8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticE((-1/a*b)^(1/2)*x,(a/b/c*d)^(1/2))*x^3*b^2*c^2+4*((d*x^2
+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*a^2*d^2*x^4-5*((d*x^2+c)*(b*x^2+a))^(1/
2)*(-1/a*b)^(1/2)*b^2*c^2*x^4+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b
)^(1/2)*a*b*c*d*x^4-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-1/a*b)^(1/2)*x^
4*b^2*c^2+5*((d*x^2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*a^2*c*d*x^2-4*((d*x^
2+c)*(b*x^2+a))^(1/2)*(-1/a*b)^(1/2)*a*b*c^2*x^2+((d*x^2+c)*(b*x^2+a))^(1/2
)*(-1/a*b)^(1/2)*a^2*c^2)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(d*x^2+c)^2/a^3/x^3
/(-1/a*b)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(\frac{e\left(b x^2+a\right)}{d x^2+c}\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

3.318 $\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=216

$$-\frac{\left(-8a^2c^2+4abc+b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16a^2d^3}+\frac{b\left(8a^2c^2+4abc+b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3}+\frac{\left(c+dx^2\right)^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3}}{6ad^3}$$

```
[Out] 1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/a/d^3+1/16*b*(8*a^2*c^2+4
*a*b*c+b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2)/d^3-
1/16*(-8*a^2*c^2+4*a*b*c+b^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a
^2/d^3-1/8*(4*a*c+b)*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^3
```

Rubi [A] time = 0.62, antiderivative size = 259, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(8a^2c^2+4abc+b^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3}+\frac{b\left(8a^2c^2+4abc+b^2\right)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{16a^{5/2}d^3\sqrt{a\left(c+dx^2\right)+b}}-\frac{(8\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((b^2 + 4*a*b*c + 8*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(16*a^2*d
^3) - ((3*b + 8*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)
))/(24*a^2*d^3) + (x^2*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^
2)))/(6*a*d^2) + (b*(b^2 + 4*a*b*c + 8*a^2*c^2)*Sqrt[c + d*x^2]*Sqrt[a + b/
(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(1
6*a^(5/2)*d^3*Sqrt[b + a*(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{x^5\sqrt{b+a\left(c+dx^2\right)}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{x^5\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{x^2\sqrt{b+ac+adx}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{x^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{6ad^2} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+ad\ldots}}{\ldots}\right.}{6ad^2\sqrt{b+a\left(c+\ldots\right.}}$$

$$= -\frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3} + \frac{x^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c\ldots\right.\right.}{6ad^2}$$

$$= \frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c\ldots\right.\right.}{24a^2d^3}$$

$$= \frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c\ldots\right.\right.}{24a^2d^3}$$

$$= \frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c\ldots\right.\right.}{24a^2d^3}$$

$$= \frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c\ldots\right.\right.}{24a^2d^3}$$

Mathematica [A] time = 0.32, size = 137, normalized size = 0.63

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2\left(c^2-cdx^2+d^2x^4\right)+2ab\left(dx^2-5c\right)-3b^2\right)+3b\left(8a^2c^2+4abc+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+\ldots}}{\sqrt{a}}\right.}{48a^{5/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b^2 + 2*a*b*
(-5*c + d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) + 3*b*(b^2 + 4*a*b*c + 8*
a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(5/2)*d^3)
```

fricas [A] time = 0.78, size = 423, normalized size = 1.96

$$\left[\frac{3\left(8\,a^2bc^2+4\,ab^2c+b^3\right)\sqrt{a}\,\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+4\left(2\,ad^2x^4+(4\,ac+b)\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2
*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c +
b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) +
4*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c^2 - 3*a*b^2*c -
 (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d
^3), -1/96*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^
2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 +
a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c
^2 - 3*a*b^2*c - (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(a^3*d^3)]
```

giac [A] time = 0.49, size = 219, normalized size = 1.01

$$\frac{1}{96}\left(2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{d}-\frac{4\,a^2cd^3-abd^3}{a^2d^5}\right)+\frac{8\,a^2c^2d^2-10\,abcd^2-3\,b^2d^2}{a^2d^5}\right)-\frac{3\,(\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2
/d - (4*a^2*c*d^3 - a*b*d^3)/(a^2*d^5)) + (8*a^2*c^2*d^2 - 10*a*b*c*d^2 - 3
*b^2*d^2)/(a^2*d^5)) - 3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*log(abs(-2*a*c*d -
 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)
)*sqrt(a)*abs(d) - b*d))/(a^(5/2)*d^2*abs(d)))*sgn(d*x^2 + c)
```

maple [B] time = 0.06, size = 533, normalized size = 2.47

$$\frac{\sqrt{\frac{a\,d\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(24a^2b\,c^2d\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-48\sqrt{a\,d^2x^4+2acd\,x^2+\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^3*(-48*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*c*a^2*d*(a*d^2)^(1/2)+24*ln(1/2*(2*a*d^2
*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2
)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*x^2*b*a*d*(a*d^2)^(1/2)+12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b
^2*c*a*d+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d+16*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*(a*d^2)^(1/2)-36*(a*d^2*x^4+2*a*c*d*x^2+b*d
```

```
*x^2+a*c^2+b*c)^(1/2)*c*b*a*(a*d^2)^(1/2)-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*b^2*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a^2/(
a*d^2)^(1/2)
```

maxima [A] time = 1.72, size = 328, normalized size = 1.52

$$-\frac{3\left(8\,a^2bc^2-4\,ab^2c-b^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6\,a^3bc^2-ab^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\left(8\,a^4bc^2+4\,a^3b^2c+a^2b^3\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48\left(a^5d^3-\frac{3\left(adx^2+ac+b\right)a^4d^3}{dx^2+c}+\frac{3\left(adx^2+ac+b\right)^2a^3d^3}{\left(dx^2+c\right)^2}-\frac{\left(adx^2+ac+b\right)^3a^2d^3}{\left(dx^2+c\right)^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(8*a^2*b*c^2 - 4*a*b^2*c - b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^
(5/2) - 8*(6*a^3*b*c^2 - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3
*(8*a^4*b*c^2 + 4*a^3*b^2*c + a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^5*d^3 - 3*(a*d*x^2 + a*c + b)*a^4*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c
+ b)^2*a^3*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a^2*d^3/(d*x^2 + c)^3)
 - 1/32*(8*a^2*c^2 + 4*a*b*c + b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/
2)*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5*(a + b/(c + d*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.319 $\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=141

$$-\frac{b(4ac+b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8ad^2}$$

```
[Out] -1/8*b*(4*a*c+b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)
/d^2+1/8*(-4*a*c+b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/4*(
d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2
```

Rubi [A] time = 0.47, antiderivative size = 181, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 80, 50, 63, 217, 206}

$$-\frac{b(4ac+b)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{8a^{3/2}d^2\sqrt{a\left(c+dx^2\right)+b}}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{4ad^2}-\frac{(4ac+b)\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -((b + 4*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(8*a*d^2) + ((c + d*x^2)
*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*a*d^2) - (b*(b + 4*a*c)*Sq
rt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqr
t[b + a*(c + d*x^2)]])/(8*a^(3/2)*d^2*Sqrt[b + a*(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\sqrt{b+a(c+dx^2)}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x\sqrt{b+ac+adx}}{\sqrt{c+dx}}dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2}-\frac{\left((b+4ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\Big(\int}{8ad\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2}-\frac{\big(b(b+}{}$$

$$=-\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2}-\frac{\big(b(b+}{}$$

$$=-\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2}-\frac{\big(b(b+}{}$$

$$=-\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2}-\frac{b(b+4}{}$$

Mathematica [A] time = 0.18, size = 97, normalized size = 0.69

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac+2adx^2+b\right)-b(4ac+b)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b - 2*a*c + 2*a
*d*x^2) - b*(b + 4*a*c)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(8*a^(3/2
)*d^2)
```

fricas [A] time = 0.81, size = 325, normalized size = 2.30

$$\left[\frac{\left(4\,abc+b^2\right)\sqrt{a}\,\log\Big(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+\right.}{32\,a^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c +
 a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2
+ b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4 +
a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*
d^2), 1/16*((4*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqr
t(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*
(2*a^2*d^2*x^4 + a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(a^2*d^2)]
```

giac [A] time = 0.44, size = 159, normalized size = 1.13

$$\frac{1}{16}\left(2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{d}-\frac{2\,acd-bd}{ad^3}\right)+\frac{\left(4\,abc+b^2\right)\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x\right.\right.\right.}{a^{\frac{3}{2}}d|\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/d - (2
*a*c*d - b*d)/(a*d^3)) + (4*a*b*c + b^2)*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d)
 - b*d))/(a^(3/2)*d*abs(d)))*sgn(d*x^2 + c)
```

maple [B] time = 0.04, size = 354, normalized size = 2.51

$$\frac{\sqrt{\frac{a d\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-4abcd\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)+4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^2*(4*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*d-4*ln(1/2*(2*a*d^2*x^2+2*
a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/
(a*d^2)^(1/2))*a*b*c*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*d^2)^(1/2)*a*c-ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*b^2*d+2*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+
b))^(1/2)/a/(a*d^2)^(1/2)
```

maxima [A] time = 1.75, size = 218, normalized size = 1.55

$$-\frac{\left(4\,abc-b^2\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc+ab^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^3d^2-\frac{2\left(adx^2+ac+b\right)a^2d^2}{dx^2+c}+\frac{\left(adx^2+ac+b\right)^2ad^2}{\left(dx^2+c\right)^2}\right)}+\frac{(4\,ac+b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*((4*a*b*c - b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c
+ a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^2 - 2*(a*d*x^2 + a*c
```

```
 + b)*a^2*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a*d^2/(d*x^2 + c)^2) + 1/
16*(4*a*c + b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sq
rt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \sqrt{a + \frac{b}{d\,x^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3*(a + b/(c + d*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{\frac{ac + adx^2 + b}{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.320 $\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=69

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

```
[Out] 1/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/d/a^(1/2)+1/2*(d*x^2+c)*(a+b/(
d*x^2+c))^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1591, 242, 47, 63, 208}

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqrt[a + b/(c + d*
x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
```

```
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx &= \frac{\text{Subst}\left(\int\sqrt{a+\frac{b}{x}}\,dx,x,c+dx^2\right)}{2d}\\ &= -\frac{\text{Subst}\left(\int\frac{\sqrt{a+bx}}{x^2}\,dx,x,\frac{1}{c+dx^2}\right)}{2d}\\ &= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}-\frac{b\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,\frac{1}{c+dx^2}\right)}{4d}\\ &= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}-\frac{\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\frac{b}{c+dx^2}}\right)}{2d}\\ &= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 1.12

$$\frac{\sqrt{a}\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}+b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] + b*ArcTanh[Sqrt
[a + b/(c + d*x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

fricas [A] time = 0.64, size = 267, normalized size = 3.87

$$\left[\frac{\sqrt{a}\,b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc\right)\sqrt{a}\sqrt{\frac{a}{}}\right.}{8\,ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a*d), -1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a*
c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c +
a*b)) - 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d)]
```

giac [B] time = 0.51, size = 127, normalized size = 1.84

$$-\frac{1}{4}\left(\frac{b\log\left(\left|-8\,a^{\frac{3}{2}}cd-8\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a|d|-4\,\sqrt{a}\,bd\right|\right)}{\sqrt{a}\,|d|}-\frac{2\,\sqrt{ad^2x^4+2\,acdx^2}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(b*log(abs(-8*a^(3/2)*c*d - 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*
c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*abs(d) - 4*sqrt(a)*b*d))/(sqrt(a)*abs(d
)) - 2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/d)*sgn(d*x^2 +
 c)
```

maple [B] time = 0.02, size = 180, normalized size = 2.61

$$\frac{\sqrt{\frac{a\,d\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(bd\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c}\right.}{4\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{a\,d^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(b*ln(1/2*(2*a*d^2*x^2+2*a*
c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a
*d^2)^(1/2))*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/d/(a*d^2)^(1/2)
```

maxima [B] time = 1.64, size = 126, normalized size = 1.83

$$-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad-\frac{\left(adx^2+ac+b\right)d}{dx^2+c}\right)}-\frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*d/(
d*x^2 + c)) - 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/
(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(sqrt(a)*d)
```

mupad [B] time = 3.01, size = 120, normalized size = 1.74

$$\frac{\sqrt{\frac{b\left(d\,x^2+c\right)+a\left(d\,x^2+c\right)^2}{\left(d\,x^2+c\right)^2}}\,\left(d\,x^2+c\right)\left(\frac{b\,\ln\left(\frac{\frac{b}{2}+a\left(d\,x^2+c\right)+\sqrt{a}\,\sqrt{b\left(d\,x^2+c\right)+a\left(d\,x^2+c\right)^2}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b\left(d\,x^2+c\right)+a\left(d\,x^2+c\right)^2}}+2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] (((b*(c + d*x^2) + a*(c + d*x^2)^2)/(c + d*x^2)^2)^(1/2)*(c + d*x^2)*((b*lo
g((b/2 + a*(c + d*x^2) + a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2))/a
^(1/2)))/(a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2)) + 2))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.321 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x} dx$

Optimal. Leaf size=96

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)-\frac{\sqrt{ac+b}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}}$$

```
[Out] arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))*a^(1/2)-arctanh(c^(1/2)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*(a*c+b)^(1/2)/c^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 184, normalized size of antiderivative = 1.92, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 105, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{\sqrt{a(c+dx^2)+b}}-\frac{\sqrt{ac+b}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a(c+dx^2)+b}}\right)}{\sqrt{c}\sqrt{a(c+dx^2)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x,x]
```

```
[Out] (Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c +
d*x^2])/Sqrt[b + a*(c + d*x^2)]])/Sqrt[b + a*(c + d*x^2)] - (Sqrt[b + a*c]*
Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x
^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$
$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$
$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{b+a(c+dx^2)}}$$
$$= -\frac{\left((-b-ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{b+a(c+dx^2)}} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{a+\right.}{}$$
$$= \frac{\left(a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx,x,\sqrt{c+dx^2}\right)}{\sqrt{b+a(c+dx^2)}} - \frac{\left((-b-ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx}\right.}{}$$
$$= -\frac{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right)}{\sqrt{c}\sqrt{b+a(c+dx^2)}} + \frac{\left(a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\right.}{\sqrt{b+a(c+\right.}}$$
$$= \frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right)}{\sqrt{b+a(c+dx^2)}} - \frac{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{c}}\right.}{\sqrt{c}\sqrt{b+a(c+dx^2)}}$$

Mathematica [A] time = 0.14, size = 80, normalized size = 0.83

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right) - \frac{\sqrt{ac+b}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x,x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]] - (Sqrt[b + a*c]*ArcTanh[(
Sqrt[c]*Sqrt[a + b/(c + d*x^2)])/Sqrt[b + a*c]])/Sqrt[c]
```

fricas [B] time = 0.84, size = 927, normalized size = 9.66

$$\left[\frac{1}{4}\sqrt{a}\log\left(8a^2d^2x^4 + 8a^2c^2 + 8\left(2a^2c+ab\right)dx^2 + 8abc + b^2 + 4\left(2ad^2x^4 + (4ac+b)dx^2 + 2ac^2 + bc\right)\sqrt{a}\sqrt{\frac{ad}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*
b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqr
```

```
t((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 1/4*sqrt((a*c + b)/c)*log(((8*a^2*c^2
 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c
^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*
c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt
((a*c + b)/c))/x^4), -1/2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(
-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 1/4*
sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16
*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^
2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d
*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4), 1/2*sqrt(-(a*c + b)/c
)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c +
 b^2)) + 1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^
2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqr
t(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))), -1/2*sqrt(-a)*arctan(1/2*(2*a*
d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^
2 + a^2*c + a*b)) + 1/2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 +
2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(
a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(d*x
^2+c)]Error: Bad Argument Type
```

maple [B] time = 0.04, size = 235, normalized size = 2.45

$$-\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,\left(d x^2+c\right)\left(-acd\ln\left(\frac{2a d^2x^2+2acd+bd+2\sqrt{a d^2x^4+2acd x^2+bd x^2+a c^2+bc}\,\sqrt{a d^2}}{2\sqrt{a d^2}}\right)+\sqrt{a c^2+bc}\,\sqrt{a d^2}\,\ln\left(2acd\right.\right.}{2\sqrt{\left(d x^2+c\right)\left(a d x^2+ac+b\right)}\,\sqrt{a d^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x,x)
```

```
[Out] -1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-ln(1/2*(2*a*d^2*x^2+2*a*
c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a
*d^2)^(1/2))*a*c*d+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c
^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(
a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c/(a*d^2)^(1/2)
```

maxima [A] time = 1.46, size = 159, normalized size = 1.66

$$\frac{(ac+b)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}}-\frac{1}{2}\sqrt{a}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*c + b)*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)
*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt((a
*c + b)*c) - 1/2*sqrt(a)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{d\,x^2+c}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x, x)
```

3.322 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx$

Optimal. Leaf size=104

$$\frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{3/2}\sqrt{ac+b}}-\frac{\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2cx^2}$$

```
[Out] 1/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^
(3/2)/(a*c+b)^(1/2)-1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^2
```

Rubi [A] time = 0.39, antiderivative size = 140, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6722, 1975, 446, 94, 93, 208}

$$\frac{bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{2c^{3/2}\sqrt{ac+b}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

```
[Out] -((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*c*x^2) + (b*d*Sqrt[c + d*x^2]*Sqr
t[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[
b + a*(c + d*x^2)])])/(2*c^(3/2)*Sqrt[b + a*c]*Sqrt[b + a*(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{x^3\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^3\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2} - \frac{\left(bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{4c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2} - \frac{\left(bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2}\,dx,x,\frac{\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{2c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2} + \frac{bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{2c^{3/2}\sqrt{b+ac}\sqrt{b+a\left(c+dx^2\right)}}$$

Mathematica [B] time = 0.47, size = 212, normalized size = 2.04

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(ac+adx^2+b\right)-2bdx^2\log(x)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}+bdx^2\sqrt{\left(c+d\right.}\cdots}{4cx^2\sqrt{c(ac+b)}\left(a\left(c+\right.\right.\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(c + d*x^2)*(b
 + a*c + a*d*x^2) - 2*b*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x]
+ b*d*x^2*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]*Log[2*a*c*(c + d*x^2) + b*(
2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]]))
/(4*c*Sqrt[c*(b + a*c)]*x^2*(b + a*(c + d*x^2)))
```

fricas [B] time = 0.84, size = 433, normalized size = 4.16

$$\left[\frac{\sqrt{ac^2+bc}\,bdx^2\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8(2a^2c^3+3abc^2+b^2c)dx^2+4((2ac+b)d^2x^4+2ac^3+(4ac^2+3bc)dx^2+2}{x^4}\right.}{8(ac^3+bc^2)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^
2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*s
qrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3
+ b*c^2)*x^2), -1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2*a*c + b)*d*x
^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3
+ b*c^2)*x^2)]
```

giac [B] time = 0.46, size = 281, normalized size = 2.70

$$-\frac{1}{2}\left(\frac{bd\arctan\left(-\frac{\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\sqrt{-ac^2-bc}\,c}+\frac{2\,a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2}\right.}{\left(ac^2-\left(\sqrt{ad^2}\,x^2-\sqrt{a}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*c) + (2*a^(3/2)*c^
2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*
c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(a*d^2)*x^2 -
 sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)*c))*sgn(d*
x^2 + c)
```

maple [B] time = 0.05, size = 454, normalized size = 4.37

$$-\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-ab\,c^2d\,x^2\ln\left(\frac{2acd\,x^2+bd\,x^2+2a\,c^2+2bc+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}{x^2}\right)-2\sqrt{a\,d^2x^4+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^3,x)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*a*d^2*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*(a*c^2+b*c)^(1/2)-ln((2*a*c*d*x^2+b*d*x
^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2))/x^2)*x^2*a*b*c^2*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*a*c*d*x^2*(a*c^2+b*c)^(1/2)-ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a
*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*b
```

```
^2*c*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*x^2*(a*c^2+b*c
)^(1/2)+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)
)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^2/(a*c+b)/x^2/(a*c^2+b*c)^(1/2)
```

maxima [A] time = 1.48, size = 156, normalized size = 1.50

$$-\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^2+bc-\frac{(adx^2+ac+b)c^2}{dx^2+c}\right)}-\frac{bd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^2 + b*c - (a*d*x^2 + a*
c + b)*c^2/(d*x^2 + c)) - 1/4*b*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a
*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^3,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**3, x)
```

3.323 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$

Optimal. Leaf size=174

$$-\frac{bd^2(4ac+3b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{5/2}(ac+b)^{3/2}}+\frac{d(4ac+5b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^2x^2(ac+b)}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2x^4}$$

```
[Out] -1/8*b*(4*a*c+3*b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a
*c+b)^(1/2))/c^(5/2)/(a*c+b)^(3/2)+1/8*(4*a*c+5*b)*d*(d*x^2+c)*((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)/c^2/x^4
```

Rubi [A] time = 0.51, antiderivative size = 218, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$-\frac{bd^2(4ac+3b)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{8c^{5/2}(ac+b)^{3/2}\sqrt{a\left(c+dx^2\right)+b}}+\frac{d(4ac+3b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2x^2(ac+b)}-\frac{\left(c+dx^2\right)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] ((3*b + 4*a*c)*d*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(8*c^2*(b + a*c)*x^2)
 - ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*c*(b + a*c)
*x^4) - (b*(3*b + 4*a*c)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTan
h[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^
(5/2)*(b + a*c)^(3/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^5\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^5\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} - \frac{\left((3b+4ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\right.}{8c(b+ac)\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{(3b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} + \frac{\left(b(3b+4a\right.}{}$$

$$= \frac{(3b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} + \frac{\left(b(3b+4a\right.}{}$$

$$= \frac{(3b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4} - \frac{b(3b+4ac}{}$$

Mathematica [A] time = 0.48, size = 278, normalized size = 1.60

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(2a^2c\left(c^2-d^2x^4\right)+ab\left(4c^2-3cdx^2-3d^2x^4\right)+b^2\left(2c-3dx^2\right)\right)-2bd^2x^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] -1/16*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*Sqrt[c*(b + a*c)]*(c + d*x^
2)*(b^2*(2*c - 3*d*x^2) + a*b*(4*c^2 - 3*c*d*x^2 - 3*d^2*x^4) + 2*a^2*c*(c^
2 - d^2*x^4)) - 2*b*(3*b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^
2))]*Log[x] + b*(3*b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]
*Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*
x^2)*(b + a*c + a*d*x^2)]]))/(c*(c*(b + a*c))^(3/2)*x^4*(b + a*(c + d*x^2))
)
```

fricas [A] time = 0.93, size = 577, normalized size = 3.32

$$\left[\frac{\left(4\,abc+3\,b^2\right)\sqrt{ac^2+bc}\,d^2x^4\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2-4\left((2\,ac+b)d^2x^4+2\,ac^3+\right.}{x^4}\right.}{32\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*
c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*
b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)
*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/
x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 + 3*b^2*c)*d^2*x^4 + 4*a*b*c^4
 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4), 1/16*((4*a*b*c + 3*b^2)*sqrt(-
a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 +
(a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 +
 3*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4)]
```

giac [B] time = 0.58, size = 713, normalized size = 4.10

$$\frac{1}{8}\left(\frac{\left(4\,abcd^2+3\,b^2d^2\right)\arctan\left(-\frac{\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\left(ac^3+bc^2\right)\sqrt{-ac^2-bc}}+\frac{8\,a^{\frac{7}{2}}c^5d|d|+16\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*((4*a*b*c*d^2 + 3*b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a*c^3 + b*c^2)*
sqrt(-a*c^2 - b*c)) + (8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2)*x^2 - sqrt(
a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2 + 8*(sqrt(a*d
^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*
c^3*d*abs(d) + 24*a^(5/2)*b*c^4*d*abs(d) + 36*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*d^2 + 8*(sqrt(a*d^2)
*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*b*c
^2*d*abs(d) + 24*a^(3/2)*b^2*c^3*d*abs(d) - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c*d^2 + 25*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*c^2*d^2
+ 8*sqrt(a)*b^3*c^2*d*abs(d) - 3*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*
d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*b^2*d^2 + 5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^3*c*d^2)/((a*c^3 + b*c^2)*(a
*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 +
b*c))^2 + b*c)^2))*sgn(d*x^2 + c)
```

maple [B] time = 0.06, size = 923, normalized size = 5.30

$$\frac{\sqrt{\frac{a d\,x^2+ac+b}{d\,x^2+c}}\;\left(d\,x^2+c\right)\left(-4a^3b\,c^5d^2x^4\ln\left(\frac{2acd\,x^2+bd\,x^2+2a\,c^2+2bc+2\sqrt{a\,c^2+bc}\;\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}{x^2}\right)-11a^2b^2c^4d^2x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^5,x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-12*a^2*d^3*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^6*c*(a*c^2+b*c)^(3/2)-4*ln((2*a*c*d*x^
2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2))/x^2)*x^4*a^3*b*c^5*d^2-10*a*d^3*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*x^6*b*(a*c^2+b*c)^(3/2)-11*ln((2*a*c*d*x^2+b*d*x^2+2
```

```
*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2))/x^2)*x^4*a^2*b^2*c^4*d^2-20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*a^2*c^2*d^2*x^4*(a*c^2+b*c)^(3/2)-10*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^
2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
)/x^2)*x^4*a*b^3*c^3*d^2-28*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
*a*c*d^2*b*x^4*(a*c^2+b*c)^(3/2)-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*
(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4
*b^4*c^2*d^2-10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b^2*d^2*x^4
*(a*c^2+b*c)^(3/2)+12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*c*d
*x^2*(a*c^2+b*c)^(3/2)+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*b
*d*x^2*(a*c^2+b*c)^(3/2)-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*
(a*c^2+b*c)^(3/2)*a*c^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(
a*c^2+b*c)^(3/2)*b*c)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^3/(a*c+b)^2/x^4/(
a*c^2+b*c)^(3/2)
```

maxima [B] time = 1.53, size = 322, normalized size = 1.85

$$\frac{\left(4\,abc+3\,b^2\right)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\left(ac^3+bc^2\right)\sqrt{(ac+b)c}}-\frac{\left(4\,abc^2+5\,b^2c\right)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc^2+7\,ab^2c+3\,b^3\right)d}{8\left(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2+\frac{\left(ac^5+bc^4\right)\left(adx^2+ac+b\right)^2}{\left(dx^2+c\right)^2}-\frac{2\left(a^2c^5+2\right.}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*(4*a*b*c + 3*b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - s
qrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)
*c)))/((a*c^3 + b*c^2)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2 + 5*b^2*c)*d^2*
((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 7*a*b^2*c + 3*b^3)
*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^
2*c^3 + b^3*c^2 + (a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 2*(
a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^5,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**5, x)
```

3.324 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx$

Optimal. Leaf size=265

$$\frac{bd^3\left(8a^2c^2+12abc+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{16c^{7/2}(ac+b)^{5/2}}-\frac{d^2\left(8a^2c^2+20abc+11b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16c^3x^2(ac+b)^2}+\frac{d(4ac+3b)}{8c\ldots}$$

```
[Out] -1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/c^2/(a*c+b)/x^6+1/16*b*(
8*a^2*c^2+12*a*b*c+5*b^2)*d^3*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(
1/2)/(a*c+b)^(1/2))/c^(7/2)/(a*c+b)^(5/2)-1/16*(8*a^2*c^2+20*a*b*c+11*b^2)*
d^2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/(a*c+b)^2/x^2+1/8*(4*a*
c+3*b)*d*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/(a*c+b)/x^4
```

Rubi [A] time = 0.61, antiderivative size = 271, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 99, 151, 12, 93, 208}

$$\frac{bd^3\left(8a^2c^2+12abc+5b^2\right)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{16c^{7/2}(ac+b)^{5/2}\sqrt{a\left(c+dx^2\right)+b}}-\frac{d^2(2ac+5b)(4ac+3b)\left(c+dx^2\right)\sqrt{a+\ldots}}{48c^3x^2(ac+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

```
[Out] -((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(6*c*x^6) + ((5*b + 4*a*c)*d*(c + d*
x^2)*Sqrt[a + b/(c + d*x^2)])/(24*c^2*(b + a*c)*x^4) - ((5*b + 2*a*c)*(3*b
+ 4*a*c)*d^2*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(48*c^3*(b + a*c)^2*x^2)
+ (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x
^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2
)])])/(16*c^(7/2)*(b + a*c)^(5/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^7\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^7\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^4\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{-\frac{1}{2}(5b+4ac)d-2ad^2x}{x^3\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{6c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subs}}{12c^2(b}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+d\right.}{48c^3(b+ac)^2x}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+d\right.}{48c^3(b+ac)^2x}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+d\right.}{48c^3(b+ac)^2x}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2\left(c+d\right.}{48c^3(b+ac)^2x}$$

Mathematica [A] time = 0.50, size = 245, normalized size = 0.92

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3bd^3\left(8a^2c^2+12abc+5b^2\right)\left(c+dx^2\right)\left(2\log(x)-\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}}+2d^3\left(8a^2c^2+\right.\right.}{96c^3(ac+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

```
[Out] -1/96*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*(15*b^2 + 26*a*b*c + 8*a^2*
c^2)*d^3 + (16*c^3*(b + a*c)^2)/x^6 - (4*b*c^2*(b + a*c)*d)/x^4 + (2*b*c*(5
*b + 8*a*c)*d^2)/x^2 + (3*b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*d^3*(c + d*x^2)*
(2*Log[x] - Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*S
qrt[(c + d*x^2)*(b + a*c + a*d*x^2)]]))/(Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)
*(b + a*(c + d*x^2))])))/(c^3*(b + a*c)^2)
```

fricas [A] time = 1.41, size = 755, normalized size = 2.85

$$\left[\frac{3\left(8\,a^2bc^2+12\,ab^2c+5\,b^3\right)\sqrt{ac^2+bc}\,d^3x^6\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2+4\left((2\right.}{x^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*log(
((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 +
 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3
 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 34*a^2*b*c^3 + 41*a*
b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*
a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2*b*c^5 + 2*a*b^2*c^4
+ b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^7 + 3*a^2*
b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6), -1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5
*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 +
2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 +
2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 +
 34*a^2*b*c^3 + 41*a*b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*
c^5 + 8*b^3*c^4 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2
*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/((a^3*c^7 + 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6)]
```

giac [B] time = 0.77, size = 1414, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*(3*(8*a^2*b*c^2*d^3 + 12*a*b^2*c*d^3 + 5*b^3*d^3)*arctan(-(sqrt(a*d^2
)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2
- b*c))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt(-a*c^2 - b*c)) + (64*a^(11/2)
*c^8*d^2*abs(d) + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))*a^5*c^7*d^3 + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(9/2)*c^6*d^2*abs(d) + 304*a^(9/2
)*b*c^7*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b
*d*x^2 + a*c^2 + b*c))^3*a^4*c^5*d^3 + 744*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^4*b*c^6*d^3 + 528*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(7/2)*b*c^
5*d^2*abs(d) + 576*a^(7/2)*b^2*c^6*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(
a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^3*b*c^4*d^3 + 1116*(s
qrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3
*b^2*c^5*d^3 + 480*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
2 + a*c^2 + b*c))^2*a^(5/2)*b^2*c^4*d^2*abs(d) + 544*a^(5/2)*b^3*c^5*d^2*ab
s(d) + 24*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
 + b*c))^5*a^2*b*c^2*d^3 - 96*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x
^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*b^2*c^3*d^3 + 801*(sqrt(a*d^2)*x^2 - sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b^3*c^4*d^3 + 144*(
sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*
a^(3/2)*b^3*c^3*d^2*abs(d) + 256*a^(3/2)*b^4*c^4*d^2*abs(d) + 36*(sqrt(a*d^
2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^5*a*b^2*c*d
```

```
^3 - 136*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
+ b*c))^3*a*b^3*c^2*d^3 + 270*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x
^2 + b*d*x^2 + a*c^2 + b*c))*a*b^4*c^3*d^3 + 48*sqrt(a)*b^5*c^3*d^2*abs(d)
+ 15*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*
c))^5*b^3*d^3 - 40*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
2 + a*c^2 + b*c))^3*b^4*c*d^3 + 33*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*
c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^5*c^2*d^3)/((a^2*c^5 + 2*a*b*c^4 + b^2*
c^3)*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a
*c^2 + b*c))^2 + b*c)^3))*sgn(d*x^2 + c)
```

maple [B] time = 0.08, size = 1518, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^7,x)
```

```
[Out] -1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-24*ln((2*a*c*d*x^2+b*d*
x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2))/x^2)*x^6*a^5*b*c^8*d^3-96*a^3*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*x^8*c^2*(a*c^2+b*c)^(5/2)-108*ln((2*a*c*d*x^2+b*d*x^2+2*
a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2))/x^2)*x^6*a^4*b^2*c^7*d^3-156*a^2*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*x^8*c*b*(a*c^2+b*c)^(5/2)-195*ln((2*a*c*d*x^2+b*d*x^2+2*a*c
^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
))/x^2)*x^6*a^3*b^3*c^6*d^3-66*a*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*x^8*b^2*(a*c^2+b*c)^(5/2)-144*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)*a^3*c^3*d^3*(a*c^2+b*c)^(5/2)*x^6-177*ln((2*a*c*d*x^2+b*d*x^2+
2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2))/x^2)*x^6*a^2*b^4*c^5*d^3-324*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*a^2*c^2*d^3*b*(a*c^2+b*c)^(5/2)*x^6-81*ln((2*a*c*d*x^2+b*d*x^2+2*
a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2))/x^2)*x^6*a*b^5*c^4*d^3-252*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*a*c*d^3*b^2*(a*c^2+b*c)^(5/2)*x^6-15*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+
2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/
x^2)*x^6*b^6*c^3*d^3+96*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a^2
*c^2*d^2*(a*c^2+b*c)^(5/2)*x^4-66*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*b^3*d^3*(a*c^2+b*c)^(5/2)*x^6+156*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*a*c*d^2*b*(a*c^2+b*c)^(5/2)*x^4+66*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(3/2)*b^2*d^2*(a*c^2+b*c)^(5/2)*x^4-48*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*a^2*c^3*d-84*(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*a*b*c^2*d-36*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*b^2*c*d+1
6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a^2*c^4
+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a*b*c
^3+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2
*c^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^4/(a*c+b)^3/x^6/(a*c^2+b*c)^(5/2)
```

maxima [B] time = 1.91, size = 557, normalized size = 2.10

$$-\frac{\left(8\,a^2bc^2+12\,ab^2c+5\,b^3\right)d^3\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{32\left(a^2c^5+2\,abc^4+b^2c^3\right)\sqrt{(ac+b)c}}-\frac{3\left(8\,a^2bc^4+20\,ab^2c^3+11\,b^3c^2\right)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6\,a^{3}\cdots\right.}{48\left(a^5c^8+5\,a^4bc^7+10\,a^3b^2c^6+10\,a^2b^3c^5+5\,ab^4c^4+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/32*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
) + sqrt((a*c + b)*c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt((a*c + b)*c))
 - 1/48*(3*(8*a^2*b*c^4 + 20*a*b^2*c^3 + 11*b^3*c^2)*d^3*((a*d*x^2 + a*c +
b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4 + 18*a^2*b^2*c^3 + 17*a*b^3*c^2 + 5*
b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^4 + 28*a^
3*b^2*c^3 + 37*a^2*b^3*c^2 + 22*a*b^4*c + 5*b^5)*d^3*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)))/(a^5*c^8 + 5*a^4*b*c^7 + 10*a^3*b^2*c^6 + 10*a^2*b^3*c^5 +
 5*a*b^4*c^4 + b^5*c^3 - (a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2 + a*c + b
)^3/(d*x^2 + c)^3 + 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*c^5)*(a*d*
x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(a^4*c^8 + 4*a^3*b*c^7 + 6*a^2*b^2*c^6 +
 4*a*b^3*c^5 + b^4*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{d\,x^2+c}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^7,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**7, x)
```

3.325 $\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=368

$$\frac{\sqrt{c}\left(-3a^2c^2+7abc+2b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{x\left(-3a^2c^2+7abc+2b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15a^2d^2}-\frac{c^{3/2}(b-3ac)\cdots}{1\cdots}$$

```
[Out] -1/15*(-3*a^2*c^2+7*a*b*c+2*b^2)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^
2+1/15*(-3*a*c+b)*x*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/5*x
^3*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-1/15*c^(3/2)*(-3*a*c+b)*(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^(5/2)/(c*
(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/15*(-3*a^2*c^2+7*a*b*c+2*b^2)*(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d
^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 478, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 478, 582, 531, 418, 492, 411}

$$-\frac{x\left(-3a^2c^2+7abc+2b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{15a^2d^2\sqrt{a\left(c+dx^2\right)+b}}+\frac{\sqrt{c}\left(-3a^2c^2+7abc+2b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\cdots\right.}{15a^2d^{5/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a\left(c+dx^2\right)+b\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -((2*b^2 + 7*a*b*c - 3*a^2*c^2)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d
*x^2)])/(15*a^2*d^2*Sqrt[b + a*(c + d*x^2)]) + ((b - 3*a*c)*x*(c + d*x^2)*S
qrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(15*a*d^2*Sqrt[b + a*(c + d
*x^2)]) + (x^3*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])
/(5*d*Sqrt[b + a*(c + d*x^2)]) + (Sqrt[c]*(2*b^2 + 7*a*b*c - 3*a^2*c^2)*Sqr
t[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], b/(b + a*c)])/(15*a^2*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a
*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) - (c^(3/2)*(b - 3*a*c)*Sqrt[b +
a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], b/(b + a*c)])/(15*a*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c +
 d*x^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\sqrt{b+a(c+dx^2)}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{x^3(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d\sqrt{b+a(c+dx^2)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2(3c(b+ac)-(b-3ac)dx}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}}{5d\sqrt{b+a(c+dx^2)}}$$

$$= \frac{(b-3ac)x(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15ad^2\sqrt{b+a(c+dx^2)}} + \frac{x^3(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+}}{5d\sqrt{b+a(c+dx^2)}}$$

$$= \frac{(b-3ac)x(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15ad^2\sqrt{b+a(c+dx^2)}} + \frac{x^3(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+}}{5d\sqrt{b+a(c+dx^2)}}$$

$$= -\frac{(2b^2+7abc-3a^2c^2)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15a^2d^2\sqrt{b+a(c+dx^2)}} + \frac{(b-3ac)x(c+dx^2)\sqrt{b+ac+a}}{15ad^2\sqrt{b+a(c+d}}$$

$$= -\frac{(2b^2+7abc-3a^2c^2)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15a^2d^2\sqrt{b+a(c+dx^2)}} + \frac{(b-3ac)x(c+dx^2)\sqrt{b+ac+a}}{15ad^2\sqrt{b+a(c+d}}$$

Mathematica [C] time = 0.93, size = 293, normalized size = 0.80

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x(c+dx^2)\sqrt{\frac{ad}{ac+b}}\left(-3a^2(c^2-d^2x^4)-2ab(c-2dx^2)+b^2\right)+ic(-3a^2c^2+7abc+2b^2)\sqrt{\frac{dx^2}{c}+1}\sqrt{}\right.}{15ad^2\sqrt{\frac{ad}{ac+b}}\left(a(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(b^2 - 2*a*b*(c - 2*d*x^2) - 3*a^2*(c^2 - d^2*x^4)) + I*c*(2*b^2 + 7*a*b*c
 - 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*b*c*(b + 9*a*c)*S
qrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(15*a*d^2*Sqrt[(a*d)/(b + a*c)]*(b
 + a*(c + d*x^2)))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4\sqrt{\frac{adx^2+ac+b}{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
[Out] integral(x^4*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{dx^2 + c}}\, x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

maple [A] time = 0.04, size = 662, normalized size = 1.80

$$\frac{\left(3\sqrt{-\frac{ad}{ac+b}}\, a^2d^3x^7 + 3\sqrt{-\frac{ad}{ac+b}}\, a^2c\,d^2x^5 + 4\sqrt{-\frac{ad}{ac+b}}\, ab\,d^2x^5 - 3\sqrt{-\frac{ad}{ac+b}}\, a^2c^2d\,x^3 + 2\sqrt{-\frac{ad}{ac+b}}\, abcd\,x^3 - 3\sqrt{-\frac{ad}{ac+}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b/(d*x^2+c))^(1/2),x)
[Out] 1/15*(3*(-a*d/(a*c+b))^(1/2)*x^7*a^2*d^3+3*(-a*d/(a*c+b))^(1/2)*x^5*a^2*c*d
^2+4*(-a*d/(a*c+b))^(1/2)*x^5*a*b*d^2-3*(-a*d/(a*c+b))^(1/2)*x^3*a^2*c^2*d+
3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*
c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^3+2*(-a*d/(a*c+b))^(1/2)*x^3*a*b*c*d
+9*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a
*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2-7*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*
a*b*c^2+(-a*d/(a*c+b))^(1/2)*x^3*b^2*d-3*(-a*d/(a*c+b))^(1/2)*x*a^2*c^3+((a
*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))
^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c-2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c-2*
(-a*d/(a*c+b))^(1/2)*x*a*b*c^2+(-a*d/(a*c+b))^(1/2)*x*b^2*c)*(d*x^2+c)*((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)/(-a*d/(a*c+b))^(1/2)/a/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{dx^2 + c}}\, x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \sqrt{a + \frac{b}{d\,x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/(c + d*x^2))^(1/2),x)
[Out] int(x^4*(a + b/(c + d*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.326 $\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=282

$$-\frac{c^{3/2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{\sqrt{c}(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{x(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3ad}+\frac{x}{}$$

```
[Out] 1/3*(-a*c+b)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d+1/3*x*(d*x^2+c)*((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)/d-1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)
^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a
*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c)
)^(1/2)-1/3*(-a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^
(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)
/(d*x^2+c))^(1/2)/a/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 370, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 478, 531, 418, 492, 411}

$$-\frac{c^{3/2}\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{c}(b-ac)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{3ad^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((b - a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(3*a*d*Sqrt[b
 + a*(c + d*x^2)]) + (x*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c +
 d*x^2)])/(3*d*Sqrt[b + a*(c + d*x^2)]) - (Sqrt[c]*(b - a*c)*Sqrt[b + a*c +
 a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(3*a*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2
))]*Sqrt[b + a*(c + d*x^2)]) - (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/
(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*d^(3/2
)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x
^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{x^2\sqrt{b+a(c+dx^2)}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{x^2\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{c(b+ac)-(b-ac)dx^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx}}}{3d\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left((b-ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{}{\sqrt{c+dx^2}\sqrt{}}}{3\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3ad\sqrt{b+a\left(c+dx^2\right)}} + \frac{x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}} - \frac{c^3}{}$$

$$= \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3ad\sqrt{b+a\left(c+dx^2\right)}} + \frac{x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}} - \frac{\sqrt{}}{}$$

Mathematica [C] time = 0.58, size = 250, normalized size = 0.89

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(ac+adx^2+b\right)+2ibc\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)+ic(\right.}{3d\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(b + a*c + a*d*x^2) + I*c*(-b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*S
qrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c
)] + (2*I)*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*d*Sqrt[(a*d)/(b
 + a*c)]*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{dx^2+c}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

maple [A] time = 0.02, size = 406, normalized size = 1.44

$$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^2 x^5 + 2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^3 + \sqrt{-\frac{ad}{ac+b}}\, bd\, x^3 + \sqrt{-\frac{ad}{ac+b}}\, a\, c^2 x - \sqrt{\frac{ad\, x^2+ac+b}{ac+b}}\, \sqrt{\frac{d\, x^2+c}{c}}\, a\, c^2\, \mathrm{EllipticE}\left(\sqrt{-\frac{ad}{ac+}}\right.\right.}{3\sqrt{a\, d^2 x^4 + 2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/3*((-1/(a*c+b)*a*d)^(1/2)*x^5*a*d^2+2*(-1/(a*c+b)*a*d)^(1/2)*x^3*a*c*d+(-
1/(a*c+b)*a*d)^(1/2)*x^3*b*d-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*a*c^2+(-1/(a*
c+b)*a*d)^(1/2)*x*a*c^2-2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*b*c+((a*d*x^2+a*
c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,
((a*c+b)/a/c)^(1/2))*b*c+(-1/(a*c+b)*a*d)^(1/2)*x*b*c)*(d*x^2+c)*((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(
-1/(a*c+b)*a*d)^(1/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{dx^2 + c}}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{a + \frac{b}{d\, x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^2*(a + b/(c + d*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\frac{ac + adx^2 + b}{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.327 $\int \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=213

$$x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}+\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}$$

```
[Out] x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+
c))^(1/2)+(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/
2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 279, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6722, 1974, 422, 418, 492, 411}

$$\frac{x\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a\left(c+dx^2\right)+b}}+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}\sqrt{a+\cdots}}{\sqrt{d}\sqrt{\frac{c\left(ac+adx^2+\cdots\right)}{(ac+b)\left(c+dx^{\cdots}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/Sqrt[b + a*(c + d*x^2)]
 - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c + a*d*x^2))
/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) + (Sqrt[c]*Sqrt[b + a*c
+ a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]
*Sqrt[b + a*(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left((b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\ldots}{\sqrt{c+\ldots}}}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+a\left(c+dx^2\right)}} + \frac{\sqrt{c}\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}\sqrt{b+a\left(c+dx^2\right)}} - \ldots$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+a\left(c+dx^2\right)}} - \frac{\sqrt{c}\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}\sqrt{b+a\left(c+dx^2\right)}} + \ldots$$

Mathematica [A] time = 0.06, size = 98, normalized size = 0.46

$$\frac{\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|\frac{ac}{b+ac}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{ac+adx^2+b}{ac+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSin
[Sqrt[-(d/c)]*x], (a*c)/(b + a*c)])/(Sqrt[-(d/c)]*Sqrt[(b + a*c + a*d*x^2)/
(b + a*c)])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{\frac{adx^2 + ac + b}{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c)), x)
```

maple [A] time = 0.02, size = 199, normalized size = 0.93

$$\frac{\left(ac \operatorname{EllipticE}\left(\sqrt{-\frac{ad}{ac+b}}\, x, \sqrt{\frac{ac+b}{ac}}\right) + b \operatorname{EllipticF}\left(\sqrt{-\frac{ad}{ac+b}}\, x, \sqrt{\frac{ac+b}{ac}}\right)\right) \sqrt{\frac{d\,x^2+c}{c}}\, \sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\, \left(d\,x^2+c\right) \sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{\sqrt{a\,d^2x^4 + 2acd\,x^2 + bd\,x^2 + a\,c^2 + bc}\, \sqrt{-\frac{ad}{ac+b}}\, \sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2),x)
[Out] (a*c*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))+EllipticF((-1/
(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*b)*((d*x^2+c)/c)^(1/2)*((a*d*x^2+
a*c+b)/(a*c+b))^(1/2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/((d*x^2+c)*(a
*d*x^2+a*c+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{b}{d\,x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2),x)
[Out] int((a + b/(c + d*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/(c + d*x**2)), x)
```

3.328 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx$

Optimal. Leaf size=265

$$\frac{dx\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}-\frac{\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{cx}+\frac{a\sqrt{c}\sqrt{d}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{}}\right.\right.}{\sqrt{c}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}$$

```
[Out] d*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c-(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/c/x-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*(1/(1+d*x^2
/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),
(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b
)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 353, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 475, 21, 422, 418, 492, 411}

$$\frac{dx\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{a\left(c+dx^2\right)+b}}-\frac{\left(c+dx^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{a\left(c+dx^2\right)+b}}+\frac{a\sqrt{c}\sqrt{d}\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}F\left(\right.}{(ac+b)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a\left(c+\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

```
[Out] (d*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*Sqrt[b + a*(c + d*
x^2)]) - ((c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*x
*Sqrt[b + a*(c + d*x^2)]) - (Sqrt[d]*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c
+ d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[c]*Sqr
t[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)])
 + (a*Sqrt[c]*Sqrt[d]*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sqrt[(c*(b + a*c
 + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{x^2\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^2\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{acd+ad^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,d\ldots}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{a\sqrt{c}\sqrt{d}\sqrt{b+a\ldots}}{(b+a\ldots}$$

$$= \frac{dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} - \frac{\sqrt{d}\sqrt{b+ac+a\ldots}}{\sqrt{c}\sqrt{\ldots}}$$

Mathematica [C] time = 0.57, size = 141, normalized size = 0.53

$$\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{iad\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)}{\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}-\frac{dx}{c}-\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

```
[Out] Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-x^(-1) - (d*x)/c - (I*a*d*Sqrt[(b +
 a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*
d)/(b + a*c)]*x], 1 + b/(a*c)])/(Sqrt[(a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

maple [A] time = 0.03, size = 272, normalized size = 1.03

$$-\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^2x^4 + 2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^2 - \sqrt{\frac{ad\, x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\, x^2+c}{c}}\, acdx\, \mathrm{EllipticE}\left(\sqrt{-\frac{ad}{ac+b}}\, x, \sqrt{\frac{ac+b}{ac}}\right) + \sqrt{-\frac{ad}{ac+b}}\, bd\, x^2 +}{\sqrt{a\, d^2x^4 + 2acd\, x^2 + bd\, x^2 + a\, c^2 + bc}\,\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\, x^2 + c\right)\left(ad\, x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^2,x)
```

```
[Out] -((-1/(a*c+b)*a*d)^(1/2)*x^4*a*d^2-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((
d*x^2+c)/c)^(1/2)*x*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))
+2*(-1/(a*c+b)*a*d)^(1/2)*x^2*a*c*d+(-1/(a*c+b)*a*d)^(1/2)*x^2*b*d+(-1/(a*c
+b)*a*d)^(1/2)*a*c^2+(-1/(a*c+b)*a*d)^(1/2)*b*c)*(d*x^2+c)*((a*d*x^2+a*c+b)
/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(a*c+
b)*a*d)^(1/2)/x/c/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{d\, x^2+c}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^2,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**2, x)
```

3.329 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx$

Optimal. Leaf size=362

$$\frac{d^{3/2}(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{d^2x(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^2(ac+b)}+\frac{d(ac+2b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^2x(ac+b)}-\ldots$$

```
[Out] -1/3*(a*c+2*b)*d^2*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)-1/3*(d*x
^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^3+1/3*(a*c+2*b)*d*(d*x^2+c)*((a
*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x+1/3*(a*c+2*b)*d^(3/2)*(1/(1+d*
x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/
2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(3/2)/(a*c+b)/(c*
(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^(3/2)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b
))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)/c^(1/2)/(c*(a*d*x^2+a*c
+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 472, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6722, 1975, 475, 583, 531, 418, 492, 411}

$$-\frac{d^2x(ac+2b)\sqrt{ac+adx^2+b}\,\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(ac+b)\sqrt{a\left(c+dx^2\right)+b}}+\frac{d^{3/2}(ac+2b)\sqrt{ac+adx^2+b}\,\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\,\sqrt{a\left(c+dx^2\right)+b}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

```
[Out] -((2*b + a*c)*d^2*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(3*c^2
*(b + a*c)*Sqrt[b + a*(c + d*x^2)]) - ((c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*
Sqrt[a + b/(c + d*x^2)])/(3*c*x^3*Sqrt[b + a*(c + d*x^2)]) + ((2*b + a*c)*d
*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(3*c^2*(b + a
*c)*x*Sqrt[b + a*(c + d*x^2)]) + ((2*b + a*c)*d^(3/2)*Sqrt[b + a*c + a*d*x^
2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*
c)])/(3*c^(3/2)*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^
2))]*Sqrt[b + a*(c + d*x^2)]) - (a*d^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a +
 b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*Sqr
t[c]*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b
 + a*(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{x^4\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^4\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(2b+ac)d-ad^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,d}{3c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)d\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx}}}{3c^2(b+ac)x\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)d\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx}}}{3c^2(b+ac)x\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(2b+ac)d^2x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(b+ac)\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b}{}$$

$$= -\frac{(2b+ac)d^2x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(b+ac)\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b}{}$$

Mathematica [C] time = 0.99, size = 314, normalized size = 0.87

$$-\frac{\sqrt{\frac{ad}{ac+b}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(a^2c\left(c^2-d^2x^4\right)+2ab\left(c^2-cdx^2-d^2x^4\right)+b^2\left(c-2dx^2\right)\right)+iabcd^2x^3\sqrt{}\right.}{3ac^2dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

```
[Out] -1/3*(Sqrt[(a*d)/(b + a*c)]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*
d)/(b + a*c)]*(c + d*x^2)*(b^2*(c - 2*d*x^2) + a^2*c*(c^2 - d^2*x^4) + 2*a*
b*(c^2 - c*d*x^2 - d^2*x^4)) - I*a*c*(2*b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*
d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a
*c)]*x], 1 + b/(a*c)] + I*a*b*c*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]
*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a
*c)]))/(a*c^2*d*x^3*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

maple [A] time = 0.04, size = 571, normalized size = 1.58

$$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a^2 c\, d^3 x^6 + 2\sqrt{-\frac{ad}{ac+b}}\, ab\, d^3 x^6 + \sqrt{-\frac{ad}{ac+b}}\, a^2 c^2 d^2 x^4 - \sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\, \sqrt{\frac{d\,x^2+c}{c}}\, a^2 c^2 d^2 x^3 \,\mathrm{EllipticE}\left(\sqrt{-\frac{ad}{ac+b}}\, x, \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^4,x)
```

```
[Out] 1/3*((-1/(a*c+b)*a*d)^(1/2)*x^6*a^2*c*d^3+2*(-1/(a*c+b)*a*d)^(1/2)*x^6*a*b*
d^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+
b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x^3*a^2*c^2*d^2+(-1/(a*c+b)*a*d)^(1/2)
*x^4*a^2*c^2*d^2+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x^3*a*b*c*d^2-2*((a*d*x^2
+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)
*x,((a*c+b)/a/c)^(1/2))*x^3*a*b*c*d^2+4*(-1/(a*c+b)*a*d)^(1/2)*x^4*a*b*c*d^
2+2*(-1/(a*c+b)*a*d)^(1/2)*x^4*b^2*d^2-(-1/(a*c+b)*a*d)^(1/2)*x^2*a^2*c^3*d
+(-1/(a*c+b)*a*d)^(1/2)*x^2*b^2*c*d-(-1/(a*c+b)*a*d)^(1/2)*a^2*c^4-2*(-1/(a
*c+b)*a*d)^(1/2)*a*b*c^3-(-1/(a*c+b)*a*d)^(1/2)*b^2*c^2)*(d*x^2+c)*((a*d*x^
2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(
-1/(a*c+b)*a*d)^(1/2)/(a*c+b)/x^3/c^2/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{d\,x^2+c}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^4,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**4, x)
```

3.330 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\, dx$

Optimal. Leaf size=466

$$-\frac{d^{5/2}\left(3a^2c^2+13abc+8b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15c^{5/2}(ac+b)^2\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{d^3x\left(3a^2c^2+13abc+8b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15c^3(ac+b)^2}-\frac{d^2\left(3a^2c^2\right.}{}$$

```
[Out] 1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*d^3*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3
/(a*c+b)^2-1/5*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^5+1/15*(3*a*
c+4*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x^3-1/15*(
3*a^2*c^2+13*a*b*c+8*b^2)*d^2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c
^3/(a*c+b)^2/x-1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*d^(5/2)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b
))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(5/2)/(a*c+b)^2/(c*(a*d*x^2+a
*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/15*a*(3*a*c+4*b)*d^(5/2)*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*
c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(3/2)/(a*c+b)^2/(c*(a*d*x^
2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 598, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6722, 1975, 475, 583, 531, 418, 492, 411}

$$\frac{d^3x\left(3a^2c^2+13abc+8b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{15c^3(ac+b)^2\sqrt{a\left(c+dx^2\right)+b}}-\frac{d^2\left(3a^2c^2+13abc+8b^2\right)\left(c+dx^2\right)\sqrt{ac+adx^2+b}\sqrt{a+}}{15c^3x(ac+b)^2\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^6,x]
```

```
[Out] ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d^3*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c
 + d*x^2)])/(15*c^3*(b + a*c)^2*Sqrt[b + a*(c + d*x^2)]) - ((c + d*x^2)*Sqr
t[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(5*c*x^5*Sqrt[b + a*(c + d*x^
2)]) + ((4*b + 3*a*c)*d*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c +
 d*x^2)])/(15*c^2*(b + a*c)*x^3*Sqrt[b + a*(c + d*x^2)]) - ((8*b^2 + 13*a*b
*c + 3*a^2*c^2)*d^2*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x
^2)])/(15*c^3*(b + a*c)^2*x*Sqrt[b + a*(c + d*x^2)]) - ((8*b^2 + 13*a*b*c +
 3*a^2*c^2)*d^(5/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(15*c^(5/2)*(b + a*c)^2*Sqrt[
(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) +
 (a*(4*b + 3*a*c)*d^(5/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(15*c^(3/2)*(b + a*c)^2
*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^
2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^6\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^6\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(4b+3ac)d-3ad^2x^2}{x^4\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(4b+3ac)d\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^2(b+ac)x^3\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(4b+3ac)d\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^2(b+ac)x^3\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(4b+3ac)d\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^2(b+ac)x^3\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(8b^2+13abc+3a^2c^2\right)d^3x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^3(b+ac)^2\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+d}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(8b^2+13abc+3a^2c^2\right)d^3x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^3(b+ac)^2\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+d}}}{5cx^5\sqrt{b+a\left(c+dx^2\right)}}$$

Mathematica [C] time = 1.09, size = 402, normalized size = 0.86

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(iacd^3x^5\left(3a^2c^2+13abc+8b^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)+\left(c+dx^2\right)\sqrt{\frac{ac}{ac+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^6,x]
```

```
[Out] -1/15*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(c + d*
x^2)*(b^3*(3*c^2 - 4*c*d*x^2 + 8*d^2*x^4) + 3*a^3*c^2*(c^3 + d^3*x^6) + a*b
^2*(9*c^3 - 8*c^2*d*x^2 + 17*c*d^2*x^4 + 8*d^3*x^6) + a^2*b*c*(9*c^3 - 4*c^
2*d*x^2 + 9*c*d^2*x^4 + 13*d^3*x^6)) + I*a*c*(8*b^2 + 13*a*b*c + 3*a^2*c^2)
*d^3*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - (2*I)*a*b*c*(2*b + 3*a*c
)*d^3*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(c^3*(b + a*c)^2*Sqrt[(
a*d)/(b + a*c)]*x^5*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
[Out] integral(sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

maple [A] time = 0.05, size = 955, normalized size = 2.05

$$-\frac{\left(3\sqrt{-\frac{ad}{ac+b}}\,a^3c^2d^4x^8+13\sqrt{-\frac{ad}{ac+b}}\,a^2bc\,d^4x^8+3\sqrt{-\frac{ad}{ac+b}}\,a^3c^3d^3x^6-3\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,a^3c^3d^3x^5\,\mathrm{EllipticE}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^6,x)
[Out] -1/15*(3*(-1/(a*c+b)*a*d)^(1/2)*x^8*a^3*c^2*d^4+13*(-1/(a*c+b)*a*d)^(1/2)*x
^8*a^2*b*c*d^4-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x^5*a^3*c^3*d^3+8*(-1/(a*
c+b)*a*d)^(1/2)*x^8*a*b^2*d^4+3*(-1/(a*c+b)*a*d)^(1/2)*x^6*a^3*c^3*d^3+6*((
a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d
)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x^5*a^2*b*c^2*d^3-13*((a*d*x^2+a*c+b)/(a*c+b
))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/
c)^(1/2))*x^5*a^2*b*c^2*d^3+22*(-1/(a*c+b)*a*d)^(1/2)*x^6*a^2*b*c^2*d^3+4*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*
d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x^5*a*b^2*c*d^3-8*((a*d*x^2+a*c+b)/(a*c+b))
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)
^(1/2))*x^5*a*b^2*c*d^3+25*(-1/(a*c+b)*a*d)^(1/2)*x^6*a*b^2*c*d^3+8*(-1/(a*
c+b)*a*d)^(1/2)*x^6*b^3*d^3+5*(-1/(a*c+b)*a*d)^(1/2)*x^4*a^2*b*c^3*d^2+9*(-
1/(a*c+b)*a*d)^(1/2)*x^4*a*b^2*c^2*d^2+3*(-1/(a*c+b)*a*d)^(1/2)*x^2*a^3*c^5
*d+4*(-1/(a*c+b)*a*d)^(1/2)*x^4*b^3*c*d^2+5*(-1/(a*c+b)*a*d)^(1/2)*x^2*a^2*
b*c^4*d+(-1/(a*c+b)*a*d)^(1/2)*x^2*a*b^2*c^3*d+3*(-1/(a*c+b)*a*d)^(1/2)*a^3
*c^6-(-1/(a*c+b)*a*d)^(1/2)*x^2*b^3*c^2*d+9*(-1/(a*c+b)*a*d)^(1/2)*a^2*b*c^
5+9*(-1/(a*c+b)*a*d)^(1/2)*a*b^2*c^4+3*(-1/(a*c+b)*a*d)^(1/2)*b^3*c^3)*(d*x
^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/(a*c+b)^2/x^5/c^3/((d*x^2+c)*(a*d*x^2+a
*c+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{d\,x^2+c}}}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^6,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^6, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**6, x)
```

3.331 $\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=249

$$-\frac{\left(-24a^2c^2+60abc+5b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48ad^3}-\frac{b\left(-24a^2c^2+12abc+b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{3/2}d^3}-\frac{bc^2\sqrt{\frac{ac+adx^2}{c+dx^2}}}{d^3}$$

```
[Out] 1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(5/2)/a/d^3-1/16*b*(-24*a^2*c^2
+12*a*b*c+b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)/d
^3-b*c^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^3-1/48*(-24*a^2*c^2+60*a*b*c+5
*b^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^3-1/24*(12*a*c+b)*(d*
x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^3
```

Rubi [A] time = 0.73, antiderivative size = 311, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 89, 80, 50, 63, 217, 206}

$$-\frac{\left(-24a^2c^2+12abc+b^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{24abd^3}-\frac{\left(-24a^2c^2+12abc+b^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((b^2 + 12*a*b*c - 24*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(16*a*
d^3) - ((b^2 + 12*a*b*c - 24*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(
b + a*(c + d*x^2)))/(24*a*b*d^3) - (c^2*Sqrt[a + b/(c + d*x^2)]*(b + a*(c +
 d*x^2))^2)/(b*d^3) + ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^
2))^2)/(6*a*d^3) - (b*(b^2 + 12*a*b*c - 24*a^2*c^2)*Sqrt[c + d*x^2]*Sqrt[a
+ b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]]
)/(16*a^(3/2)*d^3*Sqrt[b + a*(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x^2(b+ac+adx)^{3/2}}{(c+dx)^{3/2}}dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{bd^3}+\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{(b+ac+adx)^{3/2}}{\cdots}\right.}{bd^3\sqrt{b+a\left(c+dx^2\right.}}$$

$$=-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{bd^3}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{6ad^3}-\cdots$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{24abd^3}-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(b\cdots\right.}{b\cdots}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\cdots}{24abd\cdots}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\cdots}{24abd\cdots}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\cdots}{24abd\cdots}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\cdots}{24abd\cdots}$$

Mathematica [A] time = 0.41, size = 142, normalized size = 0.57

$$\frac{\sqrt{a}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2\left(c^3+d^3x^6\right)-2ab\left(47c^2+16cdx^2-7d^2x^4\right)+3b^2\left(c+dx^2\right)\right)-3b\left(-24a^2c^2+12abc+b^2\right)\cdots}{48a^{3/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b^2*(c + d*x^2) - 2*a*b*(
47*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^2*(c^3 + d^3*x^6)) - 3*b*(b^2 + 12*a
```

```
*b*c - 24*a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(3/2)*d^
3)
```

fricas [A] time = 0.99, size = 427, normalized size = 1.71

$$\left[\frac{3\left(24\,a^2bc^2-12\,ab^2c-b^3\right)\sqrt{a}\,\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+4\left(2\,ad^2x^4+(4\,ac+b)d\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a
^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c
+ b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
+ 4*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*a^2*b*c^2 + 3*a*b^2*
c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a
^2*d^3), -1/96*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*
x^2 + a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*
a^2*b*c^2 + 3*a*b^2*c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(a^2*d^3)]
```

giac [B] time = 2.10, size = 527, normalized size = 2.12

$$\frac{1}{48}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\left(\frac{4\,ax^2\mathrm{sgn}\left(dx^2+c\right)}{d}-\frac{4\,a^3cd^6\mathrm{sgn}\left(dx^2+c\right)-7\,a^2bd^6\mathrm{sgn}\left(dx^2+c\right)}{a^2d^8}\right)x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*(4*a*x^2*sgn(
d*x^2 + c)/d - (4*a^3*c*d^6*sgn(d*x^2 + c) - 7*a^2*b*d^6*sgn(d*x^2 + c))/(a
^2*d^8))*x^2 + (8*a^3*c^2*d^5*sgn(d*x^2 + c) - 46*a^2*b*c*d^5*sgn(d*x^2 + c
) + 3*a*b^2*d^5*sgn(d*x^2 + c))/(a^2*d^8)) - 1/96*(24*a^(5/2)*b*c^2*sgn(d*x
^2 + c) - 12*a^(3/2)*b^2*c*sgn(d*x^2 + c) - sqrt(a)*b^3*sgn(d*x^2 + c))*log
(abs(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
 b*d*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x
^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d - a^(3/2)*b*c^2*d
- 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c
))^3*a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
 + a*c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d
*x^2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))/(a^2*d^2*abs(d))
```

maple [B] time = 0.07, size = 1018, normalized size = 4.09

$$-\frac{\left(-72a^2b\,c^2d^2x^2\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)+48\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/96*(48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4
*a^2*c*d^2-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)
*x^4*a*b*d^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a^2*b*c^2*d^2+48*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a^2*c^2*d+
36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a*b^2*c*d^2-16*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*x^2*a*d+96*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*b*c*d+3*ln(1/2*(2*a*d^2*x
^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2))/(a*d^2)^(1/2))*x^2*b^3*d^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a^2
*b*c^3*d-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^
2*b^2*d+36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a*b^2*c^2*d+96*((d*x^2+c)*(
a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*a*b*c^2-16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*a*c+108*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*c^2+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))
*b^3*c*d-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b^
2*c)/d^3/a*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)
```

maxima [A] time = 1.71, size = 368, normalized size = 1.48

$$-\frac{bc^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^3}-\frac{3\left(8\,a^2bc^2-20\,ab^2c+b^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6\,a^3bc^2-12\,a^2b^2c-ab^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\left(8\,a^4b\right.}{48\left(a^4d^3-\frac{3\left(adx^2+ac+b\right)a^3d^3}{dx^2+c}+\frac{3\left(adx^2+ac+b\right)^2a^2d^3}{\left(dx^2+c\right)^2}-\frac{\left(adx^2+ac+b\right)^3ad^3}{\left(dx^2+c\right)^3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -b*c^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^3 - 1/48*(3*(8*a^2*b*c^2 - 2
0*a*b^2*c + b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^2 -
 12*a^2*b^2*c - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b
*c^2 - 12*a^3*b^2*c - a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*
d^3 - 3*(a*d*x^2 + a*c + b)*a^3*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c + b)^2*a
^2*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a*d^3/(d*x^2 + c)^3) - 1/32*(2
4*a^2*c^2 - 12*a*b*c - b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5*(a + b/(c + d*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.332 $\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=172

$$\frac{a\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2}+\frac{(5b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8d^2}+\frac{bc\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d^2}+\frac{3b(b-4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}\,d^2}$$

```
[Out] 3/8*b*(-4*a*c+b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/d^2/a^(
1/2)+b*c*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+1/8*(-4*a*c+5*b)*(d*x^2+c)*(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+1/4*a*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*
x^2+c))^(1/2)/d^2
```

Rubi [A] time = 0.54, antiderivative size = 222, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 78, 50, 63, 217, 206}

$$\frac{c\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)^2}{bd^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{4bd^2}+\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (3*(b - 4*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(8*d^2) + ((b - 4*a*c)*
(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*b*d^2) + (c*Sqr
t[a + b/(c + d*x^2)]*(b + a*(c + d*x^2))^2)/(b*d^2) + (3*b*(b - 4*a*c)*Sqrt
[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[
b + a*(c + d*x^2)]])/(8*Sqrt[a]*d^2*Sqrt[b + a*(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x(b+ac+adx)^{3/2}}{(c+dx)^{3/2}}dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{c\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{bd^2}+\frac{\left((b-4ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{(b+ac}{\sqrt{c}}\right.}{2bd\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4bd^2}+\frac{c\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right.\right.}{4bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right.\right.}{4bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right.\right.}{4bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right.\right.}{4bd^2}$$

Mathematica [A] time = 0.24, size = 104, normalized size = 0.60

$$\frac{\sqrt{a}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2ac^2+2ad^2x^4+13bc+5bdx^2\right)+3b(b-4ac)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(13*b*c - 2*a*c^2 + 5*b*d*x^
2 + 2*a*d^2*x^4) + 3*b*(b - 4*a*c)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]]
)/(8*Sqrt[a]*d^2)
```

fricas [A] time = 0.93, size = 335, normalized size = 1.95

$$\left[\frac{3\left(4\,abc-b^2\right)\sqrt{a}\,\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+b\right.\right.}{32\,ad^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a*b*c - b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
 + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4
+ 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/(a*d^2), 1/16*(3*(4*a*b*c - b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c +
 b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b
)) + 2*(2*a^2*d^2*x^4 + 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/(a*d^2)]
```

giac [B] time = 1.99, size = 438, normalized size = 2.55

$$\frac{1}{8}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,ax^2\mathrm{sgn}\left(dx^2+c\right)}{d}-\frac{2\,a^2cd^2\mathrm{sgn}\left(dx^2+c\right)-5\,abd^2\mathrm{sgn}\left(dx^2+c\right)}{ad^4}\right)+\frac{\left(4\,a^{\frac{3}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*a*x^2*sgn(d*x^
2 + c)/d - (2*a^2*c*d^2*sgn(d*x^2 + c) - 5*a*b*d^2*sgn(d*x^2 + c))/(a*d^4))
 + 1/16*(4*a^(3/2)*b*c*sgn(d*x^2 + c) - sqrt(a)*b^2*sgn(d*x^2 + c))*log(abs
(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d - a^(3/2)*b*c^2*d - 2*
(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3
*a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a
*c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))/(a*d*abs(d))
```

maple [B] time = 0.06, size = 593, normalized size = 3.45

$$\frac{\left(-12abc\,d^2x^2\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)+4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/16*(4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a
*d^2-12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a*b*c*d^2+3*ln(1/2*(2*a*d^
2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)
^(1/2))/(a*d^2)^(1/2))*x^2*b^2*d^2+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*d^2)^(1/2)*x^2*b*d-12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^
```

```
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a*
b*c^2*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*c
^2+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*b^2*c*d+16*((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)*(a*d^2)^(1/2)*b*c+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(a*d^2)^(1/2)*b*c)/d^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2
)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [A] time = 1.71, size = 247, normalized size = 1.44

$$\frac{bc\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^2}+\frac{3\,(4\,ac-b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,\sqrt{a}\,d^2}-\frac{\left(4\,abc-5\,b^2\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc-3\,ab^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2d^2-\frac{2\left(adx^2+ac+b\right)ad^2}{dx^2+c}+\frac{\left(adx^2+ac+b\right)^2d^2}{\left(dx^2+c\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] b*c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^2 + 3/16*(4*a*c - b)*b*log(-(sq
rt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c))))/(sqrt(a)*d^2) - 1/8*((4*a*b*c - 5*b^2)*((a*d*x^2 + a
*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c - 3*a*b^2)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c)))/(a^2*d^2 - 2*(a*d*x^2 + a*c + b)*a*d^2/(d*x^2 + c) + (a*d*x^
2 + a*c + b)^2*d^2/(d*x^2 + c)^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3*(a + b/(c + d*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.333 $\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=94

$$\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

```
[Out] 1/2*(d*x^2+c)*(a+b/(d*x^2+c))^(3/2)/d+3/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a
^(1/2))*a^(1/2)/d-3/2*b*(a+b/(d*x^2+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = $\{1591, 242, 47, 50, 63, 208\}$

$$\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (-3*b*Sqrt[a + b/(c + d*x^2)])/(2*d) + ((c + d*x^2)*(a + b/(c + d*x^2))^(3/
2))/(2*d) + (3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\text{Subst}\left(\int\left(a+\frac{b}{x}\right)^{3/2}dx,x,c+dx^2\right)}{2d}$$

$$=-\frac{\text{Subst}\left(\int\frac{(a+bx)^{3/2}}{x^2}dx,x,\frac{1}{c+dx^2}\right)}{2d}$$

$$=\frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{(3b)\,\text{Subst}\left(\int\frac{\sqrt{a+bx}}{x}dx,x,\frac{1}{c+dx^2}\right)}{4d}$$

$$=-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{(3ab)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}dx,x,\frac{1}{c+dx^2}\right)}{4d}$$

$$=-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{(3a)\,\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx,x,\sqrt{a+\frac{b}{c+dx^2}}\right)}{2d}$$

$$=-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}+\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

Mathematica [A] time = 0.10, size = 79, normalized size = 0.84

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(a\left(c+dx^2\right)-2b\right)+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b + a*(c + d*x^2)) + 3*Sqrt[a]*b
*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

fricas [A] time = 0.85, size = 269, normalized size = 2.86

$$\left[\frac{3\sqrt{a}\,b\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2+4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2+bc\right)\sqrt{a}\,\sqrt{\cdots}\right.}{8d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 +
 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c - 2*b)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/d, -1/4*(3*sqrt(-a)*b*arctan(1/2*(2*a*d*x^2
+ 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^
2*c + a*b)) - 2*(a*d*x^2 + a*c - 2*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/d]
```

giac [B] time = 1.82, size = 387, normalized size = 4.12

$$-\frac{\sqrt{a}\, b \log\left(\left|-2\, a^{\frac{5}{2}} c^3 d - 6\left(\sqrt{ad^2}\, x^2 - \sqrt{ad^2x^4 + 2\, acdx^2 + bdx^2 + ac^2 + bc}\right) a^2 c^2 |d| - 6\left(\sqrt{ad^2}\, x^2 - \sqrt{ad^2x^4 + 2\, aca\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(a)*b*log(abs(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x
^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d
- a^(3/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))^3*a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a
*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))*sgn(d*x^2
+ c)/abs(d) - 1/4*sqrt(a)*b*abs(d)*log(abs(a))*sgn(d*x^2 + c)/d^2 + 1/2*sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*a*sgn(d*x^2 + c)/d
```

maple [B] time = 0.05, size = 336, normalized size = 3.57

$$-\frac{\left(-3ab\, d^2x^2 \ln\left(\frac{2a\, d^2x^2+2acd+bd+2\sqrt{a\, d^2x^4+2acd\, x^2+bd\, x^2+a\, c^2+bc}\, \sqrt{a\, d^2}}{2\sqrt{a\, d^2}}\right) - 3abcd \ln\left(\frac{2a\, d^2x^2+2acd+bd+2\sqrt{a\, d^2x^4+2acd\, x^2+bd\, x^2+a\, c}}{2\sqrt{a\, d^2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/4*(-3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a*b*d^2-2*(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*d*x^2-3*a*b*c*d*ln(1/2*(2*
a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*
d^2)^(1/2))/(a*d^2)^(1/2))-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(a*d^2)^(1/2)*a*c+4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*b)/d*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)
^(1/2)
```

maxima [A] time = 2.24, size = 156, normalized size = 1.66

$$-\frac{ab\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad - \frac{\left(adx^2+ac+b\right)d}{dx^2+c}\right)} - \frac{3\sqrt{a}\, b \log\left(-\frac{\sqrt{a} - \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a} + \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\, d} - \frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*d
/(d*x^2 + c)) - 3/4*sqrt(a)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/d - b*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))/d
```

mupad [B] time = 3.74, size = 61, normalized size = 0.65

$$-\frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\,\left(d\,x^2+c\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\left(d\,x^2+c\right)}{b}\right)}{d\left(\frac{a\left(d\,x^2+c\right)}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] -((a + b/(c + d*x^2))^(3/2)*(c + d*x^2)*hypergeom([-3/2, -1/2], 1/2, -(a*(c
 + d*x^2))/b))/(d*((a*(c + d*x^2))/b + 1)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.334 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} dx$

Optimal. Leaf size=126

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)-\frac{(ac+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}$$

```
[Out] a^(3/2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))-(a*c+b)^(3/2)*ar
ctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(3/2)+b*((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c
```

Rubi [A] time = 0.49, antiderivative size = 206, normalized size of antiderivative = 1.63, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6722, 1975, 446, 98, 157, 63, 217, 206, 93, 208}

$$\frac{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{\sqrt{a\left(c+dx^2\right)+b}}-\frac{(ac+b)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{c^{3/2}\sqrt{a\left(c+dx^2\right)+b}}+\frac{b\sqrt{a+\cdots}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (b*Sqrt[a + b/(c + d*x^2)])/c + (a^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*
x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/Sqrt[b +
a*(c + d*x^2)] - ((b + a*c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*A
rcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/
(c^(3/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x(c+dx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\frac{1}{2}(b+ac)^2d+\frac{1}{2}a^2cd^2x}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left((b+ac)^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2c\sqrt{b+a\left(c+dx^2\right)}} + \cdots$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left(a^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx,x,\sqrt{c+dx^2}\right)}{\sqrt{b+a\left(c+dx^2\right)}} + \cdots$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} - \frac{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{c^{3/2}\sqrt{b+a\left(c+dx^2\right)}} + \cdots$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+a\left(c+dx^2\right)}} - \cdots$$

Mathematica [A] time = 0.23, size = 118, normalized size = 0.94

$$\frac{a^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)+b\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}-(ac+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (b*Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] + a^(3/2)*c^(3/2)*ArcTanh[
Sqrt[a + b/(c + d*x^2)]/Sqrt[a]] - (b + a*c)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[a
+ b/(c + d*x^2)])/Sqrt[b + a*c]])/c^(3/2)
```

fricas [A] time = 1.02, size = 1073, normalized size = 8.52

$$\left[\frac{a^{\frac{3}{2}} c \log\left(8 a^2 d^2 x^4 + 8 a^2 c^2 + 8 \left(2 a^2 c + ab\right) d x^2 + 8 abc + b^2 + 4 \left(2 a d^2 x^4 + (4 ac + b) d x^2 + 2 ac^2 + bc\right) \sqrt{a} \sqrt{\frac{ad}{}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + (a*c + b)*sqrt((a*c + b)/c)*log(((
8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8
*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c
^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c))*sqrt((a*c + b)/c))/x^4) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c
, -1/4*(2*sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt((
a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c
^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*
c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*b*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)))/c, 1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2
*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*
c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 2*(a*c + b)*sqr
t(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x
^2 + 2*a*b*c + b^2)) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c, -1/2*(
sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*
c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt(-(a*c + b)/
c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c
+ b^2)) - 2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(d*x
^2+c)]Evaluation time: 0.71Error: Bad Argument Type
```

maple [B] time = 0.06, size = 652, normalized size = 5.17

$$\frac{\left(a^2 c^2 d^2 x^2 \ln\left(\frac{2 a d^2 x^2 + 2acd + bd + 2\sqrt{a d^2 x^4 + 2acd\, x^2 + bd\, x^2 + a c^2 + bc}\ \sqrt{a d^2}}{2\sqrt{a d^2}}\right) + a^2 c^3 d \ln\left(\frac{2a d^2 x^2 + 2acd + bd + 2\sqrt{a d^2 x^4 + 2acd\, x^2 + bd\, x^2 + a c}}{2\sqrt{a d^2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x,x)
```

```
[Out] 1/2*(ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a^2*c^2*d^2-(a*d^2)^(1/2)*(a*
c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*a*c*d+ln(1/2*(2*a*d
^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2
)^(1/2))/(a*d^2)^(1/2))*a^2*c^3*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c
*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*b*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2))/x^2)*a*c^2-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2))/x^2)*b*c+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^
2)^(1/2)*b*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/c^2/((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [A] time = 2.11, size = 201, normalized size = 1.60

$$-\frac{1}{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)+\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c}+\frac{\left(a^2c^2+2\,abc+b^2\right)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\,\sqrt{(ac+b)c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a^(3/2)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a
) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))) + b*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c))/c + 1/2*(a^2*c^2 + 2*a*b*c + b^2)*log((c*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x, x)
```

3.335 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=138

$$\frac{3bd\sqrt{ac+b}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{5/2}}-\frac{3bd\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2c^2}-\frac{\left(c+dx^2\right)\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{2cx^2}$$

```
[Out] -1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/c/x^2+3/2*b*d*arctanh(c^(1
/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*(a*c+b)^(1/2)/c^(5/2)-
3/2*b*d*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2
```

Rubi [A] time = 0.53, antiderivative size = 170, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6722, 1975, 446, 94, 93, 208}

$$-\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2}+\frac{3bd\sqrt{ac+b}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{2c^{5/2}\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{a+\frac{b}{c+dx^2}}\left(a\left(c+dx^2\right)+b\right)}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (-3*b*d*Sqrt[a + b/(c + d*x^2)])/(2*c^2) - (Sqrt[a + b/(c + d*x^2)]*(b + a*
(c + d*x^2)))/(2*c*x^2) + (3*b*Sqrt[b + a*c]*d*Sqrt[c + d*x^2]*Sqrt[a + b/(
c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c
+ d*x^2)])])/(2*c^(5/2)*Sqrt[b + a*(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^3\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^3\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^2(c+dx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} - \frac{\left(3bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x(c+dx)^{3/2}}\,dx, x, x^2\right)}{4c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} - \frac{\left(3b(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Su}}{4c^2\sqrt{b+a\left(c\right.}}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} - \frac{\left(3b(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Su}}{2c^2\sqrt{b+a}}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{2cx^2} + \frac{3b\sqrt{b+ac}\,d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tan}{2c^{5/2}\sqrt{b+a\left(c+dx\right.}}$$

Mathematica [A] time = 0.58, size = 256, normalized size = 1.86

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2\sqrt{c(ac+b)}\left(a^2c\left(c+dx^2\right)^2+ab\left(2c^2+5cdx^2+3d^2x^4\right)+b^2\left(c+3dx^2\right)\right)-6bdx^2\log(x)(ac+b)\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(a^2*c*(c + d*
x^2)^2 + b^2*(c + 3*d*x^2) + a*b*(2*c^2 + 5*c*d*x^2 + 3*d^2*x^4)) - 6*b*(b
+ a*c)*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x] + 3*b*(b + a*c)*d
*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2*a*c*(c + d*x^2) + b*(2*c +
 d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]]))/(4*c
^2*Sqrt[c*(b + a*c)]*x^2*(b + a*(c + d*x^2)))
```

fricas [A] time = 1.07, size = 404, normalized size = 2.93

$$\left[\frac{3\,bdx^{2}\sqrt{\frac{ac+b}{c}}\,\log\left(\frac{\left(8\,a^{2}c^{2}+8\,abc+b^{2}\right)d^{2}x^{4}+8\,a^{2}c^{4}+16\,abc^{3}+8\,b^{2}c^{2}+8\left(2\,a^{2}c^{3}+3\,abc^{2}+b^{2}c\right)dx^{2}+4\left(\left(2\,ac^{2}+bc\right)d^{2}x^{4}+2\,ac^{4}+2\,bc^{3}+\left(4\,ac^{3}+3\,bc^{2}\right.\right.}{x^{4}}\right.}{8\,c^{2}x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*b*d*x^2*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4
+ 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*
x^2 + 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*
d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*((
a*c + 3*b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2
*x^2), -1/4*(3*b*d*x^2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2
*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a
^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 2*((a*c + 3*b)*d*x^2 + a*c
^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.07, size = 820, normalized size = 5.94

$$-\frac{\sqrt{\frac{a\,d\,x^{2}+a c+b}{d\,x^{2}+c}}\,\left(-3 a b\,c^{2} d^{2} x^{4} \ln\left(\frac{2 a c d\,x^{2}+b d\,x^{2}+2 a\,c^{2}+2 b c+2\sqrt{a\,c^{2}+b c}\,\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}}{x^{2}}\right)-2\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^3,x)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^6*a*d^3-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c
^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
))/x^2)*x^4*a*b*c^2*d^2-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*c^2+b*c)^(1/2)*x^4*a*c*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c
^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*b^2
*c*d^2-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*
x^4*b*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*a*b*c^3*d-4*(a*d^2*x^4
```

```
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^2*a*c^2*d-3*ln((2
*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*b^2*c^2*d+4*((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)*(a*c^2+b*c)^(1/2)*x^2*b*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(3/2)*(a*c^2+b*c)^(1/2)*x^2*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*(a*c^2+b*c)^(1/2)*x^2*b*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(3/2)*(a*c^2+b*c)^(1/2)*c)/(a*c^2+b*c)^(1/2)/x^2/c^3/((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)
```

maxima [A] time = 2.41, size = 202, normalized size = 1.46

$$-\frac{\left(abc+b^2\right)d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^3+bc^2-\frac{\left(adx^2+ac+b\right)c^3}{dx^2+c}\right)}-\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^2}-\frac{3\left(abc+b^2\right)d\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b*c + b^2)*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^3 + b*c^2 -
 (a*d*x^2 + a*c + b)*c^3/(d*x^2 + c)) - b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c))/c^2 - 3/4*(a*b*c + b^2)*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*
c + b)*c)))/(sqrt((a*c + b)*c)*c^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^3,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**3,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**3, x)
```

3.336 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=205

$$-\frac{3bd^2(4ac+5b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{7/2}\sqrt{ac+b}}+\frac{bd^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^3}+\frac{d(4ac+9b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^3x^2}-\frac{(ac+b)\left(c+dx^2\right.}{4c^3}$$

```
[Out] -3/8*b*(4*a*c+5*b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a
*c+b)^(1/2))/c^(7/2)/(a*c+b)^(1/2)+b*d^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/
c^3+1/8*(4*a*c+9*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^2-1
/4*(a*c+b)*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^4
```

Rubi [A] time = 0.59, antiderivative size = 260, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{3bd^2(4ac+5b)\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(ac+b)}-\frac{3bd^2(4ac+5b)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{8c^{7/2}\sqrt{ac+b}\sqrt{a\left(c+dx^2\right)+b}}+\frac{d(4ac+5b)\sqrt{a+\cdots}}{8c^2x^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (3*b*(5*b + 4*a*c)*d^2*Sqrt[a + b/(c + d*x^2)])/(8*c^3*(b + a*c)) + ((5*b +
 4*a*c)*d*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(8*c^2*(b + a*c)*x^2
) - (Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2))^2)/(4*c*(b + a*c)*x^4) - (
3*b*(5*b + 4*a*c)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt
[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^(7/2)*S
qrt[b + a*c]*Sqrt[b + a*(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^5\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^5\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^3(c+dx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{4c(b+ac)x^4} - \frac{\left((5b+4ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+a}{x^2(c+dx}\right.}{8c(b+ac)\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)^2}{4c(b+ac)x^4} + \frac{\left(3b(5b+\right.}{}$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\ (}{4c(b}$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\ (}{4c(b}$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\ (}{4c(b}$$

Mathematica [A] time = 0.33, size = 190, normalized size = 0.93

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{2c^2(ac+b)}{x^4} + \frac{3bd^2(4ac+5b)\left(c+dx^2\right)\left(2\log(x)-\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}} + d^2(2ac\right.}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((15*b + 2*a*c)*d^2 - (2*c^2*(b + a*
c))/x^4 + (5*b*c*d)/x^2 + (3*b*(5*b + 4*a*c)*d^2*(c + d*x^2)*(2*Log[x] - Lo
g[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2
)*(b + a*c + a*d*x^2)]]))/(2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*(c +
 d*x^2))])))/(8*c^3)
```

fricas [A] time = 1.52, size = 557, normalized size = 2.72

$$\left[\frac{3\left(4\,abc+5\,b^2\right)\sqrt{ac^2+bc}\,d^2x^4\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2-4\left((2\,ac+b)d^2x^4+2\,ac^3+(4\right.}{x^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a*b*c + 5*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*
b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*
a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*
c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2*c)*d^2*x^4 + 4*a*b
*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/((a*c^5 + b*c^4)*x^4), 1/16*(3*(4*a*b*c + 5*b^2)*sqrt(-a*c^2 -
b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 -
 b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2
 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2
*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^5 + b*c^4)*x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.08, size = 1653, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^5,x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a^2*c*d^4-12*ln((2*a*c*d*x^2+b*d*x^2
+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2))/x^2)*x^6*a^3*b*c^5*d^3-18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a*b*d^4-39*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*
b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^
2)*x^6*a^2*b^2*c^4*d^3-32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*c^2+b*c)^(3/2)*x^6*a^2*c^2*d^3-42*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2
*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^
6*a*b^3*c^3*d^3-12*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2
)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*a^3*b*c^6*d^2-6
2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*a*b
*c*d^3-15*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*
x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^6*b^4*c^2*d^3-39*ln((2*a*c
*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*a^2*b^2*c^5*d^2-18*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*b^2*d^3-20*(a*d^2*x^4+2*a*c*d
```

```
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a^2*c^3*d^2-42*ln((2*a*
c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*a*b^3*c^4*d^2+16*((d*x^2+c)*(a*d*x^2+a*c+b
))^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b*c^2*d^2+12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*a*c*d^2-44*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b*c^2*d^2-15*ln((2*a*c*d*x
^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2))/x^2)*x^4*b^4*c^3*d^2+16*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2
)*(a*c^2+b*c)^(3/2)*x^4*b^2*c*d^2+18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*b*d^2-18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*b^2*c*d^2+8*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*a*c^2*d+14*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*b*c*d-4*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*c^3-4*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*b*c^2)/(a*c^2+b*c)^(3/2)/x^4
/(a*c+b)/c^4/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [A] time = 2.42, size = 313, normalized size = 1.53

$$\frac{bd^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^3}+\frac{3\left(4\,abc+5\,b^2\right)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,\sqrt{(ac+b)c}\,c^3}-\frac{\left(4\,abc^2+9\,b^2c\right)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc^2+11\right.}{8\left(a^2c^5+2\,abc^4+b^2c^3+\frac{\left(adx^2+ac+b\right)^2c^5}{\left(dx^2+c\right)^2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] b*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c^3 + 3/16*(4*a*b*c + 5*b^2)*d^
2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*
c^3) - 1/8*((4*a*b*c^2 + 9*b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/
2) - (4*a^2*b*c^2 + 11*a*b^2*c + 7*b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c)))/(a^2*c^5 + 2*a*b*c^4 + b^2*c^3 + (a*d*x^2 + a*c + b)^2*c^5/(d*x^2 +
 c)^2 - 2*(a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^5,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**5,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**5, x)
```

3.337 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7} dx$

Optimal. Leaf size=292

$$\frac{bd^3\left(24a^2c^2+60abc+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{16c^{9/2}(ac+b)^{3/2}}-\frac{d^2\left(24a^2c^2+108abc+79b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48c^4x^2(ac+b)}-\frac{bd^3\sqrt{\frac{ac}{}}}{c}$$

```
[Out] -1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(5/2)/c^2/(a*c+b)/x^6+1/16*b*(
24*a^2*c^2+60*a*b*c+35*b^2)*d^3*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/(a*c+b)^(1/2))/c^(9/2)/(a*c+b)^(3/2)-b*d^3*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/c^4-1/48*(24*a^2*c^2+108*a*b*c+79*b^2)*d^2*(d*x^2+c)*((a*d*x^2+a*c
+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)/x^2+1/24*(12*a*c+11*b)*d*(d*x^2+c)^2*((a*d
*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/x^4
```

Rubi [A] time = 0.73, antiderivative size = 287, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 98, 151, 152, 12, 93, 208}

$$-\frac{d^3\left(8a^2c^2+110abc+105b^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(ac+b)}+\frac{bd^3\left(24a^2c^2+60abc+35b^2\right)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c}}{\sqrt{c}\sqrt{a(c+d}}\right)}{16c^{9/2}(ac+b)^{3/2}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] -((105*b^2 + 110*a*b*c + 8*a^2*c^2)*d^3*Sqrt[a + b/(c + d*x^2)])/(48*c^4*(b
 + a*c)) - ((b + a*c)*Sqrt[a + b/(c + d*x^2)])/(6*c*x^6) + (7*b*d*Sqrt[a +
b/(c + d*x^2)])/(24*c^2*x^4) - (b*(35*b + 32*a*c)*d^2*Sqrt[a + b/(c + d*x^2
)])/(48*c^3*(b + a*c)*x^2) + (b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*Sqrt[c
 + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(
Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(16*c^(9/2)*(b + a*c)^(3/2)*Sqrt[b + a*(
c + d*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^7\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^7\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^4(c+dx)^{3/2}}dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\frac{7}{2}b(b+ac)d+3abd^2x}{x^3(c+dx)^{3/2}\sqrt{b+ac+adx}}dx, x, x^2\right)}{6c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\frac{1}{4}b(b+ac)(35b+32\cdots}{x^2(c+dx)^{3/\cdots}}\cdots\right.}{12c^2(b+ac)\sqrt{b+a\left(c+\cdots\right.}}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35b+32ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{48c^3(b+ac)x^2} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\cdots\right.}{\cdots}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35\cdots}{\cdots}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35\cdots}{\cdots}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35\cdots}{\cdots}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35\cdots}{\cdots}$$

Mathematica [A] time = 0.52, size = 245, normalized size = 0.84

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3bd^3\left(24a^2c^2+60abc+35b^2\right)\left(c+dx^2\right)\left(2\log(x)-\log\left(2\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(ac+adx^2+b\right)}+2ac\left(c+dx^2\right)+b\left(2c+dx^2\right)\right)\right)}{\sqrt{c(ac+b)}\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}} + 2d^3\left(8a^2c^2\cdots\right.\right.}{96c^4(ac+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] -1/96*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*(105*b^2 + 110*a*b*c + 8*a^
2*c^2)*d^3 + (16*c^3*(b + a*c)^2)/x^6 - (28*b*c^2*(b + a*c)*d)/x^4 + (2*b*c
```

```
*(35*b + 32*a*c)*d^2)/x^2 + (3*b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*(c +
d*x^2)*(2*Log[x] - Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b +
a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]]))/(Sqrt[c*(b + a*c)]*Sqrt[(c +
 d*x^2)*(b + a*(c + d*x^2))])))/(c^4*(b + a*c))
```

fricas [A] time = 2.43, size = 733, normalized size = 2.51

$$\left[\frac{3\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)\sqrt{ac^2+bc}\,d^3x^6\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2+4\left(\right.}{x^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*lo
g(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2
 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c
^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 118*a^2*b*c^3 + 21
5*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4
+ (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2*b*c^5 + 2*a*
b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^7
+ 2*a*b*c^6 + b^2*c^5)*x^6), -1/96*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*
sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)
*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*
c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 + 118*a
^2*b*c^3 + 215*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5
 + 8*b^3*c^4 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2
*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/((a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^6)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.08, size = 2605, normalized size = 8.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^7,x)
```

```
[Out] -1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-738*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^8*a*b^2*c*d^4-540*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^6*a^2*b*c^3*d^3+96*(a*c^2
+b*c)^(5/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a^2*b*c^3*d^3+276*(a*d^2*
x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^6*a*b*c*d^3-56
4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^6*a*b
^2*c^2*d^3+192*(a*c^2+b*c)^(5/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a*b^
2*c^2*d^3+168*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(
```

```
5/2)*x^4*a*b*c^2*d^2-76*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*
c^2+b*c)^(5/2)*x^2*a*b*c^3*d-276*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(a*c^2+b*c)^(5/2)*x^10*a^2*b*c*d^5-816*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^8*a^2*b*c^2*d^4-144*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^6*a^3*c^4*d^3-105*ln((2*
a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^8*b^6*c^3*d^4-174*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^8*b^3*d^4-105*ln((2*a*c*d*x^2+b
*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2))/x^2)*x^6*b^6*c^4*d^3+174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^6*b^2*d^3+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a*b*c^4-927*ln((2*a*c*d*x^2+b*d*x^2+2*
a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2))/x^2)*x^6*a^2*b^4*c^6*d^3+96*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(3/2)*(a*c^2+b*c)^(5/2)*x^6*a^2*c^2*d^3-495*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^
2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
)/x^2)*x^6*a*b^5*c^5*d^3+48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)
*(a*c^2+b*c)^(5/2)*x^4*a^2*c^3*d^2-174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^6*b^3*c*d^3+96*(a*c^2+b*c)^(5/2)*((d*x^2+c)
*(a*d*x^2+a*c+b))^(1/2)*x^6*b^3*c*d^3+114*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^4*b^2*c*d^2-32*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*a^2*c^4*d-44*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*x^2*b^2*c^2*d-72*ln((2*a*c
*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2))/x^2)*x^8*a^5*b*c^8*d^4-96*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^10*a^3*c^2*d^5-396*ln((2*a*c*d*x^
2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2))/x^2)*x^8*a^4*b^2*c^7*d^4-861*ln((2*a*c*d*x^2+b*d*x^2+2*a*
c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/
2))/x^2)*x^8*a^3*b^3*c^6*d^4-72*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*
c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^6*a^
5*b*c^9*d^3-174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)
^(5/2)*x^10*a*b^2*d^5-240*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*c^2+b*c)^(5/2)*x^8*a^3*c^3*d^4-927*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+
2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x
^8*a^2*b^4*c^5*d^4-396*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^
(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^6*a^4*b^2*c^8
*d^3-495*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^8*a*b^5*c^4*d^4-861*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^6*a^3*b^3*c^7*d^3+16*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a^2*c^5+16*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2*c^3)/(a*c^2+b*c)^(5/2)/x^
6/(a*c+b)^2/c^5/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [B] time = 2.49, size = 534, normalized size = 1.83

$$-\frac{\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)d^3\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{32\left(ac^5+bc^4\right)\sqrt{(ac+b)c}}-\frac{bd^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^4}-\frac{3\left(8\,a^2bc^4+36\,ab^2c^3+29\,b^3c^2\right)d^3\Big(}{48\Big(a^4c^8+4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/32*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)) + sqrt((a*c + b)*c)))/((a*c^5 + b*c^4)*sqrt((a*c + b)*c)) - b*d^3*sqrt(
```

```
(a*d*x^2 + a*c + b)/(d*x^2 + c))/c^4 - 1/48*(3*(8*a^2*b*c^4 + 36*a*b^2*c^3
+ 29*b^3*c^2)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4
+ 30*a^2*b^2*c^3 + 41*a*b^3*c^2 + 17*b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2
 + c))^(3/2) + 3*(8*a^4*b*c^4 + 44*a^3*b^2*c^3 + 83*a^2*b^3*c^2 + 66*a*b^4*
c + 19*b^5)*d^3*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^8 + 4*a^3*b*c
^7 + 6*a^2*b^2*c^6 + 4*a*b^3*c^5 + b^4*c^4 - (a*c^8 + b*c^7)*(a*d*x^2 + a*c
 + b)^3/(d*x^2 + c)^3 + 3*(a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2 + a*c +
b)^2/(d*x^2 + c)^2 - 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*c^5)*(a*d
*x^2 + a*c + b)/(d*x^2 + c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^7,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**7,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**7, x)
```

3.338 $\int x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=405

$$-\frac{\sqrt{c}\left(a^2c^2-14abc+b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5ad^{5/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{x\left(a^2c^2-14abc+b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5ad^2}-\frac{c^{3/2}(7b-ac)\sqrt{\frac{ac+}{c}}}{5d^{5/}}$$

```
[Out] 1/5*(a^2*c^2-14*a*b*c+b^2)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/5*(-
a*c+7*b)*x*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+6/5*a*x^3*(d*x^2
+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-x^3*(a*d*x^2+a*c+b)*((a*d*x^2+a*c+b
)/(d*x^2+c))^(1/2)/d-1/5*c^(3/2)*(-a*c+7*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/
c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+
c))^(1/2)-1/5*(a^2*c^2-14*a*b*c+b^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x
^2+c))^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 526, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6722, 1975, 467, 581, 582, 531, 418, 492, 411}

$$\frac{x\left(a^2c^2-14abc+b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{5ad^2\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{c}\left(a^2c^2-14abc+b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\tan^{-1}\left(\right.\right.}{5ad^{5/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((b^2 - 14*a*b*c + a^2*c^2)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2
)])/(5*a*d^2*Sqrt[b + a*(c + d*x^2)]) + ((7*b - a*c)*x*(c + d*x^2)*Sqrt[b +
 a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(5*d^2*Sqrt[b + a*(c + d*x^2)]) +
(6*a*x^3*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(5*d*
Sqrt[b + a*(c + d*x^2)]) - (x^3*(b + a*c + a*d*x^2)^(3/2)*Sqrt[a + b/(c + d
*x^2)])/(d*Sqrt[b + a*(c + d*x^2)]) - (Sqrt[c]*(b^2 - 14*a*b*c + a^2*c^2)*S
qrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], b/(b + a*c)])/(5*a*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*
c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) - (c^(3/2)*(7*b - a*c)*Sqrt[b + a
*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, b/(b + a*c)])/(5*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x
^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
```

```
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{x^3\left(b+ac+adx^2\right)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\sqrt{b+ac+adx^2}\left(3(b+ac)+6\right.}{\sqrt{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{6ax^3\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x^3\left(b+ac+adx^2\right)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+$$

$$=\frac{(7b-ac)x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{6ax^3\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{\cdot}}{5d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{(7b-ac)x\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{6ax^3\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{\cdot}}{5d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(b^2-14abc+a^2c^2\right)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5ad^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{(7b-ac)x\left(c+dx^2\right)\sqrt{b+ac+a}}{5d^2\sqrt{b+a\left(c+dx\right.}}$$

$$=\frac{\left(b^2-14abc+a^2c^2\right)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5ad^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{(7b-ac)x\left(c+dx^2\right)\sqrt{b+ac+a}}{5d^2\sqrt{b+a\left(c+dx\right.}}$$

Mathematica [C] time = 0.85, size = 308, normalized size = 0.76

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\sqrt{\frac{ad}{ac+b}}\left(-a^2\left(c-dx^2\right)\left(c+dx^2\right)^2+3ab\left(2c^2+3cdx^2+d^2x^4\right)+b^2\left(7c+2dx^2\right)\right)-ic\left(a^2c^2-14abc+\right.\right.}{5d^2\sqrt{\frac{ad}{ac+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(-(a^2*(c -
 d*x^2)*(c + d*x^2)^2) + b^2*(7*c + 2*d*x^2) + 3*a*b*(2*c^2 + 3*c*d*x^2 + d
^2*x^4)) - I*c*(b^2 - 14*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c
)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/
(a*c)] + (8*I)*b*c*(b - a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (
```

```
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(5*d
^2*Sqrt[(a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(adx^6 + (ac+b)x^4\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*d*x^6 + (a*c + b)*x^4)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d
*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}} x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

maple [B] time = 0.06, size = 1098, normalized size = 2.71

$$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,a^2d^3x^7+\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,a^2c\,d^2x^5+3\sqrt{-\frac{ad}{ac+b}}\,\sqrt{(\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/5*((-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^7*a^2*d^3+(
-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^5*a^2*c*d^2+3*(-1
/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^5*a*b*d^2-(-1/(a*c+
b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^3*a^2*c^2*d+5*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^3*a*b*c*d+4*(
-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^3*a*b*c*d+((a*d*x
^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/
2)*x,((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^3+2*(-1/(
a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^3*b^2*d-(-1/(a*c+b)*a
*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x*a^2*c^3+8*((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b
)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2-14*((a*d*x^2+a*c+b)
/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*
c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2+5*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x*a*b*c^2+(-1/(a*c
+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x*a*b*c^2-8*((a*d*x^2+a*c+
b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((
a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c+((a*d*x^2+a*c+b)
/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*
c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c+5*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x*b^2*c+2*(-1/(a*c+b
)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x*b^2*c)*((a*d*x^2+a*c+b)/(d
*x^2+c))^(1/2)/d^2/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(a*c
+b)*a*d)^(1/2)/(a*d*x^2+a*c+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(a + \frac{b}{d\,x^2 + c}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^4*(a + b/(c + d*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(\frac{ac + adx^2 + b}{c + dx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.339 $\int x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=331

$$\frac{\sqrt{c}\,(3b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{\sqrt{c}\,(7b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{4ax\left(c+dx^2\right)\sqrt{}}{3d}$$

```
[Out] 1/3*(-a*c+7*b)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d+4/3*a*x*(d*x^2+c)*((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-x*(a*d*x^2+a*c+b)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/d-1/3*(-a*c+7*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE
(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a
*c+b)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+
1/3*(-a*c+3*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/
c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 430, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 467, 528, 531, 418, 492, 411}

$$\frac{\sqrt{c}\,(3b-ac)\sqrt{a+\frac{b}{c+dx^2}}\,\sqrt{ac+adx^2+b}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\,\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{c}\,(7b-ac)\sqrt{a+\frac{b}{c+dx^2}}\,\sqrt{ac+adx^2+b}\,E\left(\tan\right.}{3d^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\,\sqrt{a\left(c+dx^2\right)+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((7*b - a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(3*d*Sqrt[b
 + a*(c + d*x^2)]) + (4*a*x*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/
(c + d*x^2)])/(3*d*Sqrt[b + a*(c + d*x^2)]) - (x*(b + a*c + a*d*x^2)^(3/2)*
Sqrt[a + b/(c + d*x^2)])/(d*Sqrt[b + a*(c + d*x^2)]) - (Sqrt[c]*(7*b - a*c)
*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], b/(b + a*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*
c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) + (Sqrt[c]*(3*b - a*c)*Sqrt[b + a
*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, b/(b + a*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x
^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{x\left(b+ac+adx^2\right)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{\sqrt{b+ac+adx^2}\left(b+ac+4ad\right.}{\sqrt{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{4ax\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x\left(b+ac+adx^2\right)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+$$

$$=\frac{4ax\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x\left(b+ac+adx^2\right)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+$$

$$=\frac{(7b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}+\frac{4ax\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{(7b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}+\frac{4ax\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}$$

Mathematica [C] time = 0.78, size = 270, normalized size = 0.82

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\sqrt{\frac{ad}{ac+b}}\left(a^2\left(c+dx^2\right)^2-2ab\left(c+dx^2\right)-3b^2\right)+ib(5ac-3b)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\right.\right.\right.}{3d\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(-3*b^2 - 2
*a*b*(c + d*x^2) + a^2*(c + d*x^2)^2) + I*a*c*(-7*b + a*c)*Sqrt[(b + a*c +
a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b +
 a*c)]*x], 1 + b/(a*c)] + I*b*(-3*b + 5*a*c)*Sqrt[(b + a*c + a*d*x^2)/(b +
a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 +
 b/(a*c)]))/(3*d*Sqrt[(a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(adx^4+(ac+b)x^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*d*x^4 + (a*c + b)*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d
*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

maple [B] time = 0.04, size = 820, normalized size = 2.48

$$\frac{\left(\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{-\frac{ad}{ac+b}}\,a^2d^2x^5+2\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{-\frac{ad}{ac+b}}\,a^2cd\,x^3+\sqrt{\left(d\,x^2+c\right)\left(ac\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/3*(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^5*a^2*d^2+2
*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^3*a^2*c*d+((d*x
^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^3*a*b*d-((d*x^2+c)*(a
*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*a^2*c^2-3*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^3*a*b*d+((d*x
^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x*a^2*c^2-5*((d*x^2+c)*
(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*a*b*c+7*((d*x^2+c)*
(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*a*b*c+((d*x^2+c)*(a
*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x*a*b*c+3*((d*x^2+c)*(a*d*x^2+a
*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
(-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*b^2-3*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x*a*b*c-3*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x*b^2)*((a*d*x^2+a*c+
b)/(d*x^2+c))^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(
a*c+b)*a*d)^(1/2)/(a*d*x^2+a*c+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + \frac{b}{d\,x^2 + c}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^2*(a + b/(c + d*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ac + adx^2 + b}{c + dx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.340 $\int \left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

Optimal. Leaf size=260

$$-\frac{x(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}+\frac{bx\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}+\frac{a\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}$$

```
[Out] b*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c-(-a*c+b)*x*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c+(-a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^
(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c
)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(1/2
)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 348, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6722, 1974, 413, 531, 418, 492, 411}

$$-\frac{x(b-ac)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{a\left(c+dx^2\right)+b}}+\frac{bx\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{a\left(c+dx^2\right)+b}}+\frac{a\sqrt{c}\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}\,F\left(\tan^{-1}\right)}{\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}\sqrt{a\left(c+dx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*Sqrt[b + a*(c + d*
x^2)]) - ((b - a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*S
qrt[b + a*(c + d*x^2)]) + ((b - a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c
+ d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[c]*Sqr
t[d]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c +
d*x^2)]) + (a*Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c +
 a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{ac(b+ac)d-a(b-ac)d^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}dx}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(a(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}d}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} - \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} + \frac{a\sqrt{c}\sqrt{b+ac+}}{\sqrt{d}}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} - \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} + \frac{(b-ac)\sqrt{b+a}}{\sqrt{c}}$$

Mathematica [C] time = 0.55, size = 243, normalized size = 0.93

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(bx\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)-2iabc\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)-iac(ac-b)\sqrt{}\right.}{c\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*Sqrt[(a*d)/(b + a*c)]*x*(b + a*(c
 + d*x^2)) - I*a*c*(-b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - (2*
I)*a*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(c*Sqrt[(a*d)/(b + a*c)]
*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(adx^2+ac+b\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*d*x^2 + a*c + b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d*x^2 +
 c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2), x)
```

maple [A] time = 0.03, size = 515, normalized size = 1.98

$$\left(\sqrt{a\,d^2x^4 + 2acd\,x^2 + bd\,x^2 + a\,c^2 + bc}\,\sqrt{-\frac{ad}{ac+b}}\,abd\,x^3 + \sqrt{\left(d\,x^2 + c\right)\left(ad\,x^2 + ac + b\right)}\,\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,a^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] (((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*a^2*c^2
+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*a*b
*d*x^3+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))
*a*b*c-((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*a
*b*c+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)
*a*b*c*x+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(
1/2)*b^2*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/c/(a*d*x^2+a*c+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{c + dx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral((a + b/(c + d*x**2))**(3/2), x)
```

3.341 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=312

$$-\frac{\sqrt{d}\,(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{c^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{dx(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^2}-\frac{(ac+2b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^2x}+\frac{b\sqrt{\cdots}}{\cdots}$$

```
[Out] b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x+(a*c+2*b)*d*x*((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)/c^2-(a*c+2*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/
x-(a*c+2*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(
1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/c^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*(1/(1+d*x^2/
c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(1/2)/(c*(a*d
*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 422, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 468, 583, 531, 418, 492, 411}

$$\frac{dx(ac+2b)\sqrt{ac+adx^2+b}\,\sqrt{a+\frac{b}{c+dx^2}}}{c^2\sqrt{a\left(c+dx^2\right)+b}}-\frac{(ac+2b)\left(c+dx^2\right)\sqrt{ac+adx^2+b}\,\sqrt{a+\frac{b}{c+dx^2}}}{c^2x\sqrt{a\left(c+dx^2\right)+b}}-\frac{\sqrt{d}\,(ac+2b)\sqrt{ac+\cdots}}{c^{3/2}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] (b*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*x*Sqrt[b + a*(c + d*
x^2)]) + ((2*b + a*c)*d*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/
(c^2*Sqrt[b + a*(c + d*x^2)]) - ((2*b + a*c)*(c + d*x^2)*Sqrt[b + a*c + a*d
*x^2]*Sqrt[a + b/(c + d*x^2)])/(c^2*x*Sqrt[b + a*(c + d*x^2)]) - ((2*b + a*
c)*Sqrt[d]*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(c^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/
((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) + (a*Sqrt[d]*Sqrt[b + a*c
 + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
b/(b + a*c)])/(Sqrt[c]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))
]*Sqrt[b + a*(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^2\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^2\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(b+ac)(2b+ac)d-a(b+ac)d^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} - \frac{(2b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c^2x\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c\right.}}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} - \frac{(2b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c^2x\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(a(b\right.}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c^2\sqrt{b+a\left(c+dx^2\right)}} - \frac{(2b+ac)\left(c\right.}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c^2\sqrt{b+a\left(c+dx^2\right)}} - \frac{(2b+ac)\left(c\right.}{}$$

Mathematica [C] time = 0.76, size = 278, normalized size = 0.89

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\sqrt{\frac{ad}{ac+b}}\left(a^2c\left(c+dx^2\right)^2+2ab\left(c+dx^2\right)^2+b^2\left(c+2dx^2\right)\right)-iabcdx\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh\right.\right.}{c^2x\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] -((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(2*a*b*(c +
 d*x^2)^2 + a^2*c*(c + d*x^2)^2 + b^2*(c + 2*d*x^2)) + I*a*c*(2*b + a*c)*d*
x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSi
nh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*a*b*c*d*x*Sqrt[(b + a*c + a*d
*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*
c)]*x], 1 + b/(a*c)]))/(c^2*Sqrt[(a*d)/(b + a*c)]*x*(b + a*(c + d*x^2))))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(adx^2+ac+b\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^4+cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*d*x^2 + a*c + b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d*x^4 +
 c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

maple [B] time = 0.04, size = 873, normalized size = 2.80

$$-\frac{\left(\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{-\frac{ad}{ac+b}}\,a^2c\,d^2x^4+\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{-\frac{ad}{ac+b}}\,ab\,d^2x^4+\sqrt{a\,d^2x^4+2a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^2,x)
```

```
[Out] -(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^4*a^2*c*d^2+((
d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^4*a*b*d^2-((d*x^2+
c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x*a^2*c^2*d+(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^4*a*b*
d^2+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^2*a^2*c^2*
d+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x*a*b*
c*d-2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x*
a*b*c*d+3*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^2*a*b*
c*d+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*
x^2*a*b*c*d+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^2*b^
2*d+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*a^2*c^3+(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^2*b^2*d+
2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*a*b*c^2+((d*x^2+
c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*b^2*c)*((a*d*x^2+a*c+b)/(d
*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(a*c+b)*
a*d)^(1/2)/x/c^2/(a*d*x^2+a*c+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^2,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**2,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**2, x)
```

3.342 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=388

$$-\frac{ad^{3/2}(ac+4b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{d^{3/2}(ac+8b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{5/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{d^2x(ac+8b)\sqrt{}}{3c^3}$$

```
[Out] b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^3-1/3*(a*c+8*b)*d^2*x*((a*d*x^2+a*c
+b)/(d*x^2+c))^(1/2)/c^3-1/3*(a*c+4*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/c^2/x^3+1/3*(a*c+8*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
/c^3/x+1/3*(a*c+8*b)*d^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)
/(d*x^2+c))^(1/2)/c^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a
*(a*c+4*b)*d^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/c^(3/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 520, normalized size of antiderivative = 1.34, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6722, 1975, 468, 583, 531, 418, 492, 411}

$$-\frac{d^2x(ac+8b)\sqrt{ac+adx^2+b}\,\sqrt{a+\frac{b}{c+dx^2}}}{3c^3\sqrt{a\left(c+dx^2\right)+b}}-\frac{ad^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\sqrt{a+\frac{b}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a\left(c+dx^2\right)+b}}+\frac{d^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] (b*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*x^3*Sqrt[b + a*(c +
d*x^2)]) - ((8*b + a*c)*d^2*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2
)])/(3*c^3*Sqrt[b + a*(c + d*x^2)]) - ((4*b + a*c)*(c + d*x^2)*Sqrt[b + a*c
 + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(3*c^2*x^3*Sqrt[b + a*(c + d*x^2)]) +
((8*b + a*c)*d*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])
/(3*c^3*x*Sqrt[b + a*(c + d*x^2)]) + ((8*b + a*c)*d^(3/2)*Sqrt[b + a*c + a*
d*x^2]*Sqrt[a + b/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b
+ a*c)])/(3*c^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*S
qrt[b + a*(c + d*x^2)]) - (a*(4*b + a*c)*d^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sq
rt[a + b/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/
(3*c^(3/2)*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*
Sqrt[b + a*(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^4\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^4\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(b+ac)(4b+ac)d-a(3b+ac)d^2x^2}{x^4\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2x^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+\right.}}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2x^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(8b+ac}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2x^3\sqrt{b+a\left(c+dx^2\right)}} + \frac{(8b+ac}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(8b+ac)d^2x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+\right.}{3c^2}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(8b+ac)d^2x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+\right.}{3c^2}$$

Mathematica [C] time = 0.83, size = 329, normalized size = 0.85

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\sqrt{\frac{ad}{ac+b}}\left(a^2c\left(c-dx^2\right)\left(c+dx^2\right)^2+ab\left(2c^3-3c^2dx^2-13cd^2x^4-8d^3x^6\right)+b^2\left(c^2-4cdx^2-8d^2x^4\right)\right)\right.}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] -1/3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(a^2*c*(
c - d*x^2)*(c + d*x^2)^2 + b^2*(c^2 - 4*c*d*x^2 - 8*d^2*x^4) + a*b*(2*c^3 -
 3*c^2*d*x^2 - 13*c*d^2*x^4 - 8*d^3*x^6)) - I*a*c*(8*b + a*c)*d^2*x^3*Sqrt[
(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] + (4*I)*a*b*c*d^2*x^3*Sqrt[(b + a*c + a*
d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a
*c)]*x], 1 + b/(a*c)]))/(c^3*Sqrt[(a*d)/(b + a*c)]*x^3*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(adx^2+ac+b\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^6+cx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*d*x^2 + a*c + b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d*x^6 +
 c*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

maple [B] time = 0.04, size = 1039, normalized size = 2.68

$$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,a^2c\,d^3x^6+3\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{-\frac{ad}{ac+b}}\,ab\,d^3x^6+5\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^4,x)
```

```
[Out] 1/3*((-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a^2*c*d^3
+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x
^6*a*b*d^3+5*(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^6*a
*b*d^3-((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x
^3*a^2*c^2*d^2+(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4
*a^2*c^2*d^2+4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^
(1/2))*x^3*a*b*c*d^2-8*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(
a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+
b)/a/c)^(1/2))*x^3*a*b*c*d^2+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2)*(-1/(a*c+b)*a*d)^(1/2)*x^4*a*b*c*d^2+10*(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+
c)*(a*d*x^2+a*c+b))^(1/2)*x^4*a*b*c*d^2+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^4*b^2*d^2+5*(-1/(a*c+b)*a*d)^(1/2)*
((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*x^4*b^2*d^2-(-1/(a*c+b)*a*d)^(1/2)*((d*x^
2+c)*(a*d*x^2+a*c+b))^(1/2)*x^2*a^2*c^3*d+3*(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+
c)*(a*d*x^2+a*c+b))^(1/2)*x^2*a*b*c^2*d+4*(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)
*(a*d*x^2+a*c+b))^(1/2)*x^2*b^2*c*d-(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)*a^2*c^4-2*(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+
b))^(1/2)*a*b*c^3-(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*
b^2*c^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/x^3/c^3/(a*d*x^2+a*c+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{d\,x^2+c}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^4,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**4,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**4, x)
```

3.343 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=494

$$-\frac{d^{5/2}\left(a^2c^2+16abc+16b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5c^{7/2}(ac+b)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{d^3x\left(a^2c^2+16abc+16b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5c^4(ac+b)}-\frac{d^2\left(a^2c\right.}{}$$

```
[Out] b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^5+1/5*(a^2*c^2+16*a*b*c+16*b^2)*d^3
*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)-1/5*(a*c+6*b)*(d*x^2+c)*((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/x^5+1/5*(a*c+8*b)*d*(d*x^2+c)*((a*d*x^2
+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^3-1/5*(a^2*c^2+16*a*b*c+16*b^2)*d^2*(d*x^2+c
)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)/x-1/5*(a^2*c^2+16*a*b*c+16*
b^2)*d^(5/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/c^(7/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/5*a*(a*c+8*
b)*d^(5/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/
c^(5/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 1.08, antiderivative size = 648, normalized size of antiderivative = 1.31, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 468, 583, 531, 418, 492, 411}

$$\frac{d^3x\left(a^2c^2+16abc+16b^2\right)\sqrt{ac+adx^2+b}\sqrt{a+\frac{b}{c+dx^2}}}{5c^4(ac+b)\sqrt{a\left(c+dx^2\right)+b}}-\frac{d^2\left(a^2c^2+16abc+16b^2\right)\left(c+dx^2\right)\sqrt{ac+adx^2+b}\sqrt{a}}{5c^4x(ac+b)\sqrt{a\left(c+dx^2\right)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] (b*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(c*x^5*Sqrt[b + a*(c +
d*x^2)]) + ((16*b^2 + 16*a*b*c + a^2*c^2)*d^3*x*Sqrt[b + a*c + a*d*x^2]*Sqr
t[a + b/(c + d*x^2)])/(5*c^4*(b + a*c)*Sqrt[b + a*(c + d*x^2)]) - ((6*b + a
*c)*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)])/(5*c^2*x^5
*Sqrt[b + a*(c + d*x^2)]) + ((8*b + a*c)*d*(c + d*x^2)*Sqrt[b + a*c + a*d*x
^2]*Sqrt[a + b/(c + d*x^2)])/(5*c^3*x^3*Sqrt[b + a*(c + d*x^2)]) - ((16*b^2
 + 16*a*b*c + a^2*c^2)*d^2*(c + d*x^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(
c + d*x^2)])/(5*c^4*(b + a*c)*x*Sqrt[b + a*(c + d*x^2)]) - ((16*b^2 + 16*a*
b*c + a^2*c^2)*d^(5/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*c^(7/2)*(b + a*c)*Sqrt[
(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)]) +
 (a*(8*b + a*c)*d^(5/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a + b/(c + d*x^2)]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*c^(5/2)*(b + a*c)*Sqrt
[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[b + a*(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^6\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^6\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(b+ac)(6b+ac)d-a(5b+ac)d^2x^2}{x^6\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c\right.}}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(8b+}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(8b+}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{(8b+}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(16b^2+16abc+a^2c^2\right)d^3x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^4(b+ac)\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(16b^2+16abc+a^2c^2\right)d^3x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5c^4(b+ac)\sqrt{b+a\left(c+dx^2\right)}}$$

Mathematica [C] time = 1.11, size = 430, normalized size = 0.87

$$-\frac{\sqrt{\frac{ad}{ac+b}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(iacd^3x^5\left(a^2c^2+16abc+16b^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)+\sqrt{\frac{ac}{ac+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^6,x]

[Out] -1/5*(Sqrt[(a*d)/(b + a*c)]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*
d)/(b + a*c)]*(b^3*(c^3 - 2*c^2*d*x^2 + 8*c*d^2*x^4 + 16*d^3*x^6) + a^3*c^2
*(c^4 + c^3*d*x^2 + c*d^3*x^6 + d^4*x^8) + a^2*b*c*(3*c^4 + 5*c^2*d^2*x^4 +
 24*c*d^3*x^6 + 16*d^4*x^8) + a*b^2*(3*c^4 - 3*c^3*d*x^2 + 13*c^2*d^2*x^4 +
 40*c*d^3*x^6 + 16*d^4*x^8)) + I*a*c*(16*b^2 + 16*a*b*c + a^2*c^2)*d^3*x^5*
Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh
```

```
[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*a*b*c*(8*b + 7*a*c)*d^3*x^5*Sqr
t[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sq
rt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(a*c^4*d*x^5*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(adx^2 + ac + b\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{dx^8 + cx^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((a*d*x^2 + a*c + b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(d*x^8 +
 c*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

maple [B] time = 0.04, size = 1666, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^6,x)
```

```
[Out] -1/5*(7*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*
x^5*a^2*b*c^2*d^3-16*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*
c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)
/a/c)^(1/2))*x^5*a^2*b*c^2*d^3+8*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^
2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2
)*x,((a*c+b)/a/c)^(1/2))*x^5*a*b^2*c*d^3-16*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/
2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b
)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x^5*a*b^2*c*d^3+((d*x^2+c)*(a*d*x^2+a*c
+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*a^3*c^6+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
*(-1/(a*c+b)*a*d)^(1/2)*b^3*c^3+5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^8*a*b^2*d^4+5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^6*b^3*d^3+11*((d*x^2+c)*(a*d*x^
2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^6*b^3*d^3+3*((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*a^2*b*c^5+3*((d*x^2+c)*(a*d*x^2+a*c+b))^
(1/2)*(-1/(a*c+b)*a*d)^(1/2)*a*b^2*c^4+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-
1/(a*c+b)*a*d)^(1/2)*x^8*a^3*c^2*d^4+11*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(
-1/(a*c+b)*a*d)^(1/2)*x^8*a*b^2*d^4+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(
a*c+b)*a*d)^(1/2)*x^6*a^3*c^3*d^3+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*
c+b)*a*d)^(1/2)*x^2*a^3*c^5*d+8*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+
b)*a*d)^(1/2)*x^4*b^3*c*d^2+5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/
2)*(-1/(a*c+b)*a*d)^(1/2)*x^8*a^2*b*c*d^4+5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^6*a^2*b*c^2*d^3+10*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^6*a*b^2*c*d^3+11
*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^8*a^2*b*c*d^4+1
```

```
9*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^6*a^2*b*c^2*d^
3+30*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^6*a*b^2*c*d
^3+5*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^4*a^2*b*c^3
*d^2+13*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^4*a*b^2*
c^2*d^2-3*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^2*a*b^
2*c^3*d-2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^2*b^3*
c^2*d-((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x^
5*a^3*c^3*d^3)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)/(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/x^5/c^4/(a*d*x^2+a*c+b
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{d\,x^2+c}\right)^{3/2}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^6,x)
[Out] int((a + b/(c + d*x^2))^(3/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**6,x)
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**6, x)
```

3.344 $\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}} \, dx$

Optimal. Leaf size=225

$$-\frac{(8ac+5b)\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24a^2d^3}-\frac{b\left(8a^2c^2+12abc+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{7/2}d^3}+\frac{\left(8a^2c^2+12abc+5b^2\right)\left(c+dx\right.}{16a^3d^3}$$

```
[Out] -1/16*b*(8*a^2*c^2+12*a*b*c+5*b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/a^(1/2))/a^(7/2)/d^3+1/16*(8*a^2*c^2+12*a*b*c+5*b^2)*(d*x^2+c)*((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)/a^3/d^3-1/24*(8*a*c+5*b)*(d*x^2+c)^2*((a*d*x^2+a*c+
b)/(d*x^2+c))^(1/2)/a^2/d^3+1/6*x^2*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/a/d^2
```

Rubi [A] time = 0.62, antiderivative size = 267, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6722, 1975, 446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(8a^2c^2+12abc+5b^2\right)\left(a\left(c+dx^2\right)+b\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}}-\frac{b\left(8a^2c^2+12abc+5b^2\right)\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{16a^{7/2}d^3\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{(8ac+}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((5*b^2 + 12*a*b*c + 8*a^2*c^2)*(b + a*(c + d*x^2)))/(16*a^3*d^3*Sqrt[a + b
/(c + d*x^2)]) - ((5*b + 8*a*c)*(c + d*x^2)*(b + a*(c + d*x^2)))/(24*a^2*d^
3*Sqrt[a + b/(c + d*x^2)]) + (x^2*(c + d*x^2)*(b + a*(c + d*x^2)))/(6*a*d^2
*Sqrt[a + b/(c + d*x^2)]) - (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*Sqrt[b + a*(c
 + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(16*
a^(7/2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x^2\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{6ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(-c(b+ac)-\frac{1}{2}(5b+8ac)dx\right)}{\sqrt{b+ac+adx}}\,dx,x,\right.}{6ad^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{6ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(\left(-2ac(b+ac)d^2\right.\right.}{}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+d\right.}{6a}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+d\right.}{6a}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+d\right.}{6a}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2\left(c+d\right.}{6a}$$

Mathematica [A] time = 0.28, size = 140, normalized size = 0.62

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(8a^2\left(c^2-cdx^2+d^2x^4\right)+2ab\left(13c-5dx^2\right)+15b^2\right)-3b\left(8a^2c^2+12abc+5b^2\right)\tanh^{-1}\left(\right.}{48a^{7/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + 2*a*b*
(13*c - 5*d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) - 3*b*(5*b^2 + 12*a*b*c
 + 8*a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(7/2)*d^3)
```

fricas [A] time = 0.71, size = 425, normalized size = 1.89

$$\left[\frac{3\left(8a^2bc^2+12ab^2c+5b^3\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2-4\left(2ad^2x^4+(4ac+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*
a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c
 + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
 + 4*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*a*b^
2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(a^4*d^3), 1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(-a)*arctan(1/2*
(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2
*d*x^2 + a^2*c + a*b)) + 2*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 +
26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a^4*d^3)]
```

giac [A] time = 0.57, size = 274, normalized size = 1.22

$$\frac{1}{48}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{ad\mathrm{sgn}\left(dx^2+c\right)}-\frac{4\,a^2cd^3\mathrm{sgn}\left(dx^2+c\right)+5\,abd^3\mathrm{sgn}\left(dx^2+c\right)}{a^3d^5}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2/(a
*d*sgn(d*x^2 + c)) - (4*a^2*c*d^3*sgn(d*x^2 + c) + 5*a*b*d^3*sgn(d*x^2 + c)
)/(a^3*d^5)) + (8*a^2*c^2*d^2*sgn(d*x^2 + c) + 26*a*b*c*d^2*sgn(d*x^2 + c)
+ 15*b^2*d^2*sgn(d*x^2 + c))/(a^3*d^5)) + 1/32*(8*a^2*b*c^2 + 12*a*b^2*c +
5*b^3)*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(7/2)*d^2*abs(d)*sgn(d
*x^2 + c))
```

maple [B] time = 0.05, size = 533, normalized size = 2.37

$$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-24a^2b\,c^2d\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-48\sqrt{a\,d^2x^4+2acd\,x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^3/d^3*(-48*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*c*d*x^2-36*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*d*x^2-24*a^2*b*c^2*d
*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-36*a*b^2*c*d*ln(1/2*(2*a*d^2*x^2+2*a*
c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a
*d^2)^(1/2))+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/
```

```
2)*a+36*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*c
-15*b^3*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+30*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
/(a*d^2)^(1/2)
```

maxima [A] time = 2.28, size = 340, normalized size = 1.51

$$-\frac{3\left(8\,a^{2}bc^{2}+12\,ab^{2}c+5\,b^{3}\right)\left(\frac{adx^{2}+ac+b}{dx^{2}+c}\right)^{\frac{5}{2}}-8\left(6\,a^{3}bc^{2}+12\,a^{2}b^{2}c+5\,ab^{3}\right)\left(\frac{adx^{2}+ac+b}{dx^{2}+c}\right)^{\frac{3}{2}}+3\left(8\,a^{4}bc^{2}+20\,a^{3}b^{2}c+\right.}{48\left(a^{6}d^{3}-\frac{3\left(adx^{2}+ac+b\right)a^{5}d^{3}}{dx^{2}+c}+\frac{3\left(adx^{2}+ac+b\right)^{2}a^{4}d^{3}}{\left(dx^{2}+c\right)^{2}}-\frac{\left(adx^{2}+ac+b\right)^{3}a^{3}d^{3}}{\left(dx^{2}+c\right)^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c
))^(5/2) - 8*(6*a^3*b*c^2 + 12*a^2*b^2*c + 5*a*b^3)*((a*d*x^2 + a*c + b)/(d
*x^2 + c))^(3/2) + 3*(8*a^4*b*c^2 + 20*a^3*b^2*c + 11*a^2*b^3)*sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c)))/(a^6*d^3 - 3*(a*d*x^2 + a*c + b)*a^5*d^3/(d*x^2
+ c) + 3*(a*d*x^2 + a*c + b)^2*a^4*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^
3*a^3*d^3/(d*x^2 + c)^3) + 1/32*(8*a^2*c^2 + 12*a*b*c + 5*b^2)*b*log(-(sqrt
(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c))))/(a^(7/2)*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5/(a + b/(c + d*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**5/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.345 $\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=148

$$\frac{b(4ac+3b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}-\frac{(4ac+3b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^2d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4ad^2}$$

```
[Out] 1/8*b*(4*a*c+3*b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2
)/d^2-1/8*(4*a*c+3*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^2+1
/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2
```

Rubi [A] time = 0.46, antiderivative size = 189, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 80, 50, 63, 217, 206}

$$-\frac{(4ac+3b)\left(a\left(c+dx^2\right)+b\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{b(4ac+3b)\sqrt{a\left(c+dx^2\right)+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{8a^{5/2}d^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -((3*b + 4*a*c)*(b + a*(c + d*x^2)))/(8*a^2*d^2*Sqrt[a + b/(c + d*x^2)]) +
((c + d*x^2)*(b + a*(c + d*x^2)))/(4*a*d^2*Sqrt[a + b/(c + d*x^2)]) + (b*(3
*b + 4*a*c)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[
b + a*(c + d*x^2)]])/(8*a^(5/2)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((3b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{\sqrt{b+ac+adx}}\,dx,x,x^2\right.}{8ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a\left(c+d\right.}\right.}{16a^2d\sqrt{}}$$

$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a\left(c+d\right.}\right.}{8a^2d^2\sqrt{}}$$

$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a\left(c+d\right.}\right.}{8a^2d^2\sqrt{}}$$

$$= -\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{b(3b+4ac)\sqrt{b+a\left(c+dx\right.}}{8a^{5/2}d^2\sqrt{c+dx}}$$

Mathematica [A] time = 0.17, size = 101, normalized size = 0.68

$$\frac{b(4ac+3b)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)-\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(2a\left(c-dx^2\right)+3b\right)}{8a^{5/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (-(Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b + 2*a*(c
- d*x^2))) + b*(3*b + 4*a*c)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(8*a
^(5/2)*d^2)
```

fricas [A] time = 0.66, size = 333, normalized size = 2.25

$$\left[\frac{\left(4abc+3b^2\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2+4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2\right.\right.}{32a^3d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
 + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4
- 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(a^3*d^2), -1/16*((4*a*b*c + 3*b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c
 + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a
*b)) - 2*(2*a^2*d^2*x^4 - 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2
+ a*c + b)/(d*x^2 + c)))/(a^3*d^2)]
```

giac [A] time = 0.52, size = 191, normalized size = 1.29

$$\frac{1}{8}\sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\left(\frac{2\,x^2}{ad\mathrm{sgn}\left(dx^2 + c\right)} - \frac{2\,acd\mathrm{sgn}\left(dx^2 + c\right) + 3\,bd\mathrm{sgn}\left(dx^2 + c\right)}{a^2d^3}\right) - \frac{\left(4\,abc + 3\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a*d*sgn(d
*x^2 + c)) - (2*a*c*d*sgn(d*x^2 + c) + 3*b*d*sgn(d*x^2 + c))/(a^2*d^3)) - 1
/16*(4*a*b*c + 3*b^2)*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(5/2)*d
*abs(d)*sgn(d*x^2 + c))
```

maple [B] time = 0.04, size = 354, normalized size = 2.39

$$\frac{\sqrt{\frac{a d\,x^2+a c+b}{d\,x^2+c}}\;\left(d\,x^2+c\right)\left(4abcd\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\;\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)+4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^2*(4*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*d*x^2+4*a*b*c*d*ln(1/2*(2*a*d^
2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)
^(1/2))/(a*d^2)^(1/2))-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*d^2)^(1/2)*a*c+3*b^2*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-6*(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)/a^2/(a*d^2)^(1/2)
```

maxima [A] time = 2.49, size = 223, normalized size = 1.51

$$-\frac{\left(4\,abc + 3\,b^2\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} - \left(4\,a^2bc + 5\,ab^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4d^2 - \frac{2\left(adx^2+ac+b\right)a^3d^2}{dx^2+c} + \frac{\left(adx^2+ac+b\right)^2a^2d^2}{\left(dx^2+c\right)^2}\right)} - \frac{(4\,ac + 3\,b)b\log\left(-\frac{\sqrt{a} - \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a} + \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{5}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*((4*a*b*c + 3*b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*
c + 5*a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^2 - 2*(a*d*x^2 +
 a*c + b)*a^3*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a^2*d^2/(d*x^2 + c)^2
) - 1/16*(4*a*c + 3*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3/(a + b/(c + d*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.346 $\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}} \, dx$

Optimal. Leaf size=72

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

```
[Out] -1/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/2*(d*x^2+c)*(a+b/
(d*x^2+c))^(1/2)/a/d
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1591, 242, 51, 63, 208}

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*a*d) - (b*ArcTanh[Sqrt[a + b/(c +
d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
```

```
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx, x, c+dx^2\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}+\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4ad}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2ad}\\
&= \frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 70, normalized size = 0.97

$$\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}-b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)] - b*ArcTanh[Sqrt[a + b/(c + d*
x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

fricas [A] time = 0.83, size = 267, normalized size = 3.71

$$\left[\frac{\sqrt{a}\,b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc\right)\sqrt{a}\,\sqrt{\frac{a}{}}\right.}{8\,a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a^2*d), 1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a
```

```
*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c +
 a*b)) + 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d)]
```

giac [B] time = 0.51, size = 138, normalized size = 1.92

$$\frac{b\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}\,|d|-bd\right|\right)}{4\,a^{\frac{3}{2}}|d|\mathrm{sgn}\left(dx^2+c\right)}+\frac{\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{2\,ad\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
+ b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(3/2)*abs(d)*sgn(d*x^2
+ c)) + 1/2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a*d*sgn(
d*x^2 + c))
```

maple [B] time = 0.03, size = 184, normalized size = 2.56

$$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-bd\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\right)}{4\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{a\,d^2}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-b*d*ln(1/2*(2*a*d^2*x^2+2
*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))
/(a*d^2)^(1/2))+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(
1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a/d/(a*d^2)^(1/2)
```

maxima [B] time = 2.81, size = 129, normalized size = 1.79

$$-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2d-\frac{\left(adx^2+ac+b\right)ad}{dx^2+c}\right)}+\frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d - (a*d*x^2 + a*c + b)*a
*d/(d*x^2 + c)) + 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d)
```

mupad [B] time = 3.34, size = 111, normalized size = 1.54

$$\frac{\sqrt{\frac{a\left(d\,x^2+c\right)}{b}+1}\,\left(d\,x^2+c\right)\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\left(d\,x^2+c\right)}}{2\,a\left(d\,x^2+c\right)}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{d\,x^2+c}\,1\mathrm{i}}{\sqrt{b}}\right)3\mathrm{i}}{2\,a^{3/2}\left(d\,x^2+c\right)^{3/2}}\right)}{3\,d\sqrt{a+\frac{b}{d\,x^2+c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] (((a*(c + d*x^2))/b + 1)^(1/2)*(c + d*x^2)*((b^(3/2)*asin((a^(1/2)*(c + d*x
^2)^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3/2)*(c + d*x^2)^(3/2)) + (3*b^(1/2)*(b +
a*(c + d*x^2))^(1/2))/(2*a*(c + d*x^2))))/(3*d*(a + b/(c + d*x^2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.347 $\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=96

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}$$

```
[Out] arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(1/2)-arctanh(c^(1/2)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*c^(1/2)/(a*c+b)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 184, normalized size of antiderivative = 1.92, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 105, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{\sqrt{ac+b}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c +
d*x^2)]])/(Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*Sqrt
[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b
 + a*(c + d*x^2)])])/(Sqrt[b + a*c]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \cdots\right.}{2\sqrt{c+dx^2}\sqrt{a \cdots}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx,x,\sqrt{c+dx^2}\right)}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{\cdots}{-c-(-\cdots}\right.}{\sqrt{c+dx^2}\sqrt{a+\cdots}}$$

$$= -\frac{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\cdots\right.}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Mathematica [A] time = 0.12, size = 80, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]]/Sqrt[a] - (Sqrt[c]*ArcTanh[(Sqrt[c
]*Sqrt[a + b/(c + d*x^2)])/Sqrt[b + a*c]])/Sqrt[b + a*c]
```

fricas [B] time = 0.81, size = 972, normalized size = 10.12

$$\left[\frac{a\sqrt{\frac{c}{ac+b}}\log\left(\frac{\left(8a^2c^2+8abc+b^2\right)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8\left(2a^2c^3+3abc^2+b^2c\right)dx^2-4\left(\left(2a^2c^2+3abc+b^2\right)d^2x^4+2a^2c^4+4abc^3+2b^2c^2+\left(4a^2c\cdots\right.\right.}{x^4}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*
c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*
((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 +
(4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(c/(a*c + b)))/x^4) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*
a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2
*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/4*(a*sqr
t(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*
b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2
 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3
+ 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/
(a*c + b)))/x^4) - 2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)))/a, 1/4*(2*
a*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*
c)) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a
*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/2*(a*sqrt(-c/(a*c + b))*arctan(1
/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) - sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*
x^2 + a^2*c + a*b)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(c+d
*t_nostep)]Error: Bad Argument Type
```

maple [B] time = 0.04, size = 312, normalized size = 3.25

$$\frac{\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\,\left(d x^2+c\right)\left(a c d \ln\left(\frac{2 a d^2 x^2+2 a c d+b d+2\sqrt{a d^2 x^4+2 a c d x^2+b d x^2+a c^2+b c}\,\sqrt{a d^2}}{2\sqrt{a d^2}}\right)+b d \ln\left(\frac{2 a d^2 x^2+2 a c d+b d+2\sqrt{a d^2 x^4 \cdots}}{2\sqrt{\cdots}}\right)\cdots\right.}{2\sqrt{\left(d x^2+c\right)\left(a d x^2+a c \cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(a*c*d*ln(1/2*(2*a*d^2*x^2+
2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)
)/(a*d^2)^(1/2))+b*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-(a*c^2+b*c)^(1/2)
*(a*d^2)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2))/((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)/(a*c+b)/(a*d^2)^(1/2)
```

maxima [A] time = 3.34, size = 155, normalized size = 1.61

$$\frac{c \log\left(\frac{c\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}-\sqrt{(a c+b) c}}{c\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}+\sqrt{(a c+b) c}}\right)}{2\sqrt{(a c+b) c}}-\frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{\sqrt{a}+\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt((a*c + b)*
c) - 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) +
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/sqrt(a)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x*(a + b/(c + d*x^2))^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

3.348 $\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=108

$$-\frac{(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2x^2(ac+b)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2\sqrt{c}\,(ac+b)^{3/2}}$$

```
[Out] -1/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/(
a*c+b)^(3/2)/c^(1/2)-1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b
)/x^2
```

Rubi [A] time = 0.39, antiderivative size = 148, normalized size of antiderivative = 1.37, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6722, 1975, 446, 94, 93, 208}

$$-\frac{a(c+dx^2)+b}{2x^2(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{bd\sqrt{a(c+dx^2)+b}\,\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a(c+dx^2)+b}}\right)}{2\sqrt{c}\,(ac+b)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -(b + a*(c + d*x^2))/(2*(b + a*c)*x^2*Sqrt[a + b/(c + d*x^2)]) - (b*d*Sqrt[
b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b
+ a*(c + d*x^2)])])/(2*Sqrt[c]*(b + a*c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(
c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^3\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^3\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= -\frac{b+a\left(c+dx^2\right)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(bd\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{4(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= -\frac{b+a\left(c+dx^2\right)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(bd\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2}\,dx,x,\frac{\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{2(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= -\frac{b+a\left(c+dx^2\right)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{bd\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{2\sqrt{c}(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 210, normalized size = 1.94

$$-\frac{c\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(ac+adx^2+b\right)-2bdx^2\log(x)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}+bdx^2\sqrt{(c+}\right.}{4x^2(c(ac+b))^{3/2}\left(a\left(c\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -1/4*(c*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*Sqrt[c*(b + a*c)]*(c + d*x
^2)*(b + a*c + a*d*x^2) - 2*b*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*L
```

```
og[x] + b*d*x^2*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]*Log[2*a*c*(c + d*x^2)
 + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^
2)]]))/((c*(b + a*c))^(3/2)*x^2*(b + a*(c + d*x^2)))
```

fricas [B] time = 0.62, size = 451, normalized size = 4.18

$$\left[\frac{\sqrt{ac^2+bc}\,bdx^2\log\left(\frac{\left(8a^2c^2+8abc+b^2\right)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8\left(2a^2c^3+3abc^2+b^2c\right)dx^2-4\left((2ac+b)d^2x^4+2ac^3+\left(4ac^2+3bc\right)dx^2+2\right.}{x^4}\right.}{8\left(a^2c^3+2abc^2+b^2c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^
2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*s
qrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^
3 + 2*a*b*c^2 + b^2*c)*x^2), 1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2
*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) -
 2*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2)]
```

giac [B] time = 0.70, size = 315, normalized size = 2.92

$$\frac{bd\arctan\left(-\frac{\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{2\sqrt{-ac^2-bc}\left(ac\mathrm{sgn}\left(dx^2+c\right)+b\mathrm{sgn}\left(dx^2+c\right)\right)}-\frac{2\,a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+\right.}{2\left(ac^2-\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bc\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*(a*c*sgn(d*x^2 + c)
+ b*sgn(d*x^2 + c))) - 1/2*(2*a^(3/2)*c^2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*a
bs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 +
 b*c))*b*d)/((a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))^2 + b*c)*(a*c*sgn(d*x^2 + c) + b*sgn(d*x^2 + c)))
```

maple [B] time = 0.04, size = 452, normalized size = 4.19

$$-\frac{\sqrt{\frac{a\,d\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(ab\,c^2d\,x^2\ln\left(\frac{2acd\,x^2+bd\,x^2+2a\,c^2+2bc+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}{x^2}\right)-2\sqrt{a\,d^2x^4+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*a*d^2*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*(a*c^2+b*c)^(1/2)+ln((2*a*c*d*x^2+b*d*x
^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
```

```
*c)^(1/2))/x^2)*x^2*a*b*c^2*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*a*c*d*x^2*(a*c^2+b*c)^(1/2)+ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a
*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*b
^2*c*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*x^2*(a*c^2+b*c
)^(1/2)+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)
)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^2/c/x^2/(a*c^2+b*c)^(1/2)
```

maxima [A] time = 2.78, size = 173, normalized size = 1.60

$$-\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^2+2\,abc+b^2-\frac{\left(adx^2+ac+b\right)\left(ac^2+bc\right)}{dx^2+c}\right)}+\frac{bd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}\,(ac+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^2 + 2*a*b*c + b^2 - (
a*d*x^2 + a*c + b)*(a*c^2 + b*c)/(d*x^2 + c)) + 1/4*b*d*log((c*sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*(a*c + b))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b/(c + d*x^2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

3.349 $\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=177

$$\frac{bd^2(4ac+b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{3/2}(ac+b)^{5/2}}+\frac{d(4ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8cx^2(ac+b)^2}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4cx^4(ac+b)}$$

```
[Out] 1/8*b*(4*a*c+b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+
b)^(1/2))/c^(3/2)/(a*c+b)^(5/2)+1/8*(4*a*c+b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/
(d*x^2+c))^(1/2)/c/(a*c+b)^2/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/c/(a*c+b)/x^4
```

Rubi [A] time = 0.47, antiderivative size = 218, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{bd^2(4ac+b)\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a(c+dx^2)+b}}\right)}{8c^{3/2}(ac+b)^{5/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{d(4ac+b)\left(a\left(c+dx^2\right)+b\right)}{8cx^2(ac+b)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+\right.}{4cx^4(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] ((b + 4*a*c)*d*(b + a*(c + d*x^2)))/(8*c*(b + a*c)^2*x^2*Sqrt[a + b/(c + d*
x^2)]) - ((c + d*x^2)*(b + a*(c + d*x^2)))/(4*c*(b + a*c)*x^4*Sqrt[a + b/(c
 + d*x^2)]) + (b*(b + 4*a*c)*d^2*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b +
a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^(3/2)*(b + a
*c)^(5/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^5\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^5\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((b+4ac)d\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2\sqrt{b+ac+adx}}\,dx,\right.}{8c(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{(b+4ac)d\left(b+a\left(c+dx^2\right)\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(b(b+4ac)d^2\sqrt{b+a\left(c+\right.\right.}}{16c(b+}$$

$$= \frac{(b+4ac)d\left(b+a\left(c+dx^2\right)\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(b(b+4ac)d^2\sqrt{b+a\left(c+\right.\right.}}{8c(b+}$$

$$= \frac{(b+4ac)d\left(b+a\left(c+dx^2\right)\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{b(b+4ac)d^2\sqrt{b+a\left(c+\right.}}{8c^{3/2}(b+ac)^{5/2}\sqrt{}}$$

Mathematica [A] time = 0.48, size = 269, normalized size = 1.52

$$\frac{c\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(2a^2c\left(c^2-d^2x^4\right)+ab\left(4c^2+cdx^2+d^2x^4\right)+b^2\left(2c+dx^2\right)\right)-2bd^2x^4\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (c*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(c + d*x^2)*
(b^2*(2*c + d*x^2) + 2*a^2*c*(c^2 - d^2*x^4) + a*b*(4*c^2 + c*d*x^2 + d^2*x
^4)) - 2*b*(b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x]
 + b*(b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2*a*c*(c
 + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c
 + a*d*x^2)]]))/(16*(c*(b + a*c))^(5/2)*x^4*(b + a*(c + d*x^2)))
```

fricas [A] time = 0.94, size = 593, normalized size = 3.35

$$\left[\frac{\left(4\,abc+b^2\right)\sqrt{ac^2+bc}\,d^2x^4\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\left(2\,a^2c^3+3\,abc^2+b^2c\right)dx^2+4\left((2\,ac+b)d^2x^4+2\,ac^3+(4\right.}{x^4}\right.}{32\left(a^3c^5+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*c
+ b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*
c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d
*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^
4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b
^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^4), -1/16*((4*a*b*c +
b^2)*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2
*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2
*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(2*a^2*c^5 - (2*a^2*c^3 +
a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*
x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b
^2*c^3 + b^3*c^2)*x^4)]
```

giac [B] time = 10.39, size = 815, normalized size = 4.60

$$-\frac{\left(4\,abcd^2+b^2d^2\right)\arctan\left(-\frac{\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{8\left(a^2c^3\mathrm{sgn}\left(dx^2+c\right)+2\,abc^2\mathrm{sgn}\left(dx^2+c\right)+b^2c\mathrm{sgn}\left(dx^2+c\right)\right)\sqrt{-ac^2-bc}}+\frac{8\,a^{\frac{7}{2}}c^5d|d|+16\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*a*b*c*d^2 + b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*
a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a^2*c^3*sgn(d*x^2
 + c) + 2*a*b*c^2*sgn(d*x^2 + c) + b^2*c*sgn(d*x^2 + c))*sqrt(-a*c^2 - b*c)
) + 1/8*(8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*
a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2 + 8*(sqrt(a*d^2)*x^2 - sqrt
(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*c^3*d*abs(d) +
 16*a^(5/2)*b*c^4*d*abs(d) + 28*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d
*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*d^2 + 16*(sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*b*c^2*d*abs(d) +
 8*a^(3/2)*b^2*c^3*d*abs(d) + 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d
*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c*d^2 + 13*(sqrt(a*d^2)*x^2 - sqrt(a*d
^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*c^2*d^2 + 8*(sqrt(a*d^
2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b
^2*c*d*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))^3*b^2*d^2 + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
+ b*d*x^2 + a*c^2 + b*c))*b^3*c*d^2)/((a^2*c^3*sgn(d*x^2 + c) + 2*a*b*c^2*s
gn(d*x^2 + c) + b^2*c*sgn(d*x^2 + c))*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^
2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)^2)
```

maple [B] time = 0.05, size = 922, normalized size = 5.21

$$\frac{\sqrt{\frac{a\,d\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(4a^3b\,c^5d^2x^4\ln\left(\frac{2acd\,x^2+bd\,x^2+2a\,c^2+2bc+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}{x^2}\right)+9a^2b^2c^4d^2x^4\ln\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-12*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a^2*c*d^3*x^6+4*a^3*b*c^5*d^2*
x^4*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
```

```
*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a*b*d^3*x^6+9*a^2*b^2*c^4*d^2*x^4*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2))/x^2)-20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(a*c^2+b*c)^(3/2)*a^2*c^2*d^2*x^4+6*a*b^3*c^3*d^2*x^4*ln((2*a*c*d*x^2
+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2))/x^2)-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*c^2+b*c)^(3/2)*a*b*c*d^2*x^4+b^4*c^2*d^2*x^4*ln((2*a*c*d*x^2+b*d*x^2+2*a*c
^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
))/x^2)-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)
*b^2*d^2*x^4+12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)
^(3/2)*a*c*d*x^2+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b
*c)^(3/2)*b*d*x^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+
b*c)^(3/2)*a*c^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b
*c)^(3/2)*b*c)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^3/c^2/x^4/(a*c^2+b
*c)^(3/2)
```

maxima [B] time = 2.37, size = 359, normalized size = 2.03

$$-\frac{\left(4\,abc+b^2\right)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\left(a^2c^3+2\,abc^2+b^2c\right)\sqrt{(ac+b)c}}-\frac{\left(4\,abc^2+b^2c\right)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc^2+3\,a\right.}{8\left(a^4c^5+4\,a^3bc^4+6\,a^2b^2c^3+4\,ab^3c^2+b^4c+\frac{\left(a^2c^5+2\,abc^4+b^2c^3\right)\left(adx^2\right.}{\left(dx^2+c\right)^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(4*a*b*c + b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sq
rt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*
c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2 +
b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 3*a*b^2
*c - b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^5 + 4*a^3*b*c^4
 + 6*a^2*b^2*c^3 + 4*a*b^3*c^2 + b^4*c + (a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a
*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 2*(a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3
+ b^3*c^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b/(c + d*x^2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

3.350 $\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=443

$$\frac{c^{3/2}(3ac+4b)\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{x(3ac+4b)\left(ac+adx^2+b\right)}{15a^2d^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{\sqrt{c}\left(3a^2c^2+13abc+8b^2\right)\left(ac\right.}{15a^3d^{5/2}\left(c+dx^2\right)\sqrt{}}$$

```
[Out] -1/15*(3*a*c+4*b)*x*(a*d*x^2+a*c+b)/a^2/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)+1/5*x^3*(a*d*x^2+a*c+b)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/15*(3*a^
2*c^2+13*a*b*c+8*b^2)*x*(a*d*x^2+a*c+b)/a^3/d^2/(d*x^2+c)/((a*d*x^2+a*c+b)/
(d*x^2+c))^(1/2)+1/15*c^(3/2)*(3*a*c+4*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*
c+b))^(1/2))/a^2/d^(5/2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*
d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*(a*d*
x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^3/d^(5/2)/(d*x^2+c)/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 498, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 478, 582, 531, 418, 492, 411}

$$\frac{x\left(3a^2c^2+13abc+8b^2\right)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{15a^3d^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\left(3a^2c^2+13abc+8b^2\right)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx\right.}}{15a^3d^{5/2}\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -((4*b + 3*a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(15*a^2*
d^2*Sqrt[a + b/(c + d*x^2)]) + (x^3*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c +
 d*x^2)])/(5*a*d*Sqrt[a + b/(c + d*x^2)]) + ((8*b^2 + 13*a*b*c + 3*a^2*c^2)
*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(15*a^3*d^2*(c + d*x^2)
*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*(8*b^2 + 13*a*b*c + 3*a^2*c^2)*Sqrt[b
+ a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], b/(b + a*c)])/(15*a^3*d^(5/2)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))
/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)]) + (c^(3/2)*(4*b + 3*a*c)
*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], b/(b + a*c)])/(15*a^2*d^(5/2)*(c + d*x^2)*Sqrt[(c*(b + a*c + a
*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^4\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^4\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\left(3c(b+ac)+(4b+3ac)dx^2\right)}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b}\cdots}{\cdots}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(c(\cdots}{\cdots}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(8b\cdots}{\cdots}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(8b\cdots}{\cdots}$$

Mathematica [C] time = 0.79, size = 297, normalized size = 0.67

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(3a^2\left(c^2-d^2x^4\right)+ab\left(7c+dx^2\right)+4b^2\right)+ic\left(3a^2c^2+13abc+8b^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{a\cdots}\right.}{15a^2d^2\sqrt{\frac{ad}{ac+b}}\left(a\left(\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -1/15*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c +
d*x^2)*(4*b^2 + a*b*(7*c + d*x^2) + 3*a^2*(c^2 - d^2*x^4)) + I*c*(8*b^2 + 1
3*a*b*c + 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - (2*I)*b*c*(2
*b + 3*a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(a^2*d^2*Sqrt[(a*d)/(
b + a*c)]*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^6+cx^4\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{adx^2+ac+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^6 + c*x^4)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d*x^2 + a
*c + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a + \frac{b}{dx^2+c}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

maple [A] time = 0.04, size = 665, normalized size = 1.50

$$\frac{\left(3\sqrt{-\frac{ad}{ac+b}}\, a^2d^3x^7 + 3\sqrt{-\frac{ad}{ac+b}}\, a^2c\,d^2x^5 - \sqrt{-\frac{ad}{ac+b}}\, ab\,d^2x^5 - 3\sqrt{-\frac{ad}{ac+b}}\, a^2c^2d\,x^3 - 8\sqrt{-\frac{ad}{ac+b}}\, abcd\,x^3 - 3\sqrt{-\frac{ad}{ac+b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/15*(3*(-1/(a*c+b)*a*d)^(1/2)*a^2*d^3*x^7+3*(-1/(a*c+b)*a*d)^(1/2)*a^2*c*d
^2*x^5-(-1/(a*c+b)*a*d)^(1/2)*a*b*d^2*x^5-3*(-1/(a*c+b)*a*d)^(1/2)*a^2*c^2*
d*x^3-8*(-1/(a*c+b)*a*d)^(1/2)*a*b*c*d*x^3+3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2
))*a^2*c^3-4*(-1/(a*c+b)*a*d)^(1/2)*b^2*d*x^3-3*(-1/(a*c+b)*a*d)^(1/2)*a^2*
c^3*x-6*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(
a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*a*b*c^2+13*((a*d*x^2+a*c+b)/(a*c+b
))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/
c)^(1/2))*a*b*c^2-7*(-1/(a*c+b)*a*d)^(1/2)*a*b*c^2*x-4*((a*d*x^2+a*c+b)/(a*
c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)
/a/c)^(1/2))*b^2*c+8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*b^2*c-4*(-1/(a*c+b)*a
*d)^(1/2)*b^2*c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/a^2/(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/((d*x^2+
c)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a + \frac{b}{dx^2+c}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{a + \frac{b}{d\,x^2+c}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^4/(a + b/(c + d*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**4/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.351 $\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=354

$$\frac{\sqrt{c}\,(ac+2b)\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{x(ac+2b)\left(ac+adx^2+b\right)}{3a^2d\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{c^{3/2}\left(ac+adx^2+b\right)F\left(\tan^{-1}\right.}{3ad^{3/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\sqrt{}}$$

```
[Out] 1/3*x*(a*d*x^2+a*c+b)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(a*c+2*b)*x
*(a*d*x^2+a*c+b)/a^2/d/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*c^(3
/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(
1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a/d^(3/2)/(d*x^2+c)/((a*d
*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/
3*(a*c+2*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^2/d^(3/
2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(
d*x^2+c))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 398, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 478, 531, 418, 492, 411}

$$\frac{\sqrt{c}\,(ac+2b)\sqrt{ac+adx^2+b}\,\sqrt{a\left(c+dx^2\right)+b}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{x(ac+2b)\sqrt{ac+adx^2+b}\,\sqrt{a\left(c+dx^2\right)}}{3a^2d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(3*a*d*Sqrt[a + b/(c +
d*x^2)]) - ((2*b + a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/
(3*a^2*d*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) + (Sqrt[c]*(2*b + a*c)*Sqrt[b
 + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], b/(b + a*c)])/(3*a^2*d^(3/2)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))
/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)]) - (c^(3/2)*Sqrt[b + a*c
+ a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(3*a*d^(3/2)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*
c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{c(b+ac)+(2b+ac)dx^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((2b+ac)\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c^{3/2}\cdots}{\cdots}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{c}\,(\cdots}{\cdots}$$

Mathematica [C] time = 0.61, size = 253, normalized size = 0.71

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(ac+adx^2+b\right)-ibc\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)+ic(a\cdots\right.}{3ad\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(b + a*c + a*d*x^2) + I*c*(2*b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*
Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*
c)] - I*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*a*d*Sqrt[(a*d)/(b
+ a*c)]*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(dx^4+cx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{adx^2+ac+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d*x^2 + a
*c + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + \frac{b}{dx^2+c}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

maple [A] time = 0.02, size = 409, normalized size = 1.16

$$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a d^2x^5 + 2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^3 + \sqrt{-\frac{ad}{ac+b}}\, bd\, x^3 + \sqrt{-\frac{ad}{ac+b}}\, a c^2x - \sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\, \sqrt{\frac{d\,x^2+c}{c}}\, a c^2 \operatorname{EllipticE}\left(\sqrt{-\frac{ad}{ac+}}\right.\right.}{3\sqrt{a\,d^2x^4 + 2ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/3*((-1/(a*c+b)*a*d)^(1/2)*a*d^2*x^5+2*(-1/(a*c+b)*a*d)^(1/2)*a*c*d*x^3+(-
1/(a*c+b)*a*d)^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*a*c^2+(-1/(a*
c+b)*a*d)^(1/2)*a*c^2*x+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*b*c-2*((a*d*x^2+a*
c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,
((a*c+b)/a/c)^(1/2))*b*c+(-1/(a*c+b)*a*d)^(1/2)*b*c*x)*(d*x^2+c)*((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(
-1/(a*c+b)*a*d)^(1/2)/a/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + \frac{b}{dx^2+c}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a + \frac{b}{d\,x^2+c}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^2/(a + b/(c + d*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.352 $\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=286

$$\frac{c^{3/2}\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\,(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{x\left(ac+adx^2+b\right)}{a\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{\sqrt{c}\,\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a\sqrt{d}\,\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}$$

```
[Out] x*(a*d*x^2+a*c+b)/a/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a*
d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/(a*c+b)/(d*x^2+c)/d^(1/2)/((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-(a*
d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a/(d*x^2+c)/d^(1/2)/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 319, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6722, 1974, 422, 418, 492, 411}

$$\frac{c^{3/2}\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\,(ac+b)\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{x\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{a\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(a*(c + d*x^2)*Sqrt[a +
 b/(c + d*x^2)]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*Sqrt[d]*(c + d*x^2
)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x
^2)]) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sqrt[d]*(c + d*x^2)*S
qrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)
])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
```

```
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(d\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c^{3/2}\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\ldots\right.}{(b+ac)\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}\\
&= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\ldots\right.}{a\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 107, normalized size = 0.37

$$\frac{\sqrt{\frac{ac+adx^2+b}{ac+b}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right)}{\sqrt{\frac{dx^2}{c}+1}\sqrt{-\frac{ad}{ac+b}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*EllipticE[ArcSin[Sqrt[-((a*d)/(b + a*c
))]*x], 1 + b/(a*c)])/(Sqrt[-((a*d)/(b + a*c))]*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]*Sqrt[1 + (d*x^2)/c])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^2+c\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{adx^2+ac+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d*x^2 + a*c +
 b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

maple [A] time = 0.02, size = 164, normalized size = 0.57

$$\frac{\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\left(d\,x^2+c\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,c\,\mathrm{EllipticE}\left(\sqrt{-\frac{ad}{ac+b}}\,x,\sqrt{\frac{ac+b}{ac}}\right)}{\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*((d*x^2+c)/c)^(1/2)
*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*c*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/
((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(1/(a + b/(c + d*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{c+dx^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b/(c + d*x**2)), x)
```

3.353 $\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=343

$$-\frac{ac+adx^2+b}{x(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{dx\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{\sqrt{c}\sqrt{d}}{(ac+b}$$

```
[Out] (-a*d*x^2-a*c-b)/(a*c+b)/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+d*x*(a*d*x^2+a
*c+b)/(a*c+b)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(a*d*x^2+a*c+b)*(
1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2
/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/(a*c+b)/(d*x^2+c)/((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+(a*d*x^2+
a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/(a*c+b)/(d*x^2+c)/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 387, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 475, 21, 422, 418, 492, 411}

$$\frac{dx\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{(ac+b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{x(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\sqrt{c}\sqrt{d}\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)}}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{c(ac+adx}{(ac+b)(c}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -((Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/((b + a*c)*x*Sqrt[a + b
/(c + d*x^2)])) + (d*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/((b
 + a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*Sqrt[d]*Sqrt[b + a*
c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 b/(b + a*c)])/((b + a*c)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*
c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)]) + (Sqrt[c]*Sqrt[d]*Sqrt[b + a*c +
 a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/((b + a*c)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*
(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^2\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^2\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{(b+ac)d+ad^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \cdots \\
&= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{dx\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{c}\sqrt{d}\cdots}{\cdots} \\
&= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{dx\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{(b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{d}\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 151, normalized size = 0.44

$$
\frac{d\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|\frac{ac}{b+ac}\right)}{\sqrt{-\frac{d}{c}}\,(ac+b)\sqrt{\frac{ac+adx^2+b}{ac+b}}} - \frac{\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x(ac+b)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -(((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x)) + (d*S
qrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSin[S
qrt[-(d/c)]*x], (a*c)/(b + a*c)])/((b + a*c)*Sqrt[-(d/c)]*Sqrt[(b + a*c + a
*d*x^2)/(b + a*c)])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(dx^2+c\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{adx^4+(ac+b)x^2}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d*x^4 + (a*c
+ b)*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

maple [A] time = 0.03, size = 345, normalized size = 1.01

$$-\frac{\left(\sqrt{-\frac{ad}{ac+b}}\,a\,d^2x^4+2\sqrt{-\frac{ad}{ac+b}}\,acd\,x^2-\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,acdx\,\mathrm{EllipticE}\left(\sqrt{-\frac{ad}{ac+b}}\,x,\sqrt{\frac{ac+b}{ac}}\right)+\sqrt{-\frac{ad}{ac+b}}\,bd\,x^2-\right.}{\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{-\frac{ad}{ac+}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] -((-1/(a*c+b)*a*d)^(1/2)*a*d^2*x^4-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((
d*x^2+c)/c)^(1/2)*x*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))
+2*(-1/(a*c+b)*a*d)^(1/2)*a*c*d*x^2-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x*b*d+
(-1/(a*c+b)*a*d)^(1/2)*b*d*x^2+(-1/(a*c+b)*a*d)^(1/2)*a*c^2+(-1/(a*c+b)*a*d
)^(1/2)*b*c)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/x/(a*c+b)/((d*x^2+c)*(
a*d*x^2+a*c+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b/(c + d*x^2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

3.354 $\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=435

$$-\frac{a\sqrt{c}\,d^{3/2}\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{d^{3/2}(b-ac)\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3\sqrt{c}\,(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{d^2x(b}{3c(ac+b}$$

```
[Out] 1/3*(-a*d*x^2-a*c-b)/(a*c+b)/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(-a*
c+b)*d*(a*d*x^2+a*c+b)/c/(a*c+b)^2/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/3*
(-a*c+b)*d^2*x*(a*d*x^2+a*c+b)/c/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)-1/3*(-a*c+b)*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d
*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/
2))/(a*c+b)^2/(d*x^2+c)/c^(1/2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x
^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^
2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2)
,(b/(a*c+b))^(1/2))*c^(1/2)/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 486, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6722, 1975, 475, 583, 531, 418, 492, 411}

$$\frac{d^2x(b-ac)\sqrt{ac+adx^2+b}\,\sqrt{a\left(c+dx^2\right)+b}}{3c(ac+b)^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{a\sqrt{c}\,d^{3/2}\sqrt{ac+adx^2+b}\,\sqrt{a\left(c+dx^2\right)+b}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -(Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(3*(b + a*c)*x^3*Sqrt[a
+ b/(c + d*x^2)]) - ((b - a*c)*d*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*
x^2)])/(3*c*(b + a*c)^2*x*Sqrt[a + b/(c + d*x^2)]) + ((b - a*c)*d^2*x*Sqrt[
b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(3*c*(b + a*c)^2*(c + d*x^2)*Sq
rt[a + b/(c + d*x^2)]) - ((b - a*c)*d^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b
+ a*(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*Sq
rt[c]*(b + a*c)^2*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c +
d*x^2))]*Sqrt[a + b/(c + d*x^2)]) - (a*Sqrt[c]*d^(3/2)*Sqrt[b + a*c + a*d*x
^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a
*c)])/(3*(b + a*c)^2*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c
 + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^4\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^4\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{(b-ac)d-ad^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} \\
&= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} - \cdots \\
&= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} - \cdots \\
&= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} + \cdots \\
&= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}} + \cdots
\end{aligned}
$$

Mathematica [C] time = 1.02, size = 314, normalized size = 0.72

$$
\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(a^2c\left(c^2-d^2x^4\right)+ab\left(2c^2+cdx^2+d^2x^4\right)+b^2\left(c+dx^2\right)\right)+2iabcd^2x^3\sqrt{\frac{dx^2}{c}+\cdots}\right.}{3cx^3(ac+b)^2\sqrt{\cdots}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(Sqrt[(a*d)/(b + a*c)]*(c + d*x^2)
*(b^2*(c + d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + c*d*x^2 + d^2*x^4)
)) + I*a*c*(-b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] + (2*
I)*a*b*c*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*c*(b + a*c)^2
*Sqrt[(a*d)/(b + a*c)]*x^3*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{\left(dx^2+c\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{adx^6+(ac+b)x^4}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d*x^6 + (a*c
+ b)*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{dx^2+c}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

maple [A] time = 0.03, size = 596, normalized size = 1.37

$$\left(\sqrt{-\frac{ad}{ac+b}}\,a^2c\,d^3x^6 - \sqrt{-\frac{ad}{ac+b}}\,ab\,d^3x^6 + \sqrt{-\frac{ad}{ac+b}}\,a^2c^2d^2x^4 - \sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,a^2c^2d^2x^3\,\mathrm{EllipticE}\left(\sqrt{-\frac{ad}{ac+b}}\,x, \sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/3*((-1/(a*c+b)*a*d)^(1/2)*a^2*c*d^3*x^6-(-1/(a*c+b)*a*d)^(1/2)*a*b*d^3*x^
6-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)
*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x^3*a^2*c^2*d^2+(-1/(a*c+b)*a*d)^(1/2)*a
^2*c^2*d^2*x^4-2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x^3*a*b*c*d^2+((a*d*x^2+a
*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x
,((a*c+b)/a/c)^(1/2))*x^3*a*b*c*d^2-2*(-1/(a*c+b)*a*d)^(1/2)*a*b*c*d^2*x^4-
(-1/(a*c+b)*a*d)^(1/2)*b^2*d^2*x^4-(-1/(a*c+b)*a*d)^(1/2)*a^2*c^3*d*x^2-3*(
-1/(a*c+b)*a*d)^(1/2)*x^2*a*b*c^2*d-2*(-1/(a*c+b)*a*d)^(1/2)*b^2*c*d*x^2-(-
1/(a*c+b)*a*d)^(1/2)*a^2*c^4-2*(-1/(a*c+b)*a*d)^(1/2)*a*b*c^3-(-1/(a*c+b)*a
*d)^(1/2)*b^2*c^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/c/x^3/(a*c+b)^2/
((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{dx^2+c}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{a + \frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b/(c + d*x^2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

3.355 $\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=310

$$\frac{\left(6a^2c^2+12abc+7b^2\right)\left(c+dx^2\right)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a^2b^2d^3}-\frac{b\left(24a^2c^2+60abc+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{9/2}d^3}+\frac{\left(24a^2c^2+60abc\right.}{}$$

```
[Out] -1/16*b*(24*a^2*c^2+60*a*b*c+35*b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/a^(1/2))/a^(9/2)/d^3-(a*c+b)^2*(d*x^2+c)^3/a/b^2/d^3/((a*d*x^2+a*c+b)/(
d*x^2+c))^(1/2)+1/16*(24*a^2*c^2+60*a*b*c+35*b^2)*(d*x^2+c)*((a*d*x^2+a*c+b
)/(d*x^2+c))^(1/2)/a^4/d^3-1/24*(24*a^2*c^2+60*a*b*c+35*b^2)*(d*x^2+c)^2*((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^3/b/d^3+1/6*(6*a^2*c^2+12*a*b*c+7*b^2)*(d
*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/b^2/d^3
```

Rubi [A] time = 0.78, antiderivative size = 323, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 446, 89, 80, 50, 63, 217, 206}

$$-\frac{\left(24a^2c^2+60abc+35b^2\right)\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{24a^3bd^3\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(24a^2c^2+60abc+35b^2\right)\left(a\left(c+dx^2\right)+b\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}}-\frac{b\left(24a^2c^2+6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((b + a*c)^2*(c + d*x^2)^2)/(a^2*b*d^3*Sqrt[a + b/(c + d*x^2)]) + ((35*b^2
+ 60*a*b*c + 24*a^2*c^2)*(b + a*(c + d*x^2)))/(16*a^4*d^3*Sqrt[a + b/(c + d
*x^2)]) - ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(c + d*x^2)*(b + a*(c + d*x^2))
)/(24*a^3*b*d^3*Sqrt[a + b/(c + d*x^2)]) + ((c + d*x^2)^2*(b + a*(c + d*x^2
)))/(6*a^2*d^3*Sqrt[a + b/(c + d*x^2)]) - (b*(35*b^2 + 60*a*b*c + 24*a^2*c^
2)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c
+ d*x^2)]])/(16*a^(9/2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(\frac{1}{2}(b+ac)(5b+4ac)d-\frac{1}{2}abd^2x\right)}{\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{a^2bd^3\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)^2\left(b+a\left(c+dx^2\right)\right)}{6a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(\left(35b^2+60abc+24a^2c^2\right)\sqrt{b+a\left(c\right.\right.}}{12a^2bd^2\sqrt{c+}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2c^2\right)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{24a^3bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)^2\left(\right.}{6a^2d^3}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2\right.}{24a^3b}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2\right.}{24a^3b}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2\right.}{24a^3b}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a\left(c+dx^2\right)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2\right.}{24a^3b}$$

Mathematica [C] time = 11.65, size = 1215, normalized size = 3.92

$$b\left(-344c^2\,{}_4F_3\left(\frac{1}{2},2,2,2;1,1,\frac{7}{2};\frac{b}{adx^2+ac}+1\right)\left(a+\frac{b}{dx^2+c}\right)^5-192c^2\,{}_5F_4\left(\frac{1}{2},2,2,2,2;1,1,1,\frac{7}{2};\frac{b}{adx^2+ac}+1\right)\left(a+\frac{b}{dx^2+c}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*(2835*a^2*(b + a*c)^2*(a + b/(c + d*x^2)) - 3240*a^2*c*(b + a*c)*(a + b/
(c + d*x^2))^2 + 1365*a*(b + a*c)^2*(a + b/(c + d*x^2))^2 + 765*a^2*c^2*(a
```

```
+ b/(c + d*x^2))^3 + 300*a*c*(b + a*c)*(a + b/(c + d*x^2))^3 - 105*a*c^2*(a
 + b/(c + d*x^2))^4 + (300*(b + a*c)^2*(a + b/(c + d*x^2))^3*ArcTanh[Sqrt[1
 + b/(a*c + a*d*x^2)]])/(1 + b/(a*c + a*d*x^2))^(3/2) + (60*c*(b + a*c)*(a
+ b/(c + d*x^2))^4*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]])/(1 + b/(a*c + a*d*
x^2))^(3/2) + (120*a*c^2*(a + b/(c + d*x^2))^4*ArcTanh[Sqrt[1 + b/(a*c + a*
d*x^2)]])/Sqrt[1 + b/(a*c + a*d*x^2)] - 2835*a^3*(b + a*c)^2*Sqrt[1 + b/(a*
c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]] - 765*a^3*c^2*(a + b/(c
+ d*x^2))^2*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]
] - 1365*a*(b + a*c)^2*(a + b/(c + d*x^2))^2*Sqrt[1 + b/(a*c + a*d*x^2)]*Ar
cTanh[Sqrt[1 + b/(a*c + a*d*x^2)]] - 300*a*c*(b + a*c)*(a + b/(c + d*x^2))^
3*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]] + 105*a*
c^2*(a + b/(c + d*x^2))^4*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a
*c + a*d*x^2)]] + 3240*a^4*c*(b + a*c)*(1 + b/(a*c + a*d*x^2))^(3/2)*ArcTan
h[Sqrt[1 + b/(a*c + a*d*x^2)]] - 760*(b + a*c)^2*(a + b/(c + d*x^2))^3*Hype
rgeometricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] + 1040*c*
(b + a*c)*(a + b/(c + d*x^2))^4*HypergeometricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/
2}, 1 + b/(a*c + a*d*x^2)] - 344*c^2*(a + b/(c + d*x^2))^5*HypergeometricPF
Q[{1/2, 2, 2, 2}, {1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 256*(b + a*c)^2*(a
+ b/(c + d*x^2))^3*HypergeometricPFQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 +
 b/(a*c + a*d*x^2)] + 448*c*(b + a*c)*(a + b/(c + d*x^2))^4*HypergeometricP
FQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 192*c^2*(a +
 b/(c + d*x^2))^5*HypergeometricPFQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 +
b/(a*c + a*d*x^2)] - 32*(b + a*c)^2*(a + b/(c + d*x^2))^3*HypergeometricPFQ
[{1/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] + 64*c*(b
+ a*c)*(a + b/(c + d*x^2))^4*HypergeometricPFQ[{1/2, 2, 2, 2, 2, 2}, {1, 1,
 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 32*c^2*(a + b/(c + d*x^2))^5*Hypergeo
metricPFQ[{1/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)]))
/(720*a^5*d^3*(a + b/(c + d*x^2))^(5/2))
```

fricas [A] time = 1.08, size = 675, normalized size = 2.18

$$\left[\frac{3\left(24\,a^3bc^3 + 84\,a^2b^2c^2 + 95\,ab^3c + 35\,b^4 + \left(24\,a^3bc^2 + 60\,a^2b^2c + 35\,ab^3\right)dx^2\right)\sqrt{a}\,\log\left(8\,a^2d^2x^4 + 8\,a^2c^2 + 8\right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*
c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2
 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d
*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(8
*a^4*d^4*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 +
(18*a^3*b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4*
c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3), 1/96*(3*(24*a^3*b*
c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*c^2 + 60*a^2*b^2*c +
 35*a*b^3)*d*x^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(8*a^4*d^4
*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 + (18*a^3*
b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4*c^3 + 15
0*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3)]
```

giac [B] time = 1.67, size = 597, normalized size = 1.93

$$\frac{1}{48}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{a^2d\mathrm{sgn}\left(dx^2+c\right)}-\frac{4\,a^{11}cd^6\mathrm{sgn}\left(dx^2+c\right)+11\,a^{10}bd^6\mathrm{sgn}\left(dx^2+c\right)}{a^{13}d^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2/(a
^2*d*sgn(d*x^2 + c)) - (4*a^11*c*d^6*sgn(d*x^2 + c) + 11*a^10*b*d^6*sgn(d*x
^2 + c))/(a^13*d^8)) + (8*a^11*c^2*d^5*sgn(d*x^2 + c) + 62*a^10*b*c*d^5*sgn
(d*x^2 + c) + 57*a^9*b^2*d^5*sgn(d*x^2 + c))/(a^13*d^8)) + 1/96*(24*a^(5/2)
*b*c^2 + 60*a^(3/2)*b^2*c + 35*sqrt(a)*b^3)*log(abs(-2*a^(7/2)*c^3*d - 6*(s
qrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3
*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))^2*a^(5/2)*c*d - 5*a^(5/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt
(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*
c*abs(d) - 5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*
c^2 + b*c))^2*a^(3/2)*b*d - 4*a^(3/2)*b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3
*d))/(a^5*d^2*abs(d)*sgn(d*x^2 + c))
```

maple [B] time = 0.07, size = 1240, normalized size = 4.00

$$\frac{\sqrt{\frac{a\,d\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-72a^3b\,c^2d^2x^2\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-48\sqrt{a\,d^2x^4+2acd\,x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^4/d^3*(-48*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a^3*c*d^2-60*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a^2*b*d^2-72*ln(1/
2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a^3*b*c^2*d^2-48*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a^3*c^2*d-180*ln(1/2*(2*a*d^2*x
^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2))/(a*d^2)^(1/2))*x^2*a^2*b^2*c*d^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*(a*d^2)^(1/2)*x^2*a^2*d-105*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2
*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/
2))*x^2*a*b^3*d^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a^3*b*c^3*d+54*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*b^2*d-252
*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a^2*b^2*c^2*d+96*((d*x^2+c)*(a*d*x^2+
a*c+b))^(1/2)*(a*d^2)^(1/2)*a^2*b*c^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*(a*d^2)^(1/2)*a^2*c+108*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*d^2)^(1/2)*a^2*b*c^2-285*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))
*a*b^3*c*d+192*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*a*b^2*c+16*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*a*b+222*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b^2*c-105*ln(1/2*
(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*
(a*d^2)^(1/2))/(a*d^2)^(1/2))*b^4*d+96*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a
```

```
*d^2)^(1/2)*b^3+114*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)
^(1/2)*b^3)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

maxima [A] time = 2.36, size = 389, normalized size = 1.25

$$\frac{48\,a^5bc^2+96\,a^4b^2c+48\,a^3b^3-\frac{3\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)\left(adx^2+ac+b\right)^3}{\left(dx^2+c\right)^3}+\frac{8\left(24\,a^3bc^2+60\,a^2b^2c+35\,ab^3\right)\left(adx^2+ac+b\right)^2}{\left(dx^2+c\right)^2}-\frac{3\left(56\,a^4b\right.}{}}{48\left(a^7d^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-3\,a^6d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\,a^5d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-a^4d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{7}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/48*(48*a^5*b*c^2 + 96*a^4*b^2*c + 48*a^3*b^3 - 3*(24*a^2*b*c^2 + 60*a*b^2
*c + 35*b^3)*(a*d*x^2 + a*c + b)^3/(d*x^2 + c)^3 + 8*(24*a^3*b*c^2 + 60*a^2
*b^2*c + 35*a*b^3)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(56*a^4*b*c^2 +
132*a^3*b^2*c + 77*a^2*b^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^7*d^3*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c)) - 3*a^6*d^3*((a*d*x^2 + a*c + b)/(d*x^2 +
c))^(3/2) + 3*a^5*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - a^4*d^3*((a
*d*x^2 + a*c + b)/(d*x^2 + c))^(7/2)) + 1/32*(24*a^2*c^2 + 60*a*b*c + 35*b^
2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(9/2)*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5/(a + b/(c + d*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**5/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.356 $\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$\frac{3b(4ac+5b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2}-\frac{(4ac+7b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^3d^2}-\frac{b(ac+b)}{a^3d^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2d^2}$$

```
[Out] 3/8*b*(4*a*c+5*b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(7/2
)/d^2-b*(a*c+b)/a^3/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/8*(4*a*c+7*b)*(
d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^3/d^2+1/4*(d*x^2+c)^2*((a*d*x^
2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^2
```

Rubi [A] time = 0.57, antiderivative size = 242, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 446, 78, 50, 63, 217, 206}

$$\frac{(4ac+5b)\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right)}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3(4ac+5b)\left(a\left(c+dx^2\right)+b\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{3b(4ac+5b)\sqrt{a\left(c+dx^2\right)+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\ldots}{\sqrt{a(\ldots}}\right.}{8a^{7/2}d^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -(((b + a*c)*(c + d*x^2)^2)/(a*b*d^2*Sqrt[a + b/(c + d*x^2)])) - (3*(5*b +
4*a*c)*(b + a*(c + d*x^2)))/(8*a^3*d^2*Sqrt[a + b/(c + d*x^2)]) + ((5*b + 4
*a*c)*(c + d*x^2)*(b + a*(c + d*x^2)))/(4*a^2*b*d^2*Sqrt[a + b/(c + d*x^2)]
) + (3*b*(5*b + 4*a*c)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*
x^2])/Sqrt[b + a*(c + d*x^2)]])/(8*a^(7/2)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(
c + d*x^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(b+ac+adx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left((5b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2abd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(3(5b+4ac)\sqrt{b+a\left(c+a\right.\right.}}{8a^2d\sqrt{c}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx\right.\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx\right.\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx\right.\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a\left(c+dx\right.\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Mathematica [A] time = 0.27, size = 133, normalized size = 0.71

$$\frac{3b(4ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right) - \sqrt{a}\left(2a^2\left(c^2-d^2x^4\right)+ab\left(17c+5dx^2\right)+15b^2\right)}{8a^{7/2}d^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (-(Sqrt[a]*(15*b^2 + a*b*(17*c + 5*d*x^2) + 2*a^2*(c^2 - d^2*x^4))) + 3*b*(
5*b + 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*ArcTanh[Sqrt[a + b/(c +
d*x^2)]/Sqrt[a]])/(8*a^(7/2)*d^2*Sqrt[a + b/(c + d*x^2)])
```

fricas [A] time = 0.96, size = 541, normalized size = 2.89

$$\left[\frac{3\left(4\,a^2bc^2+9\,ab^2c+\left(4\,a^2bc+5\,ab^2\right)dx^2+5\,b^3\right)\sqrt{a}\,\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x^2 + 5*b^3)*sq
rt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b
^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^3*d^3*x^6 + (2*a^3*c - 5*a^2*b)*d^2*x
^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a^3*c^2 + 22*a^2*b*c + 15*a
*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^3*x^2 + (a^5*c +
 a^4*b)*d^2), -1/16*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x
^2 + 5*b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - 2*(2*a^3*d^3*x^6 +
 (2*a^3*c - 5*a^2*b)*d^2*x^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a
^3*c^2 + 22*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^5*d^3*x^2 + (a^5*c + a^4*b)*d^2)]
```

giac [B] time = 1.57, size = 510, normalized size = 2.73

$$\frac{1}{8}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{a^2d\mathrm{sgn}\left(dx^2+c\right)}-\frac{2\,a^6cd^2+7\,a^5bd^2}{a^8d^4\mathrm{sgn}\left(dx^2+c\right)}\right)-\frac{\left(4\,a^{\frac{3}{2}}bc+5\,\sqrt{a}\,b^2\right)\log\left(\left|-2\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a^2*d*sgn
(d*x^2 + c)) - (2*a^6*c*d^2 + 7*a^5*b*d^2)/(a^8*d^4*sgn(d*x^2 + c))) - 1/16
*(4*a^(3/2)*b*c + 5*sqrt(a)*b^2)*log(abs(-2*a^(7/2)*c^3*d - 6*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^2*abs(d)
 - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*
c))^2*a^(5/2)*c*d - 5*a^(5/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4
 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt(a*d^2)*x^2
 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c*abs(d) -
5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))
^2*a^(3/2)*b*d - 4*a^(3/2)*b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3*d))/(a^4*d
*abs(d)*sgn(d*x^2 + c))
```

maple [B] time = 0.06, size = 783, normalized size = 4.19

$$-\frac{\sqrt{\frac{a\,d\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-12a^2bc\,d^2x^2\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-4\sqrt{a\,d^2x^4+2acd\,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^3/d^2*(-4*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4*a^2*d^2-12*ln(1/2*(2*a
*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d
^2)^(1/2))/(a*d^2)^(1/2))*x^2*a^2*b*c*d^2-15*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*
d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^
(1/2))*x^2*a*b^2*d^2+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*
d^2)^(1/2)*a*b*d*x^2-12*a^2*b*c^2*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+4*
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*c^2-27*a*
b^2*c*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+16*((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)*(a*d^2)^(1/2)*a*b*c+18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/
2)*(a*d^2)^(1/2)*a*b*c-15*b^3*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+16*((d
*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*b^2+14*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/
2)/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

maxima [A] time = 2.31, size = 262, normalized size = 1.40

$$-\frac{8\,a^3bc+8\,a^2b^2+\frac{3\left(adx^2+ac+b\right)^2\left(4\,abc+5\,b^2\right)}{\left(dx^2+c\right)^2}-\frac{5\left(4\,a^2bc+5\,ab^2\right)\left(adx^2+ac+b\right)}{dx^2+c}}{8\left(a^5d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-2\,a^4d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+a^3d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}\right)}-\frac{3\,(4\,ac+5\,b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{7}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^3*b*c + 8*a^2*b^2 + 3*(a*d*x^2 + a*c + b)^2*(4*a*b*c + 5*b^2)/(d*
x^2 + c)^2 - 5*(4*a^2*b*c + 5*a*b^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^5*
d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - 2*a^4*d^2*((a*d*x^2 + a*c + b)/
(d*x^2 + c))^(3/2) + a^3*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2)) - 3/1
6*(4*a*c + 5*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(s
qrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(7/2)*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3/(a + b/(c + d*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**3/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.357 $\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}+\frac{3b}{2a^2d\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}}$$

```
[Out] -3/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2)/d+3/2*b/a^2/d/(a+b/(d
*x^2+c))^(1/2)+1/2*(d*x^2+c)/a/d/(a+b/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1591, 242, 51, 63, 208}

$$\frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((c + d*x^2)/(a*d*Sqrt[a + b/(c + d*x^2)])) + (3*(c + d*x^2)*Sqrt[a + b/(c
 + d*x^2)])/(2*a^2*d) - (3*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*a
^(5/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx, x, c+dx^2\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}$$

$$= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{2ad}$$

$$= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4a^2d}$$

$$= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} + \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2a^2d}$$

$$= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

Mathematica [C] time = 0.06, size = 50, normalized size = 0.50

$$\frac{b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{a+\frac{b}{dx^2+c}}{a}\right)}{a^2d\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*Hypergeometric2F1[-1/2, 2, 1/2, (a + b/(c + d*x^2))/a])/(a^2*d*Sqrt[a +
b/(c + d*x^2)])
```

fricas [B] time = 0.87, size = 395, normalized size = 3.95

$$\left[\frac{3\left(abdx^2+abc+b^2\right)\sqrt{a}\,\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-4\left(2\,ad^2x^4+(4\,ac+b)dx^2+2\right.\right.}{8\left(a^4d^2x^2+\left(a^4c+a^3b\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a*b*d*x^2 + a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8
*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2
 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a^2*d
^2*x^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d), 1/4*(3*(a*b*d*x^2 + a*
b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x
^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(a^2*d^2*x^4 + a^
2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d)]
```

giac [B] time = 1.55, size = 449, normalized size = 4.49

$$b\log\left(\left|-2\,a^{\frac{7}{2}}c^3d-6\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^3c^2|d|-6\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*a^(7/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*
c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*c*d - 5*a^(5/
2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d
*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c*abs(d) - 5*(sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*b*d - 4*a^(3/2)*
b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c
^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3*d))/(a^(5/2)*abs(d)*sgn(d*x^2 + c)) +
 1/2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a^2*d*sgn(d*x^2
 + c))
```

maple [B] time = 0.05, size = 478, normalized size = 4.78

$$\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-3ab\,d^2x^2\ln\left(\frac{2a\,d^2x^2+2acd+bd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}}{2\sqrt{a\,d^2}}\right)-3abcd\ln\left(\frac{2a\,d^2x^2+2acd+b}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^2/d*(-3*a*b*d^2*x^2*ln(1/
2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*(a*d^2)^(1/2)*a*d*x^2-3*a*b*c*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))
-3*b^2*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*c+4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*
(a*d^2)^(1/2)*b+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(
1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

maxima [A] time = 2.22, size = 161, normalized size = 1.61

$$\frac{2\,ab - \frac{3\,(adx^2+ac+b)b}{dx^2+c}}{2\left(a^3d\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - a^2d\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}\right)} + \frac{3\,b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b - 3*(a*d*x^2 + a*c + b)*b/(d*x^2 + c))/(a^3*d*sqrt((a*d*x^2 + a*
c + b)/(d*x^2 + c)) - a^2*d*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2)) + 3/4*
b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d)
```

mupad [B] time = 3.93, size = 61, normalized size = 0.61

$$\frac{\left(\frac{a\,(d\,x^2+c)}{b}+1\right)^{3/2}\,\left(d\,x^2+c\right)\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,(d\,x^2+c)}{b}\right)}{5\,d\,\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] (((a*(c + d*x^2))/b + 1)^(3/2)*(c + d*x^2)*hypergeom([3/2, 5/2], 7/2, -(a*(
c + d*x^2))/b))/(5*d*(a + b/(c + d*x^2))^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.358 $\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}}-\frac{b}{a(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[Out] arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)-c^(3/2)*arctanh(
c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/(a*c+b)^(3/2)-b/a/
(a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 214, normalized size of antiderivative = 1.60, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6722, 1975, 446, 98, 157, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a\left(c+dx^2\right)+b}}\right)}{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c^{3/2}\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{(ac+b)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{b}{a(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*Sqrt[a + b/(c + d*x^2)])) + (Sqrt[b + a*(c + d*x^2)]*ArcTa
nh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(a^(3/2)*Sqrt[c + d*
x^2]*Sqrt[a + b/(c + d*x^2)]) - (c^(3/2)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(S
qrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/((b + a*c
)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$=\frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$=\frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$=-\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\frac{1}{2}ac^2d+\frac{1}{2}(b+ac)d^2x}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$=-\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(c^2\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx,x,x^2\right)}{2(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\cdots$$

$$=-\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx,x,\sqrt{c+dx^2}\right)}{a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(c^2\sqrt{\cdots}\right.}{\cdots}$$

$$=-\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c^{3/2}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\sqrt{b+a\left(c+\cdots\right.}}{\cdots}$$

$$=-\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c^{3/2}\sqrt{b+a\left(c+d\cdots\right.}}{(b+ac)^{3/2}\cdots}$$

Mathematica [A] time = 0.46, size = 110, normalized size = 0.82

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}}-\frac{b}{a(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*Sqrt[a + b/(c + d*x^2)])) + ArcTanh[Sqrt[a + b/(c + d*x^2)
]/Sqrt[a]]/a^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x^2)])/Sqr
t[b + a*c]])/(b + a*c)^(3/2)
```

fricas [B] time = 1.22, size = 1477, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2
*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4
 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c))) + (a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2
*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a
^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4
+ 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x
^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*(a*b*
d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c
+ a^2*b^2 + (a^4*c + a^3*b)*d*x^2), -1/4*(2*(a^2*c^2 + (a^2*c + a*b)*d*x^2
+ 2*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2
+ a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2
*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*
c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 +
 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) + 4*(a*b*d*x^2 + a*b*c)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*
b)*d*x^2), 1/4*(2*(a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(-c/(a*c + b))*arct
an(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^
2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) + (a^2*c^2 + (a^2*c +
 a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a
^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*
a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) - 4*(a*b*d*x^2
+ a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*
b^2 + (a^4*c + a^3*b)*d*x^2), -1/2*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*
c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2 + a^3*c^
2 + a^2*b*c)*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2
*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 +
 a*c^2 + b*c)) + 2*(a*b*d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(c+d
*t_nostep)]Error: Bad Argument Type
```

maple [B] time = 0.06, size = 1015, normalized size = 7.57

$$-\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,\left(d x^2+c\right)\left(-a^3c^2d^2x^2\ln\left(\frac{2a d^2x^2+2acd+bd+2\sqrt{a d^2x^4+2acd x^2+bd x^2+a c^2+bc}\,\sqrt{a d^2}}{2\sqrt{a d^2}}\right)-2a^2bc\,d^2x^2\ln\left(\frac{2a d^2x^2+2a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a*(-ln(1/2*(2*a*d^2*x^2+2*
a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/
(a*d^2)^(1/2))*x^2*a^3*c^2*d^2-2*a^2*b*c*d^2*x^2*ln(1/2*(2*a*d^2*x^2+2*a*c*
```

```
d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d
^2)^(1/2))+(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+
2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/
x^2)*x^2*a^2*c*d-a*b^2*d^2*x^2*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-ln(1/2*
(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*
(a*d^2)^(1/2))/(a*d^2)^(1/2))*a^3*c^3*d-3*a^2*b*c^2*d*ln(1/2*(2*a*d^2*x^2+2
*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))
/(a*d^2)^(1/2))+(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a
*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2))/x^2)*a^2*c^2-3*a*b^2*c*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+(a*d^2)^
(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)
^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*a*b*c+2*((d*x^
2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*a*b*c-b^3*d*ln(1/2*(2*a*d^2*x^2+2
*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))
/(a*d^2)^(1/2))+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*b^2)/((d*
x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^2/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

maxima [A] time = 2.23, size = 201, normalized size = 1.50

$$\frac{c^2 \log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}\,(ac+b)} - \frac{b}{\left(a^2c+ab\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}} - \frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(
c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c +
b)*c)*(a*c + b)) - b/((a^2*c + a*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
- 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqr
t((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x*(a + b/(c + d*x^2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

3.359 $\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{3bd}{2(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{c+dx^2}{2x^2(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{3b\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2(ac+b)^{5/2}}$$

```
[Out] -3/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*c
^(1/2)/(a*c+b)^(5/2)+3/2*b*d/(a*c+b)^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/
2*(-d*x^2-c)/(a*c+b)/x^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 174, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6722, 1975, 446, 94, 93, 208}

$$\frac{3bd}{2(ac+b)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{c+dx^2}{2x^2(ac+b)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3b\sqrt{c}\,d\sqrt{a\left(c+dx^2\right)+b}\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{2(ac+b)^{5/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (3*b*d)/(2*(b + a*c)^2*Sqrt[a + b/(c + d*x^2)]) - (c + d*x^2)/(2*(b + a*c)*
x^2*Sqrt[a + b/(c + d*x^2)]) - (3*b*Sqrt[c]*d*Sqrt[b + a*(c + d*x^2)]*ArcTa
nh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(2*(
b + a*c)^(5/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3bd\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{4(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3bcd\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int\right.}{4(b+ac)^2\sqrt{c+dx}}$$

$$= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3bcd\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\right.}{2(b+ac)^2\sqrt{c+}}$$

$$= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3b\sqrt{c}\,d\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\right.}{2(b+ac)^{5/2}\sqrt{c+dx^2}\sqrt{a}}$$

Mathematica [A] time = 0.42, size = 229, normalized size = 1.57

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(ac\left(c+dx^2\right)+b\left(c-2dx^2\right)\right)+6bcdx^2\log(x)\sqrt{\left(c+dx^2\right)\left(a\left(c+dx^2\right)+b\right.}\right.}{4x^2(ac+b)^2\sqrt{c(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(c + d*x^2)*(b
*(c - 2*d*x^2) + a*c*(c + d*x^2)) + 6*b*c*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c
+ d*x^2))]*Log[x] - 3*b*c*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2
*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(
b + a*c + a*d*x^2)]]]))/(4*(b + a*c)^2*Sqrt[c*(b + a*c)]*x^2*(b + a*(c + d*x
^2)))
```

fricas [A] time = 0.67, size = 599, normalized size = 4.10

$$\left[\frac{3\left(abd^2x^4 + \left(abc + b^2\right)dx^2\right)\sqrt{\frac{c}{ac+b}}\log\left(\frac{\left(8a^2c^2+8abc+b^2\right)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8\left(2a^2c^3+3abc^2+b^2c\right)dx^2-4\left(\left(2a^2c^2+3abc+b^2\right)\right.}{x^4}\right.}{8\left(\left(a^3c^2+2a^2bc+ab^2\right)dx^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*sqrt(c/(a*c + b))*log(((8*a^2*c
^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2
*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 +
2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*((a*c -
 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*c^3 + 3*a^2*
b*c^2 + 3*a*b^2*c + b^3)*x^2), 1/4*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*s
qrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c))
 - 2*((a*c - 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*
c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.07, size = 1088, normalized size = 7.45

$$\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-3a^2b\,c^2d^2x^4\ln\left(\frac{2acd\,x^2+bd\,x^2+2a\,c^2+2bc+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}{x^2}\right)+2\sqrt{a\,d^2x^4+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(2*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^6*a^2*d^3-3*ln((2*a*c*d*x^2+b*d
*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2))/x^2)*x^4*a^2*b*c^2*d^2+6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^4*a^2*c*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^
```

```
2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
)/x^2)*x^4*a*b^2*c*d^2+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*c^2+b*c)^(1/2)*x^4*a*b*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^
2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*a^2*
b*c^3*d+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)
*x^2*a^2*c^2*d-6*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*a*b^2*c^2*d+4*((d
*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*c^2+b*c)^(1/2)*x^2*a*b*c*d-2*(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)*x^2*a*d+6*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^2*a*b*c*d-3*ln((2*
a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*b^3*c*d+4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1
/2)*(a*c^2+b*c)^(1/2)*x^2*b^2*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*(a*c^2+b*c)^(1/2)*x^2*b^2*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(3/2)*(a*c^2+b*c)^(1/2)*a*c-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(3/2)*(a*c^2+b*c)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^3/(a*
d*x^2+a*c+b)/x^2/(a*c^2+b*c)^(1/2)
```

maxima [A] time = 2.34, size = 247, normalized size = 1.69

$$\frac{3\,bcd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\left(a^2c^2+2\,abc+b^2\right)\sqrt{(ac+b)c}}+\frac{\frac{3\left(adx^2+ac+b\right)bcd}{dx^2+c}-2\left(abc+b^2\right)d}{2\left(\left(a^2c^3+2\,abc^2+b^2c\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 3/4*b*c*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))
/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/((a^2*c^2 +
 2*a*b*c + b^2)*sqrt((a*c + b)*c)) + 1/2*(3*(a*d*x^2 + a*c + b)*b*c*d/(d*x^
2 + c) - 2*(a*b*c + b^2)*d)/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*((a*d*x^2 + a*c
+ b)/(d*x^2 + c))^(3/2) - (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^3*(a + b/(c + d*x^2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

3.360 $\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{abd^2}{(ac+b)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{3bd^2(b-4ac)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8\sqrt{c}\,(ac+b)^{7/2}}-\frac{d(3b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8x^2(ac+b)^3}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+}{c}}}{4x^4(ac+b}$$

```
[Out] -3/8*b*(-4*a*c+b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*
c+b)^(1/2))/(a*c+b)^(7/2)/c^(1/2)-a*b*d^2/(a*c+b)^3/((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)-1/8*(-4*a*c+3*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(
a*c+b)^3/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^2/x^
4
```

Rubi [A] time = 0.58, antiderivative size = 246, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{3bd^2(b-4ac)}{8c(ac+b)^3\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3bd^2(b-4ac)\sqrt{a\left(c+dx^2\right)+b}\,\tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a\left(c+dx^2\right)+b}}\right)}{8\sqrt{c}\,(ac+b)^{7/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{d(b-4ac)\left(c+dx^2\right)}{8cx^2(ac+b)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{}{4cx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (3*b*(b - 4*a*c)*d^2)/(8*c*(b + a*c)^3*Sqrt[a + b/(c + d*x^2)]) - ((b - 4*a
*c)*d*(c + d*x^2))/(8*c*(b + a*c)^2*x^2*Sqrt[a + b/(c + d*x^2)]) - (c + d*x
^2)^2/(4*c*(b + a*c)*x^4*Sqrt[a + b/(c + d*x^2)]) - (3*b*(b - 4*a*c)*d^2*Sq
rt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt
[b + a*(c + d*x^2)])])/(8*Sqrt[c]*(b + a*c)^(7/2)*Sqrt[c + d*x^2]*Sqrt[a +
b/(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
```

```
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^5\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^5\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left((b-4ac)d\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(b+ac+adx)^{3/2}}\,dx,x\right.}{8c(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3b(b-4ac)d^2\sqrt{b+a\left(c+dx^2\right.}\right.}{16c(b+ac)^2\sqrt{}}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3b(b\right.}{}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3b(b\right.}{}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3b(b-}{}$$

Mathematica [A] time = 0.57, size = 281, normalized size = 1.33

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(2\sqrt{c(ac+b)}\left(c+dx^2\right)\left(2a^2c\left(c^2-d^2x^4\right)+ab\left(4c^2+5cdx^2+13d^2x^4\right)+b^2\left(2c+5dx^2\right)\right)+6bd^2x^4\,\mathrm{l}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -1/16*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*Sqrt[c*(b + a*c)]*(c + d*x^
2)*(b^2*(2*c + 5*d*x^2) + 2*a^2*c*(c^2 - d^2*x^4) + a*b*(4*c^2 + 5*c*d*x^2
+ 13*d^2*x^4)) + 6*b*(-b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^
2))]*Log[x] + 3*b*(b - 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]
*Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*
x^2)*(b + a*c + a*d*x^2)]]]))/((b + a*c)^3*Sqrt[c*(b + a*c)]*x^4*(b + a*(c +
 d*x^2)))
```

fricas [B] time = 1.26, size = 961, normalized size = 4.53

$$\left[\frac{3\left(\left(4\,a^2bc-ab^2\right)d^3x^6+\left(4\,a^2bc^2+3\,ab^2c-b^3\right)d^2x^4\right)\sqrt{ac^2+bc}\,\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2
*x^4)*sqrt(a*c^2 + b*c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^
4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((
2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^
2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) + 4*((2*a^3*c^3 - 11*a
^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5 - 6*a*b^2*c^4 + (2
*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4 - 2*b^3*c^3 - (2*
a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2*c^3 + 4*a^2*b^3*c
^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2*c^4 + 10*a^2*b^3*
c^3 + 5*a*b^4*c^2 + b^5*c)*x^4), -1/16*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4
*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*x^4)*sqrt(-a*c^2 - b*c)*arctan(1/2*((2*a*
c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*
((2*a^3*c^3 - 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5
- 6*a*b^2*c^4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4
 - 2*b^3*c^3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)
*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2
*c^3 + 4*a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2
*c^4 + 10*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.08, size = 1947, normalized size = 9.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-33*ln((2*a*c*d*x^2+b*d*
x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2))/x^2)*x^4*a^3*b^2*c^5*d^2+16*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(
a*c^2+b*c)^(3/2)*x^4*a^2*b*c^2*d^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a^2*b*c^2*d^2+16*((d*x^2+c)*(a*d*x^2+a*c+b
))^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b^2*c*d^2-10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b^2*c*d^2-2*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*a*b*c*d+2*(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*a^2*b*c*d^3+3*ln((2*
```

```
a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*b^5*c^2*d^2-6*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*b^3*d^2+6*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*b^2*d+8*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*b*c^2-3*ln((2*a*c*d*x^2+b
*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(1/2))/x^2)*x^4*a*b^4*c^3*d^2+6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*a*b*d^2-8*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^2*a^2*c^2*d+12*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a^3*c*d^4-12*ln((2*a*c*d*x^
2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2))/x^2)*x^6*a^4*b*c^5*d^3-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^8*a^2*b*d^4-21*ln((2*a*c*d*x^2+b*d*x^2+
2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2))/x^2)*x^6*a^3*b^2*c^4*d^3+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*a^3*c^2*d^3-6*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^
2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
)/x^2)*x^6*a^2*b^3*c^3*d^3-12*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^
2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*a^4*
b*c^6*d^2+3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^6*a*b^4*c^2*d^3+4*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a^2*c^3+4*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*b^2*c-12*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^6*a*b^2*d^3+
20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a^
3*c^3*d^2-27*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*a^2*b^3*c^4*d^2-12*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*x^4*a^2*c*d
^2)/c/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^4/x^4/(a*c^2+b*c)^(3/2)/(a*
d*x^2+a*c+b)
```

maxima [B] time = 2.35, size = 450, normalized size = 2.12

$$-\frac{3\left(4\,abc-b^{2}\right)d^{2}\log\left(\frac{c\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}+\sqrt{(ac+b)c}}\right)}{16\left(a^{3}c^{3}+3\,a^{2}bc^{2}+3\,ab^{2}c+b^{3}\right)\sqrt{(ac+b)c}}-\frac{8\left(a^{3}bc^{2}+2\,a^{2}b^{2}c\cdots\right.}{8\left(\left(a^{3}c^{5}+3\,a^{2}bc^{4}+3\,ab^{2}c^{3}+b^{3}c^{2}\right)\left(\frac{adx^{2}+ac+b}{dx^{2}+c}\right)^{\frac{5}{2}}-2\left(a^{4}c^{5}+4\,a^{3}b\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -3/16*(4*a*b*c - b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sq
rt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*
c)))/((a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sqrt((a*c + b)*c)) - 1/8*(8
*(a^3*b*c^2 + 2*a^2*b^2*c + a*b^3)*d^2 + 3*(4*a*b*c^2 - b^2*c)*(a*d*x^2 + a
*c + b)^2*d^2/(d*x^2 + c)^2 - 5*(4*a^2*b*c^2 + 3*a*b^2*c - b^3)*(a*d*x^2 +
a*c + b)*d^2/(d*x^2 + c))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*
((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 2*(a^4*c^5 + 4*a^3*b*c^4 + 6*a^2*
b^2*c^3 + 4*a*b^3*c^2 + b^4*c)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + (a
^5*c^5 + 5*a^4*b*c^4 + 10*a^3*b^2*c^3 + 10*a^2*b^3*c^2 + 5*a*b^4*c + b^5)*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{5}\left(a+\frac{b}{d\,x^{2}+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^5*(a + b/(c + d*x^2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(1/(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

3.361 $\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=482

$$\frac{c^{3/2}(ac+8b)\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5a^3d^{5/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{x(ac+8b)\left(ac+adx^2+b\right)}{5a^3d^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{6x^3\left(ac+adx^2+b\right)}{5a^2d\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{\sqrt{c}\left(a^2c^2\right.}{}$$

```
[Out] -x^3*(d*x^2+c)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/5*(a*c+8*b)*x*(a*d*x
^2+a*c+b)/a^3/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+6/5*x^3*(a*d*x^2+a*c+b)
/a^2/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5*(a^2*c^2+16*a*b*c+16*b^2)*x*(a
*d*x^2+a*c+b)/a^4/d^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5*c^(3/
2)*(a*c+8*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a^3/d^(5/2)/(d*x
^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c
))^(1/2)-1/5*(a^2*c^2+16*a*b*c+16*b^2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b
))^(1/2))*c^(1/2)/a^4/d^(5/2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(
c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.91, antiderivative size = 559, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6722, 1975, 467, 581, 582, 531, 418, 492, 411}

$$\frac{x\left(a^2c^2+16abc+16b^2\right)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{5a^4d^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\left(a^2c^2+16abc+16b^2\right)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx\right.}}{5a^4d^{5/2}\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((x^3*(c + d*x^2)*Sqrt[b + a*(c + d*x^2)])/(a*d*Sqrt[b + a*c + a*d*x^2]*Sq
rt[a + b/(c + d*x^2)])) - ((8*b + a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a
*(c + d*x^2)])/(5*a^3*d^2*Sqrt[a + b/(c + d*x^2)]) + (6*x^3*Sqrt[b + a*c +
a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(5*a^2*d*Sqrt[a + b/(c + d*x^2)]) + ((16*
b^2 + 16*a*b*c + a^2*c^2)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]
)/(5*a^4*d^2*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*(16*b^2 + 16*a
*b*c + a^2*c^2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*a^4*d^(5/2)*(c + d*x^2)*Sqrt[(
c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)]) +
(c^(3/2)*(8*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*a^3*d^(5/2)*(c + d*x^2)*S
qrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)
])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^4\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^4\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\sqrt{c+dx^2}\left(3c+6dx^2\right)}{\sqrt{b+ac+adx^2}}\,dx}{ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{6x^3\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5a^2d\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx \ldots}{5a^2d \ldots}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{6x^3\sqrt{b \ldots}{\ldots}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{6x^3\sqrt{b \ldots}{\ldots}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{6x^3\sqrt{b \ldots}{\ldots}$$

$$= -\frac{x^3\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{6x^3\sqrt{b \ldots}{\ldots}$$

Mathematica [C] time = 0.82, size = 296, normalized size = 0.61

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(a^2\left(c^2-d^2x^4\right)+ab\left(9c+2dx^2\right)+8b^2\right)+ic\left(a^2c^2+16abc+16b^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{a \ldots}{5a^3d^2\sqrt{\frac{ad}{ac+b}}\left(a\left(c \ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -1/5*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d
*x^2)*(8*b^2 + a*b*(9*c + 2*d*x^2) + a^2*(c^2 - d^2*x^4)) + I*c*(16*b^2 + 1
6*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*
```

```
EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*b*c*(8*b + 7
*a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(a^3*d^2*Sqrt[(a*d)/(b + a*
c)]*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^8 + 2\,cdx^6 + c^2x^4\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{a^2d^2x^4 + a^2c^2 + 2\left(a^2c + ab\right)dx^2 + 2\,abc + b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^8 + 2*c*d*x^6 + c^2*x^4)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))/(a^2*d^2*x^4 + a^2*c^2 + 2*(a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

maple [B] time = 0.06, size = 1158, normalized size = 2.40

$$\left(\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^2 + c\right)\left(ad\,x^2 + ac + b\right)}\,a^2d^3x^7 + \sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^2 + c\right)\left(ad\,x^2 + ac + b\right)}\,a^2c\,d^2x^5 - 2\sqrt{-\frac{ad}{ac+b}}\,\sqrt{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/5*((-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*d^3*x^7+(
-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c*d^2*x^5-2*(-1
/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*d^2*x^5-(-1/(a*c+
b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^2*d*x^3-6*(-1/(a*c+b)
*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c*d*x^3+((a*d*x^2+a*c+b)/
(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c
+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^3-5*(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*a*b*c*d*x^3-3*(-1/(
a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*d*x^3-(-1/(a*c+b)*a
*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^3*x-7*((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b
)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2+16*((a*d*x^2+a*c+b)
/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*
c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2-5*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^3*b^2*d-4*(-1/(a
*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2*x-8*((a*d*x^2+a*
c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,
((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c+16*((a*d*x^2+a
*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x
,((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c-5*(a*d^2*x^4+
```

```
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*a*b*c^2*x-3*(-1
/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c*x-5*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*b^2*c*x)*((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/a^3/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/(a*d*x^2+a*c+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^4/(a + b/(c + d*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**4/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.362 $\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} \, dx$

Optimal. Leaf size=409

$$\frac{\sqrt{c}\,(ac+8b)\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^3d^{3/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{x(ac+8b)\left(ac+adx^2+b\right)}{3a^3d\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{c^{3/2}(ac+4b)\left(ac+adx^2+b\right)}{3a^2d^{3/2}(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac}{}}}$$

```
[Out] -x*(d*x^2+c)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+4/3*x*(a*d*x^2+a*c+b)/a^
2/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(a*c+8*b)*x*(a*d*x^2+a*c+b)/a^3/d
/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*c^(3/2)*(a*c+4*b)*(a*d*x^2
+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a^2/(a*c+b)/d^(3/2)/(d*x^2+c)/((a*d*x
^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/3*
(a*c+8*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE
(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^3/d^(3/2)
/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*
x^2+c))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 475, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 467, 528, 531, 418, 492, 411}

$$-\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}(ac+b)\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\sqrt{c}\,(ac+8b)\sqrt{ac+adx^2+b}\sqrt{a\left(c+d\right.}}{3a^3d^{3/2}\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx\right.}{(ac+b)\left(c\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((x*(c + d*x^2)*Sqrt[b + a*(c + d*x^2)])/(a*d*Sqrt[b + a*c + a*d*x^2]*Sqrt
[a + b/(c + d*x^2)])) + (4*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)
])/(3*a^2*d*Sqrt[a + b/(c + d*x^2)]) - ((8*b + a*c)*x*Sqrt[b + a*c + a*d*x^
2]*Sqrt[b + a*(c + d*x^2)])/(3*a^3*d*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) +
 (Sqrt[c]*(8*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*a^3*d^(3/2)*(c + d*x^2)*
Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2
)]) - (c^(3/2)*(4*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*a^2*(b + a*c)*d^(3/
2)*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a
 + b/(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}\left(c+4dx^2\right)}{\sqrt{b+ac+adx^2}}dx}{ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+\right.}}{3a^2d}$$

$$= -\frac{x\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((8b+ac)\sqrt{b\right.}}{}$$

$$= -\frac{x\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b}}{3a^3d}$$

$$= -\frac{x\left(c+dx^2\right)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b}}{3a^3d}$$

Mathematica [C] time = 0.60, size = 255, normalized size = 0.62

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(x\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(ac+adx^2+4b\right)-4ibc\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)\middle|\frac{b}{ac}+1\right)+ic\right.}{3a^2d\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(4*b + a*c + a*d*x^2) + I*c*(8*b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(
a*c)] - (4*I)*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*E
llipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*a^2*d*Sqrt[(
a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^6+2\,cdx^4+c^2x^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{a^2d^2x^4+a^2c^2+2\left(a^2c+ab\right)dx^2+2\,abc+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^4 + c^2*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))/(a^2*d^2*x^4 + a^2*c^2 + 2*(a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

maple [A] time = 0.04, size = 667, normalized size = 1.63

$$\frac{\left(\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{-\frac{ad}{ac+b}}\,a\,d^2x^5+2\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{-\frac{ad}{ac+b}}\,acd\,x^3+\sqrt{\left(d\,x^2+c\right)\left(ad\,x\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/3*(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^5*a*d^2+2*(
(d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^3*a*c*d+((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^3*b*d-((d*x^2+c)*(a*d*x^2
+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*a*c^2+3*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^3*b*d+((d*x^2+c)*(a*d
*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x*a*c^2+4*((d*x^2+c)*(a*d*x^2+a*c
+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-
1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*b*c-8*((d*x^2+c)*(a*d*x^2+a*c+b
))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/
(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*b*c+((d*x^2+c)*(a*d*x^2+a*c+b))^(
1/2)*(-1/(a*c+b)*a*d)^(1/2)*x*b*c+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x*b*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^
2/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/
(a*d*x^2+a*c+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^2/(a + b/(c + d*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**2/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.363 $\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=356

$$\frac{x(ac+2b)\left(ac+adx^2+b\right)}{a^2(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{\sqrt{c}\,(ac+2b)\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a^2\sqrt{d}\,(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{c^{3/2}\left(ac+adx^2+b\right)F\left(\cdots\right.}{a\sqrt{d}\,(ac+b)\left(c+dx^2\right)\sqrt{\cdots}}$$

```
[Out] -b*x/a/(a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+(a*c+2*b)*x*(a*d*x^2+a*c+b
)/a^2/(a*c+b)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a*d*x^2+
a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a/(a*c+b)/(d*x^2+c)/d^(1/2)/((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-(a*c+2*
b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^2/(a*c+b)/(d*x^2
+c)/d^(1/2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d
*x^2+c))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 411, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6722, 1974, 413, 531, 418, 492, 411}

$$\frac{x(ac+2b)\sqrt{ac+adx^2+b}\,\sqrt{a\left(c+dx^2\right)+b}}{a^2(ac+b)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\,(ac+2b)\sqrt{ac+adx^2+b}\,\sqrt{a\left(c+dx^2\right)+b}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right.\cdots}{a^2\sqrt{d}\,(ac+b)\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(-3/2),x]
```

```
[Out] -((b*x*Sqrt[b + a*(c + d*x^2)])/(a*(b + a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[a
 + b/(c + d*x^2)])) + ((2*b + a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c
+ d*x^2)])/(a^2*(b + a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*(
2*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a^2*(b + a*c)*Sqrt[d]*(c + d*x^2)*Sqr
t[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
 + (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticF[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*(b + a*c)*Sqrt[d]*(c + d*x^2)*Sqrt
[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{c(b+ac)d+(2b+ac)d^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \cdots$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a^2(b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} + \cdots$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a^2(b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}} - \cdots$$

Mathematica [C] time = 0.52, size = 241, normalized size = 0.68

$$-\frac{\sqrt{\frac{ad}{ac+b}}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(bx\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}} - ibc\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|\frac{b}{ac}+1\right) + ic(ac+2b)\sqrt{\cdots}\right.}{a^2d\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(-3/2),x]
```

```
[Out] -((Sqrt[(a*d)/(b + a*c)]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*Sqrt[(a*d
)/(b + a*c)]*x*(c + d*x^2) + I*c*(2*b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b +
a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 +
 b/(a*c)] - I*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*E
llipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(a^2*d*(b + a*(
c + d*x^2))))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(d^2x^4+2\,cdx^2+c^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{a^2d^2x^4+a^2c^2+2\left(a^2c+ab\right)dx^2+2\,abc+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/
(a^2*d^2*x^4 + a^2*c^2 + 2*(a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

maple [A] time = 0.03, size = 466, normalized size = 1.31

$$-\frac{\left(\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{-\frac{ad}{ac+b}}\,bd\,x^3-\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,a\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -((a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*b*
d*x^3-((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*a*
c^2+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*b*c-
2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*b*c+(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*b*c*x)
/a*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/(a*c+b)/(a*d*x^2+a*c+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(1/(a + b/(c + d*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral((a + b/(c + d*x**2))**(-3/2), x)
```

3.364 $\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=410

$$\frac{c^{3/2}\sqrt{d}\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{(b-ac)\left(ac+adx^2+b\right)}{ax(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{dx(b-ac)\left(ac+adx^2+b\right)}{a(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{}{ax(}$$

```
[Out] -b/a/(a*c+b)/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+(-a*c+b)*(a*d*x^2+a*c+b)/a
/(a*c+b)^2/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(-a*c+b)*d*x*(a*d*x^2+a*c+b)
/a/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a*d*x^2+a
*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a
*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+(-a*c+b)
*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2
)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/a/(a*c+b)^2/
(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x
^2+c))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 476, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 468, 583, 531, 418, 492, 411}

$$\frac{c^{3/2}\sqrt{d}\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}+\frac{(b-ac)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{ax(ac+b)^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -((b*Sqrt[b + a*(c + d*x^2)])/(a*(b + a*c)*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[a
 + b/(c + d*x^2)])) + ((b - a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*
x^2)])/(a*(b + a*c)^2*x*Sqrt[a + b/(c + d*x^2)]) - ((b - a*c)*d*x*Sqrt[b +
a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(a*(b + a*c)^2*(c + d*x^2)*Sqrt[a +
 b/(c + d*x^2)]) + (Sqrt[c]*(b - a*c)*Sqrt[d]*Sqrt[b + a*c + a*d*x^2]*Sqrt[
b + a*(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*
(b + a*c)^2*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)
)]*Sqrt[a + b/(c + d*x^2)]) + (c^(3/2)*Sqrt[d]*Sqrt[b + a*c + a*d*x^2]*Sqrt
[b + a*(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((
b + a*c)^2*(c + d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))
]*Sqrt[a + b/(c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{c(b-ac)d-acd^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

Mathematica [C] time = 0.65, size = 268, normalized size = 0.65

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(ac\left(c+dx^2\right)+b\left(c-dx^2\right)\right)+2ibcdx\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{ac+adx^2+b}{ac+b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right.\right.\right.}{x(ac+b)^2\sqrt{\frac{ad}{ac+b}}\left(a\left(c+dx^2\right)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(c + d*x^2)
*(b*(c - d*x^2) + a*c*(c + d*x^2)) + I*c*(-b + a*c)*d*x*Sqrt[(b + a*c + a*d
*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*
c)]*x], 1 + b/(a*c)] + (2*I)*b*c*d*x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sq
rt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)
]))/((b + a*c)^2*Sqrt[(a*d)/(b + a*c)]*x*(b + a*(c + d*x^2))))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^4+2\,cdx^2+c^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{a^2d^2x^6+2\left(a^2c+ab\right)dx^4+\left(a^2c^2+2\,abc+b^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/
(a^2*d^2*x^6 + 2*(a^2*c + a*b)*d*x^4 + (a^2*c^2 + 2*a*b*c + b^2)*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

maple [A] time = 0.04, size = 685, normalized size = 1.67

$$-\frac{\left(\sqrt{\left(d\,x^2+c\right)\left(ad\,x^2+ac+b\right)}\,\sqrt{-\frac{ad}{ac+b}}\,ac\,d^2x^4-\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{-\frac{ad}{ac+b}}\,b\,d^2x^4+2\sqrt{(d\,x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] -(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^4*a*c*d^2-a*c^
2*d*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+
b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)-(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^4*b*
d^2+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^2*a*c^2*d-
2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x*b*c*
d+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x*b*c*
d+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^2*b*c*d-(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*x^2*b*c*d+
((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*a*c^3+((d*x^2+c)*(
a*d*x^2+a*c+b))^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*b*c^2)*((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-1/(a*c+b)*a*d)
^(1/2)/x/(a*c+b)^2/(a*d*x^2+a*c+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b/(c + d*x^2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

3.365 $\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=490

$$\frac{\sqrt{c}\,d^{3/2}(3b-ac)\left(ac+adx^2+b\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{\sqrt{c}\,d^{3/2}(7b-ac)\left(ac+adx^2+b\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\cdots$$

```
[Out] -b/a/(a*c+b)/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+3*b)*(a*d*x^2+
a*c+b)/a/(a*c+b)^2/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(-a*c+7*b)*d*(
a*d*x^2+a*c+b)/(a*c+b)^3/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+7*b)
*d^2*x*(a*d*x^2+a*c+b)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)-1/3*(-a*c+7*b)*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1
/2)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b
)/(a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+3*b)*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(b/(a*c+b))^(1/2))*c^(1/2)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 567, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6722, 1975, 468, 583, 531, 418, 492, 411}

$$\frac{d^2x(7b-ac)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}}{3(ac+b)^3\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\sqrt{c}\,d^{3/2}(3b-ac)\sqrt{ac+adx^2+b}\sqrt{a\left(c+dx^2\right)+b}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\cdots\right.}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\sqrt{a+\frac{b}{c+dx^2}}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -((b*Sqrt[b + a*(c + d*x^2)])/(a*(b + a*c)*x^3*Sqrt[b + a*c + a*d*x^2]*Sqrt
[a + b/(c + d*x^2)])) + ((3*b - a*c)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c
+ d*x^2)])/(3*a*(b + a*c)^2*x^3*Sqrt[a + b/(c + d*x^2)]) - ((7*b - a*c)*d*S
qrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)])/(3*(b + a*c)^3*x*Sqrt[a + b
/(c + d*x^2)]) + ((7*b - a*c)*d^2*x*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c +
 d*x^2)])/(3*(b + a*c)^3*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*(7
*b - a*c)*d^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^2)]*EllipticE
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*(b + a*c)^3*(c + d*x^2)*Sqrt
[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(c + d*x^2)])
+ (Sqrt[c]*(3*b - a*c)*d^(3/2)*Sqrt[b + a*c + a*d*x^2]*Sqrt[b + a*(c + d*x^
2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*(b + a*c)^3*(c
+ d*x^2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]*Sqrt[a + b/(
c + d*x^2)])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{c(3b-ac)d+(2b-ac)d^2x^2}{x^4\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+dx^2}}}$$

Mathematica [C] time = 0.90, size = 319, normalized size = 0.65

$$-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\left(c+dx^2\right)\sqrt{\frac{ad}{ac+b}}\left(a^2c\left(c^2-d^2x^4\right)+ab\left(2c^2+4cdx^2+7d^2x^4\right)+b^2\left(c+4dx^2\right)\right)+ibd^2x^3(3b-5ac)\sqrt{}\right.}{3x^3(ac+b)^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -1/3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(c + d*x
^2)*(b^2*(c + 4*d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + 4*c*d*x^2 + 7
*d^2*x^4)) - I*a*c*(-7*b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a
*c)] + I*b*(3*b - 5*a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1
 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/
((b + a*c)^3*Sqrt[(a*d)/(b + a*c)]*x^3*(b + a*(c + d*x^2)))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^4 + 2\,cdx^2 + c^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{a^2d^2x^8 + 2\left(a^2c + ab\right)dx^6 + \left(a^2c^2 + 2\,abc + b^2\right)x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/
(a^2*d^2*x^8 + 2*(a^2*c + a*b)*d*x^6 + (a^2*c^2 + 2*a*b*c + b^2)*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

maple [B] time = 0.04, size = 1082, normalized size = 2.21

$$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^2 + c\right)\left(ad\,x^2 + ac + b\right)}\,a^2c\,d^3x^6 - 4\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^2 + c\right)\left(ad\,x^2 + ac + b\right)}\,ab\,d^3x^6 - 3\sqrt{a\,d^2x^4 +}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b/(d*x^2+c))^(3/2),x)
```

```
[Out] 1/3*((-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c*d^3*x^6
-4*(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*d^3*x^6-((d
*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^(1/2))*x^3*a^2*c^2
*d^2-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-1/(a*c+b)*a*d)^(1/
2)*a*b*d^3*x^6+(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2
*c^2*d^2*x^4-5*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+b)/a/c)^
(1/2))*x^3*a*b*c*d^2+7*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(
a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE((-1/(a*c+b)*a*d)^(1/2)*x,((a*c+
b)/a/c)^(1/2))*x^3*a*b*c*d^2-8*(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a
*c+b))^(1/2)*a*b*c*d^2*x^4+3*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*
c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF((-1/(a*c+b)*a*d)^(1/2)*x,
((a*c+b)/a/c)^(1/2))*x^3*b^2*d^2-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(-1/(a*c+b)*a*d)^(1/2)*a*b*c*d^2*x^4-4*(-1/(a*c+b)*a*d)^(1/2)*((d*x
^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*d^2*x^4-(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*
(a*d*x^2+a*c+b))^(1/2)*a^2*c^3*d*x^2-6*(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a
*d*x^2+a*c+b))^(1/2)*a*b*c^2*d*x^2-5*(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d
*x^2+a*c+b))^(1/2)*b^2*c*d*x^2-(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a
*c+b))^(1/2)*a^2*c^4-2*(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(
1/2)*a*b*c^3-(-1/(a*c+b)*a*d)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c
^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)/(-1/(a*c+b)*a*d)^(1/2)/x^3/(a*c+b)^3/(a*d*x^2+a*c+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^4*(a + b/(c + d*x^2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(1/(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

3.366 $\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=75

$$\frac{3\sqrt{ax^{23}}\,\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}-\frac{3\sqrt{x^5+1}\,\sqrt{ax^{23}}}{20x^9}+\frac{\sqrt{x^5+1}\,\sqrt{ax^{23}}}{10x^4}$$

```
[Out] 3/20*arcsinh(x^(5/2))*(a*x^23)^(1/2)/x^(23/2)-3/20*(a*x^23)^(1/2)*(x^5+1)^(
1/2)/x^9+1/10*(a*x^23)^(1/2)*(x^5+1)^(1/2)/x^4
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 321, 329, 275, 215}

$$\frac{\sqrt{x^5+1}\,\sqrt{ax^{23}}}{10x^4}-\frac{3\sqrt{x^5+1}\,\sqrt{ax^{23}}}{20x^9}+\frac{3\sqrt{ax^{23}}\,\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

```
[Out] (-3*Sqrt[a*x^23]*Sqrt[1 + x^5])/(20*x^9) + (Sqrt[a*x^23]*Sqrt[1 + x^5])/(10
*x^4) + (3*Sqrt[a*x^23]*ArcSinh[x^(5/2)])/(20*x^(23/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{ax^{23}}\int \frac{x^{23/2}}{\sqrt{1+x^5}}\,dx}{x^{23/2}}$$
$$= \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} - \frac{\left(3\sqrt{ax^{23}}\right)\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx}{4x^{23/2}}$$
$$= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{8x^{23/2}}$$
$$= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx,x,\sqrt{x}\right)}{4x^{23/2}}$$
$$= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,x^{5/2}\right)}{20x^{23/2}}$$
$$= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{3\sqrt{ax^{23}}\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.65

$$\frac{\sqrt{ax^{23}}\left(3\sinh^{-1}\left(x^{5/2}\right)+\sqrt{x^5+1}\,\left(2x^5-3\right)x^{5/2}\right)}{20x^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
[Out] (Sqrt[a*x^23]*(x^(5/2)*Sqrt[1 + x^5]*(-3 + 2*x^5) + 3*ArcSinh[x^(5/2)]))/(2
0*x^(23/2))
```

fricas [A] time = 0.58, size = 169, normalized size = 2.25

$$\left[\frac{3\sqrt{a}\,x^9\log\left(-\frac{8\,ax^{19}+8\,ax^{14}+ax^9+4\,\sqrt{ax^{23}}\left(2\,x^5+1\right)\sqrt{x^5+1}\,\sqrt{a}}{x^9}\right)+4\,\sqrt{ax^{23}}\left(2\,x^5-3\right)\sqrt{x^5+1}}{80\,x^9},\,-\frac{3\,\sqrt{-a}\,x^9\arctan\left(\frac{\sqrt{ax^{23}}\,(2\,x^5}{2\,(ax}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
[Out] [1/80*(3*sqrt(a)*x^9*log(-(8*a*x^19 + 8*a*x^14 + a*x^9 + 4*sqrt(a*x^23)*(2*
x^5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^9) + 4*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5
+ 1))/x^9, -1/40*(3*sqrt(-a)*x^9*arctan(1/2*sqrt(a*x^23)*(2*x^5 + 1)*sqrt(x
^5 + 1)*sqrt(-a)/(a*x^19 + a*x^14)) - 2*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5 +
 1))/x^9]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)
```

maple [A] time = 0.07, size = 64, normalized size = 0.85

$$\frac{\left(2x^{5}-3\right)\sqrt{x^{5}+1}\,\sqrt{a\,x^{23}}}{20x^{9}}+\frac{3\sqrt{a\,x^{23}}\,\sqrt{\left(x^{5}+1\right)ax}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{20\sqrt{x^{5}+1}\,\sqrt{a}\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^23)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 1/20/x^9*(2*x^5-3)*(x^5+1)^(1/2)*(a*x^23)^(1/2)+3/20/a^(1/2)*arcsinh(x^(5/2
))*(a*x^23)^(1/2)/x^12*(a*x*(x^5+1))^(1/2)/(x^5+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{x^{5}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^{23}}}{\sqrt{x^{5}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^23)^(1/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] int((a*x^23)^(1/2)/(x^5 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{(x+1)\left(x^{4}-x^{3}+x^{2}-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**23)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**23)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.367 $\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{x^5+1}\,\sqrt{ax^{13}}}{5x^4}-\frac{\sqrt{ax^{13}}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}$$

```
[Out] -1/5*arcsinh(x^(5/2))*(a*x^13)^(1/2)/x^(13/2)+1/5*(a*x^13)^(1/2)*(x^5+1)^(1
/2)/x^4
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 321, 329, 275, 215}

$$\frac{\sqrt{x^5+1}\,\sqrt{ax^{13}}}{5x^4}-\frac{\sqrt{ax^{13}}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^13]*Sqrt[1 + x^5])/(5*x^4) - (Sqrt[a*x^13]*ArcSinh[x^(5/2)])/(5*x
^(13/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx &= \frac{\sqrt{ax^{13}}\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx}{x^{13/2}} \\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{2x^{13/2}} \\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{x^{13/2}} \\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{5x^{13/2}} \\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.84

$$
\frac{\sqrt{ax^{13}}\left(x^{5/2}\sqrt{x^5+1}-\sinh^{-1}\left(x^{5/2}\right)\right)}{5x^{13/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
[Out] (Sqrt[a*x^13]*(x^(5/2)*Sqrt[1 + x^5] - ArcSinh[x^(5/2)]))/(5*x^(13/2))
```

fricas [B] time = 0.58, size = 153, normalized size = 3.06

$$
\left[\frac{\sqrt{a}\,x^4\log\left(-\frac{8\,ax^{14}+8\,ax^9+ax^4-4\,\sqrt{ax^{13}}\left(2\,x^5+1\right)\sqrt{x^5+1}\,\sqrt{a}}{x^4}\right)+4\,\sqrt{ax^{13}}\,\sqrt{x^5+1}}{20\,x^4}, \frac{\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{ax^{13}}\left(2\,x^5+1\right)\sqrt{x^5+1}\,\sqrt{-a}}{2\left(ax^{14}+ax^9\right)}\right)}{10\,x^4}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
[Out] [1/20*(sqrt(a)*x^4*log(-(8*a*x^14 + 8*a*x^9 + a*x^4 - 4*sqrt(a*x^13)*(2*x^5
 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^4) + 4*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4, 1/10
*(sqrt(-a)*x^4*arctan(1/2*sqrt(a*x^13)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(-a)/(
a*x^14 + a*x^9)) + 2*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4]
```

giac [A] time = 0.32, size = 68, normalized size = 1.36

$$
\frac{a^{\frac{11}{2}}\log\left(-\sqrt{ax}\,a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)}{5\,|a|^5}+\frac{\sqrt{a^6x^5+a^6}\,\sqrt{ax}\,x^2}{5\,a^2|a|}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
[Out] 1/5*a^(11/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))/abs(a)^5 + 1
/5*sqrt(a^6*x^5 + a^6)*sqrt(a*x)*x^2/(a^2*abs(a))
```

maple [A] time = 0.06, size = 57, normalized size = 1.14

$$
\frac{\sqrt{a\,x^{13}}\,\sqrt{x^5+1}}{5x^4}-\frac{\sqrt{a\,x^{13}}\,\sqrt{\left(x^5+1\right)ax}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{5\sqrt{x^5+1}\,\sqrt{a}\,x^7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^13)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 1/5*(a*x^13)^(1/2)*(x^5+1)^(1/2)/x^4-1/5/a^(1/2)*arcsinh(x^(5/2))*(a*x^13)^
(1/2)/x^7*((x^5+1)*a*x)^(1/2)/(x^5+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^13)/sqrt(x^5 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^{13}}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^13)^(1/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] int((a*x^13)^(1/2)/(x^5 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**13)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**13)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.368 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}} dx$

Optimal. Leaf size=24

$$\frac{2\sqrt{ax^3}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

```
[Out] 2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {15, 329, 275, 215}

$$\frac{2\sqrt{ax^3}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
```

```
[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{x^{3/2}}$$
$$= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{x^{3/2}}$$
$$= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{5x^{3/2}}$$
$$= \frac{2\sqrt{ax^3}\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{2\sqrt{ax^3}\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

fricas [B] time = 0.55, size = 98, normalized size = 4.08

$$\left[\frac{1}{10}\sqrt{a}\log\left(-8\,ax^{10}-8\,ax^5-4\left(2\,x^6+x\right)\sqrt{x^5+1}\,\sqrt{ax^3}\,\sqrt{a}-a\right), -\frac{1}{5}\sqrt{-a}\arctan\left(\frac{\left(2\,x^5+1\right)\sqrt{x^5+1}\,\sqrt{ax^3}\,\sqrt{-a}}{2\left(ax^9+ax^4\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
[Out] [1/10*sqrt(a)*log(-8*a*x^10 - 8*a*x^5 - 4*(2*x^6 + x)*sqrt(x^5 + 1)*sqrt(a*
x^3)*sqrt(a) - a), -1/5*sqrt(-a)*arctan(1/2*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(
a*x^3)*sqrt(-a)/(a*x^9 + a*x^4))]
```

giac [B] time = 0.19, size = 58, normalized size = 2.42

$$-\frac{2\,a^{\frac{3}{2}}\log\left(-\sqrt{ax}\,a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)\mathrm{sgn}\left(x\right)}{5\,|a|}+\frac{2\,a^{\frac{3}{2}}\log\left(a^2|a|\right)\mathrm{sgn}\left(x\right)}{5\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
[Out] -2/5*a^(3/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))*sgn(x)/abs(a
) + 2/5*a^(3/2)*log(a^2*abs(a))*sgn(x)/abs(a)
```

maple [A] time = 0.05, size = 17, normalized size = 0.71

$$\frac{2\sqrt{a\,x^3}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{5x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^3)/sqrt(x^5 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a\,x^3}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^5 + 1)^(1/2),x)
[Out] int((a*x^3)^(1/2)/(x^5 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(x**5+1)**(1/2),x)
[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.369 $\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

```
[Out] -2/5*x*(a/x^7)^(1/2)*(x^5+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 264}

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx &= \left(\sqrt{\frac{a}{x^7}}\,x^{7/2}\right)\int \frac{1}{x^{7/2}\sqrt{1+x^5}}\,dx\\ &= -\frac{2}{5}\sqrt{\frac{a}{x^7}}\,x\sqrt{1+x^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

fricas [A] time = 0.45, size = 17, normalized size = 0.74

$$-\frac{2}{5}\sqrt{x^5+1}\,x\sqrt{\frac{a}{x^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(x^5 + 1)*x*sqrt(a/x^7)
```

giac [A] time = 0.24, size = 28, normalized size = 1.22

$$-\frac{2\,a^4\left(\frac{\sqrt{a+\frac{a}{x^5}}}{a^3}-\frac{1}{a^{\frac{5}{2}}}\right)}{5\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*a^4*(sqrt(a + a/x^5)/a^3 - 1/a^(5/2))/abs(a)
```

maple [B] time = 0.00, size = 37, normalized size = 1.61

$$-\frac{2\,(x+1)\left(x^4-x^3+x^2-x+1\right)\sqrt{\frac{a}{x^7}}\,x}{5\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^7)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] -2/5*x*(x+1)*(x^4-x^3+x^2-x+1)*(a/x^7)^(1/2)/(x^5+1)^(1/2)
```

maxima [B] time = 2.22, size = 41, normalized size = 1.78

$$-\frac{2\left(\sqrt{a}\,x^6+\sqrt{a}\,x\right)}{5\,\sqrt{x^4-x^3+x^2-x+1}\,\sqrt{x+1}\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*(sqrt(a)*x^6 + sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x + 1)*sqrt(x + 1)*x
^(7/2))
```

mupad [B] time = 2.69, size = 17, normalized size = 0.74

$$-\frac{2\,x\,\sqrt{x^5+1}\,\sqrt{\frac{a}{x^7}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^7)^(1/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] -(2*x*(x^5 + 1)^(1/2)*(a/x^7)^(1/2))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{a}{x^7}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**7)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**7)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.370 $\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}} \, dx$

Optimal. Leaf size=49

$$\frac{4}{15}x^6\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}} - \frac{2}{15}x\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

```
[Out] -2/15*x*(a/x^17)^(1/2)*(x^5+1)^(1/2)+4/15*x^6*(a/x^17)^(1/2)*(x^5+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 271, 264}

$$\frac{4}{15}x^6\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}} - \frac{2}{15}x\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^17]*x*Sqrt[1 + x^5])/15 + (4*Sqrt[a/x^17]*x^6*Sqrt[1 + x^5])/1
5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}} \, dx &= \left(\sqrt{\frac{a}{x^{17}}} x^{17/2}\right) \int \frac{1}{x^{17/2}\sqrt{1+x^5}} \, dx \\
&= -\frac{2}{15}\sqrt{\frac{a}{x^{17}}} x\sqrt{1+x^5} - \frac{1}{3}\left(2\sqrt{\frac{a}{x^{17}}} x^{17/2}\right) \int \frac{1}{x^{7/2}\sqrt{1+x^5}} \, dx \\
&= -\frac{2}{15}\sqrt{\frac{a}{x^{17}}} x\sqrt{1+x^5} + \frac{4}{15}\sqrt{\frac{a}{x^{17}}} x^6\sqrt{1+x^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.61

$$-\frac{2}{15}x\left(1-2x^5\right)\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^17]*x*(1 - 2*x^5)*Sqrt[1 + x^5])/15
```

fricas [A] time = 0.45, size = 25, normalized size = 0.51

$$\frac{2}{15}\left(2x^6 - x\right)\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*x^6 - x)*sqrt(x^5 + 1)*sqrt(a/x^17)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(t_nostep)]sym2poly/r2sym(const gen & e,const index_m & i,c
onst vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.00, size = 44, normalized size = 0.90

$$\frac{2(x+1)\left(x^4 - x^3 + x^2 - x + 1\right)\left(2x^5 - 1\right)\sqrt{\frac{a}{x^{17}}}\, x}{15\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^17)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 2/15*x*(x+1)*(x^4-x^3+x^2-x+1)*(2*x^5-1)*(a/x^17)^(1/2)/(x^5+1)^(1/2)
```

maxima [A] time = 2.87, size = 50, normalized size = 1.02

$$\frac{2\left(2\sqrt{a}x^{11} + \sqrt{a}x^6 - \sqrt{a}x\right)}{15\sqrt{x^4 - x^3 + x^2 - x + 1}\sqrt{x+1}\,x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(2*sqrt(a)*x^11 + sqrt(a)*x^6 - sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x +
 1)*sqrt(x + 1)*x^(17/2))
```

mupad [B] time = 2.67, size = 29, normalized size = 0.59

$$\frac{\sqrt{\frac{a}{x^{17}}}\left(\frac{4x^{11}}{15} + \frac{2x^6}{15} - \frac{2x}{15}\right)}{\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^17)^(1/2)/(x^5 + 1)^(1/2),x)
[Out] ((a/x^17)^(1/2)*((2*x^6)/15 - (2*x)/15 + (4*x^11)/15))/(x^5 + 1)^(1/2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**17)**(1/2)/(x**5+1)**(1/2),x)
[Out] Integral(sqrt(a/x**17)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.371 $\int \frac{\sqrt{ax^6}}{x(1-x^4)} dx$

Optimal. Leaf size=37

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

```
[Out] -1/2*arctan(x)*(a*x^6)^(1/2)/x^3+1/2*arctanh(x)*(a*x^6)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {15, 298, 203, 206}

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^6}}{x(1-x^4)} dx &= \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4} dx}{x^3} \\
&= \frac{\sqrt{ax^6}\int \frac{1}{1-x^2} dx}{2x^3} - \frac{\sqrt{ax^6}\int \frac{1}{1+x^2} dx}{2x^3} \\
&= -\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} + \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.89

$$-\frac{\sqrt{ax^6}\left(\log(1-x)-\log(x+1)+2\tan^{-1}(x)\right)}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^6]/(x*(1 - x^4)),x]
[Out] -1/4*(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/x^3
```

fricas [A] time = 0.46, size = 29, normalized size = 0.78

$$-\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

giac [A] time = 0.18, size = 29, normalized size = 0.78

$$-\frac{1}{4}\left(2\arctan(x)\mathrm{sgn}(x)-\log\left(|x+1|\right)\mathrm{sgn}(x)+\log\left(|x-1|\right)\mathrm{sgn}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a)
```

maple [A] time = 0.01, size = 28, normalized size = 0.76

$$-\frac{\sqrt{a\,x^6}\,\left(2\arctan(x)+\ln\left(x-1\right)-\ln\left(x+1\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/x/(-x^4+1),x)
[Out] -1/4*(a*x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x))/x^3
```

maxima [A] time = 2.53, size = 26, normalized size = 0.70

$$-\frac{1}{2}\sqrt{a}\arctan(x)+\frac{1}{4}\sqrt{a}\log\left(x+1\right)-\frac{1}{4}\sqrt{a}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int\frac{\sqrt{a\,x^6}}{x\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x^6)^(1/2)/(x*(x^4 - 1)),x)
[Out] -int((a*x^6)^(1/2)/(x*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ax^6}}{x^5 - x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(1/2)/x/(-x**4+1),x)
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

3.372 $\int \frac{\sqrt{ax^6}}{x-x^5} dx$

Optimal. Leaf size=37

$$\frac{\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\,\tan^{-1}(x)}{2x^3}$$

```
[Out] -1/2*arctan(x)*(a*x^6)^(1/2)/x^3+1/2*arctanh(x)*(a*x^6)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 1584, 298, 203, 206}

$$\frac{\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\,\tan^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = \frac{\sqrt{ax^6}\int \frac{x^3}{x-x^5}\,dx}{x^3}$$
$$= \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3}$$
$$= \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3} - \frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3}$$
$$= -\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} + \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.89

$$-\frac{\sqrt{ax^6}\left(\log(1-x)-\log(x+1)+2\tan^{-1}(x)\right)}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^6]/(x - x^5),x]
[Out] -1/4*(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/x^3
```

fricas [A] time = 0.45, size = 29, normalized size = 0.78

$$-\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

giac [A] time = 0.20, size = 29, normalized size = 0.78

$$-\frac{1}{4}\left(2\arctan(x)\mathrm{sgn}(x)-\log(|x+1|)\,\mathrm{sgn}(x)+\log(|x-1|)\,\mathrm{sgn}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a)
```

maple [A] time = 0.01, size = 28, normalized size = 0.76

$$-\frac{\sqrt{a\,x^6}\,\left(2\arctan(x)+\ln(x-1)-\ln(x+1)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/(-x^5+x),x)
[Out] -1/4*(a*x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1))/x^3
```

maxima [A] time = 2.08, size = 26, normalized size = 0.70

$$-\frac{1}{2}\sqrt{a}\arctan(x)+\frac{1}{4}\sqrt{a}\log(x+1)-\frac{1}{4}\sqrt{a}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a\,x^6}}{x-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/(x - x^5),x)
```

```
[Out] int((a*x^6)^(1/2)/(x - x^5), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ax^6}}{x^5-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(1/2)/(-x**5+x),x)
```

```
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

3.373 $\int \frac{(ax^6)^{3/2}}{x(1-x^4)} dx$

Optimal. Leaf size=71

$$\frac{a\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}-\frac{1}{5}ax^2\sqrt{ax^6}-\frac{a\sqrt{ax^6}}{x^2}$$

```
[Out] -a*(a*x^6)^(1/2)/x^2-1/5*a*x^2*(a*x^6)^(1/2)+1/2*a*arctan(x)*(a*x^6)^(1/2)/
x^3+1/2*a*arctanh(x)*(a*x^6)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {15, 302, 212, 206, 203}

$$-\frac{1}{5}ax^2\sqrt{ax^6}-\frac{a\sqrt{ax^6}}{x^2}+\frac{a\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] -((a*Sqrt[a*x^6])/x^2) - (a*x^2*Sqrt[a*x^6])/5 + (a*Sqrt[a*x^6]*ArcTan[x])/
(2*x^3) + (a*Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx &= \frac{\left(a\sqrt{ax^6}\right)\int \frac{x^8}{1-x^4}\,dx}{x^3} \\
&= \frac{\left(a\sqrt{ax^6}\right)\int \left(-1-x^4+\frac{1}{1-x^4}\right)dx}{x^3} \\
&= -\frac{a\sqrt{ax^6}}{x^2} - \frac{1}{5}ax^2\sqrt{ax^6} + \frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1-x^4}\,dx}{x^3} \\
&= -\frac{a\sqrt{ax^6}}{x^2} - \frac{1}{5}ax^2\sqrt{ax^6} + \frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1-x^2}\,dx}{2x^3} + \frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1+x^2}\,dx}{2x^3} \\
&= -\frac{a\sqrt{ax^6}}{x^2} - \frac{1}{5}ax^2\sqrt{ax^6} + \frac{a\sqrt{ax^6}\,\tan^{-1}(x)}{2x^3} + \frac{a\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.62

$$
-\frac{a\sqrt{ax^6}\left(4x^5+20x+5\log(1-x)-5\log(x+1)-10\tan^{-1}(x)\right)}{20x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] -1/20*(a*Sqrt[a*x^6]*(20*x + 4*x^5 - 10*ArcTan[x] + 5*Log[1 - x] - 5*Log[1
+ x]))/x^3
```

fricas [A] time = 0.43, size = 41, normalized size = 0.58

$$
-\frac{\sqrt{ax^6}\left(4\,ax^5+20\,ax-10\,a\arctan(x)-5\,a\log\left(\frac{x+1}{x-1}\right)\right)}{20\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(a*x^6)*(4*a*x^5 + 20*a*x - 10*a*arctan(x) - 5*a*log((x + 1)/(x -
 1)))/x^3
```

giac [A] time = 0.18, size = 42, normalized size = 0.59

$$
-\frac{1}{20}\left(4\,x^5\mathrm{sgn}(x)+20\,x\mathrm{sgn}(x)-10\,\arctan(x)\mathrm{sgn}(x)-5\,\log\left(|x+1|\right)\mathrm{sgn}(x)+5\,\log\left(|x-1|\right)\mathrm{sgn}(x)\right)a^{\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*(4*x^5*sgn(x) + 20*x*sgn(x) - 10*arctan(x)*sgn(x) - 5*log(abs(x + 1))
*sgn(x) + 5*log(abs(x - 1))*sgn(x))*a^(3/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.54

$$
-\frac{\left(a\,x^6\right)^{\frac{3}{2}}\left(4x^5+20x-10\arctan(x)+5\ln(x-1)-5\ln(x+1)\right)}{20x^9}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(3/2)/x/(-x^4+1),x)
```

```
[Out] -1/20*(a*x^6)^(3/2)*(4*x^5+5*ln(x-1)-5*ln(x+1)-10*arctan(x)+20*x)/x^9
```

maxima [A] time = 2.09, size = 40, normalized size = 0.56

$$-\frac{1}{5}\,a^{\frac{3}{2}}x^{5}-a^{\frac{3}{2}}x+\frac{1}{2}\,a^{\frac{3}{2}}\arctan(x)+\frac{1}{4}\,a^{\frac{3}{2}}\log\left(x+1\right)-\frac{1}{4}\,a^{\frac{3}{2}}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*a^(3/2)*x^5 - a^(3/2)*x + 1/2*a^(3/2)*arctan(x) + 1/4*a^(3/2)*log(x +
1) - 1/4*a^(3/2)*log(x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(a\,x^6\right)^{3/2}}{x\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x^6)^(3/2)/(x*(x^4 - 1)),x)
```

```
[Out] -int((a*x^6)^(3/2)/(x*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(ax^{6}\right)^{\frac{3}{2}}}{x^{5}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(3/2)/x/(-x**4+1),x)
```

```
[Out] -Integral((a*x**6)**(3/2)/(x**5 - x), x)
```

3.374 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx$

Optimal. Leaf size=49

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(a*x^6)^(1/2)/x^3-1/2*arctanh(x)
*(a*x^6)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {212, 206, 203, 15, 298}

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rubi steps

$$\int\left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right)dx=\int\frac{1}{1-x^4}\,dx-\int\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\,dx$$
$$=\frac{1}{2}\int\frac{1}{1-x^2}\,dx+\frac{1}{2}\int\frac{1}{1+x^2}\,dx-\frac{\sqrt{ax^6}\int\frac{x^2}{1-x^4}\,dx}{x^3}$$
$$=\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\int\frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{ax^6}\int\frac{1}{1+x^2}\,dx}{2x^3}$$
$$=\frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

Mathematica [A] time = 0.06, size = 29, normalized size = 0.59

$$\frac{1}{2}\left(\frac{\sqrt{ax^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] (ArcTan[x] + (Sqrt[a*x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

fricas [B] time = 0.45, size = 256, normalized size = 5.22

$$\left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log(x+1)-x^3\log(x-1)-\sqrt{ax^6}\left(\mathrm{lo}\right.}{4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/(x
^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - lo
g(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-
(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x^2))
+ x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/
x^3]
```

giac [A] time = 0.18, size = 48, normalized size = 0.98

$$\frac{1}{4}\left(2\arctan(x)\mathrm{sgn}(x)-\log(|x+1|)\,\mathrm{sgn}(x)+\log(|x-1|)\,\mathrm{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan(x)+\frac{1}{4}\log(|x+1|)-\frac{1}{4}\log(|x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*
sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 37, normalized size = 0.76

$$\frac{\mathrm{arctanh}(x)}{2}+\frac{\arctan(x)}{2}+\frac{\sqrt{a\,x^6}\;(2\arctan(x)+\ln(x-1)-\ln(x+1))}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x)
```

```
[Out] 1/2*arctanh(x)+1/2*arctan(x)+1/4*(a*x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1)
)/x^3
```

maxima [A] time = 2.70, size = 42, normalized size = 0.86

$$\frac{1}{2}\sqrt{a}\arctan(x)-\frac{1}{4}\sqrt{a}\log(x+1)+\frac{1}{4}\sqrt{a}\log(x-1)+\frac{1}{2}\arctan(x)+\frac{1}{4}\log(x+1)-\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1
/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^6}}{x\,\left(x^4-1\right)}-\frac{1}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1),x)
```

```
[Out] int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^5-x}\,dx-\int \left(-\frac{\sqrt{ax^6}}{x^5-x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/x/(-x**4+1),x)
```

```
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

3.375 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx$

Optimal. Leaf size=49

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(a*x^6)^(1/2)/x^3-1/2*arctanh(x)
*(a*x^6)^(1/2)/x^3
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {212, 206, 203, 15, 1584, 298}

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx = \int \frac{1}{1-x^4}\,dx - \int \frac{\sqrt{ax^6}}{x-x^5}\,dx$$
$$= \frac{1}{2}\int \frac{1}{1-x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \frac{\sqrt{ax^6}\int \frac{x^3}{x-x^5}\,dx}{x^3}$$
$$= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3}$$
$$= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3} + \frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3}$$
$$= \frac{1}{2}\tan^{-1}(x) + \frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.59

$$\frac{1}{2}\left(\frac{\sqrt{ax^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
[Out] (ArcTan[x] + (Sqrt[a*x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

fricas [B] time = 0.45, size = 256, normalized size = 5.22

$$\left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log(x+1)-x^3\log(x-1)-\sqrt{ax^6}\left(\log(\right.}{4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/(x
^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - lo
g(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-
(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x^2))
+ x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/
x^3]
```

giac [A] time = 0.19, size = 48, normalized size = 0.98

$$\frac{1}{4}\left(2\arctan(x)\mathrm{sgn}(x)-\log(|x+1|)\,\mathrm{sgn}(x)+\log(|x-1|)\,\mathrm{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan(x)+\frac{1}{4}\log(|x+1|)-\frac{1}{4}\log(|x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
```

```
[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*
sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 37, normalized size = 0.76

$$\frac{\operatorname{arctanh}(x)}{2}+\frac{\arctan(x)}{2}+\frac{\sqrt{a\,x^6}\,\left(2\arctan(x)+\ln\left(x-1\right)-\ln\left(x+1\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x)
```

```
[Out] 1/2*arctanh(x)+1/2*arctan(x)+1/4*(a*x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1)
)/x^3
```

maxima [A] time = 2.04, size = 42, normalized size = 0.86

$$\frac{1}{2}\sqrt{a}\arctan(x)-\frac{1}{4}\sqrt{a}\log\left(x+1\right)+\frac{1}{4}\sqrt{a}\log\left(x-1\right)+\frac{1}{2}\arctan(x)+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1
/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{1}{x^4-1}-\frac{\sqrt{a\,x^6}}{x-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5),x)
```

```
[Out] int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x^5-x}\,dx-\int\left(-\frac{\sqrt{ax^6}}{x^5-x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/(-x**5+x),x)
```

```
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

3.376 $\int \frac{\sqrt{ax^3}}{x-x^3}\,dx$

Optimal. Leaf size=44

$$\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

```
[Out] -arctan(x^(1/2))*(a*x^3)^(1/2)/x^(3/2)+arctanh(x^(1/2))*(a*x^3)^(1/2)/x^(3/
2)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {15, 1584, 329, 298, 203, 206}

$$\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/(x - x^3),x]
```

```
[Out] -((Sqrt[a*x^3]*ArcTan[Sqrt[x]])/x^(3/2)) + (Sqrt[a*x^3]*ArcTanh[Sqrt[x]])/x
^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^3}}{x-x^3}\,dx &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{x-x^3}\,dx}{x^{3/2}}\\ &= \frac{\sqrt{ax^3}\int \frac{\sqrt{x}}{1-x^2}\,dx}{x^{3/2}}\\ &= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{x}\right)}{x^{3/2}}\\ &= \frac{\sqrt{ax^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)}{x^{3/2}} - \frac{\sqrt{ax^3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)}{x^{3/2}}\\ &= -\frac{\sqrt{ax^3}\,\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}} + \frac{\sqrt{ax^3}\,\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.68

$$\frac{\sqrt{ax^3}\left(\tanh^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)\right)}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/(x - x^3),x]
[Out] (Sqrt[a*x^3]*(-ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]))/x^(3/2)
```

fricas [A] time = 0.45, size = 127, normalized size = 2.89

$$\left[-\sqrt{a}\arctan\left(\frac{\sqrt{ax^3}}{\sqrt{a}\,x}\right)+\frac{1}{2}\sqrt{a}\log\left(\frac{ax^2+ax+2\sqrt{ax^3}\sqrt{a}}{x^2-x}\right), -\sqrt{-a}\arctan\left(\frac{\sqrt{ax^3}\sqrt{-a}}{ax}\right)+\frac{1}{2}\sqrt{-a}\log\left(\frac{ax^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="fricas")
[Out] [-sqrt(a)*arctan(sqrt(a*x^3)/(sqrt(a)*x)) + 1/2*sqrt(a)*log((a*x^2 + a*x +
2*sqrt(a*x^3)*sqrt(a))/(x^2 - x)), -sqrt(-a)*arctan(sqrt(a*x^3)*sqrt(-a)/(a
*x)) + 1/2*sqrt(-a)*log((a*x^2 - a*x - 2*sqrt(a*x^3)*sqrt(-a))/(x^2 + x))]
```

giac [A] time = 0.21, size = 43, normalized size = 0.98

$$-\frac{\left(\frac{a^2\arctan\left(\frac{\sqrt{ax}}{\sqrt{-a}}\right)}{\sqrt{-a}}+a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\text{sgn}(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="giac")
[Out] -(a^2*arctan(sqrt(a*x)/sqrt(-a))/sqrt(-a) + a^(3/2)*arctan(sqrt(a*x)/sqrt(a
)))*sgn(x)/a
```

maple [A] time = 0.02, size = 43, normalized size = 0.98

$$\frac{\sqrt{a\,x^3}\,\left(\text{arctanh}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)-\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\sqrt{a}}{\sqrt{ax}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(-x^3+x),x)
```

```
[Out] (a*x^3)^(1/2)*a^(1/2)*(arctanh((a*x)^(1/2)/a^(1/2))-arctan((a*x)^(1/2)/a^(1
/2)))/x/(a*x)^(1/2)
```

maxima [A] time = 2.18, size = 32, normalized size = 0.73

$$-\sqrt{a}\arctan\left(\sqrt{x}\right)+\frac{1}{2}\sqrt{a}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\sqrt{a}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -sqrt(a)*arctan(sqrt(x)) + 1/2*sqrt(a)*log(sqrt(x) + 1) - 1/2*sqrt(a)*log(s
qrt(x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{a\,x^3}}{x-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x - x^3),x)
```

```
[Out] int((a*x^3)^(1/2)/(x - x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{ax^3}}{x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(-x**3+x),x)
```

```
[Out] -Integral(sqrt(a*x**3)/(x**3 - x), x)
```

3.377 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=44

$$\frac{\sqrt{x^2+1}\sqrt{ax^4}}{2x}-\frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}$$

```
[Out] -1/2*arcsinh(x)*(a*x^4)^(1/2)/x^2+1/2*(a*x^4)^(1/2)*(x^2+1)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 321, 215}

$$\frac{\sqrt{x^2+1}\sqrt{ax^4}}{2x}-\frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^4]*Sqrt[1 + x^2])/(2*x) - (Sqrt[a*x^4]*ArcSinh[x])/(2*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx &= \frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{1+x^2}}\,dx}{x^2}\\ &= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x}-\frac{\sqrt{ax^4}\int \frac{1}{\sqrt{1+x^2}}\,dx}{2x^2}\\ &= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x}-\frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.73

$$\frac{\sqrt{ax^4}\left(x\sqrt{x^2+1}-\sinh^{-1}(x)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
[Out] (Sqrt[a*x^4]*(x*Sqrt[1 + x^2] - ArcSinh[x]))/(2*x^2)
```

fricas [A] time = 0.44, size = 42, normalized size = 0.95

$$\frac{\sqrt{ax^4}\sqrt{x^2+1}\,x + \sqrt{ax^4}\log\left(-x+\sqrt{x^2+1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(sqrt(a*x^4)*sqrt(x^2 + 1)*x + sqrt(a*x^4)*log(-x + sqrt(x^2 + 1)))/x^2
```

giac [A] time = 0.19, size = 27, normalized size = 0.61

$$\frac{1}{2}\left(\sqrt{x^2+1}\,x + \log\left(-x+\sqrt{x^2+1}\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*(sqrt(x^2 + 1)*x + log(-x + sqrt(x^2 + 1)))*sqrt(a)
```

maple [A] time = 0.01, size = 27, normalized size = 0.61

$$\frac{\sqrt{a\,x^4}\,\left(\sqrt{x^2+1}\;x - \operatorname{arcsinh}(x)\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^2+1)^(1/2),x)
[Out] 1/2*(a*x^4)^(1/2)*(x*(x^2+1)^(1/2)-arcsinh(x))/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^4)/sqrt(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a*x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**4)/sqrt(x**2 + 1), x)
```

3.378 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{x^2+1}\,\sqrt{ax^3}}{3x}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,\sqrt{ax^3}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{3x^{3/2}\sqrt{x^2+1}}$$

```
[Out] 2/3*(a*x^3)^(1/2)*(x^2+1)^(1/2)/x-1/3*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2
)/cos(2*arctan(x^(1/2)))*EllipticF(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a*x
^3)^(1/2)*((x^2+1)/(1+x)^2)^(1/2)/x^(3/2)/(x^2+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {15, 321, 329, 220}

$$\frac{2\sqrt{x^2+1}\,\sqrt{ax^3}}{3x}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,\sqrt{ax^3}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{3x^{3/2}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x^3]*Sqrt[1 + x^2])/(3*x) - (Sqrt[a*x^3]*(1 + x)*Sqrt[(1 + x^2)/(
1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*x^(3/2)*Sqrt[1 + x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^2}}\,dx}{x^{3/2}}$$

$$= \frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\sqrt{ax^3}\int \frac{1}{\sqrt{x}\sqrt{1+x^2}}\,dx}{3x^{3/2}}$$

$$= \frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \sqrt{x}\right)}{3x^{3/2}}$$

$$= \frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\sqrt{ax^3}\,(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{3x^{3/2}\sqrt{1+x^2}}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.52

$$\frac{2\sqrt{ax^3}\left(\sqrt{x^2+1} - {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^2\right)\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
[Out] (2*Sqrt[a*x^3]*(Sqrt[1 + x^2] - Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]))/(3
*x)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ax^3}}{\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

maple [C] time = 0.03, size = 76, normalized size = 0.92

$$-\frac{\sqrt{a\,x^3}\,\left(-2x^3-2x+i\sqrt{-i\,(x+i)}\,\sqrt{-i\,(-x+i)}\,\sqrt{ix}\,\sqrt{2}\,\operatorname{EllipticF}\left(\sqrt{-i\,(x+i)}, \frac{\sqrt{2}}{2}\right)\right)}{3\sqrt{x^2+1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -1/3*(a*x^3)^(1/2)/x^2/(x^2+1)^(1/2)*(I*(-I*(x+I))^(1/2)*(-I*(-x+I))^(1/2)*
(I*x)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(1/2))*2^(1/2)-2*x^3-2*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^3}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a*x^3)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(sqrt(a*x**3)/sqrt(x**2 + 1), x)
```

3.379 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=22

$$\frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

```
[Out] (a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 261}

$$\frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}} \, dx &= \frac{\sqrt{ax^2} \int \frac{x}{\sqrt{1+x^2}} \, dx}{x} \\ &= \frac{\sqrt{ax^2}\sqrt{1+x^2}}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

fricas [A] time = 0.42, size = 18, normalized size = 0.82

$$\frac{\sqrt{ax^2}\sqrt{x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*x^2)*sqrt(x^2 + 1)/x
```

giac [A] time = 0.16, size = 19, normalized size = 0.86

$$\left(\sqrt{x^2+1}\,\mathrm{sgn}(x)-\mathrm{sgn}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] (sqrt(x^2 + 1)*sgn(x) - sgn(x))*sqrt(a)
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{\sqrt{a\,x^2}\,\sqrt{x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2)^(1/2)/(x^2+1)^(1/2),x)
[Out] (a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

maxima [A] time = 1.90, size = 19, normalized size = 0.86

$$\frac{\sqrt{a}\,x^2+\sqrt{a}}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] (sqrt(a)*x^2 + sqrt(a))/sqrt(x^2 + 1)
```

mupad [B] time = 2.67, size = 19, normalized size = 0.86

$$\frac{\sqrt{a}\,\sqrt{x^2+1}\,\sqrt{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] (a^(1/2)*(x^2 + 1)^(1/2)*(x^2)^(1/2))/x
```

sympy [A] time = 0.48, size = 20, normalized size = 0.91

$$\frac{\sqrt{a}\,\sqrt{x^2+1}\,\sqrt{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2)**(1/2)/(x**2+1)**(1/2),x)
[Out] sqrt(a)*sqrt(x**2 + 1)*sqrt(x**2)/x
```

3.380 $\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=131

$$\frac{2\sqrt{x^2+1}\,\sqrt{ax}}{x+1}+\frac{\sqrt{a}\,(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2\sqrt{a}\,(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

```
[Out] 2*(a*x)^(1/2)*(x^2+1)^(1/2)/(1+x)-2*(1+x)*(cos(2*arctan((a*x)^(1/2)/a^(1/2)
))^2)^(1/2)/cos(2*arctan((a*x)^(1/2)/a^(1/2)))*EllipticE(sin(2*arctan((a*x)
^(1/2)/a^(1/2))),1/2*2^(1/2))*a^(1/2)*((x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1/2)
+(1+x)*(cos(2*arctan((a*x)^(1/2)/a^(1/2)))^2)^(1/2)/cos(2*arctan((a*x)^(1/2
)/a^(1/2)))*EllipticF(sin(2*arctan((a*x)^(1/2)/a^(1/2))),1/2*2^(1/2))*a^(1/
2)*((x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {329, 305, 220, 1196}

$$\frac{2\sqrt{x^2+1}\,\sqrt{ax}}{x+1}+\frac{\sqrt{a}\,(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2\sqrt{a}\,(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x]*Sqrt[1 + x^2])/(1 + x) - (2*Sqrt[a]*(1 + x)*Sqrt[(1 + x^2)/(1
+ x)^2]*EllipticE[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/Sqrt[1 + x^2] + (Sqrt[
a]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[a*x]/Sqrt[a]],
 1/2])/Sqrt[1 + x^2]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right)}{a}\\
&= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right) - 2\,\text{Subst}\left(\int \frac{1-\frac{x^2}{a}}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right)\\
&= \frac{2\sqrt{ax}\,\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{a}\,(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}} + \frac{\sqrt{a}\,(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right.\right.}{\sqrt{1+x^2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.21

$$
\frac{2}{3}x\sqrt{ax}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x]/Sqrt[1 + x^2],x]
[Out] (2*x*Sqrt[a*x]*Hypergeometric2F1[1/2, 3/4, 7/4, -x^2])/3
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\sqrt{ax}}{\sqrt{x^2+1}}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*x)/sqrt(x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

maple [C] time = 0.03, size = 81, normalized size = 0.62

$$
\frac{\sqrt{ax}\,\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,\left(2\,\text{EllipticE}\left(\sqrt{-i(x+i)}, \frac{\sqrt{2}}{2}\right) - \text{EllipticF}\left(\sqrt{-i(x+i)}, \frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x^2+1}\;x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^2+1)^(1/2),x)
[Out] (a*x)^(1/2)/(x^2+1)^(1/2)*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))^(1/2)*(I*x)^
(1/2)*(2*EllipticE((-I*(x+I))^(1/2),1/2*2^(1/2))-EllipticF((-I*(x+I))^(1/2)
,1/2*2^(1/2)))/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int((a*x)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [C] time = 1.02, size = 36, normalized size = 0.27

$$\frac{\sqrt{a}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|x^2e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(a)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**2*exp_polar(I*pi))
/(2*gamma(7/4))
```

3.381 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{x}\,(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{\frac{a}{x}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

```
[Out] (1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*arctan(x^(1/2)))*EllipticF(sin
(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x)^(1/2)*x^(1/2)*((x^2+1)/(1+x)^2)^(1/2
)/(x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = $\{15, 329, 220\}$

$$\frac{\sqrt{x}\,(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{\frac{a}{x}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a/x]*Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqr
t[x]], 1/2])/Sqrt[1 + x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx &= \left(\sqrt{\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x^2}}\,dx\\
&= \left(2\sqrt{\frac{a}{x}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \sqrt{x}\right)\\
&= \frac{\sqrt{\frac{a}{x}}\sqrt{x}\,(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.50

$$2x\sqrt{\frac{a}{x}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x]/Sqrt[1 + x^2],x]
[Out] 2*Sqrt[a/x]*x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a/x)/sqrt(x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

maple [C] time = 0.04, size = 62, normalized size = 1.15

$$\frac{i\sqrt{\frac{a}{x}}\,\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,\operatorname{EllipticF}\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x)^(1/2)/(x^2+1)^(1/2),x)
[Out] I*(a/x)^(1/2)/(x^2+1)^(1/2)*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))^(1/2)*(I*x
)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int((a/x)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x)/sqrt(x**2 + 1), x)
```

3.382 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=22

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

```
[Out] -x*arctanh((x^2+1)^(1/2))*(a/x^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {15, 266, 63, 207}

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \left(\sqrt{\frac{a}{x^2}}\,x\right)\int \frac{1}{x\sqrt{1+x^2}}\,dx$$
$$= \frac{1}{2}\left(\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^2\right)$$
$$= \left(\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^2}\right)$$
$$= -\sqrt{\frac{a}{x^2}}\,x\tanh^{-1}\left(\sqrt{1+x^2}\right)$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

fricas [A] time = 0.44, size = 76, normalized size = 3.45

$$\left[x\sqrt{\frac{a}{x^2}}\log\left(\frac{\sqrt{x^2+1}-1}{x}\right), 2\sqrt{-a}\arctan\left(-\frac{\sqrt{-a}\,x^2\sqrt{\frac{a}{x^2}}-\sqrt{x^2+1}\sqrt{-a}\,x\sqrt{\frac{a}{x^2}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt(a/x^2)*log((sqrt(x^2 + 1) - 1)/x), 2*sqrt(-a)*arctan(-(sqrt(-a)*x^2
*sqrt(a/x^2) - sqrt(x^2 + 1)*sqrt(-a)*x*sqrt(a/x^2))/a)]
```

giac [A] time = 0.21, size = 30, normalized size = 1.36

$$-\frac{1}{2}\sqrt{a}\left(\log\left(\sqrt{x^2+1}+1\right)-\log\left(\sqrt{x^2+1}-1\right)\right)\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(log(sqrt(x^2 + 1) + 1) - log(sqrt(x^2 + 1) - 1))*sgn(x)
```

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$-\sqrt{\frac{a}{x^2}}\,x\,\text{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -(a/x^2)^(1/2)*x*arctanh(1/(x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a/x^2)/sqrt(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a/x^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(sqrt(a/x**2)/sqrt(x**2 + 1), x)
```

3.383 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=159

$$\frac{2\sqrt{x^2+1}\,x^2\sqrt{\frac{a}{x^3}}}{x+1}-2\sqrt{x^2+1}\,x\sqrt{\frac{a}{x^3}}+\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,x^{3/2}\sqrt{\frac{a}{x^3}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,x^{3/2}\sqrt{\frac{a}{x^3}}\,E\left(2\,t\right.}{\sqrt{x^2+1}}$$

```
[Out] -2*x*(a/x^3)^(1/2)*(x^2+1)^(1/2)+2*x^2*(a/x^3)^(1/2)*(x^2+1)^(1/2)/(1+x)-2*
x^(3/2)*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*arctan(x^(1/2)))*Ellip
ticE(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x^3)^(1/2)*((x^2+1)/(1+x)^2)^(1
/2)/(x^2+1)^(1/2)+x^(3/2)*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*arct
an(x^(1/2)))*EllipticF(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x^3)^(1/2)*((
x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {15, 325, 329, 305, 220, 1196}

$$\frac{2\sqrt{x^2+1}\,x^2\sqrt{\frac{a}{x^3}}}{x+1}-2\sqrt{x^2+1}\,x\sqrt{\frac{a}{x^3}}+\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,x^{3/2}\sqrt{\frac{a}{x^3}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,x^{3/2}\sqrt{\frac{a}{x^3}}\,E\left(2\,t\right.}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^2] + (2*Sqrt[a/x^3]*x^2*Sqrt[1 + x^2])/(1 + x)
- (2*Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticE[2*ArcT
an[Sqrt[x]], 1/2])/Sqrt[1 + x^2] + (Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x
^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/Sqrt[1 + x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \left(\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{1+x^2}}\,dx$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^2} + \left(\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{1+x^2}}\,dx$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^2} + \left(2\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \sqrt{x}\right)$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^2} + \left(2\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \sqrt{x}\right) - \left(2\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^2} + \frac{2\sqrt{\frac{a}{x^3}}\,x^2\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{\frac{a}{x^3}}\,x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,E\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}} + $$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.17

$$-2x\sqrt{\frac{a}{x^3}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
[Out] -2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -x^2]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

maple [C] time = 0.04, size = 116, normalized size = 0.73

$$\frac{\sqrt{\frac{a}{x^3}}\left(-2x^2+2\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,\operatorname{EllipticE}\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)-\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\right.}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^3)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] (a/x^3)^(1/2)*x*(2*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))^(1/2)*(I*x)^(1/2)*E
llipticE((-I*(x+I))^(1/2),1/2*2^(1/2))-(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))
^(1/2)*(I*x)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(1/2))-2*x^2-2)/(x^2+1)
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^3)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int((a/x^3)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**3)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**3)/sqrt(x**2 + 1), x)
```

3.384 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=21

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

[Out] -x*(a/x^4)^(1/2)*(x^2+1)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 264}

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a/x^4]/Sqrt[1 + x^2],x]

[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx &= \left(\sqrt{\frac{a}{x^4}}\,x^2\right)\int \frac{1}{x^2\sqrt{1+x^2}}\,dx\\ &= -\sqrt{\frac{a}{x^4}}\,x\sqrt{1+x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a/x^4]/Sqrt[1 + x^2],x]

[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])

fricas [A] time = 0.43, size = 30, normalized size = 1.43

$$-x^2\sqrt{\frac{a}{x^4}}-\sqrt{x^2+1}\,x\sqrt{\frac{a}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -x^2*sqrt(a/x^4) - sqrt(x^2 + 1)*x*sqrt(a/x^4)
```

giac [A] time = 0.18, size = 22, normalized size = 1.05

$$\frac{2\sqrt{a}}{\left(x-\sqrt{x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)/((x - sqrt(x^2 + 1))^2 - 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -x*(a/x^4)^(1/2)*(x^2+1)^(1/2)
```

maxima [A] time = 1.97, size = 23, normalized size = 1.10

$$-\frac{\sqrt{a}\,x^2+\sqrt{a}}{\sqrt{x^2+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(a)*x^2 + sqrt(a))/(sqrt(x^2 + 1)*x)
```

mupad [B] time = 2.87, size = 18, normalized size = 0.86

$$-\sqrt{a}\,x\,\sqrt{x^2+1}\,\sqrt{\frac{1}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] -a^(1/2)*x*(x^2 + 1)^(1/2)*(1/x^4)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**4)/sqrt(x**2 + 1), x)
```

3.385 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=25

$$\frac{2\sqrt{x^3+1}\,\sqrt{ax^4}}{3x^2}$$

```
[Out] 2/3*(a*x^4)^(1/2)*(x^3+1)^(1/2)/x^2
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 261}

$$\frac{2\sqrt{x^3+1}\,\sqrt{ax^4}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx &= \frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{1+x^3}}\,dx}{x^2}\\ &= \frac{2\sqrt{ax^4}\,\sqrt{1+x^3}}{3x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}\,\sqrt{ax^4}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

fricas [A] time = 0.43, size = 19, normalized size = 0.76

$$\frac{2\,\sqrt{ax^4}\,\sqrt{x^3+1}}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(a*x^4)*sqrt(x^3 + 1)/x^2
```

giac [A] time = 0.17, size = 12, normalized size = 0.48

$$\frac{2}{3}\sqrt{x^3+1}\,\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(x^3 + 1)*sqrt(a)
```

maple [A] time = 0.01, size = 31, normalized size = 1.24

$$\frac{2\,(x+1)\left(x^2-x+1\right)\sqrt{a\,x^4}}{3\sqrt{x^3+1}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^3+1)^(1/2),x)
[Out] 2/3*(x+1)*(x^2-x+1)/x^2*(a*x^4)^(1/2)/(x^3+1)^(1/2)
```

maxima [A] time = 2.26, size = 28, normalized size = 1.12

$$\frac{2\left(\sqrt{a}\,x^3+\sqrt{a}\right)}{3\,\sqrt{x^2-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] 2/3*(sqrt(a)*x^3 + sqrt(a))/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

mupad [B] time = 2.91, size = 20, normalized size = 0.80

$$\frac{2\,\sqrt{a}\,\sqrt{x^3+1}\,\sqrt{x^4}}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] (2*a^(1/2)*(x^3 + 1)^(1/2)*(x^4)^(1/2))/(3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^4}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a*x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.386 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=292

$$\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}\,\sqrt{ax^3}}{x\left(\left(1+\sqrt{3}\right)x+1\right)}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,\sqrt{ax^3}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\,(x+1}{}$$

```
[Out] (1+3^(1/2))*(a*x^3)^(1/2)*(x^3+1)^(1/2)/x/(1+x*(1+3^(1/2)))-3^(1/4)*(1+x)*(
(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^
(1/2)))*EllipticE((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))*(a*x^3)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)/x/(x^
3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)-1/6*(1+x)*((1+x*(1-3^(1/2)))
^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*EllipticF
((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))
*(1-3^(1/2))*(a*x^3)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/x/
(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 329, 308, 225, 1881}

$$\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}\,\sqrt{ax^3}}{x\left(\left(1+\sqrt{3}\right)x+1\right)}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,\sqrt{ax^3}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\,(x+1}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```

```
[Out] ((1 + Sqrt[3])*Sqrt[a*x^3]*Sqrt[1 + x^3])/(x*(1 + (1 + Sqrt[3])*x)) - (3^(1
/4)*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*Ellipti
cE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(
x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])
*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[
ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(2*3
^(1/4)*x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^3}}\,dx}{x^{3/2}}\\
&= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{x^{3/2}}\\
&= -\frac{\sqrt{ax^3}\,\text{Subst}\left(\int \frac{-1+\sqrt{3}-2x^4}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{x^{3/2}} + \frac{\left(\left(-1+\sqrt{3}\right)\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{x^{3/2}}\\
&= \frac{\left(1+\sqrt{3}\right)\sqrt{ax^3}\sqrt{1+x^3}}{x\left(1+\left(1+\sqrt{3}\right)x\right)} - \frac{\sqrt[4]{3}\sqrt{ax^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,E\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}} -
\end{aligned}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.10

$$\frac{2}{5}x\sqrt{ax^3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
[Out] (2*x*Sqrt[a*x^3]*Hypergeometric2F1[1/2, 5/6, 11/6, -x^3])/5
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ax^3}}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

maple [C] time = 0.28, size = 1521, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(a*x^3)^(1/2)/x*(x^3+1)^(1/2)*a*(I*3^(1/2)*((3+I*3^(1/2))*x/(1+I*3^(1/2)
)/(x+1))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((I*3^(1/2)-2*
x+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(x
+1))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2)
)*x^2+2*I*3^(1/2)*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(x+1))^(1/2)*((I*3^(1/2)+2
*x-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(x+1))^(1
/2)*EllipticE(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(x+1))^(1/2),((-3+I*3^(1/2))*(
1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*x+I*3^(1/2)*((3+I*3^(1/2))
*x/(1+I*3^(1/2))/(x+1))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(x+1))^(1/2)
*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(
1+I*3^(1/2))/(x+1))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*
3^(1/2)))^(1/2))-2*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(x+1))^(1/2)*((I*3^(1/2)+
2*x-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(x+1))^(
1/2)*EllipticF(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(x+1))^(1/2),((-3+I*3^(1/2))*
(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*x^2+3*((3+I*3^(1/2))*x/(1
+I*3^(1/2))/(x+1))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((I*
3^(1/2)-2*x+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticE(((3+I*3^(1/2))*x/(1+I*3
^(1/2))/(x+1))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/
2)))^(1/2))*x^2-I*3^(1/2)*x^3-4*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(x+1))^(1/2)
*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1
/2))/(x+1))^(1/2)*EllipticF(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(x+1))^(1/2),((-
3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*x+6*((3+I*3^
(1/2))*x/(1+I*3^(1/2))/(x+1))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(x+1))
^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticE(((3+I*3^(1/2
))*x/(1+I*3^(1/2))/(x+1))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)
/(3+I*3^(1/2)))^(1/2))*x+I*3^(1/2)*x^2-2*((3+I*3^(1/2))*x/(1+I*3^(1/2))/(x+
1))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((I*3^(1/2)-2*x+1)/
(1+I*3^(1/2))/(x+1))^(1/2)*EllipticF(((3+I*3^(1/2))*x/(1+I*3^(1/2))/(x+1))^
(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))+3*(
(3+I*3^(1/2))*x/(1+I*3^(1/2))/(x+1))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)
/(x+1))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticE(((3+I
*3^(1/2))*x/(1+I*3^(1/2))/(x+1))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(
1/2)-1)/(3+I*3^(1/2)))^(1/2))-I*3^(1/2)*x-3*x^3+3*x^2-3*x)/(x*(x^3+1)*a)^(1
/2)/(3+I*3^(1/2))/(-a*x*(x+1)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x^3}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] int((a*x^3)^(1/2)/(x^3 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.387 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=260

$$\frac{2\sqrt{x^3+1}\,\sqrt{ax^2}}{x\left(x+\sqrt{3}+1\right)}+\frac{2\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{ax^2}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x\cdots}{(x+\sqrt{3}\cdots}}}{x\sqrt{\cdots}}$$

```
[Out] 2*(a*x^2)^(1/2)*(x^3+1)^(1/2)/x/(1+x+3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1
/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(a*x^2)^(1/2)*((x^2-x+1)/(1+x+3^(
1/2))^2)^(1/2)*3^(3/4)/x/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-3^(1/4
)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(a*x^2)^(1/2)*
(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/x/(x^3+1)^(1/2)
/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {15, 303, 218, 1877}

$$\frac{2\sqrt{x^3+1}\,\sqrt{ax^2}}{x\left(x+\sqrt{3}+1\right)}+\frac{2\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{ax^2}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x\cdots}{(x+\sqrt{3}\cdots}}}{x\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x^2]*Sqrt[1 + x^3])/(x*(1 + Sqrt[3] + x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*Sqrt[a*x^2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE
[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(x*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2]*Sqrt[a*x^2]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
```

```
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \frac{\sqrt{ax^2}\int \frac{x}{\sqrt{1+x^3}}\,dx}{x}$$
$$= \frac{\sqrt{ax^2}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{x} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\sqrt{ax^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{x}$$
$$= \frac{2\sqrt{ax^2}\sqrt{1+x^3}}{x\left(1+\sqrt{3}+x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{ax^2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{}}{}$$

Mathematica [C] time = 0.00, size = 29, normalized size = 0.11

$$\frac{1}{2}x\sqrt{ax^2}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (x*Sqrt[a*x^2]*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/2
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ax^2}}{\sqrt{x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

[Out] integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)

maple [A] time = 0.06, size = 270, normalized size = 1.04

$$\frac{\sqrt{a\,x^2}\,\left(-3+i\sqrt{3}\right)\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{3+i\sqrt{3}}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,\left(i\sqrt{3}\ \mathrm{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{3+i\sqrt{3}}}\right)+3\,\mathrm{Ellipti}\right.}{2\sqrt{x^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2)^(1/2)/(x^3+1)^(1/2),x)

[Out] 1/2*(a*x^2)^(1/2)*(-3+I*3^(1/2))*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(3+I*3^(1/2)))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*(I*EllipticE((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(3+I*3^(1/2)))^(1/2))*3^(1/2)-I*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(3+I*3^(1/2)))^(1/2))*3^(1/2)+3*EllipticE((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(3+I*3^(1/2)))^(1/2))-EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(3+I*3^(1/2)))^(1/2)))/x/(x^3+1)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x^2}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2)^(1/2)/(x^3 + 1)^(1/2),x)

[Out] int((a*x^2)^(1/2)/(x^3 + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**2)**(1/2)/(x**3+1)**(1/2),x)

[Out] Integral(sqrt(a*x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)

3.388 $\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}\sqrt{a}\,\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

```
[Out] 2/3*arcsinh((a*x)^(3/2)/a^(3/2))*a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {329, 275, 215}

$$\frac{2}{3}\sqrt{a}\,\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(a*x)^(3/2)/a^(3/2)])/3
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^6}{a^3}}}\,dx, x, \sqrt{ax}\right)}{a}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^3}}}\,dx, x, (ax)^{3/2}\right)}{3a}\\
&= \frac{2}{3}\sqrt{a}\,\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.96

$$\frac{2\sqrt{ax}\,\sinh^{-1}\left(x^{3/2}\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x]*ArcSinh[x^(3/2)])/(3*Sqrt[x])
```

fricas [B] time = 0.50, size = 85, normalized size = 3.70

$$\left[\frac{1}{6}\sqrt{a}\log\left(-8\,ax^6-8\,ax^3-4\left(2\,x^4+x\right)\sqrt{x^3+1}\,\sqrt{ax}\,\sqrt{a}-a\right),-\frac{1}{3}\sqrt{-a}\arctan\left(\frac{2\,\sqrt{x^3+1}\,\sqrt{ax}\,\sqrt{-a}\,x}{2\,ax^3+a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a)*log(-8*a*x^6 - 8*a*x^3 - 4*(2*x^4 + x)*sqrt(x^3 + 1)*sqrt(a*x)
*sqrt(a) - a), -1/3*sqrt(-a)*arctan(2*sqrt(x^3 + 1)*sqrt(a*x)*sqrt(-a)*x/(2
*a*x^3 + a))]
```

giac [B] time = 0.19, size = 35, normalized size = 1.52

$$-\frac{2\,a^{\frac{5}{2}}\log\left(-\sqrt{ax}\,a^{\frac{3}{2}}x+\sqrt{a^4x^3+a^4}\right)}{3\,|a|^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*a^(5/2)*log(-sqrt(a*x)*a^(3/2)*x + sqrt(a^4*x^3 + a^4))/abs(a)^2
```

maple [C] time = 0.14, size = 321, normalized size = 13.96

$$-\frac{4\sqrt{ax}\,\sqrt{x^3+1}\,\left(1+i\sqrt{3}\right)\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x+1)}}\,(x+1)^2\sqrt{\frac{2x+i\sqrt{3}-1}{\left(i\sqrt{3}-1\right)(x+1)}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{\left(1+i\sqrt{3}\right)(x+1)}}\,\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x+1)}},\right.\right.}{\sqrt{\left(x^3+1\right)ax}\,\left(3+i\sqrt{3}\right)\sqrt{-(x+1)\left(2x+i\sqrt{3}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -4*(a*x)^(1/2)*(x^3+1)^(1/2)*a*(1+I*3^(1/2))*((3+I*3^(1/2))/(1+I*3^(1/2))/(
x+1)*x)^(1/2)*(x+1)^2*((2*x+I*3^(1/2)-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((-2*x+
I*3^(1/2)+1)/(1+I*3^(1/2))/(x+1))^(1/2)*(EllipticF(((3+I*3^(1/2))/(1+I*3^(1
/2))/(x+1)*x)^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2
)))^(1/2))-EllipticPi(((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2),(1+I*3^(1
/2))/(3+I*3^(1/2)),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)
))^(1/2)))/((x^3+1)*a*x)^(1/2)/(3+I*3^(1/2))/(-(x+1)*(2*x+I*3^(1/2)-1)*(-2*
x+I*3^(1/2)+1)*a*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a\,x}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^3 + 1)^(1/2),x)
```

```
[Out] int((a*x)^(1/2)/(x^3 + 1)^(1/2), x)
```

sympy [A] time = 1.15, size = 14, normalized size = 0.61

$$\frac{2\sqrt{a}\,\operatorname{asinh}\left(x^{\frac{3}{2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt(a)*asinh(x**(3/2))/3
```

3.389 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=116

$$\frac{x(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{\frac{a}{x}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] 1/3*x*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)
))*(1+x*(1+3^(1/2)))*EllipticF((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a/x)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {15, 329, 225}

$$\frac{x(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{\frac{a}{x}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

```
[Out] (Sqrt[a/x]*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[
ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(
1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \left(\sqrt{\frac{a}{x}}\,\sqrt{x}\right)\int \frac{1}{\sqrt{x}\,\sqrt{1+x^3}}\,dx$$
$$= \left(2\sqrt{\frac{a}{x}}\,\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\sqrt{\frac{a}{x}}\,x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\sqrt{1+x^3}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.23

$$2x\sqrt{\frac{a}{x}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x]/Sqrt[1 + x^3],x]
[Out] 2*Sqrt[a/x]*x*Hypergeometric2F1[1/6, 1/2, 7/6, -x^3]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a/x)/sqrt(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

maple [C] time = 0.18, size = 232, normalized size = 2.00

$$\frac{4\sqrt{\frac{a}{x}}\,\sqrt{x^3+1}\,\left(1+i\sqrt{3}\right)\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x+1)}}\,(x+1)^2\sqrt{\frac{2x+i\sqrt{3}-1}{\left(i\sqrt{3}-1\right)(x+1)}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{\left(1+i\sqrt{3}\right)(x+1)}}\,x\,\text{EllipticF}\left(\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x+1)}}, \sqrt{\frac{\left(-3+\right.}{\left(i\sqrt{3}\right.}}\right.}{\sqrt{\left(x^3+1\right)x}\,\left(3+i\sqrt{3}\right)\sqrt{-(x+1)\left(2x+i\sqrt{3}-1\right)\left(-2x+i\sqrt{3}+1\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] 4*(a/x)^(1/2)*x*(x^3+1)^(1/2)*(1+I*3^(1/2))*((3+I*3^(1/2))/(1+I*3^(1/2))/(x
+1)*x)^(1/2)*(x+1)^2*((2*x+I*3^(1/2)-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((-2*x+I
*3^(1/2)+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticF(((3+I*3^(1/2))/(1+I*3^(1/2
))/(x+1)*x)^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2))
)^(1/2))/((x^3+1)*x)^(1/2)/(3+I*3^(1/2))/(-x*(x+1)*(2*x+I*3^(1/2)-1)*(-2*x+
I*3^(1/2)+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x)^(1/2)/(x^3 + 1)^(1/2),x)
```

```
[Out] int((a/x)^(1/2)/(x^3 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.390 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}} \, dx$

Optimal. Leaf size=24

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

[Out] -2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {15, 266, 63, 207}

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^3],x]

[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \left(\sqrt{\frac{a}{x^2}}\,x\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx$$
$$= \frac{1}{3}\left(\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\left(2\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)$$
$$= -\frac{2}{3}\sqrt{\frac{a}{x^2}}\,x\tanh^{-1}\left(\sqrt{1+x^3}\right)$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

fricas [A] time = 0.48, size = 68, normalized size = 2.83

$$\left[\frac{1}{3}\,x\sqrt{\frac{a}{x^2}}\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right), \frac{2}{3}\sqrt{-a}\arctan\left(\frac{\sqrt{x^3+1}\sqrt{-a}\,x\sqrt{\frac{a}{x^2}}}{ax^3+a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] [1/3*x*sqrt(a/x^2)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3), 2/3*sqrt(-a)*arcta
n(sqrt(x^3 + 1)*sqrt(-a)*x*sqrt(a/x^2)/(a*x^3 + a))]
```

giac [A] time = 0.20, size = 31, normalized size = 1.29

$$-\frac{1}{3}\sqrt{a}\left(\log\left(\sqrt{x^3+1}+1\right)-\log\left(\left|\sqrt{x^3+1}-1\right|\right)\right)\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(a)*(log(sqrt(x^3 + 1) + 1) - log(abs(sqrt(x^3 + 1) - 1)))*sgn(x)
```

maple [A] time = 0.01, size = 19, normalized size = 0.79

$$-\frac{2\sqrt{\frac{a}{x^2}}\,x\,\text{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^3+1)^(1/2),x)
[Out] -2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^2)/sqrt(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^3 + 1)^(1/2),x)
```

```
[Out] int((a/x^2)^(1/2)/(x^3 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.391 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=312

$$-2\sqrt{x^3+1}\,x\sqrt{\frac{a}{x^3}}+\frac{2\left(1+\sqrt{3}\right)\sqrt{x^3+1}\,x^2\sqrt{\frac{a}{x^3}}}{\left(1+\sqrt{3}\right)x+1}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,x^2\sqrt{\frac{a}{x^3}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\right.}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] -2*x*(a/x^3)^(1/2)*(x^3+1)^(1/2)+2*x^2*(1+3^(1/2))*(a/x^3)^(1/2)*(x^3+1)^(1
/2)/(1+x*(1+3^(1/2)))-2*3^(1/4)*x^2*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1
/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*EllipticE((1-(1+x*(1-3^(
1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a/x^3)^(1/2)*
((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2
)))^2)^(1/2)-1/3*x^2*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/
(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*EllipticF((1-(1+x*(1-3^(1/2)))^2/(1+x*(
1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(a/x^3)^(1/2)*((x
^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {15, 325, 329, 308, 225, 1881}

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{x^3+1}\,x^2\sqrt{\frac{a}{x^3}}}{\left(1+\sqrt{3}\right)x+1}-2\sqrt{x^3+1}\,x\sqrt{\frac{a}{x^3}}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,x^2\sqrt{\frac{a}{x^3}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\right.}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^3] + (2*(1 + Sqrt[3])*Sqrt[a/x^3]*x^2*Sqrt[1 +
x^3])/(1 + (1 + Sqrt[3])*x) - (2*3^(1/4)*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1
 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3]
)*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1
+ (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 +
Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqrt[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1 - Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \left(\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{1+x^3}}\,dx$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^3}+\left(2\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\int \frac{x^{3/2}}{\sqrt{1+x^3}}\,dx$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^3}+\left(4\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^6}}\,dx,x,\sqrt{x}\right)$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^3}-\left(2\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\text{Subst}\left(\int \frac{-1+\sqrt{3}-2x^4}{\sqrt{1+x^6}}\,dx,x,\sqrt{x}\right)+\left(2\left(-1+\sqrt{3}\right)\sqrt{\frac{a}{x^3}}\,x^{3}\right.$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^3}+\frac{2\left(1+\sqrt{3}\right)\sqrt{\frac{a}{x^3}}\,x^2\sqrt{1+x^3}}{1+\left(1+\sqrt{3}\right)x}-\frac{2\sqrt[4]{3}\sqrt{\frac{a}{x^3}}\,x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,E\left(\cos^{-1}\left(\right.\right.}{\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.09

$$-2x\sqrt{\frac{a}{x^3}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
[Out] -2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/6, 1/2, 5/6, -x^3]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

maple [C] time = 0.14, size = 1784, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^3)^(1/2)/(x^3+1)^(1/2),x)
[Out] -2*(a/x^3)^(1/2)*x/(x^3+1)^(1/2)*(-2*I*3^(1/2)*((x^3+1)*x)^(1/2)*x-2*I*3^(1
/2)*((x^3+1)*x)^(1/2)*x^3+I*3^(1/2)*(-(x+1)*(2*x+I*3^(1/2)-1)*(-2*x+I*3^(1/
2)+1)*x)^(1/2)-4*((x^3+1)*x)^(1/2)*((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1
/2)*((2*x+I*3^(1/2)-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((-2*x+I*3^(1/2)+1)/(1+I*
3^(1/2))/(x+1))^(1/2)*EllipticF(((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2)
,((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*x^2+6*((
x^3+1)*x)^(1/2)*((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2)*((2*x+I*3^(1/2)
-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((-2*x+I*3^(1/2)+1)/(1+I*3^(1/2))/(x+1))^(1/
2)*EllipticE(((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2),((-3+I*3^(1/2))*(1
+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*x^2+I*3^(1/2)*(-(x+1)*(2*x+
I*3^(1/2)-1)*(-2*x+I*3^(1/2)+1)*x)^(1/2)*x^3+4*I*3^(1/2)*((x^3+1)*x)^(1/2)*
((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2)*((2*x+I*3^(1/2)-1)/(I*3^(1/2)-1
)/(x+1))^(1/2)*((-2*x+I*3^(1/2)+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticE(((3
+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3
^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*x-8*((x^3+1)*x)^(1/2)*((3+I*3^(1/2))/(1+I*3
^(1/2))/(x+1)*x)^(1/2)*((2*x+I*3^(1/2)-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((-2*x
+I*3^(1/2)+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticF(((3+I*3^(1/2))/(1+I*3^(1
/2))/(x+1)*x)^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2
)))^(1/2))*x+12*((x^3+1)*x)^(1/2)*((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/
2)*((2*x+I*3^(1/2)-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((-2*x+I*3^(1/2)+1)/(1+I*3
^(1/2))/(x+1))^(1/2)*EllipticE(((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2),
((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*x+2*I*3^(
1/2)*((x^3+1)*x)^(1/2)*((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2)*((2*x+I*
3^(1/2)-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((-2*x+I*3^(1/2)+1)/(1+I*3^(1/2))/(x+
1))^(1/2)*EllipticE(((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2),((-3+I*3^(1
/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))-4*((x^3+1)*x)^(1/2)*
```

```
((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2)*((2*x+I*3^(1/2)-1)/(I*3^(1/2)-1
)/(x+1))^(1/2)*((-2*x+I*3^(1/2)+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticF(((3
+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3
^(1/2)-1)/(3+I*3^(1/2)))^(1/2))+6*((x^3+1)*x)^(1/2)*((3+I*3^(1/2))/(1+I*3^(
1/2))/(x+1)*x)^(1/2)*((2*x+I*3^(1/2)-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((-2*x+I
*3^(1/2)+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticE(((3+I*3^(1/2))/(1+I*3^(1/2
))/(x+1)*x)^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2))
)^(1/2))+2*I*3^(1/2)*((x^3+1)*x)^(1/2)*((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x
)^(1/2)*((2*x+I*3^(1/2)-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((-2*x+I*3^(1/2)+1)/(
1+I*3^(1/2))/(x+1))^(1/2)*EllipticE(((3+I*3^(1/2))/(1+I*3^(1/2))/(x+1)*x)^(
1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))*x^2-
6*((x^3+1)*x)^(1/2)*x^3+3*(-(x+1)*(2*x+I*3^(1/2)-1)*(-2*x+I*3^(1/2)+1)*x)^(
1/2)*x^3+2*I*3^(1/2)*((x^3+1)*x)^(1/2)*x^2+6*((x^3+1)*x)^(1/2)*x^2-6*((x^3+
1)*x)^(1/2)*x+3*(-(x+1)*(2*x+I*3^(1/2)-1)*(-2*x+I*3^(1/2)+1)*x)^(1/2))/(3+I
*3^(1/2))/(-(x+1)*(2*x+I*3^(1/2)-1)*(-2*x+I*3^(1/2)+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^3)^(1/2)/(x^3 + 1)^(1/2),x)
```

```
[Out] int((a/x^3)^(1/2)/(x^3 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**3)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.392 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}} \, dx$

Optimal. Leaf size=281

$$-\sqrt{x^3+1}\,x\sqrt{\frac{a}{x^4}}+\frac{\sqrt{x^3+1}\,x^2\sqrt{\frac{a}{x^4}}}{x+\sqrt{3}+1}+\frac{\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,x^2\sqrt{\frac{a}{x^4}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-}}{}$$

```
[Out] -x*(a/x^4)^(1/2)*(x^3+1)^(1/2)+x^2*(a/x^4)^(1/2)*(x^3+1)^(1/2)/(1+x+3^(1/2)
)+1/3*x^2*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2
)*(a/x^4)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1
+x)/(1+x+3^(1/2))^2)^(1/2)-1/2*3^(1/4)*x^2*(1+x)*EllipticE((1+x-3^(1/2))/(1
+x+3^(1/2)),I*3^(1/2)+2*I)*(a/x^4)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 325, 303, 218, 1877}

$$\frac{\sqrt{x^3+1}\,x^2\sqrt{\frac{a}{x^4}}}{x+\sqrt{3}+1}-\sqrt{x^3+1}\,x\sqrt{\frac{a}{x^4}}+\frac{\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,x^2\sqrt{\frac{a}{x^4}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^3]) + (Sqrt[a/x^4]*x^2*Sqrt[1 + x^3])/(1 + Sqrt[
3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[a/x^4]*x^2*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] +
 x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
+ (Sqrt[2]*Sqrt[a/x^4]*x^2*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
```

```
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx &= \left(\sqrt{\frac{a}{x^4}}\,x^2\right)\int \frac{1}{x^2\sqrt{1+x^3}}\,dx\\
&= -\sqrt{\frac{a}{x^4}}\,x\sqrt{1+x^3}+\frac{1}{2}\left(\sqrt{\frac{a}{x^4}}\,x^2\right)\int \frac{x}{\sqrt{1+x^3}}\,dx\\
&= -\sqrt{\frac{a}{x^4}}\,x\sqrt{1+x^3}+\frac{1}{2}\left(\sqrt{\frac{a}{x^4}}\,x^2\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx+\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\sqrt{\frac{a}{x^4}}\,x^2\right)\int \frac{1}{\sqrt{1+x^3}}\,dx\\
&= -\sqrt{\frac{a}{x^4}}\,x\sqrt{1+x^3}+\frac{\sqrt{\frac{a}{x^4}}\,x^2\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{\frac{a}{x^4}}\,x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\right.}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.10

$$x\left(-\sqrt{\frac{a}{x^4}}\right)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Hypergeometric2F1[-1/3, 1/2, 2/3, -x^3])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

maple [A] time = 0.07, size = 353, normalized size = 1.26

$$\sqrt{\frac{a}{x^4}}\left(-2x^3-6\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\,\sqrt{\frac{-2x+i\sqrt{3}+1}{3+i\sqrt{3}}}\,\sqrt{\frac{2x+i\sqrt{3}-1}{-3+i\sqrt{3}}}\,x\,\mathrm{EllipticE}\left(\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{3+i\sqrt{3}}}\right)+i\sqrt{3}\,\sqrt{-\frac{2(x+1)}{-3+i\sqrt{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] 1/2*(a/x^4)^(1/2)*x*(I*3^(1/2)*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(
1/2)+1)/(3+I*3^(1/2)))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*Ellip
ticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(3+I*3^(1/2)))^(1/2))
*x-6*(-2*(x+1)/(-3+I*3^(1/2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(3+I*3^(1/2)))^(1/
2)*((2*x+I*3^(1/2)-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(x+1)/(-3+I*3^(1/
2)))^(1/2),(-(-3+I*3^(1/2))/(3+I*3^(1/2)))^(1/2))*x+3*(-2*(x+1)/(-3+I*3^(1/
2)))^(1/2)*((-2*x+I*3^(1/2)+1)/(3+I*3^(1/2)))^(1/2)*((2*x+I*3^(1/2)-1)/(-3+
I*3^(1/2)))^(1/2)*EllipticF((-2*(x+1)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2)
)/(3+I*3^(1/2)))^(1/2))*x-2*x^3-2)/(x^3+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^3 + 1)^(1/2),x)
```

```
[Out] int((a/x^4)^(1/2)/(x^3 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**4)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.393 $\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} dx$

Optimal. Leaf size=37

$$\frac{x\sqrt{ax^{2n}}\ {}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

```
[Out] x*hypergeom([1/2, 1+1/n],[2+1/n],-x^n)*(a*x^(2*n))^(1/2)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {15, 364}

$$\frac{x\sqrt{ax^{2n}}\ {}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx &= \left(x^{-n}\sqrt{ax^{2n}}\right)\int \frac{x^n}{\sqrt{1+x^n}}\,dx\\ &= \frac{x\sqrt{ax^{2n}}\ {}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{1+n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{x\sqrt{ax^{2n}}\ {}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^{2n}}}{\sqrt{x^n + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x)
```

```
[Out] int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a\,x^{2\,n}}}{\sqrt{x^n + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2),x)
```

```
[Out] int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**(2*n))/sqrt(x**n + 1), x)
```

3.394 $\int \frac{\sqrt{a x^n}}{\sqrt{1+x^n}}\,dx$

Optimal. Leaf size=48

$$\frac{2x\sqrt{ax^n}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

```
[Out] 2*x*hypergeom([1/2, 1/2+1/n],[3/2+1/n],-x^n)*(a*x^n)^(1/2)/(2+n)
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 364}

$$\frac{2x\sqrt{ax^n}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -x^n])/(2
 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx &= \left(x^{-n/2}\sqrt{ax^n}\right)\int \frac{x^{n/2}}{\sqrt{1+x^n}}\,dx\\ &= \frac{2x\sqrt{ax^n}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{2+n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.83

$$\frac{2x\sqrt{ax^n}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])/
(2 + n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

maple [A] time = 0.08, size = 35, normalized size = 0.73

$$\frac{2\sqrt{a\,x^n}\;x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{n}+\frac{1}{2}\right],\left[\frac{1}{n}+\frac{3}{2}\right],-x^n\right)}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^n)^(1/2)/(x^n+1)^(1/2),x)
[Out] 2*x*hypergeom([1/2,1/2+1/n],[3/2+1/n],-x^n)*(a*x^n)^(1/2)/(n+2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^n)^(1/2)/(x^n + 1)^(1/2),x)
[Out] int((a*x^n)^(1/2)/(x^n + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**n)**(1/2)/(1+x**n)**(1/2),x)
[Out] Integral(sqrt(a*x**n)/sqrt(x**n + 1), x)
```

3.395 $\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx$

Optimal. Leaf size=52

$$\frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{n+4}$$

```
[Out] 4*x*hypergeom([1/2, 1/4+1/n],[5/4+1/n],-x^n)*(a*x^(1/2*n))^(1/2)/(4+n)
```

Rubi [A] time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {15, 364}

$$\frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{n+4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

```
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, (1 + 4/n)/4, (5 + 4/n)/4, -x^n]
)/(4 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx &= \left(x^{-n/4}\sqrt{ax^{n/2}}\right)\int \frac{x^{n/4}}{\sqrt{1+x^n}}\,dx\\ &= \frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{4+n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.85

$$\frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}+\frac{1}{n};\frac{5}{4}+\frac{1}{n};-x^n\right)}{n+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

```
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, 1/4 + n^(-1), 5/4 + n^(-1), -x^
n])/(4 + n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{\frac{1}{2}n}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

maple [A] time = 0.09, size = 37, normalized size = 0.71

$$\frac{4\sqrt{a\,x^{\frac{n}{2}}}\;x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{n}+\frac{1}{4}\right],\left[\frac{1}{n}+\frac{5}{4}\right],-x^{n}\right)}{n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(1/2*n))^(1/2)/(x^n+1)^(1/2),x)
```

```
[Out] 4*x*hypergeom([1/2,1/4+1/n],[5/4+1/n],-x^n)*(a*x^(1/2*n))^(1/2)/(4+n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{\frac{1}{2}n}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^{n/2}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(n/2))^(1/2)/(x^n + 1)^(1/2),x)
```

```
[Out] int((a*x^(n/2))^(1/2)/(x^n + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{\frac{n}{2}}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**(1/2*n))**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**(n/2))/sqrt(x**n + 1), x)
```

3.396 $\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right) dx$

Optimal. Leaf size=34

$$\frac{2x^{1-n}\sqrt{x^n+1}\,\sqrt{ax^{2n}}}{n+2}$$

```
[Out] 2*x^(1-n)*(a*x^(2*n))^(1/2)*(1+x^n)^(1/2)/(2+n)
```

Rubi [C] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 2.35, number of steps used = 5, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15, 364, 245}

$$\frac{2x^{1-n}\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^n\right)}{n+2}+\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[1
 + x^n]),x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n) + (2*x^(1 - n)*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(
-1), -x^n])/(2 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right) dx &= \frac{2\int \frac{x^{-n}\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx}{2+n}+\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx\\ &= \left(x^{-n}\sqrt{ax^{2n}}\right)\int \frac{x^n}{\sqrt{1+x^n}}\,dx+\frac{\left(2x^{-n}\sqrt{ax^{2n}}\right)\int \frac{1}{\sqrt{1+x^n}}\,dx}{2+n}\\ &= \frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{1+n}+\frac{2x^{1-n}\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^n\right)}{2+n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.97

$$\frac{2ax^{n+1}\sqrt{x^n+1}}{(n+2)\sqrt{ax^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*
Sqrt[1 + x^n]),x]
```

```
[Out] (2*a*x^(1 + n)*Sqrt[1 + x^n])/((2 + n)*Sqrt[a*x^(2*n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}} + \frac{2\sqrt{ax^{2n}}}{(n+2)\sqrt{x^n+1}\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n + 2)*sqrt(x
^n + 1)*x^n), x)
```

maple [A] time = 0.05, size = 30, normalized size = 0.88

$$\frac{2\sqrt{x^n+1}\,\sqrt{a\,x^{2n}}\,x\,x^{-n}}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(x^n+1)^(1/2)+2*(a*x^(2*n))^(1/2)/(n+2)/(x^n)/(x^n+1)
^(1/2),x)
```

```
[Out] 2*x*(x^n+1)^(1/2)/(n+2)*(a*(x^n)^2)^(1/2)/(x^n)
```

maxima [A] time = 1.51, size = 18, normalized size = 0.53

$$\frac{2\sqrt{a}\sqrt{x^n+1}\,x}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*sqrt(x^n + 1)*x/(n + 2)
```

mupad [B] time = 2.89, size = 43, normalized size = 1.26

$$\frac{\sqrt{a\,x^{2\,n}}\,\left(\frac{2\,x}{n+2}+\frac{2\,x^{n+1}}{n+2}\right)}{x^n\,\sqrt{x^n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2) + (2*(a*x^(2*n))^(1/2))/(x^n*(x^n + 1
)^(1/2)*(n + 2)),x)
```

```
[Out] ((a*x^(2*n))^(1/2)*((2*x)/(n + 2) + (2*x^(n + 1))/(n + 2)))/(x^n*(x^n + 1)^
(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx+\int \frac{n\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx+\int \frac{2x^{-n}\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2)+2*(a*x**(2*n))**(1/2)/(2+n)/(
x**n)/(1+x**n)**(1/2),x)
```

```
[Out] (Integral(2*sqrt(a*x**(2*n))/sqrt(x**n + 1), x) + Integral(n*sqrt(a*x**(2*n
))/sqrt(x**n + 1), x) + Integral(2*x**(-n)*sqrt(a*x**(2*n))/sqrt(x**n + 1),
 x))/(n + 2)
```

3.397 $\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=114

$$\frac{2\sqrt{ax}\sqrt{df-e^2}\sqrt{\frac{e(e+fx)}{e^2-df}}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{df-e^2}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

```
[Out] 2*EllipticE(f^(1/2)*(e*x+d)^(1/2)/(d*f-e^2)^(1/2),(1-e^2/d/f)^(1/2))*(d*f-e
^2)^(1/2)*(a*x)^(1/2)*(e*(f*x+e)/(-d*f+e^2))^(1/2)/e/f^(1/2)/(-e*x/d)^(1/2)
/(f*x+e)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {114, 113}

$$\frac{2\sqrt{ax}\sqrt{df-e^2}\sqrt{\frac{e(e+fx)}{e^2-df}}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{df-e^2}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*Sqrt[-e^2 + d*f]*Sqrt[a*x]*Sqrt[(e*(e + f*x))/(e^2 - d*f)]*EllipticE[Arc
Sin[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[-e^2 + d*f]], 1 - e^2/(d*f)])/(e*Sqrt[f]*S
qrt[-((e*x)/d)]*Sqrt[e + f*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] && !LtQ[-((b*c - a*d)/d), 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-((b*c - a*d)/d), 0]
```

Rubi steps

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \frac{\left(\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}\right)\int \frac{\sqrt{-\frac{ex}{d}}}{\sqrt{d+ex}\sqrt{\frac{e^2}{e^2-df}+\frac{efx}{e^2-df}}}\,dx}{\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$
$$= \frac{2\sqrt{-e^2+df}\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{-e^2+df}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

Mathematica [C] time = 0.21, size = 106, normalized size = 0.93

$$-\frac{2ie\sqrt{ax}\sqrt{\frac{fx}{e}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{ex}{d}}\right)|\frac{df}{e^2}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{ex}{d}}\right)|\frac{df}{e^2}\right)\right)}{f\sqrt{\frac{ex}{d+ex}}\sqrt{d+ex}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((-2*I)*e*Sqrt[a*x]*Sqrt[1 + (f*x)/e]*(EllipticE[I*ArcSinh[Sqrt[(e*x)/d]],
(d*f)/e^2] - EllipticF[I*ArcSinh[Sqrt[(e*x)/d]], (d*f)/e^2]))/(f*Sqrt[(e*x)
/(d + e*x)]*Sqrt[d + e*x]*Sqrt[e + f*x])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ax}\sqrt{ex+d}\sqrt{fx+e}}{efx^2+de+\left(e^2+df\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x)*sqrt(e*x + d)*sqrt(f*x + e)/(e*f*x^2 + d*e + (e^2 + d*f)
*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

maple [A] time = 0.07, size = 191, normalized size = 1.68

$$-\frac{2\left(df\,\mathrm{EllipticE}\left(\sqrt{\frac{ex+d}{d}},\sqrt{\frac{df}{df-e^2}}\right)-e^2\,\mathrm{EllipticE}\left(\sqrt{\frac{ex+d}{d}},\sqrt{\frac{df}{df-e^2}}\right)+e^2\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{\frac{df}{df-e^2}}\right)\right)\sqrt{-\frac{ex}{d}}}{\left(efx^2+dfx+e^2x+de\right)e^2fx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] -2*(e^2*EllipticF(((e*x+d)/d)^(1/2),(d*f/(d*f-e^2))^(1/2))+EllipticE(((e*x+
d)/d)^(1/2),(d*f/(d*f-e^2))^(1/2))*d*f-EllipticE(((e*x+d)/d)^(1/2),(d*f/(d*
f-e^2))^(1/2))*e^2)*(-e*x/d)^(1/2)*(-(f*x+e)*e/(d*f-e^2))^(1/2)*((e*x+d)/d)
^(1/2)*d*(f*x+e)^(1/2)*(e*x+d)^(1/2)*(a*x)^(1/2)/f/e^2/x/(e*f*x^2+d*f*x+e^2
*x+d*e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax}}{\sqrt{ex+d}\,\sqrt{fx+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x}}{\sqrt{e+f\,x}\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/((e + f*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] int((a*x)^(1/2)/((e + f*x)^(1/2)*(d + e*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\,\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)**(1/2)/(e*x+d)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x)/(sqrt(d + e*x)*sqrt(e + f*x)), x)
```

3.398 $\int (ax^m)^r \, dx$

Optimal. Leaf size=16

$$\frac{x(ax^m)^r}{mr+1}$$

```
[Out] x*(a*x^m)^r/(m*r+1)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{x(ax^m)^r}{mr+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r,x]
```

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (ax^m)^r \, dx &= \left(x^{-mr}(ax^m)^r\right)\int x^{mr} \, dx\\ &= \frac{x(ax^m)^r}{1+mr}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x(ax^m)^r}{mr+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r,x]
```

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

fricas [A] time = 0.45, size = 20, normalized size = 1.25

$$\frac{xe^{\left(mr\log(x)+r\log(a)\right)}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

giac [**A**] time = 0.21, size = 20, normalized size = 1.25

$$\frac{xe^{(mr\log(x)+r\log(a))}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="giac")
[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

maple [**A**] time = 0.00, size = 17, normalized size = 1.06

$$\frac{x\,(a\,x^{m})^{r}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r,x)
[Out] x*(a*x^m)^r/(m*r+1)
```

maxima [**A**] time = 1.48, size = 17, normalized size = 1.06

$$\frac{a^{r}x(x^{m})^{r}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="maxima")
[Out] a^r*x*(x^m)^r/(m*r + 1)
```

mupad [**B**] time = 3.40, size = 16, normalized size = 1.00

$$\frac{x\,(a\,x^{m})^{r}}{m\,r+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r,x)
[Out] (x*(a*x^m)^r)/(m*r + 1)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^{r}x(x^{m})^{r}}{mr+1} & \text{for } m \neq -\frac{1}{r} \\ \int \left(ax^{-\frac{1}{r}}\right)^{r}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r,x)
[Out] Piecewise((a**r*x*(x**m)**r/(m*r + 1), Ne(m, -1/r)), (Integral((a*x**(-1/r)
)**r, x), True))
```

3.399 $\int (ax^m)^r (bx^n)^s \, dx$

Optimal. Leaf size=26

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

```
[Out] x*(a*x^m)^r*(b*x^n)^s/(m*r+n*s+1)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 30}

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r*(b*x^n)^s,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (ax^m)^r (bx^n)^s \, dx &= \left(x^{-mr} (ax^m)^r\right) \int x^{mr} (bx^n)^s \, dx \\
&= \left(x^{-mr-ns} (ax^m)^r (bx^n)^s\right) \int x^{mr+ns} \, dx \\
&= \frac{x (ax^m)^r (bx^n)^s}{1 + mr + ns}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r*(b*x^n)^s,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

fricas [A] time = 0.50, size = 32, normalized size = 1.23

$$\frac{xe^{\left(mr\log(x)+ns\log(x)+r\log(a)+s\log(b)\right)}}{mr + ns + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="fricas")
[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

giac [A] time = 0.24, size = 32, normalized size = 1.23

$$\frac{xe^{\left(mr\log(x)+ns\log(x)+r\log(a)+s\log(b)\right)}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="giac")
[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

maple [A] time = 0.00, size = 27, normalized size = 1.04

$$\frac{x\left(a\,x^{m}\right)^{r}\left(b\,x^{n}\right)^{s}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s,x)
[Out] x*(a*x^m)^r*(b*x^n)^s/(m*r+n*s+1)
```

maxima [A] time = 1.08, size = 32, normalized size = 1.23

$$\frac{a^{r}b^{s}xe^{\left(r\log(x^{m})+s\log(x^{n})\right)}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="maxima")
[Out] a^r*b^s*x*e^(r*log(x^m) + s*log(x^n))/(m*r + n*s + 1)
```

mupad [B] time = 2.92, size = 26, normalized size = 1.00

$$\frac{x\left(a\,x^{m}\right)^{r}\left(b\,x^{n}\right)^{s}}{m\,r+n\,s+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s,x)
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(m*r + n*s + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^{r}b^{s}x(x^{m})^{r}(x^{n})^{s}}{mr+ns+1} & \text{for}\ m\neq -\frac{ns+1}{r} \\ \int (bx^{n})^{s}\left(ax^{-\frac{1}{r}}x^{-\frac{ns}{r}}\right)^{r}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r*(b*x**n)**s,x)
[Out] Piecewise((a**r*b**s*x*(x**m)**r*(x**n)**s/(m*r + n*s + 1), Ne(m, -(n*s + 1
)/r)), (Integral((b*x**n)**s*(a*x**(-1/r)*x**(-n*s/r))**r, x), True))
```

3.400 $\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx$

Optimal. Leaf size=36

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr+ns+pt+1}$$

```
[Out] x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 30}

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr+ns+pt+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx &= \left(x^{-mr} (ax^m)^r\right) \int x^{mr} (bx^n)^s (cx^p)^t \, dx \\
&= \left(x^{-mr-ns} (ax^m)^r (bx^n)^s\right) \int x^{mr+ns} (cx^p)^t \, dx \\
&= \left(x^{-mr-ns-pt} (ax^m)^r (bx^n)^s (cx^p)^t\right) \int x^{mr+ns+pt} \, dx \\
&= \frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{1+mr+ns+pt}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr+ns+pt+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

fricas [A] time = 0.48, size = 44, normalized size = 1.22

$$\frac{xe^{\left(mr\log(x)+ns\log(x)+pt\log(x)+r\log(a)+s\log(b)+t\log(c)\right)}}{mr+ns+pt+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c))
/(m*r + n*s + p*t + 1)
```

giac [A] time = 0.24, size = 44, normalized size = 1.22

$$\frac{xe^{\left(mr\log(x)+ns\log(x)+pt\log(x)+r\log(a)+s\log(b)+t\log(c)\right)}}{mr+ns+pt+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="giac")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c))
/(m*r + n*s + p*t + 1)
```

maple [A] time = 0.00, size = 37, normalized size = 1.03

$$\frac{x\left(a\,x^{m}\right)^{r}\left(b\,x^{n}\right)^{s}\left(c\,x^{p}\right)^{t}}{mr+ns+pt+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x)
```

```
[Out] x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)
```

maxima [A] time = 1.27, size = 44, normalized size = 1.22

$$\frac{a^{r}b^{s}c^{t}xe^{\left(r\log(x^{m})+s\log(x^{n})+t\log(x^{p})\right)}}{mr+ns+pt+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="maxima")
```

```
[Out] a^r*b^s*c^t*x*e^(r*log(x^m) + s*log(x^n) + t*log(x^p))/(m*r + n*s + p*t + 1
)
```

mupad [B] time = 3.10, size = 36, normalized size = 1.00

$$\frac{x\left(a\,x^{m}\right)^{r}\left(b\,x^{n}\right)^{s}\left(c\,x^{p}\right)^{t}}{m\,r+n\,s+p\,t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x)
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(m*r + n*s + p*t + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r*(b*x**n)**s*(c*x**p)**t,x)
```

```
[Out] Timed out
```

3.401 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}-\frac{2(bx+c)^{7/2}}{7b^3(a-c)}+\frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

```
[Out] 2/3*a^2*(b*x+a)^(3/2)/b^3/(a-c)-4/5*a*(b*x+a)^(5/2)/b^3/(a-c)+2/7*(b*x+a)^(
7/2)/b^3/(a-c)-2/3*c^2*(b*x+c)^(3/2)/b^3/(a-c)+4/5*c*(b*x+c)^(5/2)/b^3/(a-c
)-2/7*(b*x+c)^(7/2)/b^3/(a-c)
```

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2104, 43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}-\frac{2(bx+c)^{7/2}}{7b^3(a-c)}+\frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(a - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(a -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(a - c)) - (2*c^2*(c + b*x)^(3/2))/(3*b^3
*(a - c)) + (4*c*(c + b*x)^(5/2))/(5*b^3*(a - c)) - (2*(c + b*x)^(7/2))/(7*
b^3*(a - c))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx &= -\frac{b\int x^2\sqrt{a+bx}\,dx}{-ab+bc}+\frac{b\int x^2\sqrt{c+bx}\,dx}{-ab+bc}\\ &= -\frac{b\int \left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{-ab+bc}+\frac{b\int \left(\frac{c^2\sqrt{c+bx}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}+\frac{(c+bx)^{5/2}}{b^2}\right)}{-ab+bc}\\ &= \frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{2c^2(c+bx)^{3/2}}{3b^3(a-c)}+\frac{4c(c+bx)^{5/2}}{5b^3(a-c)}-\end{aligned}$$

Mathematica [A] time = 0.17, size = 140, normalized size = 0.95

$$\frac{2\left(8a^3\sqrt{a+bx}-4a^2bx\sqrt{a+bx}+15b^3x^3\left(\sqrt{a+bx}-\sqrt{bx+c}\right)+3ab^2x^2\sqrt{a+bx}-3b^2cx^2\sqrt{bx+c}-8c^3\sqrt{bx}\right.}{105b^3(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*(8*a^3*Sqrt[a + b*x] - 4*a^2*b*x*Sqrt[a + b*x] + 3*a*b^2*x^2*Sqrt[a + b*
x] - 8*c^3*Sqrt[c + b*x] + 4*b*c^2*x*Sqrt[c + b*x] - 3*b^2*c*x^2*Sqrt[c + b
*x] + 15*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*x])))/(105*b^3*(a - c))
```

fricas [A] time = 0.44, size = 94, normalized size = 0.64

$$\frac{2\left(\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}-\left(15\,b^3x^3+3\,b^2cx^2-4\,bc^2x+8\,c^3\right)\sqrt{bx+c}\right)}{105\left(ab^3-b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a) - (15*b
^3*x^3 + 3*b^2*c*x^2 - 4*b*c^2*x + 8*c^3)*sqrt(b*x + c))/(a*b^3 - b^3*c)
```

giac [B] time = 0.27, size = 390, normalized size = 2.65

$$-\frac{2}{105}\left(\left(\left(3\,(bx+a)\left(\frac{5\left(a^2b^9-2\,ab^9c+b^9c^2\right)(bx+a)}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}-\frac{15\,a^3b^9-31\,a^2b^9c+17\,ab^9c^2-b^9c^3}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}\right)+\frac{45\,a^4b^9-96\,a^3b}{a^3b^{12}-3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/105*((3*(b*x + a)*(5*(a^2*b^9 - 2*a*b^9*c + b^9*c^2)*(b*x + a)/(a^3*b^12
 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3) - (15*a^3*b^9 - 31*a^2*b^9*c + 1
7*a*b^9*c^2 - b^9*c^3)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))
 + (45*a^4*b^9 - 96*a^3*b^9*c + 53*a^2*b^9*c^2 + 2*a*b^9*c^3 - 4*b^9*c^4)/(
a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*(b*x + a) - (15*a^5*b^9
 - 33*a^4*b^9*c + 17*a^3*b^9*c^2 - 3*a^2*b^9*c^3 + 12*a*b^9*c^4 - 8*b^9*c^5
)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*sqrt(b*x + c) + 2/10
5*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/(a*b
^3 - b^3*c)
```

maple [A] time = 0.00, size = 90, normalized size = 0.61

$$\frac{\frac{2(bx+a)^{\frac{3}{2}}a^2}{3}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+\frac{2(bx+a)^{\frac{7}{2}}}{7}}{(a-c)\,b^3}-\frac{2\left(\frac{(bx+c)^{\frac{3}{2}}c^2}{3}-\frac{2(bx+c)^{\frac{5}{2}}c}{5}+\frac{(bx+c)^{\frac{7}{2}}}{7}\right)}{(a-c)\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*a^2*(b*x+a)^(3/2))-2
/(a-c)/b^3*(1/7*(b*x+c)^(7/2)-2/5*(b*x+c)^(5/2)*c+1/3*c^2*(b*x+c)^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

mupad [B] time = 2.95, size = 179, normalized size = 1.22

$$\frac{2\,x^3\,\sqrt{a+b\,x}}{7\,\left(a-c\right)}-\frac{2\,x^3\,\sqrt{c+b\,x}}{7\,\left(a-c\right)}+\frac{16\,a^3\,\sqrt{a+b\,x}}{105\,b^3\,\left(a-c\right)}-\frac{16\,c^3\,\sqrt{c+b\,x}}{105\,b^3\,\left(a-c\right)}+\frac{2\,a\,x^2\,\sqrt{a+b\,x}}{35\,b\,\left(a-c\right)}-\frac{8\,a^2\,x\,\sqrt{a+b\,x}}{105\,b^2\,\left(a-c\right)}-\frac{2\,c\,x^2\,\sqrt{c+b\,x}}{35\,b\,\left(a-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x^3*(a + b*x)^(1/2))/(7*(a - c)) - (2*x^3*(c + b*x)^(1/2))/(7*(a - c)) +
 (16*a^3*(a + b*x)^(1/2))/(105*b^3*(a - c)) - (16*c^3*(c + b*x)^(1/2))/(105
*b^3*(a - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(a - c)) - (8*a^2*x*(a + b*
x)^(1/2))/(105*b^2*(a - c)) - (2*c*x^2*(c + b*x)^(1/2))/(35*b*(a - c)) + (8
*c^2*x*(c + b*x)^(1/2))/(105*b^2*(a - c))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

3.402 $\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal. Leaf size=95

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

```
[Out] -2/3*a*(b*x+a)^(3/2)/b^2/(a-c)+2/5*(b*x+a)^(5/2)/b^2/(a-c)+2/3*c*(b*x+c)^(3
/2)/b^2/(a-c)-2/5*(b*x+c)^(5/2)/b^2/(a-c)
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2104, 43}

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(a - c)
) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x)^(5/2))/(5*b^2*(a -
 c))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx &= -\frac{b\int x\sqrt{a+bx}\,dx}{-ab+bc}+\frac{b\int x\sqrt{c+bx}\,dx}{-ab+bc}\\ &= -\frac{b\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{-ab+bc}+\frac{b\int\left(-\frac{c\sqrt{c+bx}}{b}+\frac{(c+bx)^{3/2}}{b}\right)dx}{-ab+bc}\\ &= -\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}+\frac{2(a+bx)^{5/2}}{5b^2(a-c)}+\frac{2c(c+bx)^{3/2}}{3b^2(a-c)}-\frac{2(c+bx)^{5/2}}{5b^2(a-c)}\end{aligned}$$

Mathematica [A] time = 0.10, size = 95, normalized size = 1.00

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(a - c)
) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x)^(5/2))/(5*b^2*(a -
 c))
```

fricas [A] time = 0.43, size = 70, normalized size = 0.74

$$\frac{2\left(\left(3\,b^2x^2 + abx - 2\,a^2\right)\sqrt{bx + a} - \left(3\,b^2x^2 + bcx - 2\,c^2\right)\sqrt{bx + c}\right)}{15\left(ab^2 - b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a) - (3*b^2*x^2 + b*c*x - 2*c^
2)*sqrt(b*x + c))/(a*b^2 - b^2*c)
```

giac [B] time = 0.21, size = 206, normalized size = 2.17

$$-\frac{2\left(\left((bx + a)\left(\frac{3\left(ab^2 - b^2c\right)(bx + a)}{a^2b^3 - 2\,ab^3c + b^3c^2} - \frac{6\,a^2b^2 - 7\,ab^2c + b^2c^2}{a^2b^3 - 2\,ab^3c + b^3c^2}\right) + \frac{3\,a^3b^2 - 4\,a^2b^2c - ab^2c^2 + 2\,b^2c^3}{a^2b^3 - 2\,ab^3c + b^3c^2}\right)\sqrt{bx + c} - \frac{3\,(bx + a)^{\frac{5}{2}} - 5\,(bx + a)^{\frac{3}{2}}a}{ab - bc}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/15*(((b*x + a)*(3*(a*b^2 - b^2*c)*(b*x + a)/(a^2*b^3 - 2*a*b^3*c + b^3*c
^2) - (6*a^2*b^2 - 7*a*b^2*c + b^2*c^2)/(a^2*b^3 - 2*a*b^3*c + b^3*c^2)) +
(3*a^3*b^2 - 4*a^2*b^2*c - a*b^2*c^2 + 2*b^2*c^3)/(a^2*b^3 - 2*a*b^3*c + b^
3*c^2))*sqrt(b*x + c) - (3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(a*b - b*
c))/b
```

maple [A] time = 0.00, size = 66, normalized size = 0.69

$$\frac{-\frac{2(bx+a)^{\frac{3}{2}}a}{3} + \frac{2(bx+a)^{\frac{5}{2}}}{5}}{(a - c)\,b^2} - \frac{2\left(-\frac{(bx+c)^{\frac{3}{2}}c}{3} + \frac{(bx+c)^{\frac{5}{2}}}{5}\right)}{(a - c)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))-2/(a-c)/b^2*(1/5*(b*x+c
)^(5/2)-1/3*(b*x+c)^(3/2)*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx + a} + \sqrt{bx + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

mupad [B] time = 2.70, size = 129, normalized size = 1.36

$$\frac{2\,x^2\,\sqrt{a + b\,x}}{5\,(a - c)} - \frac{2\,x^2\,\sqrt{c + b\,x}}{5\,(a - c)} - \frac{4\,a^2\,\sqrt{a + b\,x}}{15\,b^2\,(a - c)} + \frac{4\,c^2\,\sqrt{c + b\,x}}{15\,b^2\,(a - c)} + \frac{2\,a\,x\,\sqrt{a + b\,x}}{15\,b\,(a - c)} - \frac{2\,c\,x\,\sqrt{c + b\,x}}{15\,b\,(a - c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x^2*(a + b*x)^(1/2))/(5*(a - c)) - (2*x^2*(c + b*x)^(1/2))/(5*(a - c)) -
 (4*a^2*(a + b*x)^(1/2))/(15*b^2*(a - c)) + (4*c^2*(c + b*x)^(1/2))/(15*b^2
*(a - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(a - c)) - (2*c*x*(c + b*x)^(1/2)
)/(15*b*(a - c))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

3.403 $\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(bx+c)^{3/2}}{3b(a-c)}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {6689}

$$\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(bx+c)^{3/2}}{3b(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
[Out] (2*(a + b*x)^(3/2))/(3*b*(a - c)) - (2*(c + b*x)^(3/2))/(3*b*(a - c))
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx &= \frac{\int\left(\sqrt{a+bx}-\sqrt{c+bx}\right)dx}{a-c}\\ &= \frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(c+bx)^{3/2}}{3b(a-c)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 35, normalized size = 0.74

$$\frac{2\left((a+bx)^{3/2}-(bx+c)^{3/2}\right)}{3b(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
[Out] (2*((a + b*x)^(3/2) - (c + b*x)^(3/2)))/(3*b*(a - c))
```

fricas [A] time = 0.44, size = 29, normalized size = 0.62

$$\frac{2\left((bx+a)^{\frac{3}{2}}-(bx+c)^{\frac{3}{2}}\right)}{3\,(ab-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
[Out] 2/3*((b*x + a)^(3/2) - (b*x + c)^(3/2))/(a*b - b*c)
```

giac [A] time = 0.21, size = 75, normalized size = 1.60

$$-\frac{2}{3}\sqrt{bx+c}\left(\frac{(bx+a)b}{ab^2-b^2c}-\frac{ab-bc}{ab^2-b^2c}\right)+\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(ab-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*x + c)*((b*x + a)*b/(a*b^2 - b^2*c) - (a*b - b*c)/(a*b^2 - b^2*
c)) + 2/3*(b*x + a)^(3/2)/(a*b - b*c)
```

maple [A] time = 0.00, size = 40, normalized size = 0.85

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(a-c)\,b}-\frac{2\,(bx+c)^{\frac{3}{2}}}{3\,(a-c)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

mupad [B] time = 2.71, size = 79, normalized size = 1.68

$$\frac{2\,x\,\sqrt{a+b\,x}}{3\,(a-c)}-\frac{2\,x\,\sqrt{c+b\,x}}{3\,(a-c)}+\frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,(a-c)}-\frac{2\,c\,\sqrt{c+b\,x}}{3\,b\,(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x*(a + b*x)^(1/2))/(3*(a - c)) - (2*x*(c + b*x)^(1/2))/(3*(a - c)) + (2*
a*(a + b*x)^(1/2))/(3*b*(a - c)) - (2*c*(c + b*x)^(1/2))/(3*b*(a - c))
```

sympy [A] time = 0.71, size = 136, normalized size = 2.89

$$\begin{cases} \frac{2a}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{4bx}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{2c}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} & \text{for}\: b\neq 0 \\ \frac{x}{\sqrt{a}+\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*a/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 4*b*x/(3*b*sqrt(a
+ b*x) + 3*b*sqrt(b*x + c)) + 2*c/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) +
 2*sqrt(a + b*x)*sqrt(b*x + c)/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)), Ne(
b, 0)), (x/(sqrt(a) + sqrt(c)), True))
```

3.404 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(a-c)+2*arctanh((b*x+c)^(1/2)/c^(
1/2))*c^(1/2)/(a-c)+2*(b*x+a)^(1/2)/(a-c)-2*(b*x+c)^(1/2)/(a-c)
```

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2104, 50, 63, 208}

$$\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*Sqrt[a + b*x])/(a - c) - (2*Sqrt[c + b*x])/(a - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(a - c) + (2*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]
)/(a - c)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = -\frac{b\int\frac{\sqrt{a+bx}}{x}\,dx}{-ab+bc}+\frac{b\int\frac{\sqrt{c+bx}}{x}\,dx}{-ab+bc}$$
$$= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}+\frac{a\int\frac{1}{x\sqrt{a+bx}}\,dx}{a-c}-\frac{c\int\frac{1}{x\sqrt{c+bx}}\,dx}{a-c}$$
$$= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}+\frac{(2a)\,\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b(a-c)}-\frac{(2c)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{a-c}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.77

$$\frac{2\left(\sqrt{a+bx}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\sqrt{bx+c}+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)}{a-c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*(Sqrt[a + b*x] - Sqrt[c + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
+ Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]))/(a - c)
```

fricas [A] time = 0.52, size = 318, normalized size = 3.28

$$\left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c},-\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(c)*log((b*x
- 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a
 - c), -(2*sqrt(-c)*arctan(sqrt(b*x + c)*sqrt(-c)/c) + sqrt(a)*log((b*x + 2
*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a -
c), (2*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(c)*log((b*x - 2*sqr
t(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a) - 2*sqrt(b*x + c))/(a - c),
2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-c)*arctan(sqrt(b*x + c
)*sqrt(-c)/c) + sqrt(b*x + a) - sqrt(b*x + c))/(a - c)]
```

giac [B] time = 0.86, size = 1016, normalized size = 10.47

$$\frac{2a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}(a-c)}-\frac{2\left(a^4c-a^3c^2-a^2c^3+ac^4+2\left(ac^2+\sqrt{ac}\,c^2\right)(a-c)^2\text{sgn}(-a+c)-2\left(ac^2+\sqrt{ac}\,ac\right)(a-c)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) - 2*(a^4*c - a^3*c^2
- a^2*c^3 + a*c^4 + 2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*sgn(-a + c) - 2*(a*
c^2 + sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c - 2*a*c^
2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) - (a^3*c - 2*a^2*c^2 + a*c^3 +
(a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) - (a^4*c - a^3*c^2 - a^2*c^3
 + a*c^4 + (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*sgn(-a + c) + (a^4 -
a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))
/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a
 - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) - 4*sqrt(-a)*a^3*c + 4*a^3*
sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-c)*c^2 - 4*sqrt(-a)*a*c^3 + 4
*a*sqrt(-c)*c^3 + sqrt(-a)*c^4 - sqrt(-c)*c^4)*abs(-a + c)) + 2*(a^4*c - a^
3*c^2 - a^2*c^3 + a*c^4 - 2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*sgn(-a + c) -
 2*(a*c^2 - sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c -
2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) + (a^3*c - 2*a^2*c^2 + a*
c^3 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) + (a^4*c - a^3*c^2 - a
^2*c^3 + a*c^4 - (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*sgn(-a + c) + (
a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*x
 + c))/sqrt(-(a^2 - c^2 - sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c
^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) - 4*sqrt(-a)*a^3*c +
4*a^3*sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-c)*c^2 - 4*sqrt(-a)*a*c
^3 + 4*a*sqrt(-c)*c^3 + sqrt(-a)*c^4 - sqrt(-c)*c^4)*abs(-a + c)) + 2*sqrt(
b*x + a)/(a - c) - 2*sqrt(b*x + c)/(a - c)
```

maple [A] time = 0.01, size = 73, normalized size = 0.75

$$\frac{-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}}{a-c}-\frac{-2\sqrt{c}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+2\sqrt{bx+c}}{a-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 1/(a-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(a-c)*
(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

mupad [B] time = 18.08, size = 2983, normalized size = 30.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)
```

```
[Out] (atan((a^2*c^(5/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^
3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/
2)*2i + a^(5/2)*c^2*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^
(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 +
 a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3
*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c -
 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^
```

(1/2)*4i - a^(3/2)*c^(5/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2
)*2i - a^(5/2)*c^(3/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i
)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2) + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2
)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/
2) - 2*a^4*c^2*(a + b*x)^(1/2) - 2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5
/2)*c^(7/2)*(a + b*x)^(1/2) + 2*a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)
c^(3/2)(a + b*x)^(1/2) + 2*a^2*c^4*(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^
(1/2) + 2*a^4*c^2*(c + b*x)^(1/2)))*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*
c^2)^(1/2)*2i - 4*a^(3/2)*c - 8*a*c^(3/2) + atan((a^2*c^(5/2)*(a*c^3 + a^3*
c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i
- a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c^3 + a^3
*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2
)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a
^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/
2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(
c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/2)*(c +
b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(c + b*x)
^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2)
+ 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^
4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)^(1/2) -
2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1/2) + 2*
a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2) + 2*a^2
c^4(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b*x)^(1/2
)))*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + 4*a^(3/2)*c^(1/2
)*(c + b*x)^(1/2) - 3*a*c^(3/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^
(1/2) - c^(1/2))) - 3*a^(3/2)*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^
(1/2) - c^(1/2))) + 8*a*c*(c + b*x)^(1/2) - c^(1/2)*atan((a^2*c^(5/2)*(a*c^
3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(
1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c
^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2
*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2
)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i
+ a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a
^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/
2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(
c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4
*c^(5/2) + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 -
2*a^2*c^4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)
^(1/2) - 2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1
/2) + 2*a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2)
+ 2*a^2*c^4*(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b
x)^(1/2)))(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^(1/2)*log(((a + b*
x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2))) + a^(3/2)*c^(1/2)*log(((a
+ b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a + b*x)^(1/2) + a^(3
/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(c
+ b*x)^(1/2) + 2*a*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^
(1/2)))*(a + b*x)^(1/2) + 2*a*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^
(1/2) - c^(1/2)))*(c + b*x)^(1/2))/(a^(1/2)*c^(1/2)*(a^(1/2) - c^(1/2))*(a^
(1/2) + c^(1/2))^2*((a + b*x)^(1/2) + (c + b*x)^(1/2) - a^(1/2) - c^(1/2)))
- (c^2*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2))) - 4*c^
(3/2)*(c + b*x)^(1/2) + 4*c^2 + atan((a^2*c^(5/2)*(a*c^3 + a^3*c - 2*a^2*c^
2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(7/2)*c*
(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c^3 + a^3*c - 2*a^2*c
^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i +
a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(3/2)*c^(5/
2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^(3/2)*(
a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(c + b*x)^(1/
2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/2)*(c + b*x)^(1/2)*(
a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(c + b*x)^(1/2)*(a*c^

```
3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2) + 2*a^(5/2)
*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^4*(a + b*x)^
(1/2) + 4*a^3*c^3*(a + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)^(1/2) - 2*a^(3/2)*c
^(9/2)*(a + b*x)^(1/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1/2) + 2*a^(7/2)*c^(5
/2)*(a + b*x)^(1/2) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2) + 2*a^2*c^4*(c + b*
x)^(1/2) - 4*a^3*c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b*x)^(1/2)))*(a*c^3 +
 a^3*c - 2*a^2*c^2)^(1/2)*2i - c^(3/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c
+ b*x)^(1/2) - c^(1/2)))*(a + b*x)^(1/2) - c^(3/2)*log(((a + b*x)^(1/2) - a
^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(c + b*x)^(1/2))/(c^(1/2)*(a^(1/2) - c
^(1/2))*(a^(1/2) + c^(1/2))^2*((a + b*x)^(1/2) + (c + b*x)^(1/2) - a^(1/2)
- c^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

3.405 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{a+bx}}{x(a-c)}+\frac{\sqrt{bx+c}}{x(a-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}\,(a-c)}$$

```
[Out] -b*arctanh((b*x+a)^(1/2)/a^(1/2))/(a-c)/a^(1/2)+b*arctanh((b*x+c)^(1/2)/c^(
1/2))/(a-c)/c^(1/2)-(b*x+a)^(1/2)/(a-c)/x+(b*x+c)^(1/2)/(a-c)/x
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2104, 47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x(a-c)}+\frac{\sqrt{bx+c}}{x(a-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}\,(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((a - c)*x)) + Sqrt[c + b*x]/((a - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)) + (b*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(
(a - c)*Sqrt[c])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = -\frac{b\int \frac{\sqrt{a+bx}}{x^2}\,dx}{-ab+bc}+\frac{b\int \frac{\sqrt{c+bx}}{x^2}\,dx}{-ab+bc}$$
$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}+\frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(a-c)}-\frac{b\int \frac{1}{x\sqrt{c+bx}}\,dx}{2(a-c)}$$
$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a-c}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{b}+\frac{x^2}{b}}\right.}{a\cdot}$$
$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)\sqrt{c}}$$

Mathematica [**A**] time = 0.27, size = 99, normalized size = 0.96

$$\frac{\frac{bx\sqrt{\frac{bx}{c}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{c}+1}\right)+bx+c}{\sqrt{bx+c}}-\frac{bx\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{\sqrt{a+bx}}}{x(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (-((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[a + b*
x]) + (c + b*x + b*x*Sqrt[1 + (b*x)/c]*ArcTanh[Sqrt[1 + (b*x)/c]])/Sqrt[c +
 b*x])/((a - c)*x)
```

fricas [**A**] time = 0.51, size = 399, normalized size = 3.87

$$\left[-\frac{\sqrt{a}\,bcx\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+ab\sqrt{c}\,x\log\left(\frac{bx-2\,\sqrt{bx+c}\,\sqrt{c}+2\,c}{x}\right)+2\,\sqrt{bx+a}\,ac-2\,\sqrt{bx+c}\,ac}{2\left(a^2c-ac^2\right)x},-\frac{2\,ab\sqrt{-c}\,x\,\text{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + a*b*sqr
t(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a)*a*c -
 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x), -1/2*(2*a*b*sqrt(-c)*x*arctan(sq
rt(b*x + c)*sqrt(-c)/c) + sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a)
+ 2*a)/x) + 2*sqrt(b*x + a)*a*c - 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x),
 1/2*(2*sqrt(-a)*b*c*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - a*b*sqrt(c)*x*log
((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a)*a*c + 2*sqrt(b*
x + c)*a*c)/((a^2*c - a*c^2)*x), (sqrt(-a)*b*c*x*arctan(sqrt(b*x + a)*sqrt(
-a)/a) - a*b*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) - sqrt(b*x + a)*a*
c + sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x)]
```

giac [**B**] time = 10.21, size = 1190, normalized size = 11.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) + (2*(a*c^2 + sqrt(a*c)
*c^2)*(a - c)^2*b*sgn(2*a - 2*c) + 2*(a*c^2 + sqrt(a*c)*a*c)*(a - c)^2*b +
(a^2*c^2 - 2*a*c^3 + c^4 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c)*
sgn(2*a - 2*c) + (a^3*c - 2*a^2*c^2 + a*c^3 + (a^2*c - 2*a*c^2 + c^3)*sqrt(
a*c))*b*abs(a - c) - (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 + (a^3*c - a^2*c^2
- a*c^3 + c^4)*sqrt(a*c))*b*sgn(2*a - 2*c) - (a^4*c - a^3*c^2 - a^2*c^3 + a
*c^4 + (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a)
 - sqrt(b*x + c))/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c +
3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4*c - a^4*sqrt(-c)*c - 4*sq
rt(-a)*a^3*c^2 + 4*a^3*sqrt(-c)*c^2 + 6*sqrt(-a)*a^2*c^3 - 6*a^2*sqrt(-c)*c
^3 - 4*sqrt(-a)*a*c^4 + 4*a*sqrt(-c)*c^4 + sqrt(-a)*c^5 - sqrt(-c)*c^5)*abs
(a - c)) - (2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*b*sgn(2*a - 2*c) - 2*(a*c^2
 + sqrt(a*c)*a*c)*(a - c)^2*b + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c - 2*a*c^2
 + c^3)*sqrt(a*c))*b*abs(a - c)*sgn(2*a - 2*c) - (a^3*c - 2*a^2*c^2 + a*c^3
 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c) - (a^4*c - a^3*c^2 - a^2
*c^3 + a*c^4 - (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*b*sgn(2*a - 2*c)
+ (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 - (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt
(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a^2 - c^2 - sqrt((
a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-
a)*a^4*c - a^4*sqrt(-c)*c - 4*sqrt(-a)*a^3*c^2 + 4*a^3*sqrt(-c)*c^2 + 6*sqr
t(-a)*a^2*c^3 - 6*a^2*sqrt(-c)*c^3 - 4*sqrt(-a)*a*c^4 + 4*a*sqrt(-c)*c^4 +
sqrt(-a)*c^5 - sqrt(-c)*c^5)*abs(a - c)) - 2*(b*(sqrt(b*x + a) - sqrt(b*x +
 c))^3 - a*b*(sqrt(b*x + a) - sqrt(b*x + c)) + b*c*(sqrt(b*x + a) - sqrt(b*
x + c)))/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqrt(b*x + a) - sqrt(b*
x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2 - 2*a*c + c^2)*(a -
 c)) - sqrt(b*x + a)/((a - c)*x)
```

maple [A] time = 0.02, size = 88, normalized size = 0.85

$$\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)b}{a-c}-\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\sqrt{bx+c}}{2bx}\right)b}{a-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-2/(a-c)*b*(-1/2*(b*x+c)^(1/2)/b/x-1/2/c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1
/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

mupad [B] time = 18.88, size = 2642, normalized size = 25.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)
```

```
[Out] (b*atan(((b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(
1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2) -
a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c
^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a
^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 -
2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(
7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a
^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c^4
 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4
*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(
1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)
*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/
2) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a
*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((
a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2
*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5
*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1
/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2
)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5
 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*
c^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*
(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/
2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a
^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2
 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))/(((a^(3/2)*b^2*c^(7/2))/2 - a^(5/2)
*b^2*c^(5/2) + (a^(7/2)*b^2*c^(3/2))/2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (
((a + b*x)^(1/2) - a^(1/2))*(a^(3/2)*b^2*c^(7/2) - 2*a^(5/2)*b^2*c^(5/2) +
a^(7/2)*b^2*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a
^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c
^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2)
- a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5
*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2
*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5
- 2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a
^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 +
 a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c
^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a
^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a
^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/
2)*c^(7/2) - a^(7/2)*c^(3/2)))))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2
) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*
c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a
^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*
a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*
b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/
2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)
*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5
- 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c
^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(
a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2
))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^
3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)))))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a
^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)))))*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3
/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))
^(1/2)*1i)/(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)) - ((
a^(1/2)*b)/(4*(a*c - a^2)) - (b*c^(1/2))/(4*(a*c - c^2)) - (((a^(1/2)*((a^2
*b)/4 - (b*c^2)/4 + (a*b*c)/4))/(a^3*c - a^2*c^2) - (c^(1/2)*((b*c^2)/4 - (
a^2*b)/4 + (a*b*c)/4))/(a*c^3 - a^2*c^2))*((a + b*x)^(1/2) - a^(1/2))^2)/((
c + b*x)^(1/2) - c^(1/2))^2 + (((a^(1/2)*((a*b)/4 - (3*b*c)/4))/(a*c^2 - a^
```

```
2*c) - (c^(1/2)*((3*a*b)/4 - (b*c)/4))/(a*c^2 - a^2*c))*((a + b*x)^(1/2) -
a^(1/2)))/((c + b*x)^(1/2) - c^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b
*x)^(1/2) - c^(1/2)) + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(
1/2))^3 - ((a + c)*((a + b*x)^(1/2) - a^(1/2))^2)/(a^(1/2)*c^(1/2)*((c + b*
x)^(1/2) - c^(1/2))^2)) - log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2)
- c^(1/2)))*(b/(2*a^(1/2)*c) - (b*(a^(1/2) + c^(1/2)))/(2*c*(a - c))) - (b*
((a + b*x)^(1/2) - a^(1/2)))/(4*a^(1/2)*c^(1/2)*(a^(1/2) - c^(1/2))*((c + b
*x)^(1/2) - c^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(\sqrt{a+bx} + \sqrt{bx+c}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

3.406 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2} \, dx$

Optimal. Leaf size=228

$$\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2}-\frac{\left(4ac-5(a+c)^2\right)\sqrt{a+bx}\sqrt{bx+c}}{32b^3(a-c)}-\frac{(4ac}{}$$

```
[Out] 1/3*(a+c)*x^3/(a-c)^2+1/2*b*x^4/(a-c)^2+5/12*(a+c)*(b*x+a)^(3/2)*(b*x+c)^(3
/2)/b^3/(a-c)^2-1/2*x*(b*x+a)^(3/2)*(b*x+c)^(3/2)/b^2/(a-c)^2-1/32*(4*a*c-5
*(a+c)^2)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b^3+1/16*(4*a*c-5*(a+c)^2)*(
b*x+a)^(3/2)*(b*x+c)^(1/2)/b^3/(a-c)^2-1/32*(4*a*c-5*(a+c)^2)*(b*x+a)^(1/2)
*(b*x+c)^(1/2)/b^3/(a-c)
```

Rubi [A] time = 0.37, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6689, 90, 80, 50, 63, 217, 206}

$$-\frac{x(a+bx)^{3/2}(bx+c)^{3/2}}{2b^2(a-c)^2}+\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2}-\frac{\left(4ac-5(a+c)\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((a + c)*x^3)/(3*(a - c)^2) + (b*x^4)/(2*(a - c)^2) - ((4*a*c - 5*(a + c)^2
)*Sqrt[a + b*x]*Sqrt[c + b*x])/(32*b^3*(a - c)) + ((4*a*c - 5*(a + c)^2)*(a
 + b*x)^(3/2)*Sqrt[c + b*x])/(16*b^3*(a - c)^2) + (5*(a + c)*(a + b*x)^(3/2
)*(c + b*x)^(3/2))/(12*b^3*(a - c)^2) - (x*(a + b*x)^(3/2)*(c + b*x)^(3/2))
/(2*b^2*(a - c)^2) - ((4*a*c - 5*(a + c)^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[c +
b*x]])/(32*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(a\left(1+\frac{c}{a}\right)x^2+2bx^3-2x^2\sqrt{a+bx}\,\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{2\int x^2\sqrt{a+bx}\,\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2}-\frac{\int \sqrt{a+bx}\,\sqrt{c+bx}\,\left(-ac-\frac{5}{2}b(a\right.}{2b^2(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2}-\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2}+$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2}+\frac{5(a+c)(a+bx)^{3}}{12b^3(a-}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)}{16b^3(}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)}{16b^3(}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)}{16b^3(}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)}{16b^3(}$$

Mathematica [A] time = 1.29, size = 361, normalized size = 1.58

$$-15a^3\sqrt{a+bx}\sqrt{bx+c}+\frac{3\sqrt{b}(c-a)^3\left(5a^2+6ac+5c^2\right)\sqrt{-\frac{bx+c}{a-c}}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)}{\sqrt{b(c-a)}\sqrt{bx+c}}+a^2\sqrt{a+bx}\sqrt{bx+c}(10bx+7c)-16b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (48*b^4*x^4 - 15*a^3*Sqrt[a + b*x]*Sqrt[c + b*x] - 15*c^3*Sqrt[a + b*x]*Sqr
t[c + b*x] + 10*b*c^2*x*Sqrt[a + b*x]*Sqrt[c + b*x] - 8*b^2*c*x^2*Sqrt[a +
b*x]*Sqrt[c + b*x] + a^2*Sqrt[a + b*x]*Sqrt[c + b*x]*(7*c + 10*b*x) - 16*b^
3*x^3*(-2*c + 3*Sqrt[a + b*x]*Sqrt[c + b*x]) - a*(-32*b^3*x^3 - 7*c^2*Sqrt[
a + b*x]*Sqrt[c + b*x] + 4*b*c*x*Sqrt[a + b*x]*Sqrt[c + b*x] + 8*b^2*x^2*Sq
rt[a + b*x]*Sqrt[c + b*x]) + (3*Sqrt[b]*(-a + c)^3*(5*a^2 + 6*a*c + 5*c^2)*
Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a + c)]
])/(Sqrt[b*(-a + c)]*Sqrt[c + b*x]))/(96*b^3*(a - c)^2)
```

fricas [A] time = 0.46, size = 196, normalized size = 0.86

$$\frac{96\,b^4x^4+64\left(ab^3+b^3c\right)x^3-2\left(48\,b^3x^3+15\,a^3-7\,a^2c-7\,ac^2+15\,c^3+8\left(ab^2+b^2c\right)x^2-2\left(5\,a^2b-2\,abc+5\right.\right.}{192\left(a^2b^3-2\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/192*(96*b^4*x^4 + 64*(a*b^3 + b^3*c)*x^3 - 2*(48*b^3*x^3 + 15*a^3 - 7*a^2
*c - 7*a*c^2 + 15*c^3 + 8*(a*b^2 + b^2*c)*x^2 - 2*(5*a^2*b - 2*a*b*c + 5*b*
c^2)*x)*sqrt(b*x + a)*sqrt(b*x + c) - 3*(5*a^4 - 4*a^3*c - 2*a^2*c^2 - 4*a*
c^3 + 5*c^4)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^3
- 2*a*b^3*c + b^3*c^2)
```

giac [B] time = 0.39, size = 797, normalized size = 3.50

$$-\frac{1}{96}\left(2\left(4\,(bx+a)\left(\frac{6\left(a^5b^9-5\,a^4b^9c+10\,a^3b^9c^2-10\,a^2b^9c^3+5\,ab^9c^4-b^9c^5\right)(bx+a)}{a^7b^{12}-7\,a^6b^{12}c+21\,a^5b^{12}c^2-35\,a^4b^{12}c^3+35\,a^3b^{12}c^4-21\,a^2b^{12}c^5+7\,ab^{12}c^6-b^{12}c^7}-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/96*(2*(4*(b*x + a)*(6*(a^5*b^9 - 5*a^4*b^9*c + 10*a^3*b^9*c^2 - 10*a^2*b
^9*c^3 + 5*a*b^9*c^4 - b^9*c^5)*(b*x + a)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5
*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*
c^6 - b^12*c^7) - (17*a^6*b^9 - 86*a^5*b^9*c + 175*a^4*b^9*c^2 - 180*a^3*b^
9*c^3 + 95*a^2*b^9*c^4 - 22*a*b^9*c^5 + b^9*c^6)/(a^7*b^12 - 7*a^6*b^12*c +
 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*
a*b^12*c^6 - b^12*c^7)) + (59*a^7*b^9 - 301*a^6*b^9*c + 615*a^5*b^9*c^2 - 6
25*a^4*b^9*c^3 + 305*a^3*b^9*c^4 - 39*a^2*b^9*c^5 - 19*a*b^9*c^6 + 5*b^9*c^
7)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^
12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*c^6 - b^12*c^7))*(b*x + a) - 3*(5*a^8*b
^9 - 24*a^7*b^9*c + 44*a^6*b^9*c^2 - 40*a^5*b^9*c^3 + 30*a^4*b^9*c^4 - 40*a
^3*b^9*c^5 + 44*a^2*b^9*c^6 - 24*a*b^9*c^7 + 5*b^9*c^8)/(a^7*b^12 - 7*a^6*b
^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c
^5 + 7*a*b^12*c^6 - b^12*c^7))*sqrt(b*x + a)*sqrt(b*x + c) + 1/6*(3*(b*x +
a)^4 - 10*(b*x + a)^3*a + 12*(b*x + a)^2*a^2 - 6*(b*x + a)*a^3 + 2*(b*x + a
)^3*c - 6*(b*x + a)^2*a*c + 6*(b*x + a)*a^2*c)/(a^2*b^3 - 2*a*b^3*c + b^3*c
```

```
^2) - 1/32*(5*a^2 + 6*a*c + 5*c^2)*log(abs(-sqrt(b*x + a) + sqrt(b*x + c)))
/b^3
```

maple [C] time = 0.02, size = 604, normalized size = 2.65

$$\frac{b\,x^4}{2\,(a-c)^2}+\frac{a\,x^3}{3\,(a-c)^2}+\frac{c\,x^3}{3\,(a-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(96\sqrt{b^2x^2+abx+bcx+ac}\,b^3x^3\mathrm{csgn}(b)+16\sqrt{b^2x^2+abx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/3*x^3/(a-c)^2*a+1/3*x^3/(a-c)^2*c+1/2*b*x^4/(a-c)^2-1/192/(a-c)^2*(b*x+a)
^(1/2)*(b*x+c)^(1/2)*(96*csgn(b)*x^3*b^3*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16
*csgn(b)*x^2*a*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16*csgn(b)*x^2*b^2*c*(b^
2*x^2+a*b*x+b*c*x+a*c)^(1/2)-20*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a
^2*b+8*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a*b*c-20*(b^2*x^2+a*b*x+b*
c*x+a*c)^(1/2)*csgn(b)*x*b*c^2+30*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a
^3-14*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a^2*c-14*(b^2*x^2+a*b*x+b*c*x
+a*c)^(1/2)*csgn(b)*a*c^2+30*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*c^3-15
*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))*a^4+
12*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))*a^
3*c+6*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))
*a^2*c^2+12*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*cs
gn(b))*a*c^3-15*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c
)*csgn(b))*c^4)*csgn(b)/b^3/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

mupad [B] time = 81.17, size = 1358, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

```
[Out] (x^3*(a + c))/(3*(a - c)^2) - ((((a + b*x)^(1/2) - a^(1/2))^15*((3*a*c)/8 +
 (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2) - c^(1/2))^15) + (((a + b*
x)^(1/2) - a^(1/2))^3*((23*a*c^3)/12 + (23*a^3*c)/12 - (115*a^4)/48 - (115*
c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^3 + b^3*c
^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^13*((23*a*c^3)/12 + (23*a^3
*c)/12 - (115*a^4)/48 - (115*c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^(1/2)
- c^(1/2))^13*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2
))^5*((3917*a*c^3)/12 + (3917*a^3*c)/12 + (383*a^4)/48 + (383*c^4)/48 + (72
79*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^3 + b^3*c^2 - 2*a*b^3
*c)) + (((a + b*x)^(1/2) - a^(1/2))^11*((3917*a*c^3)/12 + (3917*a^3*c)/12 +
 (383*a^4)/48 + (383*c^4)/48 + (7279*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/
2))^11*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^7*((
17567*a*c^3)/12 + (17567*a^3*c)/12 + (2789*a^4)/48 + (2789*c^4)/48 + (28213
*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c
```

```
)) + (((a + b*x)^(1/2) - a^(1/2))^9*((17567*a*c^3)/12 + (17567*a^3*c)/12 +
(2789*a^4)/48 + (2789*c^4)/48 + (28213*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(
1/2))^9*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))*((3
*a*c)/8 + (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2) - c^(1/2))) - (a^
(1/2)*c^(1/2)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) - a^(1/2))^4)/(((c +
 b*x)^(1/2) - c^(1/2))^4*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2)*c^(1/2
)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) - a^(1/2))^12)/(((c + b*x)^(1/2)
 - c^(1/2))^12*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2)*c^(1/2)*((a + b*
x)^(1/2) - a^(1/2))^6*((5120*a*c^2)/3 + (5120*a^2*c)/3 + 256*a^3 + 256*c^3)
)/(((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2
)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^10*((5120*a*c^2)/3 + (5120*a^2*c)/3 +
 256*a^3 + 256*c^3))/(((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b^3 + b^3*c^2 - 2
*a*b^3*c)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^8*((10112*a*c^2)/
3 + (10112*a^2*c)/3 + 512*a^3 + 512*c^3))/(((c + b*x)^(1/2) - c^(1/2))^8*(a
^2*b^3 + b^3*c^2 - 2*a*b^3*c)))/((28*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b
*x)^(1/2) - c^(1/2))^4 - (8*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2)
 - c^(1/2))^2 - (56*((a + b*x)^(1/2) - a^(1/2))^6)/((c + b*x)^(1/2) - c^(1/
2))^6 + (70*((a + b*x)^(1/2) - a^(1/2))^8)/((c + b*x)^(1/2) - c^(1/2))^8 -
(56*((a + b*x)^(1/2) - a^(1/2))^10)/((c + b*x)^(1/2) - c^(1/2))^10 + (28*((
a + b*x)^(1/2) - a^(1/2))^12)/((c + b*x)^(1/2) - c^(1/2))^12 - (8*((a + b*x
)^(1/2) - a^(1/2))^14)/((c + b*x)^(1/2) - c^(1/2))^14 + ((a + b*x)^(1/2) -
a^(1/2))^16/((c + b*x)^(1/2) - c^(1/2))^16 + 1) + (b*x^4)/(2*(a - c)^2) + (
atanh(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(6*a*c + 5*a
^2 + 5*c^2))/(16*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

3.407 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=165

$$-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{bx+c}}{4b^2(a-c)}-\frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}+\frac{2bx^3}{3(a-c)^2}$$

```
[Out] 1/2*(a+c)*x^2/(a-c)^2+2/3*b*x^3/(a-c)^2-2/3*(b*x+a)^(3/2)*(b*x+c)^(3/2)/b^2
/(a-c)^2-1/4*(a+c)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b^2+1/2*(a+c)*(b*x+
a)^(3/2)*(b*x+c)^(1/2)/b^2/(a-c)^2-1/4*(a+c)*(b*x+a)^(1/2)*(b*x+c)^(1/2)/b^
2/(a-c)
```

Rubi [A] time = 0.21, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6689, 80, 50, 63, 217, 206}

$$-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{bx+c}}{4b^2(a-c)}-\frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}+\frac{2bx^3}{3(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((a + c)*x^2)/(2*(a - c)^2) + (2*b*x^3)/(3*(a - c)^2) - ((a + c)*Sqrt[a + b
*x]*Sqrt[c + b*x])/(4*b^2*(a - c)) + ((a + c)*(a + b*x)^(3/2)*Sqrt[c + b*x]
)/(2*b^2*(a - c)^2) - (2*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(3*b^2*(a - c)^2)
 - ((a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(4*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\frac{\int \left(a\left(1+\frac{c}{a}\right)x+2bx^2-2x\sqrt{a+bx}\,\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$=\frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{2\int x\sqrt{a+bx}\,\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$=\frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)\int \sqrt{a+bx}\,\sqrt{c+bx}\,d}{b(a-c)^2}$$

$$=\frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-\frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-$$

$$=\frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}$$

$$=\frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}$$

$$=\frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}$$

$$=\frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}$$

Mathematica [A] time = 0.88, size = 229, normalized size = 1.39

$$\frac{3a^2\sqrt{a+bx}\,\sqrt{bx+c}-2a\left(bx\sqrt{a+bx}\,\sqrt{bx+c}+c\sqrt{a+bx}\,\sqrt{bx+c}-3b^2x^2\right)+(4bx+3c)\left(-2bx\sqrt{a+bx}\,\sqrt{bx\right.}{12b^2(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (3*a^2*Sqrt[a + b*x]*Sqrt[c + b*x] + (3*c + 4*b*x)*(2*b^2*x^2 + c*Sqrt[a +
b*x]*Sqrt[c + b*x] - 2*b*x*Sqrt[a + b*x]*Sqrt[c + b*x]) - 2*a*(-3*b^2*x^2 +
 c*Sqrt[a + b*x]*Sqrt[c + b*x] + b*x*Sqrt[a + b*x]*Sqrt[c + b*x]))/(12*b^2*
(a - c)^2) - (Sqrt[b*(-a + c)]*(a + c)*Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(
Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a + c)]])/(4*b^(5/2)*Sqrt[c + b*x])
```

fricas [A] time = 0.45, size = 149, normalized size = 0.90

$$\frac{16 b^3x^3 + 12 \left(ab^2 + b^2c\right)x^2 - 2 \left(8 b^2x^2 - 3 a^2 + 2 ac - 3 c^2 + 2 (ab + bc)x\right)\sqrt{bx + a} \sqrt{bx + c} + 3 \left(a^3 - a^2c - ac^2 + c\right.}{24 \left(a^2b^2 - 2 ab^2c + b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*b^3*x^3 + 12*(a*b^2 + b^2*c)*x^2 - 2*(8*b^2*x^2 - 3*a^2 + 2*a*c -
3*c^2 + 2*(a*b + b*c)*x)*sqrt(b*x + a)*sqrt(b*x + c) + 3*(a^3 - a^2*c - a*c
^2 + c^3)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^2 - 2
*a*b^2*c + b^2*c^2)
```

giac [B] time = 0.27, size = 445, normalized size = 2.70

$$-\frac{\left(2 (bx + a)\left(\frac{4 \left(a^3b^2-3 a^2b^2c+3 ab^2c^2-b^2c^3\right)(bx+a)}{a^5b^3-5 a^4b^3c+10 a^3b^3c^2-10 a^2b^3c^3+5 ab^3c^4-b^3c^5} - \frac{7 a^4b^2-22 a^3b^2c+24 a^2b^2c^2-10 ab^2c^3+b^2c^4}{a^5b^3-5 a^4b^3c+10 a^3b^3c^2-10 a^2b^3c^3+5 ab^3c^4-b^3c^5}\right) + \frac{3 \left(a^5b^2-3 a^4b^2c+2 a^3b^2\right.}{a^5b^3-5 a^4b^3c+10 a^3b^3c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/12*((2*(b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x
 + a)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3*c^
4 - b^3*c^5) - (7*a^4*b^2 - 22*a^3*b^2*c + 24*a^2*b^2*c^2 - 10*a*b^2*c^3 +
b^2*c^4)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3
*c^4 - b^3*c^5)) + 3*(a^5*b^2 - 3*a^4*b^2*c + 2*a^3*b^2*c^2 + 2*a^2*b^2*c^3
 - 3*a*b^2*c^4 + b^2*c^5)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*
b^3*c^3 + 5*a*b^3*c^4 - b^3*c^5))*sqrt(b*x + a)*sqrt(b*x + c) - 3*(a + c)*l
og(abs(-sqrt(b*x + a) + sqrt(b*x + c)))/b - 2*(4*(b*x + a)^3 - 9*(b*x + a)^
2*a + 6*(b*x + a)*a^2 + 3*(b*x + a)^2*c - 6*(b*x + a)*a*c)/(a^2*b - 2*a*b*c
 + b*c^2))/b
```

maple [C] time = 0.02, size = 431, normalized size = 2.61

$$\frac{2b x^3}{3 (a - c)^2}+\frac{a x^2}{2 (a - c)^2}+\frac{c x^2}{2 (a - c)^2}-\frac{\sqrt{bx + a} \sqrt{bx + c} \left(16\sqrt{b^2x^2 + abx + bcx + ac}\, b^2x^2\mathrm{csgn}(b) + 3a^3 \ln\left(\frac{\left(2bx+a+c+2\right.}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/2*x^2/(a-c)^2*a+1/2*x^2/(a-c)^2*c+2/3*b*x^3/(a-c)^2-1/24/(a-c)^2*(b*x+a)^
(1/2)*(b*x+c)^(1/2)*(16*csgn(b)*x^2*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+4*(
b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a*b+4*(b^2*x^2+a*b*x+b*c*x+a*c)^(1
/2)*csgn(b)*x*b*c-6*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a^2+4*(b^2*x^2+
a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a*c-6*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b
)*c^2+3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b))*csgn(b
))*a^3-3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b))*csgn(
b))*a^2*c-3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b))*cs
gn(b))*a*c^2+3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b))
*csgn(b))*c^3)*csgn(b)/b^2/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{bx + a} + \sqrt{bx + c}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

mupad [B] time = 37.52, size = 1012, normalized size = 6.13

$$\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)\left(\frac{a}{2}+\frac{c}{2}\right)}{b^2\left(\sqrt{c+b\,x}-\sqrt{c}\right)}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{11}\left(\frac{a}{2}+\frac{c}{2}\right)}{b^2\left(\sqrt{c+b\,x}-\sqrt{c}\right)^{11}}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3\left(\frac{17\,a^3}{6}+\frac{101\,a^2\,c}{2}+\frac{101\,a\,c^2}{2}+\frac{17\,c^3}{6}\right)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^3\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^9\left(\frac{17\,a^3}{6}+\frac{101\,a^2\,c}{2}+\frac{101\,a\,c}{2}\right.}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^9\left(a^2\,b^2-2\,a\,b^2\,c+b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

```
[Out] ((((a + b*x)^(1/2) - a^(1/2))*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^(1/2))
) + (((a + b*x)^(1/2) - a^(1/2))^11*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^
(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*((101*a*c^2)/2 + (101*a^2*c)/2
+ (17*a^3)/6 + (17*c^3)/6))/((((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^2 + b^2*c
^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^9*((101*a*c^2)/2 + (101*a^2
*c)/2 + (17*a^3)/6 + (17*c^3)/6))/((((c + b*x)^(1/2) - c^(1/2))^9*(a^2*b^2 +
 b^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^5*(269*a*c^2 + 269*a^
2*c + 19*a^3 + 19*c^3))/((((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^2 + b^2*c^2 -
 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^7*(269*a*c^2 + 269*a^2*c + 19*a
^3 + 19*c^3))/((((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c
)) + (16*a^(3/2)*c^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2)/((((c + b*x)^(1/2) -
 c^(1/2))^2*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) + (16*a^(3/2)*c^(3/2)*((a + b*
x)^(1/2) - a^(1/2))^10)/((((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b^2 + b^2*c^2
- 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^4*(192*a*c + 6
4*a^2 + 64*c^2))/((((c + b*x)^(1/2) - c^(1/2))^4*(a^2*b^2 + b^2*c^2 - 2*a*b^
2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^8*(192*a*c + 64*a^2 +
64*c^2))/((((c + b*x)^(1/2) - c^(1/2))^8*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) +
(a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^6*((1312*a*c)/3 + 128*a^2 + 12
8*c^2))/((((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)))/((
15*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b*x)^(1/2) - c^(1/2))^4 - (6*((a +
b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 - (20*((a + b*x)^(1/
2) - a^(1/2))^6)/((c + b*x)^(1/2) - c^(1/2))^6 + (15*((a + b*x)^(1/2) - a^(
1/2))^8)/((c + b*x)^(1/2) - c^(1/2))^8 - (6*((a + b*x)^(1/2) - a^(1/2))^10)
/((c + b*x)^(1/2) - c^(1/2))^10 + ((a + b*x)^(1/2) - a^(1/2))^12/((c + b*x)
^(1/2) - c^(1/2))^12 + 1) - (atanh(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(
1/2) - c^(1/2)))*(a + c))/(2*b^2) + (x^2*(a + c))/(2*(a - c)^2) + (2*b*x^3)
/(3*(a - c)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{a + bx} + \sqrt{bx + c}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

3.408 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2} \, dx$

Optimal. Leaf size=63

$$\frac{(a-c)^2}{8b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}$$

```
[Out] 1/2*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b+1/8*(a-c)^2/b/((b*x+a)^(1/2)+(b*
x+c)^(1/2))^4
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.81, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6689, 50, 63, 217, 206}

$$\frac{bx^2}{(a-c)^2}-\frac{(a+bx)^{3/2}\sqrt{bx+c}}{b(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{bx+c}}{2b(a-c)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}+\frac{x(a+c)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] ((a + c)*x)/(a - c)^2 + (b*x^2)/(a - c)^2 + (Sqrt[a + b*x]*Sqrt[c + b*x])/(
2*b*(a - c)) - ((a + b*x)^(3/2)*Sqrt[c + b*x])/(b*(a - c)^2) + ArcTanh[Sqrt
[a + b*x]/Sqrt[c + b*x]]/(2*b)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
```

```
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(a\left(1+\frac{c}{a}\right)+2bx-2\sqrt{a+bx}\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{2\int \sqrt{a+bx}\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\int \frac{\sqrt{a+bx}}{\sqrt{c+bx}}\,dx}{2(a-c)}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{1}{4}\int \frac{}{\sqrt{a+bx}}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\text{Subst}\left(\int \frac{}{\sqrt{-}}\right.}{}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\text{Subst}\left(\int \frac{1}{1-x}\right.}{}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{c+b}}\right.}{2b}$$

Mathematica [B] time = 0.56, size = 179, normalized size = 2.84

$$\frac{2bx\left(bx-\sqrt{a+bx}\sqrt{bx+c}\right)+a\left(2bx-\sqrt{a+bx}\sqrt{bx+c}\right)+c\left(2bx-\sqrt{a+bx}\sqrt{bx+c}\right)+\frac{\sqrt{b}(c-a)^3\sqrt{\frac{bx+c}{c-a}}\sinh^{-1}\left(\right.}{\sqrt{b(c-a)}\sqrt{bx+}}}{2b(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] (2*c^2 + 2*b*x*(b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + a*(2*b*x - Sqrt[a + b*
x]*Sqrt[c + b*x]) + c*(2*b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + (Sqrt[b]*(-a
+ c)^3*Sqrt[(c + b*x)/(-a + c)]*ArcSinh[(Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a
+ c)]])/(Sqrt[b*(-a + c)]*Sqrt[c + b*x]))/(2*b*(a - c)^2)
```

fricas [B] time = 0.46, size = 103, normalized size = 1.63

$$\frac{4\,b^2x^2-2\,(2\,bx+a+c)\sqrt{bx+a}\,\sqrt{bx+c}\,+4\,(ab+bc)x-\left(a^2-2\,ac+c^2\right)\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}\,-a\right.}{4\left(a^2b-2\,abc+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^2 - 2*(2*b*x + a + c)*sqrt(b*x + a)*sqrt(b*x + c) + 4*(a*b + b
*c)*x - (a^2 - 2*a*c + c^2)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a
- c))/(a^2*b - 2*a*b*c + b*c^2)
```

giac [B] time = 0.24, size = 189, normalized size = 3.00

$$-\frac{1}{2}\sqrt{bx+a}\sqrt{bx+c}\left(\frac{2\,(ab-bc)(bx+a)}{a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3}-\frac{a^2b-2\,abc+bc^2}{a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3}\right)+\frac{(bx+a)^2-(bx+a)}{a^2b-2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(b*x + a)*sqrt(b*x + c)*(2*(a*b - b*c)*(b*x + a)/(a^3*b^2 - 3*a^2*
b^2*c + 3*a*b^2*c^2 - b^2*c^3) - (a^2*b - 2*a*b*c + b*c^2)/(a^3*b^2 - 3*a^2
*b^2*c + 3*a*b^2*c^2 - b^2*c^3)) + ((b*x + a)^2 - (b*x + a)*a + (b*x + a)*c
)/(a^2*b - 2*a*b*c + b*c^2) - 1/2*log(abs(-sqrt(b*x + a) + sqrt(b*x + c)))/
b
```

maple [B] time = 0.01, size = 377, normalized size = 5.98

$$\frac{\sqrt{(bx+a)(bx+c)}\, a^2 \ln\left(\frac{b^2x+\frac{1}{2}ab+\frac{1}{2}bc}{\sqrt{b^2}}+\sqrt{b^2x^2+ac+(ab+bc)x}\right)}{4(a-c)^2\sqrt{bx+c}\,\sqrt{bx+a}\,\sqrt{b^2}}-\frac{\sqrt{(bx+a)(bx+c)}\, ac \ln\left(\frac{b^2x+\frac{1}{2}ab+\frac{1}{2}bc}{\sqrt{b^2}}+\sqrt{b^2x^2\right.}{2(a-c)^2\sqrt{bx+c}\,\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] x/(a-c)^2*a+x/(a-c)^2*c+b*x^2/(a-c)^2-1/(a-c)^2/b*(b*x+a)^(1/2)*(b*x+c)^(3/
2)-1/2/(a-c)^2/b*(b*x+c)^(1/2)*(b*x+a)^(1/2)*a+1/2/(a-c)^2/b*(b*x+c)^(1/2)*
(b*x+a)^(1/2)*c+1/4/(a-c)^2*((b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+a)^(
1/2)*ln((1/2*a*b+1/2*b*c+x*b^2)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/2)
)/(b^2)^(1/2)*a^2-1/2/(a-c)^2*((b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+a)
^(1/2)*ln((1/2*a*b+1/2*b*c+x*b^2)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/
2))/(b^2)^(1/2)*a*c+1/4/(a-c)^2*((b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+
a)^(1/2)*ln((1/2*a*b+1/2*b*c+x*b^2)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(
1/2))/(b^2)^(1/2)*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-2), x)
```

mupad [B] time = 0.24, size = 110, normalized size = 1.75

$$\frac{b\,x^2}{{\left(a-c\right)}^2}+\frac{x\,\left(a+c\right)}{{\left(a-c\right)}^2}+\frac{\ln\left(a+c+2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}+2\,b\,x\right)\,{\left(a\,b-b\,c\right)}^2}{4\,b^3\,{\left(a-c\right)}^2}-\frac{2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}\,\left(\frac{x}{2}+\frac{a\,b+b\,c}{4\,b^2}\right)}{{\left(a-c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

```
[Out] (b*x^2)/(a - c)^2 + (x*(a + c))/(a - c)^2 + (log(a + c + 2*(a + b*x)^(1/2)*
(c + b*x)^(1/2) + 2*b*x)*(a*b - b*c)^2)/(4*b^3*(a - c)^2) - (2*(a + b*x)^(1
/2)*(c + b*x)^(1/2)*(x/2 + (a*b + b*c)/(4*b^2)))/(a - c)^2
```

sympy [A] time = 1.04, size = 388, normalized size = 6.16

$$\begin{cases} \frac{2a\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\,\sqrt{bx+c}}+\frac{a}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\,\sqrt{bx+c}}+\frac{4bx\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\,\sqrt{bx+c}}+\frac{2bx}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\,\sqrt{bx+c}} \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*a*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c
 + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + a/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqr
t(a + b*x)*sqrt(b*x + c)) + 4*b*x*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b
 + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*b*x/(4*a*b + 8*b
**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*c*log(sqrt(a + b*x) +
sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c))
 + c/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 4*sqrt(
a + b*x)*sqrt(b*x + c)*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x
 + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(
c))**2, True))
```

3.409 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2} dx$

Optimal. Leaf size=133

$$\frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{bx+c}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

```
[Out] 2*b*x/(a-c)^2-2*(a+c)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/(a-c)^2+(a+c)*ln
(x)/(a-c)^2+4*arctanh(c^(1/2)*(b*x+a)^(1/2)/a^(1/2)/(b*x+c)^(1/2))*a^(1/2)*
c^(1/2)/(a-c)^2-2*(b*x+a)^(1/2)*(b*x+c)^(1/2)/(a-c)^2
```

Rubi [A] time = 0.23, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6689, 101, 157, 63, 217, 206, 93, 208}

$$\frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{bx+c}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (2*b*x)/(a - c)^2 - (2*Sqrt[a + b*x]*Sqrt[c + b*x])/(a - c)^2 - (2*(a + c)*
ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (4*Sqrt[a]*Sqrt[c]*ArcTan
h[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(a - c)^2 + ((a + c)*Lo
g[x])/(a - c)^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(2b+\frac{a\left(1+\frac{c}{a}\right)}{x}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{x}\right)dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{c+bx}}{x}\,dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}+\frac{2\int \frac{-ac-\frac{1}{2}b(a+c)x}{x\sqrt{a+bx}\,\sqrt{c+bx}}\,dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{(2ac)\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+bx}}\,dx}{(a-c)^2}-$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{(4ac)\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x\right.}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2}+\frac{4\sqrt{a}\,\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\,\sqrt{c}\tanh^{-1}}{(a-c)}$$

Mathematica [A] time = 1.07, size = 195, normalized size = 1.47

$$\frac{\sqrt{b}\left(-2\left(c\sqrt{a+bx}+bx\left(\sqrt{a+bx}-\sqrt{bx+c}\right)\right)+(a+c)\log(x)\sqrt{bx+c}+4\sqrt{a}\,\sqrt{c}\,\sqrt{bx+c}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{bx+c}}\right)\right.}{\sqrt{b}\,(a-c)^2\sqrt{bx+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (-2*Sqrt[b*(-a + c)]*(a + c)*Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(Sqrt[b]*Sq
rt[a + b*x])/Sqrt[b*(-a + c)]] + Sqrt[b]*(-2*(c*Sqrt[a + b*x] + b*x*(Sqrt[a
 + b*x] - Sqrt[c + b*x])) + 4*Sqrt[a]*Sqrt[c]*Sqrt[c + b*x]*ArcTanh[(Sqrt[c
]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])] + (a + c)*Sqrt[c + b*x]*Log[x]))/
(Sqrt[b]*(a - c)^2*Sqrt[c + b*x])
```

fricas [A] time = 0.50, size = 290, normalized size = 2.18

$$\left[\frac{2\,bx + (a+c)\log\left(-2\,bx + 2\,\sqrt{bx+a}\,\sqrt{bx+c} - a - c\right) + (a+c)\log(x) + 2\,\sqrt{ac}\,\log\left(\frac{2\,a^2c+2\,ac^2+2\left(2\,ac+\sqrt{ac}\,(a+c)\right)\sqrt{b\cdots}}{\cdots}\right.\cdots}{a^2 - 2\,ac + c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + (a
+ c)*log(x) + 2*sqrt(a*c)*log((2*a^2*c + 2*a*c^2 + 2*(2*a*c + sqrt(a*c)*(a
+ c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c + b*c^2)*x + 2*(2*a*c
+ (a*b + b*c)*x)*sqrt(a*c))/x) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2*a*
c + c^2), (2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a -
 c) + (a + c)*log(x) - 4*sqrt(-a*c)*arctan(-(sqrt(-a*c)*b*x - sqrt(-a*c)*sq
rt(b*x + a)*sqrt(b*x + c))/(a*c)) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2
*a*c + c^2)]
```

giac [A] time = 0.67, size = 194, normalized size = 1.46

$$\frac{4\,ac\arctan\left(\frac{\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a-c}{2\,\sqrt{-ac}}\right)}{\left(a^2-2\,ac+c^2\right)\sqrt{-ac}} - \frac{2\left(a^2-2\,ac+c^2\right)\sqrt{bx+a}\,\sqrt{bx+c}}{a^4-4\,a^3c+6\,a^2c^2-4\,ac^3+c^4} + \frac{(a+c)\log\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2\right)}{a^2-2\,ac+c^2} + \frac{(a+\cdots}{a^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 4*a*c*arctan(1/2*((sqrt(b*x + a) - sqrt(b*x + c))^2 - a - c)/sqrt(-a*c))/((
a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 2*(a^2 - 2*a*c + c^2)*sqrt(b*x + a)*sqrt(b
*x + c)/(a^4 - 4*a^3*c + 6*a^2*c^2 - 4*a*c^3 + c^4) + (a + c)*log((sqrt(b*x
 + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + (a + c)*log(abs(b*x))/(a^2
- 2*a*c + c^2) + 2*(b*x + a)/(a^2 - 2*a*c + c^2)
```

maple [C] time = 0.02, size = 258, normalized size = 1.94

$$\frac{a\ln(x)}{(a-c)^2} + \frac{2bx}{(a-c)^2} + \frac{c\ln(x)}{(a-c)^2} + \frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(2ac\,\mathrm{csgn}(b)\ln\left(\frac{abx+bcx+2ac+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}}{x}\right) - \sqrt{ac}\,a\ln\left(\frac{(2b\cdots}{\cdots}\right.\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/(a-c)^2*a*ln(x)+1/(a-c)^2*c*ln(x)+2*b*x/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(
b*x+c)^(1/2)*(2*csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+
a*c)^(1/2)+2*a*c)/x)*a*c-2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*(a*c)^(1/2)*csgn
(b)-(a*c)^(1/2)*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)
```

```
)*csgn(b))*a-(a*c)^(1/2)*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2
)*csgn(b))*csgn(b))*c)*csgn(b)/(a*c)^(1/2)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

mupad [B] time = 11.14, size = 524, normalized size = 3.94

$$\frac{2bx}{(a-c)^2} - \ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{c+bx}-\sqrt{c}}+1\right)\left(\frac{4c}{(a-c)^2}+\frac{2}{a-c}\right) - \frac{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^3(4a+4c)}{\left(\sqrt{c+bx}-\sqrt{c}\right)^3\left(a^2-2ac+c^2\right)} + \frac{\left(\sqrt{a+bx}-\sqrt{a}\right)(4a+4c)}{\left(\sqrt{c+bx}-\sqrt{c}\right)\left(a^2-2ac+c^2\right)} - \frac{16\sqrt{}}{\left(\sqrt{c+b}\right.}}{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{\left(\sqrt{c+bx}-\sqrt{c}\right)^4} - \frac{2\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{\left(\sqrt{c+bx}-\sqrt{c}\right)^2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

```
[Out] (2*b*x)/(a - c)^2 - log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1
/2)) + 1)*((4*c)/(a - c)^2 + 2/(a - c)) - ((((a + b*x)^(1/2) - a^(1/2))^3*(
4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2 - 2*a*c + c^2)) + (((a + b*
x)^(1/2) - a^(1/2))*(4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))*(a^2 - 2*a*c
+ c^2)) - (16*a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*x)^(1
/2) - c^(1/2))^2*(a^2 - 2*a*c + c^2)))/(((a + b*x)^(1/2) - a^(1/2))^4/((c +
 b*x)^(1/2) - c^(1/2))^4 - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/
2) - c^(1/2))^2 + 1) + (2*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2)
- c^(1/2)) - 1)*(a + c))/(a - c)^2 + (log(x)*(a + c))/(a^2 - 2*a*c + c^2) +
 (2*a^(1/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1
/2))))/(a - c)^2 - (2*a^(1/2)*c^(1/2)*log((a*((a + b*x)^(1/2) - a^(1/2)))/(
(c + b*x)^(1/2) - c^(1/2)) - a^(1/2)*c^(1/2) + (c*((a + b*x)^(1/2) - a^(1/2
)))/((c + b*x)^(1/2) - c^(1/2)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/
2))^2)/((c + b*x)^(1/2) - c^(1/2))^2))/(a^2 - 2*a*c + c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

3.410 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{x(a-c)^2}+\frac{2b\log(x)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{bx+c}}\right)}{\sqrt{a}\,\sqrt{c}\,(a-c)^2}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}-\frac{a+c}{x(a-c)^2}$$

```
[Out] (-a-c)/(a-c)^2/x-4*b*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/(a-c)^2+2*b*ln(x)
/(a-c)^2+2*b*(a+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/a^(1/2)/(b*x+c)^(1/2))/(a-
c)^2/a^(1/2)/c^(1/2)+2*(b*x+a)^(1/2)*(b*x+c)^(1/2)/(a-c)^2/x
```

Rubi [A] time = 0.22, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6689, 97, 157, 63, 217, 206, 93, 208}

$$\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{x(a-c)^2}+\frac{2b\log(x)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{bx+c}}\right)}{\sqrt{a}\,\sqrt{c}\,(a-c)^2}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}-\frac{a+c}{x(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] -((a + c)/((a - c)^2*x)) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/((a - c)^2*x) -
(4*b*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (2*b*(a + c)*ArcTanh
[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(Sqrt[a]*(a - c)^2*Sqrt[
c]) + (2*b*Log[x])/(a - c)^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\frac{\int \left(\frac{a\left(1+\frac{c}{a}\right)}{x^2}+\frac{2b}{x}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{x^2}\right)dx}{(a-c)^2}$$

$$=-\frac{a+c}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{c+bx}}{x^2}\,dx}{(a-c)^2}$$

$$=-\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{2\int \frac{\frac{1}{2}b(a+c)+b^2x}{x\sqrt{a+bx}\,\sqrt{c+bx}}\,dx}{(a-c)^2}$$

$$=-\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{\left(2b^2\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+bx}}\,dx}{(a-c)^2}-\frac{(b}{}$$

$$=-\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+c+x^2}}\,dx,\right.}{(a-c)^2}$$

$$=-\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{c+bx}}\right)}{\sqrt{a}\,(a-c)^2\sqrt{c}}+\frac{2b\log(x}{(a-c)^2}$$

$$=-\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\right.}{\sqrt{a}\,(a-c)^2}$$

Mathematica [A] time = 0.96, size = 205, normalized size = 1.45

$$\frac{\frac{a\left(-\sqrt{bx+c}\right)+2c\sqrt{a+bx}+2bx\sqrt{a+bx}-c\sqrt{bx+c}+2bx\log(x)\sqrt{bx+c}}{x}-4\sqrt{b}\,\sqrt{b(c-a)}\,\sqrt{-\frac{bx+c}{a-c}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)+\frac{2b(a+c)\sqrt{bx+c}\,\mathrm{ta}}{\sqrt{a}}}{(a-c)^2\sqrt{bx+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (-4*Sqrt[b]*Sqrt[b*(-a + c)]*Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(Sqrt[b]*Sq
rt[a + b*x])/Sqrt[b*(-a + c)]] + (2*b*(a + c)*Sqrt[c + b*x]*ArcTanh[(Sqrt[c
]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])]])/(Sqrt[a]*Sqrt[c]) + (2*c*Sqrt[a
+ b*x] + 2*b*x*Sqrt[a + b*x] - a*Sqrt[c + b*x] - c*Sqrt[c + b*x] + 2*b*x*Sq
rt[c + b*x]*Log[x])/x)/((a - c)^2*Sqrt[c + b*x])
```

fricas [A] time = 0.51, size = 367, normalized size = 2.60

$$\left[\frac{2\,abcx\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)+2\,abcx\log\left(x\right)+2\,abcx+\left(ab+bc\right)\sqrt{ac}\,x\log\left(\frac{2\,a^{2}c+2\,ac^{2}+2\left(2\,ac+\cdots\right.}{\cdots}\right)\cdots}{\left(a^{3}c-2\,a^{2}c^{2}+ac^{3}\right)x}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b*c*x*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x
*log(x) + 2*a*b*c*x + (a*b + b*c)*sqrt(a*c)*x*log((2*a^2*c + 2*a*c^2 + 2*(2
*a*c + sqrt(a*c)*(a + c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c +
b*c^2)*x + 2*(2*a*c + (a*b + b*c)*x)*sqrt(a*c))/x) + 2*sqrt(b*x + a)*sqrt(b
*x + c)*a*c - a^2*c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x), (2*a*b*c*x*lo
g(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x*log(x) + 2*a*
b*c*x - 2*(a*b + b*c)*sqrt(-a*c)*x*arctan(-(sqrt(-a*c)*b*x - sqrt(-a*c)*sqr
t(b*x + a)*sqrt(b*x + c))/(a*c)) + 2*sqrt(b*x + a)*sqrt(b*x + c)*a*c - a^2*
c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x)]
```

giac [B] time = 1.89, size = 311, normalized size = 2.21

$$\frac{2\,b\log\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^{2}\right)}{a^{2}-2\,ac+c^{2}}+\frac{2\,b\log\left(|bx|\right)}{a^{2}-2\,ac+c^{2}}+\frac{2\,(ab+bc)\arctan\left(\frac{\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^{2}-a-c}{2\,\sqrt{-ac}}\right)}{\left(a^{2}-2\,ac+c^{2}\right)\sqrt{-ac}}-\frac{4\left(\cdots\right.}{\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*b*log((sqrt(b*x + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + 2*b*log(ab
s(b*x))/(a^2 - 2*a*c + c^2) + 2*(a*b + b*c)*arctan(1/2*((sqrt(b*x + a) - sq
rt(b*x + c))^2 - a - c)/sqrt(-a*c))/((a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 4*(a
*b*(sqrt(b*x + a) - sqrt(b*x + c))^2 + b*c*(sqrt(b*x + a) - sqrt(b*x + c))^
2 - a^2*b + 2*a*b*c - b*c^2)/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqr
t(b*x + a) - sqrt(b*x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2
 - 2*a*c + c^2)*(a^2 - 2*a*c + c^2)) - (2*(b*x + a)*b - a*b + b*c)/((a^2 -
2*a*c + c^2)*b*x)
```

maple [C] time = 0.02, size = 274, normalized size = 1.94

$$\frac{2b\ln(x)}{(a-c)^{2}}-\frac{a}{(a-c)^{2}\,x}-\frac{c}{(a-c)^{2}\,x}+\frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(abx\,\mathrm{csgn}(b)\ln\left(\frac{abx+bcx+2ac+2\sqrt{ac}\,\sqrt{b^{2}x^{2}+abx+bcx+ac}}{x}\right)+bcx\,\mathrm{csgn}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] -1/x/(a-c)^2*a-1/x/(a-c)^2*c+2*b*ln(x)/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(b*x
+c)^(1/2)*(csgn(b)*ln((a*b*x+b*c*x+2*a*c+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x
+a*c)^(1/2))/x)*x*a*b+csgn(b)*ln((a*b*x+b*c*x+2*a*c+2*(a*c)^(1/2)*(b^2*x^2+
a*b*x+b*c*x+a*c)^(1/2))/x)*x*b*c-2*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x
+a*c)^(1/2)*csgn(b))*csgn(b))*x*b*(a*c)^(1/2)+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(
1/2)*(a*c)^(1/2)*csgn(b))*csgn(b)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)/x/(a*c)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

mupad [B] time = 28.82, size = 7637, normalized size = 54.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

```
[Out] (2*b*log(x))/(a^2 - 2*a*c + c^2) - ((((a + b*x)^(1/2) - a^(1/2))^2*((a^2*b)
/2 + (b*c^2)/2 - (3*a*b*c)/2))/(((c + b*x)^(1/2) - c^(1/2))^2*(a*c^3 + a^3*
c - 2*a^2*c^2)) - b/(2*(a^2 - 2*a*c + c^2)) + (a^(1/2)*c^(1/2)*((a*b)/2 + (
b*c)/2)*((a + b*x)^(1/2) - a^(1/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a*c^3 +
a^3*c - 2*a^2*c^2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2
)) + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))^3 - ((a + c)
*((a + b*x)^(1/2) - a^(1/2))^2)/(a^(1/2)*c^(1/2)*((c + b*x)^(1/2) - c^(1/2)
)^2)) + (b*atan(((b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^3*c^10
- 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11
*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*
c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) +
 (4*b*((4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c
^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 +
12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*
a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^1
4*c^8 + a^15*c^7) + (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a
^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2
)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/
2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8
*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^1
3 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 +
 a^15*c^7) - (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268
*a^6*c^16 - 982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 -
 8436*a^11*c^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15
*c^7 + 268*a^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) - c^(1/2
))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*
a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((a + b*
x)^(1/2) - a^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(
11/2)*b*c^(29/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 177
0*a^(17/2)*b*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) -
 1770*a^(23/2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13
/2) + 231*a^(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*c^(7/2
)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*
a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c
```

^7))))/(a - c)^2 - (4*(a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100
*a^(11/2)*b^2*c^(25/2) + 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21
/2) + 192*a^(17/2)*b^2*c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*
b^2*c^(15/2) + 285*a^(23/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a
^(27/2)*b^2*c^(9/2) + a^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^
9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*
c^8 + a^15*c^7) + (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 - 570*a^5
*b^2*c^13 + 2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^10 - 815
6*a^9*b^2*c^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^2*c^6 -
570*a^13*b^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^1
5 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 +
28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) -
a^(1/2))*(65*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*a^(11/2
)*b^3*c^(21/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/2)*b^3*c^(17/2) + 9
62*a^(17/2)*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2)*b^3*c
^(11/2) - 427*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c + b*x)^
(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a
^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7)))*4i)/(a - c
)^2 + (b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^3*c^10 - 8*a^7*b^3
*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11*b^3*c^5 +
4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a
^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(
a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2)
+ 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*
c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(2
3/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a
^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 +
70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*
((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c^12 + 504*a^
9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^
7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 5
6*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15
*c^7) - (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a^(13/2)*c^(3
1/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2)*c^(25/2) +
6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/2) + 50*a^(2
7/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8*a^(33/2)*c^
(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*
c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) -
(2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268*a^6*c^16 -
982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 - 8436*a^11*c
^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15*c^7 + 268*a
^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15
- 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 +
28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((a + b*x)^(1/2) - a
^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(11/2)*b*c^(2
9/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 1770*a^(17/2)*b
*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) - 1770*a^(23/
2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13/2) + 231*a^
(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*c^(7/2)))/(((c + b
x)^(1/2) - c^(1/2))(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 +
70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a -
c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 - 570*a^5*b^2*c^13 +
2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^10 - 8156*a^9*b^2*c
^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^2*c^6 - 570*a^13*b
^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c
^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^
9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(6
5*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*a^(11/2)*b^3*c^(21
/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/2)*b^3*c^(17/2) + 962*a^(17/2)

*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2)*b^3*c^(11/2) - 4
27*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c + b*x)^(1/2) - c^(
1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 -
56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7)))*4i)/(a - c)^2)/((8*(1
4*a^(7/2)*b^4*c^(21/2) - 14*a^(9/2)*b^4*c^(19/2) - 42*a^(11/2)*b^4*c^(17/2)
 + 42*a^(13/2)*b^4*c^(15/2) + 42*a^(15/2)*b^4*c^(13/2) - 42*a^(17/2)*b^4*c^
(11/2) - 14*a^(19/2)*b^4*c^(9/2) + 14*a^(21/2)*b^4*c^(7/2)))/(a^7*c^15 - 8*
a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^
13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11
 - 32*a^6*b^3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^
10*b^3*c^6 + 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*
a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^1
4*c^8 + a^15*c^7) + (4*b*((4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b
*c^13 - 192*a^8*b*c^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9
- 192*a^12*b*c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15
 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 +
28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11
/2)*c^(33/2) + 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^
(27/2) - 27*a^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) -
 27*a^(25/2)*c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^
(31/2)*c^(13/2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^
8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13
*c^9 - 8*a^14*c^8 + a^15*c^7) - (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18
- 47*a^5*c^17 + 268*a^6*c^16 - 982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13
 + 7404*a^10*c^12 - 8436*a^11*c^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*
a^14*c^8 - 982*a^15*c^7 + 268*a^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c +
b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 +
 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a -
 c)^2 + (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b
*c^(31/2) + 231*a^(11/2)*b*c^(29/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/
2)*b*c^(25/2) - 1770*a^(17/2)*b*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^
(21/2)*b*c^(19/2) - 1770*a^(23/2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 7
49*a^(27/2)*b*c^(13/2) + 231*a^(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) +
4*a^(33/2)*b*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14
+ 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 -
8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (4*(a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)
*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2) + 285*a^(13/2)*b^2*c^(23/2) - 390
*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*c^(19/2) + 192*a^(19/2)*b^2*c^(17
/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(23/2)*b^2*c^(13/2) - 100*a^(25/2)*
b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a^(29/2)*b^2*c^(7/2)))/(a^7*c^15 -
 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28
*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4
*b^2*c^14 - 570*a^5*b^2*c^13 + 2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090
*a^8*b^2*c^10 - 8156*a^9*b^2*c^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 +
2053*a^12*b^2*c^6 - 570*a^13*b^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2)
- c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^
11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*
((a + b*x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(2
3/2) + 1256*a^(11/2)*b^3*c^(21/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/
2)*b^3*c^(17/2) + 962*a^(17/2)*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) +
1256*a^(21/2)*b^3*c^(11/2) - 427*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(
7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 -
56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^1
5*c^7))))/(a - c)^2 - (4*b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^
3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 +
 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 5
6*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15
*c^7) + (4*b*((4*(a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(1
1/2)*b^2*c^(25/2) + 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) +

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192*a^(17/2)*b^2*c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c
^(15/2) + 285*a^(23/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/
2)*b^2*c^(9/2) + a^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^1
3 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 +
 a^15*c^7) + (4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a
^8*b*c^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*
c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14
 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 -
 8*a^14*c^8 + a^15*c^7) - (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2)
+ 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a
^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*
c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/
2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a
^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14
*c^8 + a^15*c^7) - (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17
 + 268*a^6*c^16 - 982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*
c^12 - 8436*a^11*c^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 98
2*a^15*c^7 + 268*a^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) -
c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11
 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((
a + b*x)^(1/2) - a^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 2
31*a^(11/2)*b*c^(29/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2)
 - 1770*a^(17/2)*b*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(1
9/2) - 1770*a^(23/2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b
*c^(13/2) + 231*a^(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*
c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13
 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 +
a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 -
570*a^5*b^2*c^13 + 2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^1
0 - 8156*a^9*b^2*c^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^
2*c^6 - 570*a^13*b^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(
a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12
*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(
1/2) - a^(1/2))*(65*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*
a^(11/2)*b^3*c^(21/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/2)*b^3*c^(17
/2) + 962*a^(17/2)*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2
)*b^3*c^(11/2) - 427*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c
+ b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12
 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a
 - c)^2 + (4*((a + b*x)^(1/2) - a^(1/2))*(224*a^5*b^4*c^9 - 112*a^4*b^4*c^1
0 + 112*a^6*b^4*c^8 - 448*a^7*b^4*c^7 + 112*a^8*b^4*c^6 + 224*a^9*b^4*c^5 -
 112*a^10*b^4*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 2
8*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a
^14*c^8 + a^15*c^7))))*8i)/(a - c)^2 - (log((a*((a + b*x)^(1/2) - a^(1/2)))
/((c + b*x)^(1/2) - c^(1/2)) - a^(1/2)*c^(1/2) + (c*((a + b*x)^(1/2) - a^(1
/2)))/((c + b*x)^(1/2) - c^(1/2)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(
1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2)*(a^(1/2)*b*c^(3/2) + a^(3/2)*b*c^(1
/2)))/(a*c^3 + a^3*c - 2*a^2*c^2) - (a + c)/(x*(a^2 - 2*a*c + c^2)) + (log(
((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2)
 + a^(3/2)*b*c^(1/2)))/(a*c^3 + a^3*c - 2*a^2*c^2) + (b*((a + b*x)^(1/2) -
a^(1/2)))/(2*(a - c)^2*((c + b*x)^(1/2) - c^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

3.411 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=375

$$-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3}-\frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3}$$

```
[Out] -8/3*a^3*(b*x+a)^(3/2)/b^3/(a-c)^3+2/3*a^2*(a+3*c)*(b*x+a)^(3/2)/b^3/(a-c)^
3+24/5*a^2*(b*x+a)^(5/2)/b^3/(a-c)^3-4/5*a*(a+3*c)*(b*x+a)^(5/2)/b^3/(a-c)^
3-24/7*a*(b*x+a)^(7/2)/b^3/(a-c)^3+2/7*(a+3*c)*(b*x+a)^(7/2)/b^3/(a-c)^3+8/
9*(b*x+a)^(9/2)/b^3/(a-c)^3+8/3*c^3*(b*x+c)^(3/2)/b^3/(a-c)^3-2/3*c^2*(3*a+
c)*(b*x+c)^(3/2)/b^3/(a-c)^3-24/5*c^2*(b*x+c)^(5/2)/b^3/(a-c)^3+4/5*c*(3*a+
c)*(b*x+c)^(5/2)/b^3/(a-c)^3+24/7*c*(b*x+c)^(7/2)/b^3/(a-c)^3-2/7*(3*a+c)*(
b*x+c)^(7/2)/b^3/(a-c)^3-8/9*(b*x+c)^(9/2)/b^3/(a-c)^3
```

Rubi [A] time = 0.37, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6689, 43}

$$\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}-\frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3}+\frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (-8*a^3*(a + b*x)^(3/2))/(3*b^3*(a - c)^3) + (2*a^2*(a + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(a - c)^3) + (24*a^2*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (4*a*(
a + 3*c)*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (24*a*(a + b*x)^(7/2))/(7*b^3
*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(7/2))/(7*b^3*(a - c)^3) + (8*(a + b*x
)^(9/2))/(9*b^3*(a - c)^3) + (8*c^3*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (2
*c^2*(3*a + c)*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (24*c^2*(c + b*x)^(5/2)
)/(5*b^3*(a - c)^3) + (4*c*(3*a + c)*(c + b*x)^(5/2))/(5*b^3*(a - c)^3) + (
24*c*(c + b*x)^(7/2))/(7*b^3*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(7/2))/(7*
b^3*(a - c)^3) - (8*(c + b*x)^(9/2))/(9*b^3*(a - c)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(a\left(1+\frac{3c}{a}\right)x^2\sqrt{a+bx}+4bx^3\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)x^2\sqrt{c+bx}-4bx^3\sqrt{c+\right.}{(a-c)^3}$$

$$= \frac{(4b)\int x^3\sqrt{a+bx}\,dx}{(a-c)^3} - \frac{(4b)\int x^3\sqrt{c+bx}\,dx}{(a-c)^3} - \frac{(3a+c)\int x^2\sqrt{c+bx}\,dx}{(a-c)^3} + \frac{(a}{}$$

$$= \frac{(4b)\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx}{(a-c)^3} - \frac{(4b)\int\left(-\frac{c^3\sqrt{c+bx}}{b^3}\right.}{}$$

$$= -\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3} - \frac{4a(a+3c)(a+}{5b^3(a-c)^3}$$

Mathematica [A] time = 0.40, size = 282, normalized size = 0.75

$$\frac{2\left(-40a^4\sqrt{a+bx}+4a^3\sqrt{a+bx}\,(5bx+18c)-3a^2bx\sqrt{a+bx}\,(5bx+12c)+a\left(5b^3x^3\left(13\sqrt{a+bx}-27\sqrt{bx+c}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*(-40*a^4*Sqrt[a + b*x] - 3*a^2*b*x*Sqrt[a + b*x]*(12*c + 5*b*x) + 4*a^3*
Sqrt[a + b*x]*(18*c + 5*b*x) + a*(-72*c^3*Sqrt[c + b*x] + 36*b*c^2*x*Sqrt[c
 + b*x] + 5*b^3*x^3*(13*Sqrt[a + b*x] - 27*Sqrt[c + b*x]) + 27*b^2*c*x^2*(S
qrt[a + b*x] - Sqrt[c + b*x])) + 5*(8*c^4*Sqrt[c + b*x] - 4*b*c^3*x*Sqrt[c
+ b*x] + 3*b^2*c^2*x^2*Sqrt[c + b*x] + b^3*c*x^3*(27*Sqrt[a + b*x] - 13*Sqr
t[c + b*x]) + 28*b^4*x^4*(Sqrt[a + b*x] - Sqrt[c + b*x]))))/(315*b^3*(a - c
)^3)
```

fricas [A] time = 0.43, size = 208, normalized size = 0.55

$$\frac{2\left(\left(140\,b^4x^4-40\,a^4+72\,a^3c+5\left(13\,ab^3+27\,b^3c\right)x^3-3\left(5\,a^2b^2-9\,ab^2c\right)x^2+4\left(5\,a^3b-9\,a^2bc\right)x\right)\sqrt{bx+a}-\right.}{315\left(a^3b^3-3\,a^2b^3c+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/315*((140*b^4*x^4 - 40*a^4 + 72*a^3*c + 5*(13*a*b^3 + 27*b^3*c)*x^3 - 3*(
5*a^2*b^2 - 9*a*b^2*c)*x^2 + 4*(5*a^3*b - 9*a^2*b*c)*x)*sqrt(b*x + a) - (14
0*b^4*x^4 + 72*a*c^3 - 40*c^4 + 5*(27*a*b^3 + 13*b^3*c)*x^3 + 3*(9*a*b^2*c
- 5*b^2*c^2)*x^2 - 4*(9*a*b*c^2 - 5*b*c^3)*x)*sqrt(b*x + c))/(a^3*b^3 - 3*a
^2*b^3*c + 3*a*b^3*c^2 - b^3*c^3)
```

giac [B] time = 0.81, size = 1447, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/315*(((5*(b*x + a)*(28*(a^9*b^12 - 9*a^8*b^12*c + 36*a^7*b^12*c^2 - 84*a
^6*b^12*c^3 + 126*a^5*b^12*c^4 - 126*a^4*b^12*c^5 + 84*a^3*b^12*c^6 - 36*a^
2*b^12*c^7 + 9*a*b^12*c^8 - b^12*c^9)*(b*x + a)/(a^12*b^15 - 12*a^11*b^15*c
 + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^
5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c
```

```
^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12) - (85*a^10*b^12 - 778*a
^9*b^12*c + 3177*a^8*b^12*c^2 - 7608*a^7*b^12*c^3 + 11802*a^6*b^12*c^4 - 12
348*a^5*b^12*c^5 + 8778*a^4*b^12*c^6 - 4152*a^3*b^12*c^7 + 1233*a^2*b^12*c^
8 - 202*a*b^12*c^9 + 13*b^12*c^10)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^
15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b
^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b
^15*c^10 - 12*a*b^15*c^11 + b^15*c^12)) + 3*(145*a^11*b^12 - 1361*a^10*b^12
*c + 5719*a^9*b^12*c^2 - 14151*a^8*b^12*c^3 + 22794*a^7*b^12*c^4 - 24906*a^
6*b^12*c^5 + 18606*a^5*b^12*c^6 - 9294*a^4*b^12*c^7 + 2901*a^3*b^12*c^8 - 4
69*a^2*b^12*c^9 + 11*a*b^12*c^10 + 5*b^12*c^11)/(a^12*b^15 - 12*a^11*b^15*c
 + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^
5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c
^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12))*(b*x + a) - (155*a^12*
b^12 - 1536*a^11*b^12*c + 6855*a^10*b^12*c^2 - 18170*a^9*b^12*c^3 + 31770*a
^8*b^12*c^4 - 38520*a^7*b^12*c^5 + 33222*a^6*b^12*c^6 - 20700*a^5*b^12*c^7
+ 9495*a^4*b^12*c^8 - 3320*a^3*b^12*c^9 + 915*a^2*b^12*c^10 - 186*a*b^12*c^
11 + 20*b^12*c^12)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9
*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^
5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*
b^15*c^11 + b^15*c^12))*(b*x + a) + (5*a^13*b^12 - 83*a^12*b^12*c + 543*a^1
1*b^12*c^2 - 1925*a^10*b^12*c^3 + 4070*a^9*b^12*c^4 - 4950*a^8*b^12*c^5 + 2
046*a^7*b^12*c^6 + 3894*a^6*b^12*c^7 - 8415*a^5*b^12*c^8 + 8305*a^4*b^12*c^
9 - 5005*a^3*b^12*c^10 + 1887*a^2*b^12*c^11 - 412*a*b^12*c^12 + 40*b^12*c^1
3)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*
a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495
*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15
*c^12))*sqrt(b*x + c) + 2/315*(140*(b*x + a)^(9/2) - 495*(b*x + a)^(7/2)*a
+ 630*(b*x + a)^(5/2)*a^2 - 315*(b*x + a)^(3/2)*a^3 + 135*(b*x + a)^(7/2)*c
 - 378*(b*x + a)^(5/2)*a*c + 315*(b*x + a)^(3/2)*a^2*c)/(a^3*b^3 - 3*a^2*b^
3*c + 3*a*b^3*c^2 - b^3*c^3)
```

maple [A] time = 0.01, size = 294, normalized size = 0.78

$$\frac{2\left(\frac{(bx+a)^{\frac{3}{2}}a^2}{3}-\frac{2(bx+a)^{\frac{5}{2}}a}{5}+\frac{(bx+a)^{\frac{7}{2}}}{7}\right)a}{(a-c)^3b^3}-\frac{6\left(\frac{(bx+c)^{\frac{3}{2}}c^2}{3}-\frac{2(bx+c)^{\frac{5}{2}}c}{5}+\frac{(bx+c)^{\frac{7}{2}}}{7}\right)a}{(a-c)^3b^3}+\frac{6\left(\frac{(bx+a)^{\frac{3}{2}}a^2}{3}-\frac{2(bx+a)^{\frac{5}{2}}a}{5}+\frac{(bx+a)^{\frac{7}{2}}}{7}\right)c}{(a-c)^3b^3}-\frac{2(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a/b^3*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2
))+6/(a-c)^3*c/b^3*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(
7/2))-6/(a-c)^3*a/b^3*(1/3*(b*x+c)^(3/2)*c^2-2/5*(b*x+c)^(5/2)*c+1/7*(b*x+c
)^(7/2))-2/(a-c)^3*c/b^3*(1/3*(b*x+c)^(3/2)*c^2-2/5*(b*x+c)^(5/2)*c+1/7*(b*
x+c)^(7/2))+8/(a-c)^3/b^3*(1/9*(b*x+a)^(9/2)-3/7*a*(b*x+a)^(7/2)+3/5*a^2*(b
*x+a)^(5/2)-1/3*a^3*(b*x+a)^(3/2))-8/(a-c)^3/b^3*(1/9*(b*x+c)^(9/2)-3/7*c*(
b*x+c)^(7/2)+3/5*c^2*(b*x+c)^(5/2)-1/3*c^3*(b*x+c)^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

mupad [B] time = 3.30, size = 529, normalized size = 1.41

$$\frac{x^3\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)\sqrt{c+b\,x}}{7\,b}-\frac{x^3\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)\sqrt{a+b\,x}}{7\,b}-\frac{8\,c^2\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

```
[Out] (x^3*((64*b*c)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2)
)/(7*b) - (x^3*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a +
b*x)^(1/2))/(7*b) - (8*c^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(
a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15*b^3)
- (x^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(a - c)^3) - (2*b*(3*
a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(5*b) + (8*a^2*((2*(3*a*c + a
^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3
))/(7*b))*(a + b*x)^(1/2))/(15*b^3) + (x^2*((2*(3*a*c + a^2))/(a - c)^3 + (
6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x
)^(1/2))/(5*b) + (8*b*x^4*(a + b*x)^(1/2))/(9*(a - c)^3) - (8*b*x^4*(c + b*
x)^(1/2))/(9*(a - c)^3) + (4*c*x*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c
)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15
*b^2) - (4*a*x*((2*(3*a*c + a^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3)
- (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x)^(1/2))/(15*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**3, x)
```

3.412 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=261

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8(bx+}{7b^2(a}$$

```
[Out] 8/3*a^2*(b*x+a)^(3/2)/b^2/(a-c)^3-2/3*a*(a+3*c)*(b*x+a)^(3/2)/b^2/(a-c)^3-1
6/5*a*(b*x+a)^(5/2)/b^2/(a-c)^3+2/5*(a+3*c)*(b*x+a)^(5/2)/b^2/(a-c)^3+8/7*(
b*x+a)^(7/2)/b^2/(a-c)^3-8/3*c^2*(b*x+c)^(3/2)/b^2/(a-c)^3+2/3*c*(3*a+c)*(b
*x+c)^(3/2)/b^2/(a-c)^3+16/5*c*(b*x+c)^(5/2)/b^2/(a-c)^3-2/5*(3*a+c)*(b*x+c
)^(5/2)/b^2/(a-c)^3-8/7*(b*x+c)^(7/2)/b^2/(a-c)^3
```

Rubi [A] time = 0.24, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6689, 43}

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8(bx+}{7b^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (8*a^2*(a + b*x)^(3/2))/(3*b^2*(a - c)^3) - (2*a*(a + 3*c)*(a + b*x)^(3/2))
/(3*b^2*(a - c)^3) - (16*a*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (2*(a + 3*c
)*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (8*(a + b*x)^(7/2))/(7*b^2*(a - c)^3
) - (8*c^2*(c + b*x)^(3/2))/(3*b^2*(a - c)^3) + (2*c*(3*a + c)*(c + b*x)^(3
/2))/(3*b^2*(a - c)^3) + (16*c*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (2*(3*a
 + c)*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (8*(c + b*x)^(7/2))/(7*b^2*(a -
c)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(a\left(1+\frac{3c}{a}\right)x\sqrt{a+bx}+4bx^2\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)x\sqrt{c+bx}-4bx^2\sqrt{c+b\right.}{(a-c)^3}$$

$$= \frac{(4b)\int x^2\sqrt{a+bx}\,dx}{(a-c)^3} - \frac{(4b)\int x^2\sqrt{c+bx}\,dx}{(a-c)^3} - \frac{(3a+c)\int x\sqrt{c+bx}\,dx}{(a-c)^3} + \frac{(a+}{}$$

$$= \frac{(4b)\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{(a-c)^3} - \frac{(4b)\int\left(\frac{c^2\sqrt{c+bx}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}+\frac{(c-}{}\right.}{(a-c)^3}$$

$$= \frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3} + \frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}$$

Mathematica [A] time = 0.26, size = 214, normalized size = 0.82

$$\frac{2\left(6a^3\sqrt{a+bx}-a^2\sqrt{a+bx}(3bx+14c)+20b^3x^3\left(\sqrt{a+bx}-\sqrt{bx+c}\right)+a\left(b^2x^2\left(11\sqrt{a+bx}-21\sqrt{bx+c}\right)\right.\right.}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*(6*a^3*Sqrt[a + b*x] - 6*c^3*Sqrt[c + b*x] + 3*b*c^2*x*Sqrt[c + b*x] - a
^2*Sqrt[a + b*x]*(14*c + 3*b*x) + b^2*c*x^2*(21*Sqrt[a + b*x] - 11*Sqrt[c +
 b*x]) + 20*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*x]) + a*(14*c^2*Sqrt[c + b*
x] + b^2*x^2*(11*Sqrt[a + b*x] - 21*Sqrt[c + b*x]) + 7*b*c*x*(Sqrt[a + b*x]
 - Sqrt[c + b*x]))))/(35*b^2*(a - c)^3)
```

fricas [A] time = 0.45, size = 159, normalized size = 0.61

$$\frac{2\left(\left(20\,b^3x^3+6\,a^3-14\,a^2c+\left(11\,ab^2+21\,b^2c\right)x^2-\left(3\,a^2b-7\,abc\right)x\right)\sqrt{bx+a}-\left(20\,b^3x^3-14\,ac^2+6\,c^3+\left(21\,a\right.\right.\right.}{35\left(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/35*((20*b^3*x^3 + 6*a^3 - 14*a^2*c + (11*a*b^2 + 21*b^2*c)*x^2 - (3*a^2*b
 - 7*a*b*c)*x)*sqrt(b*x + a) - (20*b^3*x^3 - 14*a*c^2 + 6*c^3 + (21*a*b^2 +
 11*b^2*c)*x^2 + (7*a*b*c - 3*b*c^2)*x)*sqrt(b*x + c))/(a^3*b^2 - 3*a^2*b^2
*c + 3*a*b^2*c^2 - b^2*c^3)
```

giac [B] time = 0.38, size = 866, normalized size = 3.32

$$-\frac{2\left(\left(\left((bx+a)\left(\frac{20\left(a^6b^3-6\,a^5b^3c+15\,a^4b^3c^2-20\,a^3b^3c^3+15\,a^2b^3c^4-6\,ab^3c^5+b^3c^6\right)(bx+a)}{a^9b^4-9\,a^8b^4c+36\,a^7b^4c^2-84\,a^6b^4c^3+126\,a^5b^4c^4-126\,a^4b^4c^5+84\,a^3b^4c^6-36\,a^2b^4c^7+9\,ab^4c^8-b^4c^9}-\frac{39\,a^7b^3-245\,a^6b^3c+6}{a^9b^4-9\,a^8b^4c+36\,a^7b^4c^2-}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/35*((((b*x + a)*(20*(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3
*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)*(b*x + a)/(a^9*b^4 - 9*a^8*b
^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5
+ 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9) - (39*a^7*b^3 -
245*a^6*b^3*c + 651*a^5*b^3*c^2 - 945*a^4*b^3*c^3 + 805*a^3*b^3*c^4 - 399*a
^2*b^3*c^5 + 105*a*b^3*c^6 - 11*b^3*c^7)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^
```

```
4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6
 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9)) + 3*(6*a^8*b^3 - 41*a^7*b^3*c +
 119*a^6*b^3*c^2 - 189*a^5*b^3*c^3 + 175*a^4*b^3*c^4 - 91*a^3*b^3*c^5 + 21*
a^2*b^3*c^6 + a*b^3*c^7 - b^3*c^8)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^4*c^2
- 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6 - 36*
a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*(b*x + a) + (a^9*b^3 - 2*a^8*b^3*c -
20*a^7*b^3*c^2 + 112*a^6*b^3*c^3 - 266*a^5*b^3*c^4 + 364*a^4*b^3*c^5 - 308*
a^3*b^3*c^6 + 160*a^2*b^3*c^7 - 47*a*b^3*c^8 + 6*b^3*c^9)/(a^9*b^4 - 9*a^8*
b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5
 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*sqrt(b*x + c)
- (20*(b*x + a)^(7/2) - 49*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 + 21*
(b*x + a)^(5/2)*c - 35*(b*x + a)^(3/2)*a*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2
- b*c^3))/b
```

maple [A] time = 0.00, size = 222, normalized size = 0.85

$$\frac{2\left(-\frac{(bx+a)^{\frac{3}{2}}a}{3}+\frac{(bx+a)^{\frac{5}{2}}}{5}\right)a}{(a-c)^{3}b^{2}}-\frac{6\left(-\frac{(bx+c)^{\frac{3}{2}}c}{3}+\frac{(bx+c)^{\frac{5}{2}}}{5}\right)a}{(a-c)^{3}b^{2}}+\frac{6\left(-\frac{(bx+a)^{\frac{3}{2}}a}{3}+\frac{(bx+a)^{\frac{5}{2}}}{5}\right)c}{(a-c)^{3}b^{2}}-\frac{2\left(-\frac{(bx+c)^{\frac{3}{2}}c}{3}+\frac{(bx+c)^{\frac{5}{2}}}{5}\right)c}{(a-c)^{3}b^{2}}+\frac{\frac{8(bx+a)^{\frac{3}{2}}a}{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a/b^2*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))+6/(a-c)^3*c/b^2*(-
1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))-6/(a-c)^3*a/b^2*(-1/3*(b*x+c)^(3/2)*
c+1/5*(b*x+c)^(5/2))-2/(a-c)^3*c/b^2*(-1/3*(b*x+c)^(3/2)*c+1/5*(b*x+c)^(5/2
))+8/(a-c)^3/b^2*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/
2))-8/(a-c)^3/b^2*(1/3*(b*x+c)^(3/2)*c^2-2/5*(b*x+c)^(5/2)*c+1/7*(b*x+c)^(7
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

mupad [B] time = 3.21, size = 385, normalized size = 1.48

$$\frac{x^{2}\left(\frac{48\,b\,c}{7\,(a-c)^{3}}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^{3}}\right)\sqrt{c+b\,x}}{5\,b}-\frac{x^{2}\left(\frac{48\,a\,b}{7\,(a-c)^{3}}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^{3}}\right)\sqrt{a+b\,x}}{5\,b}-\frac{2\,a\left(\frac{2\,a\,(a+3\,c)}{(a-c)^{3}}+\frac{4\,a\left(\frac{48\,a\,b}{7\,(a-c)^{3}}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^{3}}\right)}{5\,b}\right)\sqrt{a+}}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

```
[Out] (x^2*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2)
)/(5*b) - (x^2*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a +
b*x)^(1/2))/(5*b) - (2*a*((2*a*(a + 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a
- c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(5*b))*(a + b*x)^(1/2))/(3*b^2) + (
8*b*x^3*(a + b*x)^(1/2))/(7*(a - c)^3) + (2*c*((2*c*(3*a + c))/(a - c)^3 +
(4*c*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c + b*
x)^(1/2))/(3*b^2) - (8*b*x^3*(c + b*x)^(1/2))/(7*(a - c)^3) + (x*((2*a*(a +
```

```
 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)
^3))/(5*b))*(a + b*x)^(1/2))/(3*b) - (x*((2*c*(3*a + c))/(a - c)^3 + (4*c*(
(48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c + b*x)^(1/
2))/(3*b)
```

sympy [A] time = 2.70, size = 942, normalized size = 3.61

$$\begin{cases} \frac{12a^2}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+bx}+140b^3x\sqrt{bx+c}+105b^2c\sqrt{a+bx}+35b^2c\sqrt{bx+c}} + \frac{54ab}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+bx}+} \\ \frac{x^2}{2\left(\sqrt{a}+\sqrt{c}\right)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((12*a**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 14
0*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x
) + 35*b**2*c*sqrt(b*x + c)) + 54*a*b*x/(35*a*b**2*sqrt(a + b*x) + 105*a*b*
*2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 10
5*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 44*a*c/(35*a*b**2*sqrt(
a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x
*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 36*a
*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x
 + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sq
rt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 40*b**2*x**2/(35*a*b**2*sqrt(a + b
*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt
(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 54*b*c*x/
(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b
*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(
b*x + c)) + 30*b*x*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 1
05*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x +
c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 12*c**2/(35*a*b*
*2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 14
0*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)
) + 36*c*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*
sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b
**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)), Ne(b, 0)), (x**2/(2*(sqrt(a
) + sqrt(c))**3), True))
```

3.413 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=64

$$\frac{(a-c)^2}{10b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^5}-\frac{1}{2b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}$$

```
[Out] 1/10*(a-c)^2/b/((b*x+a)^(1/2)+(b*x+c)^(1/2))^5-1/2/b/((b*x+a)^(1/2)+(b*x+c)
^(1/2))
```

Rubi [B] time = 0.09, antiderivative size = 151, normalized size of antiderivative = 2.36, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6689, 43}

$$\frac{8(a+bx)^{5/2}}{5b(a-c)^3}+\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8a(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8(bx+c)^{5/2}}{5b(a-c)^3}+\frac{8c(bx+c)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(bx+c)^{3/2}}{3b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (-8*a*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b
*(a - c)^3) + (8*(a + b*x)^(5/2))/(5*b*(a - c)^3) + (8*c*(c + b*x)^(3/2))/(
3*b*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (8*(c + b*
x)^(5/2))/(5*b*(a - c)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\frac{\int \left(a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}+4bx\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}-4bx\sqrt{c+bx}\right)dx}{(a-c)^3}$$

$$=\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3}+\frac{(4b)\int x\sqrt{a+bx}\,dx}{(a-c)^3}-\frac{(4b)\int x\sqrt{c+}}{(a-c)}$$

$$=\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3}+\frac{(4b)\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{(a-c)^3}-$$

$$=-\frac{8a(a+bx)^{3/2}}{3b(a-c)^3}+\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}+\frac{8(a+bx)^{5/2}}{5b(a-c)^3}+\frac{8c(c+bx)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)}{3b(a}$$

Mathematica [B] time = 0.15, size = 151, normalized size = 2.36

$$\frac{8(a+bx)^{5/2}}{5b(a-c)^3}+\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8a(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8(bx+c)^{5/2}}{5b(a-c)^3}+\frac{8c(bx+c)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(bx+c)^{3/2}}{3b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (-8*a*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b
*(a - c)^3) + (8*(a + b*x)^(5/2))/(5*b*(a - c)^3) + (8*c*(c + b*x)^(3/2))/(
3*b*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (8*(c + b*
x)^(5/2))/(5*b*(a - c)^3)
```

fricas [B] time = 0.44, size = 106, normalized size = 1.66

$$\frac{2\left(\left(4\,b^2x^2 - a^2 + 5\,ac + (3\,ab + 5\,bc)x\right)\sqrt{bx+a} - \left(4\,b^2x^2 + 5\,ac - c^2 + (5\,ab + 3\,bc)x\right)\sqrt{bx+c}\right)}{5\left(a^3b - 3\,a^2bc + 3\,abc^2 - bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((4*b^2*x^2 - a^2 + 5*a*c + (3*a*b + 5*b*c)*x)*sqrt(b*x + a) - (4*b^2*x
^2 + 5*a*c - c^2 + (5*a*b + 3*b*c)*x)*sqrt(b*x + c))/(a^3*b - 3*a^2*b*c + 3
*a*b*c^2 - b*c^3)
```

giac [B] time = 0.30, size = 427, normalized size = 6.67

$$-\frac{2}{5}\left((bx+a)\left(\frac{4\left(a^3b^2 - 3\,a^2b^2c + 3\,ab^2c^2 - b^2c^3\right)(bx+a)}{a^6b^3 - 6\,a^5b^3c + 15\,a^4b^3c^2 - 20\,a^3b^3c^3 + 15\,a^2b^3c^4 - 6\,ab^3c^5 + b^3c^6} - \frac{3\left(a^4b^2 - 4\,a^3b^2\right.}{a^6b^3 - 6\,a^5b^3c + 15\,a^4b^3c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/5*((b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x + a
)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4
 - 6*a*b^3*c^5 + b^3*c^6) - 3*(a^4*b^2 - 4*a^3*b^2*c + 6*a^2*b^2*c^2 - 4*a*
b^2*c^3 + b^2*c^4)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3
 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)) - (a^5*b^2 - 5*a^4*b^2*c + 10*a
^3*b^2*c^2 - 10*a^2*b^2*c^3 + 5*a*b^2*c^4 - b^2*c^5)/(a^6*b^3 - 6*a^5*b^3*c
 + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6
))*sqrt(b*x + c) + 2/5*(4*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a + 5*(b*x +
a)^(3/2)*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2 - b*c^3)
```

maple [B] time = 0.01, size = 146, normalized size = 2.28

$$\frac{2\,(bx+a)^{\frac{3}{2}}\,a}{3\,(a-c)^3\,b} - \frac{2\,(bx+c)^{\frac{3}{2}}\,a}{(a-c)^3\,b} + \frac{2\,(bx+a)^{\frac{3}{2}}\,c}{(a-c)^3\,b} - \frac{2\,(bx+c)^{\frac{3}{2}}\,c}{3\,(a-c)^3\,b} + \frac{-\frac{8(bx+a)^{\frac{3}{2}}a}{3} + \frac{8(bx+a)^{\frac{5}{2}}}{5}}{(a-c)^3\,b} - \frac{8\left(-\frac{(bx+c)^{\frac{3}{2}}c}{3} + \frac{(bx+c)^{\frac{5}{2}}}{5}\right)}{(a-c)^3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/3*a*(b*x+a)^(3/2)/b/(a-c)^3+2/(a-c)^3*c*(b*x+a)^(3/2)/b-2/(a-c)^3*a*(b*x+
c)^(3/2)/b-2/3*c*(b*x+c)^(3/2)/b/(a-c)^3+8/(a-c)^3/b*(-1/3*(b*x+a)^(3/2)*a+
1/5*(b*x+a)^(5/2))-8/(a-c)^3/b*(-1/3*(b*x+c)^(3/2)*c+1/5*(b*x+c)^(5/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{bx+a} + \sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3), x)
```

mupad [B] time = 2.99, size = 252, normalized size = 3.94

$$\frac{\left(\frac{2\left(a^2+3\,c\,a\right)}{(a-c)^3}+\frac{2\,a\left(\frac{32\,a\,b}{5\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{3\,b}\right)\sqrt{a+b\,x}}{b}-\frac{\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{2\,c\left(\frac{32\,b\,c}{5\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{3\,b}\right)\sqrt{c+b\,x}}{b}+\frac{8\,b\,x^2\,\sqrt{a+b\,x}}{5\,(a-c)^3}-\frac{8\,b\,x^2\,\sqrt{}}{5\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

```
[Out] (((2*(3*a*c + a^2))/(a - c)^3 + (2*a*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a +
3*c))/(a - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((2*c*(3*a + c))/(a - c)^3 +
 (2*c*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(3*b))*(c + b
*x)^(1/2))/b + (8*b*x^2*(a + b*x)^(1/2))/(5*(a - c)^3) - (8*b*x^2*(c + b*x)
^(1/2))/(5*(a - c)^3) - (x*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a + 3*c))/(a -
 c)^3)*(a + b*x)^(1/2))/(3*b) + (x*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*
c))/(a - c)^3)*(c + b*x)^(1/2))/(3*b)
```

sympy [A] time = 1.82, size = 384, normalized size = 6.00

$$\begin{cases} -\frac{2a}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc\sqrt{a+bx}+5bc\sqrt{bx+c}}-\frac{4bx}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc} \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((-2*a/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqr
t(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*
x + c)) - 4*b*x/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqr
t(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*
x + c)) - 2*c/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqrt(
a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x
+ c)) - 6*sqrt(a + b*x)*sqrt(b*x + c)/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*
x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a
+ b*x) + 5*b*c*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c))**3, True))
```

3.414 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=157

$$\frac{8(a+bx)^{3/2}}{3(a-c)^3}+\frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}-\frac{8(bx+c)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3}-\frac{2\sqrt{a}\,(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}\,(3a+c}{(}$$

```
[Out] 8/3*(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3-2*(a+3*c)*arctanh((b*x+
a)^(1/2)/a^(1/2))*a^(1/2)/(a-c)^3+2*(3*a+c)*arctanh((b*x+c)^(1/2)/c^(1/2))*
c^(1/2)/(a-c)^3+2*(a+3*c)*(b*x+a)^(1/2)/(a-c)^3-2*(3*a+c)*(b*x+c)^(1/2)/(a-
c)^3
```

Rubi [A] time = 0.23, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6689, 50, 63, 208}

$$\frac{8(a+bx)^{3/2}}{3(a-c)^3}+\frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}-\frac{8(bx+c)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3}-\frac{2\sqrt{a}\,(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}\,(3a+c}{(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (2*(a + 3*c)*Sqrt[a + b*x])/(a - c)^3 + (8*(a + b*x)^(3/2))/(3*(a - c)^3) -
 (2*(3*a + c)*Sqrt[c + b*x])/(a - c)^3 - (8*(c + b*x)^(3/2))/(3*(a - c)^3)
- (2*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 + (2*Sqrt[
c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)^3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(4b\sqrt{a+bx}+\frac{a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}}{x}-4b\sqrt{c+bx}-\frac{3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}}{x}\right)dx}{(a-c)^3}$$

$$= \frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(3a+c)\int \frac{\sqrt{c+bx}}{x}\,dx}{(a-c)^3}+\frac{(a+3c)\int \frac{\sqrt{a+bx}}{x}\,dx}{(a-c)^3}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(c(3a+$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(2c(3a$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{2\sqrt{a}\,(a$$

Mathematica [A] time = 0.19, size = 142, normalized size = 0.90

$$\frac{2\left(-9a\sqrt{bx+c}+9c\sqrt{a+bx}-3\sqrt{a}\,(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+3\sqrt{c}\,(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+7a\sqrt{a+bx}+4bx$$
$$3(a-c)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (2*(7*a*Sqrt[a + b*x] + 9*c*Sqrt[a + b*x] + 4*b*x*Sqrt[a + b*x] - 9*a*Sqrt[
c + b*x] - 7*c*Sqrt[c + b*x] - 4*b*x*Sqrt[c + b*x] - 3*Sqrt[a]*(a + 3*c)*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a]] + 3*Sqrt[c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sq
rt[c]]))/(3*(a - c)^3)
```

fricas [A] time = 0.50, size = 516, normalized size = 3.29

$$\left[-\frac{3\,(a+3\,c)\sqrt{a}\,\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+3\,(3\,a+c)\sqrt{c}\,\log\left(\frac{bx-2\,\sqrt{bx+c}\,\sqrt{c}+2\,c}{x}\right)-2\,(4\,bx+7\,a+9\,c)\sqrt{bx+a}+2\,($$
$$3\left(a^3-3\,a^2c+3\,ac^2-c^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3
*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*(4*b*x
+ 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 - 3*
a^2*c + 3*a*c^2 - c^3), -1/3*(6*(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)*sqr
t(-c)/c) + 3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
 - 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c
))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 1/3*(6*sqrt(-a)*(a + 3*c)*arctan(sqrt(b
*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c
) + 2*c)/x) + 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) - 2*(4*b*x + 9*a + 7*c)*s
qrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 2/3*(3*sqrt(-a)*(a + 3*c)*ar
ctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)*
sqrt(-c)/c) + (4*b*x + 7*a + 9*c)*sqrt(b*x + a) - (4*b*x + 9*a + 7*c)*sqrt(
b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)]
```

giac [B] time = 2.91, size = 2652, normalized size = 16.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*x + c)*(4*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(b*x + a)/(a^6 - 6*a^
5*c + 15*a^4*c^2 - 20*a^3*c^3 + 15*a^2*c^4 - 6*a*c^5 + c^6) + (5*a^4 - 8*a^
3*c - 6*a^2*c^2 + 16*a*c^3 - 7*c^4)/(a^6 - 6*a^5*c + 15*a^4*c^2 - 20*a^3*c^
3 + 15*a^2*c^4 - 6*a*c^5 + c^6)) + 2*(a^2 + 3*a*c)*arctan(sqrt(b*x + a)/sqr
t(-a))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) + 2/3*(4*(b*x + a)^(3/2)*
a^6 + 3*sqrt(b*x + a)*a^7 - 24*(b*x + a)^(3/2)*a^5*c - 9*sqrt(b*x + a)*a^6*
c + 60*(b*x + a)^(3/2)*a^4*c^2 - 9*sqrt(b*x + a)*a^5*c^2 - 80*(b*x + a)^(3/
2)*a^3*c^3 + 75*sqrt(b*x + a)*a^4*c^3 + 60*(b*x + a)^(3/2)*a^2*c^4 - 135*sq
rt(b*x + a)*a^3*c^4 - 24*(b*x + a)^(3/2)*a*c^5 + 117*sqrt(b*x + a)*a^2*c^5
+ 4*(b*x + a)^(3/2)*c^6 - 51*sqrt(b*x + a)*a*c^6 + 9*sqrt(b*x + a)*c^7)/(a^
9 - 9*a^8*c + 36*a^7*c^2 - 84*a^6*c^3 + 126*a^5*c^4 - 126*a^4*c^5 + 84*a^3*
c^6 - 36*a^2*c^7 + 9*a*c^8 - c^9) - 2*(3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 -
6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 - 2*(3*
a^2*c^2 + a*c^3 + (3*a*c^2 + c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3
)^2*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^3 - (3*a^2*c +
a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 - (3*a^5*c^2 - 11*a^4*c
^3 + 14*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 - (3*a^5*c - 11*a^4*c^2 + 14*a^3*
c^3 - 6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^
3)*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - (3*a^6*c - 11*a^5*c^2 + 14*a^4*c^3
- 6*a^3*c^4 - a^2*c^5 + a*c^6 + (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - 6*a^2*
c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) + (3*a^9*
c - 14*a^8*c^2 + 22*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c
^7 - 2*a^2*c^8 + a*c^9 - (3*a^8*c - 14*a^7*c^2 + 22*a^6*c^3 - 6*a^5*c^4 - 2
0*a^4*c^5 + 22*a^3*c^6 - 6*a^2*c^7 - 2*a*c^8 + c^9)*sqrt(a*c))*sgn(a^3 - 3*
a^2*c + 3*a*c^2 - c^3) + (3*a^9 - 14*a^8*c + 22*a^7*c^2 - 6*a^6*c^3 - 20*a^
5*c^4 + 22*a^4*c^5 - 6*a^3*c^6 - 2*a^2*c^7 + a*c^8)*sqrt(a*c))*arctan(-(sqr
t(b*x + a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 + sqrt((a^
4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 +
 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2
- c^3)))/((sqrt(-a)*a^8 - a^8*sqrt(-c) - 8*sqrt(-a)*a^7*c + 8*a^7*sqrt(-c)*
c + 28*sqrt(-a)*a^6*c^2 - 28*a^6*sqrt(-c)*c^2 - 56*sqrt(-a)*a^5*c^3 + 56*a^
5*sqrt(-c)*c^3 + 70*sqrt(-a)*a^4*c^4 - 70*a^4*sqrt(-c)*c^4 - 56*sqrt(-a)*a^
3*c^5 + 56*a^3*sqrt(-c)*c^5 + 28*sqrt(-a)*a^2*c^6 - 28*a^2*sqrt(-c)*c^6 - 8
*sqrt(-a)*a*c^7 + 8*a*sqrt(-c)*c^7 + sqrt(-a)*c^8 - sqrt(-c)*c^8)*abs(-a^3
+ 3*a^2*c - 3*a*c^2 + c^3)) + 2*(3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 - 6*a^6*
c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 + 2*(3*a^2*c^
2 + a*c^3 + (3*a*c^2 + c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*sg
n(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^3 + (3*a^2*c + a*c^2)
*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 - (3*a^5*c^2 - 11*a^4*c^3 + 1
4*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 - (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 -
6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn
(a^3 - 3*a^2*c + 3*a*c^2 - c^3) + (3*a^6*c - 11*a^5*c^2 + 14*a^4*c^3 - 6*a^
3*c^4 - a^2*c^5 + a*c^6 + (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - 6*a^2*c^4 -
a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) - (3*a^9*c - 14
*a^8*c^2 + 22*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2
*a^2*c^8 + a*c^9 + (3*a^8*c - 14*a^7*c^2 + 22*a^6*c^3 - 6*a^5*c^4 - 20*a^4*
c^5 + 22*a^3*c^6 - 6*a^2*c^7 - 2*a*c^8 + c^9)*sqrt(a*c))*sgn(a^3 - 3*a^2*c
+ 3*a*c^2 - c^3) + (3*a^9 - 14*a^8*c + 22*a^7*c^2 - 6*a^6*c^3 - 20*a^5*c^4
+ 22*a^4*c^5 - 6*a^3*c^6 - 2*a^2*c^7 + a*c^8)*sqrt(a*c))*arctan(-(sqrt(b*x
+ a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 - sqrt((a^4 - 2*
a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c
^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)
```

```
))/((sqrt(-a)*a^8 - a^8*sqrt(-c) - 8*sqrt(-a)*a^7*c + 8*a^7*sqrt(-c)*c + 28
*sqrt(-a)*a^6*c^2 - 28*a^6*sqrt(-c)*c^2 - 56*sqrt(-a)*a^5*c^3 + 56*a^5*sqrt
(-c)*c^3 + 70*sqrt(-a)*a^4*c^4 - 70*a^4*sqrt(-c)*c^4 - 56*sqrt(-a)*a^3*c^5
+ 56*a^3*sqrt(-c)*c^5 + 28*sqrt(-a)*a^2*c^6 - 28*a^2*sqrt(-c)*c^6 - 8*sqrt(
-a)*a*c^7 + 8*a*sqrt(-c)*c^7 + sqrt(-a)*c^8 - sqrt(-c)*c^8)*abs(-a^3 + 3*a^
2*c - 3*a*c^2 + c^3))
```

maple [A] time = 0.01, size = 181, normalized size = 1.15

$$\frac{\left(-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\right)a}{(a-c)^3}-\frac{3\left(-2\sqrt{c}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+2\sqrt{bx+c}\right)a}{(a-c)^3}+\frac{3\left(-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right.\right.}{(a-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 1/(a-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8/3*
(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3+3/(a-c)^3*c*(2*(b*x+a)^(1/2
)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-3/(a-c)^3*a*(2*(b*x+c)^(1/2)-2*
c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))-1/(a-c)^3*c*(2*(b*x+c)^(1/2)-2*c^(1
/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

mupad [B] time = 27.72, size = 4060, normalized size = 25.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
```

```
[Out] ((((a^(1/2)*(16*a + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(16*a
 + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((a + b*x)^(1/2) - a^(1/2)))/((c
 + b*x)^(1/2) - c^(1/2)) + (((a^(1/2)*(12*a + 20*c))/(3*a*c^2 - 3*a^2*c + a
^3 - c^3) + (c^(1/2)*(20*a + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((a +
b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 + (a^(1/2)*((28*a)/3
 + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(12*a + (28*c)/3))/(3*
a*c^2 - 3*a^2*c + a^3 - c^3))/((3*((a + b*x)^(1/2) - a^(1/2)))/((c + b*x)^(
1/2) - c^(1/2)) + (3*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1
/2))^2 + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))^3 + 1) +
 (log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a*(a^(1/2)
+ 3*c^(1/2)) + c*(3*a^(1/2) + c^(1/2))))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) +
(atan(((((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/
2) + a^(3/2)*c^(1/2)))^(1/2)*((6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5
- 2*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(
9/2)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3
 - 6*a^6*c^2) + (((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(5/2)*c^(11/
2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2) - 5*a^(13/2
)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*
a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(a*c^9 +
 a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a^6*c^4 + 22
```

```
*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 +
15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((a^(1/2)*c^(
3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2))
)^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3
/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3
+ 5*a^5*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2
) + 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(
11/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 +
15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3
- 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*
1i)/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2) - (((
a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/
2)*c^(1/2)))^(1/2)*((((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(5/2)*c^
(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2) - 5*a^(
13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 -
 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(a*c
^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a^6*c^4
+ 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^
7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((a^(1/2)
*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1
/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*
c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*
c^3 + 5*a^5*c^2) - (6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5 - 2*a^5*c^(
3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(9/2)*c^2)/(
a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^
2) + (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2) + 8*a^6*
c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(11/2) - a^
(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 + 15*a^4*c^6
 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3 - 3*a^(5/2
)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*1i)/(a*c^6
 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2))/((2*(a^(1/2)*c
^(9/2) - 4*a^(3/2)*c^(7/2) + 6*a^(5/2)*c^(5/2) - 4*a^(7/2)*c^(3/2) + a^(9/2
)*c^(1/2)))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c
^3 - 6*a^6*c^2) - (((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^
(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((6*a*c^(11/2) - 6*a^(11/2)*c + 2*a
^(3/2)*c^5 - 2*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/
2) + 16*a^(9/2)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 +
 15*a^5*c^3 - 6*a^6*c^2) + (((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(
5/2)*c^(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2)
- 5*a^(13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^
3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2
))*(a*c^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a
^6*c^4 + 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6
*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((
a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/
2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) +
 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 -
10*a^4*c^3 + 5*a^5*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3
*a^7*c^(3/2) + 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^
5 + a^3*c^(11/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6
*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a
^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^
(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^
2) - (((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2)
 + a^(3/2)*c^(1/2)))^(1/2)*((((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^
(5/2)*c^(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2)
 - 5*a^(13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a
^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/
2))*(a*c^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*
```

```
a^6*c^4 + 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 -
6*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(
(a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3
/2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2)
+ 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 -
 10*a^4*c^3 + 5*a^5*c^2) - (6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5 - 2
*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(9/2
)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 -
6*a^6*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2)
+ 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(11
/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 + 15
*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3 -
3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(
a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2) + (4*((a +
 b*x)^(1/2) - a^(1/2))*(6*a^3*c^4 - a^6*c - 5*a^2*c^5 - a*c^6 + 6*a^4*c^3 -
 5*a^5*c^2 + 3*a^(3/2)*c^(11/2) + 4*a^(5/2)*c^(9/2) - 14*a^(7/2)*c^(7/2) +
4*a^(9/2)*c^(5/2) + 3*a^(11/2)*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*
c^8 - 6*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^
2))))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2)
 + a^(3/2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c
^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*2i)/(a*c^6 - a^6*c - 5*a^2*c^5 + 10
*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

3.415 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=162

$$\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{8b\sqrt{bx+c}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3}+\frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3}-\frac{3b(3a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)^3}-\frac{3b(a+3c)\tanh^{-1}}{\sqrt{c}\,(c-a)}$$

```
[Out] -3*b*(3*a+c)*arctanh((b*x+a)^(1/2)/a^(1/2))/(a-c)^3/a^(1/2)-3*b*(a+3*c)*arc
tanh((b*x+c)^(1/2)/c^(1/2))/(-a+c)^3/c^(1/2)+8*b*(b*x+a)^(1/2)/(a-c)^3-(a+3
*c)*(b*x+a)^(1/2)/(a-c)^3/x-8*b*(b*x+c)^(1/2)/(a-c)^3+(3*a+c)*(b*x+c)^(1/2)
/(a-c)^3/x
```

Rubi [A] time = 0.27, antiderivative size = 223, normalized size of antiderivative = 1.38, number of steps used = 14, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6689, 47, 63, 208, 50}

$$\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{8b\sqrt{bx+c}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3}+\frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3}-\frac{b(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)^3}-\frac{8\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a}}\right.}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (8*b*Sqrt[a + b*x])/(a - c)^3 - ((a + 3*c)*Sqrt[a + b*x])/((a - c)^3*x) - (
8*b*Sqrt[c + b*x])/(a - c)^3 + ((3*a + c)*Sqrt[c + b*x])/((a - c)^3*x) - (8
*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 - (b*(a + 3*c)*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)^3) + (8*b*Sqrt[c]*ArcTanh[Sqrt[c
+ b*x]/Sqrt[c]])/(a - c)^3 + (b*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(
(a - c)^3*Sqrt[c])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(\frac{a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}}{x^2}+\frac{4b\sqrt{a+bx}}{x}-\frac{3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}}{x^2}-\frac{4b\sqrt{c+bx}}{x}\right)dx}{(a-c)^3}$$

$$= \frac{(4b)\int \frac{\sqrt{a+bx}}{x}\,dx}{(a-c)^3}-\frac{(4b)\int \frac{\sqrt{c+bx}}{x}\,dx}{(a-c)^3}-\frac{(3a+c)\int \frac{\sqrt{c+bx}}{x^2}\,dx}{(a-c)^3}+\frac{(a+3c)\int \frac{\sqrt{a+bx}}{x^2}\,\ldots}{(a-c)^3}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}+\frac{(4ab)\int \ldots}{(a-\ldots}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}+\frac{(8a)\,\mathrm{Subs}\ldots}{\ldots}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}-\frac{8\sqrt{a}\,b\tan\ldots}{(a\ldots}$$

Mathematica [A] time = 0.59, size = 187, normalized size = 1.15

$$\frac{b\left(-\frac{(a+3c)\left(bx\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx\right)}{bx\sqrt{a+bx}}+\frac{(3a+c)\left(bx\sqrt{\frac{bx}{c}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{c}+1}\right)+bx+c\right)}{bx\sqrt{bx+c}}+8\sqrt{a+bx}-8\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\ldots\right.}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (b*(8*Sqrt[a + b*x] - 8*Sqrt[c + b*x] - 8*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqr
t[a]] + 8*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]] - ((a + 3*c)*(a + b*x + b*
x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]]))/(b*x*Sqrt[a + b*x]) + ((3*
a + c)*(c + b*x + b*x*Sqrt[1 + (b*x)/c]*ArcTanh[Sqrt[1 + (b*x)/c]]))/(b*x*S
qrt[c + b*x])))/(a - c)^3
```

fricas [A] time = 0.51, size = 675, normalized size = 4.17

$$\left[-\frac{3\left(3abc+bc^2\right)\sqrt{a}\,x\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\left(a^2b+3abc\right)\sqrt{c}\,x\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\left(8abcx-a^2c-3ac\ldots\right.}{2\left(a^4c-3a^3c^2+3a^2c^3-ac^4\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2
*a)/x) + 3*(a^2*b + 3*a*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) +
 2*c)/x) - 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3
*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x),
 -1/2*(6*(a^2*b + 3*a*b*c)*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) + 3*
(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3*a^2*c - a*
c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), 1/2*(6*(3*
a*b*c + b*c^2)*sqrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a
*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*(8*a*b*c*x
 - a^2*c - 3*a*c^2)*sqrt(b*x + a) - 2*(8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*
x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), (3*(3*a*b*c + b*c^2)*s
qrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a*b*c)*sqrt(-c)*x
*arctan(sqrt(b*x + c)*sqrt(-c)/c) + (8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x
+ a) - (8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3
*a^2*c^3 - a*c^4)*x)]
```

giac [B] time = 39.98, size = 2594, normalized size = 16.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 8*sqrt(b*x + a)*b/(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 8*sqrt(b*x + c)*b/(a^3
- 3*a^2*c + 3*a*c^2 - c^3) + 3*(3*a*b + b*c)*arctan(sqrt(b*x + a)/sqrt(-a))
/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) - 3*(2*(a^2*c^2 + 3*a*c^3 + (a*
c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b*sgn(-2*a^3 + 6*
a^2*c - 6*a*c^2 + 2*c^3) - 2*(a^2*c^2 + 3*a*c^3 - (a^2*c + 3*a*c^2)*sqrt(a*
c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b + (a^5*c^2 - a^4*c^3 - 6*a^3*c^4 +
14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 -
 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn(-2*
a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c - a^5*c^2 - 6*a^4*c^3 + 14*a^3*c^
4 - 11*a^2*c^5 + 3*a*c^6 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a
*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) - (a^9*c - 2
*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 1
4*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6*a^6*c^3 + 22*a^5*c^4 - 20*a^4*
c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*c^9)*sqrt(a*c))*b*sgn(-2*a^3 +
6*a^2*c - 6*a*c^2 + 2*c^3) + (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 -
20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^9 - 2*a^8*c
 - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a^4*c^5 + 22*a^3*c^6 - 14*a^2*c^
7 + 3*a*c^8)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a
^4 - 2*a^3*c + 2*a*c^3 - c^4 + sqrt((a^4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^
5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a
*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/((sqrt(-a)*a^8*c - a^8*sqrt
(-c)*c - 8*sqrt(-a)*a^7*c^2 + 8*a^7*sqrt(-c)*c^2 + 28*sqrt(-a)*a^6*c^3 - 28
*a^6*sqrt(-c)*c^3 - 56*sqrt(-a)*a^5*c^4 + 56*a^5*sqrt(-c)*c^4 + 70*sqrt(-a)
*a^4*c^5 - 70*a^4*sqrt(-c)*c^5 - 56*sqrt(-a)*a^3*c^6 + 56*a^3*sqrt(-c)*c^6
+ 28*sqrt(-a)*a^2*c^7 - 28*a^2*sqrt(-c)*c^7 - 8*sqrt(-a)*a*c^8 + 8*a*sqrt(-
c)*c^8 + sqrt(-a)*c^9 - sqrt(-c)*c^9)*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3))
- 3*(2*(a^2*c^2 + 3*a*c^3 + (a*c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a
*c^2 - c^3)^2*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) + 2*(a^2*c^2 + 3*a*
c^3 + (a^2*c + 3*a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b - (a
^5*c^2 - a^4*c^3 - 6*a^3*c^4 + 14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4
*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3
*a^2*c - 3*a*c^2 + c^3)*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c -
a^5*c^2 - 6*a^4*c^3 + 14*a^3*c^4 - 11*a^2*c^5 + 3*a*c^6 + (a^5*c - a^4*c^2
- 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*
c - 3*a*c^2 + c^3) - (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c
^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6
```

```
*a^6*c^3 + 22*a^5*c^4 - 20*a^4*c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*
c^9)*sqrt(a*c))*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^9*c - 2*a^8*
c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2
*c^8 + 3*a*c^9 - (a^9 - 2*a^8*c - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a
^4*c^5 + 22*a^3*c^6 - 14*a^2*c^7 + 3*a*c^8)*sqrt(a*c))*b)*arctan(-(sqrt(b*x
 + a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 - sqrt((a^4 - 2
*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*
c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3
)))/((sqrt(-a)*a^8*c - a^8*sqrt(-c)*c - 8*sqrt(-a)*a^7*c^2 + 8*a^7*sqrt(-c)
*c^2 + 28*sqrt(-a)*a^6*c^3 - 28*a^6*sqrt(-c)*c^3 - 56*sqrt(-a)*a^5*c^4 + 56
*a^5*sqrt(-c)*c^4 + 70*sqrt(-a)*a^4*c^5 - 70*a^4*sqrt(-c)*c^5 - 56*sqrt(-a)
*a^3*c^6 + 56*a^3*sqrt(-c)*c^6 + 28*sqrt(-a)*a^2*c^7 - 28*a^2*sqrt(-c)*c^7
- 8*sqrt(-a)*a*c^8 + 8*a*sqrt(-c)*c^8 + sqrt(-a)*c^9 - sqrt(-c)*c^9)*abs(-a
^3 + 3*a^2*c - 3*a*c^2 + c^3)) - 2*(3*a*b*(sqrt(b*x + a) - sqrt(b*x + c))^3
 + b*c*(sqrt(b*x + a) - sqrt(b*x + c))^3 - 3*a^2*b*(sqrt(b*x + a) - sqrt(b*
x + c)) + 2*a*b*c*(sqrt(b*x + a) - sqrt(b*x + c)) + b*c^2*(sqrt(b*x + a) -
sqrt(b*x + c)))/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqrt(b*x + a) -
sqrt(b*x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2 - 2*a*c + c^
2)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)) - (sqrt(b*x + a)*a*b + 3*sqrt(b*x + a)*
b*c)/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*b*x)
```

maple [A] time = 0.02, size = 252, normalized size = 1.56

$$\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)ab}{(a-c)^3}-\frac{6\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\sqrt{bx+c}}{2bx}\right)ab}{(a-c)^3}+\frac{6\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)bc}{(a-c)^3}-\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right.}{(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^
(1/2))+6/(a-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(1/2))-6/(a-c)^3*a*b*(-1/2*(b*x+c)^(1/2)/b/x-1/2/c^(1/2)*arctanh((b*
x+c)^(1/2)/c^(1/2)))-2/(a-c)^3*c*b*(-1/2*(b*x+c)^(1/2)/b/x-1/2/c^(1/2)*arct
anh((b*x+c)^(1/2)/c^(1/2)))+4/(a-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh(
(b*x+a)^(1/2)/a^(1/2)))-4/(a-c)^3*b*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x
+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

mupad [B] time = 33.22, size = 4681, normalized size = 28.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
```

```
[Out] (b*atan(((b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c
^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a
```

```
^5*b*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3
*a^2*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*
a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*
(6*a^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 -
48*a^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4
))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^
6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a^(5/2)*c^(19/2) - 5*a^
(7/2)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(1
1/2) + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9
 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)
- (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*
a^5*c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2)
)/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6
*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3
/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2)
+ a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2
))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)
))*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) +
 2*a^(5/2)*c^2)*3i)/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a
^6*c^3 - a^7*c^2)) + (b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c
 + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*
b*c^6 - 24*a^5*b*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/
2)*b*c^4 - 3*a^2*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a
^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2)
 - a^(1/2))*(6*a^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7
/2)*b*c^6 - 48*a^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^
(11/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5
*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) + (3*b*((a^(5/2)*c^(
19/2) - 5*a^(7/2)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a
^(13/2)*c^(11/2) + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/
2))/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4
 + a^9*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^
4*c^8 - 164*a^5*c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 +
 4*a^10*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*
c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) -
2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)
*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2
*a^(5/2)*c^2))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^
3 - a^7*c^2)))*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a
^2*c^(5/2) + 2*a^(5/2)*c^2)*3i)/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a
^5*c^4 + 5*a^6*c^3 - a^7*c^2)))/(((9*a^(3/2)*b^2*c^(11/2))/2 - 18*a^(5/2)*b
^2*c^(9/2) + 27*a^(7/2)*b^2*c^(7/2) - 18*a^(9/2)*b^2*c^(5/2) + (9*a^(11/2)*
b^2*c^(3/2))/2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5
 - 6*a^8*c^4 + a^9*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(72*a^3*b^2*c^4 - 72
*a^2*b^2*c^5 + 72*a^4*b^2*c^3 - 72*a^5*b^2*c^2 + 27*a^(3/2)*b^2*c^(11/2) +
36*a^(5/2)*b^2*c^(9/2) - 126*a^(7/2)*b^2*c^(7/2) + 36*a^(9/2)*b^2*c^(5/2) +
 27*a^(11/2)*b^2*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^
8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a
^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2
)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a^5*b*c^(9/2) +
24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3*a^2*b*c^(15/2)
 + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^
7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(6*a^(3/2)*b*c^
8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 - 48*a^5*b*c^(9/2
) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4))/(2*((c + b*x
)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*
c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a^(5/2)*c^(19/2) - 5*a^(7/2)*c^(17/2)
+ 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(11/2) + 9*a^(15/
2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9 - 6*a^4*c^8 +
```

```
15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) - (((a + b*x)^(
1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*a^5*c^7 + 200*a
^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2))/(2*((c + b*x)
^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c
^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2
*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2) + a^(7/2)*c - 3
*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2))/(2*(a^2*c^7
- 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)))*(a*c^(7/2) +
 a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2)
)/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2))
 + (3*b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/
2) + a^(3/2)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a^5*b
*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3*a^2
*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*
c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(6*a
^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 - 48*a
^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4))/(
2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^
6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) + (3*b*((a^(5/2)*c^(19/2) - 5*a^(7/2
)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(11/2)
 + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9 - 6
*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) - ((
(a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*a^5*
c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2))/(2
*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6
 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*
c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2) + a^
(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2))/(
2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)))*(
a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a
^(5/2)*c^2))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3
- a^7*c^2))))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)
*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) -
 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2)*3i)/(a^2*c^7 - 5*a^3*c^6 +
10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2) - (log(((a + b*x)^(1/2) - a^
(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(3*a^2*b*c^(1/2) + 3*a^(1/2)*b*c^2 + a*
c*(9*a^(1/2)*b + 9*b*c^(1/2))))/(2*a*c^4 - 2*a^4*c - 6*a^2*c^3 + 6*a^3*c^2)
 - ((a^(1/2)*((3*a*b)/4 + (b*c)/4))/(a*c^3 + 3*a^3*c - a^4 - 3*a^2*c^2) - (
c^(1/2)*((a*b)/4 + (3*b*c)/4))/(3*a*c^3 + a^3*c - c^4 - 3*a^2*c^2) - (((a^(
1/2)*((3*a^3*b)/4 - (b*c^3)/4 - (a*b*c^2)/2 + 17*a^2*b*c))/(a^5*c - a^2*c^4
 + 3*a^3*c^3 - 3*a^4*c^2) + (c^(1/2)*((a^3*b)/4 - (3*b*c^3)/4 - 17*a*b*c^2
+ (a^2*b*c)/2))/(a*c^5 - 3*a^2*c^4 + 3*a^3*c^3 - a^4*c^2))*((a + b*x)^(1/2)
 - a^(1/2))^3)/((c + b*x)^(1/2) - c^(1/2))^3 + (((a^(1/2)*((b*c^3)/4 - a^3*
b + (75*a*b*c^2)/4 + 15*a^2*b*c))/(a^5*c - a^2*c^4 + 3*a^3*c^3 - 3*a^4*c^2)
 - (c^(1/2)*((a^3*b)/4 - b*c^3 + 15*a*b*c^2 + (75*a^2*b*c)/4))/(a*c^5 - 3*a
^2*c^4 + 3*a^3*c^3 - a^4*c^2))*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1
/2) - c^(1/2))^2 + (((a^(1/2)*((a^2*b)/4 - 2*b*c^2 + (67*a*b*c)/4))/(a*c^4
- a^4*c - 3*a^2*c^3 + 3*a^3*c^2) + (c^(1/2)*((b*c^2)/4 - 2*a^2*b + (67*a*b*
c)/4))/(a*c^4 - a^4*c - 3*a^2*c^3 + 3*a^3*c^2))*((a + b*x)^(1/2) - a^(1/2))
)/((c + b*x)^(1/2) - c^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2
) - c^(1/2)) + ((a + b*x)^(1/2) - a^(1/2))^4/((c + b*x)^(1/2) - c^(1/2))^4
- (((a + c)/(a^(1/2)*c^(1/2)) - 1)*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x
)^(1/2) - c^(1/2))^2 - (((a + c)/(a^(1/2)*c^(1/2)) - 1)*((a + b*x)^(1/2) -
a^(1/2))^3)/((c + b*x)^(1/2) - c^(1/2))^3) - (b*((a + b*x)^(1/2) - a^(1/2))
)/(4*a^(1/2)*c^(1/2)*(a^(1/2) - c^(1/2))^3*((c + b*x)^(1/2) - c^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

3.416 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}} \, dx$

Optimal. Leaf size=21

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

```
[Out] -2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2106, 30, 32}

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x}+\sqrt{1+x}} \, dx &= -\int \sqrt{x} \, dx+\int \sqrt{1+x} \, dx \\ &= -\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

fricas [A] time = 0.45, size = 13, normalized size = 0.62

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}-\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

giac [A] time = 0.23, size = 13, normalized size = 0.62

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$-\frac{2x^{\frac{3}{2}}}{3} + \frac{2(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -2/3*x^(3/2)+2/3*(x+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

mupad [B] time = 2.97, size = 21, normalized size = 1.00

$$\frac{2x\sqrt{x+1}}{3} + \frac{2\sqrt{x+1}}{3} - \frac{2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + x^(1/2)),x)
```

```
[Out] (2*x*(x + 1)^(1/2))/3 + (2*(x + 1)^(1/2))/3 - (2*x^(3/2))/3
```

sympy [B] time = 0.94, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\sqrt{x+1}}{3\sqrt{x} + 3\sqrt{x+1}} + \frac{4x}{3\sqrt{x} + 3\sqrt{x+1}} + \frac{2}{3\sqrt{x} + 3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

3.417 $\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

```
[Out] -2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2106, 30, 32}

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx &= -\int \sqrt{-1+x}\,dx+\int \sqrt{x}\,dx\\ &= -\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

fricas [A] time = 0.43, size = 13, normalized size = 0.62

$$-\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

giac [A] time = 0.19, size = 13, normalized size = 0.62

$$-\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2x^{\frac{3}{2}}}{3}-\frac{2(x-1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x-1)^(1/2)+x^(1/2)),x)
```

```
[Out] -2/3*(x-1)^(3/2)+2/3*x^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x-1}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x - 1) + sqrt(x)), x)
```

mupad [B] time = 2.94, size = 21, normalized size = 1.00

$$\frac{2\sqrt{x-1}}{3}-\frac{2x\sqrt{x-1}}{3}+\frac{2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^(1/2) + x^(1/2)),x)
```

```
[Out] (2*(x - 1)^(1/2))/3 - (2*x*(x - 1)^(1/2))/3 + (2*x^(3/2))/3
```

sympy [B] time = 0.39, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\sqrt{x-1}}{3\sqrt{x}+3\sqrt{x-1}}+\frac{4x}{3\sqrt{x}+3\sqrt{x-1}}-\frac{2}{3\sqrt{x}+3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x - 1)/(3*sqrt(x) + 3*sqrt(x - 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x - 1)) - 2/(3*sqrt(x) + 3*sqrt(x - 1))
```

3.418 $\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

```
[Out] -1/3*(-1+x)^(3/2)+1/3*(1+x)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6689}

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] -(-1 + x)^(3/2)/3 + (1 + x)^(3/2)/3
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx &= -\left(\frac{1}{2}\int \left(\sqrt{-1+x}-\sqrt{1+x}\right)\,dx\right)\\ &= -\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] -1/3*(-1 + x)^(3/2) + (1 + x)^(3/2)/3
```

fricas [A] time = 0.45, size = 15, normalized size = 0.65

$$\frac{1}{3}\,(x+1)^{\frac{3}{2}}-\frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

giac [A] time = 0.19, size = 15, normalized size = 0.65

$$\frac{1}{3}\,(x+1)^{\frac{3}{2}}-\frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.70

$$-\frac{(x-1)^{\frac{3}{2}}}{3}+\frac{(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x-1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -1/3*(x-1)^(3/2)+1/3*(x+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + 1) + sqrt(x - 1)), x)
```

mupad [B] time = 2.84, size = 15, normalized size = 0.65

$$\frac{(x+1)^{3/2}}{3}-\frac{(x-1)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

```
[Out] (x + 1)^(3/2)/3 - (x - 1)^(3/2)/3
```

sympy [B] time = 0.41, size = 51, normalized size = 2.22

$$\frac{4x}{3\sqrt{x-1}+3\sqrt{x+1}}+\frac{2\sqrt{x-1}\sqrt{x+1}}{3\sqrt{x-1}+3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 4*x/(3*sqrt(x - 1) + 3*sqrt(x + 1)) + 2*sqrt(x - 1)*sqrt(x + 1)/(3*sqrt(x -
 1) + 3*sqrt(x + 1))
```

3.419 $\int x^3 \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

Optimal. Leaf size=38

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] 1/2*x^4-2/3*(-x^2+1)^(3/2)+2/5*(-x^2+1)^(5/2)
```

Rubi [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6742, 266, 43}

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx &= \int \left(2x^3+2x^3\sqrt{1-x^2}\right) dx\\ &= \frac{x^4}{2}+2\int x^3\sqrt{1-x^2}\, dx\\ &= \frac{x^4}{2}+\text{Subst}\left(\int \sqrt{1-x}\,x\,dx,x,x^2\right)\\ &= \frac{x^4}{2}+\text{Subst}\left(\int \left(\sqrt{1-x}-(1-x)^{3/2}\right) dx,x,x^2\right)\\ &= \frac{x^4}{2}-\frac{2}{3}\left(1-x^2\right)^{3/2}+\frac{2}{5}\left(1-x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 1.00

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

fricas [A] time = 0.48, size = 32, normalized size = 0.84

$$\frac{1}{2}x^4 + \frac{2}{15}\left(3x^4 - x^2 - 2\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^4 + 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

giac [B] time = 0.22, size = 77, normalized size = 2.03

$$\frac{1}{2}x^4+\frac{1}{60}\left(\left(2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{12}\left(\left(2\left(3x-10\right)(x+1)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*x^4 + 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 19
5)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 3
9)*sqrt(x + 1)*sqrt(-x + 1)
```

maple [A] time = 0.01, size = 33, normalized size = 0.87

$$\frac{x^4}{2} + \frac{2\sqrt{x+1}\sqrt{-x+1}\left(x^2-1\right)\left(3x^2+2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1-x)^(1/2)+(x+1)^(1/2))^2,x)
```

```
[Out] 1/2*x^4+2/15*(x+1)^(1/2)*(1-x)^(1/2)*(x^2-1)*(3*x^2+2)
```

maxima [A] time = 1.31, size = 31, normalized size = 0.82

$$\frac{1}{2}x^4 - \frac{2}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2 - \frac{4}{15}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^4 - 2/5*(-x^2 + 1)^(3/2)*x^2 - 4/15*(-x^2 + 1)^(3/2)
```

mupad [B] time = 3.02, size = 45, normalized size = 1.18

$$\frac{x^4}{2} - \frac{\sqrt{1-x}\left(-\frac{2x^5}{5} - \frac{2x^4}{5} + \frac{2x^3}{15} + \frac{2x^2}{15} + \frac{4x}{15} + \frac{4}{15}\right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] x^4/2 - ((1 - x)^(1/2)*((4*x)/15 + (2*x^2)/15 + (2*x^3)/15 - (2*x^4)/5 - (2
*x^5)/5 + 4/15))/(x + 1)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Timed out
```

3.420 $\int x^2 \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx$

Optimal. Leaf size=48

$$\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}\,x+\frac{1}{2}\sqrt{1-x^2}\,x^3+\frac{1}{4}\sin^{-1}(x)$$

```
[Out] 2/3*x^3+1/4*arcsin(x)-1/4*x*(-x^2+1)^(1/2)+1/2*x^3*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6742, 279, 321, 216}

$$\frac{1}{2}\sqrt{1-x^2}\,x^3+\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}\,x+\frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (2*x^3)/3 - (x*Sqrt[1 - x^2])/4 + (x^3*Sqrt[1 - x^2])/2 + ArcSin[x]/4
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx&=\int\left(2x^2+2x^2\sqrt{1-x^2}\right)dx\\&=\frac{2x^3}{3}+2\int x^2\sqrt{1-x^2}\,dx\\&=\frac{2x^3}{3}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}\,dx\\&=\frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{4}\int\frac{1}{\sqrt{1-x^2}}\,dx\\&=\frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{4}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.88

$$\frac{1}{12}\left(-3\sqrt{1-x^2}\,x+\left(6\sqrt{1-x^2}+8\right)x^3+3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (-3*x*Sqrt[1 - x^2] + x^3*(8 + 6*Sqrt[1 - x^2]) + 3*ArcSin[x])/12
```

fricas [A] time = 0.44, size = 51, normalized size = 1.06

$$\frac{2}{3}x^3+\frac{1}{4}\left(2x^3-x\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2/3*x^3 + 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arctan((sqrt(x + 1
)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 0.23, size = 76, normalized size = 1.58

$$\frac{2}{3}x^3+\frac{1}{12}((2(3x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{3}((2x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/3*x^3 + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(
-x + 1) + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arcsin
(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.01, size = 59, normalized size = 1.23

$$\frac{2x^3}{3}+\frac{\sqrt{x+1}\sqrt{-x+1}\left(2\sqrt{-x^2+1}\,x^3-\sqrt{-x^2+1}\,x+\arcsin(x)\right)}{4\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((-x+1)^(1/2)+(x+1)^(1/2))^2,x)
```

```
[Out] 2/3*x^3+1/4*(x+1)^(1/2)*(-x+1)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)
+arcsin(x))/(-x^2+1)^(1/2)
```

maxima [A] time = 1.40, size = 34, normalized size = 0.71

$$\frac{2}{3}x^3 - \frac{1}{2}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{1}{4}\sqrt{-x^2+1}\,x + \frac{1}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2/3*x^3 - 1/2*(-x^2 + 1)^(3/2)*x + 1/4*sqrt(-x^2 + 1)*x + 1/4*arcsin(x)
```

mupad [B] time = 14.09, size = 563, normalized size = 11.73

$$\frac{\frac{4\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{28\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{4\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+1}-\operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{3\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}+\frac{23\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}-333\left(\sqrt{1\cdots}\right.}{\frac{8\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{28\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+56\left(\sqrt{1\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] ((4*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/(
(x + 1)^(1/2) - 1)^3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (
4*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/
((x + 1)^(1/2) - 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (
4*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x
+ 1)^(1/2) - 1)^8 + 1) - atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((
3*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) + (23*((1 - x)^(1/2) - 1)^3)/((x
 + 1)^(1/2) - 1)^3 - (333*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 + (6
71*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7 - (671*((1 - x)^(1/2) - 1)^
9)/((x + 1)^(1/2) - 1)^9 + (333*((1 - x)^(1/2) - 1)^11)/((x + 1)^(1/2) - 1)
^11 - (23*((1 - x)^(1/2) - 1)^13)/((x + 1)^(1/2) - 1)^13 - (3*((1 - x)^(1/2
) - 1)^15)/((x + 1)^(1/2) - 1)^15)/((8*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2
) - 1)^2 + (28*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (56*((1 - x)^
(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + (70*((1 - x)^(1/2) - 1)^8)/((x + 1)^(
1/2) - 1)^8 + (56*((1 - x)^(1/2) - 1)^10)/((x + 1)^(1/2) - 1)^10 + (28*((1
- x)^(1/2) - 1)^12)/((x + 1)^(1/2) - 1)^12 + (8*((1 - x)^(1/2) - 1)^14)/((x
 + 1)^(1/2) - 1)^14 + ((1 - x)^(1/2) - 1)^16/((x + 1)^(1/2) - 1)^16 + 1) +
(2*x^3)/3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Timed out
```

3.421 $\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

Optimal. Leaf size=19

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] x^2-2/3*(-x^2+1)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6742, 261}

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^2 - (2*(1 - x^2)^(3/2))/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx &= \int \left(2x+2x\sqrt{1-x^2}\right) dx\\ &= x^2+2\int x\sqrt{1-x^2}\,dx\\ &= x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^2 - (2*(1 - x^2)^(3/2))/3
```

fricas [A] time = 0.49, size = 23, normalized size = 1.21

$$x^2+\frac{2}{3}\left(x^2-1\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] x^2 + 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

giac [B] time = 0.25, size = 51, normalized size = 2.68

$$(x+1)^2 + \frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1} + \sqrt{x+1}\,(x-2)\sqrt{-x+1} - 2x - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] (x + 1)^2 + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x +
 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2
```

maple [A] time = 0.00, size = 24, normalized size = 1.26

$$x^2 + \frac{2\sqrt{x+1}\,\sqrt{-x+1}\,\left(x^2-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-x+1)^(1/2)+(x+1)^(1/2))^2,x)
```

```
[Out] x^2+2/3*(x+1)^(1/2)*(-x+1)^(1/2)*(x^2-1)
```

maxima [A] time = 1.34, size = 15, normalized size = 0.79

$$x^2 - \frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] x^2 - 2/3*(-x^2 + 1)^(3/2)
```

mupad [B] time = 2.98, size = 33, normalized size = 1.74

$$x^2 - \frac{\sqrt{1-x}\,\left(-\frac{2x^3}{3} - \frac{2x^2}{3} + \frac{2x}{3} + \frac{2}{3}\right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] x^2 - ((1 - x)^(1/2)*((2*x)/3 - (2*x^2)/3 - (2*x^3)/3 + 2/3))/(x + 1)^(1/2)
```

sympy [A] time = 106.39, size = 110, normalized size = 5.79

$$-\frac{x^3}{3} - x + \frac{(x+1)^3}{3} - 4\left(\begin{cases} \frac{x\sqrt{1-x}\,\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x \ge -1 \wedge x < 1 \end{cases}\right) + 4\left(\begin{cases} \frac{x\sqrt{1-x}\,\sqrt{x+1}}{4} - \frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\operatorname{asin}\left(\right.}{}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] -x**3/3 - x + (x + 1)**3/3 - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asi
n(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) + 4*Piecewise((x*sqrt(1 -
 x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x +
 1)/2)/2, (x >= -1) & (x < 1))) - 1
```

3.422 $\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx$

Optimal. Leaf size=19

$$\sqrt{1-x^2}\,x+2x+\sin^{-1}(x)$$

```
[Out] 2*x+arcsin(x)+x*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6742, 195, 216}

$$\sqrt{1-x^2}\,x+2x+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] 2*x + x*Sqrt[1 - x^2] + ArcSin[x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx &= \int \left(2+2\sqrt{1-x^2}\right) dx\\
&= 2x+2\int \sqrt{1-x^2}\, dx\\
&= 2x+x\sqrt{1-x^2}+\int \frac{1}{\sqrt{1-x^2}}\, dx\\
&= 2x+x\sqrt{1-x^2}+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.95

$$x\left(\sqrt{1-x^2}+2\right)+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x*(2 + Sqrt[1 - x^2]) + ArcSin[x]
```

fricas [B] time = 0.44, size = 40, normalized size = 2.11

$$\sqrt{x+1}\,x\sqrt{-x+1}+2\,x-2\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] sqrt(x + 1)*x*sqrt(-x + 1) + 2*x - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/
x)
```

giac [B] time = 0.20, size = 48, normalized size = 2.53

$$\sqrt{x+1}\,(x-2)\sqrt{-x+1}+2\,x+2\,\sqrt{x+1}\,\sqrt{-x+1}+2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2*sqrt(x + 1)*sqrt(-x + 1) + 2*arc
sin(1/2*sqrt(2)*sqrt(x + 1)) + 2
```

maple [B] time = 0.01, size = 58, normalized size = 3.05

$$2x+\frac{\sqrt{(x+1)(-x+1)}\;\arcsin(x)}{\sqrt{-x+1}\;\sqrt{x+1}}-\sqrt{x+1}\;(-x+1)^{\frac{3}{2}}+\sqrt{-x+1}\;\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)^(1/2)+(x+1)^(1/2))^2,x)
```

```
[Out] 2*x-(x+1)^(1/2)*(-x+1)^(3/2)+(-x+1)^(1/2)*(x+1)^(1/2)+((x+1)*(-x+1))^(1/2)/
(-x+1)^(1/2)/(x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 1.42, size = 17, normalized size = 0.89

$$\sqrt{-x^2+1}\,x+2\,x+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1)*x + 2*x + arcsin(x)
```

mupad [B] time = 7.72, size = 206, normalized size = 10.84

$$2\,x-4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{4\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{28\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{4\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] 2*x - 4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((4*((1 - x)^(1/2)
- 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^
3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2) -
1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)
```

```
^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) -
1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 +
 1)
```

sympy [A] time = 31.43, size = 44, normalized size = 2.32

$$2x + 4\left(\begin{cases} \frac{x\sqrt{1-x}\,\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x \geq -1 \wedge x < 1 \end{cases}\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] 2*x + 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/2
)/2, (x >= -1) & (x < 1))) + 2
```

3.423 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x} \, dx$

Optimal. Leaf size=32

$$2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)$$

```
[Out] -2*arctanh((-x^2+1)^(1/2))+2*ln(x)+2*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6742, 266, 50, 63, 206}

$$2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = \int \left(\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}\right)dx$$
$$= 2\log(x) + 2\int \frac{\sqrt{1-x^2}}{x}\,dx$$
$$= 2\log(x) + \text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, x^2\right)$$
$$= 2\sqrt{1-x^2} + 2\log(x) + \text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^2\right)$$
$$= 2\sqrt{1-x^2} + 2\log(x) - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right)$$
$$= 2\sqrt{1-x^2} - 2\tanh^{-1}\left(\sqrt{1-x^2}\right) + 2\log(x)$$

Mathematica [A] time = 0.04, size = 32, normalized size = 1.00

$$2\sqrt{1-x^2} - 2\tanh^{-1}\left(\sqrt{1-x^2}\right) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

fricas [A] time = 0.46, size = 41, normalized size = 1.28

$$2\sqrt{x+1}\sqrt{-x+1} + 2\log(x) + 2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1)*sqrt(-x + 1) + 2*log(x) + 2*log((sqrt(x + 1)*sqrt(-x + 1) - 1
)/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086]2*ln(abs(sqrt(x+1)-1))+2*ln(sqrt(x+1)+1)+2*sqrt(x+1)*sq
rt(-x+1)-2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1
)+2*sqrt(2))/sqrt(x+1)))+2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1
/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))
```

maple [A] time = 0.01, size = 51, normalized size = 1.59

$$2\ln(x) + \frac{2\sqrt{x+1}\,\sqrt{-x+1}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)+\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)^(1/2)+(x+1)^(1/2))^2/x,x)
```

```
[Out] 2*ln(x)+2*(x+1)^(1/2)*(-x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1
/(-x^2+1)^(1/2)))
```

maxima [A] time = 1.53, size = 41, normalized size = 1.28

$$2\sqrt{-x^2+1}+2\log(x)-2\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 1) + 2*log(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 4.10, size = 122, normalized size = 3.81

$$2\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-2\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+2\ln(x)+\frac{16\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\left(\frac{2\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

```
[Out] 2*log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(((1 - x)^(1/
2) - 1)/((x + 1)^(1/2) - 1)) + 2*log(x) + (16*((1 - x)^(1/2) - 1)^2)/(((x +
 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1 - x
)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x, x)
```

3.424 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}-2\sin^{-1}(x)$$

```
[Out] -2/x-2*arcsin(x)-2*(-x^2+1)^(1/2)/x
```

Rubi [A] time = 0.08, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6742, 277, 216}

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}-2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

```
[Out] -2/x - (2*Sqrt[1 - x^2])/x - 2*ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx &= \int \left(\frac{2}{x^2}+\frac{2\sqrt{1-x^2}}{x^2}\right)dx\\
&= -\frac{2}{x}+2\int \frac{\sqrt{1-x^2}}{x^2}\,dx\\
&= -\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.85

$$-\frac{2\left(\sqrt{1-x^2}+x\sin^{-1}(x)+1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
[Out] (-2*(1 + Sqrt[1 - x^2] + x*ArcSin[x]))/x
```

fricas [**A**] time = 0.45, size = 44, normalized size = 1.69

$$\frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\,\sqrt{-x+1}\,-1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="fricas")
[Out] 2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1)
- 1)/x
```

giac [**B**] time = 0.26, size = 149, normalized size = 5.73

$$-2\,\pi-\frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^{2}-4}-\frac{2}{x}-4\,\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="giac")
[Out] -2*pi - 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sq
rt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2)
- sqrt(-x + 1)))^2 - 4) - 2/x - 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-
x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

maple [**B**] time = 0.02, size = 50, normalized size = 1.92

$$-\frac{2}{x}+\frac{2\left(-x\arcsin(x)-\sqrt{-x^{2}+1}\right)\sqrt{x+1}\,\sqrt{-x+1}}{\sqrt{-x^{2}+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)^(1/2)+(x+1)^(1/2))^2/x^2,x)
[Out] -2/x+2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(x+1)^(1/2)*(-x+1)^(1/2)/x/(-x^2+1)^(1
/2)
```

maxima [**A**] time = 1.26, size = 24, normalized size = 0.92

$$-\frac{2\sqrt{-x^{2}+1}}{x}-\frac{2}{x}-2\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="maxima")
[Out] -2*sqrt(-x^2 + 1)/x - 2/x - 2*arcsin(x)
```

mupad [**B**] time = 3.79, size = 120, normalized size = 4.62

$$8\,\mathrm{atan}\left(\frac{\sqrt{1-x}\,-1}{\sqrt{x+1}\,-1}\right)-\frac{\frac{5\left(\sqrt{1-x}-1\right)^{2}}{2\left(\sqrt{x+1}-1\right)^{2}}-\frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{\left(\sqrt{1-x}-1\right)^{3}}{\left(\sqrt{x+1}-1\right)^{3}}}-\frac{\sqrt{1-x}\,-1}{2\left(\sqrt{x+1}\,-1\right)}-\frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

```
[Out] 8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((5*((1 - x)^(1/2) - 1)^2
)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/((((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)
 - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3) - ((1 - x)^(1/2) - 1)/(2*((
x + 1)^(1/2) - 1)) - 2/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**2,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**2, x)
```

3.425 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3} dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -1/x^2+arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.09, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6742, 266, 47, 63, 206}

$$-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

```
[Out] -x^(-2) - Sqrt[1 - x^2]/x^2 + ArcTanh[Sqrt[1 - x^2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = \int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)dx$$
$$= -\frac{1}{x^2}+2\int \frac{\sqrt{1-x^2}}{x^3}\,dx$$
$$= -\frac{1}{x^2}+\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,x^2\right)$$
$$= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)$$
$$= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)$$
$$= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Mathematica [A] time = 0.04, size = 45, normalized size = 1.32

$$-\frac{1}{x^2\sqrt{1-x^2}}+\frac{1}{\sqrt{1-x^2}}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
[Out] -x^(-2) + 1/Sqrt[1 - x^2] - 1/(x^2*Sqrt[1 - x^2]) + ArcTanh[Sqrt[1 - x^2]]
```

fricas [A] time = 0.44, size = 44, normalized size = 1.29

$$-\frac{x^2\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\,\sqrt{-x+1}+1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="fricas")
[Out] -(x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)
/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086]-(4*(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-1/2*(-2*sqrt
(-x+1)+2*sqrt(2))/sqrt(x+1))^3+16*(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-1/
2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))/((2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt
(2))-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1))^2-4)^2+ln(abs(2*sqrt(x+1)/(-2
*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))-ln(abs(2
*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x
+1)))-1/x^2
```

maple [A] time = 0.02, size = 58, normalized size = 1.71

$$-\frac{1}{x^{2}}+\frac{\sqrt{x+1}\,\sqrt{-x+1}\,\left(x^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)-\sqrt{-x^{2}+1}\right)}{\sqrt{-x^{2}+1}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)^(1/2)+(x+1)^(1/2))^2/x^3,x)
```

```
[Out] -1/x^2+(x+1)^(1/2)*(-x+1)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/
2))/x^2/(-x^2+1)^(1/2)
```

maxima [A] time = 1.52, size = 54, normalized size = 1.59

$$-\sqrt{-x^{2}+1}-\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{x^{2}}-\frac{1}{x^{2}}+\log\left(\frac{2\sqrt{-x^{2}+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) - (-x^2 + 1)^(3/2)/x^2 - 1/x^2 + log(2*sqrt(-x^2 + 1)/abs(x
) + 2/abs(x))
```

mupad [B] time = 4.88, size = 189, normalized size = 5.56

$$\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\ln\left(\frac{\left(\sqrt{1-x}-1\right)^{2}}{\left(\sqrt{x+1}-1\right)^{2}}-1\right)+\frac{\left(\sqrt{1-x}-1\right)^{2}}{16\left(\sqrt{x+1}-1\right)^{2}}-\frac{\frac{\left(\sqrt{1-x}-1\right)^{2}}{8\left(\sqrt{x+1}-1\right)^{2}}+\frac{15\left(\sqrt{1-x}-1\right)^{4}}{16\left(\sqrt{x+1}-1\right)^{4}}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^{2}}{\left(\sqrt{x+1}-1\right)^{2}}-\frac{2\left(\sqrt{1-x}-1\right)^{4}}{\left(\sqrt{x+1}-1\right)^{4}}+\frac{\left(\sqrt{1-x}-1\right)^{6}}{\left(\sqrt{x+1}-1\right)^{6}}}-\frac{1}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

```
[Out] log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - log(((1 - x)^(1/2) - 1)^2/((
x + 1)^(1/2) - 1)^2 - 1) + ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^2)
 - (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2) -
1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1)^(1
/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x)^(1/
2) - 1)^6/((x + 1)^(1/2) - 1)^6) - 1/x^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**3,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**3, x)
```

3.426 $\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

```
[Out] 2/3*a^2*(b*x+a)^(3/2)/b^3/(b-c)-4/5*a*(b*x+a)^(5/2)/b^3/(b-c)+2/7*(b*x+a)^(
7/2)/b^3/(b-c)-2/3*a^2*(c*x+a)^(3/2)/(b-c)/c^3+4/5*a*(c*x+a)^(5/2)/(b-c)/c^
3-2/7*(c*x+a)^(7/2)/(b-c)/c^3
```

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2103, 43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(b -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a + c*x)^(3/2))/(3*(b
- c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*c^3) - (2*(a + c*x)^(7/2))/(7*
(b - c)*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx &= \frac{\int x^2\sqrt{a+bx}\,dx}{b-c}-\frac{\int x^2\sqrt{a+cx}\,dx}{b-c}\\ &= \frac{\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{b-c}-\frac{\int\left(\frac{a^2\sqrt{a+cx}}{c^2}-\frac{2a(a+cx)^{3/2}}{c^2}+\frac{(a+cx)^{5/2}}{c^2}\right)dx}{b-c}\\ &= \frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3(b-c)c^3}+\frac{4a(a+cx)^{5/2}}{5(b-c)c^3}-\frac{2(}{7}\end{aligned}$$

Mathematica [A] time = 0.22, size = 147, normalized size = 1.00

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(b -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a + c*x)^(3/2))/(3*(b
- c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*c^3) - (2*(a + c*x)^(7/2))/(7*
(b - c)*c^3)
```

fricas [A] time = 0.45, size = 122, normalized size = 0.83

$$\frac{2\left(\left(15\,b^3c^3x^3+3\,ab^2c^3x^2-4\,a^2bc^3x+8\,a^3c^3\right)\sqrt{bx+a}-\left(15\,b^3c^3x^3+3\,ab^3c^2x^2-4\,a^2b^3cx+8\,a^3b^3\right)\sqrt{cx+a}\right)}{105\left(b^4c^3-b^3c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*b^3*c^3*x^3 + 3*a*b^2*c^3*x^2 - 4*a^2*b*c^3*x + 8*a^3*c^3)*sqrt(
b*x + a) - (15*b^3*c^3*x^3 + 3*a*b^3*c^2*x^2 - 4*a^2*b^3*c*x + 8*a^3*b^3)*s
qrt(c*x + a))/(b^4*c^3 - b^3*c^4)
```

giac [B] time = 0.43, size = 451, normalized size = 3.07

$$-\frac{2}{105}\sqrt{ab^2+(bx+a)bc-abc}\left(\left(3\,(bx+a)\left(\frac{5\left(b^{17}c^5|b|-2\,b^{16}c^6|b|+b^{15}c^7|b|\right)(bx+a)}{b^{23}c^5-3\,b^{22}c^6+3\,b^{21}c^7-b^{20}c^8}+\frac{ab^{18}c^4|b|-17\,ab^{17}c^5|b|}{b^{23}c^5-3\,b^{22}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/105*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*(b*x + a)*(5*(b^17*c^5*abs(b
) - 2*b^16*c^6*abs(b) + b^15*c^7*abs(b))*(b*x + a)/(b^23*c^5 - 3*b^22*c^6 +
 3*b^21*c^7 - b^20*c^8) + (a*b^18*c^4*abs(b) - 17*a*b^17*c^5*abs(b) + 31*a*
b^16*c^6*abs(b) - 15*a*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22*c^6 + 3*b^21*c^7
 - b^20*c^8)) - (4*a^2*b^19*c^3*abs(b) - 2*a^2*b^18*c^4*abs(b) - 53*a^2*b^1
7*c^5*abs(b) + 96*a^2*b^16*c^6*abs(b) - 45*a^2*b^15*c^7*abs(b))/(b^23*c^5 -
 3*b^22*c^6 + 3*b^21*c^7 - b^20*c^8))*(b*x + a) + (8*a^3*b^20*c^2*abs(b) -
12*a^3*b^19*c^3*abs(b) + 3*a^3*b^18*c^4*abs(b) - 17*a^3*b^17*c^5*abs(b) + 3
3*a^3*b^16*c^6*abs(b) - 15*a^3*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22*c^6 + 3*
b^21*c^7 - b^20*c^8)) + 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a +
35*(b*x + a)^(3/2)*a^2)/(b^4 - b^3*c)
```

maple [A] time = 0.00, size = 90, normalized size = 0.61

$$\frac{\frac{2(bx+a)^{\frac{3}{2}}a^2}{3}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+\frac{2(bx+a)^{\frac{7}{2}}}{7}}{(b-c)\,b^3}-\frac{2\left(\frac{(cx+a)^{\frac{3}{2}}a^2}{3}-\frac{2(cx+a)^{\frac{5}{2}}a}{5}+\frac{(cx+a)^{\frac{7}{2}}}{7}\right)}{(b-c)\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)/b^3*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2))-2
/(b-c)/c^3*(1/7*(c*x+a)^(7/2)-2/5*(c*x+a)^(5/2)*a+1/3*a^2*(c*x+a)^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

mupad [B] time = 2.92, size = 179, normalized size = 1.22

$$\frac{2x^3\sqrt{a+bx}}{7(b-c)}-\frac{2x^3\sqrt{a+cx}}{7(b-c)}+\frac{16a^3\sqrt{a+bx}}{105b^3(b-c)}-\frac{16a^3\sqrt{a+cx}}{105c^3(b-c)}+\frac{2ax^2\sqrt{a+bx}}{35b(b-c)}-\frac{8a^2x\sqrt{a+bx}}{105b^2(b-c)}-\frac{2ax^2\sqrt{a+cx}}{35c(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x^3*(a + b*x)^(1/2))/(7*(b - c)) - (2*x^3*(a + c*x)^(1/2))/(7*(b - c)) +
 (16*a^3*(a + b*x)^(1/2))/(105*b^3*(b - c)) - (16*a^3*(a + c*x)^(1/2))/(105
*c^3*(b - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(b - c)) - (8*a^2*x*(a + b*
x)^(1/2))/(105*b^2*(b - c)) - (2*a*x^2*(a + c*x)^(1/2))/(35*c*(b - c)) + (8
*a^2*x*(a + c*x)^(1/2))/(105*c^2*(b - c))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

3.427 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=95

$$\frac{2(a+bx)^{5/2}}{5b^2(b-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}-\frac{2(a+cx)^{5/2}}{5c^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

```
[Out] -2/3*a*(b*x+a)^(3/2)/b^2/(b-c)+2/5*(b*x+a)^(5/2)/b^2/(b-c)+2/3*a*(c*x+a)^(3
/2)/(b-c)/c^2-2/5*(c*x+a)^(5/2)/(b-c)/c^2
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2103, 43}

$$\frac{2(a+bx)^{5/2}}{5b^2(b-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}-\frac{2(a+cx)^{5/2}}{5c^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(b - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(b - c)
) + (2*a*(a + c*x)^(3/2))/(3*(b - c)*c^2) - (2*(a + c*x)^(5/2))/(5*(b - c)*
c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx &= \frac{\int x\sqrt{a+bx}\,dx}{b-c}-\frac{\int x\sqrt{a+cx}\,dx}{b-c}\\
&= \frac{\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{b-c}-\frac{\int \left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{b-c}\\
&= -\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}+\frac{2(a+bx)^{5/2}}{5b^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3(b-c)c^2}-\frac{2(a+cx)^{5/2}}{5(b-c)c^2}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 70, normalized size = 0.74

$$\frac{2\left(\frac{3(a+bx)^{5/2}}{b^2}-\frac{5a(a+bx)^{3/2}}{b^2}-\frac{3(a+cx)^{5/2}}{c^2}+\frac{5a(a+cx)^{3/2}}{c^2}\right)}{15(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*((-5*a*(a + b*x)^(3/2))/b^2 + (3*(a + b*x)^(5/2))/b^2 + (5*a*(a + c*x)^(
3/2))/c^2 - (3*(a + c*x)^(5/2))/c^2))/(15*(b - c))
```

fricas [A] time = 0.42, size = 92, normalized size = 0.97

$$\frac{2\left(\left(3\,b^2c^2x^2+abc^2x-2\,a^2c^2\right)\sqrt{bx+a}-\left(3\,b^2c^2x^2+ab^2cx-2\,a^2b^2\right)\sqrt{cx+a}\right)}{15\left(b^3c^2-b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*b^2*c^2*x^2 + a*b*c^2*x - 2*a^2*c^2)*sqrt(b*x + a) - (3*b^2*c^2*x^
2 + a*b^2*c*x - 2*a^2*b^2)*sqrt(c*x + a))/(b^3*c^2 - b^2*c^3)
```

giac [B] time = 0.39, size = 255, normalized size = 2.68

$$-\frac{2}{15}\sqrt{ab^2+(bx+a)bc-abc}\left((bx+a)\left(\frac{3\left(b^9c^3|b|-b^8c^4|b|\right)(bx+a)}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}+\frac{ab^{10}c^2|b|-7\,ab^9c^3|b|+6\,ab^8c^4|b|}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}\right)-\frac{2\,a^2b^1}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/15*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*(3*(b^9*c^3*abs(b) - b
^8*c^4*abs(b))*(b*x + a)/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5) + (a*b^10*c^2*a
bs(b) - 7*a*b^9*c^3*abs(b) + 6*a*b^8*c^4*abs(b))/(b^14*c^3 - 2*b^13*c^4 + b
^12*c^5)) - (2*a^2*b^11*c*abs(b) - a^2*b^10*c^2*abs(b) - 4*a^2*b^9*c^3*abs(
b) + 3*a^2*b^8*c^4*abs(b))/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5)) + 2/15*(3*(b
*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(b^3 - b^2*c)
```

maple [A] time = 0.00, size = 66, normalized size = 0.69

$$\frac{-\frac{2(bx+a)^{\frac{3}{2}}a}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}}{(b-c)\,b^2}-\frac{2\left(-\frac{(cx+a)^{\frac{3}{2}}a}{3}+\frac{(cx+a)^{\frac{5}{2}}}{5}\right)}{(b-c)\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)/b^2*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))-2/(b-c)/c^2*(1/5*(c*x+
a)^(5/2)-1/3*a*(c*x+a)^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

mupad [B] time = 2.86, size = 129, normalized size = 1.36

$$\frac{2\,x^2\,\sqrt{a+b\,x}}{5\,(b-c)}-\frac{2\,x^2\,\sqrt{a+c\,x}}{5\,(b-c)}-\frac{4\,a^2\,\sqrt{a+b\,x}}{15\,b^2\,(b-c)}+\frac{4\,a^2\,\sqrt{a+c\,x}}{15\,c^2\,(b-c)}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{15\,b\,(b-c)}-\frac{2\,a\,x\,\sqrt{a+c\,x}}{15\,c\,(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x^2*(a + b*x)^(1/2))/(5*(b - c)) - (2*x^2*(a + c*x)^(1/2))/(5*(b - c)) -
 (4*a^2*(a + b*x)^(1/2))/(15*b^2*(b - c)) + (4*a^2*(a + c*x)^(1/2))/(15*c^2
*(b - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(b - c)) - (2*a*x*(a + c*x)^(1/2)
)/(15*c*(b - c))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

3.428 $\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2103, 32}

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
[Out] (2*(a + b*x)^(3/2))/(3*b*(b - c)) - (2*(a + c*x)^(3/2))/(3*(b - c)*c)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx &= \frac{\int \sqrt{a+bx}\,dx}{b-c}-\frac{\int \sqrt{a+cx}\,dx}{b-c}\\ &= \frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3(b-c)c}\end{aligned}$$

Mathematica [A] time = 0.09, size = 39, normalized size = 0.83

$$\frac{2\left(\frac{(a+bx)^{3/2}}{b}-\frac{(a+cx)^{3/2}}{c}\right)}{3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
[Out] (2*((a + b*x)^(3/2)/b - (a + c*x)^(3/2)/c))/(3*(b - c))
```

fricas [A] time = 0.44, size = 50, normalized size = 1.06

$$\frac{2\left((bcx+ac)\sqrt{bx+a}-(bcx+ab)\sqrt{cx+a}\right)}{3\left(b^2c-bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*((b*c*x + a*c)*sqrt(b*x + a) - (b*c*x + a*b)*sqrt(c*x + a))/(b^2*c - b*
c^2)
```

giac [B] time = 0.32, size = 107, normalized size = 2.28

$$-\frac{2\left(\left(\frac{(bx+a)b^2c|b|}{b^5c-b^4c^2}+\frac{ab^3|b|-ab^2c|b|}{b^5c-b^4c^2}\right)\sqrt{ab^2+(bx+a)bc-abc}-\frac{(bx+a)^{\frac{3}{2}}}{b-c}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*(((b*x + a)*b^2*c*abs(b)/(b^5*c - b^4*c^2) + (a*b^3*abs(b) - a*b^2*c*a
bs(b))/(b^5*c - b^4*c^2))*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c) - (b*x + a)^(
3/2)/(b - c))/b
```

maple [A] time = 0.00, size = 40, normalized size = 0.85

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(b-c)\,b}-\frac{2\,(cx+a)^{\frac{3}{2}}}{3\,(b-c)\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

mupad [B] time = 2.91, size = 79, normalized size = 1.68

$$\frac{2\,x\,\sqrt{a+b\,x}}{3\,\left(b-c\right)}-\frac{2\,x\,\sqrt{a+c\,x}}{3\,\left(b-c\right)}+\frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,\left(b-c\right)}-\frac{2\,a\,\sqrt{a+c\,x}}{3\,c\,\left(b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x*(a + b*x)^(1/2))/(3*(b - c)) - (2*x*(a + c*x)^(1/2))/(3*(b - c)) + (2*
a*(a + b*x)^(1/2))/(3*b*(b - c)) - (2*a*(a + c*x)^(1/2))/(3*c*(b - c))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

3.429 $\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)+2*arctanh((c*x+a)^(1/2)/a^(
1/2))*a^(1/2)/(b-c)+2*(b*x+a)^(1/2)/(b-c)-2*(c*x+a)^(1/2)/(b-c)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6690, 50, 63, 208}

$$\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (2*Sqrt[a + b*x])/(b - c) - (2*Sqrt[a + c*x])/(b - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(b - c) + (2*Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]
)/(b - c)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{\int \left(\frac{\sqrt{a+bx}}{x}-\frac{\sqrt{a+cx}}{x}\right)dx}{b-c}$$
$$= \frac{\int \frac{\sqrt{a+bx}}{x}dx}{b-c} - \frac{\int \frac{\sqrt{a+cx}}{x}dx}{b-c}$$
$$= \frac{2\sqrt{a+bx}}{b-c} - \frac{2\sqrt{a+cx}}{b-c} + \frac{a\int \frac{1}{x\sqrt{a+bx}}dx}{b-c} - \frac{a\int \frac{1}{x\sqrt{a+cx}}dx}{b-c}$$
$$= \frac{2\sqrt{a+bx}}{b-c} - \frac{2\sqrt{a+cx}}{b-c} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx, x, \sqrt{a+bx}\right)}{b(b-c)} - \frac{(2a)\,\text{Subst}\left(\int \cdots\right.}{(\cdots}$$
$$= \frac{2\sqrt{a+bx}}{b-c} - \frac{2\sqrt{a+cx}}{b-c} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c} + \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.77

$$\frac{2\left(\sqrt{a+bx}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\sqrt{a+cx}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)\right)}{b-c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (2*(Sqrt[a + b*x] - Sqrt[a + c*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
+ Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]))/(b - c)
```

fricas [A] time = 0.51, size = 158, normalized size = 1.63

$$\left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{cx+a}}{b-c}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-\cdots}}{a}\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*log((c*x
- 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(c*x + a))/(b
 - c), 2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*arctan(sqrt(
c*x + a)*sqrt(-a)/a) + sqrt(b*x + a) - sqrt(c*x + a))/(b - c)]
```

giac [B] time = 1.00, size = 1093, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*abs(b)/(b^3 - b^2*c) + 2*a*arctan(sq
rt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) + 2*sqrt(b*x + a)/(b - c) - 2*(2*(
a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(b - c) + 2*(a*b^
3 - a*b^2*c)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5 - 3*a^2*b^4*c
 + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*sqrt(-a*b*c)*abs(a*b^2 - a*b*c)*abs(b)*sgn(
b - c) + (a^2*b^6 - 3*a^2*b^5*c + 3*a^2*b^4*c^2 - a^2*b^3*c^3)*sqrt(-a)*abs
```

```
(a*b^2 - a*b*c)*abs(b) + (a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b
^3*c^5)*sqrt(-a)*abs(b)*sgn(b - c) + (a^3*b^7 - 2*a^3*b^6*c + 2*a^3*b^4*c^3
 - a^3*b^3*c^4)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqr
t(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^3 - a*b*c^2 + sqrt((a*b^3 - a*
b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*(b - c)))/
(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*
a^2*abs(a*b^2 - a*b*c)) + 2*(2*(a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt
(-a)*abs(b)*sgn(b - c) + 2*(a*b^3 - a*b^2*c)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)
*abs(b) + (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*sqrt(-a*b*c
)*abs(a*b^2 - a*b*c)*abs(b)*sgn(b - c) + (a^2*b^6 - 3*a^2*b^5*c + 3*a^2*b^4
*c^2 - a^2*b^3*c^3)*sqrt(-a)*abs(a*b^2 - a*b*c)*abs(b) + (a^3*b^7*c - 2*a^3
*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(b)*sgn(b - c) + (a^3*b
^7 - 2*a^3*b^6*c + 2*a^3*b^4*c^3 - a^3*b^3*c^4)*sqrt(-a*b*c)*abs(b))*arctan
(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*
b^3 - a*b*c^2 - sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b
^3*c^2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10
*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^2*abs(a*b^2 - a*b*c))
```

maple [A] time = 0.01, size = 73, normalized size = 0.75

$$\frac{-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}}{b-c}-\frac{-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)+2\sqrt{cx+a}}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 1/(b-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(b-c)*
(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

mupad [B] time = 4.33, size = 213, normalized size = 2.20

$$-\frac{2\sqrt{a}\,c\left(\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+\frac{\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)-2\sqrt{a}\,b\left(\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)-\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+4\right)}{(b-c)\left(b-\frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] -(2*a^(1/2)*c*((2*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))
+ (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^
(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2) - 2*a^(1/2)*b*(log(((a +
 b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))) - (2*((a + b*x)^(1/2) -
 a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 4))/((b - c)*(b - (c*((a + b*x)^(1
/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
[Out] Integral(1/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

3.430 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{a+bx}}{x(b-c)}+\frac{\sqrt{a+cx}}{x(b-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}$$

```
[Out] -b*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)/a^(1/2)+c*arctanh((c*x+a)^(1/2)/a^(
1/2))/(b-c)/a^(1/2)-(b*x+a)^(1/2)/(b-c)/x+(c*x+a)^(1/2)/(b-c)/x
```

Rubi [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2103, 47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x(b-c)}+\frac{\sqrt{a+cx}}{x(b-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((b - c)*x)) + Sqrt[a + c*x]/((b - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)) + (c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(
Sqrt[a]*(b - c))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \frac{\int \frac{\sqrt{a+bx}}{x^2}\,dx}{b-c} - \frac{\int \frac{\sqrt{a+cx}}{x^2}\,dx}{b-c}$$
$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} + \frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(b-c)} - \frac{c\int \frac{1}{x\sqrt{a+cx}}\,dx}{2(b-c)}$$
$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b-c} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\right.}{b-}$$
$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)} + \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}$$

Mathematica [A] time = 0.21, size = 135, normalized size = 1.31

$$\frac{-\frac{a}{\sqrt{a+bx}} - \frac{bx}{\sqrt{a+bx}} - \frac{bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{\sqrt{a+bx}} + \frac{a}{\sqrt{a+cx}} + \frac{cx}{\sqrt{a+cx}} + \frac{cx\sqrt{\frac{cx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx}{a}+1}\right)}{\sqrt{a+cx}}}{bx-cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] (-(a/Sqrt[a + b*x]) - (b*x)/Sqrt[a + b*x] + a/Sqrt[a + c*x] + (c*x)/Sqrt[a
+ c*x] - (b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[a + b*x] +
 (c*x*Sqrt[1 + (c*x)/a]*ArcTanh[Sqrt[1 + (c*x)/a]])/Sqrt[a + c*x])/(b*x - c
*x)
```

fricas [A] time = 0.49, size = 182, normalized size = 1.77

$$\left[-\frac{\sqrt{a}\,bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + \sqrt{a}\,cx\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right) + 2\sqrt{bx+a}\,a - 2\sqrt{cx+a}\,a}{2\,(ab-ac)x}, \frac{\sqrt{-a}\,bx\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*c
*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a)*a - 2*sqr
t(c*x + a)*a)/((a*b - a*c)*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/
a) - sqrt(-a)*c*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) - sqrt(b*x + a)*a + sqrt
(c*x + a)*a)/((a*b - a*c)*x)]
```

giac [B] time = 12.67, size = 1402, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) - 2*((sqrt(b*c)*sqrt(b*
x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b^2*c*abs(b) - (sqrt(b*c)*s
qrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b*c^2*abs(b) + (sqrt(
b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*c*abs(b))/((a^2
*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
```

```
+ (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
+ (b*x + a)*b*c - a*b*c))^4)*(b - c)) - sqrt(b*x + a)/((b - c)*x) + (2*(a*b
^3*c^2 - a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + 2*(
a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5*c - 3
*a^2*b^4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*sqrt(-a*b*c)*abs(-a*b^2 + a*b*c
)*abs(b)*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b^4*c^3 - a^2
*b^3*c^4)*sqrt(-a)*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 - 2*a^3*b^6*c^
3 + 2*a^3*b^4*c^5 - a^3*b^3*c^6)*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + (a^3*b^7
*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a*b*c)*abs(b))*arct
an(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(
a*b^3 - a*b*c^2 + sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2
*b^3*c^2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 -
10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c)) - (2*(a*b^3*c^2
- a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + 2*(a*b^3*c
 - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5*c - 3*a^2*b^
4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*sqrt(-a*b*c)*abs(-a*b^2 + a*b*c)*abs(b
)*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b^4*c^3 - a^2*b^3*c^
4)*sqrt(-a)*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 - 2*a^3*b^6*c^3 + 2*a
^3*b^4*c^5 - a^3*b^3*c^6)*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + (a^3*b^7*c - 2*
a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a*b*c)*abs(b))*arctan(-(sq
rt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^3 -
 a*b*c^2 - sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^
2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*
c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c))
```

maple [A] time = 0.01, size = 88, normalized size = 0.85

$$\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)b}{b-c}-\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{cx+a}}{2cx}\right)c}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-2/(b-c)*c*(-1/2*(c*x+a)^(1/2)/x/c-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1
/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

mupad [B] time = 10.93, size = 1637, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

```
[Out] (2*a*b - 2*a*c + a*c*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(
1/2))) - 2*a^(1/2)*b*(a + c*x)^(1/2) + 2*a^(1/2)*c*(a + b*x)^(1/2) + a*b*at
```

```
an((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1
i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a +
 b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(
1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*2i - a*c*atan((b^3*(a + b*x)^(1/2)*1i
- b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2
)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)
^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x
)^(1/2)))*2i + a*b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/
2))) + b*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b
*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i
)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2
)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*(a + c*x)^(
1/2)*2i - c*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a
+ b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)
*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(
1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*(a + c*x
)^(1/2)*2i + b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))
*(a + b*x)^(1/2)*(a + c*x)^(1/2) + c*log(((a + b*x)^(1/2) - a^(1/2))/((a +
c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2)*(a + c*x)^(1/2) - a^(1/2)*b*atan((b^
3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^
(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^
(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b
*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*2i - a^(1/2)*b*atan((b^3*(a
+ b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)
*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2)
 - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2
+ b*c^2*(a + c*x)^(1/2)))*(a + c*x)^(1/2)*2i + a^(1/2)*c*atan((b^3*(a + b*x
)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*
1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^
3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c
^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*2i + a^(1/2)*c*atan((b^3*(a + b*x)^(1/
2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i +
a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a
+ c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a
 + c*x)^(1/2)))*(a + c*x)^(1/2)*2i - a^(1/2)*b*log(((a + b*x)^(1/2) - a^(1/
2))/((a + c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2) - a^(1/2)*b*log(((a + b*x)
^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a + c*x)^(1/2) - a^(1/2)*c*
log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2
) - a^(1/2)*c*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*
(a + c*x)^(1/2))/(2*a^(1/2)*(b - c)*((a + b*x)^(1/2) - a^(1/2))*((a + c*x)^
(1/2) - a^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

3.431 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

Optimal. Leaf size=171

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{\sqrt{a+bx}}{2x^2(b-c)}+\frac{\sqrt{a+cx}}{2x^2(b-c)}-\frac{b\sqrt{a+bx}}{4ax(b-c)}+\frac{c\sqrt{a+cx}}{4ax(b-c)}$$

```
[Out] 1/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)/(b-c)-1/4*c^2*arctanh((c*x+a
)^(1/2)/a^(1/2))/a^(3/2)/(b-c)-1/2*(b*x+a)^(1/2)/(b-c)/x^2-1/4*b*(b*x+a)^(1
/2)/a/(b-c)/x+1/2*(c*x+a)^(1/2)/(b-c)/x^2+1/4*c*(c*x+a)^(1/2)/a/(b-c)/x
```

Rubi [A] time = 0.11, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2103, 47, 51, 63, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{\sqrt{a+bx}}{2x^2(b-c)}+\frac{\sqrt{a+cx}}{2x^2(b-c)}-\frac{b\sqrt{a+bx}}{4ax(b-c)}+\frac{c\sqrt{a+cx}}{4ax(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -Sqrt[a + b*x]/(2*(b - c)*x^2) - (b*Sqrt[a + b*x])/(4*a*(b - c)*x) + Sqrt[a
 + c*x]/(2*(b - c)*x^2) + (c*Sqrt[a + c*x])/(4*a*(b - c)*x) + (b^2*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(4*a^(3/2)*(b - c)) - (c^2*ArcTanh[Sqrt[a + c*x]/Sq
rt[a]])/(4*a^(3/2)*(b - c))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \frac{\int \frac{\sqrt{a+bx}}{x^3}\,dx}{b-c} - \frac{\int \frac{\sqrt{a+cx}}{x^3}\,dx}{b-c}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{b\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4(b-c)} - \frac{c\int \frac{1}{x^2\sqrt{a+cx}}\,dx}{4(b-c)}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2} - \frac{b\sqrt{a+bx}}{4a(b-c)x} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{c\sqrt{a+cx}}{4a(b-c)x} - \frac{b^2\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a(b-c)} + \frac{c^2 \dots}{8 \dots}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2} - \frac{b\sqrt{a+bx}}{4a(b-c)x} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{c\sqrt{a+cx}}{4a(b-c)x} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, \dots\right.}{4a(b-c)}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2} - \frac{b\sqrt{a+bx}}{4a(b-c)x} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{c\sqrt{a+cx}}{4a(b-c)x} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)} - \dots$$

Mathematica [C] time = 0.09, size = 75, normalized size = 0.44

$$\frac{2c^2(a+cx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx}{a}+1\right)-2b^2(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] (-2*b^2*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a] + 2*c^2
*(a + c*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/(3*a^3*(b - c
))
```

fricas [A] time = 0.49, size = 243, normalized size = 1.42

$$\left[-\frac{\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}\,c^2x^2\log\left(\frac{cx+2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\left(abx+2a^2\right)\sqrt{bx+a}-2\left(acx+2a^2\right)\sqrt{\dots}}{8\left(a^2b-a^2c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(
a)*c^2*x^2*log((c*x + 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(a*b*x + 2*a^2)
*sqrt(b*x + a) - 2*(a*c*x + 2*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2), -1
/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*c^2*x^2*ar
ctan(sqrt(c*x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a) - (a*c*x + 2
*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2)]
```

giac [B] time = 37.26, size = 1895, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/((a*b - a*c)*sqrt(-a)) - 1/2*((sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^6*c^2*abs(
b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*
b^5*c^3*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c -
a*b*c))*a^3*b^4*c^4*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
 a)*b*c - a*b*c))*a^3*b^3*c^5*abs(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*
b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^4*c^2*abs(b) - 10*(sqrt(b*c)*sqrt(b*x
 + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^3*c^3*abs(b) + 3*(sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^2*c^4*ab
s(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*
a*b^2*c^2*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
- a*b*c))^5*a*b*c^3*abs(b) + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
 a)*b*c - a*b*c))^7*c^2*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(
sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*
(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (
sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4)^2*(a*b -
a*c)) - 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((a*b - a*c)*b^2*x^
2) - 1/4*(2*(a*b^3*c^3 - a*b^2*c^4)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a)*abs(b)*s
gn(8*a*b - 8*a*c) + 2*(a*b^3*c^2 - a*b^2*c^3)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a
*b*c)*abs(b) + (a^3*b^5*c^2 - 3*a^3*b^4*c^3 + 3*a^3*b^3*c^4 - a^3*b^2*c^5)*
sqrt(-a*b*c)*abs(a^2*b^2 - a^2*b*c)*abs(b)*sgn(8*a*b - 8*a*c) + (a^3*b^6*c^
2 - 3*a^3*b^5*c^3 + 3*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(a^2*b^2 - a^2
*b*c)*abs(b) + (a^5*b^7*c^3 - 2*a^5*b^6*c^4 + 2*a^5*b^4*c^6 - a^5*b^3*c^7)*
sqrt(-a)*abs(b)*sgn(8*a*b - 8*a*c) + (a^5*b^7*c^2 - 2*a^5*b^6*c^3 + 2*a^5*b
^4*c^5 - a^5*b^3*c^6)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a)
 - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a^2*b^3 - a^2*b*c^2 + sqrt((
a^2*b^3 - a^2*b*c^2)^2 - (a^3*b^5 - 3*a^3*b^4*c + 3*a^3*b^3*c^2 - a^3*b^2*c
^3)*(a*b - a*c)))/(a*b - a*c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 +
 5*b^4*c^4 - b^3*c^5)*a^5*abs(a^2*b^2 - a^2*b*c)) + 1/4*(2*(a*b^3*c^3 - a*b
^2*c^4)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a)*abs(b)*sgn(8*a*b - 8*a*c) + 2*(a*b^3
*c^2 - a*b^2*c^3)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^3*b^5*c^2
- 3*a^3*b^4*c^3 + 3*a^3*b^3*c^4 - a^3*b^2*c^5)*sqrt(-a*b*c)*abs(a^2*b^2 - a
^2*b*c)*abs(b)*sgn(8*a*b - 8*a*c) + (a^3*b^6*c^2 - 3*a^3*b^5*c^3 + 3*a^3*b^
4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(a^2*b^2 - a^2*b*c)*abs(b) + (a^5*b^7*c^3
- 2*a^5*b^6*c^4 + 2*a^5*b^4*c^6 - a^5*b^3*c^7)*sqrt(-a)*abs(b)*sgn(8*a*b -
8*a*c) + (a^5*b^7*c^2 - 2*a^5*b^6*c^3 + 2*a^5*b^4*c^5 - a^5*b^3*c^6)*sqrt(-
a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))/sqrt(-(a^2*b^3 - a^2*b*c^2 - sqrt((a^2*b^3 - a^2*b*c^2)^2 - (a^
3*b^5 - 3*a^3*b^4*c + 3*a^3*b^3*c^2 - a^3*b^2*c^3)*(a*b - a*c)))/(a*b - a*c
)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^5*ab
s(a^2*b^2 - a^2*b*c))
```

maple [A] time = 0.01, size = 120, normalized size = 0.70

$$\frac{2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}+\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{b^{2}x^{2}}\right)b^{2}}{b-c}-\frac{2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}+\frac{-\frac{(cx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{cx+a}}{8}}{c^{2}x^{2}}\right)c^{2}}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/b^2/x^2+1/8/a^(3/2)*a
rctanh((b*x+a)^(1/2)/a^(1/2)))-2/(b-c)*c^2*((-1/8/a*(c*x+a)^(3/2)-1/8*(c*x+
a)^(1/2))/x^2/c^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

mupad [B] time = 11.85, size = 1610, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

```
[Out] ((a^(3/2)*b^3)/(16*(a^3*c^2 - a^3*b*c)) + (a^(3/2)*((a + b*x)^(1/2) - a^(1/
2))^2*((b*c^2)/4 - (7*b^2*c)/16 + b^3/4))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(
1/2) - a^(1/2))^2) - (a^(3/2)*((b^2*c)/16 + b^3/16)*((a + b*x)^(1/2) - a^(1
/2)))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(1/2) - a^(1/2))) + ((b^2/8 - c^2/8)*
((a + b*x)^(1/2) - a^(1/2))^3)/(a^(3/2)*c*((a + c*x)^(1/2) - a^(1/2))^3))/(
((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^(1/2) - a^(1/2))^4 - ((b + c)*((a
+ b*x)^(1/2) - a^(1/2))^3)/(c*((a + c*x)^(1/2) - a^(1/2))^3) + (b*((a + b*x
)^(1/2) - a^(1/2))^2)/(c*((a + c*x)^(1/2) - a^(1/2))^2)) - (((c*(b + c))/(4
*a^(3/2)*(b - c)) - (c*(b^2 - c^2))/(4*a^(3/2)*(b - c)^2))*((a + b*x)^(1/2)
 - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (log(((a + b*x)^(1/2) - a^(1/2))
/((a + c*x)^(1/2) - a^(1/2)))*(a^(3/2)*b^2 + a^(3/2)*c^2))/(8*a^3*b - 8*a^3
*c) + (atan((((b + c)*(((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 -
 a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128*
a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^
(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2))
+ ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*
b*c^2)*((a + c*x)^(1/2) - a^(1/2))))*1i)/(8*a^3) - ((b + c)*((16*a^3*b^4 +
16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2)) + ((b + c)*((64*a^6*b^3 - 64*a^6*b
*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3
 - 64*a^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*(
(a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)
^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2))))*
1i)/(8*a^3))/(((b + c)*(((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3
- a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128
*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a
^(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2))
 + ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6
*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - (b*c^4 - b^5)/(32*(a^6*c^3
 - a^6*b*c^2)) + ((b + c)*((16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b
*c^2)) + ((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) -
 (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128*a^6*b*c^2 - 12
8*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a
^3) - ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 -
a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) + (((a + b*x)^(1/2) - a^(
1/2))*(b*c^4 - b^4*c + b^2*c^3 - b^3*c^2))/(16*(a^6*c^3 - a^6*b*c^2)*((a +
c*x)^(1/2) - a^(1/2)))))*(a^(3/2)*b + a^(3/2)*c)*1i)/(4*a^3) + (c^2*((a + b
*x)^(1/2) - a^(1/2))^2)/(16*a^(3/2)*(b - c)*((a + c*x)^(1/2) - a^(1/2))^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

3.432 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=195

$$-\frac{a^3(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2(b-c)}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2}+\frac{ax^2}{(b-c)^2}-\frac{2(a+bx)^{3/2}}{3bc(b}$$

```
[Out] a*x^2/(b-c)^2+1/3*(b+c)*x^3/(b-c)^2-2/3*(b*x+a)^(3/2)*(c*x+a)^(3/2)/b/(b-c)
^2/c-1/4*a^3*(b+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))/b^(
5/2)/c^(5/2)+1/2*a*(b+c)*(b*x+a)^(3/2)*(c*x+a)^(1/2)/b^2/(b-c)^2/c+1/4*a^2*
(b+c)*(b*x+a)^(1/2)*(c*x+a)^(1/2)/b^2/(b-c)/c^2
```

Rubi [A] time = 0.35, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6690, 80, 50, 63, 217, 206}

$$\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2(b-c)}-\frac{a^3(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2}+\frac{ax^2}{(b-c)^2}-\frac{2(a+bx)^{3/2}(a}{3bc(b-}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (a*x^2)/(b - c)^2 + ((b + c)*x^3)/(3*(b - c)^2) + (a^2*(b + c)*Sqrt[a + b*x
]*Sqrt[a + c*x])/(4*b^2*(b - c)*c^2) + (a*(b + c)*(a + b*x)^(3/2)*Sqrt[a +
c*x])/(2*b^2*(b - c)^2*c) - (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*b*(b - c
)^2*c) - (a^3*(b + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x
])])/(4*b^(5/2)*c^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(2ax+b\left(1+\frac{c}{b}\right)x^2-2x\sqrt{a+bx}\,\sqrt{a+cx}\right)dx}{(b-c)^2}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}-\frac{2\int x\sqrt{a+bx}\,\sqrt{a+cx}\,dx}{(b-c)^2}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}+\frac{(a(b+c))\int \sqrt{a+bx}\,\sqrt{a+cx}\,dx}{b(b-c)^2c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}+\frac{(a}{}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}$$

Mathematica [A] time = 0.67, size = 238, normalized size = 1.22

$$\frac{\frac{3a^4(c-b)^3(b+c)\sqrt{\frac{b(a+cx)}{a(b-c)}}\sinh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a(b-c)}}\right)}{\sqrt{a(b-c)}\,\sqrt{a+cx}}+b\sqrt{c}\left(a^2\left(3b^2-2bc+3c^2\right)\sqrt{a+bx}\,\sqrt{a+cx}+4b^2c^2x^2\left(-2\sqrt{a+bx}\,\sqrt{a+cx}\right.\right.}{12b^3c^{5/2}(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (b*Sqrt[c]*(a^2*(3*b^2 - 2*b*c + 3*c^2)*Sqrt[a + b*x]*Sqrt[a + c*x] + 4*b^2
*c^2*x^2*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]) - 2*a*b*c*x*(-6*b*c*x
+ b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a + c*x])) + (3*a^4*
```

```
(-b + c)^3*(b + c)*Sqrt[(b*(a + c*x))/(a*(b - c))]*ArcSinh[(Sqrt[c]*Sqrt[a
+ b*x])/Sqrt[a*(b - c)]])/(Sqrt[a*(b - c)]*Sqrt[a + c*x]))/(12*b^3*(b - c)^
2*c^(5/2))
```

fricas [A] time = 0.52, size = 479, normalized size = 2.46

$$\left[\frac{24\,ab^3c^3x^2+8\left(b^4c^3+b^3c^4\right)x^3+3\left(a^3b^3-a^3b^2c-a^3bc^2+a^3c^3\right)\sqrt{bc}\,\log\left(ab^2+2\,abc+ac^2+2\left(2\,bc-\sqrt{bc}\,(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(24*a*b^3*c^3*x^2 + 8*(b^4*c^3 + b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*
c - a^3*b*c^2 + a^3*c^3)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c -
 sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x - 2*(
2*b*c*x + a*b + a*c)*sqrt(b*c)) - 2*(8*b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^
2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x +
 a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5), 1/12*(12*a*b^3*c^3*x^2 + 4*(b^4*c^3 +
 b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*sqrt(-b*c)*ar
ctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (8*
b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*
b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x + a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5)]
```

giac [B] time = 3.47, size = 511, normalized size = 2.62

$$-\frac{1}{12}\sqrt{ab^2+(bx+a)bc-abc}\left(2\,(bx+a)\left(\frac{4\left(b^{11}c^4|b|-3\,b^{10}c^5|b|+3\,b^9c^6|b|-b^8c^7|b|\right)(bx+a)}{b^{17}c^4-5\,b^{16}c^5+10\,b^{15}c^6-10\,b^{14}c^7+5\,b^{13}c^8-b^{12}c^9}+\frac{ab^{12}c^3|b|-1}{b^{17}c^4-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/12*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(2*(b*x + a)*(4*(b^11*c^4*abs(b)
- 3*b^10*c^5*abs(b) + 3*b^9*c^6*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^17*c^
4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 + 5*b^13*c^8 - b^12*c^9) + (a*b^
12*c^3*abs(b) - 10*a*b^11*c^4*abs(b) + 24*a*b^10*c^5*abs(b) - 22*a*b^9*c^6*
abs(b) + 7*a*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14
*c^7 + 5*b^13*c^8 - b^12*c^9)) - 3*(a^2*b^13*c^2*abs(b) - 3*a^2*b^12*c^3*ab
s(b) + 2*a^2*b^11*c^4*abs(b) + 2*a^2*b^10*c^5*abs(b) - 3*a^2*b^9*c^6*abs(b)
 + a^2*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 +
 5*b^13*c^8 - b^12*c^9))*sqrt(b*x + a) + 1/3*((b*x + a)^3*b - 3*(b*x + a)*a
^2*b + (b*x + a)^3*c - 3*(b*x + a)^2*a*c + 3*(b*x + a)*a^2*c)/(b^5 - 2*b^4*
c + b^3*c^2) + 1/4*(a^3*b*abs(b) + a^3*c*abs(b))*log(abs(-sqrt(b*c)*sqrt(b*
x + a) + sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^3*c^2)
```

maple [B] time = 0.02, size = 517, normalized size = 2.65

$$\frac{b\,x^3}{3\,(b-c)^2}+\frac{c\,x^3}{3\,(b-c)^2}+\frac{a\,x^2}{(b-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(3a^3b^3\ln\left(\frac{2bcx+ab+ac+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}}{2\sqrt{bc}}\right)-3a^3b^2c\ln\left(\frac{2bc}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/3*x^3/(b-c)^2*b+1/3*x^3/(b-c)^2*c+a*x^2/(b-c)^2-1/24/(b-c)^2*(b*x+a)^(1/2
)*(c*x+a)^(1/2)*(16*x^2*b^2*c^2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)
+3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(
```

```
b*c)^(1/2))*a^3*b^3-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*
c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b^2*c-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b*c^2+3*ln(1/2*(2*b
*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^
3*c^3+4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*x*a*b^2*c+4*(b*c*x^2+a*
b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*x*a*b*c^2-6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)*(b*c)^(1/2)*a^2*b^2+4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*a^2*b*c
-6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*a^2*c^2)/(b*c*x^2+a*b*x+a*c*
x+a^2)^(1/2)/b^2/c^2/(b*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

mupad [B] time = 18.15, size = 1107, normalized size = 5.68

$$\frac{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6\left(128\,a^3\,b^3\,c+\frac{1312\,a^3\,b^2\,c^2}{3}+128\,a^3\,b\,c^3\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^6}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^7\left(19\,a^3\,b^3\,c+269\,a^3\,b^2\,c^2+269\,a^3\,b\,c^3+19\,a^3\,c^4\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^7}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5\left(19\,a^3\,b^4\cdots\right.}{\left(\sqrt{\cdots}\right.}}{b^8-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] ((((a + b*x)^(1/2) - a^(1/2))^6*(128*a^3*b*c^3 + 128*a^3*b^3*c + (1312*a^3*
b^2*c^2)/3))/((a + c*x)^(1/2) - a^(1/2))^6 - (((a + b*x)^(1/2) - a^(1/2))^7
*(19*a^3*c^4 + 269*a^3*b*c^3 + 19*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*x)^
(1/2) - a^(1/2))^7 - (((a + b*x)^(1/2) - a^(1/2))^5*(19*a^3*b^4 + 19*a^3*b*
c^3 + 269*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^5 + (((
a + b*x)^(1/2) - a^(1/2))^4*(64*a^3*b^4 + 192*a^3*b^3*c + 64*a^3*b^2*c^2))/
((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(64*a^3*c^4
+ 192*a^3*b*c^3 + 64*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 + (16*a^3*
b^4*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (16*a^3*
c^4*((a + b*x)^(1/2) - a^(1/2))^10)/((a + c*x)^(1/2) - a^(1/2))^10 + (((a +
 b*x)^(1/2) - a^(1/2))^11*(a^3*c^6 - a^3*b*c^5 - a^3*b^2*c^4 + a^3*b^3*c^3)
)/(2*b^2*((a + c*x)^(1/2) - a^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*(
17*a^3*b^5 + 303*a^3*b^4*c + 17*a^3*b^2*c^3 + 303*a^3*b^3*c^2))/(6*c*((a +
c*x)^(1/2) - a^(1/2))^3) - (((a + b*x)^(1/2) - a^(1/2))^9*(17*a^3*c^5 + 303
*a^3*b*c^4 + 303*a^3*b^2*c^3 + 17*a^3*b^3*c^2))/(6*b*((a + c*x)^(1/2) - a^(
1/2))^9) + ((a^3*b + a^3*c)*((a + b*x)^(1/2) - a^(1/2))*(b^5 - 2*b^4*c + b^
3*c^2))/(2*c^2*((a + c*x)^(1/2) - a^(1/2))))/(b^8 - 2*b^7*c + b^6*c^2 + (((
a + b*x)^(1/2) - a^(1/2))^12*(c^8 - 2*b*c^7 + b^2*c^6))/((a + c*x)^(1/2) -
a^(1/2))^12 - (((a + b*x)^(1/2) - a^(1/2))^2*(6*b^7*c + 6*b^5*c^3 - 12*b^6*
c^2))/((a + c*x)^(1/2) - a^(1/2))^2 - (((a + b*x)^(1/2) - a^(1/2))^10*(6*b*
c^7 - 12*b^2*c^6 + 6*b^3*c^5))/((a + c*x)^(1/2) - a^(1/2))^10 + (((a + b*x)
^(1/2) - a^(1/2))^4*(15*b^4*c^4 - 30*b^5*c^3 + 15*b^6*c^2))/((a + c*x)^(1/2
) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(15*b^2*c^6 - 30*b^3*c^5 +
15*b^4*c^4))/((a + c*x)^(1/2) - a^(1/2))^8 - (((a + b*x)^(1/2) - a^(1/2))^6
*(20*b^3*c^5 - 40*b^4*c^4 + 20*b^5*c^3))/((a + c*x)^(1/2) - a^(1/2))^6) + (
x^3*(b + c))/(3*(b - c)^2) + (a*x^2)/(b - c)^2 - (a^3*atanh((c^(1/2)*((a +
b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((a + c*x)^(1/2) - a^(1/2))))*(b + c))/(2*b
^(5/2)*c^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

3.433 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx$

Optimal. Leaf size=142

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}+\frac{2ax}{(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc(b-c)}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{x^2(b+c)}{2(b-c)^2}$$

```
[Out] 2*a*x/(b-c)^2+1/2*(b+c)*x^2/(b-c)^2+1/2*a^2*arctanh(c^(1/2)*(b*x+a)^(1/2)/b
^(1/2)/(c*x+a)^(1/2))/b^(3/2)/c^(3/2)-(b*x+a)^(3/2)*(c*x+a)^(1/2)/b/(b-c)^2
-1/2*a*(b*x+a)^(1/2)*(c*x+a)^(1/2)/b/(b-c)/c
```

Rubi [A] time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6690, 50, 63, 217, 206}

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}+\frac{2ax}{(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc(b-c)}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{x^2(b+c)}{2(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (2*a*x)/(b - c)^2 + ((b + c)*x^2)/(2*(b - c)^2) - (a*Sqrt[a + b*x]*Sqrt[a +
 c*x])/(2*b*(b - c)*c) - ((a + b*x)^(3/2)*Sqrt[a + c*x])/(b*(b - c)^2) + (a
^2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(2*b^(3/2)*c^(
3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(2a+b\left(1+\frac{c}{b}\right)x-2\sqrt{a+bx}\,\sqrt{a+cx}\right)dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{2\int \sqrt{a+bx}\,\sqrt{a+cx}\,dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}-\frac{a\int \frac{\sqrt{a+bx}}{\sqrt{a+cx}}\,dx}{2b(b-c)}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\int \frac{1}{\sqrt{a+bx}\cdots}}{4b\cdots}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\,\mathrm{Subst}\left(\cdots\right)}{\cdots}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\,\mathrm{Subst}\left(\cdots\right)}{\cdots}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\tanh^{-1}\left(\cdots\right)}{2b^{3/2}\cdots}$$

Mathematica [A] time = 0.56, size = 177, normalized size = 1.25

$$\frac{b\sqrt{c}\left(bcx\left(-2\sqrt{a+bx}\,\sqrt{a+cx}+bx+cx\right)-a\left(b\sqrt{a+bx}\,\sqrt{a+cx}+c\sqrt{a+bx}\,\sqrt{a+cx}-4bcx\right)\right)+\frac{(a(b-c))^{5/2}\sqrt{\cdots}}{\cdots}}{2b^2c^{3/2}(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (b*Sqrt[c]*(b*c*x*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]) - a*(-4*b*c*x
 + b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a + c*x])) + ((a*(b
 - c))^(5/2)*Sqrt[(b*(a + c*x))/(a*(b - c))]*ArcSinh[(Sqrt[c]*Sqrt[a + b*x]
)/Sqrt[a*(b - c)]])/Sqrt[a + c*x])/(2*b^2*(b - c)^2*c^(3/2))
```

fricas [A] time = 0.47, size = 372, normalized size = 2.62

$$\left[\frac{8\,ab^2c^2x+2\left(b^3c^2+b^2c^3\right)x^2+\left(a^2b^2-2\,a^2bc+a^2c^2\right)\sqrt{bc}\,\log\left(ab^2+2\,abc+ac^2+2\left(2\,bc+\sqrt{bc}\,(b+c)\right)\sqrt{bx}\cdots\right.}{4\left(b^4c^2-2\,b^3c^3\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(8*a*b^2*c^2*x + 2*(b^3*c^2 + b^2*c^3)*x^2 + (a^2*b^2 - 2*a^2*b*c + a^
2*c^2)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c + sqrt(b*c)*(b + c)
)*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x + 2*(2*b*c*x + a*b + a*
c)*sqrt(b*c)) - 2*(2*b^2*c^2*x + a*b^2*c + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x
+ a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4), 1/2*(4*a*b^2*c^2*x + (b^3*c^2 + b^2*
c^3)*x^2 - (a^2*b^2 - 2*a^2*b*c + a^2*c^2)*sqrt(-b*c)*arctan((sqrt(-b*c)*sq
rt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (2*b^2*c^2*x + a*b^2*c
 + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x + a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4)]
```

giac [B] time = 3.28, size = 272, normalized size = 1.92

$$-\frac{1}{2}\sqrt{ab^2+(bx+a)bc-abc}\,\sqrt{bx+a}\left(\frac{2\left(b^4c^2|b|-b^3c^3|b|\right)(bx+a)}{b^9c^2-3\,b^8c^3+3\,b^7c^4-b^6c^5}+\frac{ab^5c|b|-2\,ab^4c^2|b|+ab^3c^3|b|}{b^9c^2-3\,b^8c^3+3\,b^7c^4-b^6c^5}\right)-\frac{a^2|b|\log\left(\left|-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*sqrt(b*x + a)*(2*(b^4*c^2*abs(b) -
 b^3*c^3*abs(b))*(b*x + a)/(b^9*c^2 - 3*b^8*c^3 + 3*b^7*c^4 - b^6*c^5) + (a
*b^5*c*abs(b) - 2*a*b^4*c^2*abs(b) + a*b^3*c^3*abs(b))/(b^9*c^2 - 3*b^8*c^3
 + 3*b^7*c^4 - b^6*c^5)) - 1/2*a^2*abs(b)*log(abs(-sqrt(b*c)*sqrt(b*x + a)
+ sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^2*c) + 1/2*((b*x + a)^
2*b + 2*(b*x + a)*a*b + (b*x + a)^2*c - 2*(b*x + a)*a*c)/(b^4 - 2*b^3*c + b
^2*c^2)
```

maple [B] time = 0.01, size = 385, normalized size = 2.71

$$\frac{\sqrt{(bx+a)\,(cx+a)}\;a^2b\ln\left(\frac{bcx+\frac{1}{2}ab+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^2+a^2+(ab+ac)\,x}\right)}{4\,(b-c)^2\sqrt{cx+a}\;\sqrt{bx+a}\;\sqrt{bc}\;c}+\frac{\sqrt{(bx+a)\,(cx+a)}\;a^2c\ln\left(\frac{bcx+\frac{1}{2}ab+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{b}\right.}{4\,(b-c)^2\sqrt{cx+a}\;\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/2*x^2/(b-c)^2*b+1/2*x^2/(b-c)^2*c+2*a*x/(b-c)^2-1/(b-c)^2/c*(b*x+a)^(1/2)
*(c*x+a)^(3/2)+1/2/(b-c)^2/c*(c*x+a)^(1/2)*(b*x+a)^(1/2)*a-1/2/(b-c)^2/b*(c
*x+a)^(1/2)*(b*x+a)^(1/2)*a+1/4/(b-c)^2/c*((b*x+a)*(c*x+a))^(1/2)/(c*x+a)^(
1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c
)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2*b-1/2/(b-c)^2*((b*x+a)*(c*x+a))^(1/2)/(c*x+
a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b
+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2+1/4/(b-c)^2*c/b*((b*x+a)*(c*x+a))^(1/2)
/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^
2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

mupad [B] time = 0.25, size = 129, normalized size = 0.91

$$\frac{2\,a\,x}{(b-c)^2}+\frac{x^2\,(b+c)}{2\,(b-c)^2}-\frac{2\left(\frac{x}{2}+\frac{a\,b+a\,c}{4\,b\,c}\right)\sqrt{a+b\,x}\,\sqrt{a+c\,x}}{(b-c)^2}+\frac{\ln\left(a\,b+a\,c+2\,b\,c\,x+2\,\sqrt{b}\,\sqrt{c}\,\sqrt{a+b\,x}\,\sqrt{a+c\,x}\right)\,(a\,b}{4\,b^{3/2}\,c^{3/2}\,(b-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (2*a*x)/(b - c)^2 + (x^2*(b + c))/(2*(b - c)^2) - (2*(x/2 + (a*b + a*c)/(4*
b*c))*(a + b*x)^(1/2)*(a + c*x)^(1/2))/(b - c)^2 + (log(a*b + a*c + 2*b*c*x
 + 2*b^(1/2)*c^(1/2)*(a + b*x)^(1/2)*(a + c*x)^(1/2))*(a*b - a*c)^2)/(4*b^(
3/2)*c^(3/2)*(b - c)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

3.434 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} \, dx$

Optimal. Leaf size=135

$$-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}(b-c)^2}+\frac{x(b+c)}{(b-c)^2}$$

```
[Out] (b+c)*x/(b-c)^2+4*a*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/(b-c)^2+2*a*ln(x)/
(b-c)^2-2*a*(b+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))/(b-c
)^2/b^(1/2)/c^(1/2)-2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/(b-c)^2
```

Rubi [A] time = 0.18, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6690, 101, 157, 63, 217, 206, 93, 208}

$$-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}(b-c)^2}+\frac{x(b+c)}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((b + c)*x)/(b - c)^2 - (2*Sqrt[a + b*x]*Sqrt[a + c*x])/(b - c)^2 + (4*a*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (2*a*(b + c)*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(Sqrt[b]*(b - c)^2*Sqrt[c]) + (
2*a*Log[x])/(b - c)^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
```

```
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(b\left(1+\frac{c}{b}\right)+\frac{2a}{x}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x}\right)dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{a+cx}}{x}\,dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{2\int \frac{-a^2-\frac{1}{2}a(b+c)x}{x\sqrt{a+bx}\,\sqrt{a+cx}}\,dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{\left(2a^2\right)\int \frac{1}{x\sqrt{a+bx}\,\sqrt{a+cx}}\,dx}{(b-c)^2}-\frac{(a(b+\ldots}{\ldots}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{\left(4a^2\right)\mathrm{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \frac{\sqrt{a+\ldots}}{\sqrt{a+\ldots}}\ldots\right.}{(b-c)^2}\ldots$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{(2a(b+c)\ldots}{\ldots}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{\ldots}}{\sqrt{b}\,\sqrt{\ldots}}\right.}{\sqrt{b}\,(b-c)^2\sqrt{c}}\ldots$$

Mathematica [A] time = 0.86, size = 195, normalized size = 1.44

$$\frac{\frac{2(b+c)\sqrt{a(b-c)}\,(a+cx)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a(b-c)}}\right)}{\sqrt{c}\sqrt{\frac{b(a+cx)}{a(b-c)}}} - (b-c)\left(-2cx\sqrt{a+bx} + bx\sqrt{a+cx} + 4a\sqrt{a+cx}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right) - 2a\sqrt{a+bx}\right.}{(c-b)^3\sqrt{a+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((2*Sqrt[a*(b - c)]*(b + c)*(a + c*x)*ArcSinh[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[
a*(b - c)]])/(Sqrt[c]*Sqrt[(b*(a + c*x))/(a*(b - c))]) - (b - c)*(-2*a*Sqrt
[a + b*x] - 2*c*x*Sqrt[a + b*x] + b*x*Sqrt[a + c*x] + c*x*Sqrt[a + c*x] + 4
*a*Sqrt[a + c*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]] + 2*a*Sqrt[a + c*x]*L
og[x]))/((-b + c)^3*Sqrt[a + c*x])
```

fricas [A] time = 0.63, size = 346, normalized size = 2.56

$$\left[\frac{2\,abc\log(x) - 2\,abc\log\left(-\frac{(b+c)x - 2\sqrt{bx+a}\sqrt{cx+a} + 2\,a}{x}\right) - 2\sqrt{bx+a}\sqrt{cx+a}\,bc + (ab+ac)\sqrt{bc}\,\log\left(ab^2 + 2\,abc + ac^2\right.}{b^3c - 2\,b^2c^2 +}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b*c*log(x) - 2*a*b*c*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a)
+ 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c*x + a)*b*c + (a*b + a*c)*sqrt(b*c)*log(a
*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c
*x + a) + 2*(b^2*c + b*c^2)*x - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b^2*c
 + b*c^2)*x)/(b^3*c - 2*b^2*c^2 + b*c^3), (2*a*b*c*log(x) - 2*a*b*c*log(-((
b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c
*x + a)*b*c + 2*(a*b + a*c)*sqrt(-b*c)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqr
t(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b^2*c + b*c^2)*x)/(b^3*c - 2*b^2*c^2
 + b*c^3)]
```

giac [B] time = 3.53, size = 306, normalized size = 2.27

$$\frac{\frac{\sqrt{bc}\,a(b+c)|b|\log\left(\left(\sqrt{bc}\sqrt{bx+a} - \sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{b^3c-2\,b^2c^2+bc^3} - \frac{4\sqrt{bc}\,a|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}\,ab}\right)}{\left(b^2-2\,bc+c^2\right)\sqrt{-bc}} + \frac{2\,ab\log(|bx|)}{b^2-2\,bc+c^2} - \frac{2\sqrt{ab^2+\cdots}}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] (sqrt(b*c)*a*(b + c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*
x + a)*b*c - a*b*c))^2)/(b^3*c - 2*b^2*c^2 + b*c^3) - 4*sqrt(b*c)*a*abs(b)*
arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/((b^2 - 2*b*c + c^2)*sqrt(-b*c)) + 2*
a*b*log(abs(b*x))/(b^2 - 2*b*c + c^2) - 2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c)*(b^2*abs(b) - 2*b*c*abs(b) + c^2*abs(b))*sqrt(b*x + a)/(b^5 - 4*b^4*c +
6*b^3*c^2 - 4*b^2*c^3 + b*c^4) + ((b*x + a)*b + (b*x + a)*c)/(b^2 - 2*b*c +
 c^2))/b
```

maple [C] time = 0.02, size = 266, normalized size = 1.97

$$\frac{2a\ln(x)}{(b-c)^2}+\frac{bx}{(b-c)^2}+\frac{cx}{(b-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(ab\,\mathrm{csgn}(a)\ln\left(\frac{2bcx+ab+ac+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}}{2\sqrt{bc}}\right)+ac\,\mathrm{csgn}(a)\ln\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] x/(b-c)^2*b+x/(b-c)^2*c+2*a*ln(x)/(b-c)^2-1/(b-c)^2*(b*x+a)^(1/2)*(c*x+a)^(
1/2)*(csgn(a)*ln(1/2*(2*b*c*x+a*b+a*c+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*
c)^(1/2))/(b*c)^(1/2))*a*b+csgn(a)*ln(1/2*(2*b*c*x+a*b+a*c+2*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*a*c+2*(b*c*x^2+a*b*x+a*c*x+a^2)
^(1/2)*csgn(a)*(b*c)^(1/2)-2*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a
)+b*x+c*x+2*a)/x)*(b*c)^(1/2)*a)*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/(b
*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

mupad [B] time = 19.76, size = 5098, normalized size = 37.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (2*a*log(x))/(b^2 - 2*b*c + c^2) - (((4*a*c^2 + 4*a*b*c)*((a + b*x)^(1/2) -
 a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + ((4*a*b^2 + 4*a*b*c)*((a + b*x
)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (16*a*b*c*((a + b*x)^(1/2
) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)/(b^4 - 2*b^3*c + b^2*c^2 - (
((a + b*x)^(1/2) - a^(1/2))^2*(2*b*c^3 + 2*b^3*c - 4*b^2*c^2))/((a + c*x)^(
1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^4*(c^4 - 2*b*c^3 + b^2*c^2
))/((a + c*x)^(1/2) - a^(1/2))^4) - (2*a*log((((a + b*x)^(1/2) - (a + c*x)^
(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/(
(a + c*x)^(1/2) - a^(1/2))))/(b^2 - 2*b*c + c^2) + (2*a*log(((a + b*x)^(1/2
) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))))/(b - c)^2 + (x*(b + c))/(b - c)^
2 + (a*atan(((a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a^(1/2))*(32*a^3
*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^
6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4
*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) - (4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 +
 44*a^3*b^6*c^6 + 4*a^3*b^7*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^
2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2
*b^5*c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5
 + 4*a^2*b^10*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a +
b*x)^(1/2) - a^(1/2))*(16*a^2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10
- 64*a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^
2*b^9*c^5 + 16*a^2*b^10*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c -
 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4*(a*b^4*c^12 + 7
*a*b^5*c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 +
7*a*b^10*c^6 + a*b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (
2*((a + b*x)^(1/2) - a^(1/2))*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^1
1 - 490*a*b^6*c^10 + 648*a*b^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b
^10*c^6 + 8*a*b^11*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^
14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4
*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a
 + b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6
```

```
*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11
*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3
*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b
^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2))*2i)/(b*c^3 + b^3*c - 2*b^2
*c^2) - (a*(b*c)^(1/2)*(b + c)*((4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3
*b^6*c^6 + 4*a^3*b^7*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2
*((a + b*x)^(1/2) - a^(1/2))*(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*
c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(
((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) +
(2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2*b^5*
c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*
a^2*b^10*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^
(1/2) - a^(1/2))*(16*a^2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10 - 64*
a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9
*c^5 + 16*a^2*b^10*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a
^(1/2))*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^11 - 490*a*b^6*c^10 + 6
48*a*b^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b^10*c^6 + 8*a*b^11*c^5
))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)
) - (4*(a*b^4*c^12 + 7*a*b^5*c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8
*c^8 - 27*a*b^9*c^7 + 7*a*b^10*c^6 + a*b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3
+ c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^14 -
5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12
*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*
x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12
 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7
- 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c -
4*b*c^3 + c^4 + 6*b^2*c^2))))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c
- 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2))*2i)/(b*c^3 + b^3*c - 2*b^2*c^2)
)/((4*((a + b*x)^(1/2) - a^(1/2))*(128*a^4*b^3*c^7 + 256*a^4*b^4*c^6 + 128*
a^4*b^5*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 +
 6*b^2*c^2)) - (8*(16*a^4*b^3*c^7 + 56*a^4*b^4*c^6 + 56*a^4*b^5*c^5 + 16*a^
4*b^6*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*
(b + c)*((2*((a + b*x)^(1/2) - a^(1/2))*(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 +
 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*
b^8*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b
^2*c^2)) - (4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3*b^6*c^6 + 4*a^3*b^7*
c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c
)*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2*b^5*c^9 + 12*a^2*b^6*c^8 +
12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*a^2*b^10*c^4))/(b^4 - 4
*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(16*a^
2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10 - 64*a^2*b^5*c^9 + 88*a^2*b^
6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9*c^5 + 16*a^2*b^10*c^4)
)/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))
 + (2*a*(b*c)^(1/2)*(b + c)*((4*(a*b^4*c^12 + 7*a*b^5*c^11 - 27*a*b^6*c^10
+ 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 + 7*a*b^10*c^6 + a*b^11*c^5))/
(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2)
)*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^11 - 490*a*b^6*c^10 + 648*a*b
^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b^10*c^6 + 8*a*b^11*c^5))/(((
a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2
*a*(b*c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 +
 b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4
- 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*
b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b
^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*
c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c
^2))))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 +
 b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c
)*((4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3*b^6*c^6 + 4*a^3*b^7*c^5))/(b
```

```
^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*
(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3
*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(
b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4*
(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2*b^5*c^9 + 12*a^2*b^6*c^8 + 12*a^2
*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*a^2*b^10*c^4))/(b^4 - 4*b^3*c
 - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(16*a^2*b^2*
c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10 - 64*a^2*b^5*c^9 + 88*a^2*b^6*c^8
- 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9*c^5 + 16*a^2*b^10*c^4))/(((a
 + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2*
a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a^(1/2))*(8*a*b^3*c^13 - 54*a*
b^4*c^12 + 212*a*b^5*c^11 - 490*a*b^6*c^10 + 648*a*b^7*c^9 - 490*a*b^8*c^8
+ 212*a*b^9*c^7 - 54*a*b^10*c^6 + 8*a*b^11*c^5))/(((a + c*x)^(1/2) - a^(1/2
))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) - (4*(a*b^4*c^12 + 7*a*b^5*
c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 + 7*a*b^1
0*c^6 + a*b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*
c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c
^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^
3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^
15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b^8*c^1
0 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/
(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))
/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c
 - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2)))*(b*c)^(1/2)*(b + c)*4i)/(b*c^
3 + b^3*c - 2*b^2*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

3.435 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=138

$$-\frac{2a}{x(b-c)^2}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x(b-c)^2}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

```
[Out] -2*a/(b-c)^2/x+2*(b+c)*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/(b-c)^2+(b+c)*l
n(x)/(b-c)^2-4*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))*b^(1/2)
*c^(1/2)/(b-c)^2+2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/(b-c)^2/x
```

Rubi [A] time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = $\{6690, 97, 157, 63, 217, 206, 93, 208\}$

$$-\frac{2a}{x(b-c)^2}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x(b-c)^2}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (-2*a)/((b - c)^2*x) + (2*Sqrt[a + b*x]*Sqrt[a + c*x])/((b - c)^2*x) + (2*(
b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (4*Sqrt[b]*Sqrt[c]
*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(b - c)^2 + ((b
+ c)*Log[x])/(b - c)^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(\frac{2a}{x^2}+\frac{b\left(1+\frac{c}{b}\right)}{x}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x^2}\right)dx}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{a+cx}}{x^2}\,dx}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{2\int \frac{\frac{1}{2}a(b+c)+bcx}{x\sqrt{a+bx}\,\sqrt{a+cx}}\,dx}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{(2bc)\int \frac{1}{\sqrt{a+bx}\,\sqrt{a+cx}}\,dx}{(b-c)^2}-$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{ac}{b}+\frac{cx^2}{b}}}\right.}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\,\sqrt{c}\,\tanh^{-1}}{(b-}$$

Mathematica [A] time = 0.71, size = 178, normalized size = 1.29

$$\frac{2c\sqrt{a+bx}+\frac{2a\left(\sqrt{a+bx}-\sqrt{a+cx}\right)}{x}+(b+c)\log(x)\sqrt{a+cx}-\frac{4b\sqrt{c}(a+cx)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a(b-c)}}\right)}{\sqrt{a(b-c)}\sqrt{\frac{b(a+cx)}{a(b-c)}}}+2(b+c)\sqrt{a+cx}\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+}}\right.}{(b-c)^2\sqrt{a+cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (2*c*Sqrt[a + b*x] + (2*a*(Sqrt[a + b*x] - Sqrt[a + c*x]))/x - (4*b*Sqrt[c]
*(a + c*x)*ArcSinh[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*(b - c)]])/(Sqrt[a*(b - c
)]*Sqrt[(b*(a + c*x))/(a*(b - c))]) + 2*(b + c)*Sqrt[a + c*x]*ArcTanh[Sqrt[
a + b*x]/Sqrt[a + c*x]] + (b + c)*Sqrt[a + c*x]*Log[x])/((b - c)^2*Sqrt[a +
 c*x])
```

fricas [A] time = 0.64, size = 317, normalized size = 2.30

$$\left[\frac{2(b+c)x\log(x)-2(b+c)x\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\sqrt{cx+a}+2a}{x}\right)+4\sqrt{bc}\,x\log\left(ab^2+2abc+ac^2+2\left(2bc-\sqrt{bc}(b+\right.\right.}{2\left(b^2-2bc+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*log(-((b + c)*x - 2*sqrt(b*x + a)*sq
rt(c*x + a) + 2*a)/x) + 4*sqrt(b*c)*x*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*
c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x -
2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c*x +
 a) - 4*a)/((b^2 - 2*b*c + c^2)*x), 1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*l
og(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) + 8*sqrt(-b*c)*x*a
rctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b
 + c)*x + 4*sqrt(b*x + a)*sqrt(c*x + a) - 4*a)/((b^2 - 2*b*c + c^2)*x)]
```

giac [B] time = 4.50, size = 438, normalized size = 3.17

$$\frac{2\sqrt{bc}\,|b|\log\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{b^3-2b^2c+bc^2}-\frac{2\sqrt{bc}(b+c)|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)}\right.}{2\sqrt{-bc}\,ab}\right.}{\left(b^2-2bc+c^2\right)\sqrt{-bc}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*sqrt(b*c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))^2)/(b^3 - 2*b^2*c + b*c^2) - 2*sqrt(b*c)*(b + c)*abs(b)*arctan(
1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
- a*b*c))^2)/(sqrt(-b*c)*a*b))/((b^2 - 2*b*c + c^2)*sqrt(-b*c)*b) + (b + c)
*log(abs(b*x))/(b^2 - 2*b*c + c^2) - 4*(sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a)
- sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*(b + c)*abs(b) - (b^3 - 2*b^2*c
+ b*c^2)*sqrt(b*c)*a^2*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b
^2 + (b*x + a)*b*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)*(b^2 - 2*
b*c + c^2)) - ((b*x + a)*b + a*b + (b*x + a)*c - a*c)/((b^2 - 2*b*c + c^2)*
b*x)
```

maple [C] time = 0.02, size = 272, normalized size = 1.97

$$\frac{b\ln(x)}{(b-c)^2}+\frac{c\ln(x)}{(b-c)^2}-\frac{2a}{(b-c)^2 x}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(2bcx\,\mathrm{csgn}(a)\ln\left(\frac{2bcx+ab+ac+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}}{2\sqrt{bc}}\right)-\sqrt{bc}\,bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/(b-c)^2*b*ln(x)+1/(b-c)^2*c*ln(x)-2*a/(b-c)^2/x-1/(b-c)^2*(b*x+a)^(1/2)*(
c*x+a)^(1/2)*(2*csgn(a)*ln(1/2*(2*b*c*x+a*b+a*c+2*(b*c*x^2+a*b*x+a*c*x+a^2)
^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*x*b*c-ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c
*x+a^2)^(1/2)*csgn(a))*a/x)*x*b*(b*c)^(1/2)-ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*
x+a*c*x+a^2)^(1/2)*csgn(a))*a/x)*x*c*(b*c)^(1/2)-2*(b*c*x^2+a*b*x+a*c*x+a^2
)^(1/2)*(b*c)^(1/2)*csgn(a))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x/(b*c
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-2), x)
```

mupad [B] time = 17.44, size = 4285, normalized size = 31.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (atan((((b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^
10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 -
 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^12 - 36
*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3
 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^1
2 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^
13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2)
- a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^
7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12
*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 +
 c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14
 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 17
5*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^
(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a +
b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*
c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2)
)*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^10
 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 167*
b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*
x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))*4i)/(b -
c)^2 - ((b*c)^(1/2)*((4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7
+ 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*
```

((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^
9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2
) + (4*(b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12
+ b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13
*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) -
a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*
c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c
^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c
^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^12 - 36*b^7*c^10 + 64*b^8*c^9 -
36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*
((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b
^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^
6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^
4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 -
 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73
*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6
*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 167*
b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*
x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))*4i)/(b -
c)^2)/((4*(b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*
c^10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4
 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^12 -
36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c
^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c
^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 -
b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2
) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*
b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^
12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3
 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^
14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 -
175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) -
a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a
+ b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^
7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/
2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^
10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*
b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 16
7*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a +
c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c
)^2 - (8*(14*b^5*c^9 + 42*b^6*c^8 + 42*b^7*c^7 + 14*b^8*c^6))/(b^4 - 4*b^3*
c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^10 + 24*b^6*c^
9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 +
6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^10 + 25*
b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 - 4*b^3*c
 - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(4*b^5*c
^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*
b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c
^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^
13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^
8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*
(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^12 -
 36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14 - 27*b^
4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 175*b^9*c^
8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(
b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/
2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*c^9 + 50
3*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 -

```
 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) -
a^(1/2))*(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*
c^7 + 65*b^9*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 +
c^4 + 6*b^2*c^2))))/(b - c)^2 + (4*((a + b*x)^(1/2) - a^(1/2))*(112*b^5*c^9
 + 224*b^6*c^8 + 112*b^7*c^7))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c
- 4*b*c^3 + c^4 + 6*b^2*c^2))))*(b*c)^(1/2)*8i)/(b - c)^2 - (((b*c + b^2)*(
(a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - b^2 + (((a + b*x)
^(1/2) - a^(1/2))^2*(b^2 - 3*b*c + c^2))/((a + c*x)^(1/2) - a^(1/2))^2)/(((
(a + b*x)^(1/2) - a^(1/2))^3*(2*b^2*c - 4*b*c^2 + 2*c^3))/((a + c*x)^(1/2)
- a^(1/2))^3 + (((a + b*x)^(1/2) - a^(1/2))^2*(2*b*c^2 + 2*b^2*c - 2*b^3 -
2*c^3))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))*(2*b*c
^2 - 4*b^2*c + 2*b^3))/((a + c*x)^(1/2) - a^(1/2))) + (log(((a + b*x)^(1/2)
 - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(b - c)^2 + (log(x)*(b +
c))/(b^2 - 2*b*c + c^2) - (log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c
*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2
) - a^(1/2)))*(b + c))/(b^2 - 2*b*c + c^2) - (2*a)/(x*(b^2 - 2*b*c + c^2))
+ (c*((a + b*x)^(1/2) - a^(1/2)))/(2*(b - c)^2*((a + c*x)^(1/2) - a^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-2), x)
```

3.436 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx$

Optimal. Leaf size=123

$$\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{ax^2(b-c)^2}-\frac{a}{x^2(b-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2ax(b-c)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}-\frac{b+c}{x(b-c)^2}$$

```
[Out] -a/(b-c)^2/x^2+(-b-c)/(b-c)^2/x-1/2*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/a+
(c*x+a)^(3/2)*(b*x+a)^(1/2)/a/(b-c)^2/x^2+1/2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/a
/(b-c)/x
```

Rubi [A] time = 0.20, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6690, 94, 93, 208}

$$\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{ax^2(b-c)^2}-\frac{a}{x^2(b-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2ax(b-c)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}-\frac{b+c}{x(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] -(a/((b - c)^2*x^2)) - (b + c)/((b - c)^2*x) + (Sqrt[a + b*x]*Sqrt[a + c*x]
)/(2*a*(b - c)*x) + (Sqrt[a + b*x]*(a + c*x)^(3/2))/(a*(b - c)^2*x^2) - Arc
Tanh[Sqrt[a + b*x]/Sqrt[a + c*x]]/(2*a)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(\frac{2a}{x^3}+\frac{b\left(1+\frac{c}{b}\right)}{x^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^3}\right)dx}{(b-c)^2}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}-\frac{2\int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^3}dx}{(b-c)^2}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}-\frac{\int \frac{\sqrt{a+cx}}{x^2\sqrt{a+bx}}dx}{2(b-c)}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}+\frac{1}{4}\int \frac{}{x}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}+\frac{1}{2}\text{Subs}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}-\frac{\tanh^{-1}}{}$$

Mathematica [A] time = 0.15, size = 109, normalized size = 0.89

$$\frac{-2a^2-x^2(b-c)^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)+2a\left(\sqrt{a+bx}\sqrt{a+cx}-bx-cx\right)+x(b+c)\sqrt{a+bx}\sqrt{a+cx}}{2ax^2(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-2*a^2 + (b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + 2*a*(-(b*x) - c*x + Sqrt[
a + b*x]*Sqrt[a + c*x]) - (b - c)^2*x^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]
])/(2*a*(b - c)^2*x^2)
```

fricas [A] time = 0.52, size = 126, normalized size = 1.02

$$\frac{4\left(b^2-2\,bc+c^2\right)x^2\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\,\sqrt{cx+a}+2\,a}{x}\right)+\left(b^2+2\,bc+c^2\right)x^2+8\left((b+c)x+2\,a\right)\sqrt{bx+a}\,\sqrt{cx+a}-}{16\left(ab^2-2\,abc+ac^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b^2 - 2*b*c + c^2)*x^2*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x
+ a) + 2*a)/x) + (b^2 + 2*b*c + c^2)*x^2 + 8*((b + c)*x + 2*a)*sqrt(b*x + a
)*sqrt(c*x + a) - 16*a^2 - 16*(a*b + a*c)*x)/((a*b^2 - 2*a*b*c + a*c^2)*x^2
)
```

giac [B] time = 12.25, size = 532, normalized size = 4.33

$$\frac{\sqrt{bc}\,|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\,\sqrt{-bc}\,ab}\right)}{2\,\sqrt{-bc}\,ab}-\frac{\left(b^2+6\,bc+c^2\right)\sqrt{bc}\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*c)*abs(b)*arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) -
 sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt(-b*c)*a*b)
 - ((b^2 + 6*b*c + c^2)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))^6*abs(b) - (3*b^4 + 5*b^3*c + 5*b^2*c^2 + 3*b*c^3)*s
qrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4*
a*abs(b) + (3*b^6 - 4*b^5*c + 2*b^4*c^2 - 4*b^3*c^3 + 3*b^2*c^4)*sqrt(b*c)*
(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a^2*abs(b
) - (b^8 - 3*b^7*c + 2*b^6*c^2 + 2*b^5*c^3 - 3*b^4*c^4 + b^3*c^5)*sqrt(b*c)
*a^3*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
 - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^2*(b^2 - 2*b*c + c^2)) - ((
b*x + a)*b^2 + (b*x + a)*b*c - a*b*c)/((b^2 - 2*b*c + c^2)*b^2*x^2)
```

maple [C] time = 0.01, size = 313, normalized size = 2.54

$$-\frac{b}{(b-c)^2 x}-\frac{c}{(b-c)^2 x}-\frac{a}{(b-c)^2 x^2}+\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(-b^2x^2\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)\right)a}{x}\right)+2bc\,x^2\ln\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] -1/x/(b-c)^2*b-1/x/(b-c)^2*c-a/(b-c)^2/x^2+1/4/(b-c)^2*(b*x+a)^(1/2)*(c*x+a
)^(1/2)/a*(-ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a))*a/x)
*x^2*b^2+2*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a))*a/x)*
x^2*b*c-ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a))*a/x)*x^2
*c^2+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*x*b+2*(b*c*x^2+a*b*x+a*c*x+a
^2)^(1/2)*csgn(a)*x*c+4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2))*csgn(a)/
(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

mupad [B] time = 12.32, size = 787, normalized size = 6.40

$$\frac{\ln\left(\frac{\left(\sqrt{a+b\,x}-\sqrt{a+c\,x}\right)\left(b-\frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{4\,a}-\frac{\frac{b^4}{2}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4\left(-\frac{b^4}{2}+4\,b^3\,c+\frac{3\,b^2\,c^2}{2}+4\,b\,c^3-\frac{c^4}{2}\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}-\frac{\left(2\,b^4+2\,c\cdots\right.}{\cdots}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4\left(8\,a\,b^4+16\,a\,b^3\,c-48\,a\,b^2\,c^2+16\,a\,b\,c^3+8\,a\,c^4\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3\left(16\,a\,b^4-16\,a\,b^3\,c-16\cdots\right.}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)
```

```
[Out] log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)
))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (b^4/
2 + (((a + b*x)^(1/2) - a^(1/2))^4*(4*b*c^3 + 4*b^3*c - b^4/2 - c^4/2 + (3*
b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2))^4 - ((2*b^3*c + 2*b^4)*((a + b*x)^
```

```
(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - ((b*c^3 + b^2*c^2)*((a + b*
x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/2))^5 + (((a + b*x)^(1/2) -
a^(1/2))^2*(6*b^3*c + (5*b^4)/2 + (5*b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2
))^2 - (((a + b*x)^(1/2) - a^(1/2))^3*(b*c^3 + 6*b^3*c + b^4 + 6*b^2*c^2))/
((a + c*x)^(1/2) - a^(1/2))^3)/((((a + b*x)^(1/2) - a^(1/2))^4*(8*a*b^4 + 8
*a*c^4 - 48*a*b^2*c^2 + 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2
))^4 - (((a + b*x)^(1/2) - a^(1/2))^3*(16*a*b^4 - 16*a*b^2*c^2 + 16*a*b*c^3
 - 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^3 - (((a + b*x)^(1/2) - a^(1/2)
)^5*(16*a*c^4 - 16*a*b^2*c^2 - 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2) -
 a^(1/2))^5 + (((a + b*x)^(1/2) - a^(1/2))^2*(8*a*b^4 + 8*a*b^2*c^2 - 16*a*
b^3*c))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^6*(8*a
*c^4 + 8*a*b^2*c^2 - 16*a*b*c^3))/((a + c*x)^(1/2) - a^(1/2))^6) - log(((a
+ b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (a + x*(b + c)
)/(x^2*(b^2 - 2*b*c + c^2)) - (c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(16*a*(b
- c)^2*((a + c*x)^(1/2) - a^(1/2))^2) + (c*(b + c)*((a + b*x)^(1/2) - a^(1/
2)))/(8*a*(b - c)^2*((a + c*x)^(1/2) - a^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

3.437 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\,(a+cx)^{3/2}}{2a^2x^2(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2x(b-c)}+\frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}-\frac{2a}{3x^3(b-c)}$$

```
[Out] -2/3*a/(b-c)^2/x^3+1/2*(-b-c)/(b-c)^2/x^2+2/3*(b*x+a)^(3/2)*(c*x+a)^(3/2)/a
^2/(b-c)^2/x^3+1/4*(b+c)*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/a^2-1/2*(b+c)
*(c*x+a)^(3/2)*(b*x+a)^(1/2)/a^2/(b-c)^2/x^2-1/4*(b+c)*(b*x+a)^(1/2)*(c*x+a
)^(1/2)/a^2/(b-c)/x
```

Rubi [A] time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6690, 96, 94, 93, 208}

$$\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\,(a+cx)^{3/2}}{2a^2x^2(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2x(b-c)}+\frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}-\frac{2a}{3x^3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-2*a)/(3*(b - c)^2*x^3) - (b + c)/(2*(b - c)^2*x^2) - ((b + c)*Sqrt[a + b*
x]*Sqrt[a + c*x])/(4*a^2*(b - c)*x) - ((b + c)*Sqrt[a + b*x]*(a + c*x)^(3/2
))/(2*a^2*(b - c)^2*x^2) + (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*a^2*(b -
c)^2*x^3) + ((b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(4*a^2)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(\frac{2a}{x^4}+\frac{b\left(1+\frac{c}{b}\right)}{x^3}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x^4}\right)dx}{(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{a+cx}}{x^4}\,dx}{(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}+\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3}+\frac{(b+c)\int \frac{\sqrt{a+bx}\,\sqrt{a+c}}{x^3}}{a(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,(a+cx)^{3/2}}{2a^2(b-c)^2x^2}+\frac{2(a+bx)^{3/2}(a+}{3a^2(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{a+bx}\,(a}{2a^2(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{a+bx}\,(a}{2a^2(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{a+bx}\,(a}{2a^2(b-c)^2}$$

Mathematica [A] time = 0.18, size = 153, normalized size = 0.88

$$\frac{-8a^3+a^2\left(8\sqrt{a+bx}\,\sqrt{a+cx}-6bx-6cx\right)+x^2\left(-3b^2+2bc-3c^2\right)\sqrt{a+bx}\,\sqrt{a+cx}+3x^3(b-c)^2(b+c)\tan}{12a^2x^3(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-8*a^3 + 2*a*(b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + (-3*b^2 + 2*b*c - 3*c
^2)*x^2*Sqrt[a + b*x]*Sqrt[a + c*x] + a^2*(-6*b*x - 6*c*x + 8*Sqrt[a + b*x]
*Sqrt[a + c*x]) + 3*(b - c)^2*(b + c)*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*
x]])/(12*a^2*(b - c)^2*x^3)
```

fricas [A] time = 0.53, size = 182, normalized size = 1.05

$$-\frac{12\left(b^3-b^2c-bc^2+c^3\right)x^3\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\,\sqrt{cx+a}+2a}{x}\right)+\left(5b^3+3b^2c+3bc^2+5c^3\right)x^3+64a^3+8\left(\left(3b^2-\right.\right.}{96\left(a^2b^2-2a^2bc+a^2c^2\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/96*(12*(b^3 - b^2*c - b*c^2 + c^3)*x^3*log(-((b + c)*x - 2*sqrt(b*x + a)
*sqrt(c*x + a) + 2*a)/x) + (5*b^3 + 3*b^2*c + 3*b*c^2 + 5*c^3)*x^3 + 64*a^3
 + 8*((3*b^2 - 2*b*c + 3*c^2)*x^2 - 8*a^2 - 2*(a*b + a*c)*x)*sqrt(b*x + a)*
sqrt(c*x + a) + 48*(a^2*b + a^2*c)*x)/((a^2*b^2 - 2*a^2*b*c + a^2*c^2)*x^3)
```

giac [B] time = 16.94, size = 802, normalized size = 4.61

$$-\frac{\sqrt{bc}\,(b+c)|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}\,ab}\right)}{4\sqrt{-bc}\,a^2b}+\frac{3\left(b^3-b^2c-bc^2+c^3\right)\sqrt{bc}\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(b*c)*(b + c)*abs(b)*arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt(-
b*c)*a^2*b) + 1/6*(3*(b^3 - b^2*c - b*c^2 + c^3)*sqrt(b*c)*(sqrt(b*c)*sqrt(
b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^10*abs(b) - 3*(5*b^5 + 22*b
^3*c^2 + 5*b*c^4)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
a)*b*c - a*b*c))^8*a*abs(b) + 2*(15*b^7 - b^6*c + 18*b^5*c^2 + 18*b^4*c^3 -
 b^3*c^4 + 15*b^2*c^5)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))^6*a^2*abs(b) - 6*(5*b^9 - 6*b^8*c - 5*b^7*c^2 + 12*b^
6*c^3 - 5*b^5*c^4 - 6*b^4*c^5 + 5*b^3*c^6)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x +
a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4*a^3*abs(b) + 3*(5*b^11 - 17*b^1
0*c + 21*b^9*c^2 - 9*b^8*c^3 - 9*b^7*c^4 + 21*b^6*c^5 - 17*b^5*c^6 + 5*b^4*
c^7)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^2*a^4*abs(b) - (3*b^13 - 20*b^12*c + 60*b^11*c^2 - 108*b^10*c^3 + 130*b
^9*c^4 - 108*b^8*c^5 + 60*b^7*c^6 - 20*b^6*c^7 + 3*b^5*c^8)*sqrt(b*c)*a^5*a
bs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4
- 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b
*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^3*(b^2 - 2*b*c + c^2)*a) - 1/6*(3
*(b*x + a)*b^3 + a*b^3 + 3*(b*x + a)*b^2*c - 3*a*b^2*c)/((b^2 - 2*b*c + c^2
)*b^3*x^3)
```

maple [C] time = 0.02, size = 457, normalized size = 2.63

$$-\frac{b}{2\left(b-c\right)^2x^2}-\frac{c}{2\left(b-c\right)^2x^2}-\frac{2a}{3\left(b-c\right)^2x^3}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(-3b^3x^3\ln\left(\frac{\left(bx+cx+2a+2\sqrt{bc\,x^2+abx+acx+a^2}\,\operatorname{csgn}(a)\right)a}{x}\right)+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] -1/2/x^2/(b-c)^2*b-1/2/x^2/(b-c)^2*c-2/3*a/(b-c)^2/x^3-1/24/(b-c)^2*(b*x+a)
^(1/2)*(c*x+a)^(1/2)/a^2*(-3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)*csgn(a))*a/x)*x^3*b^3+3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/
2)*csgn(a))*a/x)*x^3*b^2*c+3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)*csgn(a))*a/x)*x^3*b*c^2-3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(
1/2)*csgn(a))*a/x)*x^3*c^3+6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*x^2*b^
2-4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*x^2*b*c+6*(b*c*x^2+a*b*x+a*c*x+
a^2)^(1/2)*csgn(a)*x^2*c^2-4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*a*x*b-
4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*a*x*c-16*(b*c*x^2+a*b*x+a*c*x+a^2
)^(1/2)*a^2*csgn(a))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

mupad [B] time = 18.74, size = 1290, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)
```

```
[Out] (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(8*a
^2) - ((((a + b*x)^(1/2) - a^(1/2))^7*(3*b^5*c - 15*b*c^5 + 3*c^6 + 3*b^2*c
^4 + 3*b^3*c^3 - 15*b^4*c^2))/((a + c*x)^(1/2) - a^(1/2))^7 - (((a + b*x)^(
1/2) - a^(1/2))^5*(26*b^5*c - b*c^5 - b^6 + 26*b^2*c^4 + 4*b^3*c^3 + 4*b^4*
c^2))/((a + c*x)^(1/2) - a^(1/2))^5 - b^6/3 + ((b^5*c + b^6)*((a + b*x)^(1/
2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (((a + b*x)^(1/2) - a^(1/2))^8
*(c^6 - 6*b*c^5 + 7*b^2*c^4 - 6*b^3*c^3 + b^4*c^2))/((a + c*x)^(1/2) - a^(1
/2))^8 + (((a + b*x)^(1/2) - a^(1/2))^6*(6*b*c^5 + 6*b^5*c - (5*b^6)/3 - (5
*c^6)/3 + 30*b^2*c^4 - 24*b^3*c^3 + 30*b^4*c^2))/((a + c*x)^(1/2) - a^(1/2)
)^6 - (((17*b^6)/3 + (17*b^3*c^3)/3)*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c
*x)^(1/2) - a^(1/2))^3 + (((a + b*x)^(1/2) - a^(1/2))^2*(b^6 - 4*b^5*c + b^
4*c^2))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^4*(18*
b^5*c + 5*b^6 + 5*b^2*c^4 + 18*b^3*c^3 - 6*b^4*c^2))/((a + c*x)^(1/2) - a^(
1/2))^4)/((((a + b*x)^(1/2) - a^(1/2))^5*(96*a^2*b^5 + 96*a^2*b*c^4 + 96*a^
2*b^4*c + 96*a^2*b^2*c^3 - 384*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^5
- (((a + b*x)^(1/2) - a^(1/2))^8*(96*a^2*c^5 - 96*a^2*b*c^4 - 96*a^2*b^2*c^
3 + 96*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 - (((a + b*x)^(1/2) - a^
(1/2))^6*(32*a^2*b^5 + 32*a^2*c^5 + 224*a^2*b*c^4 + 224*a^2*b^4*c - 256*a^2
*b^2*c^3 - 256*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^6 - (((a + b*x)^(1
/2) - a^(1/2))^4*(96*a^2*b^5 - 96*a^2*b^4*c + 96*a^2*b^2*c^3 - 96*a^2*b^3*c
^2))/((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^7*(96*a^2
*c^5 + 96*a^2*b*c^4 + 96*a^2*b^4*c - 384*a^2*b^2*c^3 + 96*a^2*b^3*c^2))/((a
 + c*x)^(1/2) - a^(1/2))^7 + (((a + b*x)^(1/2) - a^(1/2))^3*(32*a^2*b^5 - 6
4*a^2*b^4*c + 32*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^3 + (((a + b*x)^
(1/2) - a^(1/2))^9*(32*a^2*c^5 - 64*a^2*b*c^4 + 32*a^2*b^2*c^3))/((a + c*x)
^(1/2) - a^(1/2))^9) - (((c*(8*b*c + 3*b^2 + 3*c^2))/(16*a^2*(b - c)^2) - (
c*(17*b*c + 4*b^2 + 4*c^2))/(32*a^2*(b - c)^2))*((a + b*x)^(1/2) - a^(1/2))
)/((a + c*x)^(1/2) - a^(1/2)) - (log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(
b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x
)^(1/2) - a^(1/2)))*(b + c))/(8*a^2) - ((2*a)/3 + x*(b/2 + c/2))/(x^3*(b^2
- 2*b*c + c^2)) + (c^3*((a + b*x)^(1/2) - a^(1/2))^3)/(96*a^2*(b - c)^2*((a
 + c*x)^(1/2) - a^(1/2))^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

3.438 $\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=277

$$\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}-\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3}+\frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3}-\frac{4a(b+3c)(a}{5b^3(b-}$$

```
[Out] -8/3*a^2*(b*x+a)^(3/2)/b^2/(b-c)^3+2/3*a^2*(b+3*c)*(b*x+a)^(3/2)/b^3/(b-c)^
3+8/5*a*(b*x+a)^(5/2)/b^2/(b-c)^3-4/5*a*(b+3*c)*(b*x+a)^(5/2)/b^3/(b-c)^3+2
/7*(b+3*c)*(b*x+a)^(7/2)/b^3/(b-c)^3+8/3*a^2*(c*x+a)^(3/2)/(b-c)^3/c^2-2/3*
a^2*(3*b+c)*(c*x+a)^(3/2)/(b-c)^3/c^3-8/5*a*(c*x+a)^(5/2)/(b-c)^3/c^2+4/5*a
*(3*b+c)*(c*x+a)^(5/2)/(b-c)^3/c^3-2/7*(3*b+c)*(c*x+a)^(7/2)/(b-c)^3/c^3
```

Rubi [A] time = 0.32, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6690, 43}

$$\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}-\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3}+\frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3}-\frac{4a(b+3c)(a}{5b^3(b-}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-8*a^2*(a + b*x)^(3/2))/(3*b^2*(b - c)^3) + (2*a^2*(b + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(b - c)^3) + (8*a*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (4*a*(b +
 3*c)*(a + b*x)^(5/2))/(5*b^3*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(7/2))/(7
*b^3*(b - c)^3) + (8*a^2*(a + c*x)^(3/2))/(3*(b - c)^3*c^2) - (2*a^2*(3*b +
 c)*(a + c*x)^(3/2))/(3*(b - c)^3*c^3) - (8*a*(a + c*x)^(5/2))/(5*(b - c)^3
*c^2) + (4*a*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^3) - (2*(3*b + c)*(a
 + c*x)^(7/2))/(7*(b - c)^3*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(4ax\sqrt{a+bx}+b\left(1+\frac{3c}{b}\right)x^2\sqrt{a+bx}-4ax\sqrt{a+cx}-3b\left(1+\frac{c}{3b}\right)x^2\sqrt{a+c\right.}{(b-c)^3}$$

$$= \frac{(4a)\int x\sqrt{a+bx}\,dx}{(b-c)^3} - \frac{(4a)\int x\sqrt{a+cx}\,dx}{(b-c)^3} - \frac{(3b+c)\int x^2\sqrt{a+cx}\,dx}{(b-c)^3} + \frac{(b+}{}$$

$$= \frac{(4a)\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{(b-c)^3} - \frac{(4a)\int \left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{(b-c)^3} - \frac{(3b+c)\int}{}$$

$$= -\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3} + \frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3} + \frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{4a(b+3c)(a+bx}{5b^3(b-c)^3}$$

Mathematica [A] time = 0.39, size = 271, normalized size = 0.98

$$\frac{2\left(8a^3\left(b^4\left(-\sqrt{a+cx}\right)+2b^3c\sqrt{a+cx}+c^4\sqrt{a+bx}-2bc^3\sqrt{a+bx}\right)+4a^2bcx\left(b^3\sqrt{a+cx}-2b^2c\sqrt{a+cx}-c^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(5*b^3*c^3*x^3*(b*Sqrt[a + b*x] + 3*c*Sqrt[a + b*x] - 3*b*Sqrt[a + c*x]
- c*Sqrt[a + c*x]) + 4*a^2*b*c*x*(2*b*c^2*Sqrt[a + b*x] - c^3*Sqrt[a + b*x]
 + b^3*Sqrt[a + c*x] - 2*b^2*c*Sqrt[a + c*x]) + 8*a^3*(-2*b*c^3*Sqrt[a + b*
x] + c^4*Sqrt[a + b*x] - b^4*Sqrt[a + c*x] + 2*b^3*c*Sqrt[a + c*x]) + a*b^2
*c^2*x^2*(3*c^2*Sqrt[a + b*x] - 3*b^2*Sqrt[a + c*x] + 29*b*c*(Sqrt[a + b*x]
 - Sqrt[a + c*x]))))/(35*b^3*(b - c)^3*c^3)
```

fricas [A] time = 0.48, size = 225, normalized size = 0.81

$$-\frac{2\left(\left(16a^3bc^3-8a^3c^4-5\left(b^4c^3+3b^3c^4\right)x^3-\left(29ab^3c^3+3ab^2c^4\right)x^2-4\left(2a^2b^2c^3-a^2bc^4\right)x\right)\sqrt{bx+a}+\left(8a^3\right.}{35\left(b^6c^3-3b^5c^4+3b^4c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -2/35*((16*a^3*b*c^3 - 8*a^3*c^4 - 5*(b^4*c^3 + 3*b^3*c^4)*x^3 - (29*a*b^3*
c^3 + 3*a*b^2*c^4)*x^2 - 4*(2*a^2*b^2*c^3 - a^2*b*c^4)*x)*sqrt(b*x + a) + (
8*a^3*b^4 - 16*a^3*b^3*c + 5*(3*b^4*c^3 + b^3*c^4)*x^3 + (3*a*b^4*c^2 + 29*
a*b^3*c^3)*x^2 - 4*(a^2*b^4*c - 2*a^2*b^3*c^2)*x)*sqrt(c*x + a))/(b^6*c^3 -
 3*b^5*c^4 + 3*b^4*c^5 - b^3*c^6)
```

giac [B] time = 5.58, size = 932, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/35*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(((b*x + a)*(5*(3*b^22*c^5*abs(b)
 - 17*b^21*c^6*abs(b) + 39*b^20*c^7*abs(b) - 45*b^19*c^8*abs(b) + 25*b^18*c
^9*abs(b) - 3*b^17*c^10*abs(b) - 3*b^16*c^11*abs(b) + b^15*c^12*abs(b))*(b*
x + a)/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 -
126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14) + (3
*a*b^23*c^4*abs(b) - 34*a*b^22*c^5*abs(b) + 126*a*b^21*c^6*abs(b) - 210*a*b
```

```
^20*c^7*abs(b) + 140*a*b^19*c^8*abs(b) + 42*a*b^18*c^9*abs(b) - 126*a*b^17*
c^10*abs(b) + 74*a*b^16*c^11*abs(b) - 15*a*b^15*c^12*abs(b))/(b^29*c^5 - 9*
b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^10 + 84*b^
23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) - (4*a^2*b^24*c^3*abs(b)
 - 26*a^2*b^23*c^4*abs(b) + 85*a^2*b^22*c^5*abs(b) - 203*a^2*b^21*c^6*abs(b
) + 385*a^2*b^20*c^7*abs(b) - 539*a^2*b^19*c^8*abs(b) + 511*a^2*b^18*c^9*ab
s(b) - 305*a^2*b^17*c^10*abs(b) + 103*a^2*b^16*c^11*abs(b) - 15*a^2*b^15*c^
12*abs(b))/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^
9 - 126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14))
*(b*x + a) + (8*a^3*b^25*c^2*abs(b) - 60*a^3*b^24*c^3*abs(b) + 187*a^3*b^23
*c^4*abs(b) - 296*a^3*b^22*c^5*abs(b) + 196*a^3*b^21*c^6*abs(b) + 112*a^3*b
^20*c^7*abs(b) - 350*a^3*b^19*c^8*abs(b) + 328*a^3*b^18*c^9*abs(b) - 164*a^
3*b^17*c^10*abs(b) + 44*a^3*b^16*c^11*abs(b) - 5*a^3*b^15*c^12*abs(b))/(b^2
9*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^
10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) + 2/35*(5*(b*x
 + a)^(7/2)*b + 14*(b*x + a)^(5/2)*a*b - 35*(b*x + a)^(3/2)*a^2*b + 15*(b*x
 + a)^(7/2)*c - 42*(b*x + a)^(5/2)*a*c + 35*(b*x + a)^(3/2)*a^2*c)/(b^6 - 3
*b^5*c + 3*b^4*c^2 - b^3*c^3)
```

maple [A] time = 0.01, size = 246, normalized size = 0.89

$$\frac{8\left(-\frac{(bx+a)^{\frac{3}{2}}a}{3}+\frac{(bx+a)^{\frac{5}{2}}}{5}\right)a}{(b-c)^{3}b^{2}}-\frac{8\left(-\frac{(cx+a)^{\frac{3}{2}}a}{3}+\frac{(cx+a)^{\frac{5}{2}}}{5}\right)a}{(b-c)^{3}c^{2}}-\frac{6\left(\frac{(cx+a)^{\frac{3}{2}}a^{2}}{3}-\frac{2(cx+a)^{\frac{5}{2}}a}{5}+\frac{(cx+a)^{\frac{7}{2}}}{7}\right)b}{(b-c)^{3}c^{3}}+\frac{\frac{2(bx+a)^{\frac{3}{2}}a^{2}}{3}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+}{(b-c)^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3/b^2*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2))
+8/(b-c)^3*a/b^2*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))-8/(b-c)^3*a/c^2*(
-1/3*(c*x+a)^(3/2)*a+1/5*(c*x+a)^(5/2))+6/(b-c)^3*c/b^3*(1/3*(b*x+a)^(3/2)*
a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2))-6/(b-c)^3*b/c^3*(1/7*(c*x+a)^(7/
2)-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)-2/(b-c)^3/c^2*(1/7*(c*x+a)^(7
/2)-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

mupad [B] time = 3.34, size = 429, normalized size = 1.55

$$\frac{x^{2}\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^{3}}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^{3}}\right)\sqrt{a+c\,x}}{5\,c}-\frac{2\,a\left(\frac{8\,a^{2}}{(b-c)^{3}}-\frac{4\,a\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^{3}}-\frac{12\,a\,\left(b^{2}+3\,c\,b\right)}{7\,b\,(b-c)^{3}}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b^{2}}+\frac{x\left(\frac{8\,a^{2}}{(b-c)^{3}}-\frac{4\,a\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^{3}}-\frac{12\,a}{7}\right.}{5\,b}\right.}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (x^2*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3)*(a + c*
x)^(1/2))/(5*c) - (2*a*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)
```

```
^3 - (12*a*(3*b*c + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b^2)
 + (x*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)^3 - (12*a*(3*b*c
 + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b) + (2*a*((8*a^2)/(b
 - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^
3))/(5*c))*(a + c*x)^(1/2))/(3*c^2) + (x^2*((2*a*(5*b + 3*c))/(b - c)^3 - (
12*a*(3*b*c + b^2))/(7*b*(b - c)^3))*(a + b*x)^(1/2))/(5*b) - (x*((8*a^2)/(
b - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)
^3))/(5*c))*(a + c*x)^(1/2))/(3*c) - (2*x^3*(3*b + c)*(a + c*x)^(1/2))/(7*(
b - c)^3) + (2*x^3*(3*b*c + b^2)*(a + b*x)^(1/2))/(7*b*(b - c)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Timed out
```

3.439 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=163

$$\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3}+\frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3c(b-c)^3}$$

```
[Out] 8/3*a*(b*x+a)^(3/2)/b/(b-c)^3-2/3*a*(b+3*c)*(b*x+a)^(3/2)/b^2/(b-c)^3+2/5*(
b+3*c)*(b*x+a)^(5/2)/b^2/(b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+2/3*a*(3*b+c
)*(c*x+a)^(3/2)/(b-c)^3/c^2-2/5*(3*b+c)*(c*x+a)^(5/2)/(b-c)^3/c^2
```

Rubi [A] time = 0.22, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6690, 43}

$$\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3}+\frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3c(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*(a + b*x)^(3/2))/(3*b*(b - c)^3) - (2*a*(b + 3*c)*(a + b*x)^(3/2))/(3*
b^2*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (8*a*(a
+ c*x)^(3/2))/(3*(b - c)^3*c) + (2*a*(3*b + c)*(a + c*x)^(3/2))/(3*(b - c)^
3*c^2) - (2*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx &= \frac{\int \left(4a\sqrt{a+bx}+b\left(1+\frac{3c}{b}\right)x\sqrt{a+bx}-4a\sqrt{a+cx}-3b\left(1+\frac{c}{3b}\right)x\sqrt{a+cx}\right)dx}{(b-c)^3}\\
&= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}-\frac{(3b+c)\int x\sqrt{a+cx}\,dx}{(b-c)^3}+\frac{(b+3c)\int x\sqrt{a+bx}\,d}{(b-c)^3}\\
&= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}-\frac{(3b+c)\int\left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{(b-c)^3}+\frac{(b+3c)\int}{}\\
&= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}+
\end{aligned}$$

Mathematica [A] time = 0.44, size = 120, normalized size = 0.74

$$\frac{2\left(\frac{3(b+3c)(a+bx)^{5/2}}{b^2}-\frac{5a(b+3c)(a+bx)^{3/2}}{b^2}-\frac{3(3b+c)(a+cx)^{5/2}}{c^2}+\frac{5a(3b+c)(a+cx)^{3/2}}{c^2}+\frac{20a(a+bx)^{3/2}}{b}-\frac{20a(a+cx)^{3/2}}{c}\right)}{15(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*((20*a*(a + b*x)^(3/2))/b - (5*a*(b + 3*c)*(a + b*x)^(3/2))/b^2 + (3*(b
+ 3*c)*(a + b*x)^(5/2))/b^2 - (20*a*(a + c*x)^(3/2))/c + (5*a*(3*b + c)*(a
+ c*x)^(3/2))/c^2 - (3*(3*b + c)*(a + c*x)^(5/2))/c^2))/(15*(b - c)^3)
```

fricas [A] time = 0.50, size = 167, normalized size = 1.02

$$\frac{2\left(\left(6\,a^2bc^2-2\,a^2c^3+\left(b^3c^2+3\,b^2c^3\right)x^2+\left(7\,ab^2c^2+abc^3\right)x\right)\sqrt{bx+a}+\left(2\,a^2b^3-6\,a^2b^2c-\left(3\,b^3c^2+b^2c^3\right)x^2-\right.}{5\left(b^5c^2-3\,b^4c^3+3\,b^3c^4-b^2c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((6*a^2*b*c^2 - 2*a^2*c^3 + (b^3*c^2 + 3*b^2*c^3)*x^2 + (7*a*b^2*c^2 +
a*b*c^3)*x)*sqrt(b*x + a) + (2*a^2*b^3 - 6*a^2*b^2*c - (3*b^3*c^2 + b^2*c^3
)*x^2 - (a*b^3*c + 7*a*b^2*c^2)*x)*sqrt(c*x + a))/(b^5*c^2 - 3*b^4*c^3 + 3*
b^3*c^4 - b^2*c^5)
```

giac [B] time = 5.23, size = 480, normalized size = 2.94

$$-\frac{2}{5}\sqrt{ab^2+(bx+a)bc-abc}\left((bx+a)\left(\frac{\left(3\,b^{12}c^3|b|-8\,b^{11}c^4|b|+6\,b^{10}c^5|b|-b^8c^7|b|\right)(bx+a)}{b^{18}c^3-6\,b^{17}c^4+15\,b^{16}c^5-20\,b^{15}c^6+15\,b^{14}c^7-6\,b^{13}c^8+b^{12}c^9}+\frac{ab^{13}}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/5*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*((3*b^12*c^3*abs(b) - 8
*b^11*c^4*abs(b) + 6*b^10*c^5*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^18*c^3
- 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*
c^9) + (a*b^13*c^2*abs(b) - 2*a*b^12*c^3*abs(b) - 2*a*b^11*c^4*abs(b) + 8*a
*b^10*c^5*abs(b) - 7*a*b^9*c^6*abs(b) + 2*a*b^8*c^7*abs(b))/(b^18*c^3 - 6*b
^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*c^9))
 - (2*a^2*b^14*c*abs(b) - 11*a^2*b^13*c^2*abs(b) + 25*a^2*b^12*c^3*abs(b) -
 30*a^2*b^11*c^4*abs(b) + 20*a^2*b^10*c^5*abs(b) - 7*a^2*b^9*c^6*abs(b) + a
^2*b^8*c^7*abs(b))/(b^18*c^3 - 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*
b^14*c^7 - 6*b^13*c^8 + b^12*c^9)) + 2/5*((b*x + a)^(5/2)*b + 5*(b*x + a)^(
3/2)*a*b + 3*(b*x + a)^(5/2)*c - 5*(b*x + a)^(3/2)*a*c)/(b^5 - 3*b^4*c + 3*
b^3*c^2 - b^2*c^3)
```

maple [A] time = 0.00, size = 172, normalized size = 1.06

$$\frac{8\,(bx+a)^{\frac{3}{2}}\,a}{3\,(b-c)^3\,b}-\frac{8\,(cx+a)^{\frac{3}{2}}\,a}{3\,(b-c)^3\,c}-\frac{6\left(-\frac{(cx+a)^{\frac{3}{2}}a}{3}+\frac{(cx+a)^{\frac{5}{2}}}{5}\right)b}{(b-c)^3\,c^2}+\frac{-\frac{2(bx+a)^{\frac{3}{2}}a}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}}{(b-c)^3\,b}+\frac{6\left(-\frac{(bx+a)^{\frac{3}{2}}a}{3}+\frac{(bx+a)^{\frac{5}{2}}}{5}\right)c}{(b-c)^3\,b^2}-\frac{2\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3/b*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))+8/3*a*(b*x+a)^(3/2)/b/
(b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+6/(b-c)^3*c/b^2*(-1/3*(b*x+a)^(3/2)*a
+1/5*(b*x+a)^(5/2))-6/(b-c)^3*b/c^2*(-1/3*(c*x+a)^(3/2)*a+1/5*(c*x+a)^(5/2)
)-2/(b-c)^3/c*(-1/3*(c*x+a)^(3/2)*a+1/5*(c*x+a)^(5/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

mupad [B] time = 3.36, size = 268, normalized size = 1.64

$$\frac{\left(\frac{8a^2}{(b-c)^3}+\frac{2a\left(\frac{8a(b+3c)}{5(b-c)^3}-\frac{2a(5b+3c)}{(b-c)^3}\right)}{3b}\right)\sqrt{a+bx}}{b}-\frac{\left(\frac{8a^2}{(b-c)^3}+\frac{2a\left(\frac{8a(3b+c)}{5(b-c)^3}-\frac{2a(3b+5c)}{(b-c)^3}\right)}{3c}\right)\sqrt{a+cx}}{c}-\frac{x\left(\frac{8a(b+3c)}{5(b-c)^3}-\frac{2a(5b+3c)}{(b-c)^3}\right)\sqrt{a}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (((8*a^2)/(b - c)^3 + (2*a*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*c
))/(b - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((8*a^2)/(b - c)^3 + (2*a*((8*a
*(3*b + c))/(5*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3))/(3*c))*(a + c*x)^
(1/2))/c - (x*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*c))/(b - c)^3)
*(a + b*x)^(1/2))/(3*b) + (x*((8*a*(3*b + c))/(5*(b - c)^3) - (2*a*(3*b + 5
*c))/(b - c)^3)*(a + c*x)^(1/2))/(3*c) + (2*x^2*(b + 3*c)*(a + b*x)^(1/2))/
(5*(b - c)^3) - (2*x^2*(3*b + c)*(a + c*x)^(1/2))/(5*(b - c)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

3.440 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\sqrt{a+bx}}{(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+}{3c(b-c)}$$

```
[Out] 2/3*(b+3*c)*(b*x+a)^(3/2)/b/(b-c)^3-2/3*(3*b+c)*(c*x+a)^(3/2)/(b-c)^3/c-8*a
^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)^3+8*a^(3/2)*arctanh((c*x+a)^(1/
2)/a^(1/2))/(b-c)^3+8*a*(b*x+a)^(1/2)/(b-c)^3-8*a*(c*x+a)^(1/2)/(b-c)^3
```

Rubi [A] time = 0.20, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6690, 50, 63, 208}

$$-\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\sqrt{a+bx}}{(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+}{3c(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*Sqrt[a + b*x])/(b - c)^3 + (2*(b + 3*c)*(a + b*x)^(3/2))/(3*b*(b - c)^
3) - (8*a*Sqrt[a + c*x])/(b - c)^3 - (2*(3*b + c)*(a + c*x)^(3/2))/(3*(b -
c)^3*c) - (8*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (8*a^(3/2)
*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}+\frac{4a\sqrt{a+bx}}{x}-3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}-\frac{4a\sqrt{a+cx}}{x}\right)dx}{(b-c)^3}$$

$$= \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c} + \frac{(4a)\int \frac{\sqrt{a+bx}}{x}dx}{(b-c)^3} - \frac{(4a)\int \frac{\sqrt{a+cx}}{x}dx}{(b-c)^3}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3} + \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a\sqrt{a+cx}}{(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c} + \frac{(4a^2)}{}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3} + \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a\sqrt{a+cx}}{(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c} + \frac{(8a^2)}{}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3} + \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a\sqrt{a+cx}}{(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c} - \frac{8a^{3/2}\,\mathrm{t}}{}$$

Mathematica [A] time = 0.27, size = 119, normalized size = 0.77

$$\frac{2\left(-12a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+12a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)+\frac{(b+3c)(a+bx)^{3/2}}{b}-\frac{(3b+c)(a+cx)^{3/2}}{c}+12a\sqrt{a+bx}-12a\sqrt{a+cx}\right)}{3(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(12*a*Sqrt[a + b*x] + ((b + 3*c)*(a + b*x)^(3/2))/b - 12*a*Sqrt[a + c*x]
 - ((3*b + c)*(a + c*x)^(3/2))/c - 12*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]
] + 12*a^(3/2)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]))/(3*(b - c)^3)
```

fricas [A] time = 0.46, size = 321, normalized size = 2.07

$$\left[-\frac{2\left(6\,a^{\frac{3}{2}}bc\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+6\,a^{\frac{3}{2}}bc\log\left(\frac{cx-2\,\sqrt{cx+a}\,\sqrt{a}+2\,a}{x}\right)-\left(13\,abc+3\,ac^2+\left(b^2c+3\,bc^2\right)x\right)\sqrt{bx+a}\,+\right.}{3\left(b^4c-3\,b^3c^2+3\,b^2c^3-bc^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-2/3*(6*a^(3/2)*b*c*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 6*a^(3/
2)*b*c*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - (13*a*b*c + 3*a*c^2 +
 (b^2*c + 3*b*c^2)*x)*sqrt(b*x + a) + (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^
2)*x)*sqrt(c*x + a))/(b^4*c - 3*b^3*c^2 + 3*b^2*c^3 - b*c^4), 2/3*(12*sqrt(
-a)*a*b*c*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 12*sqrt(-a)*a*b*c*arctan(sqrt(
c*x + a)*sqrt(-a)/a) + (13*a*b*c + 3*a*c^2 + (b^2*c + 3*b*c^2)*x)*sqrt(b*x
+ a) - (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^2)*x)*sqrt(c*x + a))/(b^4*c - 3
*b^3*c^2 + 3*b^2*c^3 - b*c^4)]
```

giac [B] time = 10.12, size = 2374, normalized size = 15.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/3*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*b^7*c*abs(b) - 8*b^6*c^2*abs(b
) + 6*b^5*c^3*abs(b) - b^3*c^5*abs(b))*(b*x + a)/(b^12*c - 6*b^11*c^2 + 15*
b^10*c^3 - 20*b^9*c^4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7) + (3*a*b^8*abs(b)
 + a*b^7*c*abs(b) - 22*a*b^6*c^2*abs(b) + 30*a*b^5*c^3*abs(b) - 13*a*b^4*c^
4*abs(b) + a*b^3*c^5*abs(b))/(b^12*c - 6*b^11*c^2 + 15*b^10*c^3 - 20*b^9*c^
4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7)) + 8*a^2*arctan(sqrt(b*x + a)/sqrt(-a
))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt(-a)) + 2/3*((b*x + a)^(3/2)*b^9 +
12*sqrt(b*x + a)*a*b^9 - 3*(b*x + a)^(3/2)*b^8*c - 72*sqrt(b*x + a)*a*b^8*c
 - 3*(b*x + a)^(3/2)*b^7*c^2 + 180*sqrt(b*x + a)*a*b^7*c^2 + 25*(b*x + a)^(
3/2)*b^6*c^3 - 240*sqrt(b*x + a)*a*b^6*c^3 - 45*(b*x + a)^(3/2)*b^5*c^4 + 1
80*sqrt(b*x + a)*a*b^5*c^4 + 39*(b*x + a)^(3/2)*b^4*c^5 - 72*sqrt(b*x + a)*
a*b^4*c^5 - 17*(b*x + a)^(3/2)*b^3*c^6 + 12*sqrt(b*x + a)*a*b^3*c^6 + 3*(b*
x + a)^(3/2)*b^2*c^7)/(b^12 - 9*b^11*c + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8
*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9) - 8*(2*
(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a
)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^
2*c^2 - a*b*c^3)^2*(a*b^3 - a*b^2*c)*sqrt(-a*b*c)*abs(b) + (a^2*b^7 - 5*a^2
*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*sqr
t(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 -
3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^8 - 5*a^2*b^7*c + 10*a^2*b^6*c^2 - 10*a^2
*b^5*c^3 + 5*a^2*b^4*c^4 - a^2*b^3*c^5)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3
*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^
3 - 14*a^3*b^8*c^4 + 14*a^3*b^6*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*
b^3*c^9)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^11 - 6
*a^3*b^10*c + 14*a^3*b^9*c^2 - 14*a^3*b^8*c^3 + 14*a^3*b^6*c^5 - 14*a^3*b^5
*c^6 + 6*a^3*b^4*c^7 - a^3*b^3*c^8)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)
*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^
4*c + 2*a*b^2*c^3 - a*b*c^4 + sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c
^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^
3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b
*c^2 - c^3)))/((b^12 - 9*b^11*c + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8*c^4 -
126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9)*a*abs(-a*b^4 +
 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)) + 8*(2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^
2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*
b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3 - a*b
^2*c)*sqrt(-a*b*c)*abs(b) + (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^
2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*
c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2
*b^8 - 5*a^2*b^7*c + 10*a^2*b^6*c^2 - 10*a^2*b^5*c^3 + 5*a^2*b^4*c^4 - a^2*
b^3*c^5)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) +
(a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^3 - 14*a^3*b^8*c^4 + 14*a^3*b^6
*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a)*abs(b)*sgn(b^
3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^11 - 6*a^3*b^10*c + 14*a^3*b^9*c^2 -
14*a^3*b^8*c^3 + 14*a^3*b^6*c^5 - 14*a^3*b^5*c^6 + 6*a^3*b^4*c^7 - a^3*b^3*
c^8)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 - s
qrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c
+ 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b
^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^12 - 9*b^11*c
 + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b
^5*c^7 + 9*b^4*c^8 - b^3*c^9)*a*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*
c^3))
```

maple [A] time = 0.00, size = 148, normalized size = 0.95

$$\frac{4\left(-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\right)a}{(b-c)^{3}}-\frac{4\left(-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)+2\sqrt{cx+a}\right)a}{(b-c)^{3}}-\frac{2\,(cx+a)^{\frac{3}{2}}\,b}{(b-c)^{3}\,c}+\frac{2\,(bx+}{(b-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/3/(b-c)^3*(b*x+a)^(3/2)+2/(b-c)^3*c*(b*x+a)^(3/2)/b-2/(b-c)^3*b*(c*x+a)^(
3/2)/c-2/3/(b-c)^3*(c*x+a)^(3/2)+4/(b-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arc
tanh((b*x+a)^(1/2)/a^(1/2)))-4/(b-c)^3*a*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh
((c*x+a)^(1/2)/a^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

mupad [B] time = 7.02, size = 762, normalized size = 4.92

$$4a^{3/2}b^4 - \frac{4a^{3/2}c^4\left(\frac{4\left(\sqrt{a+bx}-\sqrt{a}\right)^3}{\left(\sqrt{a+cx}-\sqrt{a}\right)^3} - \frac{15\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{\left(\sqrt{a+cx}-\sqrt{a}\right)^4} + \frac{24\left(\sqrt{a+bx}-\sqrt{a}\right)^5}{\left(\sqrt{a+cx}-\sqrt{a}\right)^5} + \frac{6\ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+cx}-\sqrt{a}}\right)\left(\sqrt{a+bx}-\sqrt{a}\right)^6}{\left(\sqrt{a+cx}-\sqrt{a}\right)^6}\right)}{3} - \frac{4a^{3/2}b^2c^2\left(\frac{24\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{a+cx}-\sqrt{a}} + \frac{12\left(\sqrt{a+\right.}{\left(\sqrt{a+c\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (4*a^(3/2)*b^4 - (4*a^(3/2)*c^4*((4*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*
x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2)
 - a^(1/2))^4 + (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/
2))^5 + (6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a
 + b*x)^(1/2) - a^(1/2))^6)/((a + c*x)^(1/2) - a^(1/2))^6))/3 - (4*a^(3/2)*
b^2*c^2*((24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + (12
*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (12*((a + b
*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2
) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 + (18*log(((a + b*x)^(1/2) -
a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^2)/((a +
c*x)^(1/2) - a^(1/2))^2 - 3))/3 + (4*a^(3/2)*b*c^3*((6*((a + b*x)^(1/2) - a
^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (12*((a + b*x)^(1/2) - a^(1/2))^
3)/((a + c*x)^(1/2) - a^(1/2))^3 + (66*((a + b*x)^(1/2) - a^(1/2))^4)/((a +
 c*x)^(1/2) - a^(1/2))^4 - (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1
/2) - a^(1/2))^5 + (18*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a
^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4))/3 +
 (4*a^(3/2)*b^3*c*(6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(
1/2))) - (24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + (6*
((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (4*((a + b*x
)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + 26))/3)/(c*(b - c)^3*
(b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

3.441 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx$

Optimal. Leaf size=157

$$-\frac{4a\sqrt{a+bx}}{x(b-c)^3}+\frac{4a\sqrt{a+cx}}{x(b-c)^3}+\frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}-\frac{6\sqrt{a}\,(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{6\sqrt{a}\,(b+c)\,\mathrm{t}}{(b}$$

```
[Out] -6*(b+c)*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)^3+6*(b+c)*arctanh((c*
x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)^3+2*(b+3*c)*(b*x+a)^(1/2)/(b-c)^3-4*a*(b*
x+a)^(1/2)/(b-c)^3/x-2*(3*b+c)*(c*x+a)^(1/2)/(b-c)^3+4*a*(c*x+a)^(1/2)/(b-c
)^3/x
```

Rubi [A] time = 0.22, antiderivative size = 223, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6690, 47, 63, 208, 50}

$$-\frac{4a\sqrt{a+bx}}{x(b-c)^3}+\frac{4a\sqrt{a+cx}}{x(b-c)^3}+\frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}-\frac{2\sqrt{a}\,(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}-\frac{4\sqrt{a}\,b\tanh}{(b-}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(b + 3*c)*Sqrt[a + b*x])/(b - c)^3 - (4*a*Sqrt[a + b*x])/((b - c)^3*x) -
 (2*(3*b + c)*Sqrt[a + c*x])/(b - c)^3 + (4*a*Sqrt[a + c*x])/((b - c)^3*x)
- (4*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 - (2*Sqrt[a]*(b +
3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (4*Sqrt[a]*c*ArcTanh[Sqrt[
a + c*x]/Sqrt[a]])/(b - c)^3 + (2*Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[a + c*x]/S
qrt[a]])/(b - c)^3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^2}+\frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x}-\frac{4a\sqrt{a+cx}}{x^2}-\frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x}\right)dx}{(b-c)^3}$$

$$= \frac{(4a)\int \frac{\sqrt{a+bx}}{x^2}\,dx}{(b-c)^3}-\frac{(4a)\int \frac{\sqrt{a+cx}}{x^2}\,dx}{(b-c)^3}-\frac{(3b+c)\int \frac{\sqrt{a+cx}}{x}\,dx}{(b-c)^3}+\frac{(b+3c)\int \frac{\sqrt{a+bx}}{x}\,dx}{(b-c)^3}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}+\frac{(2ab)\int \frac{}{x\sqrt{}}}{(b-}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}+\frac{(4a)\,\mathrm{Subs}}{}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}-\frac{4\sqrt{a}\,b\tan}{(b}$$

Mathematica [A] time = 0.88, size = 192, normalized size = 1.22

$$\frac{2\left(-(3b+c)\sqrt{a+cx}+(b+3c)\sqrt{a+bx}+\sqrt{a}\,(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)-\sqrt{a}\,(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{2a\left(bx\sqrt{\frac{bx}{a}+}\right.}{}\right.}{(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*((b + 3*c)*Sqrt[a + b*x] - (3*b + c)*Sqrt[a + c*x] - Sqrt[a]*(b + 3*c)*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]] - (2*a*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTa
nh[Sqrt[1 + (b*x)/a]]))/(x*Sqrt[a + b*x]) + Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[
a + c*x]/Sqrt[a]] + (2*a*(a + c*x + c*x*Sqrt[1 + (c*x)/a]*ArcTanh[Sqrt[1 +
(c*x)/a]]))/(x*Sqrt[a + c*x])))/(b - c)^3
```

fricas [A] time = 0.48, size = 260, normalized size = 1.66

$$\left[-\frac{3\sqrt{a}\,(b+c)x\log\left(\frac{bx+2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+3\sqrt{a}\,(b+c)x\log\left(\frac{cx-2\sqrt{cx+a}\,\sqrt{a}+2a}{x}\right)-2\left((b+3c)x-2a\right)\sqrt{bx+a}+2((}{\left(b^3-3b^2c+3bc^2-c^3\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-(3*sqrt(a)*(b + c)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*sqr
t(a)*(b + c)*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*((b + 3*c)*
x - 2*a)*sqrt(b*x + a) + 2*((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2
*c + 3*b*c^2 - c^3)*x), 2*(3*sqrt(-a)*(b + c)*x*arctan(sqrt(b*x + a)*sqrt(-
a)/a) - 3*sqrt(-a)*(b + c)*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) + ((b + 3*c)*
x - 2*a)*sqrt(b*x + a) - ((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2*c
 + 3*b*c^2 - c^3)*x)]
```

giac [B] time = 78.70, size = 2318, normalized size = 14.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2*(sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(3*b*abs(b) + c*abs(b))/(b^4 - 3*b^
3*c + 3*b^2*c^2 - b*c^3) + 2*sqrt(b*x + a)*a*b/((b^3 - 3*b^2*c + 3*b*c^2 -
c^3)*x) - 3*(a*b^2 + a*b*c)*arctan(sqrt(b*x + a)/sqrt(-a))/((b^3 - 3*b^2*c
+ 3*b*c^2 - c^3)*sqrt(-a)) - (sqrt(b*x + a)*b^2 + 3*sqrt(b*x + a)*b*c)/(b^3
 - 3*b^2*c + 3*b*c^2 - c^3) + 4*((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))*a^2*b^3*c*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*
b^2 + (b*x + a)*b*c - a*b*c))*a^2*b^2*c^2*abs(b) + (sqrt(b*c)*sqrt(b*x + a)
 - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a*b*c*abs(b))/((a^2*b^4 - 2*a^2*b
^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b
*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))^4)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)) + 3*(2*(a*b^4*c - a*b^2*c^3
)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqrt(-a)*abs(b)*sgn(b^3 - 3
*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(
a*b^4 - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2*b^8 - 4*a^2*b^7*c + 5*a^2*b^6
*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*c^5 - a^2*b^2*c^6)*sqrt(-a*b*c)*abs(-a*b^4
 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 -
c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5*a^2*b^7*c^2 - 5*a^2*b^5*c^4 + 4*a^2*b^4*c
^5 - a^2*b^3*c^6)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*
abs(b) + (a^3*b^12*c - 5*a^3*b^11*c^2 + 8*a^3*b^10*c^3 - 14*a^3*b^8*c^5 + 1
4*a^3*b^7*c^6 - 8*a^3*b^5*c^8 + 5*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(
b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^12 - 5*a^3*b^11*c + 8*a^3*b^
10*c^2 - 14*a^3*b^8*c^4 + 14*a^3*b^7*c^5 - 8*a^3*b^5*c^7 + 5*a^3*b^4*c^8 -
a^3*b^3*c^9)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a
*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b
*c^4 + sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^
2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b
^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 -
9*b^10*c + 36*b^9*c^2 - 84*b^8*c^3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6
 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c^9)*a^2*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c
^2 + a*b*c^3)) - 3*(2*(a*b^4*c - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^
2 - a*b*c^3)^2*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^
4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^4 - a*b^2*c^2)*sqrt(-a*b*c)*a
bs(b) + (a^2*b^8 - 4*a^2*b^7*c + 5*a^2*b^6*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*
c^5 - a^2*b^2*c^6)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*
c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5
*a^2*b^7*c^2 - 5*a^2*b^5*c^4 + 4*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs(-a
*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^12*c - 5*a^3*b^11
*c^2 + 8*a^3*b^10*c^3 - 14*a^3*b^8*c^5 + 14*a^3*b^7*c^6 - 8*a^3*b^5*c^8 + 5
*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 -
c^3) + (a^3*b^12 - 5*a^3*b^11*c + 8*a^3*b^10*c^2 - 14*a^3*b^8*c^4 + 14*a^3*
b^7*c^5 - 8*a^3*b^5*c^7 + 5*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a*b*c)*abs(b))
*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sq
rt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 - sqrt((a*b^5 - 2*a*b^4*c +
2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2
```

```
*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(
b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c^2 - 84*b^8*c^
3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c
^9)*a^2*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)))/b
```

maple [A] time = 0.01, size = 237, normalized size = 1.51

$$\frac{8\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)ab}{(b-c)^3}-\frac{8\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{cx+a}}{2cx}\right)ac}{(b-c)^3}+\frac{\left(-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\right)b}{(b-c)^3}-\frac{3\left(-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 1/(b-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8/(b
-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-8/(b-c)^3*a*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/
a^(1/2)))+3/(b-c)^3*c*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1
/2)))-3/(b-c)^3*b*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2))
)-1/(b-c)^3*c*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

mupad [B] time = 7.49, size = 559, normalized size = 3.56

$$\frac{2\sqrt{a}\,b^2\left(\sqrt{a+c\,x}-\sqrt{a}\right)\left(\frac{8\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}-\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}+\frac{3\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+1\right)-2\sqrt{a}\,c^2\left(\sqrt{a+c\,x}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (2*a^(1/2)*b^2*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2)))
/((a + c*x)^(1/2) - a^(1/2)) - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)
^(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
 a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 1) -
2*a^(1/2)*c^2*((a + c*x)^(1/2) - a^(1/2))*((2*((a + b*x)^(1/2) - a^(1/2))^2
)/((a + c*x)^(1/2) - a^(1/2))^2 - ((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^
(1/2) - a^(1/2))^4 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3) + 2
*a^(1/2)*b*c*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2)))/(
(a + c*x)^(1/2) - a^(1/2)) - (14*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^
(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (3*log
(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2)
- a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3))/((b - c)^3*((a + b*x)^(1/2) -
```

```
 a^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2)
)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

3.442 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=164

$$-\frac{2a\sqrt{a+bx}}{x^2(b-c)^3}+\frac{2a\sqrt{a+cx}}{x^2(b-c)^3}-\frac{(2b+3c)\sqrt{a+bx}}{x(b-c)^3}+\frac{(3b+2c)\sqrt{a+cx}}{x(b-c)^3}-\frac{3bc\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}+\frac{3bc\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}$$

```
[Out] -3*b*c*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)^3/a^(1/2)+3*b*c*arctanh((c*x+a)
^(1/2)/a^(1/2))/(b-c)^3/a^(1/2)-2*a*(b*x+a)^(1/2)/(b-c)^3/x^2-(2*b+3*c)*(b*
x+a)^(1/2)/(b-c)^3/x+2*a*(c*x+a)^(1/2)/(b-c)^3/x^2+(3*b+2*c)*(c*x+a)^(1/2)/
(b-c)^3/x
```

Rubi [A] time = 0.18, antiderivative size = 275, normalized size of antiderivative = 1.68, number of steps used = 16, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6690, 47, 51, 63, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}-\frac{2a\sqrt{a+bx}}{x^2(b-c)^3}+\frac{2a\sqrt{a+cx}}{x^2(b-c)^3}-\frac{b\sqrt{a+bx}}{x(b-c)^3}-\frac{(b+3c)\sqrt{a+bx}}{x(b-c)^3}+\frac{c\sqrt{a+cx}}{x(b-c)^3}+\frac{(3b$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/((b - c)^3*x^2) - (b*Sqrt[a + b*x])/((b - c)^3*x) - ((
b + 3*c)*Sqrt[a + b*x])/((b - c)^3*x) + (2*a*Sqrt[a + c*x])/((b - c)^3*x^2)
 + (c*Sqrt[a + c*x])/((b - c)^3*x) + ((3*b + c)*Sqrt[a + c*x])/((b - c)^3*x
) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (b*(b + 3*c)
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (c^2*ArcTanh[Sqrt[a
+ c*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) + (c*(3*b + c)*ArcTanh[Sqrt[a + c*x]/S
qrt[a]])/(Sqrt[a]*(b - c)^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^3}+\frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x^2}-\frac{4a\sqrt{a+cx}}{x^3}-\frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x^2}\right)dx}{(b-c)^3}$$

$$= \frac{(4a)\int \frac{\sqrt{a+bx}}{x^3}\,dx}{(b-c)^3}-\frac{(4a)\int \frac{\sqrt{a+cx}}{x^3}\,dx}{(b-c)^3}-\frac{(3b+c)\int \frac{\sqrt{a+cx}}{x^2}\,dx}{(b-c)^3}+\frac{(b+3c)\int \frac{\sqrt{a+bx}}{x^2}}{(b-c)^3}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{(3b+c)\sqrt{a+cx}}{(b-c)^3x}+\frac{(ab)\int \frac{}{x}}{(b}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{b\sqrt{a+bx}}{(b-c)^3x}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{c\sqrt{a+cx}}{(b-c)^3x}+\frac{(3}{}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{b\sqrt{a+bx}}{(b-c)^3x}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{c\sqrt{a+cx}}{(b-c)^3x}+\frac{(3}{}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{b\sqrt{a+bx}}{(b-c)^3x}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{c\sqrt{a+cx}}{(b-c)^3x}+\frac{(3}{}$$

Mathematica [C] time = 0.27, size = 182, normalized size = 1.11

$$\frac{-\frac{8b^2(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{a^2}+\frac{8c^2(a+cx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx}{a}+1\right)}{a^2}-\frac{3(b+3c)\left(bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx\right)}{x\sqrt{a+bx}}+\frac{3(3b+c)\left(cx\sqrt{\frac{cx}{a}+1}\tanh^{-1}\right.}{x\sqrt{a+c}}}{3(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] ((-3*(b + 3*c)*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])
)/(x*Sqrt[a + b*x]) + (3*(3*b + c)*(a + c*x + c*x*Sqrt[1 + (c*x)/a]*ArcTanh
[Sqrt[1 + (c*x)/a]]))/(x*Sqrt[a + c*x]) - (8*b^2*(a + b*x)^(3/2)*Hypergeome
tric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/a^2 + (8*c^2*(a + c*x)^(3/2)*Hypergeomet
ric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/a^2)/(3*(b - c)^3)
```

fricas [A] time = 0.47, size = 297, normalized size = 1.81

$$\left[-\frac{3\sqrt{a}\,bcx^2\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\sqrt{a}\,bcx^2\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\left(2a^2+(2ab+3ac)x\right)\sqrt{bx+a}-2}{2\left(ab^3-3ab^2c+3abc^2-ac^3\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(3*sqrt(a)*b*c*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*s
qrt(a)*b*c*x^2*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(2*a^2 + (2
*a*b + 3*a*c)*x)*sqrt(b*x + a) - 2*(2*a^2 + (3*a*b + 2*a*c)*x)*sqrt(c*x + a
))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2), (3*sqrt(-a)*b*c*x^2*arcta
n(sqrt(b*x + a)*sqrt(-a)/a) - 3*sqrt(-a)*b*c*x^2*arctan(sqrt(c*x + a)*sqrt(
-a)/a) - (2*a^2 + (2*a*b + 3*a*c)*x)*sqrt(b*x + a) + (2*a^2 + (3*a*b + 2*a*
c)*x)*sqrt(c*x + a))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 300, normalized size = 1.83

$$\frac{8\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}+\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{b^{2}x^{2}}\right)a\,b^{2}}{(b-c)^{3}}-\frac{8\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}+\frac{-\frac{(cx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{cx+a}}{8}}{c^{2}x^{2}}\right)a\,c^{2}}{(b-c)^{3}}+\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)b^{2}}{(b-c)^{3}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3*b^2*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^
(1/2))+8/(b-c)^3*a*b^2*((-1/8*(b*x+a)^(3/2)/a-1/8*(b*x+a)^(1/2))/b^2/x^2+1/
8/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-8/(b-c)^3*a*c^2*((-1/8*(c*x+a)^(3
/2)/a-1/8*(c*x+a)^(1/2))/x^2/c^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2))
)+6/(b-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/
a^(1/2))-6/(b-c)^3*b*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^
(1/2)/a^(1/2)))-2/(b-c)^3*c^2*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((
c*x+a)^(1/2)/a^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3), x)
```

mupad [B] time = 5.74, size = 287, normalized size = 1.75

$$\frac{c^{2}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{2}}{4\sqrt{a}\,(b-c)^{3}\left(\sqrt{a+c\,x}-\sqrt{a}\right)^{2}}-\frac{\left(\frac{\sqrt{a}\,b^{2}}{4\left(a\,b^{3}-3\,a\,b^{2}\,c+3\,a\,b\,c^{2}-a\,c^{3}\right)}-\frac{\sqrt{a}\,\left(b^{2}+c\,b\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)\left(a\,b^{3}-3\,a\,b^{2}\,c+3\,a\,b\,c^{2}-a\,c^{3}\right)}\right)\left(\sqrt{a+c\,x}-\sqrt{a}\right)^{2}}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(4*a^(1/2)*(b - c)^3*((a + c*x)^(1/2) -
 a^(1/2))^2) - (((a^(1/2)*b^2)/(4*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c))
- (a^(1/2)*(b*c + b^2)*((a + b*x)^(1/2) - a^(1/2)))/(((a + c*x)^(1/2) - a^(
1/2))*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c)))*((a + c*x)^(1/2) - a^(1/2))
^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (3*b*c*log(((a + b*x)^(1/2) - a^(1/2))/
((a + c*x)^(1/2) - a^(1/2))))/(a^(1/2)*(3*b*c^2 - 3*b^2*c + b^3 - c^3)) - (
c*(b + c)*((a + b*x)^(1/2) - a^(1/2)))/(a^(1/2)*(b - c)^3*((a + c*x)^(1/2)
- a^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-3), x)
```

3.443 $\int \sqrt{1-x} \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx$

Optimal. Leaf size=31

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}\,x+x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] x-1/2*x^2+1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6688, 195, 216}

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}\,x+x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-x} \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx &= \int \left(1-x+\sqrt{1-x^2}\right) \, dx\\
&= x-\frac{x^2}{2}+\int \sqrt{1-x^2} \, dx\\
&= x-\frac{x^2}{2}+\frac{1}{2} x \sqrt{1-x^2}+\frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} \, dx\\
&= x-\frac{x^2}{2}+\frac{1}{2} x \sqrt{1-x^2}+\frac{1}{2} \sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}\,x+x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

fricas [A] time = 0.43, size = 44, normalized size = 1.42

$$-\frac{1}{2}x^2 + \frac{1}{2}\sqrt{x+1}\,x\sqrt{-x+1} + x - \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + x - arctan((sqrt(x + 1)*sqrt(-x
 + 1) - 1)/x)
```

giac [B] time = 0.61, size = 54, normalized size = 1.74

$$-\frac{1}{2}(x-1)^2 + \frac{1}{2}(x+2)\sqrt{x+1}\sqrt{-x+1} - \sqrt{x+1}\sqrt{-x+1} - \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*(x - 1)^2 + 1/2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x
 + 1) - arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

maple [B] time = 0.00, size = 63, normalized size = 2.03

$$-\frac{x^2}{2} + x + \frac{\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{2\sqrt{-x+1}\,\sqrt{x+1}} - \frac{\sqrt{x+1}\,(-x+1)^{\frac{3}{2}}}{2} + \frac{\sqrt{-x+1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)*((-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] x-1/2*x^2-1/2*(x+1)^(1/2)*(-x+1)^(3/2)+1/2*(-x+1)^(1/2)*(x+1)^(1/2)+1/2*((x
+1)*(-x+1))^(1/2)/(-x+1)^(1/2)/(x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 1.49, size = 23, normalized size = 0.74

$$-\frac{1}{2}x^2 + \frac{1}{2}\sqrt{-x^2+1}\,x + x + \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/2*sqrt(-x^2 + 1)*x + x + 1/2*arcsin(x)
```

mupad [B] time = 8.12, size = 209, normalized size = 6.74

$$x - 2\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\frac{2\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1} - \frac{14\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3} + \frac{14\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5} - \frac{2\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2} + \frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4} + \frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6} + \frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8} + 1} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))*(1 - x)^(1/2),x)
```

```
[Out] x - 2*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((2*((1 - x)^(1/2) -
1))/((x + 1)^(1/2) - 1) - (14*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3
+ (14*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (2*((1 - x)^(1/2) - 1)
^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2
 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) - 1)
^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 + 1
) - x^2/2
```

sympy [**A**] time = 3.07, size = 48, normalized size = 1.55

$$-\frac{(1-x)^2}{2}-2\left(\begin{cases} -\frac{x\sqrt{1-x}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{1-x}}{2}\right)}{2} & \text{for } x\leq 1\wedge x>-1 \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(1 - x)**2/2 - 2*Piecewise((-x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sq
rt(1 - x)/2)/2, (x <= 1) & (x > -1)))
```

3.444 $\int x^3 \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right) dx$

Optimal. Leaf size=38

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] -1/2*x^4+2/3*(-x^2+1)^(3/2)-2/5*(-x^2+1)^(5/2)
```

Rubi [A] time = 0.32, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6688, 6742, 266, 43}

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^4/2 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right) dx &= -\int x^3 \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx\\
&= -\int \left(2x^3+2x^3\sqrt{1-x^2}\right) dx\\
&= -\frac{x^4}{2}-2\int x^3\sqrt{1-x^2}\, dx\\
&= -\frac{x^4}{2}-\text{Subst}\left(\int \sqrt{1-x}\, x\, dx, x, x^2\right)\\
&= -\frac{x^4}{2}-\text{Subst}\left(\int \left(\sqrt{1-x}-(1-x)^{3/2}\right) dx, x, x^2\right)\\
&= -\frac{x^4}{2}+\frac{2}{3}\left(1-x^2\right)^{3/2}-\frac{2}{5}\left(1-x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.00

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -1/2*x^4 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

fricas [A] time = 0.43, size = 32, normalized size = 0.84

$$-\frac{1}{2}x^4-\frac{2}{15}\left(3x^4-x^2-2\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="fricas")
```

```
[Out] -1/2*x^4 - 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

giac [B] time = 0.54, size = 77, normalized size = 2.03

$$-\frac{1}{2}x^4-\frac{1}{60}\left(\left(2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{12}\left(\left(2\left(3x-10\right)(x+1)+4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="giac")
```

```
[Out] -1/2*x^4 - 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 1
95)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) -
39)*sqrt(x + 1)*sqrt(-x + 1)
```

maple [A] time = 0.01, size = 33, normalized size = 0.87

$$-\frac{x^4}{2}-\frac{2\sqrt{x+1}\sqrt{-x+1}\left(x^2-1\right)\left(3x^2+2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -1/2*x^4-2/15*(x+1)^(1/2)*(-x+1)^(1/2)*(x^2-1)*(3*x^2+2)
```

maxima [A] time = 1.38, size = 31, normalized size = 0.82

$$-\frac{1}{2}x^4+\frac{2}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2+\frac{4}{15}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="maxima")
```

```
[Out] -1/2*x^4 + 2/5*(-x^2 + 1)^(3/2)*x^2 + 4/15*(-x^2 + 1)^(3/2)
```

mupad [B] time = 3.06, size = 42, normalized size = 1.11

$$\sqrt{1-x}\left(\frac{4\sqrt{x+1}}{15}+\frac{2x^2\sqrt{x+1}}{15}-\frac{2x^4\sqrt{x+1}}{5}\right)-\frac{x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] (1 - x)^(1/2)*((4*(x + 1)^(1/2))/15 + (2*x^2*(x + 1)^(1/2))/15 - (2*x^4*(x
+ 1)^(1/2))/5) - x^4/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

3.445 $\int x^2 \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right) dx$

Optimal. Leaf size=48

$$-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}\,x-\frac{1}{2}\sqrt{1-x^2}\,x^3-\frac{1}{4}\sin^{-1}(x)$$

```
[Out] -2/3*x^3-1/4*arcsin(x)+1/4*x*(-x^2+1)^(1/2)-1/2*x^3*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {6688, 6742, 279, 321, 216}

$$-\frac{1}{2}\sqrt{1-x^2}\,x^3-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}\,x-\frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] (-2*x^3)/3 + (x*Sqrt[1 - x^2])/4 - (x^3*Sqrt[1 - x^2])/2 - ArcSin[x]/4
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx\\ &= -\int\left(2x^2+2x^2\sqrt{1-x^2}\right)dx\\ &= -\frac{2x^3}{3}-2\int x^2\sqrt{1-x^2}\,dx\\ &= -\frac{2x^3}{3}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}dx\\ &= -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{4}\int\frac{1}{\sqrt{1-x^2}}dx\\ &= -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{4}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.90

$$\frac{1}{12}\left(3\sqrt{1-x^2}\,x-\left(\left(6\sqrt{1-x^2}+8\right)x^3\right)-3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] (3*x*Sqrt[1 - x^2] - x^3*(8 + 6*Sqrt[1 - x^2]) - 3*ArcSin[x])/12
```

fricas [A] time = 0.42, size = 51, normalized size = 1.06

$$-\frac{2}{3}x^3-\frac{1}{4}\left(2x^3-x\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="fricas")
[Out] -2/3*x^3 - 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arctan((sqrt(x +
1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 0.47, size = 76, normalized size = 1.58

$$-\frac{2}{3}x^3-\frac{1}{12}((2(3x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{3}((2x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="giac")
[Out] -2/3*x^3 - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt
(-x + 1) - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arcsi
n(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 59, normalized size = 1.23

$$-\frac{2x^3}{3}-\frac{\sqrt{x+1}\sqrt{-x+1}\left(2\sqrt{-x^2+1}\,x^3-\sqrt{-x^2+1}\,x+\arcsin(x)\right)}{4\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -2/3*x^3-1/4*(x+1)^(1/2)*(-x+1)^(1/2)*(2*(-x^2+1)^(1/2)*x^3-(-x^2+1)^(1/2)*
x+arcsin(x))/(-x^2+1)^(1/2)
```

maxima [A] time = 1.50, size = 34, normalized size = 0.71

$$-\frac{2}{3}x^3 + \frac{1}{2}\left(-x^2+1\right)^{\frac{3}{2}}x - \frac{1}{4}\sqrt{-x^2+1}\,x - \frac{1}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="maxima")
```

```
[Out] -2/3*x^3 + 1/2*(-x^2 + 1)^(3/2)*x - 1/4*sqrt(-x^2 + 1)*x - 1/4*arcsin(x)
```

mupad [B] time = 10.39, size = 381, normalized size = 7.94

$$\operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1} - \frac{35\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3} + \frac{273\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5} - \frac{715\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7} + \frac{715\left(\sqrt{1-x}-1\right)^9}{\left(\sqrt{x+1}-1\right)^9} - \frac{273\left(\sqrt{1-x}-1\right)^{11}}{\left(\sqrt{x+1}-1\right)^{11}} + \frac{35(\cdots}{(\cdots}}{\frac{8\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2} + \frac{28\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4} + \frac{56\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6} + \frac{70\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8} + \frac{56\left(\sqrt{1-x}-1\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}} + \frac{28\left(\sqrt{1-x}-1\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}} + \frac{8(\sqrt{1\cdots}}{(\sqrt{x+\cdots}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - (((1 - x)^(1/2) - 1)/((x +
1)^(1/2) - 1) - (35*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3 + (273*((1
 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (715*((1 - x)^(1/2) - 1)^7)/((x
 + 1)^(1/2) - 1)^7 + (715*((1 - x)^(1/2) - 1)^9)/((x + 1)^(1/2) - 1)^9 - (2
73*((1 - x)^(1/2) - 1)^11)/((x + 1)^(1/2) - 1)^11 + (35*((1 - x)^(1/2) - 1)
^13)/((x + 1)^(1/2) - 1)^13 - ((1 - x)^(1/2) - 1)^15/((x + 1)^(1/2) - 1)^15
)/((8*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + (28*((1 - x)^(1/2) - 1
)^4)/((x + 1)^(1/2) - 1)^4 + (56*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)
^6 + (70*((1 - x)^(1/2) - 1)^8)/((x + 1)^(1/2) - 1)^8 + (56*((1 - x)^(1/2)
- 1)^10)/((x + 1)^(1/2) - 1)^10 + (28*((1 - x)^(1/2) - 1)^12)/((x + 1)^(1/2
) - 1)^12 + (8*((1 - x)^(1/2) - 1)^14)/((x + 1)^(1/2) - 1)^14 + ((1 - x)^(1
/2) - 1)^16/((x + 1)^(1/2) - 1)^16 + 1) - (2*x^3)/3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

3.446 $\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

```
[Out] -x^2+2/3*(-x^2+1)^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {6688, 6742, 261}

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^2 + (2*(1 - x^2)^(3/2))/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx &= -\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx\\
&= -\int\left(2x+2x\sqrt{1-x^2}\right)\,dx\\
&= -x^2-2\int x\sqrt{1-x^2}\,dx\\
&= -x^2+\frac{2}{3}\left(1-x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^2 + (2*(1 - x^2)^(3/2))/3
```

fricas [A] time = 0.43, size = 25, normalized size = 1.19

$$-x^2 - \frac{2}{3}\left(x^2-1\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="fricas")
```

```
[Out] -x^2 - 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

giac [B] time = 0.39, size = 54, normalized size = 2.57

$$-(x+1)^2 - \frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1} - \sqrt{x+1}\,(x-2)\sqrt{-x+1} + 2x + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="giac")
```

```
[Out] -(x + 1)^2 - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x
+ 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2
```

maple [A] time = 0.00, size = 26, normalized size = 1.24

$$-x^2 - \frac{2\sqrt{x+1}\,\sqrt{-x+1}\,\left(x^2-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -x^2-2/3*(x+1)^(1/2)*(-x+1)^(1/2)*(x^2-1)
```

maxima [A] time = 1.47, size = 17, normalized size = 0.81

$$-x^2 + \frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="maxima")
```

```
[Out] -x^2 + 2/3*(-x^2 + 1)^(3/2)
```

mupad [B] time = 3.06, size = 25, normalized size = 1.19

$$-x^2 - \frac{2\left(x^2-1\right)\sqrt{1-x}\,\sqrt{x+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] - x^2 - (2*(x^2 - 1)*(1 - x)^(1/2)*(x + 1)^(1/2))/3
```

sympy [A] time = 101.26, size = 110, normalized size = 5.24

$$\frac{x^3}{3}+x-\frac{(x+1)^3}{3}+4\left(\begin{cases}\frac{x\sqrt{1-x}\,\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x\geq -1\wedge x<1\end{cases}\right)-4\left(\begin{cases}\frac{x\sqrt{1-x}\,\sqrt{x+1}}{4}-\frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] x**3/3 + x - (x + 1)**3/3 + 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin
(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) - 4*Piecewise((x*sqrt(1 -
x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x +
1)/2)/2, (x >= -1) & (x < 1))) + 1
```

3.447 $\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$

Optimal. Leaf size=22

$$-\sqrt{1-x^2}\,x-2x-\sin^{-1}(x)$$

```
[Out] -2*x-arcsin(x)-x*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {6688, 6742, 195, 216}

$$-\sqrt{1-x^2}\,x-2x-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -2*x - x*Sqrt[1 - x^2] - ArcSin[x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx\\
&= -\int \left(2+2\sqrt{1-x^2}\right)dx\\
&= -2x-2\int \sqrt{1-x^2}\,dx\\
&= -2x-x\sqrt{1-x^2}-\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -2x-x\sqrt{1-x^2}-\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$-x\left(\sqrt{1-x^2}+2\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -(x*(2 + Sqrt[1 - x^2])) - ArcSin[x]
```

fricas [B] time = 0.45, size = 41, normalized size = 1.86

$$-\sqrt{x+1}\,x\sqrt{-x+1}-2\,x+2\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="fricas")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1) - 2*x + 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

giac [B] time = 0.37, size = 49, normalized size = 2.23

$$-\sqrt{x+1}\,(x-2)\sqrt{-x+1}-2\,x-2\,\sqrt{x+1}\,\sqrt{-x+1}-2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="giac")
```

```
[Out] -sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2*sqrt(x + 1)*sqrt(-x + 1) - 2*ar
csin(1/2*sqrt(2)*sqrt(x + 1)) - 2
```

maple [B] time = 0.00, size = 59, normalized size = 2.68

$$-2x-\frac{\sqrt{(x+1)\,(-x+1)}\,\arcsin(x)}{\sqrt{-x+1}\,\sqrt{x+1}}+\sqrt{x+1}\,(-x+1)^{\frac{3}{2}}-\sqrt{-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -2*x+(x+1)^(1/2)*(-x+1)^(3/2)-(-x+1)^(1/2)*(x+1)^(1/2)-((x+1)*(-x+1))^(1/2)
/(-x+1)^(1/2)/(x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 1.39, size = 20, normalized size = 0.91

$$-\sqrt{-x^2+1}\,x-2\,x-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="maxima")
```

```
[Out] -sqrt(-x^2 + 1)*x - 2*x - arcsin(x)
```

mupad [B] time = 3.71, size = 205, normalized size = 9.32

$$4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-2\,x+\frac{\frac{4\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{28\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{4\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] 4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*x + ((4*((1 - x)^(1/2)
- 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^
3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2) -
1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)
^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) -
1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 +
 1)
```

sympy [A] time = 37.87, size = 46, normalized size = 2.09

$$-2x-4\left(\begin{cases}\frac{x\sqrt{1-x}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for } x\geq -1\wedge x<1\end{cases}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -2*x - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/
2)/2, (x >= -1) & (x < 1))) - 2
```

3.448 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$

Optimal. Leaf size=32

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

```
[Out] 2*arctanh((-x^2+1)^(1/2))-2*ln(x)-2*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6688, 6742, 266, 50, 63, 206}

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

```
[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx\\
&= -\int \left(\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}\right)dx\\
&= -2\log(x)-2\int \frac{\sqrt{1-x^2}}{x}\,dx\\
&= -2\log(x)-\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx,x,x^2\right)\\
&= -2\sqrt{1-x^2}-2\log(x)-\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -2\sqrt{1-x^2}-2\log(x)+2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 32, normalized size = 1.00

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

fricas [A] time = 0.45, size = 41, normalized size = 1.28

$$-2\,\sqrt{x+1}\,\sqrt{-x+1}\,-2\,\log(x)-2\,\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}\,-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="fricas")
[Out] -2*sqrt(x + 1)*sqrt(-x + 1) - 2*log(x) - 2*log((sqrt(x + 1)*sqrt(-x + 1) -
1)/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086]-2*ln(abs(sqrt(x+1)-1))-2*ln(sqrt(x+1)+1)-2*sqrt(x+1)*s
qrt(-x+1)+2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+
```

```
1)+2*sqrt(2))/sqrt(x+1)))-2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-
1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))
```

maple [A] time = 0.01, size = 51, normalized size = 1.59

$$-2\ln(x)-\frac{2\sqrt{x+1}\,\sqrt{-x+1}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)+\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2))/x,x)
```

```
[Out] -2*ln(x)-2*(x+1)^(1/2)*(-x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(
1/(-x^2+1)^(1/2)))
```

maxima [A] time = 1.95, size = 41, normalized size = 1.28

$$-2\sqrt{-x^2+1}-2\log(x)+2\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1) - 2*log(x) + 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 4.13, size = 122, normalized size = 3.81

$$2\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-2\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-2\ln(x)-\frac{16\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\left(\frac{2\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

```
[Out] 2*log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*log(((1 - x)^(1/2) - 1)^
2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(x) - (16*((1 - x)^(1/2) - 1)^2)/(((x +
 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1 - x
)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2}{x}\,dx-\int\frac{2\sqrt{1-x}\,\sqrt{x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x,x)
```

```
[Out] -Integral(2/x, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x, x)
```

3.449 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}+2\sin^{-1}(x)$$

```
[Out] 2/x+2*arcsin(x)+2*(-x^2+1)^(1/2)/x
```

Rubi [A] time = 0.21, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6688, 6742, 277, 216}

$$\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}+2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
```

```
[Out] 2/x + (2*Sqrt[1 - x^2])/x + 2*ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}dx = -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}dx$$
$$= -\int \left(\frac{2}{x^2}+\frac{2\sqrt{1-x^2}}{x^2}\right)dx$$
$$= \frac{2}{x}-2\int \frac{\sqrt{1-x^2}}{x^2}dx$$
$$= \frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\int \frac{1}{\sqrt{1-x^2}}dx$$
$$= \frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\sin^{-1}(x)$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.85

$$\frac{2\left(\sqrt{1-x^2}+x\sin^{-1}(x)+1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
```

```
[Out] (2*(1 + Sqrt[1 - x^2] + x*ArcSin[x]))/x
```

fricas [A] time = 0.49, size = 44, normalized size = 1.69

$$-\frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\,\sqrt{-x+1}-1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="fricas")
```

```
[Out] -2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1)
 - 1)/x
```

giac [B] time = 0.46, size = 149, normalized size = 5.73

$$2\,\pi+\frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4}+\frac{2}{x}+4\,\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="giac")
```

```
[Out] 2*pi + 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) -
 sqrt(-x + 1)))^2 - 4) + 2/x + 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-x
 + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

maple [B] time = 0.01, size = 50, normalized size = 1.92

$$\frac{2}{x}-\frac{2\left(-x\arcsin(x)-\sqrt{-x^2+1}\right)\sqrt{x+1}\,\sqrt{-x+1}}{\sqrt{-x^2+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2))/x^2,x)
```

```
[Out] 2/x-2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(x+1)^(1/2)*(-x+1)^(1/2)/x/(-x^2+1)^(1/
2)
```

maxima [A] time = 1.90, size = 24, normalized size = 0.92

$$\frac{2\sqrt{-x^2+1}}{x}+\frac{2}{x}+2\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 1)/x + 2/x + 2*arcsin(x)
```

mupad [B] time = 3.79, size = 118, normalized size = 4.54

$$\frac{\frac{5\left(\sqrt{1-x}-1\right)^2}{2\left(\sqrt{x+1}-1\right)^2}-\frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}}-8\,\mathrm{atan}\left(\frac{\sqrt{1-x}\ -1}{\sqrt{x+1}\ -1}\right)+\frac{\sqrt{1-x}\ -1}{2\left(\sqrt{x+1}\ -1\right)}+\frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

```
[Out] ((5*((1 - x)^(1/2) - 1)^2)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/((((1 - x)^(1/2)
 - 1)/((x + 1)^(1/2) - 1) - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3) -
8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) + ((1 - x)^(1/2) - 1)/(2*((
x + 1)^(1/2) - 1)) + 2/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2}{x^2}\,dx-\int\frac{2\sqrt{1-x}\,\sqrt{x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**2,x)
```

```
[Out] -Integral(2/x**2, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**2, x)
```

3.450 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] 1/x^2-arctanh((-x^2+1)^(1/2))+(-x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.22, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6688, 6742, 266, 47, 63, 206}

$$\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

```
[Out] x^(-2) + Sqrt[1 - x^2]/x^2 - ArcTanh[Sqrt[1 - x^2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx\\
&= -\int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)dx\\
&= \frac{1}{x^2}-2\int \frac{\sqrt{1-x^2}}{x^3}\,dx\\
&= \frac{1}{x^2}-\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.39

$$\frac{1}{x^2\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
[Out] x^(-2) - 1/Sqrt[1 - x^2] + 1/(x^2*Sqrt[1 - x^2]) - ArcTanh[Sqrt[1 - x^2]]
```

fricas [A] time = 0.45, size = 43, normalized size = 1.30

$$\frac{x^2\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\,\sqrt{-x+1}+1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="fricas")
[Out] (x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)/
x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086](4*(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-1/2*(-2*sqrt(
-x+1)+2*sqrt(2))/sqrt(x+1))^3+16*(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-1/2
```

```
*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))/((2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(
2))-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1))^2-4)^2-ln(abs(2*sqrt(x+1)/(-2*
sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))+ln(abs(2*
sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+
1)))+1/x^2
```

maple [A] time = 0.02, size = 57, normalized size = 1.73

$$\frac{1}{x^2}-\frac{\sqrt{x+1}\,\sqrt{-x+1}\,\left(x^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2))/x^3,x)
```

```
[Out] 1/x^2-(x+1)^(1/2)*(-x+1)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/2
))/x^2/(-x^2+1)^(1/2)
```

maxima [A] time = 2.02, size = 51, normalized size = 1.55

$$\sqrt{-x^2+1}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2}+\frac{1}{x^2}-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="maxima")
```

```
[Out] sqrt(-x^2 + 1) + (-x^2 + 1)^(3/2)/x^2 + 1/x^2 - log(2*sqrt(-x^2 + 1)/abs(x)
 + 2/abs(x))
```

mupad [B] time = 4.78, size = 186, normalized size = 5.64

$$\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\left(\sqrt{1-x}-1\right)^2}{16\left(\sqrt{x+1}-1\right)^2}+\frac{\frac{\left(\sqrt{1-x}-1\right)^2}{8\left(\sqrt{x+1}-1\right)^2}+\frac{15\left(\sqrt{1-x}-1\right)^4}{16\left(\sqrt{x+1}-1\right)^4}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-\frac{2\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}}+\frac{1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

```
[Out] log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2) -
 1)/((x + 1)^(1/2) - 1)) - ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^2)
 + (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2) -
1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1)^(1
/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x)^(1/
2) - 1)^6/((x + 1)^(1/2) - 1)^6) + 1/x^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2}{x^3}\,dx-\int\frac{2\sqrt{1-x}\,\sqrt{x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**3,x)
```

```
[Out] -Integral(2/x**3, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**3, x)
```

3.451 $\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=28

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)$$

```
[Out] -arctanh((-x^2+1)^(1/2))+ln(x)+(-x^2+1)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.180, Rules used = {2103, 6688, 14, 266, 50, 63, 206}

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
```

```
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx &= \frac{1}{2}\int \frac{\sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx+\frac{1}{2}\int \frac{\sqrt{1+x}\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx\\
&= \frac{1}{2}\int \frac{1-x+\sqrt{1-x^2}}{x}\,dx+\frac{1}{2}\int \frac{1+x+\sqrt{1-x^2}}{x}\,dx\\
&= \frac{1}{2}\int \left(-1+\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)dx+\frac{1}{2}\int \left(1+\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)dx\\
&= \log(x)+2\left(\frac{1}{2}\int \frac{\sqrt{1-x^2}}{x}\,dx\right)\\
&= \log(x)+2\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx,x,x^2\right)\right)\\
&= \log(x)+2\left(\frac{\sqrt{1-x^2}}{2}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\right)\\
&= \log(x)+2\left(\frac{\sqrt{1-x^2}}{2}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\right)\\
&= 2\left(\frac{\sqrt{1-x^2}}{2}-\frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)\right)+\log(x)
\end{aligned}$$

Mathematica [A] time = 0.16, size = 48, normalized size = 1.71

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)+2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+\sin^{-1}(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 - x^2] + 2*ArcSin[Sqrt[1 - x]/Sqrt[2]] + ArcSin[x] - ArcTanh[Sqrt[1
- x^2]] + Log[x]
```

fricas [A] time = 0.44, size = 36, normalized size = 1.29

$$\sqrt{x+1}\,\sqrt{-x+1}+\log(x)+\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1) + log(x) + log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086]ln(abs(sqrt(x+1)-1))+ln(sqrt(x+1)+1)+sqrt(x+1)*sqrt(-x+
1)-ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt
(2))/sqrt(x+1)))+ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqr
t(-x+1)+2*sqrt(2))/sqrt(x+1)))
```

maple [A] time = 0.01, size = 48, normalized size = 1.71

$$\ln(x)+\frac{\sqrt{x+1}\,\sqrt{-x+1}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)+\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)^(1/2)+(x+1)^(1/2))/(-(-x+1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] ln(x)+(x+1)^(1/2)*(-x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/(-x
^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}+\sqrt{-x+1}}{\sqrt{x+1}-\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="maxima")
```

```
[Out] integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-x + 1)), x)
```

mupad [B] time = 4.49, size = 93, normalized size = 3.32

$$\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+\ln(x)-\frac{8\,\left(x-2\,\sqrt{x+1}+2\right)\,\left(x+2\,\sqrt{1-x}-2\right)}{\left(2\,\sqrt{x+1}+2\,\sqrt{1-x}-4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))/((x + 1)^(1/2) - (1 - x)^(1/2)),x)
```

```
[Out] log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2) -
 1)/((x + 1)^(1/2) - 1)) + log(x) - (8*(x - 2*(x + 1)^(1/2) + 2)*(x + 2*(1
- x)^(1/2) - 2))/(2*(x + 1)^(1/2) + 2*(1 - x)^(1/2) - 4)^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{1-x}}{\sqrt{1-x}-\sqrt{x+1}}\,dx-\int \frac{\sqrt{x+1}}{\sqrt{1-x}-\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))/(-(1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -Integral(sqrt(1 - x)/(sqrt(1 - x) - sqrt(x + 1)), x) - Integral(sqrt(x + 1
)/(sqrt(1 - x) - sqrt(x + 1)), x)
```

3.452 $\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\,\sqrt{x+1}\,x+\frac{1}{2}\cosh^{-1}(x)$$

[Out] 1/2*x^2+1/2*arccosh(x)-1/2*x*(-1+x)^(1/2)*(1+x)^(1/2)

Rubi [A] time = 0.14, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2104, 6742, 38, 52}

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\,\sqrt{x+1}\,x+\frac{1}{2}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]

[Out] x^2/2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2104

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\left(\frac{1}{2}\int \sqrt{-1+x}\left(-\sqrt{-1+x}+\sqrt{1+x}\right)dx\right)+\frac{1}{2}\int \sqrt{1+x}\left(-\sqrt{-1+x}+\sqrt{1+\right.$$
$$= \frac{1}{2}\int \left(1+x-\sqrt{-1+x}\sqrt{1+x}\right)dx - \frac{1}{2}\int \left(1-x+\sqrt{-1+x}\sqrt{1+x}\right)dx$$
$$= \frac{x^2}{2}-2\left(\frac{1}{2}\int \sqrt{-1+x}\sqrt{1+x}\,dx\right)$$
$$= \frac{x^2}{2}-2\left(\frac{1}{4}\sqrt{-1+x}\,x\sqrt{1+x}-\frac{1}{4}\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx\right)$$
$$= \frac{x^2}{2}-2\left(\frac{1}{4}\sqrt{-1+x}\,x\sqrt{1+x}-\frac{1}{4}\cosh^{-1}(x)\right)$$

Mathematica [A] time = 0.16, size = 58, normalized size = 1.76

$$\frac{1}{2}\left(x^2-\sqrt{x-1}\sqrt{x+1}\,x+\frac{2(x-1)\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{\sqrt{-(x-1)^2}}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

```
[Out] (1 + x^2 - Sqrt[-1 + x]*x*Sqrt[1 + x] + (2*(-1 + x)*ArcSin[Sqrt[1 - x]/Sqrt
[2]])/Sqrt[-(-1 + x)^2])/2
```

fricas [A] time = 0.43, size = 37, normalized size = 1.12

$$-\frac{1}{2}\sqrt{x+1}\sqrt{x-1}\,x+\frac{1}{2}x^2-\frac{1}{2}\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="fricas")
```

```
[Out] -1/2*sqrt(x + 1)*sqrt(x - 1)*x + 1/2*x^2 - 1/2*log(sqrt(x + 1)*sqrt(x - 1)
- x)
```

giac [A] time = 0.38, size = 41, normalized size = 1.24

$$\frac{1}{2}(x+1)^2-\frac{1}{2}\sqrt{x+1}\sqrt{x-1}\,x-x-\log\left(\sqrt{x+1}-\sqrt{x-1}\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="giac")
```

```
[Out] 1/2*(x + 1)^2 - 1/2*sqrt(x + 1)*sqrt(x - 1)*x - x - log(sqrt(x + 1) - sqrt(
x - 1)) - 1
```

maple [B] time = 0.01, size = 62, normalized size = 1.88

$$\frac{x^2}{2}+\frac{\sqrt{(x-1)(x+1)}\,\ln\left(x+\sqrt{x^2-1}\right)}{2\sqrt{x+1}\,\sqrt{x-1}}-\frac{\sqrt{x-1}\,(x+1)^{\frac{3}{2}}}{2}+\frac{\sqrt{x-1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x-1)^(1/2)+(x+1)^(1/2))/((x-1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] -1/2*(x-1)^(1/2)*(x+1)^(3/2)+1/2*(x-1)^(1/2)*(x+1)^(1/2)+1/2*((x-1)*(x+1))^
(1/2)/(x+1)^(1/2)/(x-1)^(1/2)*ln(x+(x^2-1)^(1/2))+1/2*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="maxima")
```

```
[Out] integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1)), x)
```

mupad [B] time = 10.85, size = 200, normalized size = 6.06

$$\operatorname{acosh}(x)-2\operatorname{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)+\frac{\frac{14\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{14\left(\sqrt{x-1}-\mathrm{i}\right)^5}{\left(\sqrt{x+1}-1\right)^5}+\frac{2\left(\sqrt{x-1}-\mathrm{i}\right)^7}{\left(\sqrt{x+1}-1\right)^7}+\frac{2\left(\sqrt{x-1}-\mathrm{i}\right)}{\sqrt{x+1}-1}}{1+\frac{6\left(\sqrt{x-1}-\mathrm{i}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^8}{\left(\sqrt{x+1}-1\right)^8}-\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x - 1)^(1/2) - (x + 1)^(1/2))/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

```
[Out] acosh(x) - 2*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)) + ((14*((x - 1
)^(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (14*((x - 1)^(1/2) - 1i)^5)/((x +
1)^(1/2) - 1)^5 + (2*((x - 1)^(1/2) - 1i)^7)/((x + 1)^(1/2) - 1)^7 + (2*((x
 - 1)^(1/2) - 1i))/((x + 1)^(1/2) - 1))/((6*((x - 1)^(1/2) - 1i)^4)/((x + 1
)^(1/2) - 1)^4 - (4*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - (4*((x
- 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + ((x - 1)^(1/2) - 1i)^8/((x + 1)
^(1/2) - 1)^8 + 1) + x^2/2
```

sympy [A] time = 31.39, size = 226, normalized size = 6.85

$$-\frac{(x-1)^{\frac{5}{2}}}{4\sqrt{x+1}}-\frac{3(x-1)^{\frac{3}{2}}}{4\sqrt{x+1}}-\frac{\sqrt{x-1}}{2\sqrt{x+1}}+\frac{(x-1)^2}{4}+2\left(\begin{cases}\frac{(x+1)^2}{8}+\frac{\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{5}{2}}}{8\sqrt{x-1}}+\frac{3(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}-\frac{\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1\\ \frac{(x+1)^2}{8}-\frac{i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{5}{2}}}{8\sqrt{1-x}}-\frac{3i(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}+\frac{i\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)**(1/2)+(1+x)**(1/2))/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(x - 1)**(5/2)/(4*sqrt(x + 1)) - 3*(x - 1)**(3/2)/(4*sqrt(x + 1)) - sqrt(x
 - 1)/(2*sqrt(x + 1)) + (x - 1)**2/4 + 2*Piecewise(((x + 1)**2/8 + acosh(sq
rt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(5/2)/(8*sqrt(x - 1)) + 3*(x + 1)**(3/2)/
(8*sqrt(x - 1)) - sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), ((x + 1)*
*2/8 - I*asin(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(5/2)/(8*sqrt(1 - x)) -
 3*I*(x + 1)**(3/2)/(8*sqrt(1 - x)) + I*sqrt(x + 1)/(4*sqrt(1 - x)), True))
 + asinh(sqrt(2)*sqrt(x - 1)/2)/2
```

3.453 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=121

$$\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}{d}\right)}{2d^2e(n+1)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[Out] 1/2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)+1/2*a*f^2*hypergeom([2, 1
+n],[2+n],(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))/d)*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)
)^(1+n)/d^2/e/(1+n)
```

Rubi [A] time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2117, 947, 64}

$$\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}{d}\right)}{2d^2e(n+1)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (a*f^2*(d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n,
(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d])/(2*d^2*e*(1 + n))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\text{Subst}\left(\int \frac{x^n(d^2+af^2-2dx+x^2)}{(d-x)^2} dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$
$$= \frac{\text{Subst}\left(\int \left(x^n+\frac{af^2x^n}{(d-x)^2}\right) dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$
$$= \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{(af^2)\,\text{Subst}\left(\int \frac{x^n}{(d-x)^2} dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right.}{2e}$$
$$= \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n} {}_2F_1\left(2,1+n;2+\right.}{2d^2e(1+n)}$$

Mathematica [A] time = 0.12, size = 86, normalized size = 0.71

$$\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}\left(af^2\,{}_2F_1\left(2,n+1;n+2;\frac{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}{d}\right)+d^2\right)}{2d^2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
[Out] ((d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*(d^2 + a*f^2*Hypergeometric2
F1[2, 1 + n, 2 + n, (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d]))/(2*d^2*e*(1
+ n))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a}\,f\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^n,x)
```

```
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**n, x)
```

3.454 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$

Optimal. Leaf size=175

$$-\frac{ad^3f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{3ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{4e}$$

```
[Out] 3/2*a*d^2*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*d^3*f^2/e/(e*x+f*(a+e
^2*x^2/f^2)^(1/2))+a*d*f^2*(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e+1/4*a*f^2*(d+e*x
+f*(a+e^2*x^2/f^2)^(1/2))^2/e+1/8*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^4/e
```

Rubi [A] time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2117, 893}

$$-\frac{ad^3f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{3ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] -(a*d^3*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*d*f^2*(e*x + f*Sq
rt[a + (e^2*x^2)/f^2]))/e + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2)
/(4*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4/(8*e) + (3*a*d^2*f^2*Log[e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx = \frac{\text{Subst}\left(\int \frac{x^3(d^2+af^2-2dx+x^2)}{(d-x)^2}dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(2adf^2+\frac{ad^3f^2}{(d-x)^2}-\frac{3ad^2f^2}{d-x}+af^2x+x^3\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right.}{2e}$$

$$= -\frac{ad^3f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{adf^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e} + \frac{af^2\left(d+ex+f\sqrt{a+\right.}{4e}$$

Mathematica [A] time = 0.32, size = 158, normalized size = 0.90

$$\frac{-\frac{4ad^3f^2}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}+12ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)+\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4+2af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2+}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] ((-4*a*d^3*f^2)/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + 8*a*d*f^2*(e*x + f*Sqrt
[a + (e^2*x^2)/f^2]) + 2*a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2 + (d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4 + 12*a*d^2*f^2*Log[e*x + f*Sqrt[a + (
e^2*x^2)/f^2]])/(8*e)
```

fricas [A] time = 0.42, size = 161, normalized size = 0.92

$$\frac{2e^4x^4+4de^3x^3-3ad^2f^2\log\left(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+3\left(ae^2f^2+d^2e^2\right)x^2+2\left(3adef^2+d^3e\right)x+\left(2e^3fx^3+4\right.}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^4*x^4 + 4*d*e^3*x^3 - 3*a*d^2*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f
^2)/f^2)) + 3*(a*e^2*f^2 + d^2*e^2)*x^2 + 2*(3*a*d*e*f^2 + d^3*e)*x + (2*e^
3*f*x^3 + 4*d*e^2*f*x^2 + 4*a*d*f^3 + (2*a*e*f^3 + 3*d^2*e*f)*x)*sqrt((e^2*
x^2 + a*f^2)/f^2))/e
```

giac [A] time = 0.45, size = 163, normalized size = 0.93

$$-\frac{3}{2}ad^2f|f|e^{(-1)}\log\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)+\frac{3}{2}af^2x^2e+3adf^2x+x^4e^3+2dx^3e^2+\frac{3}{2}d^2x^2e+d^3x+\frac{1}{2}\left(4adf|f|e^{(-1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/2*a*d^2*f*abs(f)*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) + 3/2*a*f
^2*x^2*e + 3*a*d*f^2*x + x^4*e^3 + 2*d*x^3*e^2 + 3/2*d^2*x^2*e + d^3*x + 1/
2*(4*a*d*f*abs(f)*e^(-1) + (2*(x*abs(f)*e^2/f + 2*d*abs(f)*e/f)*x + (2*a*f^
4*abs(f)*e^4 + 3*d^2*f^2*abs(f)*e^4)*e^(-4)/f^3)*x)*sqrt(a*f^2 + x^2*e^2)
```

maple [A] time = 0.01, size = 175, normalized size = 1.00

$$e^3x^4+\frac{3aef^2x^2}{2}+2d\,e^2x^3+\frac{3a\,d^2f\ln\left(\frac{e^2x}{\sqrt{\frac{e^2}{f^2}}\,f^2}+\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2\sqrt{\frac{e^2}{f^2}}}+3ad\,f^2x+\frac{3d^2e\,x^2}{2}+d^3x+\frac{3\sqrt{\frac{e^2x^2}{f^2}+a}\,d^2fx}{2}+\left(\frac{e^2x^2}{f^2}+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^3,x)
```

```
[Out] f^3*x*(e^2/f^2*x^2+a)^(3/2)+e^3*x^4+2*d*e^2*x^3+3/2*f^2*a*e*x^2+3*f^2*a*d*x
+2*d/e*f^3*((e^2*x^2+a*f^2)/f^2)^(3/2)+3/2*f*d^2*x*(e^2/f^2*x^2+a)^(1/2)+3/
2*f*d^2*a*ln(e^2*x/f^2/(e^2/f^2)^(1/2)+(e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/
2)+3/2*e*x^2*d^2+d^3*x+1/4/e*d^4
```

maxima [A] time = 0.94, size = 266, normalized size = 1.52

$$\frac{1}{4}e^3x^4+\frac{3\left(\frac{e^2x^2}{f^2}+a\right)^2f^4}{4e}-\frac{3}{8}\left(\frac{a^2f^3\operatorname{arsinh}\left(\frac{ex}{\sqrt{a}f}\right)}{e^3}-\frac{2\left(\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}f^2x}{e^2}+\frac{\sqrt{\frac{e^2x^2}{f^2}+a}\,af^2x}{e^2}\right)e^2f+\frac{1}{8}\left(\frac{3a^2f\operatorname{arsinh}\left(\frac{ex}{\sqrt{a}\right.}\right.}{e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/4*e^3*x^4 + 3/4*(e^2*x^2/f^2 + a)^2*f^4/e - 3/8*(a^2*f^3*arcsinh(e*x/(sqr
t(a)*f))/e^3 - 2*(e^2*x^2/f^2 + a)^(3/2)*f^2*x/e^2 + sqrt(e^2*x^2/f^2 + a)*
a*f^2*x/e^2)*e^2*f + 1/8*(3*a^2*f*arcsinh(e*x/(sqrt(a)*f))/e + 2*(e^2*x^2/f
^2 + a)^(3/2)*x + 3*sqrt(e^2*x^2/f^2 + a)*a*x)*f^3 + d^3*x + 3/2*(e*x^2 + (
a*f*arcsinh(e*x/(sqrt(a)*f))/e + sqrt(e^2*x^2/f^2 + a)*x)*f)*d^2 + (e^2*x^3
 + 2*(e^2*x^2/f^2 + a)^(3/2)*f^3/e + (e^2*x^3/f^2 + 3*a*x)*f^2)*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)
```

sympy [A] time = 10.46, size = 279, normalized size = 1.59

$$\frac{a^{\frac{3}{2}}f^3x\sqrt{1+\frac{e^2x^2}{af^2}}}{2}+\frac{a^{\frac{3}{2}}f^3x}{2\sqrt{1+\frac{e^2x^2}{af^2}}}+\frac{3\sqrt{a}\,d^2fx\sqrt{1+\frac{e^2x^2}{af^2}}}{2}+\frac{3\sqrt{a}\,e^2fx^3}{2\sqrt{1+\frac{e^2x^2}{af^2}}}+\frac{3ad^2f^2\operatorname{asinh}\left(\frac{ex}{\sqrt{a}f}\right)}{2e}+3adf^2x+\frac{3aef^2x^2}{2}+d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] a**(3/2)*f**3*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a**(3/2)*f**3*x/(2*sqrt(1
+ e**2*x**2/(a*f**2))) + 3*sqrt(a)*d**2*f*x*sqrt(1 + e**2*x**2/(a*f**2))/2
```

```
+ 3*sqrt(a)*e**2*f*x**3/(2*sqrt(1 + e**2*x**2/(a*f**2))) + 3*a*d**2*f**2*as
inh(e*x/(sqrt(a)*f))/(2*e) + 3*a*d*f**2*x + 3*a*e*f**2*x**2/2 + d**3*x + 3*
d**2*e*x**2/2 + 2*d*e**2*x**3 + 6*d*e*f*Piecewise((sqrt(a)*x**2/2, Eq(e**2,
 0)), (f**2*(a + e**2*x**2/f**2)**(3/2)/(3*e**2), True)) + e**3*x**4 + e**4
*x**5/(sqrt(a)*f*sqrt(1 + e**2*x**2/(a*f**2)))
```

3.455 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx$

Optimal. Leaf size=136

$$-\frac{ad^2f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}+\frac{adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

```
[Out] a*d*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*d^2*f^2/e/(e*x+f*(a+e^2*x^2
/f^2)^(1/2))+1/2*a*f^2*(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e+1/6*(d+e*x+f*(a+e^2*
x^2/f^2)^(1/2))^3/e
```

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2117, 893}

$$-\frac{ad^2f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}+\frac{adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] -(a*d^2*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*(e*x + f*Sqrt
[a + (e^2*x^2)/f^2]))/(2*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/(6*e)
 + (a*d*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^2(d^2+af^2-2dx+x^2)}{(d-x)^2}dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(af^2+\frac{ad^2f^2}{(d-x)^2}-\frac{2adf^2}{d-x}+x^2\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{ad^2f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right.}{6e}$$

Mathematica [A] time = 0.20, size = 128, normalized size = 0.94

$$\frac{\frac{ad^2f^2}{f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex}+\frac{1}{3}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3+2adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)+af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] ((a*d^2*f^2)/(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(e*x + f*Sqrt[a +
 (e^2*x^2)/f^2]) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/3 + 2*a*d*f^2*Lo
g[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

fricas [A] time = 0.42, size = 114, normalized size = 0.84

$$\frac{2\,e^3x^3+3\,de^2x^2-3\,adf^2\log\left(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+3\left(aef^2+d^2e\right)x+\left(2\,e^2fx^2+2\,af^3+3\,defx\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*e^3*x^3 + 3*d*e^2*x^2 - 3*a*d*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2
)/f^2)) + 3*(a*e*f^2 + d^2*e)*x + (2*e^2*f*x^2 + 2*a*f^3 + 3*d*e*f*x)*sqrt(
(e^2*x^2 + a*f^2)/f^2))/e
```

giac [A] time = 0.52, size = 103, normalized size = 0.76

$$-adf|f|e^{(-1)}\log\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)+af^2x+\frac{2}{3}x^3e^2+dx^2e+d^2x+\frac{1}{3}\left(2\,af|f|e^{(-1)}+\left(\frac{2\,x|f|e}{f}+\frac{3\,d|f|}{f}\right)x\right)\sqrt{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -a*d*f*abs(f)*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) + a*f^2*x + 2/3
*x^3*e^2 + d*x^2*e + d^2*x + 1/3*(2*a*f*abs(f)*e^(-1) + (2*x*abs(f)*e/f + 3
*d*abs(f)/f)*x)*sqrt(a*f^2 + x^2*e^2)
```

maple [A] time = 0.01, size = 126, normalized size = 0.93

$$\frac{2e^2x^3}{3}+\frac{adf\ln\left(\frac{e^2x}{\sqrt{\frac{e^2}{f^2}}\,f^2}+\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{\sqrt{\frac{e^2}{f^2}}}+a f^2x+dex^2+d^2x+\sqrt{\frac{e^2x^2}{f^2}+a}\;dfx+\frac{d^3}{3e}+\frac{2\left(\frac{e^2x^2+af^2}{f^2}\right)^{\frac{3}{2}}f^3}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^2,x)
```

```
[Out] f^2*a*x+2/3*e^2*x^3+2/3/e*f^3*((e^2*x^2+a*f^2)/f^2)^(3/2)+f*d*x*(e^2/f^2*x^
2+a)^(1/2)+f*d*a*ln(1/(e^2/f^2)^(1/2)*e^2/f^2*x+(e^2/f^2*x^2+a)^(1/2))/(e^2
/f^2)^(1/2)+e*x^2*d+x*d^2+1/3/e*d^3
```

maxima [A] time = 0.89, size = 99, normalized size = 0.73

$$\frac{1}{3}e^2x^3+\frac{2\left(\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}f^3}{3e}+\frac{1}{3}\left(\frac{e^2x^3}{f^2}+3ax\right)f^2+d^2x+\left(ex^2+\left(\frac{af\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{a}f}\right)}{e}+\sqrt{\frac{e^2x^2}{f^2}+a}\,x\right)f\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/3*e^2*x^3 + 2/3*(e^2*x^2/f^2 + a)^(3/2)*f^3/e + 1/3*(e^2*x^3/f^2 + 3*a*x)
*f^2 + d^2*x + (e*x^2 + (a*f*arcsinh(e*x/(sqrt(a)*f))/e + sqrt(e^2*x^2/f^2
+ a)*x)*f)*d
```

mupad [B] time = 4.66, size = 210, normalized size = 1.54

$$\begin{cases} x\left(d+\sqrt{a}\,f\right)^2 \\ x\left(d^2+af^2\right)+\frac{2e^2x^3}{3}+dex^2+\frac{2af^3\sqrt{a+\frac{e^2x^2}{f^2}}}{e}-\frac{2f\sqrt{a+\frac{e^2x^2}{f^2}}\left(2af^2-e^2x^2\right)}{3e}+dfx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{2adf\ln\left(x\sqrt{\frac{e^2}{f^2}}+\sqrt{a+\ldots}\right.}{\sqrt{\frac{e^2}{f^2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)
```

```
[Out] piecewise(e == 0, x*(d + a^(1/2)*f)^2, e ~= 0, x*(a*f^2 + d^2) + (2*e^2*x^3
)/3 + d*e*x^2 + (2*a*f^3*(a + (e^2*x^2)/f^2)^(1/2))/e - (2*f*(a + (e^2*x^2)
/f^2)^(1/2)*(2*a*f^2 - e^2*x^2))/(3*e) + d*f*x*(a + (e^2*x^2)/f^2)^(1/2) +
(2*a*d*f*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2)^(1/2)))/(e^2/f^2)^(1/2
) - (a*d*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e^2*x^2)/f^2)^(1/2)))/(f*(e^
2/f^2)^(3/2)))
```

sympy [A] time = 4.48, size = 116, normalized size = 0.85

$$\sqrt{a}dfx\sqrt{1+\frac{e^2x^2}{af^2}}+\frac{adf^2\,\mathrm{asinh}\left(\frac{ex}{\sqrt{a}f}\right)}{e}+af^2x+d^2x+dex^2+\frac{2e^2x^3}{3}+2ef\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for } e^2=0\\ \frac{f^2\left(a+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] sqrt(a)*d*f*x*sqrt(1 + e**2*x**2/(a*f**2)) + a*d*f**2*asinh(e*x/(sqrt(a)*f)
)/e + a*f**2*x + d**2*x + d*e*x**2 + 2*e**2*x**3/3 + 2*e*f*Piecewise((sqrt(
a)*x**2/2, Eq(e**2, 0)), (f**2*(a + e**2*x**2/f**2)**(3/2)/(3*e**2), True))
```

3.456 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)dx$

Optimal. Leaf size=68

$$\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+dx+\frac{ex^2}{2}$$

```
[Out] d*x+1/2*e*x^2+1/2*a*f^2*arctanh(e*x/f/(a+e^2*x^2/f^2)^(1/2))/e+1/2*f*x*(a+e
^2*x^2/f^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {195, 217, 206}

$$\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTanh[(e*x)/(f
*Sqrt[a + (e^2*x^2)/f^2])])/(2*e)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)dx = dx+\frac{ex^2}{2}+f\int \sqrt{a+\frac{e^2x^2}{f^2}}\,dx$$
$$= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{1}{2}(af)\int \frac{1}{\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$
$$= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{1}{2}(af)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{e^2x^2}{f^2}}\,dx,x,\frac{x}{\sqrt{a+\frac{e^2x^2}{f^2}}}\right)$$
$$= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.19

$$\frac{1}{2}fx\sqrt{\frac{af^2+e^2x^2}{f^2}}+\frac{af^2\log\left(ef\sqrt{\frac{af^2+e^2x^2}{f^2}}+e^2x\right)}{2e}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[(a*f^2 + e^2*x^2)/f^2])/2 + (a*f^2*Log[e^2*x +
e*f*Sqrt[(a*f^2 + e^2*x^2)/f^2]])/(2*e)
```

fricas [A] time = 0.41, size = 74, normalized size = 1.09

$$\frac{e^2x^2-af^2\log\left(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+efx\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\,dex}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(e^2*x^2 - a*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)) + e*f*x*sqrt
((e^2*x^2 + a*f^2)/f^2) + 2*d*e*x)/e
```

giac [A] time = 0.35, size = 65, normalized size = 0.96

$$\frac{1}{2}\,x^2e+dx-\frac{\left(af^2e^{(-1)}\log\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)-\sqrt{af^2+x^2e^2}\,x\right)|f|}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*e + d*x - 1/2*(a*f^2*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2)))
- sqrt(a*f^2 + x^2*e^2)*x)*abs(f)/f
```

maple [A] time = 0.00, size = 75, normalized size = 1.10

$$\frac{af\ln\left(\frac{e^2x}{\sqrt{\frac{e^2}{f^2}}\,f^2}+\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2\sqrt{\frac{e^2}{f^2}}}+\frac{e\,x^2}{2}+dx+\frac{\sqrt{\frac{e^2x^2}{f^2}+a}\;fx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f,x)
```

```
[Out] d*x+1/2*e*x^2+1/2*f*x*(e^2/f^2*x^2+a)^(1/2)+1/2*f*a*ln(1/(e^2/f^2)^(1/2)*e^
2/f^2*x+(e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)
```

maxima [A] time = 0.87, size = 46, normalized size = 0.68

$$\frac{1}{2}\,ex^2+\frac{1}{2}\left(\frac{af\,\operatorname{arsinh}\left(\frac{ex}{\sqrt{a}\,f}\right)}{e}+\sqrt{\frac{e^2x^2}{f^2}+a}\,x\right)f+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^2 + 1/2*(a*f*arcsinh(e*x/(sqrt(a)*f))/e + sqrt(e^2*x^2/f^2 + a)*x)*
f + d*x
```

mupad [B] time = 3.89, size = 136, normalized size = 2.00

$$\begin{cases} x\left(d+\sqrt{a}\,f\right) & \text{if } e=0\\ d\,x+\frac{e\,x^2}{2}+\frac{f\,x\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}{2}+\frac{a\,e^2\,\ln\left(x\,\sqrt{\frac{e^2}{f^2}}+\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{f\,{\left(\frac{e^2}{f^2}\right)}^{3/2}}-\frac{a\,e^2\,\ln\left(2\,x\,\sqrt{\frac{e^2}{f^2}}+2\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{2\,f\,{\left(\frac{e^2}{f^2}\right)}^{3/2}} & \text{if } e\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, x*(d + a^(1/2)*f), e ~= 0, d*x + (e*x^2)/2 + (f*x*(a + (e
^2*x^2)/f^2)^(1/2))/2 + (a*e^2*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2)^
(1/2)))/(f*(e^2/f^2)^(3/2)) - (a*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e^2*
x^2)/f^2)^(1/2)))/(2*f*(e^2/f^2)^(3/2)))
```

sympy [A] time = 2.21, size = 54, normalized size = 0.79

$$dx+\frac{ex^2}{2}+f\left(\frac{\sqrt{a}\,x\sqrt{1+\frac{e^2x^2}{af^2}}}{2}+\frac{af\,\operatorname{asinh}\left(\frac{ex}{\sqrt{a}\,f}\right)}{2e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] d*x + e*x**2/2 + f*(sqrt(a)*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a*f*asinh(e*
x/(sqrt(a)*f))/(2*e))
```

3.457 $\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=117

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e}+\frac{\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e}-\frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

```
[Out] -1/2*a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^2/e+1/2*(1+a*f^2/d^2)*ln(d+e*x
+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*f^2/d/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2117, 893}

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e}+\frac{\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e}-\frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -(a*f^2)/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqr
t[a + (e^2*x^2)/f^2]])/(2*d^2*e) + ((1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqrt[
a + (e^2*x^2)/f^2]])/(2*e)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d(d-x)^2}+\frac{af^2}{d^2(d-x)}+\frac{d^2+af^2}{d^2x}\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^2e} + \frac{\left(1+\frac{af^2}{d^2}\right)\log\left(d+ex+\right.}{2e}$$

Mathematica [A] time = 0.14, size = 109, normalized size = 0.93

$$\frac{-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^2}+\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)+\frac{af^2}{d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] ((a*f^2)/(d*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt
[a + (e^2*x^2)/f^2]])/d^2 + (1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqrt[a + (e^2
*x^2)/f^2]])/(2*e)
```

fricas [A] time = 0.45, size = 187, normalized size = 1.60

$$\frac{2\,dex-2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+\left(af^2+d^2\right)\log\left(af^2-dex+df\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+\left(af^2+d^2\right)\log\left(-af^2+2\,dex+d^2\right)-\left(af\right.}{4\,d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*e*x - 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + (a*f^2 + d^2)*log(a*f^2
- d*e*x + d*f*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a*f^2 + d^2)*log(-a*f^2 + 2*d
*e*x + d^2) - (a*f^2 + d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d) +
 (a*f^2 - d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)))/(d^2*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\left[undef, +\infty, 1\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, +Infinity, 1]
```

maple [B] time = 0.04, size = 1325, normalized size = 11.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f),x)
```

```
[Out] -1/4*f/d/e*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1
/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)-1/4*f/d^2*ln(
(1/2*e*(a*f^2-d^2)/f^2/d+e^2/f^2*(x+1/2*(-a*f^2+d^2)/d/e))/(e^2/f^2)^(1/2)+
(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)
/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(e^2/f^2)^(1/2)*a+1/4/f
*ln((1/2*e*(a*f^2-d^2)/f^2/d+e^2/f^2*(x+1/2*(-a*f^2+d^2)/d/e))/(e^2/f^2)^(1
/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+
d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(e^2/f^2)^(1/2)+1/4
*f^3/d^3/e/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d
^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*
f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4
*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2
/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^2+1/2*f/d/e/((a^2*f^4+2*a*d^2*f^2+
d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)
/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/
2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f
^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/
d/e))*a+1/4/f*d/e/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^
4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2
*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^
2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d
^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))+1/2*ln(a*f^2-2*d*e*x-d^2)/e+1
/2/d*x+1/4/d^2/e*ln(-a*f^2+2*d*e*x+d^2)*a*f^2-1/4/e*ln(-a*f^2+2*d*e*x+d^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)),x)
```

```
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] Integral(1/(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

3.458 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

Optimal. Leaf size=151

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e}+\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3e}-\frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+\right.}$$

```
[Out] -a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^3/e+a*f^2*ln(d+e*x+f*(a+e^2*x^2/f^
2)^(1/2))/d^3/e-1/2*a*f^2/d^2/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))+1/2*(-1-a*f^2
/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2117, 893}

$$-\frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e}+\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+\right.}{d^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -(a*f^2)/(2*d^2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(2
*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt[a + (e^
2*x^2)/f^2]])/(d^3*e) + (a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(d
^3*e)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^2(d-x)^2}+\frac{2af^2}{d^3(d-x)}+\frac{d^2+af^2}{d^2x^2}+\frac{2af^2}{d^3x}\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{af^2\log\left(ex+f\sqrt{a\right.}}{d^3e}$$

Mathematica [A] time = 0.29, size = 141, normalized size = 0.93

$$-\frac{\frac{2af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3}-\frac{2af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3}+\frac{af^2}{d^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\frac{af^2}{d^2}+1}{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -1/2*((a*f^2)/(d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (1 + (a*f^2)/d^2)/(
d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (2*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x
^2)/f^2]])/d^3 - (2*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^3)/e
```

fricas [B] time = 0.50, size = 284, normalized size = 1.88

$$a^2f^4-2\,d^2e^2x^2+ad^2f^2-2\,d^3ex+\left(a^2f^4-2\,adef^2x-ad^2f^2\right)\log\left(-aef^2x+2\,de^2x^2+adf^2+\left(af^3-2\,defx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*f^4 - 2*d^2*e^2*x^2 + a*d^2*f^2 - 2*d^3*e*x + (a^2*f^4 - 2*a*d*e*f
^2*x - a*d^2*f^2)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e*f
*x)*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*lo
g(-a*f^2 + 2*d*e*x + d^2) - (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*log(-e*x
+ f*sqrt((e^2*x^2 + a*f^2)/f^2) - d) - 2*(a*d*f^3 - d^2*e*f*x)*sqrt((e^2*x^
2 + a*f^2)/f^2))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 4136, normalized size = 27.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^2,x)
```

```
[Out] 1/2/d^2*x-1/4*f/e/d^2*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(
x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)+1/
4/f/d*ln(((x+1/2*(-a*f^2+d^2)/d/e)*e^2/f^2+1/2*(a*f^2-d^2)/d*e/f^2)/(e^2/f^
2)^(1/2)+((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2
)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2)^(1/2
)+1/4/f/e/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2*
(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*f
^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*
(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/
f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))+1/2/e/d^3*ln(-a*f^2+2*d*e*x+d^2)*a*f^
2-1/2*a*f^2/(-a*f^2+2*d*e*x+d^2)/d/e+1/4*d^2*f/e/(a^2*f^4+2*a*d^2*f^2+d^4)*
(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e
)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)-1/4*d^3/f/(a^2*f^4+2*a*d
^2*f^2+d^4)*ln(((x+1/2*(-a*f^2+d^2)/d/e)*e^2/f^2+1/2*(a*f^2-d^2)/d*e/f^2)/(
e^2/f^2)^(1/2)+((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f
^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2
)^(1/2)-d*f/(a^2*f^4+2*a*d^2*f^2+d^4)*((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(
a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d
^2/f^2)^(1/2)*x-1/4/e/d^3/(-a*f^2+2*d*e*x+d^2)*a^2*f^4-1/4*f/d^3*ln(((x+1/2
*(-a*f^2+d^2)/d/e)*e^2/f^2+1/2*(a*f^2-d^2)/d*e/f^2)/(e^2/f^2)^(1/2)+((x+1/2
*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1
/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*a-1/4*d/(-a*f^
2+2*d*e*x+d^2)/e+d*f^3/e^2/(a^2*f^4+2*a*d^2*f^2+d^4)/(x-1/2/e/d*a*f^2+1/2*d
/e)*((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e
)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(3/2)-3/4/d*f^3/(a^2*f^4+2
*a*d^2*f^2+d^4)*ln(((x+1/2*(-a*f^2+d^2)/d/e)*e^2/f^2+1/2*(a*f^2-d^2)/d*e/f^
2)/(e^2/f^2)^(1/2)+((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(
-a*f^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2
/f^2)^(1/2)*a^2-3/4*d*f/(a^2*f^4+2*a*d^2*f^2+d^4)*ln(((x+1/2*(-a*f^2+d^2)/d
/e)*e^2/f^2+1/2*(a*f^2-d^2)/d*e/f^2)/(e^2/f^2)^(1/2)+((x+1/2*(-a*f^2+d^2)/d
/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a
*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*a-1/4*d^4/f/e/(a^2*f^4+2*a*d^
2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2
*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*
f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4
*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2
/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))+1/4*f^3/e/d^4/((a^2*f^4+2*a*d^2*f^2+
d^4)/d^2/f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a
^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/
2)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/
d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/
d/e))*a^2+1/2*f/e/d^2/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln(((a*f^2-
d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2
+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^
2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f
^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a-1/4*f^5/e/d^2/(a^2*f^4+
2*a*d^2*f^2+d^4)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2
*(-a*f^2+d^2)/d/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*a^2-1/4
*f^5/d^3/(a^2*f^4+2*a*d^2*f^2+d^4)*ln(((x+1/2*(-a*f^2+d^2)/d/e)*e^2/f^2+1/2
*(a*f^2-d^2)/d*e/f^2)/(e^2/f^2)^(1/2)+((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(
a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d
^2/f^2)^(1/2))/(e^2/f^2)^(1/2)*a^3-f^3/d/(a^2*f^4+2*a*d^2*f^2+d^4)*((x+1/2*
(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/
4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*x*a+f^5/e^2/d/(a^2*f^4+2*a*d^2*f
^2+d^4)/(x-1/2/e/d*a*f^2+1/2*d/e)*((x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+(a*f^
2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f
```

```
^2)^(3/2)*a+1/4*f^7/e/d^4/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d
^4)/d^2/f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^
2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2
)*(4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d
/e)/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d
/e))*a^4+1/2/d^2*f^5/e/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)
/d^2/f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^2*f
^4+2*a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(
4*(x+1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)
/d*e/f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e)
)*a^3-1/2*d^2*f/e/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/
f^2)^(1/2)*ln(((a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/f^2+1/2*(a^2*f^4+2*
a*d^2*f^2+d^4)/d^2/f^2+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4*(x+
1/2*(-a*f^2+d^2)/d/e)^2*e^2/f^2+4*(a*f^2-d^2)*(x+1/2*(-a*f^2+d^2)/d/e)/d*e/
f^2+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)
```

```
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-2), x)
```

3.459 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

Optimal. Leaf size=193

$$-\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e}+\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2d^4e}-\frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

```
[Out] -3/2*a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^4/e+3/2*a*f^2*ln(d+e*x+f*(a+e^
2*x^2/f^2)^(1/2))/d^4/e-1/2*a*f^2/d^3/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))+1/4*(
-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2-a*f^2/d^3/e/(d+e*x+f*(a+e
^2*x^2/f^2)^(1/2))
```

Rubi [A] time = 0.13, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2117, 893}

$$-\frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{\frac{af^2}{d^2}+1}{4e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}-\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -(a*f^2)/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(4
*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (a*f^2)/(d^3*e*(d + e*x + f*S
qrt[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(
2*d^4*e) + (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^3}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^3(d-x)^2}+\frac{3af^2}{d^4(d-x)}+\frac{d^2+af^2}{d^2x^3}+\frac{2af^2}{d^3x^2}+\frac{3af^2}{d^4x}\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{4e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2} - \frac{af^2}{d^3e\left(d+ex+f\sqrt{\ldots}\right.}$$

Mathematica [A] time = 0.57, size = 180, normalized size = 0.93

$$\frac{-\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^4}+\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^4}+\frac{af^2}{d^3\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{2af^2}{d^3\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{\frac{af^2}{d^2}+1}{2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] ((a*f^2)/(d^3*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(2*
(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (2*a*f^2)/(d^3*(d + e*x + f*Sqrt
[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^4
+ (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^4)/(2*e)
```

fricas [B] time = 0.78, size = 536, normalized size = 2.78

$$\frac{5\,a^3f^6+8\,d^3e^3x^3-6\,a^2d^2f^4-3\,ad^4f^2+2\left(ad^2e^2f^2+5\,d^4e^2\right)x^2-2\left(7\,a^2def^4+ad^3ef^2-2\,d^5e\right)x+3\left(a^3f^6+\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a^3*f^6 + 8*d^3*e^3*x^3 - 6*a^2*d^2*f^4 - 3*a*d^4*f^2 + 2*(a*d^2*e^2
*f^2 + 5*d^4*e^2)*x^2 - 2*(7*a^2*d*e*f^4 + a*d^3*e*f^2 - 2*d^5*e)*x + 3*(a^
3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 -
a*d^3*e*f^2)*x)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e*f*x
)*sqrt((e^2*x^2 + a*f^2)/f^2)) + 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d
^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-a*f^2 + 2*d*e*x
+ d^2) - 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(
a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d)
 - 2*(3*a^2*d*f^5 + 4*d^3*e^2*f*x^2 - 5*a*d^3*f^3 - 3*(3*a*d^2*e*f^3 - d^4*
e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))/(a^2*d^4*e*f^4 + 4*d^6*e^3*x^2 - 2*a*d
^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.05, size = 9721, normalized size = 50.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3), x)
```

3.460 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

Optimal. Leaf size=225

$$-\frac{5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}-\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}+\frac{af^2\left(f\sqrt{a-\right.}}{}$$

```
[Out] -5/2*a*d^(3/2)*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/e
+1/3*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)/e+1/7*(d+e*x+f*(a+e^2*x^2/
f^2)^(1/2))^(7/2)/e+2*a*d*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/2*a
*d^2*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e/(e*x+f*(a+e^2*x^2/f^2)^(1/
2))
```

Rubi [A] time = 0.19, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2117, 897, 1257, 1810, 206}

$$-\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}+\frac{af^2\left(f\sqrt{a-\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (2*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d^2*f^2*Sqrt[d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])
) + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(3*e) + (d + e*x +
f*Sqrt[a + (e^2*x^2)/f^2])^(7/2)/(7*e) - (5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
```

```
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{\text{Subst}\left(\int \frac{x^{5/2}\left(d^2+af^2-2dx+x^2\right)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6\left(d^2+af^2-2dx^2+x^4\right)}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{-ad^2f^2-2adf^2x^2-2af^2x^4+2dx^6-2x^8}{d-x^2}\,dx, x\right.}{2e}$$

$$= -\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(4adf^2+2af^2x^2+2x^6-\frac{5ad^2f^2}{d-x^2}\right)\right.}{2e}$$

$$= \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(d+ex\right.}{}$$

$$= \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(d+ex\right.}{}$$

Mathematica [A] time = 0.34, size = 213, normalized size = 0.95

$$\frac{-5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)-\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}+\frac{2}{7}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}+\frac{2}{3}af^2\left(f\sqrt{a+\frac{e^2}{f}}\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (4*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] - (a*d^2*f^2*Sqrt[d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (2*a*
f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/3 + (2*(d + e*x + f*Sqrt[a
 + (e^2*x^2)/f^2])^(7/2))/7 - 5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt
[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

fricas [A] time = 0.57, size = 416, normalized size = 1.85

$$\left[\frac{105\,ad^{\frac{3}{2}}f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}\,ex-\sqrt{d}\,f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(24\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/84*(105*a*d^(3/2)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)
/f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x +
f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(24*e^3*x^3 + 36*d*e^2*x^2 + 116*a*
d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 + 36*d
*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 +
 a*f^2)/f^2) + d))/e, 1/42*(105*a*sqrt(-d)*d*f^2*arctan(sqrt(e*x + f*sqrt((
e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (24*e^3*x^3 + 36*d*e^2*x^2 + 116*a
*d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 + 36*
d*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2
+ a*f^2)/f^2) + d))/e]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a}\,f\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
```

```
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(5/2), x)
```

3.461 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

Optimal. Leaf size=183

$$\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}-\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{3a\sqrt{d}\,f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{\cdots}}}{\cdots}\right)}{2e}$$

```
[Out] -3/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))*d^(1/2)/e
+1/5*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2)/e+a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)
^(1/2))^(1/2)/e-1/2*a*d*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e/(e*x+f*
(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1257, 1810, 206}

$$\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}-\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{3a\sqrt{d}\,f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{\cdots}}}{\cdots}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d*f^2*Sqrt[d + e*x
 + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2)/(5*e) - (3*a*Sqrt[d]*f^2*ArcTanh[
Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
```

```
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}dx=\frac{\text{Subst}\left(\int \frac{x^{3/2}(d^2+af^2-2dx+x^2)}{(d-x)^2}dx,x,d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$=\frac{\text{Subst}\left(\int \frac{x^4(d^2+af^2-2dx^2+x^4)}{(d-x^2)^2}dx,x,\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$=-\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{\text{Subst}\left(\int \frac{adf^2+2af^2x^2-2dx^4+2x^6}{d-x^2}dx,x,\sqrt{d+ex}\right.}{2e}$$

$$=-\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{\text{Subst}\left(\int\left(-2af^2-2x^4+\frac{3adf^2}{d-x^2}\right)dx,x,\sqrt{d\right.}{2e}$$

$$=\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e}-\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\left(d+ex+f\sqrt{\right.}{5e}$$

$$=\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e}-\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\left(d+ex+f\sqrt{\right.}{5e}$$

Mathematica [A] time = 0.24, size = 175, normalized size = 0.96

$$\frac{\frac{2}{5}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}+2af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}-\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}-3a\sqrt{d}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (2*a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] - (a*d*f^2*Sqrt[d + e*x
+ f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (2*(d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2))/5 - 3*a*Sqrt[d]*f^2*ArcTanh[Sqrt[d +
 e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

fricas [A] time = 0.55, size = 337, normalized size = 1.84

$$\left[\frac{15\,a\sqrt{d}\,f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}\,ex-\sqrt{d}\,f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(4\,e^2\right.}{20\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/20*(15*a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/
f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f
*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(4*e^2*x^2 + 12*a*f^2 + 9*d*e*x + 2*
d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x
^2 + a*f^2)/f^2) + d))/e, 1/10*(15*a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt(
(e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (4*e^2*x^2 + 12*a*f^2 + 9*d*e*x +
 2*d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^
2*x^2 + a*f^2)/f^2) + d))/e]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int\left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a}\;f\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
```

```
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f \sqrt{a + \frac{e^{2} x^{2}}{f^{2}}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(3/2), x)
```

3.462 $\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=147

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2\sqrt{d}\,e}$$

```
[Out] -1/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/e/d^(1/2)
+1/3*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)/e-1/2*a*f^2*(d+e*x+f*(a+e^2*x^2/
f^2)^(1/2))^(1/2)/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1257, 1153, 206}

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2\sqrt{d}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] -(a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (
e^2*x^2)/f^2])) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)/(3*e) - (a*f^
2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*Sqrt[d]*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}\left(d^2+af^2-2dx+x^2\right)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(d^2+af^2-2dx^2+x^4\right)}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{-af^2+2dx^2-2x^4}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\right.}}}{2e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(2x^2-\frac{af^2}{d-x^2}\right)dx, x, \sqrt{d+ex+f\sqrt{a+\right.}}}{2e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{\left(af^2\right)\text{Subst}\left(\int \frac{1}{d-\right.}}{}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{af^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{}\right.}{2\sqrt{d}\,e}$$

Mathematica [A] time = 0.33, size = 139, normalized size = 0.95

$$-\frac{\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}-\frac{2}{3}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}+\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] -1/2*((a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (
e^2*x^2)/f^2]) - (2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/3 + (a*f^2
*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqrt[d])/e
```

fricas [A] time = 0.60, size = 301, normalized size = 2.05

$$\left[\frac{3\,a\sqrt{d}\,f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}\,ex-\sqrt{d}\,f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(5\,dex\right.}{12\,de}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f
^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*
sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(5*d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)
/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d*e), 1/6*(3
*a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-
d)/d) + (5*d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sq
rt((e^2*x^2 + a*f^2)/f^2) + d))/(d*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int\sqrt{ex+d+\sqrt{\frac{e^2x^2}{f^2}+a}\,f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
[Out] Integral(sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

3.463 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$

Optimal. Leaf size=147

$$\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

```
[Out] 1/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(3/2)/e+
(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/2*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^
(1/2))^(1/2)/d/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] time = 0.11, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1157, 388, 206}

$$\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/e - (a*f^2*Sqrt[d + e*x + f*Sqrt[
a + (e^2*x^2)/f^2]])/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*Arc
Tanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2\sqrt{x}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d^2-af^2+2dx^2}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+}}\right.}{2de}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(af^2\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,d\right.}{}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{}}}{\sqrt{d}}\right.}{2d^{3/2}e}$$

Mathematica [A] time = 0.25, size = 143, normalized size = 0.97

$$\frac{\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}} + \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)} + \sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] (Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + (a*f^2*Sqrt[d + e*x + f*Sqrt[a
 + (e^2*x^2)/f^2]])/(2*d*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*Arc
Tanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)))/e
```

fricas [A] time = 0.55, size = 298, normalized size = 2.03

$$\left[\frac{a\sqrt{d}\,f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\left(\sqrt{d}\,ex-\sqrt{d}\,f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(dex-\right.}{4\,d^2e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2)
 - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqr
t((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2)
 + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d^2*e), -1/2*(a*s
qrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d
) - (d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^
2*x^2 + a*f^2)/f^2) + d))/(d^2*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex+d+\sqrt{\frac{e^2x^2}{f^2}+a}\;f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

```
[Out] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

3.464 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}$$

```
[Out] 3/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(5/2)/e+
(-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)-1/2*a*f^2*(d+e*x+f*(
a+e^2*x^2/f^2)^(1/2))^(1/2)/d^2/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] time = 0.16, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1259, 453, 206}

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}+\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] -((1 + (a*f^2)/d^2)/(e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])) - (a*f^2
*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e*(e*x + f*Sqrt[a + (e^2
*x^2)/f^2])) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/S
qrt[d]])/(2*d^(5/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^{3/2}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{x^2\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d\left(d^2+af^2\right)+\left(2d^2-af^2\right)x^2}{x^2\left(d-x^2\right)}\,dx, x, \sqrt{d+ex}\right.}{2d^2e}$$

$$= -\frac{d^2+af^2}{d^2e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(3af^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{d^2+af^2}{d^2e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{3af^2\tanh^{-1}\left(\right.}{2d}$$

Mathematica [**A**] time = 0.43, size = 167, normalized size = 1.06

$$\frac{\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{-2d^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)-af^2\left(3f\sqrt{a+\frac{e^2x^2}{f^2}}+d+3ex\right)}{d^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] ((-2*d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - a*f^2*(d + 3*e*x + 3*f*Sqrt[a
+ (e^2*x^2)/f^2]))/(d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])*Sqrt[d + e*x + f*
Sqrt[a + (e^2*x^2)/f^2]]) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2
*x^2)/f^2]]/Sqrt[d]])/d^(5/2))/(2*e)
```

fricas [**A**] time = 0.52, size = 487, normalized size = 3.08

$$\left[\frac{3\left(a^2f^4-2\,adef^2x-ad^2f^2\right)\sqrt{d}\,\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\left(\sqrt{d}\,ex-\sqrt{d}\,f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e}{}}}\right.}{4\left(ad\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(d)*log(a*f^2 - 2*d*e*x +
 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x
^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) - 2*(2*d^2
*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 - (3*a*d*e*f^2 + d^3*e)*x + (3*a*d*f^3 - 2*d
^2*e*f*x + d^3*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 +
 a*f^2)/f^2) + d))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e), -1/2*(3*(a^2*f^4 -
2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(-d)*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f
^2)/f^2) + d)*sqrt(-d)/d) + (2*d^2*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 - (3*a*d*e
*f^2 + d^3*e)*x + (3*a*d*f^3 - 2*d^2*e*f*x + d^3*f)*sqrt((e^2*x^2 + a*f^2)/
f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a*d^3*e*f^2 - 2*d^4*e
^2*x - d^5*e)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, cho
osing root of [1,0,%%%{-2,[1,2,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0,2,
1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%%{1
,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%%%}+%%%{1,[0,0,4,2]%%%}] at parameters valu
es [-49,85.3561567818,-64,-30]Warning, choosing root of [1,0,%%%{-2,[1,2,0,
```

```
0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%{
-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%%
%}+%%%{1,[0,0,4,2]%%%}] at parameters values [0,61.7937478349,0,0]Warning,
choosing root of [1,0,%%%{-2,[1,2,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0
,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%
%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%%%}+%%%{1,[0,0,4,2]%%%}] at parameters v
alues [56,62.4600259969,-13,46]Warning, choosing root of [1,0,%%%{-2,[1,2,0
,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%
{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%
%%}+%%%{1,[0,0,4,2]%%%}] at parameters values [-6,25.8388736797,81,18]Warni
ng, choosing root of [1,0,%%%{-2,[1,2,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,
[0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%
}+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%%%}+%%%{1,[0,0,4,2]%%%}] at paramete
rs values [63,31.8503101398,2,62]Warning, choosing root of [1,0,%%%{-2,[1,2
,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%
%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1
]%%%}+%%%{1,[0,0,4,2]%%%}] at parameters values [0,10.4309062702,0,0]Warnin
g, choosing root of [1,0,%%%{-2,[1,2,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[
0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+%%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}
+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,1]%%%}+%%%{1,[0,0,4,2]%%%}] at parameter
s values [65,39.1803401988,28,-44]Warning, choosing root of [1,0,%%%{-2,[1,
2,0,0]%%%}+%%%{-2,[0,1,0,0]%%%}+%%%{-2,[0,0,2,1]%%%},0,%%%{1,[2,4,0,0]%%%}+
%%%{-2,[1,3,0,0]%%%}+%%%{2,[1,2,2,1]%%%}+%%%{1,[0,2,0,0]%%%}+%%%{-2,[0,1,2,
1]%%%}+%%%{1,[0,0,4,2]%%%}] at parameters values [91,88.2886286299,-21,88]E
valuation time: 0.64Done
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a}\, f\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
[Out] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3/2), x)
```

3.465 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{7/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+\dots\right.}$$

```
[Out] 5/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(7/2)/e+
1/3*(-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)-2*a*f^2/d^3/e/(d
+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)-1/2*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2
))^(1/2)/d^3/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] time = 0.18, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2117, 897, 1259, 1261, 206}

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}+\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+\dots}}{\sqrt{\dots}}\right)}{2d^{7/2}e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] -(1 + (a*f^2)/d^2)/(3*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)) - (2*a
*f^2)/(d^3*e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]) - (a*f^2*Sqrt[d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])
) + (5*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2
*d^(7/2)*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
```

```
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^{5/2}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{x^4\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{2d^2\left(d^2+af^2\right)-2d\left(d^2-af^2\right)x^2+af^2x^4}{x^4\left(d-x^2\right)}\,dx, x\right.}{2d^3e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(\frac{2\left(d^3+adf^2\right)}{x^4}+\frac{4af^2}{x^2}+\frac{5af^2}{d-x^2}\right)dx, x,\right.}{2d^3e}$$

$$= -\frac{d^2+af^2}{3d^2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+}}{2d^3e\left(e\right.}$$

$$= -\frac{d^2+af^2}{3d^2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+}}{2d^3e\left(e\right.}$$

Mathematica [A] time = 0.60, size = 186, normalized size = 0.93

$$-\frac{\frac{2d\left(af^2+d^2\right)}{3\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}+\frac{4af^2}{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{2d^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] -1/2*((2*d*(d^2 + a*f^2))/(3*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)) +
 (4*a*f^2)/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + (a*f^2*Sqrt[d + e*x
+ f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - (5*a*f^2*
ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqrt[d])/(d^3*e
)
```

fricas [B] time = 0.60, size = 812, normalized size = 4.08

$$\left[\frac{15\left(a^3f^6+4\,ad^2e^2f^2x^2-2\,a^2d^2f^4+ad^4f^2-4\left(a^2def^4-ad^3ef^2\right)x\right)\sqrt{d}\,\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\left(\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a
^2*d*e*f^4 - a*d^3*e*f^2)*x)*sqrt(d)*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*
x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))
*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(12*d^3*e^3*x^3 + 10*a^
2*d^2*f^4 - 16*a*d^4*f^2 - 2*d^6 - 8*(5*a*d^2*e^2*f^2 - d^4*e^2)*x^2 + (15*
a^2*d*e*f^4 - 46*a*d^3*e*f^2 - d^5*e)*x - (15*a^2*d*f^5 + 12*d^3*e^2*f*x^2
- 22*a*d^3*f^3 - d^5*f - 8*(5*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f
^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a^2*d^4*e*f^4 + 4
*d^6*e^3*x^2 - 2*a*d^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x), -1/6
*(15*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*
e*f^4 - a*d^3*e*f^2)*x)*sqrt(-d)*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)
/f^2) + d)*sqrt(-d)/d) - (12*d^3*e^3*x^3 + 10*a^2*d^2*f^4 - 16*a*d^4*f^2 -
2*d^6 - 8*(5*a*d^2*e^2*f^2 - d^4*e^2)*x^2 + (15*a^2*d*e*f^4 - 46*a*d^3*e*f^
2 - d^5*e)*x - (15*a^2*d*f^5 + 12*d^3*e^2*f*x^2 - 22*a*d^3*f^3 - d^5*f - 8*
(5*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt
((e^2*x^2 + a*f^2)/f^2) + d))/(a^2*d^4*e*f^4 + 4*d^6*e^3*x^2 - 2*a*d^6*e*f^
2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex+d+\sqrt{\frac{e^{2}x^{2}}{f^{2}}+a}\, f\right)^{\frac{5}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
[Out] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex+\sqrt{\frac{e^{2}x^{2}}{f^{2}}+a}\, f+d\right)^{\frac{5}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-5/2), x)
```

3.466 $\int \sqrt{x-\sqrt{-4+x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{1}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{4}{\sqrt{x-\sqrt{x^2-4}}}$$

[Out] 1/3*(x-(x^2-4)^(1/2))^(3/2)+4/(x-(x^2-4)^(1/2))^(1/2)

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2117, 14}

$$\frac{1}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{4}{\sqrt{x-\sqrt{x^2-4}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x - Sqrt[-4 + x^2]],x]

[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x-\sqrt{-4+x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{-4+x^2}{x^{3/2}}\,dx,x,x-\sqrt{-4+x^2}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{4}{x^{3/2}}+\sqrt{x}\right)dx,x,x-\sqrt{-4+x^2}\right)\\
&= \frac{4}{\sqrt{x-\sqrt{-4+x^2}}}+\frac{1}{3}\left(x-\sqrt{-4+x^2}\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.98

$$\frac{2x^2-2\sqrt{x^2-4}\,x+8}{3\sqrt{x-\sqrt{x^2-4}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x - Sqrt[-4 + x^2]],x]

```
[Out] (8 + 2*x^2 - 2*x*Sqrt[-4 + x^2])/(3*Sqrt[x - Sqrt[-4 + x^2]])
```

fricas [A] time = 0.45, size = 26, normalized size = 0.63

$$\frac{2}{3}\left(2x+\sqrt{x^2-4}\right)\sqrt{x-\sqrt{x^2-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*x + sqrt(x^2 - 4))*sqrt(x - sqrt(x^2 - 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2-4)^(1/2))^(1/2),x)
[Out] int((x-(x^2-4)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (x^2 - 4)^(1/2))^(1/2),x)
[Out] int((x - (x^2 - 4)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2-4)**(1/2))**(1/2),x)
[Out] Integral(sqrt(x - sqrt(x**2 - 4)), x)
```

3.467 $\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=69

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}}{3a}-\frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

```
[Out] 1/3*(a*x+b*(c+a^2*x^2/b^2)^(1/2))^(3/2)/a-b^2*c/a/(a*x+b*(c+a^2*x^2/b^2)^(1
/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2117, 14}

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}}{3a}-\frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (a
^2*x^2)/b^2])^(3/2)/(3*a)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2c+x^2}{x^{3/2}}\,dx, x, ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)}{2a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b^2c}{x^{3/2}}+\sqrt{x}\right)dx, x, ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)}{2a}\\
&= -\frac{b^2c}{a\sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}}+\frac{\left(ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)^{3/2}}{3a}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 0.97

$$\frac{2\left(abx\sqrt{\frac{a^2x^2}{b^2}+c}+a^2x^2+b^2(-c)\right)}{3a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] (2*(-(b^2*c) + a^2*x^2 + a*b*x*Sqrt[c + (a^2*x^2)/b^2]))/(3*a*Sqrt[a*x + b*
Sqrt[c + (a^2*x^2)/b^2]])
```

fricas [A] time = 0.43, size = 59, normalized size = 0.86

$$\frac{2\left(2\,ax-b\sqrt{\frac{a^2x^2+b^2c}{b^2}}\right)\sqrt{ax+b\sqrt{\frac{a^2x^2+b^2c}{b^2}}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*a*x - b*sqrt((a^2*x^2 + b^2*c)/b^2))*sqrt(a*x + b*sqrt((a^2*x^2 + b^
2*c)/b^2))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ax+\sqrt{\frac{a^2x^2}{b^2}+c}\,b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\sqrt{ax+\sqrt{\frac{a^2x^2}{b^2}+c}\;b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ax+\sqrt{\frac{a^2x^2}{b^2}+c}\,b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a\,x+b\,\sqrt{c+\frac{a^2\,x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2),x)
[Out] int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+b\sqrt{\frac{a^2x^2}{b^2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a**2*x**2/b**2)**(1/2))**(1/2),x)
[Out] Integral(sqrt(a*x + b*sqrt(a**2*x**2/b**2 + c)), x)
```

3.468 $\int \sqrt{1+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=45

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}}-\frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

```
[Out] -2/3*x^3/(1+(-x^2+1)^(1/2))^(3/2)+2*x/(1+(-x^2+1)^(1/2))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2129}

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}}-\frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 - x^2]],x]
[Out] (-2*x^3)/(3*(1 + Sqrt[1 - x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 - x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx=-\frac{2x^3}{3\left(1+\sqrt{1-x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1-x^2}}}$$

Mathematica [A] time = 0.10, size = 35, normalized size = 0.78

$$\frac{2x\left(\sqrt{1-x^2}+2\right)}{3\sqrt{\sqrt{1-x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 - x^2]],x]
[Out] (2*x*(2 + Sqrt[1 - x^2]))/(3*Sqrt[1 + Sqrt[1 - x^2]])
```

fricas [A] time = 0.46, size = 34, normalized size = 0.76

$$\frac{2\left(x^2-\sqrt{-x^2+1}+1\right)\sqrt{\sqrt{-x^2+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 - sqrt(-x^2 + 1) + 1)*sqrt(sqrt(-x^2 + 1) + 1)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{-x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

maple [C] time = 0.04, size = 60, normalized size = 1.33

$$\frac{i\left(\frac{32i\sqrt{\pi}\,\sqrt{2}\,x^3\cos\left(\frac{3\arcsin(x)}{2}\right)}{3}-\frac{8i\sqrt{\pi}\,\sqrt{2}\left(-\frac{4}{3}x^4+\frac{2}{3}x^2+\frac{2}{3}\right)\sin\left(\frac{3\arcsin(x)}{2}\right)}{\sqrt{-x^2+1}}\right)}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(-x^2+1)^(1/2))^(1/2),x)
```

```
[Out] 1/8*I/Pi^(1/2)*(32/3*I*Pi^(1/2)*2^(1/2)*x^3*cos(3/2*arcsin(x))-8*I*Pi^(1/2)
*2^(1/2)*(-4/3*x^4+2/3*x^2+2/3)*sin(3/2*arcsin(x))/(-x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{-x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{1-x^2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^2)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(((1 - x^2)^(1/2) + 1)^(1/2), x)
```

sympy [B] time = 1.28, size = 418, normalized size = 9.29

$$\begin{cases} -\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{-12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12\pi\sqrt{i\sqrt{x^2-1}+1}}+\frac{3\sqrt{2}ix\sqrt{x^2-1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{-12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12\pi\sqrt{i\sqrt{x^2-1}+1}}+\frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{-12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12\pi\sqrt{i\sqrt{x^2-1}+1}} \\ \frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}}-\frac{3\sqrt{2}x\sqrt{1-x^2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}}-\frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(-12*I*pi*sqrt(x**2 - 1)*sq
rt(I*sqrt(x**2 - 1) + 1) - 12*pi*sqrt(I*sqrt(x**2 - 1) + 1)) + 3*sqrt(2)*I*
```

```
x*sqrt(x**2 - 1)*gamma(-1/4)*gamma(1/4)/(-12*I*pi*sqrt(x**2 - 1)*sqrt(I*sqr
t(x**2 - 1) + 1) - 12*pi*sqrt(I*sqrt(x**2 - 1) + 1)) + 3*sqrt(2)*x*gamma(-1
/4)*gamma(1/4)/(-12*I*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) - 12*pi*
sqrt(I*sqrt(x**2 - 1) + 1)), Abs(x**2) > 1), (sqrt(2)*x**3*gamma(-1/4)*gamm
a(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 -
 x**2) + 1)) - 3*sqrt(2)*x*sqrt(1 - x**2)*gamma(-1/4)*gamma(1/4)/(12*pi*sqr
t(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)) - 3*
sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2)
+ 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)), True))
```

3.469 $\int \sqrt{1+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}+\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}$$

```
[Out] 2/3*x^3/((x^2+1)^(1/2)+1)^(3/2)+2*x/((x^2+1)^(1/2)+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2129}

$$\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}+\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]],x]
[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx=\frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 1.07

$$\frac{2\left(\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}\left(\sqrt{x^2+1}+2\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]],x]
[Out] (2*(-1 + Sqrt[1 + x^2])*Sqrt[1 + Sqrt[1 + x^2]]*(2 + Sqrt[1 + x^2]))/(3*x)
```

fricas [A] time = 0.45, size = 28, normalized size = 0.68

$$\frac{2\left(x^2+\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqrt(x^2 + 1) + 1)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

maple [C] time = 0.03, size = 55, normalized size = 1.34

$$-\frac{-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^3\cosh\left(\frac{3\operatorname{arcsinh}(x)}{2}\right)}{3}-\frac{8\sqrt{\pi}\,\sqrt{2}\left(-\frac{4}{3}x^4-\frac{2}{3}x^2+\frac{2}{3}\right)\sinh\left(\frac{3\operatorname{arcsinh}(x)}{2}\right)}{\sqrt{x^2+1}}}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))^(1/2),x)
[Out] -1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(x))-8*Pi^(1/2)*2
^(1/2)*(-4/3*x^4-2/3*x^2+2/3)*sinh(3/2*arcsinh(x))/(x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)^(1/2),x)
[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2), x)
```

sympy [B] time = 1.18, size = 197, normalized size = 4.80

$$-\frac{\sqrt{2}\,x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}-\frac{3\sqrt{2}\,x\sqrt{x^2+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}-\frac{}{12\pi\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)**(1/2)+1)**(1/2),x)
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 +
 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*sqrt(x**2 + 1)*gam
ma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*
sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(
x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1))
```

3.470 $\int \sqrt{5+\sqrt{25+x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}+\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}$$

```
[Out] 2/3*x^3/(5+(x^2+25)^(1/2))^(3/2)+10*x/(5+(x^2+25)^(1/2))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2129}

$$\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}+\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (2*x^3)/(3*(5 + Sqrt[25 + x^2])^(3/2)) + (10*x)/Sqrt[5 + Sqrt[25 + x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\frac{2x^3}{3\left(5+\sqrt{25+x^2}\right)^{3/2}}+\frac{10x}{\sqrt{5+\sqrt{25+x^2}}}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 1.07

$$\frac{2\left(\sqrt{x^2+25}-5\right)\sqrt{\sqrt{x^2+25}+5}\left(\sqrt{x^2+25}+10\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (2*(-5 + Sqrt[25 + x^2])*Sqrt[5 + Sqrt[25 + x^2]]*(10 + Sqrt[25 + x^2]))/(3
*x)
```

fricas [A] time = 0.48, size = 30, normalized size = 0.73

$$\frac{2\left(x^2+5\sqrt{x^2+25}-25\right)\sqrt{\sqrt{x^2+25}+5}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + 5*sqrt(x^2 + 25) - 25)*sqrt(sqrt(x^2 + 25) + 5)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2+25}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

maple [C] time = 0.02, size = 64, normalized size = 1.56

$$-\frac{5\sqrt{5}\left(-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^3\cosh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{375}-\frac{8\sqrt{\pi}\,\sqrt{2}\left(-\frac{4}{1875}x^4-\frac{2}{75}x^2+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{\sqrt{\frac{x^2}{25}+1}}\right)}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+(x^2+25)^(1/2))^(1/2),x)
[Out] -5/8*5^(1/2)/Pi^(1/2)*(-32/375*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(1/5*x)
)-8*Pi^(1/2)*2^(1/2)*(-4/1875*x^4-2/75*x^2+2/3)*sinh(3/2*arcsinh(1/5*x))/(1
/25*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2+25}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{x^2+25}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 25)^(1/2) + 5)^(1/2),x)
[Out] int(((x^2 + 25)^(1/2) + 5)^(1/2), x)
```

sympy [B] time = 1.22, size = 197, normalized size = 4.80

$$-\frac{\sqrt{2}\,x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}-\frac{15\sqrt{2}\,x\sqrt{x^2+25}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x**2+25)**(1/2))**(1/2),x)
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2
+ 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 15*sqrt(2)*x*sqrt(x**2 + 25
)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) +
 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 75*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12
*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25)
+ 5))
```

3.471 $\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$

Optimal. Leaf size=66

$$\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}+\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}$$

```
[Out] 2/3*b^2*c*x^3/(a+b*(1/b^2*a^2+c*x^2)^(1/2))^(3/2)+2*a*x/(a+b*(1/b^2*a^2+c*x
^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2129}

$$\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}+\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

```
[Out] (2*b^2*c*x^3)/(3*(a + b*Sqrt[a^2/b^2 + c*x^2])^(3/2)) + (2*a*x)/Sqrt[a + b*
Sqrt[a^2/b^2 + c*x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx=\frac{2b^2cx^3}{3\left(a+b\sqrt{\frac{a^2}{b^2}+cx^2}\right)^{3/2}}+\frac{2ax}{\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}}$$

Mathematica [A] time = 0.23, size = 55, normalized size = 0.83

$$\frac{2bx\sqrt{\frac{a^2}{b^2}+cx^2}+4ax}{3\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

```
[Out] (4*a*x + 2*b*x*Sqrt[a^2/b^2 + c*x^2])/(3*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]])
```

fricas [A] time = 0.57, size = 70, normalized size = 1.06

$$\frac{2\left(b^2cx^2+ab\sqrt{\frac{b^2cx^2+a^2}{b^2}}-a^2\right)\sqrt{b\sqrt{\frac{b^2cx^2+a^2}{b^2}}+a}}{3\,b^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*c*x^2 + a*b*sqrt((b^2*c*x^2 + a^2)/b^2) - a^2)*sqrt(b*sqrt((b^2*c*
x^2 + a^2)/b^2) + a)/(b^2*c*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^2 + \frac{a^2}{b^2}}\,b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \sqrt{a + \sqrt{c\,x^2 + \frac{a^2}{b^2}}\,b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^2 + \frac{a^2}{b^2}}\,b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a + b\,\sqrt{c\,x^2 + \frac{a^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sqrt{\frac{a^2}{b^2} + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(a**2/b**2+c*x**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(a**2/b**2 + c*x**2)), x)
```

3.472 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n\,dx$

Optimal. Leaf size=166

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{2de-bf^2}\right)}{2e(n+1)\left(2de-bf^2\right)^2}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right.}{2e(n+1}$$

```
[Out] 1/2*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)+1/2*f^2*(-b^2*f^2+4*a
*e^2)*hypergeom([2, 1+n],[2+n],2*e*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*
f^2+2*d*e))*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1+n)/e/(-b*f^2+2*d*e)^2/(1
+n)
```

Rubi [A] time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2116, 947, 64}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{2de-bf^2}\right)}{2e(n+1)\left(2de-bf^2\right)^2}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right.}{2e(n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (f^2*(4
*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]))/(2*d*e - b*f^2)])/(2*e*(2*d*e - b*f^2)^2*(1 + n))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 947

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = 2\,\text{Subst}\left(\int \frac{x^n\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}dx, x, d+ex+\right.$$

$$= 2\,\text{Subst}\left(\int \left(\frac{x^n}{4e}+\frac{\left(4ae^2f^2-b^2f^4\right)x^n}{4e\left(2de-bf^2-2ex\right)^2}\right)dx, x, d+ex+f\sqrt{a+bx+\right.$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}+\frac{\left(4ae^2f^2-b^2f^4\right)\text{Subst}\left(\int \frac{x^n}{\left(2de-bf^2-2\right.}\right.}{}$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(d+ex+f\sqrt{a+\right.}{2}$$

Mathematica [A] time = 0.32, size = 134, normalized size = 0.81

$$\frac{\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^{n+1}\left(\left(4ae^2f^2-b^2f^4\right){}_2F_1\left(2,n+1;n+2;\frac{2e\left(d+ex+f\sqrt{a+x\left(\frac{xe^2}{f^2}+b\right)}\right)}{2de-bf^2}\right)+\left(bf^2-2de\right)^2\right.}{2e(n+1)\left(bf^2-2de\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(1 + n)*((-2*d*e + b*f^2)^2 +
(4*a*e^2*f^2 - b^2*f^4)*Hypergeometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x +
f*Sqrt[a + x*(b + (e^2*x)/f^2)]))/(2*d*e - b*f^2)]))/(2*e*(-2*d*e + b*f^2)^
2*(1 + n))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(ex+d+\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\, f\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+e^2/f^2*x^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*(a+b*x+e^2/f^2*x^2)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex+\sqrt{bx+\frac{e^{2}x^{2}}{f^{2}}+a}\, f+d\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^n,x)
```

```
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^{2}x^{2}}{f^{2}}}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**n, x)
```

3.473 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$

Optimal. Leaf size=303

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^3}{32e^5\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^2\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+\right.}{32e^5}$$

```
[Out] 3/32*f^2*(-b*f^2+2*d*e)^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^
2+e^2*x)/f^2)^(1/2)))/e^5+1/8*f^2*(-b*f^2+2*d*e)*(-b^2*f^2+4*a*e^2)*(e*x+f*
(a+b*x+e^2*x^2/f^2)^(1/2))/e^4+1/16*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+
e^2*x^2/f^2)^(1/2))^2/e^3+1/8*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^4/e-1/32*
f^2*(-b*f^2+2*d*e)^3*(-b^2*f^2+4*a*e^2)/e^5/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e
^2*x)/f^2)^(1/2)))
```

Rubi [A] time = 0.38, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2116, 893}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{16e^3}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^4}-\frac{\quad}{32e^5\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]))/(8*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*
x^2)/f^2])^2)/(16*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^4/(8*e
) - (f^2*(2*d*e - b*f^2)^3*(4*a*e^2 - b^2*f^2))/(32*e^5*(b*f^2 + 2*e*(e*x +
 f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(2*d*e - b*f^2)^2*(4*a*e^2
 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(
32*e^5)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3dx=2\,\text{Subst}\left(\int\frac{x^3\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}dx,x,d+ex+f\right.$$
$$=2\,\text{Subst}\left(\int\left(\frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)}{16e^4}+\frac{f^2\left(4ae^2-b^2f^2\right)x}{16e^3}+\frac{x^3}{4e}+\frac{f}{}\right.\right.$$
$$=\frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^4}+\frac{f^2\left(4ae^2-b^2f^2\right)}{}$$

Mathematica [A] time = 0.55, size = 276, normalized size = 0.91

$$2e^2f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^2+4ef^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (4*e*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + x*(b + (e^2*
x)/f^2)]) + 2*e^2*f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + x*(b + (e^2
*x)/f^2)])^2 + 4*e^4*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^4 - (f^2*(
-2*d*e + b*f^2)^3*(-4*a*e^2 + b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b
 + (e^2*x)/f^2)])) - 3*(-4*a*e^2 + b^2*f^2)*(-2*d*e*f + b*f^3)^2*Log[-(b*f^
2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/(32*e^5)
```

fricas [A] time = 0.48, size = 345, normalized size = 1.14

$$32\,e^8x^4+32\left(be^6f^2+2\,de^7\right)x^3+48\left(d^2e^6+\left(bde^5+ae^6\right)f^2\right)x^2+32\left(3\,ade^5f^2+d^3e^5\right)x+3\left(b^4f^8-16\,ad^2e^4f^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(32*e^8*x^4 + 32*(b*e^6*f^2 + 2*d*e^7)*x^3 + 48*(d^2*e^6 + (b*d*e^5 +
a*e^6)*f^2)*x^2 + 32*(3*a*d*e^5*f^2 + d^3*e^5)*x + 3*(b^4*f^8 - 16*a*d^2*e^
4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*lo
g(-b*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(3*b^
3*e*f^7 + 16*e^7*f*x^3 - 4*(3*b^2*d*e^2 + 2*a*b*e^3)*f^5 + 4*(3*b*d^2*e^3 +
 8*a*d*e^4)*f^3 + 8*(b*e^5*f^3 + 4*d*e^6*f)*x^2 - 2*(b^2*e^3*f^5 - 12*d^2*e
^5*f - 4*(b*d*e^4 + 2*a*e^5)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))
/e^5
```

giac [A] time = 0.37, size = 373, normalized size = 1.23

$$bf^2x^3e+\frac{3}{2}\,bdf^2x^2+\frac{3}{2}\,af^2x^2e+3\,adf^2x+x^4e^3+2\,dx^3e^2+\frac{3}{2}\,d^2x^2e+d^3x+\frac{3}{32}\left(b^4f^7|f|-4\,b^3df^5|f|e-4\,ab^2f^5|f|e^2+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] b*f^2*x^3*e + 3/2*b*d*f^2*x^2 + 3/2*a*f^2*x^2*e + 3*a*d*f^2*x + x^4*e^3 + 2
*d*x^3*e^2 + 3/2*d^2*x^2*e + d^3*x + 3/32*(b^4*f^7*abs(f) - 4*b^3*d*f^5*abs
(f)*e - 4*a*b^2*f^5*abs(f)*e^2 + 4*b^2*d^2*f^3*abs(f)*e^2 + 16*a*b*d*f^3*ab
s(f)*e^3 - 16*a*d^2*f*abs(f)*e^4)*e^(-5)*log(abs(-b*f^2 - 2*(x*e - sqrt(b*f
^2*x + a*f^2 + x^2*e^2))*e)) + 1/16*sqrt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*(
2*x*abs(f)*e^2/f + (b*f^4*abs(f)*e^6 + 4*d*f^2*abs(f)*e^7)*e^(-6)/f^3)*x -
(b^2*f^6*abs(f)*e^4 - 4*b*d*f^4*abs(f)*e^5 - 8*a*f^4*abs(f)*e^6 - 12*d^2*f^
2*abs(f)*e^6)*e^(-6)/f^3)*x + (3*b^3*f^8*abs(f)*e^2 - 12*b^2*d*f^6*abs(f)*e
^3 - 8*a*b*f^6*abs(f)*e^4 + 12*b*d^2*f^4*abs(f)*e^4 + 32*a*d*f^4*abs(f)*e^5
)*e^(-6)/f^3)
```

maple [B] time = 0.02, size = 685, normalized size = 2.26

$$-\frac{3b^{4} f^{7} \ln\left(\frac{\frac{b}{2}+\frac{e^{2} x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{bx+\frac{e^{2} x^{2}}{f^{2}}+a}\right)}{32\sqrt{\frac{e^{2}}{f^{2}}}\, e^{4}}+\frac{3a\, b^{2} f^{5} \ln\left(\frac{\frac{b}{2}+\frac{e^{2} x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{bx+\frac{e^{2} x^{2}}{f^{2}}+a}\right)}{8\sqrt{\frac{e^{2}}{f^{2}}}\, e^{2}}+\frac{3b^{3} d\, f^{5} \ln\left(\frac{\frac{b}{2}+\frac{e^{2} x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{bx+\frac{e^{2} x^{2}}{f^{2}}}\right.}{8\sqrt{\frac{e^{2}}{f^{2}}}\, e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^3,x)
```

```
[Out] 3/2*d^2*e*x^2+d^3*x+e^3*x^4-3/2*d/e*f^3*b*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1
/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*a+1/4*d^4/e+2*d/e*f^3*(b*x+e
^2/f^2*x^2+a)^(3/2)+3/2*f*d^2*(b*x+e^2/f^2*x^2+a)^(1/2)*x+3/8*f^5/e^2*a*ln(
(1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2
)*b^2-1/2*f^5/e^2*(b*x+e^2/f^2*x^2+a)^(3/2)*b+3/16*f^7/e^4*b^3*(b*x+e^2/f^2
*x^2+a)^(1/2)+f^2*x^3*b*e+3/2*f^2*x^2*b*d+2*d*e^2*x^3+3/2*a*e*f^2*x^2+3*a*d
*f^2*x+f^3*(b*x+e^2/f^2*x^2+a)^(3/2)*x-3/4*d/e^3*f^5*b^2*(b*x+e^2/f^2*x^2+a
)^(1/2)+3/4*d^2/e^2*f^3*(b*x+e^2/f^2*x^2+a)^(1/2)*b+3/2*f*d^2*ln((1/2*b+e^2
/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*a-3/2*d/
e*f^3*b*(b*x+e^2/f^2*x^2+a)^(1/2)*x+3/8*d/e^3*f^5*b^3*ln((1/2*b+e^2/f^2*x)/
(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)-3/8*d^2/e^2*f^3*
ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(
1/2)*b^2+3/8*f^5/e^2*b^2*(b*x+e^2/f^2*x^2+a)^(1/2)*x-3/32*f^7/e^4*b^4*ln((1
/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume?`
for more details)Is b^2*f^2-4*a*e^2    positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**3, x)
```

3.474 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$

Optimal. Leaf size=237

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^2}{16e^4\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+b\right.}{8e^4}$$

```
[Out] 1/8*f^2*(-b*f^2+2*d*e)*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e
^2*x)/f^2)^(1/2)))/e^4+1/8*f^2*(-b^2*f^2+4*a*e^2)*(e*x+f*(a+b*x+e^2*x^2/f^2
)^(1/2))/e^3+1/6*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3/e-1/16*f^2*(-b*f^2+2
*d*e)^2*(-b^2*f^2+4*a*e^2)/e^4/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1
/2)))
```

Rubi [A] time = 0.24, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^2}{16e^4\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+b\right.}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))/(8*e^3) +
 (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3/(6*e) - (f^2*(2*d*e - b*f^2)
^2*(4*a*e^2 - b^2*f^2))/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 +
e^2*x))/f^2]))) + (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(8*e^4)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx = 2\,\text{Subst}\left(\int \frac{x^2\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}dx, x, d+ex+f$$

$$= 2\,\text{Subst}\left(\int \left(\frac{4ae^2f^2-b^2f^4}{16e^3}+\frac{x^2}{4e}+\frac{\left(4ae^2-b^2f^2\right)\left(2def-bf^3\right)^2}{16e^3\left(2de-bf^2-2ex\right)^2}-\frac{f^2\left(2c\right.}{8e}\right.\right.$$

$$= \frac{f^2\left(4ae^2-b^2f^2\right)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{6e}$$

Mathematica [A] time = 0.32, size = 213, normalized size = 0.90

$$\frac{6f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)\log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-bf^2\right)+\frac{3\left(b^2f^2-4ae^2\right)\left(bf^3-2def\right)^2}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+6ef^2\left(4ae^2\right.}{48e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (6*e*f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 8*e^
3*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^3 + (3*(-4*a*e^2 + b^2*f^2)*(
-2*d*e*f + b*f^3)^2)/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))
+ 6*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*S
qrt[a + x*(b + (e^2*x)/f^2)])])/(48*e^4)
```

fricas [A] time = 0.56, size = 219, normalized size = 0.92

$$\frac{16\,e^6x^3+12\left(be^4f^2+2\,de^5\right)x^2+24\left(ae^4f^2+d^2e^4\right)x-3\left(b^3f^6+8\,ade^3f^2-2\left(b^2de+2\,abe^2\right)f^4\right)\log\left(-bf^2-2\,e^2\right.}{24\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*e^6*x^3 + 12*(b*e^4*f^2 + 2*d*e^5)*x^2 + 24*(a*e^4*f^2 + d^2*e^4)*
x - 3*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*log(-b*f^2 -
2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 2*(3*b^2*e*f^5 - 8
*e^5*f*x^2 - 2*(3*b*d*e^2 + 4*a*e^3)*f^3 - 2*(b*e^3*f^3 + 6*d*e^4*f)*x)*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^4
```

giac [A] time = 0.30, size = 224, normalized size = 0.95

$$\frac{1}{2}bf^2x^2+af^2x+\frac{2}{3}x^3e^2+dx^2e+d^2x-\frac{1}{8}\left(b^3f^5|f|-2\,b^2df^3|f|e-4\,abf^3|f|e^2+8\,adf|f|e^3\right)e^{(-4)}\log\left(\left|-bf^2-2\left(xe-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*f^2*x^2 + a*f^2*x + 2/3*x^3*e^2 + d*x^2*e + d^2*x - 1/8*(b^3*f^5*abs(
f) - 2*b^2*d*f^3*abs(f)*e - 4*a*b*f^3*abs(f)*e^2 + 8*a*d*f*abs(f)*e^3)*e^(-
```

```
4)*log(abs(-b*f^2 - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e)) + 1/12*sq
rt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*x*abs(f)*e/f + (b*f^3*abs(f)*e^3 + 6*d*
f*abs(f)*e^4)*e^(-4)/f^2)*x - (3*b^2*f^5*abs(f)*e - 6*b*d*f^3*abs(f)*e^2 -
8*a*f^3*abs(f)*e^3)*e^(-4)/f^2)
```

maple [A] time = 0.01, size = 409, normalized size = 1.73

$$\frac{b^{3} f^{5} \ln \left(\frac{\frac{b}{2}+\frac{e^{2} x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{b x+\frac{e^{2} x^{2}}{f^{2}}+a}\right)}{8 \sqrt{\frac{e^{2}}{f^{2}}}\, e^{3}}-\frac{a b\, f^{3} \ln \left(\frac{\frac{b}{2}+\frac{e^{2} x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{b x+\frac{e^{2} x^{2}}{f^{2}}+a}\right)}{2 \sqrt{\frac{e^{2}}{f^{2}}}\, e}-\frac{b^{2} d\, f^{3} \ln \left(\frac{\frac{b}{2}+\frac{e^{2} x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{b x+\frac{e^{2} x^{2}}{f^{2}}+a}\right)}{4 \sqrt{\frac{e^{2}}{f^{2}}}\, e^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^2,x)
```

```
[Out] a*f^2*x+1/2*f^2*b*x^2+2/3*e^2*x^3+2/3*(b*x+e^2/f^2*x^2+a)^(3/2)/e*f^3-1/2/e
*f^3*b*(b*x+e^2/f^2*x^2+a)^(1/2)*x-1/4/e^3*f^5*b^2*(b*x+e^2/f^2*x^2+a)^(1/2
)-1/2/e*f^3*b*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2
))/(e^2/f^2)^(1/2)*a+1/8/e^3*f^5*b^3*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(
b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)+f*d*(b*x+e^2/f^2*x^2+a)^(1/2)*x+1
/2*d/e^2*f^3*(b*x+e^2/f^2*x^2+a)^(1/2)*b+f*d*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)
^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*a-1/4*d/e^2*f^3*ln((1/2*b
+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*b^2+
d*e*x^2+d^2*x+1/3*d^3/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume?`
for more details)Is b^2*f^2-4*a*e^2    positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)
```

```
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^{2} x^{2}}{f^{2}}}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**2, x)
```

3.475 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)dx$

Optimal. Leaf size=118

$$\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+dx+\frac{ex^2}{2}$$

```
[Out] d*x+1/2*e*x^2+1/8*f^2*(-b^2*f^2+4*a*e^2)*arctanh(1/2*(b*f^2+2*e^2*x)/e/f/(a
+b*x+e^2*x^2/f^2)^(1/2))/e^3+1/4*f*(b*f^2+2*e^2*x)*(a+b*x+e^2*x^2/f^2)^(1/2
)/e^2
```

Rubi [A] time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {612, 621, 206}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*(b*f^2 + 2*e^2*x)*Sqrt[a + b*x + (e^2*x^2)/f^2])/(4*e^
2) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(b*f^2 + 2*e^2*x)/(2*e*f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])])/(8*e^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)dx = dx+\frac{ex^2}{2}+f\int \sqrt{a+bx+\frac{e^2x^2}{f^2}}\,dx$$

$$= dx+\frac{ex^2}{2}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{1}{8}\left(f\left(4a-\frac{b^2f^2}{e^2}\right)\right)\int \frac{}{\sqrt{a}}$$

$$= dx+\frac{ex^2}{2}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{1}{4}\left(f\left(4a-\frac{b^2f^2}{e^2}\right)\right)\text{Subst}$$

$$= dx+\frac{ex^2}{2}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\right.}{8e^3}$$

Mathematica [**A**] time = 0.21, size = 120, normalized size = 1.02

$$\frac{1}{8}\left(\frac{\left(4ae^2f^2-b^2f^4\right)\log\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}{e^3}+4fx\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+\frac{2bf^3\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right.}}{e^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] (8*d*x + 4*e*x^2 + (2*b*f^3*Sqrt[a + x*(b + (e^2*x)/f^2)])/e^2 + 4*f*x*Sqrt
[a + x*(b + (e^2*x)/f^2)] + ((4*a*e^2*f^2 - b^2*f^4)*Log[b*f^2 + 2*e*(e*x +
 f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/e^3)/8
```

fricas [**A**] time = 0.57, size = 123, normalized size = 1.04

$$\frac{4\,e^4x^2+8\,de^3x+\left(b^2f^4-4\,ae^2f^2\right)\log\left(-bf^2-2\,e^2x+2\,ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right)+2\left(bef^3+2\,e^3fx\right)\sqrt{\frac{bf^2x+e^2x^2+af}{f^2}}}{8\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*e^4*x^2 + 8*d*e^3*x + (b^2*f^4 - 4*a*e^2*f^2)*log(-b*f^2 - 2*e^2*x +
 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(b*e*f^3 + 2*e^3*f*x)*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^3
```

giac [**A**] time = 0.22, size = 111, normalized size = 0.94

$$\frac{1}{2}\,x^2e+dx+\frac{\left(\left(b^2f^4-4\,af^2e^2\right)e^{(-3)}\log\left(\left|-bf^2-2\left(xe-\sqrt{bf^2x+af^2+x^2e^2}\right)e\right|\right)+2\,\sqrt{bf^2x+af^2+x^2e^2}\left(bf^2e^{(}\right.}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*e + d*x + 1/8*((b^2*f^4 - 4*a*f^2*e^2)*e^(-3)*log(abs(-b*f^2 - 2*(x
*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e)) + 2*sqrt(b*f^2*x + a*f^2 + x^2*e^
2)*(b*f^2*e^(-2) + 2*x))*abs(f)/f
```

maple [A] time = 0.01, size = 173, normalized size = 1.47

$$-\frac{b^{2} f^{3} \ln\left(\frac{\frac{b}{2}+\frac{e^{2} x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{b x+\frac{e^{2} x^{2}}{f^{2}}+a}\right)}{8 \sqrt{\frac{e^{2}}{f^{2}}}\, e^{2}}+\frac{a f \ln\left(\frac{\frac{b}{2}+\frac{e^{2} x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{b x+\frac{e^{2} x^{2}}{f^{2}}+a}\right)}{2 \sqrt{\frac{e^{2}}{f^{2}}}}+\frac{e x^{2}}{2}+\frac{\sqrt{b x+\frac{e^{2} x^{2}}{f^{2}}+a}\, b f^{3}}{4 e^{2}}+d x+\frac{\sqrt{b x+\frac{e}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f,x)
```

```
[Out] d*x+1/2*e*x^2+1/2*f*(b*x+e^2/f^2*x^2+a)^(1/2)*x+1/4/e^2*f^3*(b*x+e^2/f^2*x^
2+a)^(1/2)*b+1/2*f*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)
^(1/2))/(e^2/f^2)^(1/2)*a-1/8/e^2*f^3*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+
(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume?`
for more details)Is b^2*f^2-4*a*e^2    positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2),x)
```

```
[Out] int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^{2} x^{2}}{f^{2}}}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] Integral(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2), x)
```

3.476 $\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=215

$$-\frac{f^2\left(4ae^2-b^2f^2\right)}{2e\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right.}{2e\left(2de-bf^2\right)^2}$$

```
[Out] 2*(a*e*f^2-b*d*f^2+d^2*e)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2*d
*e)^2-1/2*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2
)^(1/2)))/e/(-b*f^2+2*d*e)^2-1/2*f^2*(-b^2*f^2+4*a*e^2)/e/(-b*f^2+2*d*e)/(b
*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$-\frac{f^2\left(4ae^2-b^2f^2\right)}{2e\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right.}{2e\left(2de-bf^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -(f^2*(4*a*e^2 - b^2*f^2))/(2*e*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqrt[
a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d + e*
x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^2 - (f^2*(4*a*e^2 - b
^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*e*
(2*d*e - b*f^2)^2)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a+bx\right.$$

$$= 2\,\text{Subst}\left(\int \left(\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2x}+\frac{4ae^2f^2-b^2f^4}{2\left(2de-bf^2\right)\left(2de-bf^2-2ex\right)^2}+\frac{}{2\left(2de\right.}\right.\right.$$

$$= -\frac{f^2\left(4ae^2-b^2f^2\right)}{2e\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}+\frac{2\left(d^2e-bdf^2+aef^2\right)}{\left(\right.}$$

Mathematica [A] time = 0.20, size = 187, normalized size = 0.87

$$\frac{-\frac{f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)}{e\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}+\frac{f^2\left(b^2f^2-4ae^2\right)\log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)-bf^2\right)}{e}+4\left(aef^2-bdf^2+d^2e\right)\log\left(f\sqrt{a+x\left(b+\right.}\right.}{2\left(bf^2-2de\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] (-((f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(e*(b*f^2 + 2*e*(e*x + f*Sqr
t[a + x*(b + (e^2*x)/f^2)])))) + 4*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d + e*x
+ f*Sqrt[a + x*(b + (e^2*x)/f^2)]] + (f^2*(-4*a*e^2 + b^2*f^2)*Log[-(b*f^2)
 - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/e)/(2*(-2*d*e + b*f^2)^2)
```

fricas [A] time = 4.68, size = 371, normalized size = 1.73

$$-\frac{2\left(be^2f^2-2\,de^3\right)x-2\left(d^2e^2-\left(bde-ae^2\right)f^2\right)\log\left((bd-2\,ae)f^2-\left(bef^2-2\,de^2\right)x+\left(bf^3-2\,def\right)\sqrt{\frac{bf^2x+e^2x^2+af}{f^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*e^2*f^2 - 2*d*e^3)*x - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*log((b*
d - 2*a*e)*f^2 - (b*e*f^2 - 2*d*e^2)*x + (b*f^3 - 2*d*e*f)*sqrt((b*f^2*x +
e^2*x^2 + a*f^2)/f^2)) - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*log(a*f^2 - d^2
+ (b*f^2 - 2*d*e)*x) + (b^2*f^4 + 2*d^2*e^2 - 2*(b*d*e + a*e^2)*f^2)*log(-b
*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(d^2*e^2
- (b*d*e - a*e^2)*f^2)*log(-e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) -
 d) - 2*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(b^2*e
*f^4 - 4*b*d*e^2*f^2 + 4*d^2*e^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\left[undef, +\infty, 1\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, +Infinity, 1]
```

maple [B] time = 0.06, size = 4918, normalized size = 22.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f),x)
```

```
[Out] f/(b*f^2-2*d*e)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*
f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+
(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^
2)/f^2/(b*f^2-2*d*e)^2)^(1/2)+1/2*f^3/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+2*
a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2/f^2*(x+(a*f^2-d^2)/(
b*f^2-2*d*e)))/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b
^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/
(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d
^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(e^2/f^2)^(1/2)*b^2-f/(b*f^2-
2*d*e)^2*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2
*d*e)+e^2/f^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2
-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2
/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2
*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))
/(e^2/f^2)^(1/2)*a*e^2-f/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b
*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2/f^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))
)/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^
2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e)
)+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*
e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(e^2/f^2)^(1/2)*b*d*e+1/f/(b*f^2-2*d*e)^2*
ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2
/f^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))/(e^2/f^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*
f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2
*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+
2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(e^2/f^2
)^(1/2)*d^2*e^2-f^3/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4
+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a
^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)
/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^
2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^
2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^
2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d
^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*
d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e
)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*a^2*e^2+2*f^3/(b*f^2-2*d*e)^3/((
a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2
)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2
*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2
*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))
+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^
4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-
(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^
2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*
b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*
e)))*a*b*d*e-f^3/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*
a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*
e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^
2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2
*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f
^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(
x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*
```

```
e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e
*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2
)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*b^2*d^2-2*f/(b*f^2-2*d*e)^3/((a^2*e
^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2
/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^
2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+
2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((
a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2
)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2
*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b
*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3
*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*
a*d^2*e^2+2*f/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d
^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2
*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(
b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*
e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+
2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(
a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2
)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^
4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(
1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*b*d^3*e-1/f/(b*f^2-2*d*e)^3/((a^2*e^2*
f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b
*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e
^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b
*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2
*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f
^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^
4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^
2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*
f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*d^4
*e^2-d*ln((b*f^2-2*d*e)*x+a*f^2-d^2)/(b*f^2-2*d*e)-e/(b*f^2-2*d*e)*x+e/(b*f
^2-2*d*e)^2*ln(b*f^2*x+a*f^2-2*d*e*x-d^2)*a*f^2-e/(b*f^2-2*d*e)^2*ln(b*f^2*
x+a*f^2-2*d*e*x-d^2)*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)),x)
```

```
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] Integral(1/(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

3.477 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

Optimal. Leaf size=266

$$\frac{2f^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^3}-\frac{f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{2f^2\left(4ae^2\right.}{}$$

```
[Out] 2*f^2*(-b^2*f^2+4*a*e^2)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2*d*
e)^3-2*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(
1/2)))/(-b*f^2+2*d*e)^3-2*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*x+f
*(a+b*x+e^2*x^2/f^2)^(1/2))-f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e)^2/(b*f^2+
2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] time = 0.23, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2116, 893}

$$\frac{2f^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^3}-\frac{f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{2f^2\left(4ae^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (-2*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b
*x + (e^2*x^2)/f^2])) - (f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^2*(b*f^2
 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*f^2*(4*a*e^2 - b^
2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^3 -
(2*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^
2*x))/f^2])])/(2*d*e - b*f^2)^3
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^2\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{\right.$$

$$= 2\,\text{Subst}\left(\int\left(\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2x^2}+\frac{4ae^2f^2-b^2f^4}{\left(2de-bf^2\right)^3x}+\frac{4ae^3f^2-b}{\left(2de-bf^2\right)^2\left(2de-\right.}\right.\right.$$

$$= -\frac{2\left(d^2e-bdf^2+aef^2\right)}{\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}-\frac{f^2\left(4ae^2\right.}{\left(2de-bf^2\right)^2\left(bf^2+2e\left(\right.\right.}$$

Mathematica [A] time = 0.32, size = 237, normalized size = 0.89

$$-\frac{\frac{f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+2f^2\left(b^2f^2-4ae^2\right)\log\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)-2f^2\left(b^2f^2-4ae^2\right)\log\left(-2e\right.}{\left(2de-bf^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -(((2*(2*d*e - b*f^2)*(d^2*e - b*d*f^2 + a*e*f^2))/(d + e*x + f*Sqrt[a + x*
(b + (e^2*x)/f^2)]) + (f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(b*f^2 +
2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 2*f^2*(-4*a*e^2 + b^2*f^2)*L
og[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] - 2*f^2*(-4*a*e^2 + b^2*f^2)*
Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/(2*d*e - b*f^2
)^3)
```

fricas [B] time = 2.56, size = 826, normalized size = 3.11

$$-\frac{ab^2f^6+\left(3\,b^2d^2-14\,abde+8\,a^2e^2\right)f^4-2\left(bd^3e-4\,ad^2e^2\right)f^2-4\left(b^2e^2f^4-4\,bde^3f^2+4\,d^2e^4\right)x^2+\left(b^3f^6-8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*f^6 + (3*b^2*d^2 - 14*a*b*d*e + 8*a^2*e^2)*f^4 - 2*(b*d^3*e - 4
*a*d^2*e^2)*f^2 - 4*(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)*x^2 + (b^3*f^
6 - 8*b^2*d*e*f^4 + 20*b*d^2*e^2*f^2 - 16*d^3*e^3)*x - 2*(a*b^2*f^6 + 4*a*d
^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*
d*e + 2*a*b*e^2)*f^4)*x)*log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*
e^3*f^2 - 2*d*e^4)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2
*f^5 - 4*(b*d*e - a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^2*x +
 e^2*x^2 + a*f^2)/f^2)) - 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2
*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*log(
a*f^2 - d^2 + (b*f^2 - 2*d*e)*x) + 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^
2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4
)*x)*log(-e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) - 4*((b^2*d -
2*a*b*e)*f^5 - 2*(b*d^2*e - 2*a*d*e^2)*f^3 - (b^2*e*f^5 - 4*b*d*e^2*f^3 + 4
*d^2*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(a*b^3*f^8 + 8*d^5*e^
```

```
3 - (b^3*d^2 + 6*a*b^2*d*e)*f^6 + 6*(b^2*d^3*e + 2*a*b*d^2*e^2)*f^4 - 4*(3*
b*d^4*e^2 + 2*a*d^3*e^3)*f^2 + (b^4*f^8 - 8*b^3*d*e*f^6 + 24*b^2*d^2*e^2*f^
4 - 32*b*d^3*e^3*f^2 + 16*d^4*e^4)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.05, size = 58067, normalized size = 218.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-2), x)
```

3.478 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

Optimal. Leaf size=330

$$-\frac{2f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}+\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^4}-\frac{}{\left(2de-bf^2\right.}$$

```
[Out] 6*e*f^2*(-b^2*f^2+4*a*e^2)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2*
d*e)^4-6*e*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^
2)^(1/2)))/(-b*f^2+2*d*e)^4+(-a*e*f^2+b*d*f^2-d^2*e)/(-b*f^2+2*d*e)^2/(d+e*
x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2-2*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e)^3/
(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))-2*e*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*
e)^3/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] time = 0.29, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2116, 893}

$$-\frac{2f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}+\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^4}-\frac{}{\left(2de-bf^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -((d^2*e - b*d*f^2 + a*e*f^2)/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b*x
+ (e^2*x^2)/f^2])^2)) - (2*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*(d +
 e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])) - (2*e*f^2*(4*a*e^2 - b^2*f^2))/((
2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])))
 + (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]])/(2*d*e - b*f^2)^4 - (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x
+ f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*d*e - b*f^2)^4
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}dx = 2\,\mathrm{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^3\left(-2de+bf^2+2ex\right)^2}dx, x, d+ex+f\sqrt{a+\right.$$

$$= 2\,\mathrm{Subst}\left(\int \left(\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2x^3}+\frac{4ae^2f^2-b^2f^4}{\left(2de-bf^2\right)^3x^2}+\frac{3\left(4ae^3f^2-b^2ef^4\right)}{\left(2de-bf^2\right)^4x}+\right.\right.$$

$$= -\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}-\frac{2f^2\left(4ae^2-b^2\right.}{\left(2de-bf^2\right)^3\left(d+ex+f\sqrt{\right.}}$$

Mathematica [A] time = 0.69, size = 300, normalized size = 0.91

$$-\frac{\frac{2f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)}{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}+\frac{2ef^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}-6ef^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)+6ef^2\left(\right.}{\left(bf^2-2de\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -((((-2*d*e + b*f^2)^2*(d^2*e - b*d*f^2 + a*e*f^2))/(d + e*x + f*Sqrt[a + x
*(b + (e^2*x)/f^2)])^2 + (2*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(d +
 e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + (2*e*f^2*(2*d*e - b*f^2)*(4*a*e^2
 - b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 6*e*f^
2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] + 6*e*
f^2*(4*a*e^2 - b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)
/f^2)])])/(-2*d*e + b*f^2)^4)
```

fricas [B] time = 19.38, size = 1954, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] ((3*a*b^3*d - 4*a^2*b^2*e)*f^8 - (b^3*d^3 + 4*a*b^2*d^2*e + 10*a^2*b*d*e^2
- 20*a^3*e^3)*f^6 - 4*(b^2*d^4*e - 8*a*b*d^3*e^2 + 6*a^2*d^2*e^3)*f^4 - 4*(
b^3*e^3*f^6 - 6*b^2*d*e^4*f^4 + 12*b*d^2*e^5*f^2 - 8*d^3*e^6)*x^3 + 2*(b*d^
5*e^2 - 6*a*d^4*e^3)*f^2 - (b^4*e*f^8 - 2*a*b^2*e^3*f^6 - 40*d^4*e^5 - 2*(1
1*b^2*d^2*e^3 - 4*a*b*d*e^4)*f^4 + 8*(7*b*d^3*e^4 - a*d^2*e^5)*f^2)*x^2 + (
16*d^5*e^4 + (3*b^4*d - 5*a*b^3*e)*f^8 - (7*b^3*d^2*e + 10*a*b^2*d*e^2 - 28
*a^2*b*e^3)*f^6 + 2*(5*b^2*d^3*e^2 + 22*a*b*d^2*e^3 - 28*a^2*d*e^4)*f^4 - 8
*(3*b*d^4*e^3 + a*d^3*e^4)*f^2)*x - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*
(a*b^2*d^2*e + 2*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^
8 - 16*a*d^2*e^5*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a
*b*d*e^4)*f^4)*x^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^
2*d*e^2 + 4*a^2*b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*
f^4)*x)*log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*e^3*f^2 - 2*d*e^4
)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2*f^5 - 4*(b*d*e -
 a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)
/f^2)) - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2*a^3*e^3)*f
```

```
^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5*f^2 - 4*(b
^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x^2 + 2*(a*b
^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*b*e^3)*f^6
+ 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(a*f^2 - d^2 + (
b*f^2 - 2*d*e)*x) + 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2
*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5
*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x
^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*
b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(-e*x
 + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) - 2*(a*b^3*f^9 - 3*(a*b^2*d
*e + 2*a^2*b*e^2)*f^7 - 3*(b^2*d^3*e - 4*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^5 + 2
*(3*b*d^4*e^2 - 10*a*d^3*e^3)*f^3 - 2*(b^3*e^2*f^7 - 6*b^2*d*e^3*f^5 + 12*b
*d^2*e^4*f^3 - 8*d^3*e^5*f)*x^2 + (b^4*f^9 + 12*d^4*e^4*f - 3*(b^3*d*e + 3*
a*b^2*e^2)*f^7 + 3*(b^2*d^2*e^2 + 12*a*b*d*e^3)*f^5 - 4*(2*b*d^3*e^3 + 9*a*
d^2*e^4)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(a^2*b^4*f^12 + 16*
d^8*e^4 - 2*(a*b^4*d^2 + 4*a^2*b^3*d*e)*f^10 + (b^4*d^4 + 16*a*b^3*d^3*e +
24*a^2*b^2*d^2*e^2)*f^8 - 8*(b^3*d^5*e + 6*a*b^2*d^4*e^2 + 4*a^2*b*d^3*e^3)
*f^6 + 8*(3*b^2*d^6*e^2 + 8*a*b*d^5*e^3 + 2*a^2*d^4*e^4)*f^4 - 32*(b*d^7*e^
3 + a*d^6*e^4)*f^2 + (b^6*f^12 - 12*b^5*d*e*f^10 + 60*b^4*d^2*e^2*f^8 - 160
*b^3*d^3*e^3*f^6 + 240*b^2*d^4*e^4*f^4 - 192*b*d^5*e^5*f^2 + 64*d^6*e^6)*x^
2 + 2*(a*b^5*f^12 + 32*d^7*e^5 - (b^5*d^2 + 10*a*b^4*d*e)*f^10 + 10*(b^4*d^
3*e + 4*a*b^3*d^2*e^2)*f^8 - 40*(b^3*d^4*e^2 + 2*a*b^2*d^3*e^3)*f^6 + 80*(b
^2*d^5*e^3 + a*b*d^4*e^4)*f^4 - 16*(5*b*d^6*e^4 + 2*a*d^5*e^5)*f^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.16, size = 295147, normalized size = 894.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^3,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3), x)
```

3.479 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

Optimal. Leaf size=370

$$-\frac{5f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{16\sqrt{2}\,e^{9/2}}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+}}}{4e^4}$$

```
[Out] -5/32*f^2*(-b*f^2+2*d*e)^(3/2)*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(
d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(9/2)*2^(1
/2)+1/12*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2)/e
^3+1/7*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(7/2)/e+1/4*f^2*(-b*f^2+2*d*e)*(
-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^4-1/16*f^2*(-
b*f^2+2*d*e)^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)
/e^4/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] time = 0.60, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1257, 1810, 208}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{12e^3}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^4}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e
^2*x^2)/f^2]])/(4*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])^(3/2))/(12*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2])^(7/2)/(7*e) - (f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x
 + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a +
 (x*(b*f^2 + e^2*x))/f^2]))) - (5*f^2*(2*d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*
f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2
]])/Sqrt[2*d*e - b*f^2]])/(16*Sqrt[2]*e^(9/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}dx = 2\,\text{Subst}\left(\int \frac{x^{5/2}\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}dx,x,d+e\right.$$

$$= 4\,\text{Subst}\left(\int \frac{x^6\left(d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4\right)}{\left(-2de+bf^2+2ex^2\right)^2}dx,x,\sqrt{d}\right.$$

$$= -\frac{f^2\left(2de-bf^2\right)^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \text{Subs}$$

$$= -\frac{f^2\left(2de-bf^2\right)^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \text{Subs}$$

$$= \frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4} + \frac{f^2\left(4ae\right.}{}$$

$$= \frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4} + \frac{f^2\left(4ae\right.}{}$$

Mathematica [A] time = 1.06, size = 357, normalized size = 0.96

$$\frac{\frac{4}{3}e^2f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^{3/2}+4ef^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (4*e*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + x*(b
 + (e^2*x)/f^2)]] + (4*e^2*f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + x*
(b + (e^2*x)/f^2)])^(3/2))/3 + (16*e^4*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)
/f^2)])^(7/2))/7 - ((-2*d*e + b*f^2)^2*(4*a*e^3*f^2 - b^2*e*f^4)*Sqrt[d + e
*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b
+ (e^2*x)/f^2)])) - (5*Sqrt[e]*f^2*(2*d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*f^2
)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])
/Sqrt[2*d*e - b*f^2]])/Sqrt[2])/(16*e^5)
```

fricas [A] time = 0.70, size = 923, normalized size = 2.49

$$\left[\frac{105\sqrt{\frac{1}{2}}\left(b^3f^6+8\,ade^3f^2-2\left(b^2de+2\,abe^2\right)f^4\right)\sqrt{-\frac{bf^2-2\,de}{e}}\,\log\left(-b^2f^4+4\left(bde-ae^2\right)f^2-4\left(be^2f^2-2\,de^3\right)x+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*sqrt(1/2)*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^
4)*sqrt(-(b*f^2 - 2*d*e)/e)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2
*f^2 - 2*d*e^3)*x + 4*(2*sqrt(1/2)*e^2*f*sqrt(-(b*f^2 - 2*d*e)/e)*sqrt((b*f
^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(1/2)*(b*e*f^2 + 2*e^3*x)*sqrt(-(b*f^2 -
 2*d*e)/e))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*
e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(105*b^3*f^6
+ 192*e^6*x^3 + 48*d^3*e^3 - 56*(5*b^2*d*e + 6*a*b*e^2)*f^4 + 4*(21*b*d^2*e
^2 + 232*a*d*e^3)*f^2 + 144*(b*e^4*f^2 + 2*d*e^5)*x^2 + 2*(7*b^2*e^2*f^4 +
156*d^2*e^4 - 4*(3*b*d*e^3 - 32*a*e^4)*f^2)*x - 2*(35*b^2*e*f^5 - 96*e^5*f*
x^2 + 12*d^2*e^3*f - 4*(21*b*d*e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^3 + 6*d*e^
4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x +
 e^2*x^2 + a*f^2)/f^2) + d))/e^4, -1/336*(105*sqrt(1/2)*(b^3*f^6 + 8*a*d*e^
3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*sqrt((b*f^2 - 2*d*e)/e)*arctan(2*sqrt(
1/2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*e*sqrt((b*f^2
- 2*d*e)/e)/(b*f^2 - 2*d*e)) - (105*b^3*f^6 + 192*e^6*x^3 + 48*d^3*e^3 - 56
*(5*b^2*d*e + 6*a*b*e^2)*f^4 + 4*(21*b*d^2*e^2 + 232*a*d*e^3)*f^2 + 144*(b*
e^4*f^2 + 2*d*e^5)*x^2 + 2*(7*b^2*e^2*f^4 + 156*d^2*e^4 - 4*(3*b*d*e^3 - 32
*a*e^4)*f^2)*x - 2*(35*b^2*e*f^5 - 96*e^5*f*x^2 + 12*d^2*e^3*f - 4*(21*b*d*
e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^3 + 6*d*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^
4]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(ex+d+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

```
[Out] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(5/2), x)
```

3.480 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

Optimal. Leaf size=302

$$-\frac{3f^2\left(4ae^2-b^2f^2\right)\sqrt{2de-bf^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{8\sqrt{2}e^{7/2}}+\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+e}}{4e^3}$$

```
[Out] -3/16*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^
2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))*(-b*f^2+2*d*e)^(1/2)/e^(7/2)*2^(1
/2)+1/5*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2)/e+1/4*f^2*(-b^2*f^2+4*a*e
^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^3-1/8*f^2*(-b*f^2+2*d*e)*(-
b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^3/(b*f^2+2*e*(
e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] time = 0.41, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2116, 897, 1257, 1810, 208}

$$\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^3}-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{8e^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}-\frac{3f^2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(
4*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2)/(5*e) - (f^2*(2*
d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/(8*e^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) -
(3*f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqr
t[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(8*Sqrt
[2]*e^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = 2\,\text{Subst}\left(\int \frac{x^{3/2}\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2} dx, x, d+e$$

$$= 4\,\text{Subst}\left(\int \frac{x^4\left(d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4\right)}{\left(-2de+bf^2+2ex^2\right)^2} dx, x, \sqrt{d$$

$$= -\frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}}{}$$

$$= -\frac{f^2\left(2de-bf^2\right)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}}{}$$

$$= \frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3} + \frac{\left(d+ex+f\sqrt{a+b\right.}}{5e}$$

$$= \frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3} + \frac{\left(d+ex+f\sqrt{a+b\right.}}{5e}$$

Mathematica [A] time = 0.57, size = 291, normalized size = 0.96

$$\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}-\frac{3\sqrt{e}f^2\left(4ae^2-b^2f^2\right)\sqrt{2de-bf^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}}-\frac{\left(4ae^3f\right.}{}}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (2*e*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]
] + (8*e^3*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(5/2))/5 - ((2*d*e -
 b*f^2)*(4*a*e^3*f^2 - b^2*e*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/
f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - (3*Sqrt[e]*
f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d
 + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e - b*f^2]])/Sqrt[2])/(
8*e^4)
```

fricas [A] time = 0.69, size = 657, normalized size = 2.18

$$\left[-\frac{15\sqrt{\frac{1}{2}}\left(b^2f^4-4\,ae^2f^2\right)\sqrt{-\frac{bf^2-2\,de}{e}}\log\left(-b^2f^4+4\left(bde-ae^2\right)f^2-4\left(be^2f^2-2\,de^3\right)x+4\left(2\sqrt{\frac{1}{2}}e^2f\sqrt{-\frac{bf^2-2\,de}{e}}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/80*(15*sqrt(1/2)*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-(b*f^2 - 2*d*e)/e)*log(-
b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 4*(2*sqrt(1/2
)*e^2*f*sqrt(-(b*f^2 - 2*d*e)/e)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sq
rt(1/2)*(b*e*f^2 + 2*e^3*x)*sqrt(-(b*f^2 - 2*d*e)/e))*sqrt(e*x + f*sqrt((b*
f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x
+ e^2*x^2 + a*f^2)/f^2)) + 2*(15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*
d*e + 24*a*e^2)*f^2 + 2*(b*e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x
 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f
^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^3, 1/40*(15*sqrt(1/2)*(b^2*f^4 - 4*a*e
^2*f^2)*sqrt((b*f^2 - 2*d*e)/e)*arctan(2*sqrt(1/2)*sqrt(e*x + f*sqrt((b*f^2
*x + e^2*x^2 + a*f^2)/f^2) + d)*e*sqrt((b*f^2 - 2*d*e)/e)/(b*f^2 - 2*d*e))
- (15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*d*e + 24*a*e^2)*f^2 + 2*(b*
e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x - 2*d*e^2*f)*sqrt((b*f^2*x
 + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2
) + d))/e^3]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(ex+d+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\; f\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
[Out] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\, f+d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(3/2), x)
```

3.481 $\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=233

$$-\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}e^{5/2}\sqrt{2de-bf^2}}+\frac{\left(f\sqrt{a+bx\cdots\right.}{}$$

```
[Out] -1/8*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2
/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(5/2)*2^(1/2)/(-b*f^2+2*d*e)^(1/
2)+1/3*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2)/e-1/4*f^2*(4*a-b^2*f^2/e^2
)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e
^2*x)/f^2)^(1/2)))
```

Rubi [A] time = 0.30, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1257, 1153, 208}

$$-\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}e^{5/2}\sqrt{2de-bf^2}}+\frac{\left(f\sqrt{a+bx\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)/(3*e) - (f^2*(4*a - (b^2*
f^2)/e^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*(b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (f^2*(4*a*e^2 - b^2*f^2)*Ar
cTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqr
t[2*d*e - b*f^2]])/(4*Sqrt[2]*e^(5/2)*Sqrt[2*d*e - b*f^2])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{x}\,\left(d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2\right)}{\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex\right.$$

$$= 4\,\text{Subst}\left(\int \frac{x^2\left(d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4\right)}{\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d+\right.$$

$$= -\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \frac{-ef^2\left(4ae^2-b^2f^2\right.}{-2d}\right.}{}$$

$$= -\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \left(-4e^2x^2-\frac{ef}{-2}\right.\right.}{}$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right.}{f^2}}\right.\right.}$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right.}{f^2}}\right.\right.}$$

Mathematica [A] time = 0.39, size = 223, normalized size = 0.96

$$\frac{-\frac{\sqrt{e}f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{4de-2bf^2}}+\frac{(b^2ef^4-4ae^3f^2)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+\frac{4}{3}e^2\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+\right.}{4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] ((4*e^2*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(3/2))/3 + ((-4*a*e^3*f
^2 + b^2*e*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(b*f^2 + 2
*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - (Sqrt[e]*f^2*(4*a*e^2 - b^2*f
^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]
])/Sqrt[2*d*e - b*f^2]])/Sqrt[4*d*e - 2*b*f^2])/(4*e^3)
```

fricas [A] time = 0.71, size = 692, normalized size = 2.97

$$\left[-\frac{3\left(b^2f^4-4ae^2f^2\right)\sqrt{-2bef^2+4de^2}\log\left(-b^2f^4+4\left(bde-ae^2\right)f^2-4\left(be^2f^2-2de^3\right)x-2\left(2\sqrt{-2bef^2+4de^2}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4 +
 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sqrt(-2*b*e*f^2 +
 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 + 4*d
*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)
 + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 4*
(3*b^2*e*f^4 - 2*b*d*e^2*f^2 - 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x - 2*(
b*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*
sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4), 1/24*(3*
(b^2*f^4 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(e*x + f*s
qrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e^2)*f*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*x + d))/(a
*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(3*b^2*e*f^4 - 2*b*d*e^2*f^2 -
 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x - 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b
*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^
2)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{ex + d + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

```
[Out] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d + e\, x + f\, \sqrt{a + b\, x + \frac{e^2\, x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
```

```
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

3.482 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$

Optimal. Leaf size=244

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\frac{\sqrt{f\sqrt{\ldots}}}{\ldots}$$

```
[Out] 1/4*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2/
f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(3/2)/(-b*f^2+2*d*e)^(3/2)*2^(1/2
)+(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/2*f^2*(4*a*e-b^2*f^2/e)*(d+
e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)/(b*f^2+2*e*(e*x+f*(a+
x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] time = 0.29, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1157, 388, 208}

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\frac{\sqrt{f\sqrt{\ldots}}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/e - (f^2*(4*a*e - (b^2*f^2)
/e)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*(2*d*e - b*f^2)*(b*
f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (f^2*(4*a*e^2 - b
^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/Sqrt[2*d*e - b*f^2]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{\sqrt{x}\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a\right.$$

$$= 4\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4}{\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d+ex+\right.$$

$$= -\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}+\frac{2\,\text{Subst}\left(\int \frac{\frac{1}{4}\left(-8d^2\right.}{}\right.}{}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e}-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x}{}}\right.\right.}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e}-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x}{}}\right.\right.}$$

Mathematica [A] time = 0.57, size = 238, normalized size = 0.98

$$\frac{f^2\left(b^2f^2-4ae^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\sqrt{f\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]/e + (f^2*(-4*a*e^2 + b^2*f^
2)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(2*e*(2*d*e - b*f^2)*(b
*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) + (f^2*(4*a*e^2 - b^2*
f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)
]])/Sqrt[2*d*e - b*f^2]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

fricas [A] time = 0.71, size = 716, normalized size = 2.93

$$\left[\frac{\left(b^2f^4-4\,ae^2f^2\right)\sqrt{-2\,bef^2+4\,de^2}\,\log\left(-b^2f^4+4\left(bde-ae^2\right)f^2-4\left(be^2f^2-2\,de^3\right)x+2\left(2\sqrt{-2\,bef^2+4\,de^2}\,ef\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/8*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4 + 4*(
b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 2*(2*sqrt(-2*b*e*f^2 + 4*d
*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 + 4*d*e^2
)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d
) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 4*(b^2
*e*f^4 - 6*b*d*e^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f
^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b
*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e
^4), 1/4*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt
(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e
^2)*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*
x + d))/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(b^2*e*f^4 - 6*b*d*e
^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f^3 - 2*d*e^3*f)*
sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex + d + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\ f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
[Out] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

3.483 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\left(2de-bf^2\right)^{5/2}}-\frac{\ \ }{\left(2de-\right.}$$

```
[Out] 3/2*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2/
f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/(-b*f^2+2*d*e)^(5/2)*2^(1/2)/e^(1/2
)-4*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(
1/2))^(1/2)-f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2
)/(-b*f^2+2*d*e)^2/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] time = 0.37, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1259, 453, 208}

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\left(2de-bf^2\right)^{5/2}}-\frac{\ \ }{\left(2de-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*Sqrt[d + e*x + f*Sqrt[a
 + b*x + (e^2*x^2)/f^2]]) - (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[
a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a
+ (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]
*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^
2]])/(Sqrt[2]*Sqrt[e]*(2*d*e - b*f^2)^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^{3/2}\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{\ldots}\right.$$

$$= 4\,\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4}{x^2\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d+ex\ldots}\right.$$

$$= -\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\left(2de-bf^2\right)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \frac{8e^2\left(2de\ldots\right.}{\ldots}\right.}{\ldots}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{\left(2de-bf^2\right)^2\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d\ldots}}{\left(2de-bf^2\right)^2\left(bf^2+2e\ldots\right.}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{\left(2de-bf^2\right)^2\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{d\ldots}}{\left(2de-bf^2\right)^2\left(bf^2+2e\ldots\right.}$$

Mathematica [A] time = 0.49, size = 257, normalized size = 0.96

$$\frac{-\frac{2e^2\left(4ae^2f^2-b^2f^4\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}+\frac{3e^{3/2}f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{de-\frac{bf^2}{2}}}-\frac{8e^2\left(aef^2-bdf^2+d^2e\right)}{\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}}{2e^2\left(bf^2-2de\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] ((-8*e^2*(d^2*e - b*d*f^2 + a*e*f^2))/Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2
*x)/f^2)]] - (2*e^2*(4*a*e^2*f^2 - b^2*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b
+ (e^2*x)/f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + (
3*e^(3/2)*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f
*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e - b*f^2]])/Sqrt[d*e - (b*f^2)/2
])/(2*e^2*(-2*d*e + b*f^2)^2)
```

fricas [B] time = 0.86, size = 1456, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*(3*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6
 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*sqrt(-2*b*e*f^2 + 4*d*e^
2)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*
sqrt(-2*b*e*f^2 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt
(-2*b*e*f^2 + 4*d*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*
x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 +
a*f^2)/f^2)) + 4*(a*b^2*e*f^6 - 8*d^4*e^3 - (5*b^2*d^2*e - 2*a*b*d*e^2)*f^4
 + 2*(7*b*d^3*e^2 - 4*a*d^2*e^3)*f^2 + 2*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d
^2*e^5)*x^2 + (b^3*e*f^6 - 4*d^3*e^4 - 2*(4*b^2*d*e^2 - 3*a*b*e^3)*f^4 + 2*
(7*b*d^2*e^3 - 6*a*d*e^4)*f^2)*x + 2*(2*d^3*e^3*f + (2*b^2*d*e - 3*a*b*e^2)
*f^5 - (5*b*d^2*e^2 - 6*a*d*e^3)*f^3 - (b^2*e^2*f^5 - 4*b*d*e^3*f^3 + 4*d^2
*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*
x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*e*f^8 + 8*d^5*e^4 - (b^3*d^2*e + 6*a
*b^2*d*e^2)*f^6 + 6*(b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(3*b*d^4*e^3 + 2*
a*d^3*e^4)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e^3*f^4 - 32*b*d
^3*e^4*f^2 + 16*d^4*e^5)*x), -1/2*(3*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^
2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4
)*x)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(e*x + f*sqrt((b*f^2*x + e^2*
x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e^2)*f*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*x + d))/(a*e*f^2 - d^2*e + (b
*e*f^2 - 2*d*e^2)*x)) - 2*(a*b^2*e*f^6 - 8*d^4*e^3 - (5*b^2*d^2*e - 2*a*b*d
*e^2)*f^4 + 2*(7*b*d^3*e^2 - 4*a*d^2*e^3)*f^2 + 2*(b^2*e^3*f^4 - 4*b*d*e^4*
f^2 + 4*d^2*e^5)*x^2 + (b^3*e*f^6 - 4*d^3*e^4 - 2*(4*b^2*d*e^2 - 3*a*b*e^3)
*f^4 + 2*(7*b*d^2*e^3 - 6*a*d*e^4)*f^2)*x + 2*(2*d^3*e^3*f + (2*b^2*d*e - 3
*a*b*e^2)*f^5 - (5*b*d^2*e^2 - 6*a*d*e^3)*f^3 - (b^2*e^2*f^5 - 4*b*d*e^3*f^
3 + 4*d^2*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*e*f^8 + 8*d^5*e^4 - (b^3*d^
2*e + 6*a*b^2*d*e^2)*f^6 + 6*(b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(3*b*d^4
*e^3 + 2*a*d^3*e^4)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e^3*f^4
 - 32*b*d^3*e^4*f^2 + 16*d^4*e^5)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, cho
osing root of [1,0,%%%{-2,[1,0,2,0,0]%%%}+%%%{-2,[0,1,2,1,0]%%%}+%%%{-2,[0,
0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%%},0,%%%{1,[2,0,4,0,0]%%%}+%%%{2,[1,1,4,1,
0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%%%}+%
%%{-2,[0,1,3,1,0]%%%}+%%%{2,[0,1,2,3,1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{-2,[0
,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%%}] at parameters values [-49,-86,61.79374
78349,-30,70]Warning, choosing root of [1,0,%%%{-2,[1,0,2,0,0]%%%}+%%%{-2,[
0,1,2,1,0]%%%}+%%%{-2,[0,0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%%},0,%%%{1,[2,0,4
,0,0]%%%}+%%%{2,[1,1,4,1,0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%
}+%%%{1,[0,2,4,2,0]%%%}+%%%{-2,[0,1,3,1,0]%%%}+%%%{2,[0,1,2,3,1]%%%}+%%%{1,
[0,0,2,0,0]%%%}+%%%{-2,[0,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%%}] at parameters
 values [0,0,71.707969239,0,0]Warning, choosing root of [1,0,%%%{-2,[1,0,2,
0,0]%%%}+%%%{-2,[0,1,2,1,0]%%%}+%%%{-2,[0,0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%
%},0,%%%{1,[2,0,4,0,0]%%%}+%%%{2,[1,1,4,1,0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%
{2,[1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%%%}+%%%{-2,[0,1,3,1,0]%%%}+%%%{2,[0,1,
2,3,1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{-2,[0,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%
%}] at parameters values [-9,-13,47.3295757947,24,49]Warning, choosing root
 of [1,0,%%%{-2,[1,0,2,0,0]%%%}+%%%{-2,[0,1,2,1,0]%%%}+%%%{-2,[0,0,1,0,0]%%
%}+%%%{-2,[0,0,0,2,1]%%%},0,%%%{1,[2,0,4,0,0]%%%}+%%%{2,[1,1,4,1,0]%%%}+%%%
{-2,[1,0,3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%%%}+%%%{-2,[0,1
,3,1,0]%%%}+%%%{2,[0,1,2,3,1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{-2,[0,0,1,2,1]%
%%}+%%%{1,[0,0,0,4,2]%%%}] at parameters values [81,18,54.7579903365,-33,-7
0]Warning, choosing root of [1,0,%%%{-2,[1,0,2,0,0]%%%}+%%%{2,[0,1,2,1,0]%%
%}+%%%{-2,[0,0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%%},0,%%%{1,[2,0,4,0,0]%%%}+%%
%{-2,[1,1,4,1,0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%}+%%%{1,[0,
2,4,2,0]%%%}+%%%{2,[0,1,3,1,0]%%%}+%%%{-2,[0,1,2,3,1]%%%}+%%%{1,[0,0,2,0,0]
%%%}+%%%{-2,[0,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%%}] at parameters values [62
,-37,8.05231268331,-23,65]Warning, choosing root of [1,0,%%%{-2,[1,0,2,0,0]
%%%}+%%%{2,[0,1,2,1,0]%%%}+%%%{-2,[0,0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%%},0,
%%%{1,[2,0,4,0,0]%%%}+%%%{-2,[1,1,4,1,0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%{2,[
1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%%%}+%%%{2,[0,1,3,1,0]%%%}+%%%{-2,[0,1,2,3,
1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{-2,[0,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%%}]
at parameters values [0,0,64.3995612673,0,0]Warning, choosing root of [1,0,
%%%{-2,[1,0,2,0,0]%%%}+%%%{2,[0,1,2,1,0]%%%}+%%%{-2,[0,0,1,0,0]%%%}+%%%{-2,
[0,0,0,2,1]%%%},0,%%%{1,[2,0,4,0,0]%%%}+%%%{-2,[1,1,4,1,0]%%%}+%%%{-2,[1,0,
3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%%%}+%%%{2,[0,1,3,1,0]%%%
}+%%%{-2,[0,1,2,3,1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{-2,[0,0,1,2,1]%%%}+%%%{1
,[0,0,0,4,2]%%%}] at parameters values [-22,93,94.1262030317,31,-21]Warning
, choosing root of [1,0,%%%{-2,[1,0,2,0,0]%%%}+%%%{2,[0,1,2,1,0]%%%}+%%%{-2
,[0,0,1,0,0]%%%}+%%%{-2,[0,0,0,2,1]%%%},0,%%%{1,[2,0,4,0,0]%%%}+%%%{-2,[1,1
,4,1,0]%%%}+%%%{-2,[1,0,3,0,0]%%%}+%%%{2,[1,0,2,2,1]%%%}+%%%{1,[0,2,4,2,0]%
%%}+%%%{2,[0,1,3,1,0]%%%}+%%%{-2,[0,1,2,3,1]%%%}+%%%{1,[0,0,2,0,0]%%%}+%%%{
-2,[0,0,1,2,1]%%%}+%%%{1,[0,0,0,4,2]%%%}] at parameters values [-66,66,6.82
230772497,-23,79]Evaluation time: 1.2Done
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex+d+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\, f\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
```

```
[Out] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\, f + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3/2), x)
```

3.484 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

Optimal. Leaf size=335

$$-\frac{4f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{5\sqrt{2}\sqrt{e}f^2\left(4\right.}{}$$

```
[Out] 5*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2/f^
2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))*2^(1/2)*e^(1/2)/(-b*f^2+2*d*e)^(7/2)-
4/3*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(
1/2))^(3/2)-4*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e)^3/(d+e*x+f*(a+b*x+e^2*x
^2/f^2)^(1/2))^(1/2)-2*e*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2
)^(1/2))^(1/2)/(-b*f^2+2*d*e)^3/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(
1/2)))
```

Rubi [A] time = 0.50, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 897, 1259, 1261, 208}

$$-\frac{4f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{5\sqrt{2}\sqrt{e}f^2\left(4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a +
 b*x + (e^2*x^2)/f^2])^(3/2)) - (4*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2
)^3*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]) - (2*e*f^2*(4*a*e^2 -
b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^
3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (5*Sqrt[2]*S
qrt[e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sq
rt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(2*d*e - b*f^2)^(7/2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-\left(2de-bf^2\right)x+ex^2}{x^{5/2}\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{\ldots}\right.$$

$$= 4\,\mathrm{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+\left(-2de+bf^2\right)x^2+ex^4}{x^4\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d+ex\ldots}\right.$$

$$= -\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\left(2de-bf^2\right)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\mathrm{Subst}\left(\int \frac{8e^2(2de\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{2ef^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\left(2de-bf^2\right)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)} - \frac{\mathrm{Subst}\left(\int\left(-\frac{8e^2(\ldots}{\ldots}\right.\right.}{\ldots}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{3\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{4f^2\left(4a\ldots\right.}{\left(2de-bf^2\right)^3\sqrt{d+\ldots}}$$

$$= -\frac{4\left(d^2e-bdf^2+aef^2\right)}{3\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{4f^2\left(4a\ldots\right.}{\left(2de-bf^2\right)^3\sqrt{d+\ldots}}$$

Mathematica [A] time = 0.86, size = 315, normalized size = 0.94

$$\frac{\frac{8f^2\left(b^2f^2-4ae^2\right)}{\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}+\frac{10\sqrt{2}\sqrt{e}f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2de-bf^2}}-\frac{4\left(4ae^3f^2-b^2ef^4\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2}-\ldots}{2\left(2de-bf^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] ((-8*(2*d*e - b*f^2)*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(d + e*x + f*Sqrt[a +
x*(b + (e^2*x)/f^2)])^(3/2)) + (8*f^2*(-4*a*e^2 + b^2*f^2))/Sqrt[d + e*x +
f*Sqrt[a + x*(b + (e^2*x)/f^2)]] - (4*(4*a*e^3*f^2 - b^2*e*f^4)*Sqrt[d + e*
x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b +
 (e^2*x)/f^2)])) + (10*Sqrt[2]*Sqrt[e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqr
t[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e -
b*f^2]])/Sqrt[2*d*e - b*f^2])/(2*(2*d*e - b*f^2)^3)
```

fricas [B] time = 1.38, size = 2514, normalized size = 7.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas"
)
```

```
[Out] [-1/6*(15*sqrt(2)*(a^2*b^2*f^8 - 4*a*d^4*e^2*f^2 - 2*(a*b^2*d^2 + 2*a^3*e^2
)*f^6 + (b^2*d^4 + 8*a^2*d^2*e^2)*f^4 + (b^4*f^8 - 16*a*d^2*e^4*f^2 - 4*(b^
3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*x^2 + 2*(a*b^3*
f^8 - 8*a*d^3*e^3*f^2 - (b^3*d^2 + 2*a*b^2*d*e + 4*a^2*b*e^2)*f^6 + 2*(b^2*
d^3*e + 2*a*b*d^2*e^2 + 4*a^2*d*e^3)*f^4)*x)*sqrt(-e/(b*f^2 - 2*d*e))*log(-
b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sqrt(2)*
(b*e*f^3 - 2*d*e^2*f)*sqrt(-e/(b*f^2 - 2*d*e))*sqrt((b*f^2*x + e^2*x^2 + a*
f^2)/f^2) - sqrt(2)*(b^2*f^4 - 2*b*d*e*f^2 + 2*(b*e^2*f^2 - 2*d*e^3)*x)*sqr
t(-e/(b*f^2 - 2*d*e)))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) +
 d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 4*(4
*d^5*e^2 + (8*a*b^2*d - 5*a^2*b*e)*f^6 - 2*(2*b^2*d^3 + a*b*d^2*e + 10*a^2*
d*e^2)*f^4 - 6*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^3 - (9*b*d^4*e -
 32*a*d^3*e^2)*f^2 + (3*b^3*e*f^6 - 16*d^3*e^4 + 4*(b^2*d*e^2 - 10*a*b*e^3)
*f^4 - 4*(3*b*d^2*e^3 - 20*a*d*e^4)*f^2)*x^2 + 2*(d^4*e^3 + (4*b^3*d - a*b^
2*e)*f^6 - (7*b^2*d^2*e + 6*a*b*d*e^2 + 15*a^2*e^3)*f^4 - 2*(5*b*d^3*e^2 -
23*a*d^2*e^3)*f^2)*x - 2*(3*a*b^2*f^7 + d^4*e^2*f - (b^2*d^2 + 2*a*b*d*e +
15*a^2*e^2)*f^5 - 2*(3*b*d^3*e - 11*a*d^2*e^2)*f^3 - 3*(b^2*e^2*f^5 - 4*b*d
*e^3*f^3 + 4*d^2*e^4*f)*x^2 + (3*b^3*f^7 + 40*a*d*e^3*f^3 - 8*d^3*e^3*f - 4
*(b^2*d*e + 5*a*b*e^2)*f^5)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(
e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a^2*b^3*f^10 - 8*d^7*e
^3 - 2*(a*b^3*d^2 + 3*a^2*b^2*d*e)*f^8 + (b^3*d^4 + 12*a*b^2*d^3*e + 12*a^2
*b*d^2*e^2)*f^6 - 2*(3*b^2*d^5*e + 12*a*b*d^4*e^2 + 4*a^2*d^3*e^3)*f^4 + 4*
(3*b*d^6*e^2 + 4*a*d^5*e^3)*f^2 + (b^5*f^10 - 10*b^4*d*e*f^8 + 40*b^3*d^2*e
^2*f^6 - 80*b^2*d^3*e^3*f^4 + 80*b*d^4*e^4*f^2 - 32*d^5*e^5)*x^2 + 2*(a*b^4
*f^10 - 16*d^6*e^4 - (b^4*d^2 + 8*a*b^3*d*e)*f^8 + 8*(b^3*d^3*e + 3*a*b^2*d
^2*e^2)*f^6 - 8*(3*b^2*d^4*e^2 + 4*a*b*d^3*e^3)*f^4 + 16*(2*b*d^5*e^3 + a*d
^4*e^4)*f^2)*x), 1/3*(15*sqrt(2)*(a^2*b^2*f^8 - 4*a*d^4*e^2*f^2 - 2*(a*b^2*
d^2 + 2*a^3*e^2)*f^6 + (b^2*d^4 + 8*a^2*d^2*e^2)*f^4 + (b^4*f^8 - 16*a*d^2*
e^4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*
x^2 + 2*(a*b^3*f^8 - 8*a*d^3*e^3*f^2 - (b^3*d^2 + 2*a*b^2*d*e + 4*a^2*b*e^2
)*f^6 + 2*(b^2*d^3*e + 2*a*b*d^2*e^2 + 4*a^2*d*e^3)*f^4)*x)*sqrt(e/(b*f^2 -
 2*d*e))*arctan(1/2*(sqrt(2)*(b*f^3 - 2*d*e*f)*sqrt(e/(b*f^2 - 2*d*e))*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2)*(b*d*f^2 - 2*d^2*e + (b*e*f^2 -
 2*d*e^2)*x)*sqrt(e/(b*f^2 - 2*d*e)))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2
+ a*f^2)/f^2) + d)/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(4*d^5*e^
2 + (8*a*b^2*d - 5*a^2*b*e)*f^6 - 2*(2*b^2*d^3 + a*b*d^2*e + 10*a^2*d*e^2)*
f^4 - 6*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^3 - (9*b*d^4*e - 32*a*d
^3*e^2)*f^2 + (3*b^3*e*f^6 - 16*d^3*e^4 + 4*(b^2*d*e^2 - 10*a*b*e^3)*f^4 -
4*(3*b*d^2*e^3 - 20*a*d*e^4)*f^2)*x^2 + 2*(d^4*e^3 + (4*b^3*d - a*b^2*e)*f^
6 - (7*b^2*d^2*e + 6*a*b*d*e^2 + 15*a^2*e^3)*f^4 - 2*(5*b*d^3*e^2 - 23*a*d^
2*e^3)*f^2)*x - 2*(3*a*b^2*f^7 + d^4*e^2*f - (b^2*d^2 + 2*a*b*d*e + 15*a^2*
e^2)*f^5 - 2*(3*b*d^3*e - 11*a*d^2*e^2)*f^3 - 3*(b^2*e^2*f^5 - 4*b*d*e^3*f^
3 + 4*d^2*e^4*f)*x^2 + (3*b^3*f^7 + 40*a*d*e^3*f^3 - 8*d^3*e^3*f - 4*(b^2*d
*e + 5*a*b*e^2)*f^5)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f
*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a^2*b^3*f^10 - 8*d^7*e^3 - 2*
(a*b^3*d^2 + 3*a^2*b^2*d*e)*f^8 + (b^3*d^4 + 12*a*b^2*d^3*e + 12*a^2*b*d^2*
e^2)*f^6 - 2*(3*b^2*d^5*e + 12*a*b*d^4*e^2 + 4*a^2*d^3*e^3)*f^4 + 4*(3*b*d^
6*e^2 + 4*a*d^5*e^3)*f^2 + (b^5*f^10 - 10*b^4*d*e*f^8 + 40*b^3*d^2*e^2*f^6
- 80*b^2*d^3*e^3*f^4 + 80*b*d^4*e^4*f^2 - 32*d^5*e^5)*x^2 + 2*(a*b^4*f^10 -
 16*d^6*e^4 - (b^4*d^2 + 8*a*b^3*d*e)*f^8 + 8*(b^3*d^3*e + 3*a*b^2*d^2*e^2)
*f^6 - 8*(3*b^2*d^4*e^2 + 4*a*b*d^3*e^3)*f^4 + 16*(2*b*d^5*e^3 + a*d^4*e^4)
*f^2)*x)]
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex+d+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
[Out] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima"
)
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-5/2), x)
```

3.485 $\int \left(a+x^2\right)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=164

$$-\frac{a^5\left(\sqrt{a+x^2}+x\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(\sqrt{a+x^2}+x\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(\sqrt{a+x^2}+x\right)^{n+1}}{16(n+1)}+\frac{5a\left(\sqrt{a+x^2}+x\right.}{32(n+3)}$$

```
[Out] -1/32*a^5*(x+(x^2+a)^(1/2))^(-5+n)/(5-n)-5/32*a^4*(x+(x^2+a)^(1/2))^(-3+n)/
(3-n)-5/16*a^3*(x+(x^2+a)^(1/2))^(-1+n)/(1-n)+5/16*a^2*(x+(x^2+a)^(1/2))^(1
+n)/(1+n)+5/32*a*(x+(x^2+a)^(1/2))^(3+n)/(3+n)+1/32*(x+(x^2+a)^(1/2))^(5+n)
/(5+n)
```

Rubi [A] time = 0.11, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2122, 270}

$$-\frac{a^5\left(\sqrt{a+x^2}+x\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(\sqrt{a+x^2}+x\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(\sqrt{a+x^2}+x\right)^{n+1}}{16(n+1)}+\frac{5a\left(\sqrt{a+x^2}+x\right.}{32(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^5*(x + Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x + Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x + Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x + Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x + Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x + Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+x^2\right)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx = \frac{1}{32}\text{Subst}\left(\int x^{-6+n}\left(a+x^2\right)^5 \, dx, x, x+\sqrt{a+x^2}\right)$$

$$= \frac{1}{32}\text{Subst}\left(\int \left(a^5x^{-6+n}+5a^4x^{-4+n}+10a^3x^{-2+n}+10a^2x^n+5ax^{2+n}+x^{4+n}\right) dx\right.$$

$$= -\frac{a^5\left(x+\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)}-\frac{5a^4\left(x+\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)}-\frac{5a^3\left(x+\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)}+$$

Mathematica [A] time = 0.38, size = 138, normalized size = 0.84

$$\frac{1}{32}\left(\sqrt{a+x^2}+x\right)^{n-5}\left(\frac{a^5}{n-5}+\frac{5a^4\left(\sqrt{a+x^2}+x\right)^2}{n-3}+\frac{10a^3\left(\sqrt{a+x^2}+x\right)^4}{n-1}+\frac{10a^2\left(\sqrt{a+x^2}+x\right)^6}{n+1}+\frac{\left(\sqrt{a+x\right.}}{n}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x + Sqrt[a + x^2])^2)
/(-3 + n) + (10*a^3*(x + Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x + Sqrt[a +
 x^2])^6)/(1 + n) + (5*a*(x + Sqrt[a + x^2])^8)/(3 + n) + (x + Sqrt[a + x^2
])^10/(5 + n)))/32
```

fricas [A] time = 0.48, size = 158, normalized size = 0.96

$$-\frac{\left(5\left(n^4-10\,n^2+9\right)x^5+10\left(an^4-16\,an^2+15\,a\right)x^3+5\left(a^2n^4-22\,a^2n^2+45\,a^2\right)x-\left(a^2n^5-30\,a^2n^3+\left(n^5-\right.\right.\right.}{n^6-35\,n^4+259\,n^2-225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
- 22*a^2*n^2 + 45*a^2)*x - (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4
 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x + sqrt(
x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)^2\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

maple [C] time = 0.09, size = 216, normalized size = 1.32

$$\frac{a2^{n+1}x^{n+3}\,\mathrm{hypergeom}\left(\left[-\frac{n}{2},-\frac{n}{2}+\frac{1}{2},-\frac{n}{2}-\frac{3}{2}\right],\left[-n+1,-\frac{n}{2}-\frac{1}{2}\right],-\frac{a}{x^2}\right)}{n+3}+\frac{2^n x^{n+5}\,\mathrm{hypergeom}\left(\left[-\frac{n}{2},-\frac{n}{2}-\frac{5}{2},-\frac{n}{2}\right.\right.}{n+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 2^n/(5+n)*x^(5+n)*hypergeom([-1/2*n,-1/2*n-5/2,1/2-1/2*n],[1-n,-3/2-1/2*n],
-a/x^2)+2^(n+1)*a/(3+n)*x^(3+n)*hypergeom([-1/2*n,1/2-1/2*n,-3/2-1/2*n],[1-
n,-1/2-1/2*n],-a/x^2)+1/4*a^(5/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(n+1)/n*x^(n
+1)*a^(-1/2-1/2*n)*(a/x^2*n+n-1)/(2*n-2)*((1+a/x^2)^(1/2)+1)^(n-1)+4*Pi^(1/
2)/(n+1)/n*x^(n+1)*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(n-1)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)^2\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+a\right)^2 \left(x+\sqrt{x^2+a}\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^2*(x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((a + x^2)^2*(x + (a + x^2)^(1/2))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**2*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

3.486 $\int \left(a+x^2\right) \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=108

$$-\frac{a^3 \left(\sqrt{a+x^2}+x\right)^{n-3}}{8(3-n)}-\frac{3a^2 \left(\sqrt{a+x^2}+x\right)^{n-1}}{8(1-n)}+\frac{3a \left(\sqrt{a+x^2}+x\right)^{n+1}}{8(n+1)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+3}}{8(n+3)}$$

```
[Out] -1/8*a^3*(x+(x^2+a)^(1/2))^(-3+n)/(3-n)-3/8*a^2*(x+(x^2+a)^(1/2))^(-1+n)/(1
-n)+3/8*a*(x+(x^2+a)^(1/2))^(1+n)/(1+n)+1/8*(x+(x^2+a)^(1/2))^(3+n)/(3+n)
```

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2122, 270}

$$-\frac{a^3 \left(\sqrt{a+x^2}+x\right)^{n-3}}{8(3-n)}-\frac{3a^2 \left(\sqrt{a+x^2}+x\right)^{n-1}}{8(1-n)}+\frac{3a \left(\sqrt{a+x^2}+x\right)^{n+1}}{8(n+1)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^3*(x + Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x + Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x + Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x + Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+x^2\right) \left(x+\sqrt{a+x^2}\right)^n \, dx &= \frac{1}{8} \text{Subst}\left(\int x^{-4+n} \left(a+x^2\right)^3 \, dx, x, x+\sqrt{a+x^2}\right)\\
&= \frac{1}{8} \text{Subst}\left(\int \left(a^3 x^{-4+n}+3a^2 x^{-2+n}+3a x^n+x^{2+n}\right) \, dx, x, x+\sqrt{a+x^2}\right)\\
&= -\frac{a^3 \left(x+\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)}-\frac{3a^2 \left(x+\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)}+\frac{3a \left(x+\sqrt{a+x^2}\right)^{1+n}}{8(1+n)}+
\end{aligned}$$

Mathematica [A] time = 0.14, size = 92, normalized size = 0.85

$$\frac{1}{8} \left(\sqrt{a+x^2}+x\right)^{n-3} \left(\frac{a^3}{n-3}+\frac{3a^2 \left(\sqrt{a+x^2}+x\right)^2}{n-1}+\frac{\left(\sqrt{a+x^2}+x\right)^6}{n+3}+\frac{3a \left(\sqrt{a+x^2}+x\right)^4}{n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x + Sqrt[a + x^2])^2)
/(-1 + n) + (3*a*(x + Sqrt[a + x^2])^4)/(1 + n) + (x + Sqrt[a + x^2])^6/(3
+ n)))/8
```

fricas [A] time = 0.47, size = 78, normalized size = 0.72

$$-\frac{\left(3\left(n^2-1\right)x^3+3\left(an^2-3a\right)x-\left(an^3+\left(n^3-n\right)x^2-7an\right)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)^n}{n^4-10n^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x - (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqr
t(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

maple [C] time = 0.02, size = 167, normalized size = 1.55

$$\frac{2^n x^{n+3}\,\mathrm{hypergeom}\left(\left[-\frac{n}{2},-\frac{n}{2}+\frac{1}{2},-\frac{n}{2}-\frac{3}{2}\right],\left[-n+1,-\frac{n}{2}-\frac{1}{2}\right],-\frac{a}{x^2}\right)}{n+3}+\frac{\left(\frac{8\sqrt{\pi}\left(n+\frac{an}{x^2}-1\right)a^{-\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{n-1}}{(n+1)(2n-2)n}+\frac{4\sqrt{\pi}\ \sqrt{}}{}\right.}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 2^n/(n+3)*x^(n+3)*hypergeom([-1/2*n,-1/2*n+1/2,-1/2*n-3/2],[-n+1,-1/2*n-1/2
],-a/x^2)+1/4*a^(3/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(n+1)*(n+a*n/x^2-1)/(2*n
-2)/n*a^(-1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1)+4*Pi^(1/2)/(n+1)*(a/
x^2+1)^(1/2)/n*a^(-1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(x^2+a\right)\,\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)*(x + (a + x^2)^(1/2))^n,x)
[Out] int((a + x^2)*(x + (a + x^2)^(1/2))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)*(x+(x**2+a)**(1/2))**n,x)
[Out] Timed out
```

3.487 $\int \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

```
[Out] -1/2*a*(x+(x^2+a)^(1/2))^(-1+n)/(1-n)+1/2*(x+(x^2+a)^(1/2))^(1+n)/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2117, 14}

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x + Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(x+\sqrt{a+x^2}\right)^n \, dx &= \frac{1}{2}\text{Subst}\left(\int x^{-2+n}\left(a+x^2\right) \, dx,x,x+\sqrt{a+x^2}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+n}+x^n\right) \, dx,x,x+\sqrt{a+x^2}\right)\\
&= -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.83

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n-1}\left((n-1)x\left(\sqrt{a+x^2}+x\right)+an\right)}{n^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-1 + n)*(a*n + (-1 + n)*x*(x + Sqrt[a + x^2])))/(-1 +
n^2)
```

fricas [A] time = 0.46, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^2+a}\,n-x\right)\left(x+\sqrt{x^2+a}\right)^n}{n^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + a)*n - x)*(x + sqrt(x^2 + a))^n/(n^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

maple [B] time = 0.02, size = 120, normalized size = 2.31

$$\frac{\left(\frac{8\sqrt{\pi}\left(n+\frac{an}{x^2}-1\right)a^{-\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{n-1}}{(n+1)(2n-2)n}+\frac{4\sqrt{\pi}\,\sqrt{\frac{a}{x^2}+1}\;a^{-\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{n-1}}{(n+1)n}\right)n\,a^{\frac{n}{2}+\frac{1}{2}}}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 1/4*a^(1/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(n+1)*(n+a*n/x^2-1)/(2*n-2)/n*a^(-
1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1)+4*Pi^(1/2)/(n+1)*(a/x^2+1)^(1/
2)/n*a^(-1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((x + (a + x^2)^(1/2))^n, x)
```

sympy [B] time = 2.75, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(n/2)*n**2*x*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a
)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2)
 + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) +
a**(9/2)*a**(n/2)*n*x*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2
*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1
- n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - a**(7/2)*a**(n/2)*n**2*x**3*sqrt
(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1
- n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
2*a**(7/2)*x**2*gamma(1 - n/2)) + a**(7/2)*a**(n/2)*n*x**3*cosh(n*asinh(x/s
qrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 -
 n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2
)) + 2*a**5*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1
- n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(
7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**5*a*
*(n/2)*n*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(
1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 -
n/2)) - 2*a**4*a**(n/2)*n*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + a
sinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2
)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*ga
mma(1 - n/2)) + 4*a**4*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sq
rt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1
 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n
/2)) - 2*a**4*a**(n/2)*n*x**2*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2
) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**
(7/2)*x**2*gamma(1 - n/2)) - 2*a**4*a**(n/2)*x**2*sqrt(a/x**2 + 1)*sinh(n*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1
 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
 2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**4*a**(n/2)*x**2*cosh(n*asinh(x/sqrt
(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2
*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)
*x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*n*x**4*sqrt(a/x**2 + 1)*sinh(n*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 -
n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*
a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*x**4*cosh(n*asinh(x/sqrt(
a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*
a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*
x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*x**4*sqrt(a/x**2 + 1)*sinh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2
) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**
(7/2)*x**2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*x**4*cosh(n*asinh(x/sqrt(a)) +
 asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9
/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*
gamma(1 - n/2)), Abs(x**2/a) > 1), (-2*a**(5/2)*a**(n/2)*n*x*sqrt(1 + x**2/
a)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n
**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)) + a**(5/2)*a**(n/2)*n*x*cos
h(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5
/2)*gamma(1 - n/2)) - 2*a**(5/2)*a**(n/2)*x*sqrt(1 + x**2/a)*sinh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2
) - 2*a**(5/2)*gamma(1 - n/2)) - a**3*a**(n/2)*n**2*sqrt(1 + x**2/a)*sinh(n
*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)
*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt
(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 -
 n/2)) + 2*a**2*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))
*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)
) + 2*a**2*a**(n/2)*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(
1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)), True
```

```
))
```

3.488 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2} \, dx$

Optimal. Leaf size=59

$$\frac{2\left(\sqrt{a+x^2}+x\right)^{n+1} {}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

```
[Out] 2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(
1/2))^(1+n)/a/(1+n)
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2122, 364}

$$\frac{2\left(\sqrt{a+x^2}+x\right)^{n+1} {}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2} \, dx &= 2 \operatorname{Subst}\left(\int \frac{x^n}{a+x^2} \, dx, x, x+\sqrt{a+x^2}\right)\\
&= \frac{2\left(x+\sqrt{a+x^2}\right)^{1+n} {}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.03

$$\frac{2\left(\sqrt{a+x^2}+x\right)^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/
2, -((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2),x)
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a),x)
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2), x)
```

3.489 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} \, dx$

Optimal. Leaf size=59

$$\frac{8\left(\sqrt{a+x^2}+x\right)^{n+3} {}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

```
[Out] 8*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(
1/2))^(3+n)/a^3/(3+n)
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2122, 364}

$$\frac{8\left(\sqrt{a+x^2}+x\right)^{n+3} {}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} \, dx &= 8 \operatorname{Subst}\left(\int \frac{x^{2+n}}{\left(a+x^2\right)^3} \, dx, x, x+\sqrt{a+x^2}\right)\\
&= \frac{8\left(x+\sqrt{a+x^2}\right)^{3+n} {}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.03

$$\frac{8\left(\sqrt{a+x^2}+x\right)^{n+3}\,{}_2F_1\left(3,\frac{n+3}{2};\frac{n+3}{2}+1;-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, 1 + (3 + n)/
2, -((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x+\sqrt{x^2+a}\right)^n}{x^4+2\,ax^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

```
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^2,x)
```

```
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**2,x)
```

```
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**2, x)
```

3.490 $\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n\,dx$

Optimal. Leaf size=176

$$-\frac{a^5\left(x-\sqrt{a+x^2}\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(x-\sqrt{a+x^2}\right)^{n+1}}{16(n+1)}+\frac{5a\left(x-\sqrt{a+x^2}\right)^{n+3}}{32(n+3)}$$

```
[Out] -1/32*a^5*(x-(x^2+a)^(1/2))^(-5+n)/(5-n)-5/32*a^4*(x-(x^2+a)^(1/2))^(-3+n)/
(3-n)-5/16*a^3*(x-(x^2+a)^(1/2))^(-1+n)/(1-n)+5/16*a^2*(x-(x^2+a)^(1/2))^(1
+n)/(1+n)+5/32*a*(x-(x^2+a)^(1/2))^(3+n)/(3+n)+1/32*(x-(x^2+a)^(1/2))^(5+n)
/(5+n)
```

Rubi [A] time = 0.11, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2122, 270}

$$-\frac{a^5\left(x-\sqrt{a+x^2}\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(x-\sqrt{a+x^2}\right)^{n+1}}{16(n+1)}+\frac{5a\left(x-\sqrt{a+x^2}\right)^{n+3}}{32(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^5*(x - Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x - Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x - Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x - Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x - Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x - Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n\,dx &= \frac{1}{32}\text{Subst}\left(\int x^{-6+n}\left(a+x^2\right)^5\,dx, x, x-\sqrt{a+x^2}\right)\\
&= \frac{1}{32}\text{Subst}\left(\int \left(a^5x^{-6+n}+5a^4x^{-4+n}+10a^3x^{-2+n}+10a^2x^n+5ax^{2+n}+x^{4+n}\right)dx\right.\\
&= -\frac{a^5\left(x-\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)}+
\end{aligned}$$

Mathematica [A] time = 0.39, size = 150, normalized size = 0.85

$$\frac{1}{32}\left(x-\sqrt{a+x^2}\right)^{n-5}\left(\frac{a^5}{n-5}+\frac{5a^4\left(x-\sqrt{a+x^2}\right)^2}{n-3}+\frac{10a^3\left(x-\sqrt{a+x^2}\right)^4}{n-1}+\frac{10a^2\left(x-\sqrt{a+x^2}\right)^6}{n+1}+\frac{\left(x-\sqrt{a\right.}}{n-}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x - Sqrt[a + x^2])^2)
/(-3 + n) + (10*a^3*(x - Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x - Sqrt[a +
 x^2])^6)/(1 + n) + (5*a*(x - Sqrt[a + x^2])^8)/(3 + n) + (x - Sqrt[a + x^2
])^10/(5 + n)))/32
```

fricas [A] time = 0.50, size = 159, normalized size = 0.90

$$-\frac{\left(5\left(n^4-10\,n^2+9\right)x^5+10\left(an^4-16\,an^2+15\,a\right)x^3+5\left(a^2n^4-22\,a^2n^2+45\,a^2\right)x+\left(a^2n^5-30\,a^2n^3+\left(n^5-\right.\right.\right.}{n^6-35\,n^4+259\,n^2-225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
- 22*a^2*n^2 + 45*a^2)*x + (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4
 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x - sqrt(
x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)^2\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)^2\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)^2\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x-\sqrt{x^2+a}\right)^n \left(x^2+a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^2,x)
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**2*(x-(x**2+a)**(1/2))**n,x)
[Out] Integral((a + x**2)**2*(x - sqrt(a + x**2))**n, x)
```

3.491 $\int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n\,dx$

Optimal. Leaf size=116

$$-\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-3}}{8(3-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n-1}}{8(1-n)}+\frac{3a\left(x-\sqrt{a+x^2}\right)^{n+1}}{8(n+1)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+3}}{8(n+3)}$$

```
[Out] -1/8*a^3*(x-(x^2+a)^(1/2))^(-3+n)/(3-n)-3/8*a^2*(x-(x^2+a)^(1/2))^(-1+n)/(1
-n)+3/8*a*(x-(x^2+a)^(1/2))^(1+n)/(1+n)+1/8*(x-(x^2+a)^(1/2))^(3+n)/(3+n)
```

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2122, 270}

$$-\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-3}}{8(3-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n-1}}{8(1-n)}+\frac{3a\left(x-\sqrt{a+x^2}\right)^{n+1}}{8(n+1)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^3*(x - Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x - Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x - Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x - Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n\,dx &= \frac{1}{8}\text{Subst}\left(\int x^{-4+n}\left(a+x^2\right)^3\,dx,x,x-\sqrt{a+x^2}\right)\\ &= \frac{1}{8}\text{Subst}\left(\int \left(a^3x^{-4+n}+3a^2x^{-2+n}+3ax^n+x^{2+n}\right)dx,x,x-\sqrt{a+x^2}\right)\\ &= -\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)}+\frac{3a\left(x-\sqrt{a+x^2}\right)^{1+n}}{8(1+n)}+\end{aligned}$$

Mathematica [A] time = 0.13, size = 100, normalized size = 0.86

$$\frac{1}{8}\left(x-\sqrt{a+x^2}\right)^{n-3}\left(\frac{a^3}{n-3}+\frac{3a^2\left(x-\sqrt{a+x^2}\right)^2}{n-1}+\frac{\left(x-\sqrt{a+x^2}\right)^6}{n+3}+\frac{3a\left(x-\sqrt{a+x^2}\right)^4}{n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x - Sqrt[a + x^2])^2)
/(-1 + n) + (3*a*(x - Sqrt[a + x^2])^4)/(1 + n) + (x - Sqrt[a + x^2])^6/(3
+ n)))/8
```

fricas [A] time = 0.46, size = 79, normalized size = 0.68

$$-\frac{\left(3\left(n^2-1\right)x^3+3\left(an^2-3a\right)x+\left(an^3+\left(n^3-n\right)x^2-7an\right)\sqrt{x^2+a}\right)\left(x-\sqrt{x^2+a}\right)^n}{n^4-10n^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x + (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqr
t(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(x-\sqrt{x^2+a}\right)^n\,\left(x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2),x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)*(x - sqrt(a + x**2))**n, x)
```

3.492 $\int \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=56

$$\frac{\left(x-\sqrt{a+x^2}\right)^{n+1}}{2(n+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n-1}}{2(1-n)}$$

```
[Out] -1/2*a*(x-(x^2+a)^(1/2))^(-1+n)/(1-n)+1/2*(x-(x^2+a)^(1/2))^(1+n)/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2117, 14}

$$\frac{\left(x-\sqrt{a+x^2}\right)^{n+1}}{2(n+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x - Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(x-\sqrt{a+x^2}\right)^n \, dx &= \frac{1}{2}\text{Subst}\left(\int x^{-2+n}\left(a+x^2\right) \, dx,x,x-\sqrt{a+x^2}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+n}+x^n\right) \, dx,x,x-\sqrt{a+x^2}\right)\\
&= -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.89

$$\frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{n-1}\left(\frac{\left(x-\sqrt{a+x^2}\right)^2}{n+1}+\frac{a}{n-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^(-1 + n)*(a/(-1 + n) + (x - Sqrt[a + x^2])^2/(1 + n)))
/2
```

fricas [A] time = 0.48, size = 33, normalized size = 0.59

$$-\frac{\left(\sqrt{x^2+a}\,n+x\right)\left(x-\sqrt{x^2+a}\right)^n}{n^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] -(sqrt(x^2 + a)*n + x)*(x - sqrt(x^2 + a))^n/(n^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n,x)
[Out] int((x-(x^2+a)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n,x)
[Out] int((x - (a + x^2)^(1/2))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x-\sqrt{a+x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n,x)
[Out] Integral((x - sqrt(a + x**2))**n, x)
```

3.493 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

```
[Out] 2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(
1/2))^(1+n)/a/(1+n)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2122, 364}

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -
((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^n}{a+x^2}\,dx,x,x-\sqrt{a+x^2}\right)$$

$$=\frac{2\left(x-\sqrt{a+x^2}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.03

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{n+1}\,_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2),x]
[Out] (2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/
2, -((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="fricas")
[Out] integral((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2),x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a),x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2), x)
```

3.494 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} dx$

Optimal. Leaf size=63

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{n+3} {}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

```
[Out] 8*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(
1/2))^(3+n)/a^3/(3+n)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 364}

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{n+3} {}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, -
((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} dx &= 8\,\text{Subst}\left(\int \frac{x^{2+n}}{\left(a+x^2\right)^3} dx, x, x-\sqrt{a+x^2}\right)\\
&= \frac{8\left(x-\sqrt{a+x^2}\right)^{3+n} {}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.03

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{n+3} {}_2F_1\left(3,\frac{n+3}{2};\frac{n+3}{2}+1;-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, 1 + (3 + n)/
2, -((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x-\sqrt{x^2+a}\right)^n}{x^4+2\,ax^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^2,x)
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**2,x)
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**2, x)
```

3.495 $\int \left(a+x^2\right)^{5/2} \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=187

$$-\frac{a^6\left(\sqrt{a+x^2}+x\right)^{n-6}}{64(6-n)}-\frac{3a^5\left(\sqrt{a+x^2}+x\right)^{n-4}}{32(4-n)}-\frac{15a^4\left(\sqrt{a+x^2}+x\right)^{n-2}}{64(2-n)}+\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n}}{16n}+\frac{15a^2\left(\sqrt{a+x^2}+x\right.}{64(n+2)}$$

```
[Out] -1/64*a^6*(x+(x^2+a)^(1/2))^(-6+n)/(6-n)-3/32*a^5*(x+(x^2+a)^(1/2))^(-4+n)/
(4-n)-15/64*a^4*(x+(x^2+a)^(1/2))^(-2+n)/(2-n)+5/16*a^3*(x+(x^2+a)^(1/2))^n
/n+15/64*a^2*(x+(x^2+a)^(1/2))^(2+n)/(2+n)+3/32*a*(x+(x^2+a)^(1/2))^(4+n)/(
4+n)+1/64*(x+(x^2+a)^(1/2))^(6+n)/(6+n)
```

Rubi [A] time = 0.12, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 270}

$$-\frac{a^6\left(\sqrt{a+x^2}+x\right)^{n-6}}{64(6-n)}-\frac{3a^5\left(\sqrt{a+x^2}+x\right)^{n-4}}{32(4-n)}-\frac{15a^4\left(\sqrt{a+x^2}+x\right)^{n-2}}{64(2-n)}+\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n}}{16n}+\frac{15a^2\left(\sqrt{a+x^2}+x\right.}{64(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^6*(x + Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) - (3*a^5*(x + Sqrt[a + x^2
])^(-4 + n))/(32*(4 - n)) - (15*a^4*(x + Sqrt[a + x^2])^(-2 + n))/(64*(2 -
n)) + (5*a^3*(x + Sqrt[a + x^2])^n)/(16*n) + (15*a^2*(x + Sqrt[a + x^2])^(2
 + n))/(64*(2 + n)) + (3*a*(x + Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) + (x +
 Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+x^2\right)^{5/2} \left(x+\sqrt{a+x^2}\right)^n \, dx &= \frac{1}{64}\text{Subst}\left(\int x^{-7+n}\left(a+x^2\right)^6 \, dx, x, x+\sqrt{a+x^2}\right)\\
&= \frac{1}{64}\text{Subst}\left(\int \left(a^6x^{-7+n}+6a^5x^{-5+n}+15a^4x^{-3+n}+20a^3x^{-1+n}+15a^2x^{1+n}+6a\right.\right.\\
&= -\frac{a^6\left(x+\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)}-\frac{3a^5\left(x+\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)}-\frac{15a^4\left(x+\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 157, normalized size = 0.84

$$\frac{1}{64}\left(\sqrt{a+x^2}+x\right)^n\left(\frac{a^6}{(n-6)\left(\sqrt{a+x^2}+x\right)^6}+\frac{6a^5}{(n-4)\left(\sqrt{a+x^2}+x\right)^4}+\frac{15a^4}{(n-2)\left(\sqrt{a+x^2}+x\right)^2}+\frac{20a^3}{n}+\frac{15}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((20*a^3)/n + a^6/((-6 + n)*(x + Sqrt[a + x^2])^6) +
 (6*a^5)/((-4 + n)*(x + Sqrt[a + x^2])^4) + (15*a^4)/((-2 + n)*(x + Sqrt[a
+ x^2])^2) + (15*a^2*(x + Sqrt[a + x^2])^2)/(2 + n) + (6*a*(x + Sqrt[a + x^
2])^4)/(4 + n) + (x + Sqrt[a + x^2])^6/(6 + n)))/64
```

fricas [A] time = 0.46, size = 201, normalized size = 1.07

$$\frac{\left(a^3n^6-50\,a^3n^4+\left(n^6-20\,n^4+64\,n^2\right)x^6+544\,a^3n^2+3\left(an^6-30\,an^4+104\,an^2\right)x^4-720\,a^3+3\left(a^2n^6-40\,\right.\right.}{n^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n^
6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^2
*n^2)*x^2 - 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*x
^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2 + a
))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2 + a\right)^{5/2} \left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + x^2\right)^{\frac{5}{2}} \left(x + \sqrt{a + x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(5/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(5/2)*(x + sqrt(a + x**2))**n, x)
```

3.496 $\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=131

$$-\frac{a^4 \left(\sqrt{a+x^2}+x\right)^{n-4}}{16(4-n)}-\frac{a^3 \left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{3a^2 \left(\sqrt{a+x^2}+x\right)^n}{8n}+\frac{a \left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+4}}{16(n+4)}$$

```
[Out] -1/16*a^4*(x+(x^2+a)^(1/2))^(-4+n)/(4-n)-1/4*a^3*(x+(x^2+a)^(1/2))^(-2+n)/(
2-n)+3/8*a^2*(x+(x^2+a)^(1/2))^n/n+1/4*a*(x+(x^2+a)^(1/2))^(2+n)/(2+n)+1/16
*(x+(x^2+a)^(1/2))^(4+n)/(4+n)
```

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2122, 270}

$$-\frac{a^4 \left(\sqrt{a+x^2}+x\right)^{n-4}}{16(4-n)}-\frac{a^3 \left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{3a^2 \left(\sqrt{a+x^2}+x\right)^n}{8n}+\frac{a \left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^4*(x + Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) - (a^3*(x + Sqrt[a + x^2])
^(-2 + n))/(4*(2 - n)) + (3*a^2*(x + Sqrt[a + x^2])^n)/(8*n) + (a*(x + Sqrt
[a + x^2])^(2 + n))/(4*(2 + n)) + (x + Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx &= \frac{1}{16} \text{Subst}\left(\int x^{-5+n} \left(a+x^2\right)^4 \, dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{1}{16} \text{Subst}\left(\int \left(a^4 x^{-5+n}+4a^3 x^{-3+n}+6a^2 x^{-1+n}+4a x^{1+n}+x^{3+n}\right) dx, x, x+\\ &= -\frac{a^4 \left(x+\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)}-\frac{a^3 \left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}+\frac{3a^2 \left(x+\sqrt{a+x^2}\right)^n}{8n}+\end{aligned}$$

Mathematica [A] time = 0.23, size = 111, normalized size = 0.85

$$\frac{1}{16} \left(\sqrt{a+x^2}+x\right)^n \left(\frac{a^4}{(n-4)\left(\sqrt{a+x^2}+x\right)^4}+\frac{4a^3}{(n-2)\left(\sqrt{a+x^2}+x\right)^2}+\frac{6a^2}{n}+\frac{4a\left(\sqrt{a+x^2}+x\right)^2}{n+2}+\frac{\left(\sqrt{a+x\right.}{n-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((6*a^2)/n + a^4/((-4 + n)*(x + Sqrt[a + x^2])^4) +
(4*a^3)/((-2 + n)*(x + Sqrt[a + x^2])^2) + (4*a*(x + Sqrt[a + x^2])^2)/(2 +
 n) + (x + Sqrt[a + x^2])^4/(4 + n)))/16
```

fricas [A] time = 0.46, size = 110, normalized size = 0.84

$$\frac{\left(a^2n^4 + \left(n^4 - 4\,n^2\right)x^4 - 16\,a^2n^2 + 2\left(an^4 - 10\,an^2\right)x^2 + 24\,a^2 - 4\left(\left(n^3 - 4\,n\right)x^3 + \left(an^3 - 10\,an\right)x\right)\sqrt{x^2 + a}\right)\left(x + \right.}{n^5 - 20\,n^3 + 64\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*a
^2 - 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2
+ a))^n/(n^5 - 20*n^3 + 64*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2 + a\right)^{\frac{3}{2}}\left(x + \sqrt{x^2 + a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(x^2 + a\right)^{\frac{3}{2}}\left(x + \sqrt{x^2 + a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2 + a\right)^{\frac{3}{2}}\left(x + \sqrt{x^2 + a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x^2 + a\right)}^{3/2}\,{\left(x + \sqrt{x^2 + a}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^{\frac{3}{2}}\left(x+\sqrt{a+x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(3/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(3/2)*(x + sqrt(a + x**2))**n, x)
```

3.497 $\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx$

Optimal. Leaf size=75

$$-\frac{a^2\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{a\left(\sqrt{a+x^2}+x\right)^n}{2n}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}$$

```
[Out] -1/4*a^2*(x+(x^2+a)^(1/2))^(-2+n)/(2-n)+1/2*a*(x+(x^2+a)^(1/2))^n/n+1/4*(x+
(x^2+a)^(1/2))^(2+n)/(2+n)
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 270}

$$-\frac{a^2\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{a\left(\sqrt{a+x^2}+x\right)^n}{2n}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^2*(x + Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) + (a*(x + Sqrt[a + x^2])^n)
/(2*n) + (x + Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx &= \frac{1}{4}\text{Subst}\left(\int x^{-3+n}\left(a+x^2\right)^2\,dx,x,x+\sqrt{a+x^2}\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(a^2x^{-3+n}+2ax^{-1+n}+x^{1+n}\right)dx,x,x+\sqrt{a+x^2}\right)\\ &= -\frac{a^2\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}+\frac{a\left(x+\sqrt{a+x^2}\right)^n}{2n}+\frac{\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.87

$$\frac{1}{4}\left(\sqrt{a+x^2}+x\right)^n\left(\frac{a^2}{(n-2)\left(\sqrt{a+x^2}+x\right)^2}+\frac{\left(\sqrt{a+x^2}+x\right)^2}{n+2}+\frac{2a}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((2*a)/n + a^2/((-2 + n)*(x + Sqrt[a + x^2])^2) + (x
 + Sqrt[a + x^2])^2/(2 + n)))/4
```

fricas [A] time = 0.46, size = 48, normalized size = 0.64

$$\frac{\left(n^{2}x^{2} + an^{2} - 2\,\sqrt{x^{2} + a}\,nx - 2\,a\right)\left(x + \sqrt{x^{2} + a}\right)^{n}}{n^{3} - 4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (n^2*x^2 + a*n^2 - 2*sqrt(x^2 + a)*n*x - 2*a)*(x + sqrt(x^2 + a))^n/(n^3 -
4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{2} + a}\left(x + \sqrt{x^{2} + a}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{x^{2}+a}\,\left(x+\sqrt{x^{2}+a}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{2} + a}\left(x + \sqrt{x^{2} + a}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^2+a}\,\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + x^{2}}\,\left(x + \sqrt{a + x^{2}}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(1/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + x**2)*(x + sqrt(a + x**2))**n, x)
```

3.498 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

[Out] (x+(x^2+a)^(1/2))^n/n

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 30}

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]

[Out] (x + Sqrt[a + x^2])^n/n

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx &= \text{Subst}\left(\int x^{-1+n}\,dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{\left(x+\sqrt{a+x^2}\right)^n}{n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]

[Out] (x + Sqrt[a + x^2])^n/n

fricas [A] time = 0.47, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + a))^n/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

mupad [B] time = 3.00, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(1/2),x)
[Out] (x + (a + x^2)^(1/2))^n/n
```

sympy [B] time = 2.64, size = 311, normalized size = 18.29

$$\begin{cases} -\frac{\sqrt{a}\,a^{\frac{n}{2}}\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{nx\sqrt{\frac{a}{x^2}+1}} - \frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)} + \frac{a^{\frac{n}{2}}x\cosh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\,n} - \frac{a^{\frac{n}{2}}x\sinh\left(-\right.}{} \\ -\frac{a^{\frac{n}{2}}\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{n\sqrt{1+\frac{x^2}{a}}} - \frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)} - \frac{a^{\frac{n}{2}}x^2\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{an\sqrt{1+\frac{x^2}{a}}} + \frac{a^{\frac{n}{2}}x\cosh\left(-n\,\mathrm{a}\right.}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(a)*a**(n/2)*sinh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(
n*x*sqrt(a/x**2 + 1)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt(a)))*gamma(1 - n/2)/
(n**2*gamma(-n/2)) + a**(n/2)*x*cosh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a))
)/(sqrt(a)*n) - a**(n/2)*x*sinh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sq
rt(a)*n*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (-a**(n/2)*sinh(-n*asinh(x/sqr
t(a)) + asinh(x/sqrt(a)))/(n*sqrt(1 + x**2/a)) - 2*a**(n/2)*cosh(n*asinh(x/
sqrt(a)))*gamma(1 - n/2)/(n**2*gamma(-n/2)) - a**(n/2)*x**2*sinh(-n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))/(a*n*sqrt(1 + x**2/a)) + a**(n/2)*x*cosh(-n*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*n), True))
```

3.499 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}} dx$

Optimal. Leaf size=59

$$\frac{4\left(\sqrt{a+x^2}+x\right)^{n+2} {}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

```
[Out] 4*hypergeom([2, 1+1/2*n],[2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(1/2)
)^(2+n)/a^2/(2+n)
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2122, 364}

$$\frac{4\left(\sqrt{a+x^2}+x\right)^{n+2} {}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}} dx &= 4\,\text{Subst}\left(\int \frac{x^{1+n}}{\left(a+x^2\right)^2} dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{4\left(x+\sqrt{a+x^2}\right)^{2+n} {}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.03

$$\frac{4\left(\sqrt{a+x^2}+x\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+2}{2}+1;-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]

[Out] (4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, 1 + (2 + n)/2, -((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n}{x^4+2\,ax^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)

[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="maxima")

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x + \sqrt{x^2 + a}\right)^n}{\left(x^2 + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(3/2),x)
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x + \sqrt{a + x^2}\right)^n}{\left(a + x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**(3/2), x)
```

3.500 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=59

$$\frac{16\left(\sqrt{a+x^2}+x\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

```
[Out] 16*hypergeom([4, 2+1/2*n],[3+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(1/2
))^(4+n)/a^4/(4+n)
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 364}

$$\frac{16\left(\sqrt{a+x^2}+x\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2,
-((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx &= 16\,\text{Subst}\left(\int \frac{x^{3+n}}{\left(a+x^2\right)^4}\,dx,x,x+\sqrt{a+x^2}\right)\\ &= \frac{16\left(x+\sqrt{a+x^2}\right)^{4+n}\,{}_2F_1\left(4,\frac{4+n}{2};\frac{6+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.03

$$\frac{16\left(\sqrt{a+x^2}+x\right)^{n+4}{}_2F_1\left(4,\frac{n+4}{2};\frac{n+4}{2}+1;-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]

[Out] (16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, 1 + (4 + n)/2, -((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n}{x^6+3\,ax^4+3\,a^2x^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n/(x^6 + 3*a*x^4 + 3*a^2*x^2 + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)

[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="maxima")

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(5/2),x)
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**(5/2), x)
```

3.501 $\int \left(a+x^2\right)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=201

$$\frac{a^6 \left(x-\sqrt{a+x^2}\right)^{n-6}}{64(6-n)}+\frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{n-4}}{32(4-n)}+\frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{n-2}}{64(2-n)}-\frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{n}}{16n}-\frac{15a^2 \left(x-\sqrt{a+x^2}\right)^{n}}{64(n+2)}$$

```
[Out] 1/64*a^6*(x-(x^2+a)^(1/2))^(-6+n)/(6-n)+3/32*a^5*(x-(x^2+a)^(1/2))^(-4+n)/(
4-n)+15/64*a^4*(x-(x^2+a)^(1/2))^(-2+n)/(2-n)-5/16*a^3*(x-(x^2+a)^(1/2))^n/
n-15/64*a^2*(x-(x^2+a)^(1/2))^(2+n)/(2+n)-3/32*a*(x-(x^2+a)^(1/2))^(4+n)/(4
+n)-1/64*(x-(x^2+a)^(1/2))^(6+n)/(6+n)
```

Rubi [A] time = 0.11, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2122, 270}

$$\frac{a^6 \left(x-\sqrt{a+x^2}\right)^{n-6}}{64(6-n)}+\frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{n-4}}{32(4-n)}+\frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{n-2}}{64(2-n)}-\frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{n}}{16n}-\frac{15a^2 \left(x-\sqrt{a+x^2}\right)^{n}}{64(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^6*(x - Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) + (3*a^5*(x - Sqrt[a + x^2]
)^(-4 + n))/(32*(4 - n)) + (15*a^4*(x - Sqrt[a + x^2])^(-2 + n))/(64*(2 - n
)) - (5*a^3*(x - Sqrt[a + x^2])^n)/(16*n) - (15*a^2*(x - Sqrt[a + x^2])^(2
+ n))/(64*(2 + n)) - (3*a*(x - Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) - (x -
Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+x^2\right)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx &= -\left(\frac{1}{64} \text{Subst}\left(\int x^{-7+n} \left(a+x^2\right)^6 \, dx, x, x-\sqrt{a+x^2}\right)\right)\\ &= -\left(\frac{1}{64} \text{Subst}\left(\int \left(a^6 x^{-7+n}+6a^5 x^{-5+n}+15a^4 x^{-3+n}+20a^3 x^{-1+n}+15a^2 x^{1+n}+\right.\right.\right.\\ &= \frac{a^6 \left(x-\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)}+\frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)}+\frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)}-\end{aligned}$$

Mathematica [A] time = 0.49, size = 173, normalized size = 0.86

$$\frac{1}{64}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{a^6}{(n-6)\left(x-\sqrt{a+x^2}\right)^6}-\frac{6a^5}{(n-4)\left(x-\sqrt{a+x^2}\right)^4}-\frac{15a^4}{(n-2)\left(x-\sqrt{a+x^2}\right)^2}-\frac{20a^3}{n}-\frac{15}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^n*((-20*a^3)/n - a^6/((-6 + n)*(x - Sqrt[a + x^2])^6)
- (6*a^5)/((-4 + n)*(x - Sqrt[a + x^2])^4) - (15*a^4)/((-2 + n)*(x - Sqrt[a
 + x^2])^2) - (15*a^2*(x - Sqrt[a + x^2])^2)/(2 + n) - (6*a*(x - Sqrt[a + x
^2])^4)/(4 + n) - (x - Sqrt[a + x^2])^6/(6 + n)))/64
```

fricas [A] time = 0.48, size = 204, normalized size = 1.01

$$-\frac{\left(a^3n^6-50\,a^3n^4+\left(n^6-20\,n^4+64\,n^2\right)x^6+544\,a^3n^2+3\left(an^6-30\,an^4+104\,an^2\right)x^4-720\,a^3+3\left(a^2n^6-40\right.\right.}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n
^6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^
2*n^2)*x^2 + 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*
x^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2 +
a))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2+a\right)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(x-\sqrt{x^2+a}\right)^n\,{\left(x^2+a\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2),x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+x^{2}\right)^{\frac{5}{2}}\left(x-\sqrt{a+x^{2}}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(5/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(5/2)*(x - sqrt(a + x**2))**n, x)
```

3.502 $\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=141

$$\frac{a^4\left(x-\sqrt{a+x^2}\right)^{n-4}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+4}}{16(n+4)}$$

```
[Out] 1/16*a^4*(x-(x^2+a)^(1/2))^(-4+n)/(4-n)+1/4*a^3*(x-(x^2+a)^(1/2))^(-2+n)/(2
-n)-3/8*a^2*(x-(x^2+a)^(1/2))^n/n-1/4*a*(x-(x^2+a)^(1/2))^(2+n)/(2+n)-1/16*
(x-(x^2+a)^(1/2))^(4+n)/(4+n)
```

Rubi [A] time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2122, 270}

$$\frac{a^4\left(x-\sqrt{a+x^2}\right)^{n-4}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^4*(x - Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) + (a^3*(x - Sqrt[a + x^2])^
(-2 + n))/(4*(2 - n)) - (3*a^2*(x - Sqrt[a + x^2])^n)/(8*n) - (a*(x - Sqrt[
a + x^2])^(2 + n))/(4*(2 + n)) - (x - Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+x^2\right)^{3/2}\left(x-\sqrt{a+x^2}\right)^n\,dx &= -\left(\frac{1}{16}\operatorname{Subst}\left(\int x^{-5+n}\left(a+x^2\right)^4\,dx,x,x-\sqrt{a+x^2}\right)\right)\\
&= -\left(\frac{1}{16}\operatorname{Subst}\left(\int \left(a^4x^{-5+n}+4a^3x^{-3+n}+6a^2x^{-1+n}+4ax^{1+n}+x^{3+n}\right)dx,x,\right.\right.\\
&= \frac{a^4\left(x-\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a}{}-
\end{aligned}$$

Mathematica [A] time = 0.25, size = 123, normalized size = 0.87

$$\frac{1}{16}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{a^4}{(n-4)\left(x-\sqrt{a+x^2}\right)^4}-\frac{4a^3}{(n-2)\left(x-\sqrt{a+x^2}\right)^2}-\frac{6a^2}{n}-\frac{4a\left(x-\sqrt{a+x^2}\right)^2}{n+2}-\frac{\left(x-\sqrt{a\right.}}{n}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^n*((-6*a^2)/n - a^4/((-4 + n)*(x - Sqrt[a + x^2])^4) -
 (4*a^3)/((-2 + n)*(x - Sqrt[a + x^2])^2) - (4*a*(x - Sqrt[a + x^2])^2)/(2
+ n) - (x - Sqrt[a + x^2])^4/(4 + n)))/16
```

fricas [A] time = 0.48, size = 113, normalized size = 0.80

$$-\frac{\left(a^2n^4+\left(n^4-4\,n^2\right)x^4-16\,a^2n^2+2\left(an^4-10\,an^2\right)x^2+24\,a^2+4\left(\left(n^3-4\,n\right)x^3+\left(an^3-10\,an\right)x\right)\sqrt{x^2+a}\right)\left(x-\right.}{n^5-20\,n^3+64\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*
a^2 + 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2
 + a))^n/(n^5 - 20*n^3 + 64*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x-\sqrt{x^2+a}\right)^n\,\left(x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2),x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^{\frac{3}{2}} \left(x-\sqrt{a+x^2}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(3/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(3/2)*(x - sqrt(a + x**2))**n, x)
```

3.503 $\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx$

Optimal. Leaf size=81

$$\frac{a^2\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}$$

```
[Out] 1/4*a^2*(x-(x^2+a)^(1/2))^(-2+n)/(2-n)-1/2*a*(x-(x^2+a)^(1/2))^n/n-1/4*(x-(
x^2+a)^(1/2))^(2+n)/(2+n)
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2122, 270}

$$\frac{a^2\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^2*(x - Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) - (a*(x - Sqrt[a + x^2])^n)/
(2*n) - (x - Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx &= -\left(\frac{1}{4}\operatorname{Subst}\left(\int x^{-3+n}\left(a+x^2\right)^2\,dx,x,x-\sqrt{a+x^2}\right)\right)\\ &= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \left(a^2x^{-3+n}+2ax^{-1+n}+x^{1+n}\right)dx,x,x-\sqrt{a+x^2}\right)\right)\\ &= \frac{a^2\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 0.90

$$\frac{1}{4}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{a^2}{(n-2)\left(x-\sqrt{a+x^2}\right)^2}-\frac{\left(x-\sqrt{a+x^2}\right)^2}{n+2}-\frac{2a}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^n*((-2*a)/n - a^2/((-2 + n)*(x - Sqrt[a + x^2])^2) - (
x - Sqrt[a + x^2])^2/(2 + n)))/4
```

fricas [A] time = 0.49, size = 51, normalized size = 0.63

$$-\frac{\left(n^2x^2 + an^2 + 2\sqrt{x^2 + a}\,nx - 2\,a\right)\left(x - \sqrt{x^2 + a}\right)^n}{n^3 - 4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(n^2*x^2 + a*n^2 + 2*sqrt(x^2 + a)*n*x - 2*a)*(x - sqrt(x^2 + a))^n/(n^3 -
 4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 + a}\left(x - \sqrt{x^2 + a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 + a}\,\left(x - \sqrt{x^2 + a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 + a}\left(x - \sqrt{x^2 + a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x - \sqrt{x^2 + a}\right)^n\,\sqrt{x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2),x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + x^2}\,\left(x - \sqrt{a + x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(1/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + x**2)*(x - sqrt(a + x**2))**n, x)
```

3.504 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

[Out] -(x-(x^2+a)^(1/2))^n/n

Rubi [A] time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2122, 30}

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]

[Out] -((x - Sqrt[a + x^2])^n/n)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx &= -\text{Subst}\left(\int x^{-1+n}\,dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]

[Out] -((x - Sqrt[a + x^2])^n/n)

fricas [A] time = 0.43, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] -(x - sqrt(x^2 + a))^n/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

mupad [B] time = 3.19, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(1/2),x)
[Out] -(x - (a + x^2)^(1/2))^n/n
```

sympy [**A**] time = 1.57, size = 36, normalized size = 1.80

$$\begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n} & \text{for } n \neq 0 \\ \begin{cases} \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for } a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for } a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-(x - sqrt(a + x**2))**n/n, Ne(n, 0)), (Piecewise((asinh(x*sqrt(
1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))
```

3.505 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

```
[Out] -4*hypergeom([2, 1+1/2*n],[2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(1/2
))^(2+n)/a^2/(2+n)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2122, 364}

$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2,
-((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{x^{1+n}}{\left(a+x^2\right)^2}\,dx,x,x-\sqrt{a+x^2}\right)\right)\\ &= -\frac{4\left(x-\sqrt{a+x^2}\right)^{2+n}{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.03

$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+2}{2}+1;-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
[Out] (-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, 1 + (2 + n)
/2, -((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n}{x^4+2\,ax^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x - \sqrt{x^2 + a}\right)^n}{\left(x^2 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(3/2),x)
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{\left(a + x^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**(3/2), x)
```

3.506 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

```
[Out] -16*hypergeom([4, 2+1/2*n],[3+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(1/
2))^(4+n)/a^4/(4+n)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2122, 364}

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2,
 -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx &= -\left(16\,\text{Subst}\left(\int \frac{x^{3+n}}{\left(a+x^2\right)^4}\,dx,x,x-\sqrt{a+x^2}\right)\right)\\ &= -\frac{16\left(x-\sqrt{a+x^2}\right)^{4+n}\,{}_2F_1\left(4,\frac{4+n}{2};\frac{6+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.03

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{n+4}{}_2F_1\left(4,\frac{n+4}{2};\frac{n+4}{2}+1;-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, 1 + (4 + n
)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n}{x^6+3\,ax^4+3\,a^2x^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n/(x^6 + 3*a*x^4 + 3*a^2*x^2 + a
^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(5/2),x)
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**(5/2), x)
```

3.507 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=365

$$\frac{\left(d^2-af^2\right)^5\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-5}}{32ef^4(5-n)}-\frac{5\left(d^2-af^2\right)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{32ef^4(3-n)}+\frac{5\left(d^2-af^2\right)^3\left(f\ldots\right.}{}$$

```
[Out] 1/32*(-a*f^2+d^2)^5*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-5+n)/e/f^
4/(5-n)-5/32*(-a*f^2+d^2)^4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-3
+n)/e/f^4/(3-n)+5/16*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1
/2))^(-1+n)/e/f^4/(1-n)+5/16*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^(1+n)/e/f^4/(1+n)-5/32*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^
2*x^2/f^2)^(1/2))^(3+n)/e/f^4/(3+n)+1/32*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^
2)^(1/2))^(5+n)/e/f^4/(5+n)
```

Rubi [A] time = 0.47, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2121, 12, 270}

$$\frac{\left(d^2-af^2\right)^5\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-5}}{32ef^4(5-n)}-\frac{5\left(d^2-af^2\right)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{32ef^4(3-n)}+\frac{5\left(d^2-af^2\right)^3\left(f\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f
^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)^5*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-5
+ n))/(32*e*f^4*(5 - n)) - (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^(-3 + n))/(32*e*f^4*(3 - n)) + (5*(d^2 - a*f^2)^3*
(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(16*e*f^4*(
1 - n)) + (5*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2
)/f^2])^(1 + n))/(16*e*f^4*(1 + n)) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n))/(32*e*f^4*(3 + n)) + (d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(5 + n)/(32*e*f^4*(5 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
```

```
&& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-6+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^5}{64e^6}dx,x,d+\right.}{f^4}$$

$$=\frac{\text{Subst}\left(\int x^{-6+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^5\right.}{32e^6f^4}$$

$$=\frac{\text{Subst}\left(\int \left(-e^5\left(d^2-af^2\right)^5x^{-6+n}+5e^5\left(d^2-af\right.\right.\right.}{}$$

$$=\frac{\left(d^2-af^2\right)^5\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-5+n}}{32ef^4(5-n)}$$

Mathematica [A] time = 2.91, size = 280, normalized size = 0.77

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-5}\left(-\frac{5\left(d^2-af^2\right)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^8}{n+3}+\frac{10\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^6}{n+1}-\frac{10\left(d^2-af^2\right)^3\left(f\sqrt{\right.}}{}\right.}{32ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*
e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-5 + n)*(-((d^2 - a*f^2)^5/
(-5 + n)) + (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2)/(-3 + n) - (10*(d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^4)/(-1 + n) + (10*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d +
e*x))/f^2])^6)/(1 + n) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d +
 e*x))/f^2])^8)/(3 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^10/
(5 + n)))/(32*e*f^4)
```

fricas [A] time = 0.53, size = 654, normalized size = 1.79

$$-\frac{\left(5\,a^2df^4n^4+225\,a^2df^4-300\,ad^3f^2+5\left(e^5n^4-10\,e^5n^2+9\,e^5\right)x^5+120\,d^5+25\left(de^4n^4-10\,de^4n^2+9\,de^4\right)x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*a^2*d*f^4*n^4 + 225*a^2*d*f^4 - 300*a*d^3*f^2 + 5*(e^5*n^4 - 10*e^5*n^2
 + 9*e^5)*x^5 + 120*d^5 + 25*(d*e^4*n^4 - 10*d*e^4*n^2 + 9*d*e^4)*x^4 + 10*
(15*a*e^3*f^2 + 30*d^2*e^3 + (a*e^3*f^2 + 4*d^2*e^3)*n^4 - 2*(8*a*e^3*f^2 +
 17*d^2*e^3)*n^2)*x^3 - 10*(11*a^2*d*f^4 - 6*a*d^3*f^2)*n^2 + 10*(45*a*d*e^
```

```
2*f^2 + (3*a*d*e^2*f^2 + 2*d^3*e^2)*n^4 - 2*(24*a*d*e^2*f^2 + d^3*e^2)*n^2)
*x^2 + 5*(45*a^2*e*f^4 + (a^2*e*f^4 + 4*a*d^2*e*f^2)*n^4 - 2*(11*a^2*e*f^4
+ 26*a*d^2*e*f^2 - 12*d^4*e)*n^2)*x - (a^2*f^5*n^5 + (e^4*f*n^5 - 10*e^4*f*
n^3 + 9*e^4*f*n)*x^4 - 10*(3*a^2*f^5 - 2*a*d^2*f^3)*n^3 + 4*(d*e^3*f*n^5 -
10*d*e^3*f*n^3 + 9*d*e^3*f*n)*x^3 + 2*((a*e^2*f^3 + 2*d^2*e^2*f)*n^5 - 10*(
2*a*e^2*f^3 + d^2*e^2*f)*n^3 + (19*a*e^2*f^3 + 8*d^2*e^2*f)*n)*x^2 + (149*a
^2*f^5 - 260*a*d^2*f^3 + 120*d^4*f)*n + 4*(a*d*e*f^3*n^5 - 10*(2*a*d*e*f^3
- d^3*e*f)*n^3 + (19*a*d*e*f^3 - 10*d^3*e*f)*n)*x)*sqrt((e^2*x^2 + a*f^2 +
2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*f^4
*n^6 - 35*e*f^4*n^4 + 259*e*f^4*n^2 - 225*e*f^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^2 \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2
*d*e*x/f^2)*f + d)^n, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}\right)^2 \left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}\,f\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e*x/f^2+e^2/f^2*x^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2/f^2*x^2)^(1/2
))^n,x)
```

```
[Out] int((a+2*d*e*x/f^2+e^2/f^2*x^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2/f^2*x^2)^(1/2
))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^2 \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2
*d*e*x/f^2)*f + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + f\,\sqrt{a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}} + e\,x\right)^n \left(a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2,x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**2*(d+e*x+f*(a+2*d*e*x/f**2+e**2*
x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

3.508 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=239

$$\frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{8ef^2(3-n)}-\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{8ef^2(1-n)}-\frac{3(d^2-af^2)\left(f\sqrt{\right.}\right.}{}$$

```
[Out] 1/8*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-3+n)/e/f^2
/(3-n)-3/8*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1+n
)/e/f^2/(1-n)-3/8*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^
(1+n)/e/f^2/(1+n)+1/8*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(3+n)/e/f
^2/(3+n)
```

Rubi [A] time = 0.25, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2121, 12, 270}

$$\frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{8ef^2(3-n)}-\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{8ef^2(1-n)}-\frac{3(d^2-af^2)\left(f\sqrt{\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-3
+ n))/(8*e*f^2*(3 - n)) - (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(8*e*f^2*(1 - n)) - (3*(d^2 - a*f^2)*(d +
 e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n))/(8*e*f^2*(1 + n)
) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)/(8*e*f^2*
(3 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-4+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}{16e^4}dx,x,d+e\right.}{f^2}$$

$$= \frac{\text{Subst}\left(\int x^{-4+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3 d\right.}{8e^4f^2}$$

$$= \frac{\text{Subst}\left(\int \left(-e^3\left(d^2-af^2\right)^3x^{-4+n}+3e^3\left(d^2-af^2\right.\right.\right.}{}$$

$$= \frac{\left(d^2-af^2\right)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-3+n}}{8ef^2(3-n)} -$$

Mathematica [A] time = 0.61, size = 186, normalized size = 0.78

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-3}\left(-\frac{3(d^2-af^2)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^4}{n+1}+\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n-1}-\frac{(d^2-af^2)^3}{n-3}+\frac{\left(f\sqrt{\right.}}{}\right.}{8ef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-3 + n)*(-((d^2 - a*f^2)^3/
(-3 + n)) + (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2)/(-1 + n) - (3*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^
2])^4)/(1 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^6/(3 + n)))/
(8*e*f^2)
```

fricas [A] time = 0.48, size = 239, normalized size = 1.00

$$-\frac{\left(3\,adf^2n^2-9\,adf^2+3\left(e^3n^2-e^3\right)x^3+6\,d^3+9\left(de^2n^2-de^2\right)x^2-3\left(3\,aef^2-\left(aef^2+2\,d^2e\right)n^2\right)x-\left(af^3n^3+\right.\right.}{ef^2n^4-10\,ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*a*d*f^2*n^2 - 9*a*d*f^2 + 3*(e^3*n^2 - e^3)*x^3 + 6*d^3 + 9*(d*e^2*n^2
- d*e^2)*x^2 - 3*(3*a*e*f^2 - (a*e*f^2 + 2*d^2*e)*n^2)*x - (a*f^3*n^3 + (e^
2*f*n^3 - e^2*f*n)*x^2 - (7*a*f^3 - 6*d^2*f)*n + 2*(d*e*f*n^3 - d*e*f*n)*x)
*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*
d*e*x)/f^2) + d)^n/(e*f^2*n^4 - 10*e*f^2*n^2 + 9*e*f^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d
*e*x/f^2)*f + d)^n, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}\right)\left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\, f\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e^2/f^2*x^2+a+2*d*e/f^2*x)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)
^n,x)
```

```
[Out] int((e^2/f^2*x^2+a+2*d*e/f^2*x)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)
^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\, f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d
*e*x/f^2)*f + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\,\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**
2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

3.509 $\int \left(d+e x+f \sqrt{a+\frac{2 d e x}{f^2}+\frac{e^2 x^2}{f^2}}\right)^n \, dx$

Optimal. Leaf size=107

$$\frac{\left(d^2-a f^2\right)\left(f \sqrt{a+\frac{2 d e x}{f^2}+\frac{e^2 x^2}{f^2}}+d+e x\right)^{n-1}}{2 e(1-n)}+\frac{\left(f \sqrt{a+\frac{2 d e x}{f^2}+\frac{e^2 x^2}{f^2}}+d+e x\right)^{n+1}}{2 e(n+1)}$$

```
[Out] 1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1+n)/e/(1-n)
+1/2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2116, 12, 14}

$$\frac{\left(d^2-a f^2\right)\left(f \sqrt{a+\frac{2 d e x}{f^2}+\frac{e^2 x^2}{f^2}}+d+e x\right)^{n-1}}{2 e(1-n)}+\frac{\left(f \sqrt{a+\frac{2 d e x}{f^2}+\frac{e^2 x^2}{f^2}}+d+e x\right)^{n+1}}{2 e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = 2\,\text{Subst}\left(\int \frac{x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)}{4e^2}dx,x,d+ex+f\sqrt{a+\right.$$

$$=\frac{\text{Subst}\left(\int x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)dx,x,d+ex+f\sqrt{a+\frac{2c}{f}}\right.}{2e^2}$$

$$=\frac{\text{Subst}\left(\int\left(-e\left(d^2-af^2\right)x^{-2+n}+ex^n\right)dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x}{f^2}}\right.}{2e^2}$$

$$=\frac{\left(d^2-af^2\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)}+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}}\right.}{2e(1+n)}$$

Mathematica [A] time = 0.39, size = 89, normalized size = 0.83

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-1}\left(\frac{af^2-d^2}{n-1}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+1}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

fricas [A] time = 0.47, size = 80, normalized size = 0.75

$$\frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}-ex-d\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{en^2-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="frica
s")
```

```
[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac"
)
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int\left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\,f\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d + f\,\sqrt{a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}} + e\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n,x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

3.510 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

```
[Out] -2*f^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1
+n)/e/(-a*f^2+d^2)/(1+n)
```

Rubi [A] time = 0.29, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2121, 364}

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \left(2f^2\right)\text{Subst}\left(\int \frac{x^n}{d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2}\,dx, x, d+ex+f\sqrt{a+\frac{2d}{f}}\right.$$

$$= -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2d}{f^2}}\right.}{d^2-af^2}\right.}{e\left(d^2-af^2\right)(1+n)}$$

Mathematica [A] time = 0.14, size = 112, normalized size = 0.92

$$-\frac{2f^2\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(1 + n)*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^2}{e^2x^2+af^2+2\,dex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^2/(e^2*x^2
 + a*f^2 + 2*d*e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\, f\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x),x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\, f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2),x)
```

```
[Out] Timed out
```

3.511 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{8f^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}\,{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+3)\left(d^2-af^2\right)^3}$$

```
[Out] -8*f^4*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(3
+n)/e/(-a*f^2+d^2)^3/(3+n)
```

Rubi [A] time = 0.27, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2121, 12, 364}

$$-\frac{8f^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}\,{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+3)\left(d^2-af^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2)^2,x]
```

```
[Out] (-8*f^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)*Hyper
geometric2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^3*(3 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}dx = \left(2f^4\right)\text{Subst}\left(\int \frac{4e^2x^{2+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}dx, x, d+ex+f\sqrt{a+\frac{2d}{f}}\right.$$

$$= \left(8e^2f^4\right)\text{Subst}\left(\int \frac{x^{2+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}dx, x, d+ex+f\sqrt{a+}\right.$$

$$= -\frac{8f^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n}{}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\right.}{d^2-af^2}\right.}{e\left(d^2-af^2\right)^3(3+n)}$$

Mathematica [A] time = 0.19, size = 112, normalized size = 0.92

$$-\frac{8f^4\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n+3}{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e(n+3)\left(d^2-af^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2)^2,x]
```

```
[Out] (-8*f^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(3 + n)*Hypergeometri
c2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^3*(3 + n))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^4}{e^4x^4+4\,de^3x^3+a^2f^4+4\,adef^2x+2\left(ae^2f^2+2\,d^2e^2\right)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4/(e^4*x^4
 + 4*d*e^3*x^3 + a^2*f^4 + 4*a*d*e*f^2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*x^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^2, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}\, f\right)^n}{\left(\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x)^2,x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\, f + d\right)^n}{\left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d + f\,\sqrt{a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}} + e\,x\right)^n}{\left(a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2,x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**2,x)
```

```
[Out] Timed out
```

3.512 $\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx$

Optimal. Leaf size=107

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[Out] 1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1+n)/e/(1-n)
+1/2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)
```

Rubi [A] time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2118, 2116, 12, 14}

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2118

```
Int[((g_.) + (h_.)*((u_) + (f_.)*Sqrt[v_])^(n_))^(p_.), x_Symbol] :> Int[(g
 + h*(ExpandToSum[u, x] + f*Sqrt[ExpandToSum[v, x]])^n)^p, x] /; FreeQ[{f,
g, h, n}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !(LinearMatchQ[u, x]
&& QuadraticMatchQ[v, x]) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x,
 2]*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx = \int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= 2\,\text{Subst}\left(\int \frac{x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)}{4e^2}dx, x, d+ex+f\sqrt{\ldots}\right.$$
$$= \frac{\text{Subst}\left(\int x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)dx, x, d+ex+f\sqrt{a\ldots}\right.}{2e^2}$$
$$= \frac{\text{Subst}\left(\int \left(-e\left(d^2-af^2\right)x^{-2+n}+ex^n\right)dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}\ldots}\right.}{2e^2}$$
$$= \frac{\left(d^2-af^2\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)} + \frac{\left(d+ex+f\sqrt{a+\ldots}\right.}{2e(1\ldots}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.83

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n-1}\left(\frac{af^2-d^2}{n-1}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+1}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

fricas [A] time = 0.48, size = 80, normalized size = 0.75

$$\frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}-ex-d\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{en^2-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="frica
s")
```

```
[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="giac"
)
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{\frac{a f^2 + (ex + 2d) ex}{f^2}}\, f\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{af^2 + (ex + 2\,d)ex}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d + e\,x + f\,\sqrt{\frac{a\,f^2 + e\,x\,\left(2\,d + e\,x\right)}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n,x)
```

```
[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

3.513 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

```
[Out] -2*f^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1
+n)/e/(-a*f^2+d^2)/(1+n)
```

Rubi [A] time = 0.50, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2127, 2121, 364}

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2127

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
```

```
2, 0]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx=\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$$

$$=\left(2f^2\right)\text{Subst}\left(\int \frac{x^n}{d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\right.$$

$$=-\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right.}{d^2-af^2}\right.}{e\left(d^2-af^2\right)(1+n)}$$

Mathematica [A] time = 0.04, size = 112, normalized size = 0.92

$$-\frac{2f^2\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(1 + n)*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^2}{e^2x^2+af^2+2\,dex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^2/(e^2*x^2
 + a*f^2 + 2*d*e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="giac")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d+\sqrt{\frac{a f^2+(ex+2d)ex}{f^2}}\, f\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x),x)
```

```
[Out] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d+\sqrt{af^2+(ex+2\,d)ex}\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/(e^2*x^2/f^2 + a + 2*
d*e*x/f^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,(2\,d+e\,x)}{f^2}}\right)^n}{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/(a + (e^2*x^2)/f^
2 + (2*d*e*x)/f^2),x)
```

```
[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/(a + (e^2*x^2)/f^
2 + (2*d*e*x)/f^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/(a+2*d*e*x/f**2
+e**2*x**2/f**2),x)
```

```
[Out] Timed out
```

3.514 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2} \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=297

$$-\frac{\left(d^2-af^2\right)^4 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-4}}{16ef^3(4-n)}+\frac{\left(d^2-af^2\right)^3 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef^3(2-n)}+\frac{3\left(d^2-af^2\right)^2\left(f\right.}{}$$

```
[Out] -1/16*(-a*f^2+d^2)^4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-4+n)/e/f
^3/(4-n)+1/4*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-2
+n)/e/f^3/(2-n)+3/8*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/
2))^n/e/f^3/n-1/4*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^
(2+n)/e/f^3/(2+n)+1/16*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(4+n)/e/
f^3/(4+n)
```

Rubi [A] time = 0.42, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2121, 12, 270}

$$-\frac{\left(d^2-af^2\right)^4 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-4}}{16ef^3(4-n)}+\frac{\left(d^2-af^2\right)^3 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef^3(2-n)}+\frac{3\left(d^2-af^2\right)^2\left(f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -((d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-4
 + n))/(16*e*f^3*(4 - n)) + ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f^3*(2 - n)) + (3*(d^2 - a*f^2)^2*(d
 + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(8*e*f^3*n) - ((d^2
- a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n))/(4*
e*f^3*(2 + n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(4 +
 n)/(16*e*f^3*(4 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-5+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^4}{32e^5}dx,x,d\right.}{f^3}$$

$$= \frac{\text{Subst}\left(\int x^{-5+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^4\right.}{16e^5f^3}$$

$$= \frac{\text{Subst}\left(\int \left(e^4\left(d^2-af^2\right)^4x^{-5+n}-4e^4\left(d^2-af\right.\right.\right.}{}$$

$$= -\frac{\left(d^2-af^2\right)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-4}}{16ef^3(4-n)}$$

Mathematica [A] time = 1.20, size = 228, normalized size = 0.77

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n\left(\frac{\left(d^2-af^2\right)^4}{(n-4)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^4}-\frac{4\left(d^2-af^2\right)^3}{(n-2)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}-\frac{4\left(d^2-af^2\right)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right.}{n+2}\right.}{16ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (
2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((6*(d^2 - a*f^2)^2)/n + (
d^2 - a*f^2)^4/((-4 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4) -
 (4*(d^2 - a*f^2)^3)/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2) - (4*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2)/(2
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4/(4 + n)))/(16*e*f^3
)
```

fricas [A] time = 0.48, size = 377, normalized size = 1.27

$$\frac{\left(a^2f^4n^4+24\,a^2f^4-48\,ad^2f^2+\left(e^4n^4-4\,e^4n^2\right)x^4+24\,d^4+4\left(de^3n^4-4\,de^3n^2\right)x^3-4\left(4\,a^2f^4-3\,ad^2f^2\right)n^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^2*f^4*n^4 + 24*a^2*f^4 - 48*a*d^2*f^2 + (e^4*n^4 - 4*e^4*n^2)*x^4 + 24*d
^4 + 4*(d*e^3*n^4 - 4*d*e^3*n^2)*x^3 - 4*(4*a^2*f^4 - 3*a*d^2*f^2)*n^2 + 2*
((a*e^2*f^2 + 2*d^2*e^2)*n^4 - 2*(5*a*e^2*f^2 + d^2*e^2)*n^2)*x^2 + 4*(a*d*
e*f^2*n^4 - 2*(5*a*d*e*f^2 - 3*d^3*e)*n^2)*x - 4*(a*d*f^3*n^3 + (e^3*f*n^3
- 4*e^3*f*n)*x^3 + 3*(d*e^2*f*n^3 - 4*d*e^2*f*n)*x^2 - 2*(5*a*d*f^3 - 3*d^3
*f)*n + ((a*e*f^3 + 2*d^2*e*f)*n^3 - 2*(5*a*e*f^3 + d^2*e*f)*n)*x)*sqrt((e^
2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^
2) + d)^n/(e*f^3*n^5 - 20*e*f^3*n^3 + 64*e*f^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^{\frac{3}{2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a
 + 2*d*e*x/f^2)*f + d)^n, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}\right)^{\frac{3}{2}} \left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}\,f\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(3/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1
/2)*f)^n,x)
```

```
[Out] int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(3/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1
/2)*f)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^{\frac{3}{2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a
 + 2*d*e*x/f^2)*f + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + f\,\sqrt{a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}} + e\,x\right)^n \left(a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(3/2)*(d+e*x+f*(a+2*d*e*x/f**2+e
**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

3.515 $\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}} \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n \, dx$

Optimal. Leaf size=171

$$-\frac{\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)}-\frac{\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)}$$

```
[Out] -1/4*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-2+n)/e/f/
(2-n)-1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/f/n+
1/4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)/e/f/(2+n)
```

Rubi [A] time = 0.32, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2121, 12, 270}

$$-\frac{\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)}-\frac{\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)
/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -((d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2
 + n))/(4*e*f*(2 - n)) - ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^n)/(2*e*f*n) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^
2*x^2)/f^2])^(2 + n)/(4*e*f*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\,\text{Subst}\left(\int \frac{x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}{8e^3}dx,x,d+ex\right.}{f}$$

$$= \frac{\text{Subst}\left(\int x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2 dx,\right.}{4e^3f}$$

$$= \frac{\text{Subst}\left(\int \left(e^2\left(d^2-af^2\right)^2x^{-3+n}-2e^2\left(d^2-af^2\right)x^-\right.\right.}{4e^3}$$

$$= -\frac{\left(d^2-af^2\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef(2-n)} -$$

Mathematica [A] time = 0.35, size = 135, normalized size = 0.79

$$\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n\left(\frac{\left(d^2-af^2\right)^2}{(n-2)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}+\frac{2\left(af^2-d^2\right)}{n}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+2}\right)}{4ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((2*(-d^2 + a*f^2))/n + (d
^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2) +
(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(2 + n)))/(4*e*f)
```

fricas [A] time = 0.45, size = 122, normalized size = 0.71

$$\frac{\left(e^2n^2x^2+af^2n^2+2\,den^2x-2\,af^2+2\,d^2-2\left(efnx+dfn\right)\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{efn^3-4\,efn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (e^2*n^2*x^2 + a*f^2*n^2 + 2*d*e*n^2*x - 2*a*f^2 + 2*d^2 - 2*(e*f*n*x + d*f
*n)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 +
 2*d*e*x)/f^2) + d)^n/(e*f*n^3 - 4*e*f*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a +
 2*d*e*x/f^2)*f + d)^n, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\, f\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1
/2)*f)^n,x)
```

```
[Out] int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1
/2)*f)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\, f+d\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a +
 2*d*e*x/f^2)*f + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)}^n\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/f**2+e
**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2)*(d + e*x + f*sqrt(a + 2*d*
e*x/f**2 + e**2*x**2/f**2))**n, x)
```

3.516 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=41

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

```
[Out] f*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/n
```

Rubi [A] time = 0.25, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2121, 12, 30}

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx = (2f)\,\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$= \frac{f\,\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e}$$

$$= \frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 0.88

$$\frac{f\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a +
(2*d*e*x)/f^2 + (e^2*x^2)/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

fricas [A] time = 0.46, size = 41, normalized size = 1.00

$$\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/
f^2 + a + 2*d*e*x/f^2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\,f\right)^n}{\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x)^(1/2),x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n}{\sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/
f^2 + a + 2*d*e*x/f^2), x)
```

mupad [B] time = 3.08, size = 41, normalized size = 1.00

$$\frac{f\left(d + e\,x + f\,\sqrt{\frac{e^2\,x^2+2\,d\,e\,x+a\,f^2}{f^2}}\right)^n}{e\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2),x)
```

```
[Out] (f*(d + e*x + f*((a*f^2 + e^2*x^2 + 2*d*e*x)/f^2)^(1/2))^n)/(e*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/sqrt(a +
2*d*e*x/f**2 + e**2*x**2/f**2), x)
```

3.517 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{4f^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2}$$

```
[Out] 4*f^3*hypergeom([2, 1+1/2*n],[2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)/e
/(-a*f^2+d^2)^2/(2+n)
```

Rubi [A] time = 0.30, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2121, 12, 364}

$$\frac{4f^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hyperg
eometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
(e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx = \left(2f^3\right)\text{Subst}\left(\int \frac{2ex^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx, x, d+ex+f\sqrt{a+\frac{2d}{f}}\right.$$

$$= \left(4ef^3\right)\text{Subst}\left(\int \frac{x^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx, x, d+ex+f\sqrt{a+\frac{2}{}}\right.$$

$$= \frac{4f^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2}{f}}\right.}{d^2-af^2}\right.}{e\left(d^2-af^2\right)^2(2+n)}$$

Mathematica [A] time = 0.15, size = 112, normalized size = 0.92

$$\frac{4f^3\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(2 + n)*Hypergeometric
2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^
2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 + n))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^4\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^4x^4+4\,de^3x^3+a^2f^4+4\,adef^2x+2\left(ae^2f^2+2\,d^2e^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4*sqrt((e^
2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^4*x^4 + 4*d*e^3*x^3 + a^2*f^4 + 4*a*d*e*f^
2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^(3/2), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d+\sqrt{\frac{e^{2}x^{2}}{f^{2}}+a+\frac{2dex}{f^{2}}}\, f\right)^{n}}{\left(\frac{e^{2}x^{2}}{f^{2}}+a+\frac{2dex}{f^{2}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x)^(3/2),x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*
x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+\sqrt{\frac{e^{2}x^{2}}{f^{2}}+a+\frac{2\,dex}{f^{2}}}\, f+d\right)^{n}}{\left(\frac{e^{2}x^{2}}{f^{2}}+a+\frac{2\,dex}{f^{2}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)}^n}{{\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^{2}} + \frac{e^{2} x^{2}}{f^{2}}}\right)^{n}}{\left(a + \frac{2 d e x}{f^{2}} + \frac{e^{2} x^{2}}{f^{2}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/(a + 2*d*
e*x/f**2 + e**2*x**2/f**2)**(3/2), x)
```

3.518 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx$

Optimal. Leaf size=41

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

```
[Out] f*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/n
```

Rubi [A] time = 0.44, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2127, 2121, 12, 30}

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 + e*x*(
2*d + e*x))/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2127

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx=\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$$

$$=(2f)\,\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$=\frac{f\,\text{Subst}\left(\int x^{-1+n}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e}$$

$$=\frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.88

$$\frac{f\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 +
 e*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

fricas [A] time = 0.43, size = 41, normalized size = 1.00

$$\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2 +
 (e*x + 2*d)*e*x)/f^2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + d + \sqrt{\frac{a\,f^2 + (ex + 2d)ex}{f^2}}\,f\right)^n}{\sqrt{\frac{a\,f^2 + (ex + 2d)ex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2+(e*x+2*d)*e*x)/f^
2)^(1/2),x)
```

```
[Out] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2+(e*x+2*d)*e*x)/f^
2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$f \int \frac{\left(ex + d + \sqrt{af^2 + (ex + 2\,d)ex}\right)^n}{\sqrt{af^2 + (ex + 2\,d)ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] f*integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/sqrt(a*f^2 + (e*x +
 2*d)*e*x), x)
```

mupad [B] time = 3.08, size = 39, normalized size = 0.95

$$\frac{f\left(d + e\,x + f\,\sqrt{\frac{a\,f^2 + e\,x\,(2\,d + e\,x)}{f^2}}\right)^n}{e\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2 + e*x*(2*
d + e*x))/f^2)^(1/2),x)
```

```
[Out] (f*(d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n)/(e*n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2+e*x*(e
*x+2*d))/f**2)**(1/2),x)
```

```
[Out] Timed out
```

3.519 $\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n d$

Optimal. Leaf size=327

$$-\frac{(d^2-af^2)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}-\frac{(d^2-af^2)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2de}{f^2}}\right.}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

```
[Out] -1/4*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-2+n)*(a*g
+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)/e/f/(2-n)/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(
1/2)-1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n*(a*g+2*
d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)/e/f/n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)+1/
4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)*(a*g+2*d*e*g*x/f^2+e^2*
g*x^2/f^2)^(1/2)/e/f/(2+n)/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2123, 2121, 12, 270}

$$-\frac{(d^2-af^2)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}-\frac{(d^2-af^2)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2de}{f^2}}\right.}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -((d^2 - a*f^2)^2*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f*(2 - n)*Sqrt[a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) - ((d^2 - a*f^2)*Sqrt[a*g + (2*d*e*g*x)/f
^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
^n)/(2*e*f*n*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) + (Sqrt[a*g + (2*d*e*
g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)
/f^2])^(2 + n))/(4*e*f*(2 + n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
```

```
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2123

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[((i/c)^(m
- 1/2)*Sqrt[g + h*x + i*x^2])/Sqrt[a + b*x + c*x^2], Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && IGtQ[m + 1/2, 0] &&  !GtQ[i/c, 0]
```

Rubi steps

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d \cdots\right.}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2 \cdots}{\cdots}}}$$

$$= \frac{\left(2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{-3+n}\left(d^2e-\cdots\right.}{\cdots}\right.}{f\sqrt{a \cdots}}$$

$$= \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\ \text{Subst}\left(\int x^{-3+n}\left(d^2e-\cdots\right.\right.}{4e^3 \cdots}$$

$$= \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\ \text{Subst}\left(\int \left(e^2\left(d^2-af^2 \cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+\cdots\right.}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\cdots}}$$

Mathematica [A] time = 0.23, size = 175, normalized size = 0.54

$$\frac{\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n\left(\frac{\left(d^2-af^2\right)^2}{(n-2)\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}+\frac{2\left(af^2-d^2\right)}{n}+\frac{\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^2}{n+2}\right)}{4ef\sqrt{a+\frac{ex(2d+ex)}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a
 + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x)
)/f^2])^n*((2*(-d^2 + a*f^2))/n + (d^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sq
```

```
rt[a + (e*x*(2*d + e*x))/f^2])^2) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^2/(2 + n)))/(4*e*f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
```

fricas [A] time = 0.48, size = 231, normalized size = 0.71

$$-\frac{\left(2 e^{3} n x^{3}+6 d e^{2} n x^{2}+2 a d f^{2} n+2\left(a e f^{2}+2 d^{2} e\right) n x-\left(e^{2} f n^{2} x^{2}+a f^{3} n^{2}+2 d e f n^{2} x-2 a f^{3}+2 d^{2} f\right) \sqrt{\frac{e^{2} x^{2}+a f^{2}}{f^{2}}}\right.}{a e f^{2} n^{3}-4 a e f^{2} n+\left(e^{3} n^{3}-4 e^{3} n\right) x^{2}+2\left(d e^{2} n^{3}-4 d e^{2} n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(2*e^3*n*x^3 + 6*d*e^2*n*x^2 + 2*a*d*f^2*n + 2*(a*e*f^2 + 2*d^2*e)*n*x - (
e^2*f*n^2*x^2 + a*f^3*n^2 + 2*d*e*f*n^2*x - 2*a*f^3 + 2*d^2*f)*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) +
 d)^n*sqrt((e^2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)/(a*e*f^2*n^3 - 4*a*e*f^2*
n + (e^3*n^3 - 4*e^3*n)*x^2 + 2*(d*e^2*n^3 - 4*d*e^2*n)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^{2} g x^{2}}{f^{2}}+a g+\frac{2 \, d e g x}{f^{2}}}\left(e x+\sqrt{\frac{e^{2} x^{2}}{f^{2}}+a+\frac{2 \, d e x}{f^{2}}} f+d\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^{2} g \,x^{2}}{f^{2}}+a g +\frac{2 d e g x}{f^{2}}}\, \left(e x +d +\sqrt{\frac{e^{2} x^{2}}{f^{2}}+a +\frac{2 d e x}{f^{2}}}\, f \right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2
*x)^(1/2)*f)^n,x)
```

```
[Out] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2
*x)^(1/2)*f)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^{2} g x^{2}}{f^{2}}+a g+\frac{2 \, d e g x}{f^{2}}}\left(e x+\sqrt{\frac{e^{2} x^{2}}{f^{2}}+a+\frac{2 \, d e x}{f^{2}}} f+d\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)}^n\,\sqrt{a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{g \left(a + \frac{2dex}{f^{2}} + \frac{e^{2}x^{2}}{f^{2}}\right)} \left(d + ex + f \sqrt{a + \frac{2dex}{f^{2}} + \frac{e^{2}x^{2}}{f^{2}}}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f**2+e**2*g*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/
f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(g*(a + 2*d*e*x/f**2 + e**2*x**2/f**2))*(d + e*x + f*sqrt(a +
2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

3.520 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$

Optimal. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1
/2))^n/e/n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2125, 2121, 12, 30}

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*d
*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2125

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[((i/c)^(m
+ 1/2)*Sqrt[a + b*x + c*x^2])/Sqrt[g + h*x + i*x^2], Int[(a + b*x + c*x^2)^
```

```
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx = \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\cdots}\right)}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\cdots}\right)}{e\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Mathematica [A] time = 0.10, size = 76, normalized size = 0.82

$$\frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g
+ (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

```
[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/
f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

fricas [A] time = 0.42, size = 117, normalized size = 1.26

$$\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^3\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^3gnx^2+aef^2gn+2\,de^2gnx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2 +
 a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n*x
^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n}{\sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^
2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}\,f\right)^n}{\sqrt{\frac{e^2g\,x^2}{f^2} + ag + \frac{2degx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*g*x^2+a*g+2*d*e/
f^2*g*x)^(1/2),x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*g*x^2+a*g+2*d*e/
f^2*g*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n}{\sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^
2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d + f\,\sqrt{a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}} + e\,x\right)^n}{\sqrt{a\,g + \frac{e^2\,g\,x^2}{f^2} + \frac{2\,d\,e\,g\,x}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{g\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*
x/f**2+e**2*g*x**2/f**2)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/sqrt(g*(a
 + 2*d*e*x/f**2 + e**2*x**2/f**2)), x)
```

3.521 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{eg(n+2)\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] 4*f^3*hypergeom([2, 1+1/2*n],[2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^2/(-a*f^2+d^2))*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e
*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)/e/(-a*f^2+d^2)^2/g/(2+n)/(a*g+2*d*e*g*x/f^
2+e^2*g*x^2/f^2)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2125, 2121, 12, 364}

$$\frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{eg(n+2)\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (2*d*e*g
*x)/f^2 + (e^2*g*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2
, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(
e*(d^2 - a*f^2)^2*g*(2 + n)*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
```

```
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2125

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[((i/c)^(m
+ 1/2)*Sqrt[a + b*x + c*x^2])/Sqrt[g + h*x + i*x^2], Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(2f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{2ex^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx,x,d+ex+\cdots\right)}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(4ef^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx,x,d+ex+\cdots\right)}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}{}_2F_1\left(2,\frac{2+n}{2};\cdots\right)}{e\left(d^2-af^2\right)^2g(2+n)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Mathematica **[A]** time = 0.19, size = 152, normalized size = 0.86

$$\frac{4f^3\left(a+\frac{ex(2d+ex)}{f^2}\right)^{3/2}\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2\left(g\left(a+\frac{ex(2d+ex)}{f^2}\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (2
*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*(a + (e*x*(2*d + e*x))/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (e*x*(2*d +
e*x))/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f
*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 +
 n)*(g*(a + (e*x*(2*d + e*x))/f^2))^(3/2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^4\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}}{e^4g^2x^4+4\,de^3g^2x^3+a^2f^4g^2+4\,adef^2g^2x+2\left(ae^2f^2+2\,d^2e^2\right)g^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4*sqrt((e^
2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)/(e^4*g^2*x^4 + 4*d*e^3*g^2*x^3 + a^2*f^
4*g^2 + 4*a*d*e*f^2*g^2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*g^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*g*x^2/f^
2 + a*g + 2*d*e*g*x/f^2)^(3/2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int\frac{\left(ex+d+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2dex}{f^2}}\,f\right)^n}{\left(\frac{e^2g\,x^2}{f^2}+ag+\frac{2degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*g*x^2+a*g+2*d*e/
f^2*g*x)^(3/2),x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*g*x^2+a*g+2*d*e/
f^2*g*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*g*x^2/f^
2 + a*g + 2*d*e*g*x/f^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d + f\,\sqrt{a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}} + e\,x\right)^n}{\left(a\,g + \frac{e^2\,g\,x^2}{f^2} + \frac{2\,d\,e\,g\,x}{f^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(3/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*
x/f**2+e**2*g*x**2/f**2)**(3/2),x)
```

```
[Out] Timed out
```

3.522 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx$

Optimal. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1
/2))^n/e/n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2127, 2125, 2121, 12, 30}

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g + e*g
*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2125

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[((i/c)^(m
+ 1/2)*Sqrt[a + b*x + c*x^2])/Sqrt[g + h*x + i*x^2], Int[(a + b*x + c*x^2)^
```

```
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rule 2127

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx=\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$$

$$=\frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$=\frac{\left(2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$=\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int x^{-1+n}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$=\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 0.82

$$\frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(f\sqrt{a+\frac{ex(2d+ex)}{f^2}}+d+ex\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g
 + e*g*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/
f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

fricas [A] time = 0.45, size = 117, normalized size = 1.26

$$\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^3\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^3gnx^2+aef^2gn+2\,de^2gnx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2 +
 a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n*x
^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\sqrt{\frac{af^2g+(ex+2\,d)egx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2*g
 + (e*x + 2*d)*e*g*x)/f^2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d+\sqrt{\frac{a\,f^2+(ex+2d)ex}{f^2}}\,f\right)^n}{\sqrt{\frac{a\,f^2g+(ex+2d)egx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2*g+e*g*x*(e*x+2*d)
)/f^2)^(1/2),x)
```

```
[Out] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2*g+e*g*x*(e*x+2*d)
)/f^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$f\int \frac{\left(ex+d+\sqrt{af^2+(ex+2\,d)ex}\right)^n}{\sqrt{af^2g+(ex+2\,d)egx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] f*integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/sqrt(a*f^2*g + (e*x
 + 2*d)*e*g*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,\left(2\,d+e\,x\right)}{f^2}}\right)^n}{\sqrt{\frac{a\,g\,f^2+e\,g\,x\,\left(2\,d+e\,x\right)}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*x
*(2*d + e*x))/f^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*x
*(2*d + e*x))/f^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2*g+e*g*
x*(e*x+2*d))/f**2)**(1/2),x)
```

```
[Out] Timed out
```

3.523 $\int \frac{1}{(a+bx)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(-\frac{b^2c}{a^2d};\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^2}\sqrt{a^2f+b^2e}}{\sqrt{e+fx^2}\sqrt{a^2d+b^2c}}\right)}{\sqrt{a^2d+b^2c}\sqrt{a^2f+b^2e}}$$

```
[Out] -b*arctanh((a^2*f+b^2*e)^(1/2)*(d*x^2+c)^(1/2)/(a^2*d+b^2*c)^(1/2)/(f*x^2+e
)^(1/2))/(a^2*d+b^2*c)^(1/2)/(a^2*f+b^2*e)^(1/2)+EllipticPi(x*d^(1/2)/(-c)^
(1/2),-b^2*c/a^2/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e
)^(1/2)/a/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2113, 538, 537, 571, 93, 208}

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(-\frac{b^2c}{a^2d};\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^2}\sqrt{a^2f+b^2e}}{\sqrt{e+fx^2}\sqrt{a^2d+b^2c}}\right)}{\sqrt{a^2d+b^2c}\sqrt{a^2f+b^2e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[b^2*e + a^2*f]*Sqrt[c + d*x^2])/(Sqrt[b^2*c + a^2*d]*Sqr
t[e + f*x^2])])/(Sqrt[b^2*c + a^2*d]*Sqrt[b^2*e + a^2*f])) + (Sqrt[-c]*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b^2*c)/(a^2*d)), ArcSin[(
Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = a\int \frac{1}{\left(a^2-b^2x^2\right)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx - b\int \frac{x}{\left(a^2-b^2x^2\right)\sqrt{c+dx^2}\sqrt{e+\cdots}}$$

$$= -\left(\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\left(a^2-b^2x\right)\sqrt{c+dx}\sqrt{e+fx}}\,dx, x, x^2\right)\right) + \frac{\left(a\sqrt{1+\frac{dx^2}{c}}\right)\cdots}{\cdots}$$

$$= -\left(b\,\text{Subst}\left(\int \frac{1}{b^2c+a^2d-\left(b^2e+a^2f\right)x^2}\,dx, x, \frac{\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\right)\right) + \frac{\left(a\sqrt{1+\cdots}\right)\cdots}{\cdots}$$

$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{b^2e+a^2f}\sqrt{c+dx^2}}{\sqrt{b^2c+a^2d}\sqrt{e+fx^2}}\right)}{\sqrt{b^2c+a^2d}\sqrt{b^2e+a^2f}} + \frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{b^2c}{a^2d};\sin^{-1}\left(\cdots\right.\right.}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Mathematica [C] time = 1.62, size = 772, normalized size = 4.04

$$\frac{2\sqrt{d}\left(\sqrt{c}+i\sqrt{d}x\right)\left(\sqrt{e}+i\sqrt{f}x\right)\sqrt{\frac{\left(\sqrt{d}x+i\sqrt{c}\right)\left(\sqrt{d}\sqrt{e}-\sqrt{c}\sqrt{f}\right)}{\left(\sqrt{d}x-i\sqrt{c}\right)\left(\sqrt{c}\sqrt{f}+\sqrt{d}\sqrt{e}\right)}}\sqrt{\frac{\sqrt{c}\sqrt{d}\left(\sqrt{f}x+i\sqrt{e}\right)}{\left(\sqrt{d}x-i\sqrt{c}\right)\left(\sqrt{c}\sqrt{f}-\sqrt{d}\sqrt{e}\right)}}\left(\left(b\sqrt{c}+ia\sqrt{d}\right)F\left(\sin^{-1}\cdots\right.\right.}{\sqrt{c+dx^2}\sqrt{e+fx^2}\left(b\sqrt{c}-ia\sqrt{d}\right)\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (2*Sqrt[d]*(Sqrt[c] + I*Sqrt[d]*x)*Sqrt[((Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f]
)*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])*((-I)*Sqrt[
c] + Sqrt[d]*x))]*(Sqrt[e] + I*Sqrt[f]*x)*Sqrt[(Sqrt[c]*Sqrt[d]*(I*Sqrt[e]
+ Sqrt[f]*x))/((-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])*((-I)*Sqrt[c] + Sqrt[
d]*x))]*((b*Sqrt[c] + I*a*Sqrt[d])*EllipticF[ArcSin[Sqrt[((Sqrt[d]*Sqrt[e]
- Sqrt[c]*Sqrt[f])*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqr
t[f])*((-I)*Sqrt[c] + Sqrt[d]*x))]], (Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])^2/
(Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f])^2] - 2*b*Sqrt[c]*EllipticPi[((b*Sqrt[c]
 - I*a*Sqrt[d])*(Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f]))/((b*Sqrt[c] + I*a*Sqrt
[d])*(-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])), ArcSin[Sqrt[((Sqrt[d]*Sqrt[e]
 - Sqrt[c]*Sqrt[f])*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sq
rt[f])*((-I)*Sqrt[c] + Sqrt[d]*x))]], (Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])^2
/(Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f])^2]))/((b*Sqrt[c] - I*a*Sqrt[d])*(b*Sqr
```

```
t[c] + I*a*Sqrt[d])*(-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])*Sqrt[(Sqrt[c]*Sq
rt[d]*(Sqrt[e] + I*Sqrt[f]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])*(Sqrt[c
] + I*Sqrt[d]*x))]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.08, size = 353, normalized size = 1.85

$$\frac{\left(-\sqrt{-\frac{d}{c}}\, \sqrt{d f\, x^{4}+c f\, x^{2}+d e\, x^{2}+c e}\; a \operatorname{arctanh}\left(\frac{2 a^{2} d f\, x^{2}+b^{2} c f\, x^{2}+b^{2} d e\, x^{2}+a^{2} c f+a^{2} d e+2 b^{2} c e}{2 \sqrt{\frac{a^{4} d f+a^{2} b^{2} c f+a^{2} b^{2} d e+b^{4} c e}{b^{4}}}\, \sqrt{d f\, x^{4}+c f\, x^{2}+d e\, x^{2}+c e}\; b^{2}}\right)+2 \sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}}{2 \sqrt{\frac{a^{4} d f+a^{2} b^{2} c f+a^{2} b^{2} d e+b^{4} c e}{b^{4}}}\, \sqrt{-\frac{d}{c}}\, \left(d f\, x^{4}+c f\, x^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/2*(2*b*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-1/c*d)^(1/2
),-b^2*c/a^2/d,(-f/e)^(1/2)/(-1/c*d)^(1/2))*((a^4*d*f+a^2*b^2*c*f+a^2*b^2*d
*e+b^4*c*e)/b^4)^(1/2)-arctanh(1/2*(2*a^2*d*f*x^2+b^2*c*f*x^2+b^2*d*e*x^2+a
^2*c*f+a^2*d*e+2*b^2*c*e)/b^2/((a^4*d*f+a^2*b^2*c*f+a^2*b^2*d*e+b^4*c*e)/b^
4)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2))*(-1/c*d)^(1/2)*a*(d*f*x^4+c*f
*x^2+d*e*x^2+c*e)^(1/2))*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/b/a/((a^4*d*f+a^2*
b^2*c*f+a^2*b^2*d*e+b^4*c*e)/b^4)^(1/2)/(-1/c*d)^(1/2)/(d*f*x^4+c*f*x^2+d*e
*x^2+c*e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^2 + c}\,\sqrt{fx^2 + e}\,(bx + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)*(a + b*x)),x)
[Out] int(1/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(1/((a + b*x)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.524 $\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx$

Optimal. Leaf size=81

$$\frac{\log\left(2\sqrt{-d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}-\frac{\log\left(-2\sqrt{-d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}$$

```
[Out] -1/4*ln(e+2*f*x^2-2*x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+1/4*ln(e+2*f*x
^2+2*x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6, 1164, 628}

$$\frac{\log\left(2\sqrt{-d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}-\frac{\log\left(-2\sqrt{-d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -Log[e - 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f]) + Log[e + 2*S
qrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx &= \int \frac{e-2fx^2}{e^2+4(d+e)fx^2+4f^2x^4}\,dx\\
&= -\frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}}+2x}{-\frac{e}{2f}-\frac{\sqrt{-d}\,x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{-d}\,\sqrt{f}}-\frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}}-2x}{-\frac{e}{2f}+\frac{\sqrt{-d}\,x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{-d}\,\sqrt{f}}\\
&= -\frac{\log\left(e-2\sqrt{-d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}+\frac{\log\left(e+2\sqrt{-d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}
\end{aligned}$$

Mathematica [B] time = 0.12, size = 191, normalized size = 2.36

$$\frac{-\frac{\left(\sqrt{d}\sqrt{d+2e}-d-2e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-\sqrt{d}\sqrt{d+2e}+d+e}}\right)}{\sqrt{-\sqrt{d}\sqrt{d+2e}+d+e}}-\frac{\left(\sqrt{d}\sqrt{d+2e}+d+2e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{d}\sqrt{d+2e}+d+e}}\right)}{\sqrt{\sqrt{d}\sqrt{d+2e}+d+e}}}{2\sqrt{2}\sqrt{d}\sqrt{f}\sqrt{d+2e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] (-(((-d - 2*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d +
e - Sqrt[d]*Sqrt[d + 2*e]]])/Sqrt[d + e - Sqrt[d]*Sqrt[d + 2*e]]) - ((d + 2
*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d + e + Sqrt[d]
*Sqrt[d + 2*e]]])/Sqrt[d + e + Sqrt[d]*Sqrt[d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*S
qrt[d + 2*e]*Sqrt[f])
```

fricas [A] time = 0.75, size = 141, normalized size = 1.74

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4f^2x^4-4(d-e)fx^2+e^2+4\left(2fx^3+ex\right)\sqrt{-df}}{4f^2x^4+4(d+e)fx^2+e^2}\right)}{4df},-\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{d}\right)-\sqrt{df}\arctan\left(\frac{\left(2fx^3+(2d+e)x\right)\sqrt{df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x)
*sqrt(-d*f))/(4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(d*f)*ar
ctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^3 + (2*d + e)*x)*sqrt(d*f)/(d
*e)))/(d*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueDone
```

maple [B] time = 0.05, size = 394, normalized size = 4.86

$$-\frac{\sqrt{2}\,df\operatorname{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{-df-ef+\sqrt{(d+2e)d\,f^2}}}\right)}{4\sqrt{(d+2e)\,d\,f^2}\,\sqrt{-df-ef+\sqrt{(d+2e)\,d\,f^2}}}-\frac{\sqrt{2}\,df\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{df+ef+\sqrt{(d+2e)d\,f^2}}}\right)}{4\sqrt{(d+2e)\,d\,f^2}\,\sqrt{df+ef+\sqrt{(d+2e)\,d\,f^2}}}-\frac{\sqrt{2}\,ef\,\mathrm{arc}}{2\sqrt{(d+2e)\,d\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x)
```

```
[Out] -1/4*f/(d*f^2*(d+2*e))^(1/2)*2^(1/2)/(d*f+f*e+(d*f^2*(d+2*e))^(1/2))^(1/2)*
arctan(f*x*2^(1/2)/(d*f+f*e+(d*f^2*(d+2*e))^(1/2))^(1/2))*d-1/2*f/(d*f^2*(d
```

```
+2*e))^(1/2)*2^(1/2)/(d*f+f*e+(d*f^2*(d+2*e))^(1/2))^(1/2)*arctan(f*x*2^(1/
2)/(d*f+f*e+(d*f^2*(d+2*e))^(1/2))^(1/2))*e-1/4*2^(1/2)/(d*f+f*e+(d*f^2*(d+
2*e))^(1/2))^(1/2)*arctan(f*x*2^(1/2)/(d*f+f*e+(d*f^2*(d+2*e))^(1/2))^(1/2)
)-1/4*f/(d*f^2*(d+2*e))^(1/2)*2^(1/2)/(-d*f-f*e+(d*f^2*(d+2*e))^(1/2))^(1/2
)*arctanh(f*x*2^(1/2)/(-d*f-f*e+(d*f^2*(d+2*e))^(1/2))^(1/2))*d-1/2*f/(d*f^
2*(d+2*e))^(1/2)*2^(1/2)/(-d*f-f*e+(d*f^2*(d+2*e))^(1/2))^(1/2)*arctanh(f*x
*2^(1/2)/(-d*f-f*e+(d*f^2*(d+2*e))^(1/2))^(1/2))*e+1/4*2^(1/2)/(-d*f-f*e+(d
*f^2*(d+2*e))^(1/2))^(1/2)*arctanh(f*x*2^(1/2)/(-d*f-f*e+(d*f^2*(d+2*e))^(1
/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\, f x^2 - e}{4\, f^2 x^4 + 4\, d f x^2 + 4\, e f x^2 + e^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)
```

mupad [B] time = 3.11, size = 50, normalized size = 0.62

$$\frac{\mathrm{atan}\left(\frac{2\, f^{3/2}\, x^3 + 2\, d\, \sqrt{f}\, x + e\, \sqrt{f}\, x}{\sqrt{d}\, e}\right) - \mathrm{atan}\left(\frac{\sqrt{f}\, x}{\sqrt{d}}\right)}{2\, \sqrt{d}\, \sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2),x)
```

```
[Out] (atan((2*f^(3/2)*x^3 + 2*d*f^(1/2)*x + e*f^(1/2)*x)/(d^(1/2)*e)) - atan((f^
(1/2)*x)/d^(1/2)))/(2*d^(1/2)*f^(1/2))
```

sympy [A] time = 0.56, size = 70, normalized size = 0.86

$$\frac{\sqrt{-\frac{1}{df}} \log\left(-dx\sqrt{-\frac{1}{df}} + \frac{e}{2f} + x^2\right)}{4} - \frac{\sqrt{-\frac{1}{df}} \log\left(dx\sqrt{-\frac{1}{df}} + \frac{e}{2f} + x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4+4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4 - sqrt(-1/(d*f))
*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4
```

3.525 $\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx$

Optimal. Leaf size=73

$$\frac{\log\left(2\sqrt{d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}-\frac{\log\left(-2\sqrt{d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}$$

```
[Out] -1/4*ln(e+2*f*x^2-2*x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)+1/4*ln(e+2*f*x^2+2*x
*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6, 1164, 628}

$$\frac{\log\left(2\sqrt{d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}-\frac{\log\left(-2\sqrt{d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -Log[e - 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f]) + Log[e + 2*Sqr
t[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx &= \int \frac{e-2fx^2}{e^2+(-4d+4e)fx^2+4f^2x^4}\,dx\\
&= -\frac{\int \frac{\frac{\sqrt{d}}{\sqrt{f}}+2x}{-\frac{e}{2f}-\frac{\sqrt{d}\,x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{d}\,\sqrt{f}}-\frac{\int \frac{\frac{\sqrt{d}}{\sqrt{f}}-2x}{-\frac{e}{2f}+\frac{\sqrt{d}\,x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{d}\,\sqrt{f}}\\
&= -\frac{\log\left(e-2\sqrt{d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}+\frac{\log\left(e+2\sqrt{d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}
\end{aligned}$$

Mathematica [C] time = 0.14, size = 233, normalized size = 3.19

$$\frac{-\frac{\left(\sqrt{d}\sqrt{2e-d}-id+2ie\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-i\sqrt{d}\sqrt{2e-d}-d+e}}\right)}{\sqrt{-i\sqrt{d}\sqrt{2e-d}-d+e}}-\frac{\left(\sqrt{d}\sqrt{2e-d}+id-2ie\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{i\sqrt{d}\sqrt{2e-d}-d+e}}\right)}{\sqrt{i\sqrt{d}\sqrt{2e-d}-d+e}}}{2\sqrt{2}\sqrt{d}\sqrt{f}\sqrt{2e-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] (-((((-I)*d + (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/
Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d +
 2*e]]) - ((I*d - (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]
*x)/Sqrt[-d + e + I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e + I*Sqrt[d]*Sqrt[
-d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*Sqrt[-d + 2*e]*Sqrt[f])
```

fricas [A] time = 0.75, size = 146, normalized size = 2.00

$$\left[\frac{\sqrt{df}\,\log\left(\frac{4f^2x^4+4(d+e)fx^2+e^2+4\left(2fx^3+ex\right)\sqrt{df}}{4f^2x^4-4(d-e)fx^2+e^2}\right)}{4df},\,-\frac{\sqrt{-df}\,\arctan\left(\frac{\sqrt{-df}\,x}{d}\right)-\sqrt{-df}\,\arctan\left(\frac{\left(2fx^3-(2d-e)x\right)\sqrt{-df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x)*s
qrt(d*f))/(4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(-d*f)*arct
an(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^3 - (2*d - e)*x)*sqrt(-d*f)/(
d*e)))/(d*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc inde
x_m operator + Error: Bad Argument ValueDone
```

maple [B] time = 0.05, size = 394, normalized size = 5.40

$$\frac{\sqrt{2}\,df\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{df-ef+\sqrt{(d-2e)df^2}}}\right)}{4\sqrt{(d-2e)df^2}\,\sqrt{df-ef+\sqrt{(d-2e)df^2}}}+\frac{\sqrt{2}\,df\,\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{-df+ef+\sqrt{(d-2e)df^2}}}\right)}{4\sqrt{(d-2e)df^2}\,\sqrt{-df+ef+\sqrt{(d-2e)df^2}}}-\frac{\sqrt{2}\,ef\,\operatorname{arctanh}}{2\sqrt{(d-2e)df^2}\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4*f/(d*f^2*(d-2*e))^(1/2)*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*a
rctanh(f*x*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*d-1/2*f/(d*f^2*(d
```

```
-2*e))^(1/2)*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*arctanh(f*x*2^(1
/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*e+1/4*2^(1/2)/(d*f-e*f+(d*f^2*(d
-2*e))^(1/2))^(1/2)*arctanh(f*x*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/
2))+1/4*f/(d*f^2*(d-2*e))^(1/2)*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1
/2)*arctan(f*x*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*d-1/2*f/(d*f
^2*(d-2*e))^(1/2)*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*arctan(f*x
*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*e-1/4*2^(1/2)/(-d*f+e*f+(d
*f^2*(d-2*e))^(1/2))^(1/2)*arctan(f*x*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/
2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,fx^2 - e}{4\,f^2x^4 - 4\,dfx^2 + 4\,efx^2 + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)
```

mupad [B] time = 3.14, size = 28, normalized size = 0.38

$$\frac{\operatorname{atanh}\left(\frac{2\,\sqrt{d}\,\sqrt{f}\,x}{2\,f\,x^2+e}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2),x)
```

```
[Out] atanh((2*d^(1/2)*f^(1/2)*x)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))
```

sympy [A] time = 0.58, size = 63, normalized size = 0.86

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}} + \frac{e}{2f} + x^2\right)}{4} + \frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}} + \frac{e}{2f} + x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4-4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4 + sqrt(1/(d*f))*l
og(d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4
```

3.526 $\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2093, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx &= \left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2+16de^2fx^2}\,dx, x, \frac{x}{2e+4fx^3}\right)\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.06, size = 87, normalized size = 2.29

$$-\frac{\text{RootSum}\left[4\#1^6f^2+4\#1^3ef+4\#1^2df+e^2\&,\frac{4\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^5f+3\#1^2e+2\#1d}\&\right]}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] -1/4*RootSum[e^2 + 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + 4*f*Log[x - #1]*#1^3)/(2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/f
```

fricas [B] time = 0.74, size = 153, normalized size = 4.03

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4f^2x^6+4efx^3-4dfx^2+e^2+4(2fx^4+ex)\sqrt{-df}}{4f^2x^6+4efx^3+4dfx^2+e^2}\right)}{4df}, -\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x^2}{d}\right)-\sqrt{df}\arctan\left(\frac{(2fx^5+ex^2+2dx)\sqrt{df}}{de}\right)}{2df}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2 + 4*(2*f*x^4
+ e*x)*sqrt(-d*f))/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2))/(d*f), -1/2*(
sqrt(d*f)*arctan(sqrt(d*f)*x^2/d) - sqrt(d*f)*arctan((2*f*x^5 + e*x^2 + 2*d
*x)*sqrt(d*f)/(d*e)))/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{4fx^3-e}{4f^2x^6+4efx^3+4dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2), x)
```

maple [C] time = 0.01, size = 70, normalized size = 1.84

$$\frac{\left(-4\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2\right)^3 f+e\right)\ln\left(-\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z\right.\right.}{4f\left(6f\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2\right)^5+3e\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2\right)^2+2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x)
```

```
[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e+2*_R*d)*ln(-_R+x),_R=RootOf(4*_Z
^6*f^2+4*_Z^3*e*f+4*_Z^2*d*f+e^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4fx^3-e}{4f^2x^6+4efx^3+4dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2), x)
```

mupad [B] time = 3.32, size = 54, normalized size = 1.42

$$\frac{\mathrm{atan}\left(\frac{2f^{3/2}x^5+2d\sqrt{f}\,x+e\sqrt{f}\,x^2}{\sqrt{d}\,e}\right)-\mathrm{atan}\left(\frac{\sqrt{f}\,x^2}{\sqrt{d}}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 + 4*d*f*x^2 + 4*e*f*x^3),x)
```

```
[Out] (atan((2*f^(3/2)*x^5 + 2*d*f^(1/2)*x + e*f^(1/2)*x^2)/(d^(1/2)*e)) - atan((
f^(1/2)*x^2)/d^(1/2)))/(2*d^(1/2)*f^(1/2))
```

sympy [**B**] time = 0.75, size = 70, normalized size = 1.84

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3+4*d*f*x**2+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*f))
*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

3.527 $\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2093, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx &= \left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2-16de^2fx^2}\,dx,x,\frac{x}{2e+4fx^3}\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.06, size = 87, normalized size = 2.29

$$-\frac{\text{RootSum}\left[4\#1^6f^2+4\#1^3ef-4\#1^2df+e^2\&,\frac{4\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^5f+3\#1^2e-2\#1d}\&\right]}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] -1/4*RootSum[e^2 - 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + 4*f*Log[x - #1]*#1^3)/(-2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/f
```

fricas [B] time = 0.71, size = 155, normalized size = 4.08

$$\left[\frac{\sqrt{df}\log\left(\frac{4f^2x^6+4efx^3+4dfx^2+e^2+4\left(2fx^4+ex\right)\sqrt{df}}{4f^2x^6+4efx^3-4dfx^2+e^2}\right)}{4df}, -\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x^2}{d}\right)-\sqrt{-df}\arctan\left(\frac{\left(2fx^5+ex^2-2dx\right)\sqrt{-df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2 + 4*(2*f*x^4 +
e*x)*sqrt(d*f))/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2))/(d*f), -1/2*(sqr
t(-d*f)*arctan(sqrt(-d*f)*x^2/d) - sqrt(-d*f)*arctan((2*f*x^5 + e*x^2 - 2*d
*x)*sqrt(-d*f)/(d*e)))/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{4fx^3-e}{4f^2x^6+4efx^3-4dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2), x)
```

maple [C] time = 0.01, size = 70, normalized size = 1.84

$$\frac{\left(-4\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2\right)^3f+e\right)\ln\left(-\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-\right.\right.}{4f\left(6f\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2\right)^5+3e\,\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2\right)^2-2d\,\mathrm{Roo}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x)
```

```
[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e-2*_R*d)*ln(-_R+x),_R=RootOf(4*_Z
^6*f^2+4*_Z^3*e*f-4*_Z^2*d*f+e^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4fx^3-e}{4f^2x^6+4efx^3-4dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2), x)
```

mupad [B] time = 3.12, size = 67, normalized size = 1.76

$$-\frac{\mathrm{atanh}\left(\frac{-32fd^2x+27e^3+54fe^2x^3}{16d^{3/2}e\sqrt{f}+32d^{3/2}f^{3/2}x^3-54\sqrt{d}e^2\sqrt{f}x}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 - 4*d*f*x^2 + 4*e*f*x^3),x)

[Out] -atanh((27*e^3 - 32*d^2*f*x + 54*e^2*f*x^3)/(16*d^(3/2)*e*f^(1/2) + 32*d^(3/2)*f^(3/2)*x^3 - 54*d^(1/2)*e^2*f^(1/2)*x))/(2*d^(1/2)*f^(1/2))

sympy [A] time = 0.74, size = 63, normalized size = 1.66

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3-4*d*f*x**2+e**2),x)

[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f))*log(d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4

3.528 $\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

[Out] 1/2*arctan(2*x*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)

Rubi [A] time = 0.10, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2093, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]

[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx = -\left(\left(e^2(1-n)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-1+n)^2x^2}\,dx, x, \frac{x}{e(-1+n)+2f(-1+\right.\right.$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]

[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

fricas [A] time = 0.74, size = 144, normalized size = 3.79

$$\left[-\frac{\sqrt{-df}\log\left(-\frac{4\,dfx^2-4\,f^2x^{2\,n}-4\,\sqrt{-df}\,ex-e^2-4\,\left(2\,\sqrt{-df}\,fx+ef\right)x^n}{4\,dfx^2+4\,f^2x^{2\,n}+4\,efx^n+e^2}\right)}{4\,df}, -\frac{\sqrt{df}\,\arctan\left(\frac{2\,\sqrt{df}\,fx^n+\sqrt{df}\,e}{2\,dfx}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*sqrt(-d*f)*e*x - e^2 -
 4*(2*sqrt(-d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*f*x^n + sqrt(d*f)*e)/(d*
f*x))/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,f(n-1)x^n-e}{4\,dfx^2+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] integrate(-(2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2), x)
```

maple [B] time = 0.07, size = 78, normalized size = 2.05

$$\frac{\ln\left(x^n+\frac{-2dfx+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}-\frac{\ln\left(x^n+\frac{2dfx+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e-2*f*(n-1)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+e*(-d*f)^(1/2))/(-d*f)^(1/2)/f)+1/4/(
-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x+e*(-d*f)^(1/2))/(-d*f)^(1/2)/f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,f(n-1)x^n-e}{4\,dfx^2+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] -integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2), x)
```

mupad [B] time = 3.42, size = 196, normalized size = 5.16

$$\frac{\ln\left(-\frac{e+2\,f\,x^n-2\,f\,n\,x^n}{4\,f^2}-\frac{e^2\,n-4\,d\,f\,x^2+4\,d\,f\,n\,x^2+2\,e\,f\,n\,x^n}{8\,\sqrt{-d}\,f^{5/2}\,x}\right)}{4\,\sqrt{-d}\,\sqrt{f}}-\frac{\mathrm{atan}\left(\frac{x\,\left(8\,d\,f\,n^2-16\,d\,f\,n+8\,d\,f\right)}{4\,\sqrt{d}\,\sqrt{f}\,\left(e\,n-e\,n^2\right)}\right)}{2\,\sqrt{d}\,\sqrt{f}}-\frac{\ln\left(\frac{e^2\,n-4\,d\,f\,x^2+4\,d\,f\,n\,x^2+2\,e\,f\,n\,x^n}{8\,\sqrt{-d}\,f^{5/2}\,x}\right.}{4\,\sqrt{-d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^2 + 4*e*f*x^n),x)
```

```
[Out] log(- (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2) - (e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2
+ 2*e*f*n*x^n)/(8*(-d)^(1/2)*f^(5/2)*x))/(4*(-d)^(1/2)*f^(1/2)) - atan((x*(
8*d*f - 16*d*f*n + 8*d*f*n^2))/(4*d^(1/2)*f^(1/2)*(e*n - e*n^2)))/(2*d^(1/2
)*f^(1/2)) - log((e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2 + 2*e*f*n*x^n)/(8*(-d)^(1
/2)*f^(5/2)*x) - (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2))/(4*(-d)^(1/2)*f^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e - 2fnx^n + 2fx^n}{4dfx^2 + e^2 + 4efx^n + 4f^2x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2+4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),
x)
```

```
[Out] Integral((e - 2*f*n*x**n + 2*f*x**n)/(4*d*f*x**2 + e**2 + 4*e*f*x**n + 4*f*
*2*x**(2*n)), x)
```

3.529 $\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

[Out] 1/2*arctanh(2*x*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)

Rubi [A] time = 0.10, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2093, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2093

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx = -\left(\left(e^2(1-n)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-1+n)^2x^2}\,dx,x,\frac{x}{e(-1+n)+2f(-}\right.\right.$$
$$=\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]

[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

fricas [A] time = 0.74, size = 144, normalized size = 3.79

$$\left[\frac{\sqrt{df}\log\left(-\frac{4\,dfx^2+4\,f^2x^{2\,n}+4\,\sqrt{df}\,ex+e^2+4\,\left(2\,\sqrt{df}\,fx+ef\right)x^n}{4\,dfx^2-4\,f^2x^{2\,n}-4\,efx^n-e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\,\arctan\left(\frac{2\,\sqrt{-df}\,fx^n+\sqrt{-df}\,e}{2\,dfx}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*sqrt(d*f)*e*x + e^2 + 4*
(2*sqrt(d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2))
/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*x^n + sqrt(-d*f)*e)/(d*f
*x))/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,f(n-1)x^n-e}{4\,dfx^2-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^
2), x)
```

maple [B] time = 0.07, size = 72, normalized size = 1.89

$$-\frac{\ln\left(x^n+\frac{-2dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}+\frac{\ln\left(x^n+\frac{2dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e-2*f*(n-1)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+e*(d*f)^(1/2))/(d*f)^(1/2)/f)-1/4/(d*f)
^(1/2)*ln(x^n+1/2*(-2*d*f*x+e*(d*f)^(1/2))/(d*f)^(1/2)/f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,f(n-1)x^n-e}{4\,dfx^2-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^
2), x)
```

mupad [B] time = 5.30, size = 139, normalized size = 3.66

$$\frac{\ln\left(\frac{\left(e+2\,f\,x^n+2\,\sqrt{d}\,\sqrt{f}\,x\right)\left(e\,n+2\,\sqrt{d}\,\sqrt{f}\,x-2\,\sqrt{d}\,\sqrt{f}\,n\,x\right)}{x}\right)}{4\,\sqrt{d}\,\sqrt{f}}-\frac{\ln\left(\frac{\left(e+2\,f\,x^n-2\,\sqrt{d}\,\sqrt{f}\,x\right)\left(e\,n-2\,\sqrt{d}\,\sqrt{f}\,x+2\,\sqrt{d}\,\sqrt{f}\,n\,x\right)}{x}\right)}{4\,\sqrt{d}\,\sqrt{f}}-\frac{\mathrm{atan}\left(\frac{2\,\sqrt{d}\,\sqrt{f}\,x(}{e\,n}\right.}{2\,\sqrt{d}\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^2 + 4*e*f*x^n),x)
```

```
[Out] log(((e + 2*f*x^n + 2*d^(1/2)*f^(1/2)*x)*(e*n + 2*d^(1/2)*f^(1/2)*x - 2*d^(
1/2)*f^(1/2)*n*x))/x)/(4*d^(1/2)*f^(1/2)) - log(((e + 2*f*x^n - 2*d^(1/2)*f
^(1/2)*x)*(e*n - 2*d^(1/2)*f^(1/2)*x + 2*d^(1/2)*f^(1/2)*n*x))/x)/(4*d^(1/2
)*f^(1/2)) - (atan((2*d^(1/2)*f^(1/2)*x*(n*1i - 1i))/(e*n))*1i)/(2*d^(1/2)*
f^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{4dfx^2-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \frac{2fx^n}{4dfx^2-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \left(-\frac{2fnx^n}{4dfx^2-e^2-4efx^n-4f^2x^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2-4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),
x)
```

```
[Out] -Integral(e/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integr
al(2*f*x**n/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integr
al(-2*f*n*x**n/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x)
```

3.530 $\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^2(d+f)+e\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

```
[Out] 1/4*arctan((e+2*(d+f)*x^2)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6, 1107, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^2(d+f)+e\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx = \int \frac{x}{e^2+4efx^2+4\left(df+f^2\right)x^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{e^2+4efx+4\left(df+f^2\right)x^2}\,dx,x,x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{-16de^2f-x^2}\,dx,x,4f\left(e+2(d+f)x^2\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}\left(e+2(d+f)x^2\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^2(d+f)+e\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

fricas [A] time = 0.48, size = 155, normalized size = 3.69

$$\left[-\frac{\sqrt{-df}\,\log\left(\frac{4\left(d^2f+2\,df^2+f^3\right)x^4-de^2+e^2f+4\left(def+ef^2\right)x^2-2\left(2\left(de+ef\right)x^2+e^2\right)\sqrt{-df}}{4\left(df+f^2\right)x^4+4\,efx^2+e^2}\right)}{8\,def}, \frac{\sqrt{df}\,\arctan\left(\frac{\left(2\left(d+f\right)x^2+e\right)\sqrt{df}}{de}\right)}{4\,def}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
[Out] [-1/8*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^4 - d*e^2 + e^2*f + 4*(d*
e*f + e*f^2)*x^2 - 2*(2*(d*e + e*f)*x^2 + e^2)*sqrt(-d*f))/(4*(d*f + f^2)*x
^4 + 4*e*f*x^2 + e^2))/(d*e*f), 1/4*sqrt(d*f)*arctan((2*(d + f)*x^2 + e)*sq
rt(d*f)/(d*e))/(d*e*f)]
```

giac [A] time = 10.58, size = 38, normalized size = 0.90

$$\frac{\arctan\left(\frac{\left(2\,dfx^2+2\,f^2x^2+fe\right)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{4\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
[Out] 1/4*arctan((2*d*f*x^2 + 2*f^2*x^2 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)
```

maple [A] time = 0.00, size = 42, normalized size = 1.00

$$\frac{\arctan\left(\frac{4ef+2\left(4df+4f^2\right)x^2}{4\sqrt{df}\;e}\right)}{4\sqrt{df}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^2+4*e*f)/(d*f)^(1/2)/e)
```

maxima [A] time = 2.17, size = 36, normalized size = 0.86

$$\frac{\arctan\left(\frac{2\left(df+f^2\right)x^2+ef}{\sqrt{df}\,e}\right)}{4\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
```

```
[Out] 1/4*arctan((2*(d*f + f^2)*x^2 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

mupad [B] time = 3.10, size = 42, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{e\sqrt{f}+2f^{3/2}x^2+2d\sqrt{f}\,x^2}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e^2 + 4*f^2*x^4 + 4*d*f*x^4 + 4*e*f*x^2),x)
```

```
[Out] atan((e*f^(1/2) + 2*f^(3/2)*x^2 + 2*d*f^(1/2)*x^2)/(d^(1/2)*e))/(4*d^(1/2)*
e*f^(1/2))
```

sympy [B] time = 0.62, size = 78, normalized size = 1.86

$$\frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] (-sqrt(-1/(d*f))*log(x**2 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8 + sqrt
(-1/(d*f))*log(x**2 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8)/e
```

3.531 $\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^2(d-f)\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

[Out] -1/4*arctanh((e-2*(d-f)*x^2)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^2(d-f)\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]

[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx=\int \frac{x}{e^2+4efx^2+\left(-4df+4f^2\right)x^4}\,dx$$
$$=\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{e^2+4efx+\left(-4df+4f^2\right)x^2}\,dx,x,x^2\right)$$
$$=-\text{Subst}\left(\int \frac{1}{16de^2f-x^2}\,dx,x,4f\left(e-2(d-f)x^2\right)\right)$$
$$=-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e+2(-d+f)x^2\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.05

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(-2dx^2+e+2fx^2\right)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] -1/4*ArcTanh[(Sqrt[f]*(e - 2*d*x^2 + 2*f*x^2))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt
[f])
```

fricas [A] time = 0.44, size = 168, normalized size = 3.82

$$\left[\frac{\sqrt{df}\,\log\left(-\frac{4\left(d^2f-2\,df^2+f^3\right)x^4+de^2+e^2f-4\left(def-ef^2\right)x^2+2\left(2\left(de-ef\right)x^2-e^2\right)\sqrt{df}}{4\left(df-f^2\right)x^4-4\,efx^2-e^2}\right)}{8\,def},\ \frac{\sqrt{-df}\,\arctan\left(-\frac{\left(2\left(d-f\right)x^2-e\right)\sqrt{-df}}{de}\right)}{4\,def}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^4 + d*e^2 + e^2*f - 4*(d*e
*f - e*f^2)*x^2 + 2*(2*(d*e - e*f)*x^2 - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x^4
 - 4*e*f*x^2 - e^2))/(d*e*f), 1/4*sqrt(-d*f)*arctan(-(2*(d - f)*x^2 - e)*sq
rt(-d*f)/(d*e))/(d*e*f)]
```

giac [A] time = 10.05, size = 41, normalized size = 0.93

$$-\frac{\arctan\left(\frac{2\,dfx^2-2\,f^2x^2-fe}{\sqrt{-dfe^2}}\right)}{4\,\sqrt{-dfe^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
```

```
[Out] -1/4*arctan((2*d*f*x^2 - 2*f^2*x^2 - f*e)/sqrt(-d*f*e^2))/sqrt(-d*f*e^2)
```

maple [A] time = 0.00, size = 42, normalized size = 0.95

$$\frac{\operatorname{arctanh}\left(\frac{-4ef+2\left(4df-4f^2\right)x^2}{4\sqrt{df}\,e}\right)}{4\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4/(d*f)^(1/2)/e*arctanh(1/4*(2*(4*d*f-4*f^2)*x^2-4*e*f)/(d*f)^(1/2)/e)
```

maxima [A] time = 1.87, size = 67, normalized size = 1.52

$$\frac{\log\left(\frac{2\left(df-f^2\right)x^2-ef+\sqrt{df}\,e}{2\left(df-f^2\right)x^2-ef-\sqrt{df}\,e}\right)}{8\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
```

```
[Out] 1/8*log((2*(d*f - f^2)*x^2 - e*f + sqrt(d*f)*e)/(2*(d*f - f^2)*x^2 - e*f - sqrt(d*f)*e))/(sqrt(d*f)*e)
```

mupad [B] time = 3.11, size = 199, normalized size = 4.52

$$\frac{\operatorname{atanh}\left(\frac{16\,d^{3/2}\,f^{3/2}\,x^2}{\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}}-\frac{32\,\sqrt{d}\,f^{5/2}\,x^2}{\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}}+\frac{1}{\sqrt{d}\,\left(\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\right.}\right.}{4\,\sqrt{d}\,e\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e^2 + 4*f^2*x^4 - 4*d*f*x^4 + 4*e*f*x^2),x)
```

```
[Out] atanh((16*d^(3/2)*f^(3/2)*x^2)/((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*
f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) - (32*d^(1/2)*f^(5/2)*x^2)/((8*e*f^3)/d
 - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) + (16*f
^(7/2)*x^2)/(d^(1/2)*((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 +
8*d*e*f + (16*f^4*x^2)/d)))/(4*d^(1/2)*e*f^(1/2))
```

sympy [A] time = 0.66, size = 75, normalized size = 1.70

$$-\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}-\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -(sqrt(1/(d*f))*log(x**2 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8 - sqrt(1
/(d*f))*log(x**2 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8)/e
```

3.532 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^3*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(3e+2fx^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx &= \left(3e^2\right)\text{Subst}\left(\int \frac{1}{e^2+36de^2fx^2}\,dx, x, \frac{x^3}{3e+6fx^2}\right)\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[4\#1^6df+4\#1^4f^2+4\#1^2ef+e^2\&,\frac{2\#1^3f\log(x-\#1)+3\#1e\log(x-\#1)}{3\#1^4d+2\#1^2f+e}\&\right]}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x
]
```

[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 + 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1 + 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 + 3*d*#1^4) &]/(8*f)

fricas [B] time = 0.42, size = 208, normalized size = 5.20

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4dfx^6-4f^2x^4-4efx^2-e^2-4(2fx^5+ex^3)\sqrt{-df}}{4dfx^6+4f^2x^4+4efx^2+e^2}\right)}{4df}, \frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{f}\right)-\sqrt{df}\arctan\left(\frac{2(2dfx^5-(de-2f^2)x^3+ef}{de^2}\right.}{2df}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")

[Out] [-1/4*sqrt(-d*f)*log((4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2 - 4*(2*f*x^5 + e*x^3)*sqrt(-d*f))/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2))/(d*f), 1/2*(sqrt(d*f)*arctan(sqrt(d*f)*x/f) - sqrt(d*f)*arctan(2*(2*d*f*x^5 - (d*e - 2*f^2)*x^3 + e*f*x)*sqrt(d*f)/(d*e^2)) + sqrt(d*f)*arctan((2*d*f*x^3 - (d*e - 2*f^2)*x)*sqrt(d*f)/(d*e*f)))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2fx^2+3e)x^2}{4dfx^6+4f^2x^4+4efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")

[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2), x)

maple [C] time = 0.15, size = 74, normalized size = 1.85

$$\frac{\left(2\,\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)^4 f+3\,\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)^2 e\right)\ln\left(-\mathrm{R}\right.}{8f\left(3d\,\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)^5+2f\,\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)^3+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x)

[Out] 1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d+2*_R^3*f+_R*e)*ln(-_R+x),_R=RootOf(4*_Z^6*d*f+4*_Z^4*f^2+4*_Z^2*e*f+e^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2fx^2+3e)x^2}{4dfx^6+4f^2x^4+4efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")

[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2), x)

mupad [B] time = 3.21, size = 278, normalized size = 6.95

$$\frac{\mathrm{atan}\left(\frac{2f^2x+2dfx^3-dex}{\sqrt{d}\,e\,\sqrt{f}}\right)-\mathrm{atan}\left(\frac{1984d^{3/2}f^{9/2}x^3}{432d^2e^2f^2-128def^4}+\frac{1728d^{5/2}f^{7/2}x^5}{432d^2e^2f^2-128def^4}+\frac{512\sqrt{d}\,f^{13/2}x^3}{128de^2f^4-432d^2e^3f^2}+\frac{512d^{3/2}f^{11/2}x^5}{128de^2f^4-432d^2e^3}\right.}{2\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 + 4*d*f*x^6 + 4*e*f*x^2),x)
```

```
[Out] (atan((2*f^2*x + 2*d*f*x^3 - d*e*x)/(d^(1/2)*e*f^(1/2))) - atan((1984*d^(3/
2)*f^(9/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (1728*d^(5/2)*f^(7/2)*x^5
)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (512*d^(1/2)*f^(13/2)*x^3)/(128*d*e^2*f
^4 - 432*d^2*e^3*f^2) + (512*d^(3/2)*f^(11/2)*x^5)/(128*d*e^2*f^4 - 432*d^2
*e^3*f^2) - (256*d^(1/2)*f^(11/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (864
*d^(3/2)*e*f^(7/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) - (864*d^(5/2)*e*f^(5
/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4)) + atan((d^(1/2)*x)/f^(1/2)))/(2*d
^(1/2)*f^(1/2))
```

sympy [B] time = 1.13, size = 90, normalized size = 2.25

$$-\frac{\sqrt{-\frac{1}{df}}\log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2}-fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}+\frac{\sqrt{-\frac{1}{df}}\log\left(\frac{e\sqrt{-\frac{1}{df}}}{2}+fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*f*x**2+3*e)/(4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(-1/(d*f))*log(-e*sqrt(-1/(d*f))/2 - f*x**2*sqrt(-1/(d*f)) + x**3)/4 +
 sqrt(-1/(d*f))*log(e*sqrt(-1/(d*f))/2 + f*x**2*sqrt(-1/(d*f)) + x**3)/4
```

3.533 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^3*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(3e+2fx^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx &= \left(3e^2\right)\text{Subst}\left(\int \frac{1}{e^2-36de^2fx^2}\,dx, x, \frac{x^3}{3e+6fx^2}\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[-4\#1^6df+4\#1^4f^2+4\#1^2ef+e^2\&,\frac{2\#1^3f\log(x-\#1)+3\#1e\log(x-\#1)}{-3\#1^4d+2\#1^2f+e}\&\right]}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x
]
```

```
[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 - 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1
+ 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 - 3*d*#1^4) & ]/(8*f)
```

fricas [B] time = 0.42, size = 213, normalized size = 5.32

$$\left[\frac{\sqrt{df}\log\left(\frac{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2+4\,(2\,fx^5+ex^3)\sqrt{df}}{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\,\arctan\left(\frac{\sqrt{-df}\,x}{f}\right)-\sqrt{-df}\,\arctan\left(\frac{2\,(2\,dfx^5-(de+2\,f^2)x^3-efx}{de^2}\right.}{2\,df}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2 + 4*(2*f*x^5 +
e*x^3)*sqrt(d*f))/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2))/(d*f), -1/2*(s
qrt(-d*f)*arctan(sqrt(-d*f)*x/f) - sqrt(-d*f)*arctan(2*(2*d*f*x^5 - (d*e +
2*f^2)*x^3 - e*f*x)*sqrt(-d*f)/(d*e^2)) + sqrt(-d*f)*arctan((2*d*f*x^3 - (d
*e + 2*f^2)*x)*sqrt(-d*f)/(d*e*f)))/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,fx^2+3\,e)x^2}{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="giac")
```

```
[Out] integrate(-(2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2), x
)
```

maple [C] time = 0.14, size = 77, normalized size = 1.92

$$-\frac{\left(2\,\mathrm{RootOf}\left(4df\,\_Z^6-4f^2\_Z^4-4ef\,\_Z^2-e^2\right)^4 f+3\,\mathrm{RootOf}\left(4df\,\_Z^6-4f^2\_Z^4-4ef\,\_Z^2-e^2\right)^2 e\right)\ln\left(-\mathrm{Roo}\right.}{8f\left(3d\,\mathrm{RootOf}\left(4df\,\_Z^6-4f^2\_Z^4-4ef\,\_Z^2-e^2\right)^5-2f\,\mathrm{RootOf}\left(4df\,\_Z^6-4f^2\_Z^4-4ef\,\_Z^2-e^2\right)^3-e\,\mathrm{Ro}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] -1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d-2*_R^3*f-_R*e)*ln(-_R+x),_R=RootOf
(4*_Z^6*d*f-4*_Z^4*f^2-4*_Z^2*e*f-e^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,fx^2+3\,e)x^2}{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="maxima")
```

```
[Out] -integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2), x
)
```

mupad [B] time = 3.14, size = 30, normalized size = 0.75

$$\frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{2\,f\,x^2+e}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 - 4*d*f*x^6 + 4*e*f*x^2),x)
```

```
[Out] atanh((2*d^(1/2)*f^(1/2)*x^3)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))
```

sympy [B] time = 1.12, size = 80, normalized size = 2.00

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-\frac{e\sqrt{\frac{1}{df}}}{2}-fx^2\sqrt{\frac{1}{df}}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(\frac{e\sqrt{\frac{1}{df}}}{2}+fx^2\sqrt{\frac{1}{df}}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*f*x**2+3*e)/(-4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-e*sqrt(1/(d*f))/2 - f*x**2*sqrt(1/(d*f)) + x**3)/4 + sq
rt(1/(d*f))*log(e*sqrt(1/(d*f))/2 + f*x**2*sqrt(1/(d*f)) + x**3)/4
```

3.534 $\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,(1-m^2)x^{m+1}}{(1-m)(m+1)\left(e+2fx^2\right)}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(1 + m)*(e + 2*f*x^
2))]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx &= -\left(\left(e^2(1-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-1+m)^2(1+m)^2x^2}\,dx, x, \right.\right.\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,(1-m^2)x^{1+m}}{(1-m)(1+m)\left(e+2fx^2\right)}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [A] time = 0.32, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4
 + 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

fricas [A] time = 0.45, size = 146, normalized size = 3.48

$$\left[-\frac{\sqrt{-df}\log\left(-\frac{4f^2x^4-4dfx^2x^{2m}+4efx^2+4\left(2fx^3+ex\right)\sqrt{-df}x^m+e^2}{4f^2x^4+4dfx^2x^{2m}+4efx^2+e^2}\right)}{4df}, -\frac{\sqrt{df}\arctan\left(\frac{\left(2fx^2+e\right)\sqrt{df}}{2dfxx^m}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + 4*(2*f*x
^3 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^2
+ e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(d*f)/(d*f*x*x^m
))/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2f(m-1)x^2+e(m+1)\right)x^m}{4f^2x^4+4efx^2+4dfx^{2m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

maple [B] time = 0.09, size = 78, normalized size = 1.86

$$\frac{\ln\left(x^m-\frac{\left(2fx^2+e\right)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}-\frac{\ln\left(x^m+\frac{\left(2fx^2+e\right)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)),
x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/
2)*ln(x^m-1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2f(m-1)x^2+e(m+1)\right)x^m}{4f^2x^4+4efx^2+4dfx^{2m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(2\,f\,\left(m-1\right)\,x^2+e\,\left(m+1\right)\right)}{e^2+4\,f^2\,x^4+4\,e\,f\,x^2+4\,d\,f\,x^{2\,m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d*
f*x^(2*m + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4+4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

3.535 $\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\left(1-m^2\right)x^{m+1}}{(1-m)(m+1)\left(e+2fx^2\right)}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(1 + m)*(e + 2*f*x
^2))]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = -\left(\left(e^2(1-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-1+m)^2(1+m)^2x^2}\,dx,\right.\right.$$

$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\left(1-m^2\right)x^{1+m}}{(1-m)(1+m)\left(e+2fx^2\right)}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4
 - 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

fricas [A] time = 0.47, size = 146, normalized size = 3.48

$$\left[\frac{\sqrt{df}\log\left(-\frac{4f^2x^4+4dfx^2x^{2m}+4efx^2+4\left(2fx^3+ex\right)\sqrt{df}x^m+e^2}{4f^2x^4-4dfx^2x^{2m}+4efx^2+e^2}\right)}{4df}, -\frac{\sqrt{-df}\arctan\left(\frac{\left(2fx^2+e\right)\sqrt{-df}}{2dfxx^m}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + 4*(2*f*x^3
 + e*x)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + e
^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(-d*f)/(d*f*x*x^m)
)/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2f(m-1)x^2 + e(m+1)\right)x^m}{4f^2x^4 + 4efx^2 - 4dfx^{2m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

maple [B] time = 0.09, size = 74, normalized size = 1.76

$$-\frac{\ln\left(x^m - \frac{\left(2fx^2+e\right)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}} + \frac{\ln\left(x^m + \frac{\left(2fx^2+e\right)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)),
x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*l
n(x^m-1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2f(m-1)x^2 + e(m+1)\right)x^m}{4f^2x^4 + 4efx^2 - 4dfx^{2m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(2\,f\,\left(m-1\right)\,x^2+e\,\left(m+1\right)\right)}{e^2+4\,f^2\,x^4+4\,e\,f\,x^2-4\,d\,f\,x^{2\,m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d*
f*x^(2*m + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4-4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

3.536 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^2*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(2e-2fx^3\right)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx &= -\left(\left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2+16de^2fx^2}\,dx, x, \frac{x^2}{-2e-4fx^3}\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.05, size = 86, normalized size = 2.15

$$-\frac{\text{RootSum}\left[4\#1^6f^2+4\#1^4df+4\#1^3ef+e^2\&,\frac{\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^4f+4\#1^2d+3\#1e}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] -1/2*RootSum[e^2 + 4*e*f*#1^3 + 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + f*Log[x - #1]*#1^3)/(3*e*#1 + 4*d*#1^2 + 6*f*#1^4) & ]/f
```

fricas [B] time = 0.43, size = 153, normalized size = 3.82

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4f^2x^6-4dfx^4+4efx^3+e^2+4\left(2fx^5+ex^2\right)\sqrt{-df}}{4f^2x^6+4dfx^4+4efx^3+e^2}\right)}{4df}, -\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{d}\right)-\sqrt{df}\arctan\left(\frac{\left(2fx^4+2dx^2+ex\right)\sqrt{df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5
+ e*x^2)*sqrt(-d*f))/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1/2
*(sqrt(d*f)*arctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^4 + 2*d*x^2 + e
*x)*sqrt(d*f)/(d*e)))/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\left(fx^3-e\right)x}{4f^2x^6+4dfx^4+4efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="giac")
```

```
[Out] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

maple [C] time = 0.01, size = 74, normalized size = 1.85

$$-\frac{\left(\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)^4 f-\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)e\right)\ln\left(-\mathrm{Ro}\right.}{2f\left(6f\,\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)^5+4d\,\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)^3+3e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x)
```

```
[Out] -1/2/f*sum((_R^4*f-_R*e)/(6*_R^5*f+4*_R^3*d+3*_R^2*e)*ln(-_R+x),_R=RootOf(4
*_Z^6*f^2+4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\int \frac{\left(fx^3-e\right)x}{4f^2x^6+4dfx^4+4efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="maxima")
```

```
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

mupad [B] time = 3.49, size = 233, normalized size = 5.82

$$\frac{\mathrm{atan}\left(\frac{128d^{7/2}\sqrt{f}x^2}{64d^3e+729fe^3}-\frac{216d^{3/2}e^2\sqrt{f}}{64d^3e+729fe^3}+\frac{128d^{5/2}f^{3/2}x^4}{64d^3e+729fe^3}+\frac{216d^{3/2}e\sqrt{f}}{64d^3+729fe^2}+\frac{729d^{3/2}e^2f^{3/2}x}{64d^5+729fd^2e^2}+\frac{1458d^{3/2}ef^{5/2}x^4}{64d^5+729fd^2e^2}+\frac{64d^{5/2}e\sqrt{f}}{64d^3e+729f}\right.}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*e - 2*f*x^3))/(e^2 + 4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3),x)
```

```
[Out] (atan((128*d^(7/2)*f^(1/2)*x^2)/(64*d^3*e + 729*e^3*f) - (216*d^(3/2)*e^2*f
^(1/2))/(64*d^3*e + 729*e^3*f) + (128*d^(5/2)*f^(3/2)*x^4)/(64*d^3*e + 729*
e^3*f) + (216*d^(3/2)*e*f^(1/2))/(729*e^2*f + 64*d^3) + (729*d^(3/2)*e^2*f^
(3/2)*x)/(64*d^5 + 729*d^2*e^2*f) + (1458*d^(3/2)*e*f^(5/2)*x^4)/(64*d^5 +
729*d^2*e^2*f) + (64*d^(5/2)*e*f^(1/2)*x)/(64*d^3*e + 729*e^3*f) + (1458*d^
(3/2)*e*f^(3/2)*x^2)/(64*d^4 + 729*d*e^2*f)) - atan((f^(1/2)*x)/d^(1/2)))/(
2*d^(1/2)*f^(1/2))
```

sympy [B] time = 1.12, size = 73, normalized size = 1.82

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx^2\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx^2\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6+4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*
f))*log(d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

3.537 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^2*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(2e-2fx^3\right)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx &= -\left(\left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2-16de^2fx^2}\,dx, x, \frac{x^2}{-2e-4fx^3}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] time = 0.05, size = 86, normalized size = 2.15

$$-\frac{\text{RootSum}\left[4\#1^6f^2-4\#1^4df+4\#1^3ef+e^2\&,\frac{\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^4f-4\#1^2d+3\#1e}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] -1/2*RootSum[e^2 + 4*e*f*#1^3 - 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + f*Log[x - #1]*#1^3)/(3*e*#1 - 4*d*#1^2 + 6*f*#1^4) & ]/f
```

fricas [B] time = 0.42, size = 155, normalized size = 3.88

$$\left[\frac{\sqrt{df}\log\left(\frac{4f^2x^6+4dfx^4+4efx^3+e^2+4\left(2fx^5+ex^2\right)\sqrt{df}}{4f^2x^6-4dfx^4+4efx^3+e^2}\right)}{4df}, -\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}\,x}{d}\right)-\sqrt{-df}\arctan\left(\frac{\left(2fx^4-2dx^2+ex\right)\sqrt{-df}}{de}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5 +
e*x^2)*sqrt(d*f))/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1/2*(s
qrt(-d*f)*arctan(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^4 - 2*d*x^2 + e
*x)*sqrt(-d*f)/(d*e)))/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\left(fx^3-e\right)x}{4f^2x^6-4dfx^4+4efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="giac")
```

```
[Out] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

maple [C] time = 0.01, size = 74, normalized size = 1.85

$$-\frac{\left(\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)^4 f-\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)e\right)\ln\left(-\mathrm{RootC}\right.}{2f\left(6f\,\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)^5-4d\,\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)^3+3e\,\mathrm{Rc}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x)
```

```
[Out] -1/2/f*sum((_R^4*f-_R*e)/(6*_R^5*f-4*_R^3*d+3*_R^2*e)*ln(-_R+x),_R=RootOf(4
*_Z^6*f^2-4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\int \frac{\left(fx^3-e\right)x}{4f^2x^6-4dfx^4+4efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="maxima")
```

```
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

mupad [B] time = 3.38, size = 67, normalized size = 1.68

$$-\frac{\mathrm{atanh}\left(\frac{27e^2\sqrt{f}+54ef^{3/2}x^3-16d^2\sqrt{f}\,x^2}{8d^{3/2}e+16d^{3/2}fx^3-54\sqrt{d}\,efx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*e - 2*f*x^3))/(e^2 + 4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3),x)
```

```
[Out] -atanh((27*e^2*f^(1/2) + 54*e*f^(3/2)*x^3 - 16*d^2*f^(1/2)*x^2)/(8*d^(3/2)*
e + 16*d^(3/2)*f*x^3 - 54*d^(1/2)*e*f*x^2))/(2*d^(1/2)*f^(1/2))
```

sympy [A] time = 1.12, size = 66, normalized size = 1.65

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6-4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f)
)*log(d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

3.538 $\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^3(d+f)+e\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

```
[Out] 1/6*arctan((e+2*(d+f)*x^3)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6, 1352, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^3(d+f)+e\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx = \int \frac{x^2}{e^2+4efx^3+4\left(df+f^2\right)x^6}\,dx$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{e^2+4efx+4\left(df+f^2\right)x^2}\,dx, x, x^3\right)$$
$$= -\left(\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-16de^2f-x^2}\,dx, x, 4f\left(e+2(d+f)x^3\right)\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}\left(e+2(d+f)x^3\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^3(d+f)+e\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

fricas [A] time = 0.41, size = 155, normalized size = 3.69

$$\left[-\frac{\sqrt{-df}\,\log\left(\frac{4\left(d^2f+2\,df^2+f^3\right)x^6+4\left(def+ef^2\right)x^3-de^2+e^2f-2\left(2\left(de+ef\right)x^3+e^2\right)\sqrt{-df}}{4\left(df+f^2\right)x^6+4\,efx^3+e^2}\right)}{12\,def}, \frac{\sqrt{df}\,\arctan\left(\frac{\left(2\left(d+f\right)x^3+e\right)\sqrt{df}}{de}\right)}{6\,def}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")
[Out] [-1/12*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^6 + 4*(d*e*f + e*f^2)*x^
3 - d*e^2 + e^2*f - 2*(2*(d*e + e*f)*x^3 + e^2)*sqrt(-d*f))/(4*(d*f + f^2)*
x^6 + 4*e*f*x^3 + e^2))/(d*e*f), 1/6*sqrt(d*f)*arctan((2*(d + f)*x^3 + e)*s
qrt(d*f)/(d*e))/(d*e*f)]
```

giac [A] time = 10.13, size = 38, normalized size = 0.90

$$\frac{\arctan\left(\frac{\left(2\,dfx^3+2\,f^2x^3+fe\right)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{6\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
[Out] 1/6*arctan((2*d*f*x^3 + 2*f^2*x^3 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)
```

maple [A] time = 0.00, size = 42, normalized size = 1.00

$$\frac{\arctan\left(\frac{2\left(4df+4f^2\right)x^3+4ef}{4\sqrt{df}\;e}\right)}{6\sqrt{df}\;e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x)
```

```
[Out] 1/6/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^3+4*e*f)/(d*f)^(1/2)/e)
```

maxima [A] time = 1.42, size = 36, normalized size = 0.86

$$\frac{\arctan\left(\frac{2\left(df+f^2\right)x^3+ef}{\sqrt{df}\,e}\right)}{6\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")
```

```
[Out] 1/6*arctan((2*(d*f + f^2)*x^3 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

mupad [B] time = 3.43, size = 42, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{e\sqrt{f}+2f^{3/2}x^3+2d\sqrt{f}\,x^3}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e^2 + 4*f^2*x^6 + 4*d*f*x^6 + 4*e*f*x^3),x)
```

```
[Out] atan((e*f^(1/2) + 2*f^(3/2)*x^3 + 2*d*f^(1/2)*x^3)/(d^(1/2)*e))/(6*d^(1/2)*
e*f^(1/2))
```

sympy [B] time = 0.79, size = 78, normalized size = 1.86

$$\frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] (-sqrt(-1/(d*f))*log(x**3 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12 + sqr
t(-1/(d*f))*log(x**3 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12)/e
```

3.539 $\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^3(d-f)\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

```
[Out] -1/6*arctanh((e-2*(d-f)*x^3)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6, 1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2x^3(d-f)\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx = \int \frac{x^2}{e^2+4efx^3+\left(-4df+4f^2\right)x^6}\,dx$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{e^2+4efx+\left(-4df+4f^2\right)x^2}\,dx, x, x^3\right)$$
$$= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{16de^2f-x^2}\,dx, x, 4f\left(e-2(d-f)x^3\right)\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2(d-f)x^3\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.05

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(-2dx^3+e+2fx^3\right)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
[Out] -1/6*ArcTanh[(Sqrt[f]*(e - 2*d*x^3 + 2*f*x^3))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt
[f])
```

fricas [A] time = 0.45, size = 168, normalized size = 3.82

$$\left[\frac{\sqrt{df}\log\left(-\frac{4\left(d^2f-2df^2+f^3\right)x^6-4\left(def-ef^2\right)x^3+de^2+e^2f+2\left(2\left(de-ef\right)x^3-e^2\right)\sqrt{df}}{4\left(df-f^2\right)x^6-4efx^3-e^2}\right)}{12\,def}, \frac{\sqrt{-df}\arctan\left(-\frac{\left(2\left(d-f\right)x^3-e\right)\sqrt{-df}}{de}\right)}{6\,def}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")
[Out] [1/12*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^6 - 4*(d*e*f - e*f^2)*x^3
 + d*e^2 + e^2*f + 2*(2*(d*e - e*f)*x^3 - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x^
6 - 4*e*f*x^3 - e^2))/(d*e*f), 1/6*sqrt(-d*f)*arctan(-(2*(d - f)*x^3 - e)*s
qrt(-d*f)/(d*e))/(d*e*f)]
```

giac [A] time = 9.22, size = 41, normalized size = 0.93

$$-\frac{\arctan\left(\frac{2\,dfx^3-2\,f^2x^3-fe}{\sqrt{-dfe^2}}\right)}{6\,\sqrt{-dfe^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
[Out] -1/6*arctan((2*d*f*x^3 - 2*f^2*x^3 - f*e)/sqrt(-d*f*e^2))/sqrt(-d*f*e^2)
```

maple [A] time = 0.00, size = 42, normalized size = 0.95

$$\frac{\operatorname{arctanh}\left(\frac{2\left(4df-4f^2\right)x^3-4ef}{4\sqrt{df}\,e}\right)}{6\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x)

[Out] 1/6/(d*f)^(1/2)/e*arctanh(1/4*(2*(4*d*f-4*f^2)*x^3-4*e*f)/(d*f)^(1/2)/e)

maxima [A] time = 1.54, size = 67, normalized size = 1.52

$$\frac{\log\left(\frac{2\left(df-f^2\right)x^3-ef+\sqrt{df}\,e}{2\left(df-f^2\right)x^3-ef-\sqrt{df}\,e}\right)}{12\sqrt{df}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")

[Out] 1/12*log((2*(d*f - f^2)*x^3 - e*f + sqrt(d*f)*e)/(2*(d*f - f^2)*x^3 - e*f - sqrt(d*f)*e))/(sqrt(d*f)*e)

mupad [B] time = 3.53, size = 923, normalized size = 20.98

$$\operatorname{atan}\left(\frac{\left(\frac{\left(x^3\left(32d^3f^3-96d^2f^4+96df^5-32f^6\right)+\frac{x^3\left(-64ed^3f^4+192ed^2f^5-192edf^6+64ef^7\right)+16e^2f^6-48de^2f^5+48d^2e^2f^4-16d^3e^2f^3-\frac{x^3\left(-384d^3e^2f^5+1152d^2\cdots\right.}{\cdots}}{\sqrt{d}\,e\,\sqrt{f}}\right.}{\sqrt{d}\,e\,\sqrt{f}}\cdots}{\frac{x^3\left(32d^3f^3-96d^2f^4+96df^5-32f^6\right)+\frac{x^3\left(-64ed^3f^4+192ed^2f^5-192edf^6+64ef^7\right)+16e^2f^6-48de^2f^5+48d^2e^2f^4-16d^3e^2f^3-\frac{x^3\left(-384d^3e^2f^5+1152\cdots\right.}{\cdots}}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e^2 + 4*f^2*x^6 - 4*d*f*x^6 + 4*e*f*x^3),x)

[Out] (atan((((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) + (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))*1i)/(d^(1/2)*e*f^(1/2)) + ((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) - (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 + ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))*1i)/(d^(1/2)*e*f^(1/2)))/((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) + (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)) - (x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) - (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 + ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2))))*1i)/(6*d^(1/2)*e*f^(1/2))

sympy [A] time = 0.85, size = 75, normalized size = 1.70

$$-\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}-\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] -(sqrt(1/(d*f))*log(x**3 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12 - sqrt(
1/(d*f))*log(x**3 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12)/e
```

3.540 $\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 + 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx &= -\left(\left(e^2(2-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-2+m)^2(1+m)^2x^2}\,dx,\right.\right.\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [A] time = 0.32, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6
 + 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

fricas [A] time = 0.46, size = 146, normalized size = 3.48

$$\left[-\frac{\sqrt{-df}\log\left(-\frac{4f^2x^6-4dfx^2x^{2m}+4efx^3+4\left(2fx^4+ex\right)\sqrt{-df}x^m+e^2}{4f^2x^6+4dfx^2x^{2m}+4efx^3+e^2}\right)}{4df}, -\frac{\sqrt{df}\arctan\left(\frac{\left(2fx^3+e\right)\sqrt{df}}{2dfxx^m}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*x
^4 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^3
+ e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(d*f)/(d*f*x*x^m
))/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2f(m-2)x^3+e(m+1)\right)x^m}{4f^2x^6+4efx^3+4dfx^{2m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

maple [B] time = 0.05, size = 78, normalized size = 1.86

$$\frac{\ln\left(x^m-\frac{\left(2fx^3+e\right)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}-\frac{\ln\left(x^m+\frac{\left(2fx^3+e\right)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)),
x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/
2)*ln(x^m-1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2f(m-2)x^3+e(m+1)\right)x^m}{4f^2x^6+4efx^3+4dfx^{2m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m \left(2 f (m-2) x^3 + e (m+1)\right)}{e^2 + 4 f^2 x^6 + 4 e f x^3 + 4 d f x^{2m+2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d*
f*x^(2*m + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6+4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

3.541 $\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = -\left(\left(e^2(2-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-2+m)^2(1+m)^2x^2}\,dx, x, -\right.\right.$$

$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\,x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6
 - 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

fricas [A] time = 0.51, size = 146, normalized size = 3.48

$$\left[\frac{\sqrt{df}\log\left(-\frac{4f^2x^6+4dfx^2x^{2m}+4efx^3+4\left(2fx^4+ex\right)\sqrt{df}\,x^m+e^2}{4f^2x^6-4dfx^2x^{2m}+4efx^3+e^2}\right)}{4df}, -\frac{\sqrt{-df}\arctan\left(\frac{\left(2fx^3+e\right)\sqrt{-df}}{2dfxx^m}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*x^4
 + e*x)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + e
^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(-d*f)/(d*f*x*x^m)
)/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2f(m-2)x^3 + e(m+1)\right)x^m}{4f^2x^6 + 4efx^3 - 4dfx^{2m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

maple [B] time = 0.06, size = 74, normalized size = 1.76

$$-\frac{\ln\left(x^m - \frac{\left(2fx^3+e\right)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}} + \frac{\ln\left(x^m + \frac{\left(2fx^3+e\right)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+2*m)),
x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*l
n(x^m-1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2f(m-2)x^3 + e(m+1)\right)x^m}{4f^2x^6 + 4efx^3 - 4dfx^{2m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m \left(2 f (m-2) x^3 + e (m+1)\right)}{e^2 + 4 f^2 x^6 + 4 e f x^3 - 4 d f x^{2m+2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d*
f*x^(2*m + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6-4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

3.542 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m) + 4*e*
f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx &= \left(e^2(1+m)(1+m-n)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(1+m)^2(1+m-n)^2x^2}\right.\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

fricas [A] time = 0.45, size = 165, normalized size = 3.93

$$\left[-\frac{\sqrt{-df}\log\left(-\frac{4\,dfx^2x^{2\,m}-4\,\sqrt{-df}\,exx^m-4\,f^2x^{2\,n}-e^2-4\left(2\,\sqrt{-df}\,fxx^m+ef\right)x^n}{4\,dfx^2x^{2\,m}+4\,f^2x^{2\,n}+4\,efx^n+e^2}\right)}{4\,df}, -\frac{\sqrt{df}\arctan\left(\frac{2\,\sqrt{df}\,fx^n+\sqrt{df}\,e}{2\,dfxx^m}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2*x^(2*m) - 4*sqrt(-d*f)*e*x*x^m - 4*f^2*x^(
2*n) - e^2 - 4*(2*sqrt(-d*f)*f*x*x^m + e*f)*x^n)/(4*d*f*x^2*x^(2*m) + 4*f^2
*x^(2*n) + 4*e*f*x^n + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*
f*x^n + sqrt(d*f)*e)/(d*f*x*x^m))/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,f(m-n+1)x^n + e(m+1)\right)x^m}{4\,dfx^{2\,m+2} + 4\,f^2x^{2\,n} + 4\,efx^n + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*
x^(2*n) + 4*e*f*x^n + e^2), x)
```

maple [B] time = 0.09, size = 84, normalized size = 2.00

$$\frac{\ln\left(x^n + \frac{-2dfx\,x^m+\sqrt{-df}\;e}{2\sqrt{-df}\;f}\right)}{4\sqrt{-df}} - \frac{\ln\left(x^n + \frac{2dfx\,x^m+\sqrt{-df}\;e}{2\sqrt{-df}\;f}\right)}{4\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2
*n)),x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)+1
/4/(-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,f(m-n+1)x^n + e(m+1)\right)x^m}{4\,dfx^{2\,m+2} + 4\,f^2x^{2\,n} + 4\,efx^n + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*
x^(2*n) + 4*e*f*x^n + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(e\,\left(m+1\right)+2\,f\,x^n\,\left(m-n+1\right)\right)}{e^2+4\,f^2\,x^{2\,n}+4\,d\,f\,x^{2\,m+2}+4\,e\,f\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^
(2*m + 2) + 4*e*f*x^n),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^
(2*m + 2) + 4*e*f*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(em + e + 2fmx^n - 2fnx^n + 2fx^n\right)}{4dfx^2x^{2m} + e^2 + 4efx^n + 4f^2x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2+4*d*f*x**(2+2*m)+4*e*f*x**n
+4*f**2*x**(2*n)),x)
```

```
[Out] Integral(x**m*(e*m + e + 2*f*m*x**n - 2*f*n*x**n + 2*f*x**n)/(4*d*f*x**2*x*
*(2*m) + e**2 + 4*e*f*x**n + 4*f**2*x**(2*n)), x)
```

3.543 $\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m) + 4*e*
f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx &= \left(e^2(1+m)(1+m-n)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(1+m)^2(1+m-n)^2x^2}\,dx\right.\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

fricas [A] time = 0.47, size = 165, normalized size = 3.93

$$\left[\frac{\sqrt{df}\log\left(-\frac{4\,dfx^2x^{2\,m}+4\,\sqrt{df}\,exx^m+4\,f^2x^{2\,n}+e^2+4\left(2\,\sqrt{df}\,fxx^m+ef\right)x^n}{4\,dfx^2x^{2\,m}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\arctan\left(\frac{2\,\sqrt{-df}\,fx^n+\sqrt{-df}\,e}{2\,dfxx^m}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2*x^(2*m) + 4*sqrt(d*f)*e*x*x^m + 4*f^2*x^(2*n
) + e^2 + 4*(2*sqrt(d*f)*f*x*x^m + e*f)*x^n)/(4*d*f*x^2*x^(2*m) - 4*f^2*x^(
2*n) - 4*e*f*x^n - e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*
x^n + sqrt(-d*f)*e)/(d*f*x*x^m))/(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(2\,f(m-n+1)x^n+e(m+1)\right)x^m}{4\,dfx^{2\,m+2}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(-(2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2
*x^(2*n) - 4*e*f*x^n - e^2), x)
```

maple [B] time = 0.08, size = 78, normalized size = 1.86

$$-\frac{\ln\left(x^n+\frac{-2dfx\,x^m+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}+\frac{\ln\left(x^n+\frac{2dfx\,x^m+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(m+1)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2
*n)),x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)-1/4/(
d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(2\,f(m-n+1)x^n+e(m+1)\right)x^m}{4\,dfx^{2\,m+2}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2
*x^(2*n) - 4*e*f*x^n - e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^m\,\left(e\,(m+1)+2\,f\,x^n\,(m-n+1)\right)}{e^2+4\,f^2\,x^{2\,n}-4\,d\,f\,x^{2\,m+2}+4\,e\,f\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^
(2*m + 2) + 4*e*f*x^n),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^
(2*m + 2) + 4*e*f*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^m}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \frac{emx^m}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \frac{2fx^mx^n}{4dfx^2x^{2m}-e^2-4efx^n-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2-4*d*f*x**(2+2*m)+4*e*f*x**n
+4*f**2*x**(2*n)),x)
```

```
[Out] -Integral(e*x**m/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)
), x) - Integral(e*m*x**m/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2
*x**(2*n)), x) - Integral(2*f*x**m*x**n/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f
*x**n - 4*f**2*x**(2*n)), x) - Integral(2*f*m*x**m*x**n/(4*d*f*x**2*x**(2*m
) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integral(-2*f*n*x**m*x**n/(4
*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x)
```

3.544 $\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=134

$$-\frac{d\left(a+bx^2\right)^{3/2}}{3b^3c^2}+\frac{\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}+\frac{d\sqrt{a+bx^2}\left(2ac^2-d^2\right)}{b^3c^4}+\frac{\left(a+bx^2\right)^2}{4b^3c}-\frac{x^2\left(2ac^2-d^2\right)}{2b^2c^3}$$

```
[Out] -1/2*(2*a*c^2-d^2)*x^2/b^2/c^3-1/3*d*(b*x^2+a)^(3/2)/b^3/c^2+1/4*(b*x^2+a)^
2/b^3/c+(a*c^2-d^2)^2*ln(d+c*(b*x^2+a)^(1/2))/b^3/c^5+d*(2*a*c^2-d^2)*(b*x^
2+a)^(1/2)/b^3/c^4
```

Rubi [A] time = 0.37, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{x^2\left(2ac^2-d^2\right)}{2b^2c^3}+\frac{d\sqrt{a+bx^2}\left(2ac^2-d^2\right)}{b^3c^4}+\frac{\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}-\frac{d\left(a+bx^2\right)^{3/2}}{3b^3c^2}+\frac{\left(a+bx^2\right)^2}{4b^3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] -((2*a*c^2 - d^2)*x^2)/(2*b^2*c^3) + (d*(2*a*c^2 - d^2)*Sqrt[a + b*x^2])/(b
^3*c^4) - (d*(a + b*x^2)^(3/2))/(3*b^3*c^2) + (a + b*x^2)^2/(4*b^3*c) + ((a
*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(b^3*c^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^2\right)\\&=\frac{\text{Subst}\left(\int \frac{\left(a-x^2\right)^2}{d+cx}\,dx,x,\sqrt{a+bx^2}\right)}{b^3}\\&=\frac{\text{Subst}\left(\int \left(\frac{2ac^2d-d^3}{c^4}-\frac{\left(2ac^2-d^2\right)x}{c^3}-\frac{dx^2}{c^2}+\frac{x^3}{c}+\frac{\left(ac^2-d^2\right)^2}{c^4(d+cx)}\right)dx,x,\sqrt{a+bx^2}\right)}{b^3}\\&=-\frac{\left(2ac^2-d^2\right)x^2}{2b^2c^3}+\frac{d\left(2ac^2-d^2\right)\sqrt{a+bx^2}}{b^3c^4}-\frac{d\left(a+bx^2\right)^{3/2}}{3b^3c^2}+\frac{\left(a+bx^2\right)^2}{4b^3c}+\end{aligned}$$

Mathematica [A] time = 0.22, size = 126, normalized size = 0.94

$$\frac{c\left(a\left(20c^2d\sqrt{a+bx^2}-6bc^3x^2\right)+2bcdx^2\left(3d-2c\sqrt{a+bx^2}\right)-12d^3\sqrt{a+bx^2}+3b^2c^3x^4\right)+12\left(d^2-ac^2\right)^2\log}{12b^3c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] (c*(3*b^2*c^3*x^4 - 12*d^3*Sqrt[a + b*x^2] + 2*b*c*d*x^2*(3*d - 2*c*Sqrt[a
+ b*x^2]) + a*(-6*b*c^3*x^2 + 20*c^2*d*Sqrt[a + b*x^2])) + 12*(-(a*c^2) + d
^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(12*b^3*c^5)
```

fricas [A] time = 0.49, size = 233, normalized size = 1.74

$$\frac{3\,b^2c^4x^4 - 6\left(abc^4 - bc^2d^2\right)x^2 + 6\left(a^2c^4 - 2\,ac^2d^2 + d^4\right)\log\left(bc^2x^2 + ac^2 - d^2\right) + 3\left(a^2c^4 - 2\,ac^2d^2 + d^4\right)\log\left(-\frac{bc^2}{}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4 - 6*(a*b*c^4 - b*c^2*d^2)*x^2 + 6*(a^2*c^4 - 2*a*c^2*d^
2 + d^4)*log(b*c^2*x^2 + a*c^2 - d^2) + 3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log
(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 3*(a^2*c^4 - 2*a
*c^2*d^2 + d^4)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2)
 - 4*(b*c^3*d*x^2 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^2 + a))/(b^3*c^5)
```

giac [A] time = 0.35, size = 155, normalized size = 1.16

$$\frac{\left(a^2c^4 - 2\,ac^2d^2 + d^4\right)\log\left(\left|\sqrt{bx^2+a}\,c + d\right|\right)}{b^3c^5} + \frac{3\left(bx^2+a\right)^2b^9c^3 - 12\left(bx^2+a\right)ab^9c^3 - 4\left(bx^2+a\right)^{\frac{3}{2}}b^9c^2d + 24\,\sqrt{bx}}{12\,b^{12}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] (a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^2 + a)*c + d))/(b^3*c^5) + 1
/12*(3*(b*x^2 + a)^2*b^9*c^3 - 12*(b*x^2 + a)*a*b^9*c^3 - 4*(b*x^2 + a)^(3/
2)*b^9*c^2*d + 24*sqrt(b*x^2 + a)*a*b^9*c^2*d + 6*(b*x^2 + a)*b^9*c*d^2 - 1
2*sqrt(b*x^2 + a)*b^9*d^3)/(b^12*c^4)
```

maple [B] time = 0.09, size = 4947, normalized size = 36.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -a/c^3/b^3*d^2*ln(b*c^2*x^2+a*c^2-d^2)-1/3*d*(b*x^2+a)^(3/2)/b^3/c^2-1/b^2/
((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^
2)*b*c^2)^(1/2))*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*
b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)*a*d^
3+1/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(
a*c^2-d^2)*b*c^2)^(1/2))/c^2*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2
-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(
1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)
/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x-(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2))*a+1/2/b^3/c^5*d^4*ln(b*c^2*x^2+a*c^2-d^2)-1/2*a/c/b^
2*x^2+1/2*a^2/c/b^3*ln(b*c^2*x^2+a*c^2-d^2)+1/2/b^2/c^3*d^2*x^2-1/2/b^2/((-
a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*
b*c^2)^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1
/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x-(-(a
*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^
2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/
```

```
b/c^2))*a^2-1/2/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(
1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^4*d^7/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^
2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(
1/c^2*d^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*
b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x-
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-1/2*d/b^2*c^2*a^2/((-a*b)^(1/2)*c^2+(-(a
*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x
+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d/b^2*c^2*a
^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2
-d^2)*b*c^2)^(1/2))*(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b))^(1/2)+1/2*d/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(
1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*c^2*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b
/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)+1/c^2*d^2)^(1/2)*a^2-1/2*d/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2
)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)
^(1/2)*ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2))/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b
*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*a^2-
1/2/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2
-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^4*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((-(-(a*c^2-d
^2)*b*c^2)^(1/2)/c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b*(x+
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*d^5-1/2/b^2/((-a*b)^(1/2)*
c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2
))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b
*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a^2
-1/2/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-
(a*c^2-d^2)*b*c^2)^(1/2))/c^4*d^7/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^
(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2
)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x+(-(a*c^2-d^
2)*b*c^2)^(1/2)/b/c^2))+1/2*d/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1
/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*c^2*(b*(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)*a^2+1/2*d/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*
c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c
^2-d^2)*b*c^2)^(1/2)*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-
(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2
)^(1/2))*a^2+1/2/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a
*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^4*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln
(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^
(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2
)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*d^5+1/4/b/c*x^
4+1/2*d/b^(5/2)*c^2*a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*
b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln((b*(x+(-a*b)^(1/2)
/b)-(-a*b)^(1/2))/b^(1/2)+(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b))^(1/2))-1/2*d/b^(5/2)*c^2*a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^
2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln((b*
(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b
*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*(-(a*c^2-d^2
)*b*c^2)^(1/2)*ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x+(-(a*c^2-d^2)*b*c^2
)^(1/2)/b/c^2))/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/
2))*a*d^3+1/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)
*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*
```

```
(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^
2*d^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^
2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x+(-(a
*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a-1/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*
b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*(-(a*c^2-d^
2)*b*c^2)^(1/2)*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x-(-(a*c^2-d^2)*b*c^2
)^(1/2)/b/c^2))/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/
2))*a*d^3+1/2/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/
2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c
^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)
+1/c^2*d^2)^(1/2)*d^5-1/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(
(-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/
2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2)+1/c^2*d^2)^(1/2)*a*d^3+1/2/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2
)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*(b*(x-(-(a*c^2-d
^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)
*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)*d^5
```

maxima [A] time = 0.65, size = 125, normalized size = 0.93

$$\frac{\frac{3\left(bx^2+a\right)^2c^3-4\left(bx^2+a\right)^{\frac{3}{2}}c^2d-6\left(2\,ac^3-cd^2\right)\left(bx^2+a\right)+12\left(2\,ac^2d-d^3\right)\sqrt{bx^2+a}}{c^4}+\frac{12\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{bx^2+a}\,c+d\right)}{c^5}}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*(b*x^2 + a)^2*c^3 - 4*(b*x^2 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2
)*(b*x^2 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^2 + a))/c^4 + 12*(a^2*c^4 - 2
*a*c^2*d^2 + d^4)*log(sqrt(b*x^2 + a)*c + d)/c^5)/b^3
```

mupad [B] time = 3.64, size = 167, normalized size = 1.25

$$\frac{x^4}{4\,b\,c}-\sqrt{b\,x^2+a}\,\left(\frac{d^3}{b^3\,c^4}-\frac{2\,a\,d}{b^3\,c^2}\right)-\frac{d\,{\left(b\,x^2+a\right)}^{3/2}}{3\,b^3\,c^2}-\frac{x^2\,\left(a\,c^2-d^2\right)}{2\,b^2\,c^3}+\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{b\,x^2+a}}{d}\right)\,{\left(a\,c^2-d^2\right)}^2}{b^3\,c^5}+\frac{\ln\left(b\,c^2\,x^2+a\,c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] x^4/(4*b*c) - (a + b*x^2)^(1/2)*(d^3/(b^3*c^4) - (2*a*d)/(b^3*c^2)) - (d*(a
 + b*x^2)^(3/2))/(3*b^3*c^2) - (x^2*(a*c^2 - d^2))/(2*b^2*c^3) + (atanh((c*
(a + b*x^2)^(1/2))/d)*(a*c^2 - d^2)^2)/(b^3*c^5) + (log(a*c^2 - d^2 + b*c^2
*x^2)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(2*b^3*c^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x**5/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

3.545 $\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=69

$$-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3}+\frac{x^2}{2bc}$$

```
[Out] 1/2*x^2/b/c-(a*c^2-d^2)*ln(d+c*(b*x^2+a)^(1/2))/b^2/c^3-d*(b*x^2+a)^(1/2)/b
^2/c^2
```

Rubi [A] time = 0.21, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3}-\frac{d\sqrt{a+bx^2}}{b^2c^2}+\frac{x^2}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x^2/(2*b*c) - (d*Sqrt[a + b*x^2])/(b^2*c^2) - ((a*c^2 - d^2)*Log[d + c*Sqrt
[a + b*x^2]])/(b^2*c^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{-a+x^2}{d+cx}\,dx,x,\sqrt{a+bx^2}\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{d}{c^2}+\frac{x}{c}+\frac{-ac^2+d^2}{c^2(d+cx)}\right)dx,x,\sqrt{a+bx^2}\right)}{b^2}\\
&= \frac{x^2}{2bc}-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{\left(ac^2-d^2\right)\log\left(d+c\sqrt{a+bx^2}\right)}{b^2c^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 0.94

$$\frac{-\frac{d\sqrt{a+bx^2}}{c^2}-\frac{\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^2}+d\right)}{c^3}+\frac{bx^2}{2c}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b*x^2)/(2*c) - (d*Sqrt[a + b*x^2])/c^2 - ((a*c^2 - d^2)*Log[d + c*Sqrt[a
+ b*x^2]])/c^3)/b^2
```

fricas [B] time = 0.44, size = 161, normalized size = 2.33

$$\frac{2\,bc^2x^2 - 4\,\sqrt{bx^2 + a}\,cd - 2\left(ac^2 - d^2\right)\log\left(bc^2x^2 + ac^2 - d^2\right) - \left(ac^2 - d^2\right)\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right) + \left(ac^2 - \right.}{4\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c^2*x^2 - 4*sqrt(b*x^2 + a)*c*d - 2*(a*c^2 - d^2)*log(b*c^2*x^2 +
a*c^2 - d^2) - (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*
d + d^2)/x^2) + (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c
*d + d^2)/x^2))/(b^2*c^3)
```

giac [A] time = 0.40, size = 72, normalized size = 1.04

$$-\frac{\frac{2\left(ac^2-d^2\right)\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{bc^3} - \frac{\left(bx^2+a\right)bc-2\,\sqrt{bx^2+a}\,bd}{b^2c^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^2 + a)*c + d))/(b*c^3) - ((b*x^2 + a
)*b*c - 2*sqrt(b*x^2 + a)*b*d)/(b^2*c^2))/b
```

maple [B] time = 0.04, size = 3410, normalized size = 49.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(
1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-a*b)^(1/2)*l
n((b*(x+(-a*b)^(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+(b*(x+(-a*b)^(1/2)/b)^2-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x-
(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)-1/2*d*c^2*a/((-a
*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*
b*c^2)^(1/2))/b^(3/2)*(-a*b)^(1/2)*ln((b*(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b
^(1/2)+(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/
2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c
^2-d^2)*b*c^2)^(1/2))/b*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/
2)*a*c^2-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-
2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2
*d^2)^(1/2)*d^3-1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b
)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-(a*c^2-d^2)*b*c^2)^(1/2)*
ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))
```

```
/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(
1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*a+1/2/((-a*
b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b
*c^2)^(1/2))/b^(3/2)*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^2-d^2)*b*c^
2)^(1/2)/c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b*(x+(-(a*c^2
-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^
2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*d^3-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*d^3/(1
/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-
d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/
2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2)+1/c^2*d^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a+1/2/((-a*b
)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*
c^2)^(1/2))/b/c^2*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2
)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x+
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2))+1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1
/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+
1/c^2*d^2)^(1/2)*a*c^2-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-
(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x-(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)+1/c^2*d^2)^(1/2)*d^3+1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^
(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-(a*c^2-d^2)
*b*c^2)^(1/2)*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x-(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2))/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-
d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)
)*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-
(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln(((-(a*c
^2-d^2)*b*c^2)^(1/2)/c^2+b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b
*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x
-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))*d^3-1/2/((-a*b)^(1/2)*
c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/
2))/b*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*
(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x-(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b
*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*
a+1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a
*c^2-d^2)*b*c^2)^(1/2))/b/c^2*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^
(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2
)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x-(-(a*c^2-d^
2)*b*c^2)^(1/2)/b/c^2))-1/2*a/c/b^2*ln(b*c^2*x^2+a*c^2-d^2)+1/2*x^2/b/c+1/2
/b^2/c^3*d^2*ln(b*c^2*x^2+a*c^2-d^2)
```

maxima [A] time = 0.69, size = 62, normalized size = 0.90

$$\frac{\frac{(bx^2+a)c-2\sqrt{bx^2+a}\,d}{c^2}-\frac{2\left(ac^2-d^2\right)\log\left(\sqrt{bx^2+a}\,c+d\right)}{c^3}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x^2 + a)*c - 2*sqrt(b*x^2 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b
*x^2 + a)*c + d)/c^3)/b^2
```

mupad [B] time = 3.50, size = 123, normalized size = 1.78

$$\frac{x^2}{2\,b\,c}-\frac{d\,\sqrt{b\,x^2+a}}{b^2\,c^2}+\frac{\mathrm{atanh}\left(\frac{c\,\left(a\,c^2-d^2\right)\,\sqrt{b\,x^2+a}}{d^3-a\,c^2\,d}\right)\,\left(a\,c^2-d^2\right)}{b^2\,c^3}-\frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)\,\left(a\,c^2-d^2\right)}{2\,b^2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] x^2/(2*b*c) - (d*(a + b*x^2)^(1/2))/(b^2*c^2) + (atanh((c*(a*c^2 - d^2)*(a
+ b*x^2)^(1/2))/(d^3 - a*c^2*d))*(a*c^2 - d^2))/(b^2*c^3) - (log(a*c^2 - d^
2 + b*c^2*x^2)*(a*c^2 - d^2))/(2*b^2*c^3)
```

sympy [A] time = 6.51, size = 88, normalized size = 1.28

$$\begin{cases} \frac{\frac{a+bx^2}{2bc}-\frac{d\sqrt{a+bx^2}}{bc^2}-\frac{\left(ac^2-d^2\right)\left(\begin{cases} \frac{\sqrt{a+bx^2}}{d} & \text{for}\ c=0 \\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise} \end{cases}\right)}{bc^2}}{b} & \text{for}\ b\neq 0 \\ \frac{x^4}{2\left(2\sqrt{a}\,d+2ac\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Piecewise((((a + b*x**2)/(2*b*c) - d*sqrt(a + b*x**2)/(b*c**2) - (a*c**2 -
d**2)*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d
)/c, True))/(b*c**2))/b, Ne(b, 0)), (x**4/(2*(2*sqrt(a)*d + 2*a*c)), True))
```

3.546 $\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

```
[Out] ln(d+c*(b*x^2+a)^(1/2))/b/c
```

Rubi [A] time = 0.09, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2155, 31}

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\ &= \frac{\log\left(d+c\sqrt{a+bx^2}\right)}{bc}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

fricas [B] time = 0.42, size = 105, normalized size = 4.57

$$\frac{2 \log\left(bc^2x^2 + ac^2 - d^2\right) + \log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right) - \log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(b*c^2*x^2 + a*c^2 - d^2) + log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^
2 + a)*c*d + d^2)/x^2) - log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d +
d^2)/x^2))/(b*c)
```

giac [A] time = 0.32, size = 22, normalized size = 0.96

$$\frac{\log\left(\left|\sqrt{bx^2 + a}\,c + d\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(b*x^2 + a)*c + d))/(b*c)
```

maple [B] time = 0.04, size = 1931, normalized size = 83.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+
(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b))^(1/2)-1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(
-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln((b*(x+(-a*b)^
(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b))^(1/2))/b^(1/2)+1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c
^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-(-a*b)^(1/2
)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2/((-a*b)^(1/2)*c^2
+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))
*(-a*b)^(1/2)*ln((b*(x-(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+(b*(x-(-a*b)^(
1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))/b^(1/2)-1/2*d*c^2/((-a*
b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b
*c^2)^(1/2))*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^
2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)+1/2*d/((
-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2
)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/
c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/b^(1/2)+1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)
*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1/c^2*d^
2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*
c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+
1/c^2*d^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-1/2*d*c^2/((-a*b)^(
1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2
)^(1/2))*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(
1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)-1/2*d/((-a*b
)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*
```

```
c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+b
*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)+1/c^2*d^2)^(1/2))/b^(1/2)+1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^
2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1/c^2*d^2)^(1
/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+
2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2
*d^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+1/2/b/c*ln(b*c^2*x^2+a*c
^2-d^2)
```

maxima [A] time = 0.71, size = 21, normalized size = 0.91

$$\frac{\log\left(\sqrt{bx^2+a}\,c+d\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(b*x^2 + a)*c + d)/(b*c)
```

mupad [B] time = 3.47, size = 45, normalized size = 1.96

$$\frac{\operatorname{atanh}\left(\frac{c\,\sqrt{b\,x^2+a}}{d}\right)+\frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)}{2}}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] (atanh((c*(a + b*x^2)^(1/2))/d) + log(a*c^2 - d^2 + b*c^2*x^2)/2)/(b*c)
```

sympy [A] time = 4.48, size = 29, normalized size = 1.26

$$\frac{\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for}\ c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d)/c, T
rue))/b
```

3.547 $\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=88

$$-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

```
[Out] c*ln(x)/(a*c^2-d^2)-c*ln(d+c*(b*x^2+a)^(1/2))/(a*c^2-d^2)+d*arctanh((b*x^2+
a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 706, 31, 635, 207, 260}

$$-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x])/(
a*c^2 - d^2) - (c*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
```

```
b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx, x, x^2\right)$$
$$= \operatorname{Subst}\left(\int \frac{1}{(d+cx)\left(-a+x^2\right)}\,dx, x, \sqrt{a+bx^2}\right)$$
$$= -\frac{c^2\operatorname{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^2}\right)}{ac^2-d^2} + \frac{\operatorname{Subst}\left(\int \frac{d-cx}{-a+x^2}\,dx, x, \sqrt{a+bx^2}\right)}{-ac^2+d^2}$$
$$= -\frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2} + \frac{c\operatorname{Subst}\left(\int \frac{x}{-a+x^2}\,dx, x, \sqrt{a+bx^2}\right)}{ac^2-d^2} - \frac{d\operatorname{Subst}\left(\right.}{}$$
$$= \frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)} + \frac{c\log(x)}{ac^2-d^2} - \frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2}$$

Mathematica [A] time = 0.13, size = 107, normalized size = 1.22

$$\frac{\left(\sqrt{a}c-d\right)\log\left(\sqrt{a}-\sqrt{a+bx^2}\right)+\left(\sqrt{a}c+d\right)\log\left(\sqrt{a+bx^2}+\sqrt{a}\right)-2\sqrt{a}c\log\left(c\sqrt{a+bx^2}+d\right)}{2\sqrt{a}\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] ((Sqrt[a]*c - d)*Log[Sqrt[a] - Sqrt[a + b*x^2]] + (Sqrt[a]*c + d)*Log[Sqrt[
a] + Sqrt[a + b*x^2]] - 2*Sqrt[a]*c*Log[d + c*Sqrt[a + b*x^2]])/(2*Sqrt[a]*
(a*c^2 - d^2))
```

fricas [A] time = 0.52, size = 316, normalized size = 3.59

$$\left[-\frac{2\,ac\log\left(bc^2x^2+ac^2-d^2\right)-4\,ac\log(x)+ac\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-ac\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right.}{4\left(a^2c^2-ad^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*
x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c
^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) + 2*sqrt(a)*d*log(-(b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a^2*c^2 - a*d^2), -1/4*(2*a*c*log(b*c^2*x^2
 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2
 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d
 + d^2)/x^2) + 4*sqrt(-a)*d*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a^2*c^2 - a*
d^2)]
```

giac [A] time = 0.34, size = 94, normalized size = 1.07

$$-\frac{c^2 \log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{ac^3-cd^2}+\frac{c\log\left(bx^2\right)}{2\left(ac^2-d^2\right)}-\frac{d\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\left(ac^2-d^2\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -c^2*log(abs(sqrt(b*x^2 + a)*c + d))/(a*c^3 - c*d^2) + 1/2*c*log(b*x^2)/(a*
c^2 - d^2) - d*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt(-a))
```

maple [B] time = 0.04, size = 2175, normalized size = 24.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+c*ln(x)/(a*c^2-d^2)+1/2*
c/d^2*ln(b*c^2*x^2+a*c^2-d^2)+1/2*d*b*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)
*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/
2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d*b^(1/2)*c^2/a/((-a
*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b
*c^2)^(1/2))*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((
x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))+1/2*d*b*c^2
/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2
-d^2)*b*c^2)^(1/2))*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b))^(1/2)+1/2*d*b^(1/2)*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))
/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln(((x-(-a*b)^(
1/2)/b)*b+(-a*b)^(1/2))/b^(1/2)+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b))^(1/2))-1/2*d*b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d
^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-(a*c^
2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-
(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)+1/2*d*b^(1/2)*c^2/(a*c^2-d^2)/((
-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)
*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)
/b/c^2)*b-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*
c^2)^(1/2)/b/c^2)/c^2)^(1/2))+1/2*b*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*
c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1
/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)
*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)
/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+d/a^(1/2)/(a
*c^2-d^2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-d/a/(a*c^2-d^2)*(b*x^2+a)^(
1/2)-1/2*d*b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/
((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x-(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)/c^2)^(1/2)-1/2*d*b^(1/2)*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-
(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(
a*c^2-d^2)*b*c^2)^(1/2)*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2
-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1
/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/
c^2)^(1/2))+1/2*b*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1
/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln
((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)/c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*
```

```
d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^
(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^2 + ac + \sqrt{bx^2 + a}\, d\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x), x)
```

mupad [B] time = 3.95, size = 1270, normalized size = 14.43

$$\frac{c\,\ln(x)}{a\,c^2-d^2} - \frac{c\,\mathrm{atan}\left(\frac{\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}+\frac{c\left(4\,c^4\,d^5-8\,a\,c^6\,d^3+4\,a^2\,c^8\,d-\frac{c\,\sqrt{b\,x^2+a}\,\left(8\,a^3\,c^{10}-8\,a^2\,c^8\,d^2-8\,a\,c^6\,d^4+8\,c^4\,d^6\right)}{2\left(a\,c^2-d^2\right)}\right)}{2\left(a\,c^2-d^2\right)}\right)1\mathrm{i}}{2\left(a\,c^2-d^2\right)}+\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}-\frac{c\left(4\,c^4\,d^5-8\,a\,c^6 \cdots\right.}{\cdots}\right.}{\cdots}}{\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}+\frac{c\left(4\,c^4\,d^5-8\,a\,c^6\,d^3+4\,a^2\,c^8\,d-\frac{c\,\sqrt{b\,x^2+a}\,\left(8\,a^3\,c^{10}-8\,a^2\,c^8\,d^2-8\,a\,c^6\,d^4+8\,c^4\,d^6\right)}{2\left(a\,c^2-d^2\right)}\right)}{2\left(a\,c^2-d^2\right)}\right)}{2\left(a\,c^2-d^2\right)}-\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}-\frac{c\left(4\,c^4\,d^5-8\,a\,c^6 \cdots\right.}{\cdots}\right.}{\cdots}}\right. \cdots}{a\,c^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
```

```
[Out] (c*log(x))/(a*c^2 - d^2) - (c*atan(((c*(4*c^6*d^2*(a + b*x^2)^(1/2) + (c*(4
*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d - (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8
*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^
2)))*1i)/(2*(a*c^2 - d^2)) + (c*(4*c^6*d^2*(a + b*x^2)^(1/2) - (c*(4*c^4*d^
5 - 8*a*c^6*d^3 + 4*a^2*c^8*d + (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^
6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2)))*1i
)/(2*(a*c^2 - d^2)))/((c*(4*c^6*d^2*(a + b*x^2)^(1/2) + (c*(4*c^4*d^5 - 8*a
*c^6*d^3 + 4*a^2*c^8*d - (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a
*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2))))/(2*(a*c^
2 - d^2)) - (c*(4*c^6*d^2*(a + b*x^2)^(1/2) - (c*(4*c^4*d^5 - 8*a*c^6*d^3 +
 4*a^2*c^8*d + (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 -
 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2)))
)*1i)/(a*c^2 - d^2) - (c*log(a*c^2 - d^2 + b*c^2*x^2))/(2*a*c^2 - 2*d^2) -
(d*atan(((d*(4*c^6*d^2*(a + b*x^2)^(1/2) + (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*
a^2*c^8*d - (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*
a^2*c^8*d^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2)))*1i
)/(a^(1/2)*(2*a*c^2 - 2*d^2)) + (d*(4*c^6*d^2*(a + b*x^2)^(1/2) - (d*(4*c^4
*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d + (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4
*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)
*(2*a*c^2 - 2*d^2)))*1i)/(a^(1/2)*(2*a*c^2 - 2*d^2)))/((d*(4*c^6*d^2*(a + b
*x^2)^(1/2) + (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d - (d*(a + b*x^2)^(1
/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(a^(1/2)*(2*a*c
^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2)) -
(d*(4*c^6*d^2*(a + b*x^2)^(1/2) - (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d
 + (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d
^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(
2*a*c^2 - 2*d^2))))*1i)/(a^(1/2)*(a*c^2 - d^2))
```

sympy [A] time = 10.50, size = 88, normalized size = 1.00

$$-\frac{c^2\left(\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{ac^2-d^2}-\frac{-\frac{c\log\left(-bx^2\right)}{2}+\frac{d\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{ac^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] -c**2*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d
)/c, True))/(a*c**2 - d**2) - (-c*log(-b*x**2)/2 + d*atan(sqrt(a + b*x**2)/
sqrt(-a))/sqrt(-a))/(a*c**2 - d**2)
```

3.548 $\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=151

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^2}}{2ax^2\left(ac^2-d^2\right)}+\frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

```
[Out] -1/2*b*d*(3*a*c^2-d^2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/(a*c^2-d^2)
^2-b*c^3*ln(x)/(a*c^2-d^2)^2+b*c^3*ln(d+c*(b*x^2+a)^(1/2))/(a*c^2-d^2)^2+1/
2*(-a*c+d*(b*x^2+a)^(1/2))/a/(a*c^2-d^2)/x^2
```

Rubi [A] time = 0.35, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 741, 801, 635, 206, 260}

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^2}}{2ax^2\left(ac^2-d^2\right)}+\frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -(a*c - d*Sqrt[a + b*x^2])/(2*a*(a*c^2 - d^2)*x^2) - (b*d*(3*a*c^2 - d^2)*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[x
])/(a*c^2 - d^2)^2 + (b*c^3*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)^2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx,x,x^2\right)$$

$$= b\,\text{Subst}\left(\int \frac{1}{(d+cx)\left(a-x^2\right)^2}\,dx,x,\sqrt{a+bx^2}\right)$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{b\,\text{Subst}\left(\int \frac{-2ac^2+d^2+cdx}{(d+cx)\left(a-x^2\right)}\,dx,x,\sqrt{a+bx^2}\right)}{2a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{b\,\text{Subst}\left(\int \left(-\frac{2ac^4}{\left(ac^2-d^2\right)(d+cx)}+\frac{3ac^2d-d^3-2ac^3x}{\left(ac^2-d^2\right)\left(a-x^2\right)}\right)dx,x,\sqrt{a+\right.}{2a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{3ac^2d-d^3-2ac^3x}{a-x^2}\,dx\right.}{2a\left(ac^2-d^2\right)^2}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2} + \frac{\left(bc^3\right)\text{Subst}\left(\int \frac{x}{a-x^2}\,dx,x,\right.}{\left(ac^2-d^2\right)^2}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2} - \frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2} + \frac{bc^3\log}{}$$

Mathematica [A] time = 1.08, size = 291, normalized size = 1.93

$$\frac{\frac{\sqrt{a}\left(-a^2c^3\sqrt{a+bx^2}+a^2c^2d+2abc^3x^2\sqrt{a+bx^2}\tanh^{-1}\left(\frac{c\sqrt{a+bx^2}}{d}\right)-2abc^3x^2\log(x)\sqrt{a+bx^2}+bdx^2\sqrt{\frac{bx^2}{a}+1}\left(ac^2-d^2\right)\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+abc^2dx^2+abc^3x^2\right.}{x^2\sqrt{a+bx^2}}}{2a^{3/2}\left(d^2-ac^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (2*b*d*(-2*a*c^2 + d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]] + (Sqrt[a]*(a^2*c^
2*d - a*d^3 + a*b*c^2*d*x^2 - b*d^3*x^2 - a^2*c^3*Sqrt[a + b*x^2] + a*c*d^2
*Sqrt[a + b*x^2] + 2*a*b*c^3*x^2*Sqrt[a + b*x^2]*ArcTanh[(c*Sqrt[a + b*x^2]
)/d] + b*d*(a*c^2 - d^2)*x^2*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a
]] - 2*a*b*c^3*x^2*Sqrt[a + b*x^2]*Log[x] + a*b*c^3*x^2*Sqrt[a + b*x^2]*Log
[a*c^2 - d^2 + b*c^2*x^2]))/(x^2*Sqrt[a + b*x^2]))/(2*a^(3/2)*(-(a*c^2) + d
^2)^2)
```

fricas [A] time = 1.10, size = 530, normalized size = 3.51

$$\left[\frac{2\,a^2bc^3x^2\log\left(bc^2x^2+ac^2-d^2\right)-4\,a^2bc^3x^2\log(x)+a^2bc^3x^2\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-a^2bc^3x^2\log\left(-\right.}{4\left(a^4c^4-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a^2*b*c^3*x^2*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x)
 + a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2
) - a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^
2) - 2*a^3*c^3 + 2*a^2*c*d^2 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^2*log(-(b*x^
2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a^2*c^2*d - a*d^3)*sqrt(b*x^
2 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^2), 1/4*(2*a^2*b*c^3*x^2*log
(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x) + a^2*b*c^3*x^2*log(-(b*
c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a^2*b*c^3*x^2*log(-(b
*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 2*a^3*c^3 + 2*a^2*c*
d^2 + 2*(3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a))
 + 2*(a^2*c^2*d - a*d^3)*sqrt(b*x^2 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d
^4)*x^2)]
```

giac [A] time = 0.37, size = 210, normalized size = 1.39

$$\frac{bc^4\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{a^2c^5-2\,ac^3d^2+cd^4}-\frac{bc^3\log\left(-bx^2\right)}{2\left(a^2c^4-2\,ac^2d^2+d^4\right)}+\frac{\left(3\,abc^2d-bd^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\left(a^3c^4-2\,a^2c^2d^2+ad^4\right)\sqrt{-a}}-\frac{a^2bc^3-abcd^2-\left(abc^2d\right.}{2\left(ac^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] b*c^4*log(abs(sqrt(b*x^2 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4) - 1/2
*b*c^3*log(-b*x^2)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/2*(3*a*b*c^2*d - b*d^3
)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*sqrt(
-a)) - 1/2*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*sqrt(b*x^2 + a))/((
a*c^2 - d^2)^2*a*b*x^2)
```

maple [B] time = 0.05, size = 2459, normalized size = 16.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2*a*c^5*b/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^2+a*c^2-d^2)-1/2*c/(a*c^2-d^2)/x^2
-2*b*c^3*ln(x)/(a*c^2-d^2)^2+1/a*c*b/(a*c^2-d^2)^2*ln(x)*d^2-1/2*b*c^3/(a*c
^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+b*c/a/(a*c^2-d^2)*ln(x)-1/2*d*b^2*c^2/a
^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2
-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b))^(1/2)+1/2*d*b^(3/2)*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2
))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln(((x+(-a*b)
^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b))^(1/2))-1/2*d*b^2*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*
b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x-(-a*b)^(1/2
)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)-1/2*d*b^(3/2)*c^2/a^2/((-
a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*
```

```
b*c^2)^(1/2))*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1/2)+(
(x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/2*d/a^2/
(a*c^2-d^2)/x^2*(b*x^2+a)^(3/2)+1/2*d/a^(3/2)/(a*c^2-d^2)*b*ln((2*a+2*(b*x^
2+a)^(1/2)*a^(1/2))/x)-1/2*d/a^2/(a*c^2-d^2)*b*(b*x^2+a)^(1/2)+1/2*d*b^2*c^
6/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)
*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+
1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)
/c^2)^(1/2)-1/2*d*b^(3/2)*c^4/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)
*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*
b*c^2)^(1/2)*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b-(-(a*c^2-d^2)*b*c^2
)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*
(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))
-1/2*b^2*c^4/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((
-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2
*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+
2*(1/c^2*d^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(
-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/
(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-2*d*b/a^(1/2)/(a*c^2-d^2)^2*ln((2*a+2
*(b*x^2+a)^(1/2)*a^(1/2))/x)*c^2+2*d*b/a/(a*c^2-d^2)^2*(b*x^2+a)^(1/2)*c^2+
b/a^(3/2)/(a*c^2-d^2)^2*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)*d^3-b/a^2/(a*
c^2-d^2)^2*(b*x^2+a)^(1/2)*d^3+1/2*d*b^2*c^6/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^
2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))
*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)
^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)+1/2*d*b^(3/2)*c^4/(a
*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2
-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln(((x-(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^
2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-
(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))-1/2*b^2*c^4/(a*c^2-d^2)^2/((-a*
b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*
c^2)^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2
)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2
-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/
c^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^2 + ac + \sqrt{bx^2 + a}\, d\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^3), x)
```

mupad [B] time = 5.56, size = 4602, normalized size = 30.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
```

```
[Out] (atan(((((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^
5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 + 3*
a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c
^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*
d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8
 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4 + a^2
```

```
*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 -
4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c
^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(16*(
a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) + ((a +
 b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a
^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*
b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6
*d^2))^(1/2)*1i)/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6
*c^6*d^2) - (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 -
18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6
 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*
b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5
*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^
6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4
+ a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d
^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(
a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/
(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) -
((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(
32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9
*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^
6*c^6*d^2))^(1/2)*1i)/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 -
4*a^6*c^6*d^2))/(((b^3*c^8*d^5)/2 - (3*a*b^3*c^10*d^3)/2)/(a^5*c^6 - a^2*d^
6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2) + (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d
^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*
d)/(16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - ((a + b*x^2)^
(1/2)*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^
8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16
*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*
c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*
a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d
^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4
- 4*a^6*c^6*d^2))^(1/2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4
*d^4 - 4*a^6*c^6*d^2)) + ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4
+ 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*
d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d
^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*
d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2) + (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8
*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^1
4*d)/(16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + ((a + b*x^2
)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*
d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 +
16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^
6*c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 -
4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2
*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^
4 - 4*a^6*c^6*d^2))^(1/2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c
^4*d^4 - 4*a^6*c^6*d^2)) - ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^
4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^
2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2
*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(a^7*c^8 + a^3*d^8 - 4*a^4*c^
2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9
*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^
6*c^6*d^2))^(1/2)*1i)/(8*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4
 - 4*a^6*c^6*d^2)) - c/(2*x^2*(a*c^2 - d^2)) - (b*c^3*log(x))/(d^4 + a^2*c^
4 - 2*a*c^2*d^2) + (b*c^3*log(a*c^2 - d^2 + b*c^2*x^2))/(2*d^4 + 2*a^2*c^4
- 4*a*c^2*d^2) - (d*(a + b*x^2)^(1/2))/(2*x^2*(a*d^2 - a^2*c^2)) + (b*c^3*a
tan(((c^3*(((a + b*x^2)^(1/2)*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2
*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) + (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d
```

```
 - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(
4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - (c^3*(a + b*x^2)^(
1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*
a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*
a^3*c^2*d^2))))/(2*(a*c^2 - d^2)^2))*1i)/(2*(a*c^2 - d^2)^2) + (c^3*(((a +
b*x^2)^(1/2)*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d
^4 - 2*a^3*c^2*d^2)) - (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9
 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*
d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + (c^3*(a + b*x^2)^(1/2)*(16*a^7*c^14
 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48
*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2))))/(
2*(a*c^2 - d^2)^2))*1i)/(2*(a*c^2 - d^2)^2))/((c^8*d^5 - 3*a*c^10*d^3)/(2*(
a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - (c^3*(((a + b*x^2)^(1
/2)*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^
3*c^2*d^2)) + (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^
3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a
^3*c^2*d^4 - 3*a^4*c^4*d^2)) - (c^3*(a + b*x^2)^(1/2)*(16*a^7*c^14 + 16*a^2
*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12
*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2))))/(2*(a*c^2
- d^2)^2)))/(2*(a*c^2 - d^2)^2) + (c^3*(((a + b*x^2)^(1/2)*(c^6*d^6 - 6*a*c
^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) - (c^3*(
(8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*
c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c
^4*d^2)) + (c^3*(a + b*x^2)^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c
^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d
^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2))))/(2*(a*c^2 - d^2)^2)))/(2*(a*c^
2 - d^2)^2)))*1i)/(a*c^2 - d^2)^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**3*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

3.549 $\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}+\frac{x}{bc}$$

```
[Out] x/b/c-d*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)/c^2-arctan(c*x*b^(1/2)/(
a*c^2-d^2)^(1/2))*(a*c^2-d^2)^(1/2)/b^(3/2)/c^2+arctan(d*x*b^(1/2)/(a*c^2-d
^2)^(1/2)/(b*x^2+a)^(1/2))*(a*c^2-d^2)^(1/2)/b^(3/2)/c^2
```

Rubi [A] time = 0.24, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2156, 321, 205, 483, 217, 206, 377}

$$\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}+\frac{x}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x/(b*c) - (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(b^(3
/2)*c^2) + (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[
a + b*x^2]])])/(b^(3/2)*c^2) - (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(b^(
3/2)*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = (ac)\int \frac{x^2}{a^2c^2-ad^2+abc^2x^2}\,dx-(ad)\int \frac{x^2}{\sqrt{a+bx^2}\,\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx$$

$$= \frac{x}{bc}-\frac{d\int \frac{1}{\sqrt{a+bx^2}}\,dx}{bc^2}-\frac{\left(a\left(ac^2-d^2\right)\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx}{bc}+\frac{\left(ad\left(ac^2-d^2\right)\right)\int \frac{}{\sqrt{a+b}}}{bc^{}}$$

$$= \frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{bc^2}+\frac{\left(ad\left(ac^2-\right.\right.}{}$$

$$= \frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}+\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\,\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}}{b^{3/}}$$

Mathematica [A] time = 0.27, size = 157, normalized size = 1.07

$$\frac{\sqrt{ac^2-d^2}\left(\sqrt{b}cx-d\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\right)+\left(ac^2-d^2\right)\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)+\left(d^2-ac^2\right)\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] ((-(a*c^2) + d^2)*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] + (a*c^2 - d^2)*A
rcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])] + Sqrt[a*c^2 - d^2
]*(Sqrt[b]*c*x - d*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]))/(b^(3/2)*c^2*Sqrt[a
*c^2 - d^2])
```

fricas [A] time = 0.65, size = 1168, normalized size = 7.95

$$\left[\frac{4\,bcx+2\,\sqrt{b}\,d\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+b\sqrt{-\frac{ac^2-d^2}{b}}\,\log\left(\frac{a^4c^4-2\,a^3c^2d^2+a^2d^4+\left(a^2b^2c^4-8\,ab^2c^2d^2+8\,b^2d^4\right)x^4+2\,(a}{b^2c^4x^4}\right.}{4\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*c*x + 2*sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b
```

```
^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2
+ 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^
3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*
c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*c^2 - d^2)/b)*
log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2)/(b*c^2*x^2 +
 a*c^2 - d^2)))/(b^2*c^2), 1/4*(4*b*c*x + 4*sqrt(-b)*d*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^
2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*
a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*
c^2*d - a*b*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 +
a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*
c^2 - d^2)/b)*log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2
)/(b*c^2*x^2 + a*c^2 - d^2)))/(b^2*c^2), 1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d
^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b)/(a*c^2 - d^2)) - b*sqrt((a*c^2 -
 d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2
+ a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d - a*d^3)*x
)) + sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(b^2*c^2),
1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b
)/(a*c^2 - d^2)) + 2*sqrt(-b)*d*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - b*sqrt
((a*c^2 - d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sq
rt(b*x^2 + a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d -
 a*d^3)*x)))/(b^2*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [B] time = 0.04, size = 3485, normalized size = 23.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2*d*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-
a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2
)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x+(-a*b
)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b))^(1/2))/b^(1/2)-1/2*d*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^
2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)
)*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)-1/2*d*c^
2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c
^2-d^2)*b*c^2)^(1/2))*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1/2)+((x-(-
a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))/b^(1/2)-1/2*d*c
^4/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)/c^2)^(1/2)*a+1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)
^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)
^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*
b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)*d^3+1/2*d*c^2/
```

```
((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d
^2)*b*c^2)^(1/2))*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b-(-(a*c^2-d^2)*
b*c^2)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d
^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(
1/2))/b^(1/2)*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^
(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c
^2)*b-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)/c^2)^(1/2))/b^(1/2)*d^3+1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^
2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^
2)*b*c^2)^(1/2)*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^
(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x+(-(a
*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x
+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2))*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2
)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^5/(1/c^2*d^2
)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^
2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/
c^2)/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+1/2*d*c^4/((-a*b)^(1
/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)
^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*
b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)/c^2)^(1/2)*a-1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a
*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x-(-
(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*
(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)*d^3+1/2*d*c^2/((-a*b)^(1/2)
*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1
/2))*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2-d^2)*b*c^2)^(1/2)/
c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2
-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/b^(1/2)
*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(
a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+
1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)
/c^2)^(1/2))/b^(1/2)*d^3-1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/
2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/
2)*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a
*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a+1
/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^5/(1/c^2*d^2)^(1/2)*ln((2
/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/
c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2
+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/
2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-a/b/((a*c^2-d^2)*b)^(1/2)*arctan(
x*b*c/((a*c^2-d^2)*b)^(1/2))+x/b/c+1/b/c^2*d^2/((a*c^2-d^2)*b)^(1/2)*arctan
(x*b*c/((a*c^2-d^2)*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{bcx^2 + ac + \sqrt{bx^2 + a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{a\,c+d\,\sqrt{b\,x^2+a}\,+b\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
[Out] int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
[Out] Integral(x**2/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

3.550 $\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}$$

```
[Out] arctan(c*x*b^(1/2)/(a*c^2-d^2)^(1/2))/b^(1/2)/(a*c^2-d^2)^(1/2)-arctan(d*x*
b^(1/2)/(a*c^2-d^2)^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)/(a*c^2-d^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2156, 205, 377}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]]/(Sqrt[b]*Sqrt[a*c^2 - d^2]) - ArcTa
n[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])]/(Sqrt[b]*Sqrt[a*c^2 -
d^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx &= (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx-(ad)\int \frac{1}{\sqrt{a+bx^2}\,\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}-(ad)\,\text{Subst}\left(\int \frac{1}{a^2c^2-ad^2-\left(-a^2bc^2+b\left(a^2c^2-ad^2\right)\right)x^2}\,dx,x\right.\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{ac^2-d^2}\,\sqrt{a+bx^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}\end{aligned}$$

Mathematica [A] time = 0.13, size = 83, normalized size = 0.81

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)-\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] (ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] - ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2
 - d^2]*Sqrt[a + b*x^2])])/(Sqrt[b]*Sqrt[a*c^2 - d^2])
```

fricas [B] time = 0.48, size = 510, normalized size = 4.95

$$\left[-\frac{\sqrt{-abc^2+bd^2}\log\left(\frac{a^4c^4-2a^3c^2d^2+a^2d^4+\left(a^2b^2c^4-8ab^2c^2d^2+8b^2d^4\right)x^4+2\left(a^3bc^4-5a^2bc^2d^2+4abd^4\right)x^2-4\sqrt{-abc^2+bd^2}\left(\left(abc^2d-2bd^3\right)x^3\right.}{b^2c^4x^4+a^2c^4-2ac^2d^2+d^4+2\left(abc^4-bc^2d^2\right)x^2}\right.}{4\left(abc^2-bd^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b*c^2 + b*d^2)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2
*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^
2 + 4*a*b*d^4)*x^2 - 4*sqrt(-a*b*c^2 + b*d^2)*((a*b*c^2*d - 2*b*d^3)*x^3 +
(a^2*c^2*d - a*d^3)*x)*sqrt(b*x^2 + a))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^
2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*sqrt(-a*b*c^2 + b*d^2)*log((b*c
^2*x^2 - a*c^2 - 2*sqrt(-a*b*c^2 + b*d^2)*c*x + d^2)/(b*c^2*x^2 + a*c^2 - d
^2)))/(a*b*c^2 - b*d^2), -1/2*(2*sqrt(a*b*c^2 - b*d^2)*arctan(-sqrt(a*b*c^2
 - b*d^2)*c*x/(a*c^2 - d^2)) - sqrt(a*b*c^2 - b*d^2)*arctan(1/2*(a^2*c^2 -
a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(a*b*c^2 - b*d^2)*sqrt(b*x^2 + a)/((a*
b^2*c^2*d - b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^3)*x)))/(a*b*c^2 - b*d^2)]
```

giac [A] time = 0.36, size = 107, normalized size = 1.04

$$\frac{\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}}+\frac{\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2c^2+ac^2-2d^2}{2\sqrt{ac^2-d^2}d}\right)}{\sqrt{ac^2-d^2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/sqrt(a*b*c^2 - b*d^2) + arctan(1/2*((sq
rt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2*d^2)/(sqrt(a*c^2 - d^2)*d))/(s
qrt(a*c^2 - d^2)*sqrt(b))
```

maple [B] time = 0.04, size = 1995, normalized size = 19.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2*d*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(
-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2*d*b^(1/2)*c^2/((-a*b)^(1/2)*c^2+(-(a*
c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((
```

```
x+(-a*b)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))+1/2*d*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2
+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))
*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d*b^(
1/2)*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(
-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1/2)+(
(x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/2*d*b*c^
4/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/
2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2
)^(1/2)/b/c^2)/c^2)^(1/2)-1/2*d*b^(1/2)*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)
*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x+(-(a*c
^2-d^2)*b*c^2)^(1/2)/b/c^2)*b-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x+(
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)
*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))-1/2*b*c^2/((-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2
))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^
2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^
(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b
*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/(x+(-(a*c^2-d^
2)*b*c^2)^(1/2)/b/c^2))-1/2*d*b*c^4/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^
(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^
(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b
*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)-1/2*d*b^(1/2)*c
^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2-d^
2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^
2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2
)^(1/2))+1/2*b*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(
1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^3/(1/c^2*
d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)
/b/c^2)/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+1/((a*c^2-d^2)*b)
^(1/2)*arctan(x*b*c/((a*c^2-d^2)*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bcx^2 + ac + \sqrt{bx^2 + a}\, d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2-b\,d^2}}\right)}{\sqrt{a\,b\,c^2-b\,d^2}}-\frac{d\,x}{\sqrt{a}\,\left(a\,c^2-d^2\right)} & \text{if } b=0\vee d=0 \\ \frac{\operatorname{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2-b\,d^2}}\right)}{\sqrt{a\,b\,c^2-b\,d^2}}-\frac{d\operatorname{atan}\left(\frac{x\,\sqrt{a\,b\,c^2-b\,\left(a\,c^2-d^2\right)}}{\sqrt{a\,c^2-d^2}\,\sqrt{b\,x^2+a}}\right)}{\sqrt{-\left(a\,c^2-d^2\right)\,\left(b\,\left(a\,c^2-d^2\right)-a\,b\,c^2\right)}} & \text{if } 0<b\,d^2 \\ \frac{\operatorname{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2-b\,d^2}}\right)}{\sqrt{a\,b\,c^2-b\,d^2}}-\frac{d\,\ln\left(\frac{\sqrt{\left(a\,c^2-d^2\right)\,\left(b\,x^2+a\right)}+x\,\sqrt{b\,\left(a\,c^2-d^2\right)-a\,b\,c^2}}{\sqrt{\left(a\,c^2-d^2\right)\,\left(b\,x^2+a\right)}-x\,\sqrt{b\,\left(a\,c^2-d^2\right)-a\,b\,c^2}}\right)}{2\,\sqrt{\left(a\,c^2-d^2\right)\,\left(b\,\left(a\,c^2-d^2\right)-a\,b\,c^2\right)}} & \text{if } b\,d^2<0 \\ \int \frac{1}{a\,c+d\,\sqrt{b\,x^2+a}+b\,c\,x^2}\,dx & \text{if } b\,d^2\notin \mathbb{R} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] piecewise(b == 0 | d == 0, atan((b*c*x)/(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2
 + a*b*c^2)^(1/2) - (d*x)/(a^(1/2)*(a*c^2 - d^2)), 0 < b*d^2, atan((b*c*x)/
(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*atan((x*(- b*(a*c
^2 - d^2) + a*b*c^2)^(1/2))/((a*c^2 - d^2)^(1/2)*(a + b*x^2)^(1/2))))/(-(a*
c^2 - d^2)*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2), b*d^2 < 0, atan((b*c*x)/(- b
*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*log((((a*c^2 - d^2)*(
a + b*x^2))^(1/2) + x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))/(((a*c^2 - d^2)*(a
 + b*x^2))^(1/2) - x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))))/(2*((a*c^2 - d^2)
*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2)), ~in(b*d^2, 'real'), int(1/(a*c + d*(a
 + b*x^2)^(1/2) + b*c*x^2), x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

3.551 $\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=160

$$\frac{d\sqrt{a+bx^2}}{ax\left(ac^2-d^2\right)}+\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{c}{x\left(ac^2-d^2\right)}$$

```
[Out] -c/(a*c^2-d^2)/x-c^2*arctan(c*x*b^(1/2)/(a*c^2-d^2)^(1/2))*b^(1/2)/(a*c^2-d
^2)^(3/2)+c^2*arctan(d*x*b^(1/2)/(a*c^2-d^2)^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)
/(a*c^2-d^2)^(3/2)+d*(b*x^2+a)^(1/2)/a/(a*c^2-d^2)/x
```

Rubi [A] time = 0.24, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2156, 325, 205, 480, 12, 377}

$$\frac{d\sqrt{a+bx^2}}{ax\left(ac^2-d^2\right)}+\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{c}{x\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x) - (Sqrt[b]
*c^2*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2)^(3/2) + (Sqrt[b
]*c^2*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*c^2 - d
^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
```

```
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = (ac)\int \frac{1}{x^2\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx-(ad)\int \frac{1}{x^2\sqrt{a+bx^2}\,\left(a^2c^2-ad^2+a\right.}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\left(abc^3\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx}{ac^2-d^2}+\frac{d\int \frac{}{\sqrt{a+bx^2}}}{a\left(\right.}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\left(abc^2d\right)\int \frac{}{\sqrt{a+b}}}{a}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\left(abc^2d\right)\text{Subst}}{}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\sqrt{b}\,c^2\tan^{-1}\left(-\right.}{\left(ac^2-\right.}$$

Mathematica [A] time = 0.34, size = 139, normalized size = 0.87

$$\frac{\sqrt{ac^2-d^2}\left(d\sqrt{a+bx^2}-ac\right)+a\sqrt{b}\,c^2x\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)-a\sqrt{b}\,c^2x\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{ax\left(ac^2-d^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (Sqrt[a*c^2 - d^2]*(-(a*c) + d*Sqrt[a + b*x^2]) - a*Sqrt[b]*c^2*x*ArcTan[(S
qrt[b]*c*x)/Sqrt[a*c^2 - d^2]] + a*Sqrt[b]*c^2*x*ArcTan[(Sqrt[b]*d*x)/(Sqrt
[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*(a*c^2 - d^2)^(3/2)*x)
```

fricas [A] time = 0.56, size = 581, normalized size = 3.63

$$\left[-\frac{ac^2x\sqrt{-\frac{b}{ac^2-d^2}}\,\log\left(\frac{a^4c^4-2\,a^3c^2d^2+a^2d^4+\left(a^2b^2c^4-8\,ab^2c^2d^2+8\,b^2d^4\right)x^4+2\left(a^3bc^4-5\,a^2bc^2d^2+4\,abd^4\right)x^2+4\left(\left(a^2bc^4d-3\,abc^2d^3+2\,bd^5\right)x^3+\right.}{b^2c^4x^4+a^2c^4-2\,ac^2d^2+d^4+2\left(abc^4-bc^2d^2\right)x^2}\right.}{4\left(a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^
4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*
b*c^2*d^2 + 4*a*b*d^4)*x^2 + 4*((a^2*b*c^4*d - 3*a*b*c^2*d^3 + 2*b*d^5)*x^3
 + (a^3*c^4*d - 2*a^2*c^2*d^3 + a*d^5)*x)*sqrt(b*x^2 + a)*sqrt(-b/(a*c^2 -
d^2)))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)
*x^2)) + 2*a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((b*c^2*x^2 - a*c^2 + 2*(a*c^3
 - c*d^2)*x*sqrt(-b/(a*c^2 - d^2)) + d^2)/(b*c^2*x^2 + a*c^2 - d^2)) + 4*a*
c - 4*sqrt(b*x^2 + a)*d)/((a^2*c^2 - a*d^2)*x), -1/2*(2*a*c^2*x*sqrt(b/(a*c
^2 - d^2))*arctan(c*x*sqrt(b/(a*c^2 - d^2))) - a*c^2*x*sqrt(b/(a*c^2 - d^2)
)*arctan(-1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2 + a)*s
qrt(b/(a*c^2 - d^2))/(b^2*d*x^3 + a*b*d*x)) + 2*a*c - 2*sqrt(b*x^2 + a)*d)/
((a^2*c^2 - a*d^2)*x)]
```

giac [A] time = 0.46, size = 211, normalized size = 1.32

$$-b^{\frac{3}{2}}d\left(\frac{c^2\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2c^2+ac^2-2\,d^2}{2\,\sqrt{ac^2-d^2}\,d}\right)}{\left(abc^2-bd^2\right)\sqrt{ac^2-d^2}\,d}+\frac{2}{\left(abc^2-bd^2\right)\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)}\right)-\frac{bc^2\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}\left(ac^2-d^2\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -b^(3/2)*d*(c^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2
*d^2)/(sqrt(a*c^2 - d^2)*d))/((a*b*c^2 - b*d^2)*sqrt(a*c^2 - d^2)*d) + 2/((
a*b*c^2 - b*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a))) - b*c^2*arctan(b*c
*x/sqrt(a*b*c^2 - b*d^2))/(sqrt(a*b*c^2 - b*d^2)*(a*c^2 - d^2)) - c/((a*c^2
 - d^2)*x)
```

maple [B] time = 0.05, size = 2289, normalized size = 14.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] b*c^2/d^2/((a*c^2-d^2)*b)^(1/2)*arctan(1/((a*c^2-d^2)*b)^(1/2)*b*c*x)-c/(a*
c^2-d^2)/x-a*c^4/(a*c^2-d^2)*b/d^2/((a*c^2-d^2)*b)^(1/2)*arctan(1/((a*c^2-d
^2)*b)^(1/2)*b*c*x)-1/2*d*b^2*c^2/a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b
)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2*d*b^(3/2)*c^2/a
/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d
^2)*b*c^2)^(1/2))*ln(((x+(-a*b)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)
^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))+d/a^2/(a*c^2-d^2)/x
*(b*x^2+a)^(3/2)-d/a^2/(a*c^2-d^2)*b*x*(b*x^2+a)^(1/2)-d/a/(a*c^2-d^2)*b^(1
/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*d*b^2*c^2/a/(-a*b)^(1/2)/((-a*b)^(1/2
)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1
/2))*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d
*b^(3/2)*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*
c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1
/2)+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/2*d
*b^2*c^6/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^
(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x+(-(a*c
^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)-1/2*d*b^(3/2)*c^4/(a*c^2-d^2)/(
(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2
```

```
)*b*c^2)^(1/2))*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b-(-(a*c^2-d^2)*b*
c^2)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2
-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/
2))-1/2*b^2*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(
(-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^3
/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d
^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-1/2*d*b^2
*c^6/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2
)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((x-(-(a*c^2-d
^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*
c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)-1/2*d*b^(3/2)*c^4/(a*c^2-d^2)/((-a*
b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*
c^2)^(1/2))*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2-d^2)*b*c^2)
^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(
-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))+
1/2*b^2*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*
b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^3/(1/
c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2)*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^2 + ac + \sqrt{bx^2 + a}\,d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(a\,c+d\,\sqrt{b\,x^2+a}+b\,c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
[Out] int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(ac + bcx^2 + d\sqrt{a + bx^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
[Out] Integral(1/(x**2*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

3.552 $\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=140

$$-\frac{2d\left(a+bx^3\right)^{3/2}}{9b^3c^2}+\frac{2\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}+\frac{2d\sqrt{a+bx^3}\left(2ac^2-d^2\right)}{3b^3c^4}+\frac{\left(a+bx^3\right)^2}{6b^3c}-\frac{x^3\left(2ac^2-d^2\right)}{3b^2c^3}$$

```
[Out] -1/3*(2*a*c^2-d^2)*x^3/b^2/c^3-2/9*d*(b*x^3+a)^(3/2)/b^3/c^2+1/6*(b*x^3+a)^
2/b^3/c+2/3*(a*c^2-d^2)^2*ln(d+c*(b*x^3+a)^(1/2))/b^3/c^5+2/3*d*(2*a*c^2-d^
2)*(b*x^3+a)^(1/2)/b^3/c^4
```

Rubi [A] time = 0.30, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{x^3\left(2ac^2-d^2\right)}{3b^2c^3}+\frac{2d\sqrt{a+bx^3}\left(2ac^2-d^2\right)}{3b^3c^4}+\frac{2\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}-\frac{2d\left(a+bx^3\right)^{3/2}}{9b^3c^2}+\frac{\left(a+bx^3\right)^2}{6b^3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -((2*a*c^2 - d^2)*x^3)/(3*b^2*c^3) + (2*d*(2*a*c^2 - d^2)*Sqrt[a + b*x^3])/
(3*b^3*c^4) - (2*d*(a + b*x^3)^(3/2))/(9*b^3*c^2) + (a + b*x^3)^2/(6*b^3*c)
 + (2*(a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^3]])/(3*b^3*c^5)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^3\right)\\ &= \frac{2\,\text{Subst}\left(\int \frac{\left(a-x^2\right)^2}{d+cx}\,dx,x,\sqrt{a+bx^3}\right)}{3b^3}\\ &= \frac{2\,\text{Subst}\left(\int \left(\frac{2ac^2d-d^3}{c^4}-\frac{\left(2ac^2-d^2\right)x}{c^3}-\frac{dx^2}{c^2}+\frac{x^3}{c}+\frac{\left(ac^2-d^2\right)^2}{c^4(d+cx)}\right)dx,x,\sqrt{a+bx^3}\right)}{3b^3}\\ &= -\frac{\left(2ac^2-d^2\right)x^3}{3b^2c^3}+\frac{2d\left(2ac^2-d^2\right)\sqrt{a+bx^3}}{3b^3c^4}-\frac{2d\left(a+bx^3\right)^{3/2}}{9b^3c^2}+\frac{\left(a+bx^3\right)^2}{6b^3c}+\frac{2(}{}\end{aligned}$$

Mathematica [A] time = 0.20, size = 126, normalized size = 0.90

$$\frac{c\left(a\left(20c^2d\sqrt{a+bx^3}-6bc^3x^3\right)+2bcdx^3\left(3d-2c\sqrt{a+bx^3}\right)-12d^3\sqrt{a+bx^3}+3b^2c^3x^6\right)+12\left(d^2-ac^2\right)^2\log\left(c\right.}{18b^3c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (c*(3*b^2*c^3*x^6 - 12*d^3*Sqrt[a + b*x^3] + 2*b*c*d*x^3*(3*d - 2*c*Sqrt[a
+ b*x^3]) + a*(-6*b*c^3*x^3 + 20*c^2*d*Sqrt[a + b*x^3])) + 12*(-(a*c^2) + d
^2)^2*Log[d + c*Sqrt[a + b*x^3]])/(18*b^3*c^5)
```

fricas [A] time = 0.42, size = 191, normalized size = 1.36

$$\frac{3\,b^2c^4x^6-6\left(abc^4-bc^2d^2\right)x^3+6\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(bc^2x^3+ac^2-d^2\right)+6\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{\right.}{18\,b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*(3*b^2*c^4*x^6 - 6*(a*b*c^4 - b*c^2*d^2)*x^3 + 6*(a^2*c^4 - 2*a*c^2*d^
2 + d^4)*log(b*c^2*x^3 + a*c^2 - d^2) + 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log
(sqrt(b*x^3 + a)*c + d) - 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 +
a)*c - d) - 4*(b*c^3*d*x^3 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^3 + a))/(b^3*c^5
)
```

giac [A] time = 0.39, size = 156, normalized size = 1.11

$$\frac{2\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{3\,b^3c^5}+\frac{3\left(bx^3+a\right)^2b^9c^3-12\left(bx^3+a\right)ab^9c^3-4\left(bx^3+a\right)^{\frac{3}{2}}b^9c^2d+24}{18\,b^{12}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^3 + a)*c + d))/(b^3*c^5)
 + 1/18*(3*(b*x^3 + a)^2*b^9*c^3 - 12*(b*x^3 + a)*a*b^9*c^3 - 4*(b*x^3 + a)
^(3/2)*b^9*c^2*d + 24*sqrt(b*x^3 + a)*a*b^9*c^2*d + 6*(b*x^3 + a)*b^9*c*d^2
 - 12*sqrt(b*x^3 + a)*b^9*d^3)/(b^12*c^4)
```

maple [C] time = 0.18, size = 1473, normalized size = 10.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -2/9*d*(b*x^3+a)^(3/2)/b^3/c^2+4/3*d/b^3/c^2*(b*x^3+a)^(1/2)*a-2/3/b^3/c^4*
d^3*(b*x^3+a)^(1/2)-1/3*I/b^5/d*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/
b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/
b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2
*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(
1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I
*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)
/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))*a^2+2/3*I*d/b^5/c
^2*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(
-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2
)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)
+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^
```

```
2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^
(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/
b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=R
ootOf(_Z^3*b*c^2+a*c^2-d^2))*a-1/3*I/b^5/c^4*d^3*2^(1/2)*sum((-a*b^2)^(1/3)
*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3
))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1
/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(
-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-a*b^2)^(1/3)*3^(
1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2
/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))-
1/3*a/c/b^2*x^3+1/3*a^2/c/b^3*ln(b*c^2*x^3+a*c^2-d^2)-2/3*a/c^3/b^3*d^2*ln(
b*c^2*x^3+a*c^2-d^2)+1/6/b/c*x^6+1/3/b^2/c^3*x^3*d^2+1/3/b^3/c^5*d^4*ln(b*c
^2*x^3+a*c^2-d^2)
```

maxima [A] time = 0.78, size = 125, normalized size = 0.89

$$\frac{\frac{3\left(bx^3+a\right)^2c^3-4\left(bx^3+a\right)^{\frac{3}{2}}c^2d-6\left(2\,ac^3-cd^2\right)\left(bx^3+a\right)+12\left(2\,ac^2d-d^3\right)\sqrt{bx^3+a}}{c^4}+\frac{12\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{bx^3+a}\,c+d\right)}{c^5}}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/18*((3*(b*x^3 + a)^2*c^3 - 4*(b*x^3 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2
)*(b*x^3 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^3 + a))/c^4 + 12*(a^2*c^4 - 2
*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c + d)/c^5)/b^3
```

mupad [B] time = 3.74, size = 200, normalized size = 1.43

$$\frac{\left(\frac{2\,d\,\left(a\,c^2-d^2\right)}{b^2\,c^4}+\frac{4\,a\,d}{3\,b^2\,c^2}\right)\,\sqrt{b\,x^3+a}}{3\,b}+\frac{x^6}{6\,b\,c}-\frac{x^3\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}+\frac{\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)\,{\left(a\,c^2-d^2\right)}^2}{3\,b^3\,c^5}+\frac{\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)\,\left(a^2\,c\right.}{3\,b^3\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] (((2*d*(a*c^2 - d^2))/(b^2*c^4) + (4*a*d)/(3*b^2*c^2))*(a + b*x^3)^(1/2))/(
3*b) + x^6/(6*b*c) - (x^3*(a*c^2 - d^2))/(3*b^2*c^3) + (log((d + c*(a + b*x
^3)^(1/2))/(d - c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2)^2)/(3*b^3*c^5) + (log(a
*c^2 - d^2 + b*c^2*x^3)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(3*b^3*c^5) - (2*d*x
^3*(a + b*x^3)^(1/2))/(9*b^2*c^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

3.553 $\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}-\frac{2\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3}+\frac{x^3}{3bc}$$

```
[Out] 1/3*x^3/b/c-2/3*(a*c^2-d^2)*ln(d+c*(b*x^3+a)^(1/2))/b^2/c^3-2/3*d*(b*x^3+a)
^(1/2)/b^2/c^2
```

Rubi [A] time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2155, 697}

$$-\frac{2\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3}-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}+\frac{x^3}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] x^3/(3*b*c) - (2*d*Sqrt[a + b*x^3])/(3*b^2*c^2) - (2*(a*c^2 - d^2)*Log[d +
c*Sqrt[a + b*x^3]])/(3*b^2*c^3)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^3\right)\\ &= \frac{2\,\text{Subst}\left(\int \frac{-a+x^2}{d+cx}\,dx,x,\sqrt{a+bx^3}\right)}{3b^2}\\ &= \frac{2\,\text{Subst}\left(\int \left(-\frac{d}{c^2}+\frac{x}{c}+\frac{-ac^2+d^2}{c^2(d+cx)}\right)dx,x,\sqrt{a+bx^3}\right)}{3b^2}\\ &= \frac{x^3}{3bc}-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}-\frac{2\left(ac^2-d^2\right)\log\left(d+c\sqrt{a+bx^3}\right)}{3b^2c^3}\end{aligned}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.86

$$\frac{\left(2d^2-2ac^2\right)\log\left(c\sqrt{a+bx^3}+d\right)+c\left(bcx^3-2d\sqrt{a+bx^3}\right)}{3b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (c*(b*c*x^3 - 2*d*Sqrt[a + b*x^3]) + (-2*a*c^2 + 2*d^2)*Log[d + c*Sqrt[a +
b*x^3]])/(3*b^2*c^3)
```

fricas [A] time = 0.41, size = 118, normalized size = 1.62

$$\frac{bc^2x^3 - 2\sqrt{bx^3+a}\,cd - \left(ac^2-d^2\right)\log\left(bc^2x^3+ac^2-d^2\right) - \left(ac^2-d^2\right)\log\left(\sqrt{bx^3+a}\,c+d\right) + \left(ac^2-d^2\right)\log\left(\sqrt{bx^3+a}\,c-d\right)}{3\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*c^2*x^3 - 2*sqrt(b*x^3 + a)*c*d - (a*c^2 - d^2)*log(b*c^2*x^3 + a*c^
2 - d^2) - (a*c^2 - d^2)*log(sqrt(b*x^3 + a)*c + d) + (a*c^2 - d^2)*log(sqr
t(b*x^3 + a)*c - d))/(b^2*c^3)
```

giac [A] time = 0.33, size = 72, normalized size = 0.99

$$-\frac{\frac{2\left(ac^2-d^2\right)\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{bc^3} - \frac{\left(bx^3+a\right)bc-2\sqrt{bx^3+a}\,bd}{b^2c^2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c^3) - ((b*x^3 + a
)*b*c - 2*sqrt(b*x^3 + a)*b*d)/(b^2*c^2))/b
```

maple [C] time = 0.03, size = 943, normalized size = 12.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -2/3*d*(b*x^3+a)^(1/2)/b^2/c^2+1/3*I/b^4/d*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*
I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)
*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2
)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*
b)^(1/2)/(b*x^3+a)^(1/2)*(2*_alpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-
(-a*b^2)^(1/3)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(
1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*
3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/
b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))*a-1/3*I*d/b^
4/c^2*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*
b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(
1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1
/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(2*_alpha^2*
b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*b^2)^(2
/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)
/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/2*(2*
I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(2/3)*3^
(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^(1/3)/(
-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=Root
```

```
Of(_Z^3*b*c^2+a*c^2-d^2))-1/3*a/c/b^2*ln(b*c^2*x^3+a*c^2-d^2)+1/3*x^3/b/c+1
/3/b^2/c^3*d^2*ln(b*c^2*x^3+a*c^2-d^2)
```

maxima [A] time = 0.58, size = 62, normalized size = 0.85

$$\frac{\frac{\left(bx^3+a\right)c-2\sqrt{bx^3+a}\,d}{c^2}-\frac{2\left(ac^2-d^2\right)\log\left(\sqrt{bx^3+a}\,c+d\right)}{c^3}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x^3 + a)*c - 2*sqrt(b*x^3 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b
*x^3 + a)*c + d)/c^3)/b^2
```

mupad [B] time = 3.56, size = 119, normalized size = 1.63

$$\frac{x^3}{3\,b\,c}-\frac{2\,d\,\sqrt{b\,x^3+a}}{3\,b^2\,c^2}+\frac{\ln\left(\frac{d-c\,\sqrt{b\,x^3+a}}{d+c\,\sqrt{b\,x^3+a}}\right)\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}-\frac{\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] x^3/(3*b*c) - (2*d*(a + b*x^3)^(1/2))/(3*b^2*c^2) + (log((d - c*(a + b*x^3)
^(1/2))/(d + c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2))/(3*b^2*c^3) - (log(a*c^2
- d^2 + b*c^2*x^3)*(a*c^2 - d^2))/(3*b^2*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

3.554 $\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=26

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

```
[Out] 2/3*ln(d+c*(b*x^3+a)^(1/2))/b/c
```

Rubi [A] time = 0.11, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2155, 31}

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\ &= \frac{2\log\left(d+c\sqrt{a+bx^3}\right)}{3bc}\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 1.00

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

fricas [B] time = 0.43, size = 61, normalized size = 2.35

$$\frac{\log\left(bc^2x^3 + ac^2 - d^2\right) + \log\left(\sqrt{bx^3 + a}\,c + d\right) - \log\left(\sqrt{bx^3 + a}\,c - d\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(log(b*c^2*x^3 + a*c^2 - d^2) + log(sqrt(b*x^3 + a)*c + d) - log(sqrt(b
*x^3 + a)*c - d))/(b*c)
```

giac [A] time = 0.32, size = 23, normalized size = 0.88

$$\frac{2\,\log\left(\left|\sqrt{bx^3 + a}\,c + d\right|\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c)
```

maple [C] time = 0.03, size = 455, normalized size = 17.50

$$\frac{\ln\left(b\,c^2x^3 + a\,c^2 - d^2\right)}{3bc} - \frac{i\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)b}{-3\left(-a\,b^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{2\left(-a\,b^2\right)^{\frac{1}{3}}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*I/d/b^3*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/
2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a
*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)
+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(2*_a
lpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*
b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2
)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-
1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(
2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^
(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)),_alp
ha=RootOf(_Z^3*b*c^2+a*c^2-d^2))+1/3/b/c*ln(b*c^2*x^3+a*c^2-d^2)
```

maxima [A] time = 0.56, size = 22, normalized size = 0.85

$$\frac{2\,\log\left(\sqrt{bx^3 + a}\,c + d\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(b*x^3 + a)*c + d)/(b*c)
```

mupad [B] time = 3.51, size = 60, normalized size = 2.31

$$\frac{\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)+\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] (log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))) + log(a*c^2 - d^2
 + b*c^2*x^3))/(3*b*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

3.555 $\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=93

$$-\frac{2c\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)}+\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

```
[Out] c*ln(x)/(a*c^2-d^2)-2/3*c*ln(d+c*(b*x^3+a)^(1/2))/(a*c^2-d^2)+2/3*d*arctanh
((b*x^3+a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 706, 31, 635, 207, 260}

$$-\frac{2c\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)}+\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x
])/(a*c^2 - d^2) - (2*c*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
```

```
b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx,x,x^3\right)$$
$$= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{(d+cx)\left(-a+x^2\right)}\,dx,x,\sqrt{a+bx^3}\right)$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{d-cx}{-a+x^2}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)} - \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{d+cx}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}$$
$$= -\frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)} + \frac{(2c)\,\text{Subst}\left(\int \frac{x}{-a+x^2}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)} - (2d)\,\text{Sub}$$
$$= \frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)} + \frac{c\log(x)}{ac^2-d^2} - \frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}$$

Mathematica [A] time = 0.11, size = 107, normalized size = 1.15

$$\frac{\left(\sqrt{a}\,c-d\right)\log\left(\sqrt{a}-\sqrt{a+bx^3}\right)+\left(\sqrt{a}\,c+d\right)\log\left(\sqrt{a+bx^3}+\sqrt{a}\right)-2\sqrt{a}\,c\log\left(c\sqrt{a+bx^3}+d\right)}{3\sqrt{a}\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] ((Sqrt[a]*c - d)*Log[Sqrt[a] - Sqrt[a + b*x^3]] + (Sqrt[a]*c + d)*Log[Sqrt[
a] + Sqrt[a + b*x^3]] - 2*Sqrt[a]*c*Log[d + c*Sqrt[a + b*x^3]])/(3*Sqrt[a]*
(a*c^2 - d^2))
```

fricas [A] time = 0.47, size = 232, normalized size = 2.49

$$\left[-\frac{ac\log\left(bc^2x^3+ac^2-d^2\right)+ac\log\left(\sqrt{bx^3+a}\,c+d\right)-ac\log\left(\sqrt{bx^3+a}\,c-d\right)-3\,ac\log(x)-\sqrt{a}\,d\log\left(\frac{bx^3+2\sqrt{b}}{\ldots}\right.}{3\left(a^2c^2-ad^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/3*(a*c*log(b*c^2*x^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) -
a*c*log(sqrt(b*x^3 + a)*c - d) - 3*a*c*log(x) - sqrt(a)*d*log((b*x^3 + 2*sq
rt(b*x^3 + a)*sqrt(a) + 2*a)/x^3))/(a^2*c^2 - a*d^2), -1/3*(a*c*log(b*c^2*x
^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) - a*c*log(sqrt(b*x^3 + a
)*c - d) - 3*a*c*log(x) + 2*sqrt(-a)*d*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a))/
(a^2*c^2 - a*d^2)]
```

giac [A] time = 0.35, size = 94, normalized size = 1.01

$$-\frac{2\,c^2\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{3\left(ac^3-cd^2\right)}+\frac{c\log\left(bx^3\right)}{3\left(ac^2-d^2\right)}-\frac{2\,d\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\left(ac^2-d^2\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*c^2*log(abs(sqrt(b*x^3 + a)*c + d))/(a*c^3 - c*d^2) + 1/3*c*log(b*x^3)
/(a*c^2 - d^2) - 2/3*d*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt
(-a))
```

maple [C] time = 0.06, size = 636, normalized size = 6.84

$$-\frac{a c^3 \ln\left(b c^2 x^3+a c^2-d^2\right)}{3\left(a c^2-d^2\right) d^2}+\frac{c \ln(x)}{a c^2-d^2}+\frac{2 d \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3\left(a c^2-d^2\right) \sqrt{a}}+\frac{2 \sqrt{b x^3+a}\, c^2}{3\left(a c^2-d^2\right) d}-\frac{2 \sqrt{b x^3+a}\, d}{3\left(a c^2-d^2\right) a}+\frac{i c^2 \left(-a b^2\right)^{\frac{1}{3}} \sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] 1/(a*c^2-d^2)*c*ln(x)-1/3*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-d^2)+1/3
*c/d^2*ln(b*c^2*x^3+a*c^2-d^2)-2/3/a/d*(b*x^3+a)^(1/2)-2/3*d/a/(a*c^2-d^2)*
(b*x^3+a)^(1/2)+2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)+
2/3*c^2/(a*c^2-d^2)/d*(b*x^3+a)^(1/2)+1/3*I/b^2*c^2/(a*c^2-d^2)/d*2^(1/2)*s
um((-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/
(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*
(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1
/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(2*_alpha^2*b^2+I*(-a*b^2)^
(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a
*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/2*(2*I*(-a*b^2)^(1/3
)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha-3*
(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(
1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a
*c^2-d^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}\, d\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x), x)
```

mupad [B] time = 4.31, size = 156, normalized size = 1.68

$$\frac{c\,\ln(x)}{a\,c^2-d^2}+\frac{c\,\ln\left(\frac{d-c\,\sqrt{b\,x^3+a}}{d+c\,\sqrt{b\,x^3+a}}\right)}{3\,\left(a\,c^2-d^2\right)}-\frac{c\,\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,a\,c^2-3\,d^2}+\frac{d\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)}{3\,\sqrt{a}\,\left(a\,c^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] (c*log(x))/(a*c^2 - d^2) + (c*log((d - c*(a + b*x^3)^(1/2))/(d + c*(a + b*x
^3)^(1/2))))/(3*(a*c^2 - d^2)) - (c*log(a*c^2 - d^2 + b*c^2*x^3))/(3*a*c^2
- 3*d^2) + (d*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/
2))^3)/x^6))/(3*a^(1/2)*(a*c^2 - d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

3.556 $\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=154

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^3}}{3ax^3\left(ac^2-d^2\right)}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

```
[Out] -1/3*b*d*(3*a*c^2-d^2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)/(a*c^2-d^2)
^2-b*c^3*ln(x)/(a*c^2-d^2)^2+2/3*b*c^3*ln(d+c*(b*x^3+a)^(1/2))/(a*c^2-d^2)^
2+1/3*(-a*c+d*(b*x^3+a)^(1/2))/a/(a*c^2-d^2)/x^3
```

Rubi [A] time = 0.30, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2155, 741, 801, 635, 206, 260}

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^3}}{3ax^3\left(ac^2-d^2\right)}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -(a*c - d*Sqrt[a + b*x^3])/(3*a*(a*c^2 - d^2)*x^3) - (b*d*(3*a*c^2 - d^2)*A
rcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[x
])/(a*c^2 - d^2)^2 + (2*b*c^3*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2)^
2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx,x,x^3\right)$$

$$= \frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{1}{(d+cx)\left(a-x^2\right)^2}\,dx,x,\sqrt{a+bx^3}\right)$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{b\,\text{Subst}\left(\int \frac{-2ac^2+d^2+cdx}{(d+cx)\left(a-x^2\right)}\,dx,x,\sqrt{a+bx^3}\right)}{3a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{b\,\text{Subst}\left(\int \left(-\frac{2ac^4}{\left(ac^2-d^2\right)(d+cx)}+\frac{3ac^2d-d^3-2ac^3x}{\left(ac^2-d^2\right)\left(a-x^2\right)}\right)dx,x,\sqrt{a+\right.}{3a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} + \frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{3ac^2d-d^3-2ac^3x}{a-x^2}\,d\right.}{3a\left(ac^2-d^2\right.}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} + \frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)^2} + \frac{\left(2bc^3\right)\text{Subst}\left(\int \frac{x}{a-x^2}\,dx,x\right.}{3\left(ac^2-d^2\right)^2}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2} - \frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2} + \frac{2bc^3\,\text{l}}{}$$

Mathematica [A] time = 0.67, size = 307, normalized size = 1.99

$$\frac{\sqrt{a}\left(-a^2c^3\sqrt{a+bx^3}+a^2c^2d+2abc^3x^3\sqrt{a+bx^3}\tanh^{-1}\left(\frac{c\sqrt{a+bx^3}}{d}\right)-3abc^3x^3\log(x)\sqrt{a+bx^3}+bdx^3\sqrt{\frac{bx^3}{a}+1}\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (-2*b*d*(2*a*c^2 - d^2)*x^3*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]
] + Sqrt[a]*(a^2*c^2*d - a*d^3 + a*b*c^2*d*x^3 - b*d^3*x^3 - a^2*c^3*Sqrt[a
 + b*x^3] + a*c*d^2*Sqrt[a + b*x^3] + 2*a*b*c^3*x^3*Sqrt[a + b*x^3]*ArcTanh
[(c*Sqrt[a + b*x^3])/d] + b*d*(a*c^2 - d^2)*x^3*Sqrt[1 + (b*x^3)/a]*ArcTanh
[Sqrt[1 + (b*x^3)/a]] - 3*a*b*c^3*x^3*Sqrt[a + b*x^3]*Log[x] + a*b*c^3*x^3*
Sqrt[a + b*x^3]*Log[a*c^2 - d^2 + b*c^2*x^3]))/(3*a^(3/2)*(-(a*c^2) + d^2)^
2*x^3*Sqrt[a + b*x^3])
```

fricas [A] time = 0.55, size = 445, normalized size = 2.89

$$\left[\frac{2\,a^2bc^3x^3\log\left(bc^2x^3+ac^2-d^2\right)+2\,a^2bc^3x^3\log\left(\sqrt{bx^3+a}\,c+d\right)-2\,a^2bc^3x^3\log\left(\sqrt{bx^3+a}\,c-d\right)-6\,a^2bc^3x}{6\left(a^4c^4-2\,a^3c^2d^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b*c^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + 2*a^2*b*c^3*x^3*log(sq
rt(b*x^3 + a)*c + d) - 2*a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 6*a^2*b
*c^3*x^3*log(x) - 2*a^3*c^3 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^3*log((b*x^3
+ 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*a^2*c*d^2 + 2*(a^2*c^2*d - a*d^
3)*sqrt(b*x^3 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^3), 1/3*(a^2*b*c
^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c +
 d) - a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 3*a^2*b*c^3*x^3*log(x) - a
^3*c^3 + (3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + a^2*c*d^2 + (a^2*c^2*d - a*d^3)*sqrt(b*x^3 + a))/((a^4*c^4 - 2*a^3*c^
2*d^2 + a^2*d^4)*x^3)]
```

giac [A] time = 0.34, size = 211, normalized size = 1.37

$$\frac{2\,bc^4\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{3\left(a^2c^5-2\,ac^3d^2+cd^4\right)}-\frac{bc^3\log\left(-bx^3\right)}{3\left(a^2c^4-2\,ac^2d^2+d^4\right)}+\frac{\left(3\,abc^2d-bd^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\left(a^3c^4-2\,a^2c^2d^2+ad^4\right)\sqrt{-a}}-\frac{a^2bc^3-abcd^2-\left(abc^2\right.}{3\left(ac^2-d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*b*c^4*log(abs(sqrt(b*x^3 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4) -
 1/3*b*c^3*log(-b*x^3)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/3*(3*a*b*c^2*d - b
*d^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*s
qrt(-a)) - 1/3*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*sqrt(b*x^3 + a)
)/((a*c^2 - d^2)^2*a*b*x^3)
```

maple [C] time = 0.07, size = 863, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*c/(a*c^2-d^2)/x^3-2/(a*c^2-d^2)^2*b*c^3*ln(x)+1/(a*c^2-d^2)^2/a*b*c*d^
2*ln(x)+1/3*a*c^5*b/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^3+a*c^2-d^2)+1/(a*c^2-d^2)
/a*b*c*ln(x)-1/3*b*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-d^2)+2/3*b/d/a^2*
(b*x^3+a)^(1/2)+1/3*d/a/(a*c^2-d^2)*(b*x^3+a)^(1/2)/x^3+1/3*d/a^(3/2)/(a*c^
2-d^2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))+4/3*d*b/a/(a*c^2-d^2)^2*(b*x^3+a)
^(1/2)*c^2-2/3*b/a^2/(a*c^2-d^2)^2*(b*x^3+a)^(1/2)*d^3-4/3*d*b/a^(1/2)/(a*c
^2-d^2)^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))*c^2+2/3*b/a^(3/2)/(a*c^2-d^2)^2*
arctanh((b*x^3+a)^(1/2)/a^(1/2))*d^3-2/3*b*c^4/(a*c^2-d^2)^2/d*(b*x^3+a)^(1
/2)-1/3*I/b*c^4/(a*c^2-d^2)^2/d*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*(2*x+((-a
*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^
2)^(1/3)/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(
2*x+((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b
*x^3+a)^(1/2)*(2*_alpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1
/3)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2
```

```
)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2),-1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b
-3*a*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(
I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}\,d\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^4), x)
```

mupad [B] time = 5.46, size = 248, normalized size = 1.61

$$\frac{b\,c^3\,\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,a^2\,c^4-6\,a\,c^2\,d^2+3\,d^4}-\frac{b\,c^3\,\ln(x)}{a^2\,c^4-2\,a\,c^2\,d^2+d^4}-\frac{c}{3\,x^3\,\left(a\,c^2-d^2\right)}+\frac{b\,c^3\,\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)}{3\,\left(a\,c^2-d^2\right)^2}+\frac{d\,\sqrt{b\,x^3+a}}{3\,a\,x^3\,\left(a\,c^2-d^2\right)}+\frac{b\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] (b*c^3*log(a*c^2 - d^2 + b*c^2*x^3))/(3*d^4 + 3*a^2*c^4 - 6*a*c^2*d^2) - (b
*c^3*log(x))/(d^4 + a^2*c^4 - 2*a*c^2*d^2) - c/(3*x^3*(a*c^2 - d^2)) + (b*c
^3*log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))))/(3*(a*c^2 - d^
2)^2) + (d*(a + b*x^3)^(1/2))/(3*a*x^3*(a*c^2 - d^2)) + (b*d*log((((a + b*x
^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(3*a*c^2 - d^2))
/(6*a^(3/2)*(a*c^2 - d^2)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

3.557 $\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=311

$$-\frac{dx^4\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\left(ac^2-d^2\right)}+\frac{\sqrt[3]{ac^2-d^2}\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}-$$

```
[Out] x/b/c-1/3*(a*c^2-d^2)^(1/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(4/3)
/c^(5/3)+1/6*(a*c^2-d^2)^(1/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^2-
d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/b^(4/3)/c^(5/3)+1/3*(a*c^2-d^2)^(1/3)*arc
tan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(4/3)/c^(5/3)*
3^(1/2)-1/4*d*x^4*AppellF1(4/3,1/2,1,7/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2))*(
1+b*x^3/a)^(1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2156, 321, 200, 31, 634, 617, 204, 628, 511, 510}

$$-\frac{dx^4\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\left(ac^2-d^2\right)}+\frac{\sqrt[3]{ac^2-d^2}\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] x/(b*c) - (d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a
), -((b*c^2*x^3)/(a*c^2 - d^2))])/(4*(a*c^2 - d^2)*Sqrt[a + b*x^3]) + ((a*c
^2 - d^2)^(1/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt
[3]])/(Sqrt[3]*b^(4/3)*c^(5/3)) - ((a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(1
/3) + b^(1/3)*c^(2/3)*x])/(3*b^(4/3)*c^(5/3)) + ((a*c^2 - d^2)^(1/3)*Log[(a
*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)
*x^2])/(6*b^(4/3)*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = (ac)\int \frac{x^3}{a^2c^2-ad^2+abc^2x^3}\,dx - (ad)\int \frac{x^3}{\sqrt{a+bx^3}\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx$$

$$= \frac{x}{bc} - \frac{\left(a\left(ac^2-d^2\right)\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^3}\,dx}{bc} - \frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{x^3}{\sqrt{1+\frac{bx^3}{a}}\left(a^2c^2-ad^2+abc^2\right.}}{\sqrt{a+bx^3}}$$

$$= \frac{x}{bc} - \frac{dx^4\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}} - \frac{\left(\sqrt[3]{a}\sqrt[3]{ac^2-d^2}\right)\int \frac{}{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}}}{3bc}$$

$$= \frac{x}{bc} - \frac{dx^4\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}} - \frac{\sqrt[3]{ac^2-d^2}\,\log\left(\sqrt[3]{ac^2-d^2}+\right.}{3b^{4/3}c^{5/3}}$$

$$= \frac{x}{bc} - \frac{dx^4\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}} - \frac{\sqrt[3]{ac^2-d^2}\,\log\left(\sqrt[3]{ac^2-d^2}+\right.}{3b^{4/3}c^{5/3}}$$

$$= \frac{x}{bc} - \frac{dx^4\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}} + \frac{\sqrt[3]{ac^2-d^2}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}c^{5/3}}$$

Mathematica [A] time = 0.55, size = 296, normalized size = 0.95

$$\frac{\sqrt[3]{ac^2-d^2}\,\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)-2\sqrt[3]{ac^2-d^2}\,\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right)-}{6b^{4/3}c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -((d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -((b*
c^2*x^3)/(a*c^2 - d^2))])/((4*a*c^2 - 4*d^2)*Sqrt[a + b*x^3])) + (6*b^(1/3)
*c^(2/3)*x - 2*Sqrt[3]*(a*c^2 - d^2)^(1/3)*ArcTan[(-1 + (2*b^(1/3)*c^(2/3)*
x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*(a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^
(1/3) + b^(1/3)*c^(2/3)*x] + (a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(2/3) -
b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(4/3)*c^
(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{bcx^3+ac+\sqrt{bx^3+a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

maple [C] time = 0.08, size = 1544, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] 2/3*I*d/b^2/c^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*
(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*
b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
+1/3*I/b^4/d*2^(1/2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1
/3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)
/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a
*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^
(1/2)*(2*_alpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alp
ha*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)
*b)^(1/2),-1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I
*(-a*b^2)^(2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2
)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^
(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))*a-1/3*I*d/b^4/c^2*2^(1/2)*sum(1
/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/
3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1/3)+I*3
^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/2)*(-a*
b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(2*_alpha^2*b^2+I*(-
a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1
/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/2*(2*I*(-a*b^
2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(2/3)*3^(1/2)*_a
lpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a
*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*
b*c^2+a*c^2-d^2))-1/3*a/c/b^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b
/c^2)^(1/3))+1/6*a/c/b^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^
2)^(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))-1/3*a/c/b^2/((a*c^2-d^2)/b/c^2)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))+x/b/c+1/3/b^
2/c^3*d^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))-1/6/b^2
/c^3*d^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c
^2-d^2)/b/c^2)^(2/3))+1/3/b^2/c^3*d^2/((a*c^2-d^2)/b/c^2)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{bcx^3 + ac + \sqrt{bx^3 + a}\, d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
[Out] int(x^3/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
[Out] Timed out
```

3.558 $\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=304

$$-\frac{dx^2\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,\left(ac^2-d^2\right)}+\frac{\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\,\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-\right.}}{3b^{2/3}\sqrt[3]{c}}$$

```
[Out] -1/3*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(1
/3)+1/6*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*c^
(4/3)*x^2)/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(1/3)-1/3*arctan(1/3*(1-2*b^(1/3)*c^
(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(1/3)*3^(1/
2)-1/2*d*x^2*AppellF1(2/3,1/2,1,5/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2))*(1+b*x
^3/a)^(1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2156, 292, 31, 634, 617, 204, 628, 511, 510}

$$-\frac{dx^2\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,\left(ac^2-d^2\right)}+\frac{\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\,\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-\right.}}{3b^{2/3}\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -(d*x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*c
^2*x^3)/(a*c^2 - d^2))])/(2*(a*c^2 - d^2)*Sqrt[a + b*x^3]) - ArcTan[(1 - (2
*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*c^(1/3)*
(a*c^2 - d^2)^(1/3)) - Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(3*b^(2
/3)*c^(1/3)*(a*c^2 - d^2)^(1/3)) + Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3
)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]/(6*b^(2/3)*c^(1/3)*(a*c^2 -
d^2)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = (ac)\int \frac{x}{a^2c^2-ad^2+abc^2x^3}\,dx-(ad)\int \frac{x}{\sqrt{a+bx^3}\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx$$

$$= -\frac{\left(\sqrt[3]{a}\sqrt[3]{c}\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}\,dx}{3\sqrt[3]{b}\sqrt[3]{ac^2-d^2}}+\frac{\left(\sqrt[3]{a}\sqrt[3]{c}\right)\int \frac{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^2}{a^{2/3}\left(ac^2-d^2\right)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}}{3\sqrt[3]{b}\sqrt[3]{ac^2-d^2}}$$

$$= -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}+\frac{\left(a^{2/3}\right.}{}$$

$$= -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}+\frac{\log\left(\right.}{}$$

$$= -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2\right.}{3b^{2/3}\sqrt[3]{}}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
[Out] Integrate[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{bcx^3+ac+\sqrt{bx^3+a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
[Out] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

maple [C] time = 0.08, size = 619, normalized size = 2.04

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a c^{2}-d^{2}}{b c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a c^{2}-d^{2}}{b c^{2}}\right)^{\frac{1}{3}} bc}-\frac{\ln\left(x+\left(\frac{a c^{2}-d^{2}}{b c^{2}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a c^{2}-d^{2}}{b c^{2}}\right)^{\frac{1}{3}} bc}+\frac{\ln\left(x^{2}-\left(\frac{a c^{2}-d^{2}}{b c^{2}}\right)^{\frac{1}{3}} x+\left(\frac{a c^{2}-d^{2}}{b c^{2}}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a c^{2}-d^{2}}{b c^{2}}\right)^{\frac{1}{3}} bc}-\frac{i\left(-a b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{\cdots}\right)}{\left(-\cdots\right)}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*I/d/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3
)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b
^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1
/2)*(2*_alpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha
*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b
)^(1/2),-1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(
-a*b^2)^(2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2)*
(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1
/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))-1/3/b/c/((a*c^2-d^2)/b/c^2)^(1/3)
*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))+1/6/b/c/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x^2-(
(a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))+1/3/b/c*3^(1/2)/((a*c
^2-d^2)/b/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{bcx^{3}+ac+\sqrt{bx^{3}+a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{a\,c+d\,\sqrt{b\,x^{3}+a}+b\,c\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] int(x/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

3.559 $\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=300

$$-\frac{dx\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2-d^2\right)}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{\ldots}\right.}{3\sqrt[3]{\ldots}}$$

```
[Out] 1/3*c^(1/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(1/3)/(a*c^2-d^2)^(2/
3)-1/6*c^(1/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^2-d^2)^(1/3)*x+b^(
2/3)*c^(4/3)*x^2)/b^(1/3)/(a*c^2-d^2)^(2/3)-1/3*c^(1/3)*arctan(1/3*(1-2*b^(
1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(1/3)/(a*c^2-d^2)^(2/3)*3^(1/2
)-d*x*AppellF1(1/3,1/2,1,4/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2))*(1+b*x^3/a)^(
1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2156, 200, 31, 634, 617, 204, 628, 430, 429}

$$-\frac{dx\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2-d^2\right)}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{\ldots}\right.}{3\sqrt[3]{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
```

```
[Out] -((d*x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3])) - (c^(1/3)*ArcTan[
(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)*
(a*c^2 - d^2)^(2/3)) + (c^(1/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x
])/(3*b^(1/3)*(a*c^2 - d^2)^(2/3)) - (c^(1/3)*Log[(a*c^2 - d^2)^(2/3) - b^(
1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(1/3)*(a*c^
2 - d^2)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^3}\,dx-(ad)\int \frac{1}{\sqrt{a+bx^3}\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx$$

$$= \frac{\left(\sqrt[3]{a}\,c\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}\,c^{2/3}x}\,dx}{3\left(ac^2-d^2\right)^{2/3}}+\frac{\left(\sqrt[3]{a}\,c\right)\int \frac{2\sqrt[3]{a}\sqrt[3]{ac^2-d^2}-\sqrt[3]{a}\sqrt[3]{b}\,c^{2/3}x}{a^{2/3}\left(ac^2-d^2\right)^{2/3}-a^{2/3}\sqrt[3]{b}\,c^{2/3}\sqrt[3]{ac^2-d^2}\,x+a^{2/3}b}}{3\left(ac^2-d^2\right)^{2/3}}$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right)}{3\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}-$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right)}{3\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}-$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{a}\right.}{3\sqrt[3]{b}}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
[Out] Integrate[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bcx^3+ac+\sqrt{bx^3+a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
[Out] integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

maple [C] time = 0.03, size = 619, normalized size = 2.06

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a\,c^2-d^2}{b\,c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a\,c^2-d^2}{b\,c^2}\right)^{\frac{2}{3}}bc}+\frac{\ln\left(x+\left(\frac{a\,c^2-d^2}{b\,c^2}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a\,c^2-d^2}{b\,c^2}\right)^{\frac{2}{3}}bc}-\frac{\ln\left(x^2-\left(\frac{a\,c^2-d^2}{b\,c^2}\right)^{\frac{1}{3}}x+\left(\frac{a\,c^2-d^2}{b\,c^2}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a\,c^2-d^2}{b\,c^2}\right)^{\frac{2}{3}}bc}-\frac{i\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{\cdots}\right)}{\left(-\cdots\right.}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*I/d/b^3*2^(1/2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1
/3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)
/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a
*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^
(1/2)*(2*_alpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alp
ha*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)
*b)^(1/2),-1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I
*(-a*b^2)^(2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2
)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^
(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))+1/3/b/c/((a*c^2-d^2)/b/c^2)^(2/
3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))-1/6/b/c/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2
-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))+1/3/b/c/((a*c^2-d^2
)/b/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bcx^3 + ac + \sqrt{bx^3 + a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{a\,c + d\,\sqrt{b\,x^3 + a} + b\,c\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] int(1/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

3.560 $\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=319

$$\frac{d\sqrt{\frac{bx^3}{a}+1}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\left(ac^2-d^2\right)}-\frac{\sqrt[3]{b}\,c^{5/3}\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\left(ac^2-d^2\right)^{4/3}}+\frac{\sqrt[3]{b}\,c^{5/3}}{}$$

```
[Out] -c/(a*c^2-d^2)/x+1/3*b^(1/3)*c^(5/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x
)/(a*c^2-d^2)^(4/3)-1/6*b^(1/3)*c^(5/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3
)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/(a*c^2-d^2)^(4/3)+1/3*b^(1/3)*c^
(5/3)*arctan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/(a*c^2-
d^2)^(4/3)*3^(1/2)+d*AppellF1(-1/3,1/2,1,2/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2
))*(1+b*x^3/a)^(1/2)/(a*c^2-d^2)/x/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2156, 325, 292, 31, 634, 617, 204, 628, 511, 510}

$$\frac{d\sqrt{\frac{bx^3}{a}+1}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\left(ac^2-d^2\right)}-\frac{\sqrt[3]{b}\,c^{5/3}\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\left(ac^2-d^2\right)^{4/3}}+\frac{\sqrt[3]{b}\,c^{5/3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-1/3, 1/2, 1, 2/3,
 -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*x*Sqrt[a + b*x
^3]) + (b^(1/3)*c^(5/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/
3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2)^(4/3)) + (b^(1/3)*c^(5/3)*Log[(a*c^2 -
 d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(4/3)) - (b^(1/3)*c^(5/3
)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)
*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=(ac)\int \frac{1}{x^2\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx-(ad)\int \frac{1}{x^2\sqrt{a+bx^3}\,\left(a^2c^2-ad^2+abc^2\right.}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}-\frac{\left(abc^3\right)\int \frac{x}{a^2c^2-ad^2+abc^2x^3}\,dx}{ac^2-d^2}-\frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{1}{x^2\sqrt{1+\frac{bx^3}{a}}\,\left(a^2c^2-\right.}}{\sqrt{a+bx^3}}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}}+\frac{\left(\sqrt[3]{a}\,b^{2/3}c^{7/3}\right)\int}{3\left(\right.}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}}+\frac{\sqrt[3]{b}\,c^{5/3}\log\left(\sqrt[3]{a}\right.}{3\left(ac^2\right.}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}}+\frac{\sqrt[3]{b}\,c^{5/3}\log\left(\sqrt[3]{a}\right.}{3\left(ac^2\right.}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}}+\frac{\sqrt[3]{b}\,c^{5/3}\tan^{-1}\left(\right.}{\sqrt{3}\left(ac^2-\right.}$$

Mathematica [A] time = 0.69, size = 496, normalized size = 1.55

$$-6b^2c^2dx^6\sqrt{\frac{bx^3}{a}+1}\,\sqrt[3]{ac^2-d^2}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)+15bdx^3\sqrt{\frac{bx^3}{a}+1}\,\sqrt[3]{ac^2-d^2}\,\left(ac^2+d^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (15*b*d*(a*c^2 - d^2)^(1/3)*(a*c^2 + d^2)*x^3*Sqrt[1 + (b*x^3)/a]*AppellF1[
2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 6*b^2*c^2*d
*(a*c^2 - d^2)^(1/3)*x^6*Sqrt[1 + (b*x^3)/a]*AppellF1[5/3, 1/2, 1, 8/3, -((
b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 10*(a*c^2 - d^2)*(-6*a*d*(a*c^2
- d^2)^(1/3) - 6*b*d*(a*c^2 - d^2)^(1/3)*x^3 + 6*a*c*(a*c^2 - d^2)^(1/3)*Sq
rt[a + b*x^3] + 2*Sqrt[3]*a*b^(1/3)*c^(5/3)*x*Sqrt[a + b*x^3]*ArcTan[(-1 +
(2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*a*b^(1/3)*c^(5/3)*x
*Sqrt[a + b*x^3]*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + a*b^(1/3)*c
^(5/3)*x*Sqrt[a + b*x^3]*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 -
 d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]))/(60*a*(a*c^2 - d^2)^(7/3)*x*Sqrt[a +
 b*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}\,d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

maple [C] time = 0.07, size = 3560, normalized size = 11.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/(a*c^2-d^2)*c/x+1/3*a*c^3/(a*c^2-d^2)/d^2/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x
+((a*c^2-d^2)/b/c^2)^(1/3))-1/6*a*c^3/(a*c^2-d^2)/d^2/((a*c^2-d^2)/b/c^2)^(
1/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))-1/3*a*c^
3/(a*c^2-d^2)/d^2*3^(1/2)/((a*c^2-d^2)/b/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(
(a*c^2-d^2)/b/c^2)^(1/3)*x-1))-1/3*c/d^2/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x+((a
*c^2-d^2)/b/c^2)^(1/3))+1/6*c/d^2/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x^2-((a*c^2-
d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))+1/3*c/d^2*3^(1/2)/((a*c^2-d^
2)/b/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))-2/3*I
/b*c^2/(a*c^2-d^2)/d*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1
/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x
+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3
)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b
-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)
*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(
1/2))-1/a/d*(-a*b^2)^(2/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3
+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(
-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))/b+2/3*I/a/d
*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/
b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)
^(1/2)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-
3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+d/a/(a*c^2-d
^2)*(b*x^3+a)^(1/2)/x-3/2*I*d/a/(a*c^2-d^2)*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/
2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b
)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*
(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1
/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)/b)^(1/2))/b-3/2*d/a/(a*c^2-d^2)*(-a*b^2)^(2/3)*(I*(x+1/2
*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)
^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
```

```
*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(
x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/
3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)/b)^(1/2))/b+I*d/a/(a*c^2-d^2)*3^(1/2)*(-a*b^2)^(2/3)*(I*(
x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/
3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*
b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/
3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)/b*EllipticF(1/3*3^(1/
2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+I/b*c^2/(a*c^2-d^2)/d*3^(1/2)*(-a*b^2)^(
2/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a
*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/
3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2
)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+1/b*c^2/(a*c^2-d^2)/d*(-a*b^2)^(2
/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*
b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*
b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3
*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1
/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+1/3*I/b^2*c^2/(a*c^2-d^2)/d*2^(1/2
)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2
)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1/3
)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/2)
*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(2*_alpha^2*b^2
+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*b^2)^(2/3)
*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/2*(2*I*(
-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(2/3)*3^(1/
2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/
2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(
_Z^3*b*c^2+a*c^2-d^2))-I/a/d*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/2*(-a*b^2)^(1/3
)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a
*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2
)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)
^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I
*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)/b)^(1/2))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}\, d\right)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] int(1/(x^2*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

3.561 $\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=324

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\left(ac^2-d^2\right)}+\frac{b^{2/3}c^{7/3}\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\left(ac^2-d^2\right)^{5/3}}-\frac{b^{2/3}c^{7/3}}{\cdots}$$

```
[Out] -1/2*c/(a*c^2-d^2)/x^2-1/3*b^(2/3)*c^(7/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(
2/3)*x)/(a*c^2-d^2)^(5/3)+1/6*b^(2/3)*c^(7/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*
c^(2/3)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/(a*c^2-d^2)^(5/3)+1/3*b^(2
/3)*c^(7/3)*arctan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/(
a*c^2-d^2)^(5/3)*3^(1/2)+1/2*d*AppellF1(-2/3,1/2,1,1/3,-b*x^3/a,-b*c^2*x^3/
(a*c^2-d^2))*(1+b*x^3/a)^(1/2)/(a*c^2-d^2)/x^2/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2156, 325, 200, 31, 634, 617, 204, 628, 511, 510}

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\left(ac^2-d^2\right)}+\frac{b^{2/3}c^{7/3}\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\left(ac^2-d^2\right)^{5/3}}-\frac{b^{2/3}c^{7/3}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -c/(2*(a*c^2 - d^2)*x^2) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-2/3, 1/2, 1, 1/
3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(2*(a*c^2 - d^2)*x^2*Sqrt[a
 + b*x^3]) + (b^(2/3)*c^(7/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^
2)^(1/3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2)^(5/3)) - (b^(2/3)*c^(7/3)*Log[(a
*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(5/3)) + (b^(2/3)*
c^(7/3)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b
^(2/3)*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = (ac)\int \frac{1}{x^3\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx - (ad)\int \frac{1}{x^3\sqrt{a+bx^3}\left(a^2c^2-ad^2+abc^2\right.}$$

$$= -\frac{c}{2\left(ac^2-d^2\right)x^2} - \frac{\left(abc^3\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^3}\,dx}{ac^2-d^2} - \frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{}{x^3\sqrt{1+\frac{bx^3}{a}}\,(a}}{\sqrt{a+bx^3}}$$

$$= -\frac{c}{2\left(ac^2-d^2\right)x^2} + \frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}} - \frac{\left(\sqrt[3]{a}\,bc^3\right)\int -}{3(}$$

$$= -\frac{c}{2\left(ac^2-d^2\right)x^2} + \frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}} - \frac{b^{2/3}c^{7/3}\log(}{3(}$$

$$= -\frac{c}{2\left(ac^2-d^2\right)x^2} + \frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}} - \frac{b^{2/3}c^{7/3}\log(}{3(}$$

$$= -\frac{c}{2\left(ac^2-d^2\right)x^2} + \frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}} + \frac{b^{2/3}c^{7/3}\tan^{-1}}{\sqrt{3}\left(ac^2\right.}$$

Mathematica [A] time = 5.29, size = 604, normalized size = 1.86

$$\frac{b^2c^2dx^4\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{16a\sqrt{a+bx^3}\left(d^2-ac^2\right)^2} + \frac{2bdx\left(d^2-\right.}{\sqrt{a+bx^3}\left(ac^2+bc^2x^3-d^2\right)\left(3bx^3\left(2ac^2F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (b^2*c^2*d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a),
 -((b*c^2*x^3)/(a*c^2 - d^2))])/(16*a*(-(a*c^2) + d^2)^2*Sqrt[a + b*x^3]) +
 (2*b*d*(-5*a*c^2 + d^2)*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))])/(Sqrt[a + b*x^3]*(a*c^2 - d^2 + b*c^2*x^3)*(8*a*(-(
a*c^2) + d^2)*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2
 - d^2))] + 3*b*x^3*(2*a*c^2*AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*
c^2*x^3)/(a*c^2 - d^2))] + (a*c^2 - d^2)*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^
3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))]))) + (-3*a*c*(a*c^2 - d^2)^(2/3) + 3*d
*(a*c^2 - d^2)^(2/3)*Sqrt[a + b*x^3] - 2*Sqrt[3]*a*b^(2/3)*c^(7/3)*x^2*ArcT
an[(-1 + (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*a*b^(2/3)*
c^(7/3)*x^2*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + a*b^(2/3)*c^(7/3
)*x^2*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(
2/3)*c^(4/3)*x^2]])/(6*a*(a*c^2 - d^2)^(5/3)*x^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}\,d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

maple [C] time = 0.06, size = 1789, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
[Out] 1/3*c/d^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))-1/6*c/d
^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2
)/b/c^2)^(2/3))+1/3*c/d^2/((a*c^2-d^2)/b/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))-1/2/(a*c^2-d^2)*c/x^2-1/3*a*c^3/(a*c
^2-d^2)/d^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))+1/6*a
*c^3/(a*c^2-d^2)/d^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(
1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))-1/3*a*c^3/(a*c^2-d^2)/d^2/((a*c^2-d^2)/b/
c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))+2/
3*I/a/d*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*
(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/
3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+1/2*d/a
/(a*c^2-d^2)/x^2*(b*x^3+a)^(1/2)+1/2*I*d/a/(a*c^2-d^2)*3^(1/2)*(-a*b^2)^(1/
3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/
(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))-2/3*I/(a*c^2-d^2)*c^2/d*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1
/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(
1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+1/3*I/(a*c^2-d^2)/b^2*c^2/
d*2^(1/2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/
2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a
*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)
+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(2*_a
lpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*
b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2
)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-
1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(
```

```
2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^
(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)),_alp
ha=RootOf(_Z^3*b*c^2+a*c^2-d^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}\,d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] int(1/(x^3*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

3.562 $\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=135

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{dx\sqrt{\frac{bx^n}{a}+1}\,F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^n}}$$

```
[Out] c*x*hypergeom([1, 1/n],[1+1/n],-b*c^2*x^n/(a*c^2-d^2))/(a*c^2-d^2)-d*x*Appe
llF1(1/n,1/2,1,1+1/n,-b*x^n/a,-b*c^2*x^n/(a*c^2-d^2))*(1+b*x^n/a)^(1/2)/(a*
c^2-d^2)/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2156, 245, 430, 429}

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{dx\sqrt{\frac{bx^n}{a}+1}\,F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```

```
[Out] -((d*x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a
), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^n])) + (c*x*H
ypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c^2*x^n)/(a*c^2 - d^2))])/(a*c
^2 - d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^n}\,dx-(ad)\int \frac{1}{\sqrt{a+bx^n}\left(a^2c^2-ad^2+abc^2x^n\right)}\,dx$$

$$= \frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{\left(ad\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{1}{\sqrt{1+\frac{bx^n}{a}}\left(a^2c^2-ad^2+abc^2x^n\right)}\,dx}{\sqrt{a+bx^n}}$$

$$= -\frac{dx\sqrt{1+\frac{bx^n}{a}}\,F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)\sqrt{a+bx^n}}+\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}$$

Mathematica [B] time = 0.60, size = 320, normalized size = 2.37

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{2ad(n+1)x\left(ac^2-d^2\right)\ldots}{\sqrt{a+bx^n}\left(ac^2+bc^2x^n-d^2\right)\left(\left(ac^2-d^2\right)\left(2a(n+1)F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)\ldots\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```

```
[Out] (-2*a*d*(a*c^2 - d^2)*(1 + n)*x*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x
^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/(Sqrt[a + b*x^n]*(a*c^2 - d^2 + b*c^
2*x^n)*(-2*a*b*c^2*n*x^n*AppellF1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)
/a), -((b*c^2*x^n)/(a*c^2 - d^2))] + (a*c^2 - d^2)*(-(b*n*x^n*AppellF1[1 +
n^(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))]) +
2*a*(1 + n)*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((b*c^2*x^n
)/(a*c^2 - d^2))]))) + (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c
^2*x^n)/(a*c^2 - d^2))])/(a*c^2 - d^2)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bcx^n+ac-\sqrt{bx^n+a}\,d}{b^2c^2x^{2n}+a^2c^2-ad^2+\left(2\,abc^2-bd^2\right)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((b*c*x^n + a*c - sqrt(b*x^n + a)*d)/(b^2*c^2*x^(2*n) + a^2*c^2 - a
*d^2 + (2*a*b*c^2 - b*d^2)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bcx^n+ac+\sqrt{bx^n+a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{bc\,x^n+ac+\sqrt{b\,x^n+a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

```
[Out] int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bcx^n + ac + \sqrt{bx^n + a}\, d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a\,c + d\,\sqrt{a + b\,x^n} + b\,c\,x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
```

```
[Out] int(1/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ac + bcx^n + d\sqrt{a + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] Integral(1/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

3.563 $\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=167

$$\frac{cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)}-\frac{dx^{m+1}\sqrt{\frac{bx^n}{a}+1}\,F_1\left(\frac{m+1}{n};\frac{1}{2},1;\frac{m+n+1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)\sqrt{a+bx^n}}$$

```
[Out] c*x^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*c^2*x^n/(a*c^2-d^2))/(a*c^2
-d^2)/(1+m)-d*x^(1+m)*AppellF1((1+m)/n,1/2,1,(1+m+n)/n,-b*x^n/a,-b*c^2*x^n/
(a*c^2-d^2))*(1+b*x^n/a)^(1/2)/(a*c^2-d^2)/(1+m)/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2156, 364, 511, 510}

$$\frac{cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)}-\frac{dx^{m+1}\sqrt{\frac{bx^n}{a}+1}\,F_1\left(\frac{m+1}{n};\frac{1}{2},1;\frac{m+n+1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] -((d*x^(1 + m)*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)/
n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m)*Sqrt
[a + b*x^n])) + (c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
 -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx=(ac)\int \frac{x^m}{a^2c^2-ad^2+abc^2x^n}\,dx-(ad)\int \frac{x^m}{\sqrt{a+bx^n}\left(a^2c^2-ad^2+abc^2x^n\right)}\,dx$$

$$=\frac{cx^{1+m}\,_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)(1+m)}-\frac{\left(ad\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{x^m}{\sqrt{1+\frac{bx^n}{a}}\left(a^2c^2-ad^2+abc^2x^n\right)}\,d}{\sqrt{a+bx^n}}$$

$$=-\frac{dx^{1+m}\sqrt{1+\frac{bx^n}{a}}\,F_1\left(\frac{1+m}{n};\frac{1}{2},1;\frac{1+m+n}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)(1+m)\sqrt{a+bx^n}}+\frac{cx^{1+m}\,_2F_1\left(1,\frac{1+m}{n};\frac{1+r}{r}\right.}{\left(ac^2-d^2\right)(1}$$

Mathematica [A] time = 0.29, size = 156, normalized size = 0.93

$$\frac{x^{m+1}\left(c\sqrt{a+bx^n}\,_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)-d\sqrt{\frac{bx^n}{a}+1}\,F_1\left(\frac{m+1}{n};\frac{1}{2},1;\frac{m+n+1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)\right)}{(m+1)\left(ac^2-d^2\right)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (x^(1 + m)*(-(d*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)
/n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))]) + c*Sqrt[a + b*x^n]*Hyperg
eometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*c^2*x^n)/(a*c^2 - d^2))]))/((
a*c^2 - d^2)*(1 + m)*Sqrt[a + b*x^n])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bcx^mx^n+acx^m-\sqrt{bx^n+a}\,dx^m}{b^2c^2x^{2\,n}+a^2c^2-ad^2+\left(2\,abc^2-bd^2\right)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((b*c*x^m*x^n + a*c*x^m - sqrt(b*x^n + a)*d*x^m)/(b^2*c^2*x^(2*n) +
 a^2*c^2 - a*d^2 + (2*a*b*c^2 - b*d^2)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bcx^n+ac+\sqrt{bx^n+a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bc\,x^n+ac+\sqrt{b\,x^n+a}\;d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*c*x^n+a*c+(b*x^n+a)^(1/2)*d),x)
```

```
[Out] int(x^m/(b*c*x^n+a*c+(b*x^n+a)^(1/2)*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bcx^n + ac + \sqrt{bx^n + a}\, d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{a\,c + d\,\sqrt{a + b\,x^n} + b\,c\,x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
[Out] int(x^m/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{ac + bcx^n + d\sqrt{a + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
[Out] Integral(x**m/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

3.564 $\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=27

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

```
[Out] 2*ln(d+c*(a+b*x^n)^(1/2))/b/c/n
```

Rubi [A] time = 0.11, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2155, 31}

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^n\right)}{n}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^n}\right)}{bn}\\ &= \frac{2\log\left(d+c\sqrt{a+bx^n}\right)}{bcn}\end{aligned}$$

Mathematica [A] time = 0.06, size = 27, normalized size = 1.00

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

fricas [A] time = 0.45, size = 25, normalized size = 0.93

$$\frac{2\,\log\left(\sqrt{bx^n+a}\,c+d\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*log(sqrt(b*x^n + a)*c + d)/(b*c*n)
```

giac [A] time = 0.37, size = 26, normalized size = 0.96

$$\frac{2 \log\left(\left|\sqrt{bx^n + a}\,c + d\right|\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*log(abs(sqrt(b*x^n + a)*c + d))/(b*c*n)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{n-1}}{bc\,x^n + ac + \sqrt{b\,x^n + a}\;d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)/(b*c*x^n+a*c+(b*x^n+a)^(1/2)*d),x)
```

```
[Out] int(x^(n-1)/(b*c*x^n+a*c+(b*x^n+a)^(1/2)*d),x)
```

maxima [B] time = 0.65, size = 61, normalized size = 2.26

$$-\frac{\log\left(\frac{bx^n+a}{b}\right)}{bcn} + \frac{2 \log\left(\frac{bcx^n+ac+\sqrt{bx^n+a}\,d}{d}\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log((b*x^n + a)/b)/(b*c*n) + 2*log((b*c*x^n + a*c + sqrt(b*x^n + a)*d)/d)/
(b*c*n)
```

mupad [B] time = 3.35, size = 25, normalized size = 0.93

$$\frac{2\,\ln\left(d+c\,\sqrt{a+b\,x^n}\right)}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
```

```
[Out] (2*log(d + c*(a + b*x^n)^(1/2)))/(b*c*n)
```

sympy [A] time = 33.63, size = 32, normalized size = 1.19

$$\frac{2\left(\begin{cases} \frac{\sqrt{a+bx^n}}{d} & \text{for}\ c = 0 \\ \frac{\log\left(c\sqrt{a+bx^n}+d\right)}{c} & \text{otherwise} \end{cases}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] 2*Piecewise((sqrt(a + b*x**n)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**n) + d)/c,
 True))/(b*n)
```

3.565 $\int \frac{1}{\sqrt{x}+4x^{3/2}} \, dx$

Optimal. Leaf size=8

$$\tan^{-1}\left(2\sqrt{x}\right)$$

```
[Out] arctan(2*x^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1593, 63, 203}

$$\tan^{-1}\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

```
[Out] ArcTan[2*Sqrt[x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}+4x^{3/2}} \, dx &= \int \frac{1}{\sqrt{x}(1+4x)} \, dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1+4x^2} \, dx, x, \sqrt{x}\right)\\ &= \tan^{-1}\left(2\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\tan^{-1}\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

```
[Out] ArcTan[2*Sqrt[x]]
```

fricas [A] time = 0.41, size = 6, normalized size = 0.75

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="fricas")
[Out] arctan(2*sqrt(x))
```

giac [A] time = 0.32, size = 6, normalized size = 0.75

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="giac")
[Out] arctan(2*sqrt(x))
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^(3/2)+x^(1/2)),x)
[Out] arctan(2*x^(1/2))
```

maxima [A] time = 1.28, size = 6, normalized size = 0.75

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="maxima")
[Out] arctan(2*sqrt(x))
```

mupad [B] time = 0.05, size = 6, normalized size = 0.75

$$\mathrm{atan}\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + 4*x^(3/2)),x)
[Out] atan(2*x^(1/2))
```

sympy [A] time = 0.22, size = 7, normalized size = 0.88

$$\operatorname{atan}\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**(3/2)+x**(1/2)),x)
[Out] atan(2*sqrt(x))
```

3.566 $\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 329, 212, 206, 203}

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] - x^(5/2))^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx &= \int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] - x^(5/2))^(-1),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

fricas [B] time = 0.44, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\,\log\left(\sqrt{x}+1\right)-\frac{1}{2}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="fricas")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

giac [B] time = 0.35, size = 22, normalized size = 1.69

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\,\log\left(\sqrt{x}+1\right)-\frac{1}{2}\,\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="giac")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

maple [A] time = 0.01, size = 10, normalized size = 0.77

$$\operatorname{arctanh}\left(\sqrt{x}\right)+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(5/2)+x^(1/2)),x)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

maxima [B] time = 1.41, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\,\log\left(\sqrt{x}+1\right)-\frac{1}{2}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

mupad [B] time = 3.08, size = 9, normalized size = 0.69

$$\operatorname{atan}\left(\sqrt{x}\right)+\operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) - x^(5/2)),x)
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

sympy [B] time = 0.40, size = 26, normalized size = 2.00

$$-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(5/2)+x**(1/2)),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

3.567 $\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=27

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

```
[Out] 4*x^(1/4)+4*ln(1-x^(1/4))+2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1593, 266, 43}

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx &= \int \frac{1}{\left(-1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx\\
&= 4\,\text{Subst}\left(\int \frac{x^2}{-1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\,\text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x\right)dx, x, \sqrt[4]{x}\right)\\
&= 4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1-\sqrt[4]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(1/4) + Sqrt[x])^(-1),x]
```

[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]

fricas [A] time = 0.45, size = 19, normalized size = 0.70

$$2\sqrt{x} + 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="fricas")

[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)

giac [A] time = 0.34, size = 20, normalized size = 0.74

$$2\sqrt{x} + 4x^{\frac{1}{4}} + 4\log\left(\left|x^{\frac{1}{4}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="giac")

[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(abs(x^(1/4) - 1))

maple [A] time = 0.01, size = 20, normalized size = 0.74

$$4\ln\left(x^{\frac{1}{4}} - 1\right) + 2\sqrt{x} + 4x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^(1/4)+x^(1/2)),x)

[Out] 2*x^(1/2)+4*x^(1/4)+4*ln(x^(1/4)-1)

maxima [A] time = 0.65, size = 19, normalized size = 0.70

$$2\sqrt{x} + 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="maxima")

[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)

mupad [B] time = 3.10, size = 19, normalized size = 0.70

$$4\ln\left(x^{1/4} - 1\right) + 2\sqrt{x} + 4x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2) - x^(1/4)),x)

[Out] 4*log(x^(1/4) - 1) + 2*x^(1/2) + 4*x^(1/4)

sympy [A] time = 0.24, size = 22, normalized size = 0.81

$$4\sqrt[4]{x} + 2\sqrt{x} + 4\log\left(\sqrt[4]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**(1/4)+x**(1/2)),x)

[Out] 4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) - 1)

3.568 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

```
[Out] 6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx &= \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx\\ &= 6\,\text{Subst}\left(\int \frac{x^3}{1+x}\,dx,x,\sqrt[6]{x}\right)\\ &= 6\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)dx,x,\sqrt[6]{x}\right)\\ &= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

fricas [A] time = 0.43, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

giac [A] time = 0.33, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

maple [B] time = 0.03, size = 92, normalized size = 2.88

$$-\ln(x-1)+\ln\left(\sqrt{x}-1\right)-\ln\left(\sqrt{x}+1\right)-2\ln\left(x^{\frac{1}{6}}+1\right)-2\ln\left(x^{\frac{1}{3}}-1\right)+2\ln\left(x^{\frac{1}{6}}-1\right)+\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-\ln\left(x^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+x^(1/2)),x)
[Out] 2*ln(x^(1/6)-1)-ln(x^(1/3)+x^(1/6)+1)+ln(1-x^(1/6)+x^(1/3))-2*ln(1+x^(1/6))
+2*x^(1/2)+ln(-1+x^(1/2))-ln(1+x^(1/2))+6*x^(1/6)-ln(x-1)-2*ln(x^(1/3)-1)+l
n(x^(2/3)+x^(1/3)+1)-3*x^(1/3)
```

maxima [A] time = 0.70, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.75

$$2\sqrt{x} - 6\ln\left(x^{1/6} + 1\right) - 3x^{1/3} + 6x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(1/3)),x)
[Out] 2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

3.569 $\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=25

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

```
[Out] -4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx &= \int \frac{1}{\left(1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx\\ &= 4\,\text{Subst}\left(\int \frac{x^2}{1+x}\,dx,x,\sqrt[4]{x}\right)\\ &= 4\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)dx,x,\sqrt[4]{x}\right)\\ &= -4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1+\sqrt[4]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

fricas [A] time = 0.42, size = 19, normalized size = 0.76

$$2\sqrt{x} - 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

giac [A] time = 0.34, size = 19, normalized size = 0.76

$$2\sqrt{x} - 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$4\ln\left(x^{\frac{1}{4}} + 1\right) + 2\sqrt{x} - 4x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/4)+x^(1/2)),x)
[Out] -4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)
```

maxima [A] time = 0.69, size = 19, normalized size = 0.76

$$2\sqrt{x} - 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$4\ln\left(x^{1/4} + 1\right) + 2\sqrt{x} - 4x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(1/4)),x)
[Out] 4*log(x^(1/4) + 1) + 2*x^(1/2) - 4*x^(1/4)
```

sympy [A] time = 0.24, size = 22, normalized size = 0.88

$$-4\sqrt[4]{x} + 2\sqrt{x} + 4\log\left(\sqrt[4]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/4)+x**(1/2)),x)
[Out] -4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) + 1)
```

3.570 $\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx$

Optimal. Leaf size=20

$$3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

```
[Out] 3*x^(1/3)+3*ln(1-x^(1/3))
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1593, 266, 43}

$$3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(1/3) + x^(2/3))^(-1),x]
```

```
[Out] 3*x^(1/3) + 3*Log[1 - x^(1/3)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx &= \int \frac{1}{\left(-1+\sqrt[3]{x}\right)\sqrt[3]{x}}\,dx\\
&= 3\,\text{Subst}\left(\int \frac{x}{-1+x}\,dx,x,\sqrt[3]{x}\right)\\
&= 3\,\text{Subst}\left(\int \left(1+\frac{1}{-1+x}\right)dx,x,\sqrt[3]{x}\right)\\
&= 3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.90

$$3\left(\sqrt[3]{x}+\log\left(1-\sqrt[3]{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(1/3) + x^(2/3))^(-1),x]
```

[Out] 3*(x^(1/3) + Log[1 - x^(1/3)])

fricas [A] time = 0.42, size = 14, normalized size = 0.70

$$3\,x^{\frac{1}{3}} + 3\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="fricas")

[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)

giac [A] time = 0.43, size = 15, normalized size = 0.75

$$3\,x^{\frac{1}{3}} + 3\,\log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="giac")

[Out] 3*x^(1/3) + 3*log(abs(x^(1/3) - 1))

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$3\ln\left(x^{\frac{1}{3}} - 1\right) + 3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^(1/3)+x^(2/3)),x)

[Out] 3*x^(1/3)+3*ln(x^(1/3)-1)

maxima [A] time = 0.70, size = 14, normalized size = 0.70

$$3\,x^{\frac{1}{3}} + 3\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="maxima")

[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)

mupad [B] time = 0.08, size = 14, normalized size = 0.70

$$3\,\ln\left(x^{1/3} - 1\right) + 3\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(x^(1/3) - x^(2/3)),x)

[Out] 3*log(x^(1/3) - 1) + 3*x^(1/3)

sympy [A] time = 0.16, size = 15, normalized size = 0.75

$$3\sqrt[3]{x} + 3\log\left(\sqrt[3]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**(1/3)+x**(2/3)),x)

[Out] 3*x**(1/3) + 3*log(x**(1/3) - 1)

3.571 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))+4/3*arctan(1/3*(1-2*x^(1/4))*3^
(1/2))*3^(1/2)+2*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1593, 341, 321, 292, 31, 634, 618, 204, 628}

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1 + x^(1/4
)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx &= \int \frac{\sqrt[4]{x}}{1+x^{3/4}}\,dx\\
&= 4\,\mathrm{Subst}\left(\int \frac{x^4}{1+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x} - 4\,\mathrm{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x} + \frac{4}{3}\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[4]{x}\right) - \frac{4}{3}\mathrm{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\mathrm{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right) - 2\,\mathrm{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x,\right.\\
&= 2\sqrt{x} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right) + 4\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[4]{x}\right)\\
&= 2\sqrt{x} + \frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.39

$$-2\sqrt{x}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-x^{3/4}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -x^(3/4)])
```

fricas [A] time = 0.44, size = 47, normalized size = 0.76

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/4) - 1/3*sqrt(3)) + 2*sqrt(x) - 2/3*lo
g(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

giac [A] time = 0.33, size = 45, normalized size = 0.73

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

maple [A] time = 0.01, size = 46, normalized size = 0.74

$$-\frac{4\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}+\frac{4\ln\left(x^{\frac{1}{4}}+1\right)}{3}-\frac{2\ln\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)}{3}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/4)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)*arctan(1/3*(2*x^(1/4)-1)*3^
(1/2))+4/3*ln(x^(1/4)+1)
```

maxima [A] time = 1.53, size = 45, normalized size = 0.73

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

mupad [B] time = 3.08, size = 73, normalized size = 1.18

$$\frac{4\ln\left(16x^{1/4}+16\right)}{3}+\ln\left(9\left(-\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)^2+16x^{1/4}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)-\ln\left(9\left(\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)^2+16x^{1/4}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,2i}{3}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + 1/x^(1/4)),x)
```

```
[Out] (4*log(16*x^(1/4) + 16))/3 + log(9*((3^(1/2)*2i)/3 - 2/3)^2 + 16*x^(1/4))*(
(3^(1/2)*2i)/3 - 2/3) - log(9*((3^(1/2)*2i)/3 + 2/3)^2 + 16*x^(1/4))*((3^(1
/2)*2i)/3 + 2/3) + 2*x^(1/2)
```

sympy [A] time = 0.64, size = 68, normalized size = 1.10

$$2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-4\sqrt[4]{x}+4\sqrt{x}+4\right)}{3}-\frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[4]{x}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/4)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-4*x**(1/4) + 4*sqrt(x) + 4)/3 -
4*sqrt(3)*atan(2*sqrt(3)*x**(1/4)/3 - sqrt(3)/3)/3
```

3.572 $\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=73

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)+12*ln(1+x^(1/12))+2*x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1/12)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx &= \int \frac{1}{\left(1+\sqrt[12]{x}\right)\sqrt[4]{x}}\,dx\\
&= 12\,\text{Subst}\left(\int \frac{x^8}{1+x}\,dx, x, \sqrt[12]{x}\right)\\
&= 12\,\text{Subst}\left(\int \left(-1+x-x^2+x^3-x^4+x^5-x^6+x^7+\frac{1}{1+x}\right)dx, x, \sqrt[12]{x}\right)\\
&= -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}+12\log\left(1+\sqrt[12]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.00

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1/12)]
```

fricas [A] time = 0.43, size = 49, normalized size = 0.67

$$\frac{3}{2}x^{\frac{2}{3}} - \frac{12}{7}x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}x^{\frac{5}{12}} + 3x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 6x^{\frac{1}{6}} - 12x^{\frac{1}{12}} + 12\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

giac [A] time = 0.32, size = 49, normalized size = 0.67

$$\frac{3}{2}x^{\frac{2}{3}} - \frac{12}{7}x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}x^{\frac{5}{12}} + 3x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 6x^{\frac{1}{6}} - 12x^{\frac{1}{12}} + 12\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

maple [B] time = 0.12, size = 173, normalized size = 2.37

$$\ln(x-1) + \ln\left(\sqrt{x}-1\right) - \ln\left(\sqrt{x}+1\right) + 2\ln\left(x^{\frac{1}{4}}+1\right) - 2\ln\left(x^{\frac{1}{6}}+1\right) + 4\ln\left(x^{\frac{1}{12}}+1\right) + 2\ln\left(x^{\frac{1}{3}}-1\right) - 2\ln\left(x^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/4)+x^(1/3)),x)
```

```
[Out] ln(x-1)-2*ln(x^(1/6)+1)+ln(x^(1/3)-x^(1/6)+1)+2*ln(x^(1/6)-1)-ln(x^(1/3)+x^
(1/6)+1)+ln(x^(1/2)-1)-ln(x^(1/2)+1)-12/7*x^(7/12)-12/5*x^(5/12)-12*x^(1/12
)-2*ln(x^(1/4)-1)+2*ln(x^(1/4)+1)+6*x^(1/6)+3/2*x^(2/3)+3*x^(1/3)+2*x^(1/2)
-4*x^(1/4)-2*ln(1-x^(1/12)+x^(1/6))-ln(x^(2/3)+x^(1/3)+1)+4*ln(1+x^(1/12))+
2*ln(x^(1/3)-1)-4*ln(x^(1/12)-1)+2*ln(x^(1/6)+x^(1/12)+1)
```

maxima [A] time = 0.70, size = 49, normalized size = 0.67

$$\frac{3}{2}x^{\frac{2}{3}} - \frac{12}{7}x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}x^{\frac{5}{12}} + 3x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 6x^{\frac{1}{6}} - 12x^{\frac{1}{12}} + 12\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

mupad [B] time = 0.04, size = 49, normalized size = 0.67

$$12\ln\left(x^{1/12}+1\right) + 2\sqrt{x} + 3x^{1/3} - 4x^{1/4} + \frac{3x^{2/3}}{2} + 6x^{1/6} - 12x^{1/12} - \frac{12x^{5/12}}{5} - \frac{12x^{7/12}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3) + x^(1/4)),x)
```

```
[Out] 12*log(x^(1/12) + 1) + 2*x^(1/2) + 3*x^(1/3) - 4*x^(1/4) + (3*x^(2/3))/2 +
6*x^(1/6) - 12*x^(1/12) - (12*x^(5/12))/5 - (12*x^(7/12))/7
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x} + \sqrt[3]{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/4)+x**(1/3)),x)
```

```
[Out] Integral(1/(x**(1/4) + x**(1/3)), x)
```

3.573 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} \, dx$

Optimal. Leaf size=130

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-$$

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} \, dx &= \int \frac{\sqrt[3]{x}}{1+\sqrt[12]{x}} \, dx\\
&= 12 \operatorname{Subst}\left(\int \frac{x^{15}}{1+x} \, dx, x, \sqrt[12]{x}\right)\\
&= 12 \operatorname{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2-x^3+x^4-x^5+x^6-x^7+x^8-x^9+x^{10}-x^{11}+x^{12}-x^1\right.\right.\\
&= 12\sqrt[12]{x}-6\sqrt[6]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}+\frac{12x^{5/12}}{5}-2\sqrt{x}+\frac{12x^{7/12}}{7}-\frac{3x^{2/3}}{2}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{11/1}}{11}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 130, normalized size = 1.00

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

fricas [A] time = 0.44, size = 76, normalized size = 0.58

$$\frac{4}{5}(x+5)x^{\frac{1}{4}}-\frac{6}{7}(x+7)x^{\frac{1}{6}}+\frac{12}{13}(x+13)x^{\frac{1}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3x^{\frac{1}{3}}-12\log\left(x^{\frac{1}{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="fricas")
```

```
[Out] 4/5*(x + 5)*x^(1/4) - 6/7*(x + 7)*x^(1/6) + 12/13*(x + 13)*x^(1/12) - x + 1
2/11*x^(11/12) - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) -
2*sqrt(x) + 12/5*x^(5/12) - 3*x^(1/3) - 12*log(x^(1/12) + 1)
```

giac [A] time = 0.36, size = 82, normalized size = 0.63

$$\frac{4}{5}x^{\frac{5}{4}}-\frac{6}{7}x^{\frac{7}{6}}+\frac{12}{13}x^{\frac{13}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-6x^{\frac{1}{6}}+12x^{\frac{1}{12}}-12\log\left(x^{\frac{1}{12}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="giac")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

maple [A] time = 0.00, size = 83, normalized size = 0.64

$$\frac{4x^{\frac{5}{4}}}{5}-\frac{6x^{\frac{7}{6}}}{7}+\frac{12x^{\frac{13}{12}}}{13}-x-12\ln\left(x^{\frac{1}{12}}+1\right)+\frac{12x^{\frac{11}{12}}}{11}-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{3}{4}}}{3}-\frac{3x^{\frac{2}{3}}}{2}+\frac{12x^{\frac{7}{12}}}{7}-2\sqrt{x}+\frac{12x^{\frac{5}{12}}}{5}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-6x^{\frac{1}{6}}+12x^{\frac{1}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+1/x^(1/4)),x)
```

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(x^(1/12)+1)-2*x^(1/2)
```

maxima [A] time = 0.73, size = 82, normalized size = 0.63

$$\frac{4}{5}x^{\frac{5}{4}}-\frac{6}{7}x^{\frac{7}{6}}+\frac{12}{13}x^{\frac{13}{12}}-x+\frac{12}{11}x^{\frac{11}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}+\frac{12}{5}x^{\frac{5}{12}}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-6x^{\frac{1}{6}}+12x^{\frac{1}{12}}-12\log\left(x^{\frac{1}{12}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="maxima")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

mupad [B] time = 0.15, size = 82, normalized size = 0.63

$$4\,x^{1/4}-12\,\ln\left(x^{1/12}+1\right)-2\,\sqrt{x}-3\,x^{1/3}-x-\frac{3\,x^{2/3}}{2}-6\,x^{1/6}+\frac{4\,x^{3/4}}{3}+\frac{4\,x^{5/4}}{5}-\frac{6\,x^{5/6}}{5}+12\,x^{1/12}-\frac{6\,x^{7/6}}{7}+\frac{12\,x^{5/12}}{5}+\frac{12\,x^{7/1}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3) + 1/x^(1/4)),x)
```

```
[Out] 4*x^(1/4) - 12*log(x^(1/12) + 1) - 2*x^(1/2) - 3*x^(1/3) - x - (3*x^(2/3))/
2 - 6*x^(1/6) + (4*x^(3/4))/3 + (4*x^(5/4))/5 - (6*x^(5/6))/5 + 12*x^(1/12)
 - (6*x^(7/6))/7 + (12*x^(5/12))/5 + (12*x^(7/12))/7 + (12*x^(11/12))/11 +
(12*x^(13/12))/13
```

sympy [A] time = 3.08, size = 121, normalized size = 0.93

$$\frac{12x^{\frac{13}{12}}}{13}+\frac{12x^{\frac{11}{12}}}{11}+\frac{12x^{\frac{7}{12}}}{7}+\frac{12x^{\frac{5}{12}}}{5}+12\sqrt[12]{x}-\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-6\sqrt[6]{x}+\frac{4x^{\frac{5}{4}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{3x^{\frac{2}{3}}}{2}-3\sqrt[3]{x}-2\sqrt{x}-x-12\log\left(\sqrt[12]{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

```
[Out] 12*x**(13/12)/13 + 12*x**(11/12)/11 + 12*x**(7/12)/7 + 12*x**(5/12)/5 + 12*
x**(1/12) - 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 4*x**(5/4)/5 + 4*x**
(3/4)/3 + 4*x**(1/4) - 3*x**(2/3)/2 - 3*x**(1/3) - 2*sqrt(x) - x - 12*log(x
**(1/12) + 1)
```

3.574 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=200

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$

```
[Out] 6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(-5^(1/2)+1)
-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*(5^(1/2)+1)+2*x^(1/2)+3/5*arc
tan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-3/5*ar
ctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1593, 341, 321, 294, 634, 618, 204, 628, 31}

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[
2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/
10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqr
t[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])
*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[
((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r^(m + 1)*Int[1/(r - s*x), x])/(a*n*s^
m) - Dist[(2*(-r)^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /;
 FreeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && N
egQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx=\int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx$$

$$=6\,\text{Subst}\left(\int \frac{x^7}{-1+x^5}\,dx,x,\sqrt[6]{x}\right)$$

$$=2\sqrt{x}+6\,\text{Subst}\left(\int \frac{x^2}{-1+x^5}\,dx,x,\sqrt[6]{x}\right)$$

$$=2\sqrt{x}-\frac{6}{5}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[6]{x}\right)-\frac{12}{5}\text{Subst}\left(\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx,x,\sqrt[6]{}\right.$$

$$=2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)+\frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx,x,\sqrt[6]{x}\right)}{\sqrt{5}}-\frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x}\right.}{\sqrt{5}}$$

$$=2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log$$

$$=2\sqrt{x}+6\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}}\left(5+\sqrt{5}\right.\right.$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.11

$$-2\sqrt{x}\left({}_2F_1\left(\frac{3}{5},1;\frac{8}{5};x^{5/6}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[3/5, 1, 8/5, x^(5/6)])
```

fricas [B] time = 1.52, size = 638, normalized size = 3.19

$$\frac{1}{10}\left(3\sqrt{5}-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 +
9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) -
sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2
)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) -
 5) + sqrt(5) + 1)^2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 3*
sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(
sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/
4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5
) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) + 1/10*(3*sqrt(5) + s
qrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(s
qrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4
*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5)
 + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 - 3*sqrt(-27/4*(sqrt(2)
*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqr
t(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90
)*(sqrt(5) - 1) + 72*x^(1/6) + 36) - 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) + 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 36*x^(1/6))
 + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-9/4*(sqrt(2)*sqrt(sq
rt(5) - 5) - sqrt(5) - 1)^2 + 36*x^(1/6)) + 2*sqrt(x) + 6/5*log(x^(1/6) - 1
)
```

giac [A] time = 1.12, size = 139, normalized size = 0.70

$$\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2*
sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
- 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) -
3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1/6) -
 1))
```

maple [A] time = 0.04, size = 175, normalized size = 0.88

$$\frac{12\sqrt{5}\,\arctan\left(\frac{4x^{\frac{1}{6}}+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{12\sqrt{5}\,\arctan\left(\frac{4x^{\frac{1}{6}}+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}-\frac{3\sqrt{5}\,\ln\left(2x^{\frac{1}{3}}+x^{\frac{1}{6}}-\sqrt{5}\,x^{\frac{1}{6}}+2\right)}{10}-\frac{3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)+6/5*ln(x^(1/6)-1)+3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*5^
(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))-12/5/(10-2*5^(1/2))^(1/2
)*arctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)-3/10*ln(2+x^(1
/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*
5^(1/2))+12/5/(10+2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/
2))^(1/2))*5^(1/2)
```

maxima [B] time = 1.39, size = 272, normalized size = 1.36

$$-\frac{6}{5}(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)-\frac{6\sqrt{5}\,(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}+\frac{6\sqrt{5}\,(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)}\right)}{5\sqrt{-2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5)*log((sqr
t(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))
/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(
1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^(1/5
) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1
)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(
-2*sqrt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) - 6/5*
log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(
sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

mupad [B] time = 0.12, size = 223, normalized size = 1.12

$$\frac{6\ln\left(1296\,x^{1/6}-1296\right)}{5}-\ln\left(-750\,x^{1/6}\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] (6*log(1296*x^(1/6) - 1296))/5 - log(- 750*x^(1/6)*((3*2^(1/2)*(- 5^(1/2) -
 5)^(1/2))/10 - (3*5^(1/2))/10 + 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2) - 5
)^(1/2))/10 - (3*5^(1/2))/10 + 3/10) + log(750*x^(1/6)*((3*2^(1/2)*(- 5^(1/
2) - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2)
 - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))/1
0 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10 -
(3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))
/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10
+ (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) + 2*x^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x)
 + 1)), x)
```

3.575 $\int \frac{\sqrt{x}}{x+x^2}\,dx$

Optimal. Leaf size=8

$$2\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*arctan(x^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {647, 63, 203}

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x + x^2),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{x+x^2}\,dx &= \int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x + x^2),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

fricas [A] time = 0.46, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

giac [A] time = 0.29, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="giac")
[Out] 2*arctan(sqrt(x))
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+x),x)
[Out] 2*arctan(x^(1/2))
```

maxima [A] time = 2.00, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

mupad [B] time = 0.14, size = 6, normalized size = 0.75

$$2 \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + x^2),x)
[Out] 2*atan(x^(1/2))
```

sympy [A] time = 0.31, size = 7, normalized size = 0.88

$$2 \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+x),x)
[Out] 2*atan(sqrt(x))
```

3.576 $\int \frac{x}{4\sqrt{x}+x}\,dx$

Optimal. Leaf size=19

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

[Out] x+32*ln(4+x^(1/2))-8*x^(1/2)

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 43}

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

[In] Int[x/(4*Sqrt[x] + x),x]

[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{4\sqrt{x}+x}\,dx &= \int \frac{\sqrt{x}}{4+\sqrt{x}}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{x^2}{4+x}\,dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-4+x+\frac{16}{4+x}\right)dx,x,\sqrt{x}\right)\\
&= -8\sqrt{x}+x+32\log\left(4+\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(4*Sqrt[x] + x),x]

```
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

fricas [A] time = 0.43, size = 15, normalized size = 0.79

$$x - 8\sqrt{x} + 32\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="fricas")
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

giac [A] time = 0.39, size = 15, normalized size = 0.79

$$x - 8\sqrt{x} + 32\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="giac")
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$x + 32\ln\left(\sqrt{x} + 4\right) - 8\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+4*x^(1/2)),x)
[Out] x+32*ln(4+x^(1/2))-8*x^(1/2)
```

maxima [A] time = 0.87, size = 15, normalized size = 0.79

$$x - 8\sqrt{x} + 32\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="maxima")
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

mupad [B] time = 0.04, size = 15, normalized size = 0.79

$$x + 32\ln\left(\sqrt{x} + 4\right) - 8\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + 4*x^(1/2)),x)
[Out] x + 32*log(x^(1/2) + 4) - 8*x^(1/2)
```

sympy [A] time = 0.17, size = 17, normalized size = 0.89

$$-8\sqrt{x} + x + 32\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x**(1/2)),x)
[Out] -8*sqrt(x) + x + 32*log(sqrt(x) + 4)
```

3.577 $\int \frac{\sqrt{x}}{\sqrt[3]{x}+x} \, dx$

Optimal. Leaf size=108

$$2\sqrt{x}-\frac{3\log\left(\sqrt[3]{x}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\log\left(\sqrt[3]{x}+\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\tan^{-1}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}$$

```
[Out] -3/2*arctan(-1+x^(1/6)*2^(1/2))*2^(1/2)-3/2*arctan(1+x^(1/6)*2^(1/2))*2^(1/
2)-3/4*ln(1+x^(1/3)-x^(1/6)*2^(1/2))*2^(1/2)+3/4*ln(1+x^(1/3)+x^(1/6)*2^(1/
2))*2^(1/2)+2*x^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1584, 341, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$2\sqrt{x}-\frac{3\log\left(\sqrt[3]{x}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\log\left(\sqrt[3]{x}+\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\tan^{-1}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] 2*Sqrt[x] + (3*ArcTan[1 - Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]
*x^(1/6)])/Sqrt[2] - (3*Log[1 - Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2]) + (
3*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
```

```
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = \int \frac{\sqrt[6]{x}}{1+x^{2/3}}\,dx$$
$$= 3\,\mathrm{Subst}\left(\int \frac{x^{5/2}}{1+x^2}\,dx, x, \sqrt[3]{x}\right)$$
$$= 2\sqrt{x} - 3\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \sqrt[3]{x}\right)$$
$$= 2\sqrt{x} - 6\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right)$$
$$= 2\sqrt{x} + 3\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right) - 3\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right)$$
$$= 2\sqrt{x} - \frac{3}{2}\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{3}{2}\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{3\,\mathrm{Su}}{}$$
$$= 2\sqrt{x} - \frac{3\log\left(1-\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{\right.}}{\sqrt{2}}$$
$$= 2\sqrt{x} + \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3\log(1+}{}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.22

$$-2\sqrt{x}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-x^{2/3}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -x^(2/3)])
```

fricas [A] time = 0.48, size = 120, normalized size = 1.11

$$3\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1}-\sqrt{2}x^{\frac{1}{6}}-1\right)+3\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}x^{\frac{1}{6}}+4x^{\frac{1}{3}}+4}-\sqrt{2}x^{\frac{1}{6}}+1\right)+\frac{3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="fricas")
```

```
[Out] 3*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*x^(1/6) + x^(1/3) + 1) - sqrt(2)*x^(1
/6) - 1) + 3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*x^(1/6) + 4*x^(1/3)
 + 4) - sqrt(2)*x^(1/6) + 1) + 3/4*sqrt(2)*log(4*sqrt(2)*x^(1/6) + 4*x^(1/3
) + 4) - 3/4*sqrt(2)*log(-4*sqrt(2)*x^(1/6) + 4*x^(1/3) + 4) + 2*sqrt(x)
```

giac [A] time = 0.44, size = 83, normalized size = 0.77

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right)-\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right)+\frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^
(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

maple [A] time = 0.00, size = 71, normalized size = 0.66

$$-\frac{3\sqrt{2}\ \arctan\left(\sqrt{2}\ x^{\frac{1}{6}}-1\right)}{2}-\frac{3\sqrt{2}\ \arctan\left(\sqrt{2}\ x^{\frac{1}{6}}+1\right)}{2}-\frac{3\sqrt{2}\ \ln\left(\frac{x^{\frac{1}{3}}-\sqrt{2}\ x^{\frac{1}{6}}+1}{x^{\frac{1}{3}}+\sqrt{2}\ x^{\frac{1}{6}}+1}\right)}{4}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/3)+x),x)
```

```
[Out] 2*x^(1/2)-3/2*arctan(1+x^(1/6)*2^(1/2))*2^(1/2)-3/2*arctan(-1+x^(1/6)*2^(1/
2))*2^(1/2)-3/4*2^(1/2)*ln((1+x^(1/3)-x^(1/6)*2^(1/2))/(1+x^(1/3)+x^(1/6)*2
^(1/2)))
```

maxima [A] time = 1.96, size = 83, normalized size = 0.77

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right)-\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right)+\frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^
(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

mupad [B] time = 0.08, size = 42, normalized size = 0.39

$$2\sqrt{x}+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x^{1/6}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x^{1/6}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + x^(1/3)),x)
```

```
[Out] 2*x^(1/2) - 2^(1/2)*atan(2^(1/2)*x^(1/6)*(1/2 + 1i/2))*(3/2 + 3i/2) - 2^(1/
2)*atan(2^(1/2)*x^(1/6)*(1/2 - 1i/2))*(3/2 - 3i/2)
```

sympy [A] time = 2.18, size = 110, normalized size = 1.02

$$2\sqrt{x}-\frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt[6]{x}+4\sqrt[3]{x}+4\right)}{4}+\frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt[6]{x}+4\sqrt[3]{x}+4\right)}{4}-\frac{3\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\sqrt[6]{x}-1\right)}{2}-\frac{3\sqrt{2}\ \mathrm{atan}\left(\sqrt{\right.}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**(1/3)+x),x)
```

```
[Out] 2*sqrt(x) - 3*sqrt(2)*log(-4*sqrt(2)*x**(1/6) + 4*x**(1/3) + 4)/4 + 3*sqrt(
2)*log(4*sqrt(2)*x**(1/6) + 4*x**(1/3) + 4)/4 - 3*sqrt(2)*atan(sqrt(2)*x**(
1/6) - 1)/2 - 3*sqrt(2)*atan(sqrt(2)*x**(1/6) + 1)/2
```

3.578 $\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=76

$$\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7}+3\sqrt[3]{x}-12\sqrt[12]{x}+6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)-4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

```
[Out] -12*x^(1/12)+3*x^(1/3)-12/7*x^(7/12)+6/5*x^(5/6)+6*ln(1+x^(1/12))-2*ln(1+x^
(1/4))-4*arctan(1/3*(1-2*x^(1/12))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1584, 341, 50, 58, 618, 204, 31}

$$\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7}+3\sqrt[3]{x}-12\sqrt[12]{x}+6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)-4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sqrt[3]*ArcT
an[(1 - 2*x^(1/12))/Sqrt[3]] + 6*Log[1 + x^(1/12)] - 2*Log[1 + x^(1/4)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx &= \int \frac{\sqrt[12]{x}}{1+\sqrt[4]{x}}\,dx\\
&= 4\,\text{Subst}\left(\int \frac{x^{10/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= \frac{6x^{5/6}}{5} - 4\,\text{Subst}\left(\int \frac{x^{7/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= -\frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4\,\text{Subst}\left(\int \frac{x^{4/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\,\text{Subst}\left(\int \frac{\sqrt[3]{x}}{1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4\,\text{Subst}\left(\int \frac{1}{x^{2/3}(1+x)}\,dx, x, \sqrt[4]{x}\right)\\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 2\log\left(1+\sqrt[4]{x}\right) + 6\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[12]{x}\right) + 6\,\text{Subs}\\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 6\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1+\sqrt[4]{x}\right) - 12\,\text{Subst}\left(\int \frac{1}{-3-x^{2}}\right.\\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right) + 6\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1+\sqrt[4]{x}\right.
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.09

$$\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7}+3\sqrt[3]{x}-12\sqrt[12]{x}+4\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[12]{x}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 + 4*Sqrt[3]*ArcT
an[(-1 + 2*x^(1/12))/Sqrt[3]] + 4*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) +
x^(1/6)]
```

fricas [A] time = 0.46, size = 62, normalized size = 0.82

$$4\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{12}}-\frac{1}{3}\sqrt{3}\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 6/5*x^(5/6) - 12/7*x
^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x
^(1/12) + 1)
```

giac [A] time = 0.34, size = 60, normalized size = 0.79

$$4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12
) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12
) + 1)
```

maple [A] time = 0.01, size = 61, normalized size = 0.80

$$4\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{12}}-1\right)\sqrt{3}}{3}\right)+4\ln\left(x^{\frac{1}{12}}+1\right)-2\ln\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(x^(1/4)+x^(1/2)),x)
```

```
[Out] 6/5*x^(5/6)-12/7*x^(7/12)+3*x^(1/3)-12*x^(1/12)-2*ln(x^(1/6)-x^(1/12)+1)+4*
3^(1/2)*arctan(1/3*(2*x^(1/12)-1)*3^(1/2))+4*ln(x^(1/12)+1)
```

maxima [A] time = 1.36, size = 60, normalized size = 0.79

$$4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12
) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12
) + 1)
```

mupad [B] time = 3.08, size = 78, normalized size = 1.03

$$4\ln\left(144x^{1/12}+144\right)-\ln\left(18-36x^{1/12}+\sqrt{3}\,18\mathrm{i}\right)\left(2+\sqrt{3}\,2\mathrm{i}\right)+\ln\left(36x^{1/12}-18+\sqrt{3}\,18\mathrm{i}\right)\left(-2+\sqrt{3}\,2\mathrm{i}\right)+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(x^(1/2) + x^(1/4)),x)
```

```
[Out] 4*log(144*x^(1/12) + 144) - log(3^(1/2)*18i - 36*x^(1/12) + 18)*(3^(1/2)*2i
 + 2) + log(3^(1/2)*18i + 36*x^(1/12) - 18)*(3^(1/2)*2i - 2) + 3*x^(1/3) +
(6*x^(5/6))/5 - 12*x^(1/12) - (12*x^(7/12))/7
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(x**(1/4)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/(x**(1/4) + sqrt(x)), x)
```

3.579 $\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}} \, dx$

Optimal. Leaf size=119

$$\frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{\vphantom{x}}\right.$$

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13*x^(13/12)+6/7*x^(7/
6)+12*ln(1+x^(1/12))+2*x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1584, 266, 43}

$$\frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{\vphantom{x}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 + (6*x^(5/6))/5 - (12
*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x^(7/6))/7 + 12*Log[1 + x^(1/12
)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}} \, dx &= \int \frac{\sqrt[4]{x}}{1+\sqrt[12]{x}} \, dx \\
&= 12\,\text{Subst}\left(\int \frac{x^{14}}{1+x} \, dx, x, \sqrt[12]{x}\right) \\
&= 12\,\text{Subst}\left(\int \left(-1+x-x^2+x^3-x^4+x^5-x^6+x^7-x^8+x^9-x^{10}+x^{11}-x^{12}+x^{13}+\frac{1}{1+x}\right)\right. \\
&= -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}-\frac{4x^{3/4}}{3}+\frac{6x^{5/6}}{5}-\frac{12x^{11/12}}{11}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 119, normalized size = 1.00

$$\frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 + (6*x^(5/6))/5 - (12
*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x^(7/6))/7 + 12*Log[1 + x^(1/12
)]
```

fricas [A] time = 0.48, size = 71, normalized size = 0.60

$$\frac{6}{7}(x+7)x^{\frac{1}{6}}-\frac{12}{13}(x+13)x^{\frac{1}{12}}+x-\frac{12}{11}x^{\frac{11}{12}}+\frac{6}{5}x^{\frac{5}{6}}-\frac{4}{3}x^{\frac{3}{4}}+\frac{3}{2}x^{\frac{2}{3}}-\frac{12}{7}x^{\frac{7}{12}}+2\sqrt{x}-\frac{12}{5}x^{\frac{5}{12}}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+12\log\left(x^{\frac{1}{12}}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

```
[Out] 6/7*(x + 7)*x^(1/6) - 12/13*(x + 13)*x^(1/12) + x - 12/11*x^(11/12) + 6/5*x
^(5/6) - 4/3*x^(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/
12) + 3*x^(1/3) - 4*x^(1/4) + 12*log(x^(1/12) + 1)
```

giac [A] time = 0.46, size = 75, normalized size = 0.63

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{12}{13}x^{\frac{13}{12}}+x-\frac{12}{11}x^{\frac{11}{12}}+\frac{6}{5}x^{\frac{5}{6}}-\frac{4}{3}x^{\frac{3}{4}}+\frac{3}{2}x^{\frac{2}{3}}-\frac{12}{7}x^{\frac{7}{12}}+2\sqrt{x}-\frac{12}{5}x^{\frac{5}{12}}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+6x^{\frac{1}{6}}-12x^{\frac{1}{12}}+12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="giac")
```

```
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(
3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3)
- 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

maple [A] time = 0.00, size = 76, normalized size = 0.64

$$\frac{6x^{\frac{7}{6}}}{7}-\frac{12x^{\frac{13}{12}}}{13}+x+12\ln\left(x^{\frac{1}{12}}+1\right)-\frac{12x^{\frac{11}{12}}}{11}+\frac{6x^{\frac{5}{6}}}{5}-\frac{4x^{\frac{3}{4}}}{3}+\frac{3x^{\frac{2}{3}}}{2}-\frac{12x^{\frac{7}{12}}}{7}+2\sqrt{x}-\frac{12x^{\frac{5}{12}}}{5}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+6x^{\frac{1}{6}}-12x^{\frac{1}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/4)+x^(1/3)),x)
```

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13*x^(13/12)+6/7*x^(7/
6)+12*ln(x^(1/12)+1)+2*x^(1/2)
```

maxima [A] time = 0.53, size = 75, normalized size = 0.63

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{12}{13}x^{\frac{13}{12}}+x-\frac{12}{11}x^{\frac{11}{12}}+\frac{6}{5}x^{\frac{5}{6}}-\frac{4}{3}x^{\frac{3}{4}}+\frac{3}{2}x^{\frac{2}{3}}-\frac{12}{7}x^{\frac{7}{12}}+2\sqrt{x}-\frac{12}{5}x^{\frac{5}{12}}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+6x^{\frac{1}{6}}-12x^{\frac{1}{12}}+12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
```

```
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(
3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3)
- 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

mupad [B] time = 0.15, size = 75, normalized size = 0.63

$$x+12\ln\left(x^{1/12}+1\right)+2\sqrt{x}+3x^{1/3}-4x^{1/4}+\frac{3x^{2/3}}{2}+6x^{1/6}-\frac{4x^{3/4}}{3}+\frac{6x^{5/6}}{5}-12x^{1/12}+\frac{6x^{7/6}}{7}-\frac{12x^{5/12}}{5}-\frac{12x^{7/12}}{7}-\frac{12x^{11/}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/3) + x^(1/4)),x)
```

```
[Out] x + 12*log(x^(1/12) + 1) + 2*x^(1/2) + 3*x^(1/3) - 4*x^(1/4) + (3*x^(2/3))/
2 + 6*x^(1/6) - (4*x^(3/4))/3 + (6*x^(5/6))/5 - 12*x^(1/12) + (6*x^(7/6))/7
 - (12*x^(5/12))/5 - (12*x^(7/12))/7 - (12*x^(11/12))/11 - (12*x^(13/12))/1
3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**(1/4)+x**(1/3)),x)
```

```
[Out] Integral(sqrt(x)/(x**(1/4) + x**(1/3)), x)
```

3.580 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=201

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}\right.$$

```
[Out] 6*x^(1/6)+x+6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*
(-5^(1/2)+1)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(5^(1/2)+1)-3/5*a
rctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2
)-3/5*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/
2)
```

Rubi [A] time = 0.22, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1584, 341, 302, 202, 634, 618, 204, 628, 31}

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/S
qrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[
5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 -
 Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt
[5])*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s*Cos[(
(2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r
*Int[1/(r - s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}],
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx &= \int \frac{x^{5/6}}{-1+x^{5/6}}\,dx \\
&= 6\,\text{Subst}\left(\int \frac{x^{10}}{-1+x^5}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\,\text{Subst}\left(\int \left(1+x^5+\frac{1}{-1+x^5}\right)dx, x, \sqrt[6]{x}\right) \\
&= 6\sqrt[6]{x}+x+6\,\text{Subst}\left(\int \frac{1}{-1+x^5}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\sqrt[6]{x}+x-\frac{6}{5}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right)-\frac{12}{5}\,\text{Subst}\left(\int \frac{1+\frac{1}{4}\left(1-\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)- \\
&= 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{1}{10}\left(3\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)- \\
&= 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log \\
&= 6\sqrt[6]{x}+x-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right.}\right.
\end{aligned}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.14

$$-6\sqrt[6]{x}\,{}_2F_1\left(\frac{1}{5},1;\frac{6}{5};x^{5/6}\right)+x+6\sqrt[6]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - 6*x^(1/6)*Hypergeometric2F1[1/5, 1, 6/5, x^(5/6)]
```

fricas [B] time = 1.38, size = 547, normalized size = 2.72

$$-\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(\frac{3}{2}\sqrt{2}\sqrt{\sqrt{5}-5}+\frac{3}{2}\sqrt{5}+6x^{\frac{1}{6}}+\frac{3}{2}\right)+\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(3/2*sqrt(2)*sqrt(sqrt(5
) - 5) + 3/2*sqrt(5) + 6*x^(1/6) + 3/2) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) -
 sqrt(5) - 1)*log(-3/2*sqrt(2)*sqrt(sqrt(5) - 5) + 3/2*sqrt(5) + 6*x^(1/6)
+ 3/2) + 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5)
+ 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5
) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 +
18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(-3*sqrt(5) + sqrt(
-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(
5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sq
rt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 1
8*sqrt(5) - 90) + 12*x^(1/6) + 3) + 1/10*(3*sqrt(5) + sqrt(-27/4*(sqrt(2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5
) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90)
- 3)*log(-3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5
) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sq
rt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) + 3) + x + 6*x^(1/6
) + 6/5*log(x^(1/6) - 1)
```

giac [A] time = 1.20, size = 140, normalized size = 0.70

$$-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2
*sqrt(5) + 10)) - 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
 + 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + x + 6*x^(1/
6) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1
/6) - 1))
```

maple [A] time = 0.03, size = 242, normalized size = 1.20

$$x-\frac{6\arctan\left(\frac{4x^{\frac{1}{6}}+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{\sqrt{10+2\sqrt{5}}}-\frac{6\sqrt{5}\,\arctan\left(\frac{4x^{\frac{1}{6}}+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{6\arctan\left(\frac{4x^{\frac{1}{6}}+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{10-2\sqrt{5}}}+\frac{6\sqrt{5}\,\arctan\left(\frac{4x^{\frac{1}{6}}+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{6\ln\left(x^{\frac{1}{6}}\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] x+6*x^(1/6)+6/5*ln(x^(1/6)-1)-3/10*5^(1/2)*ln(2*x^(1/3)+x^(1/6)+5^(1/2)*x^(
1/6)+2)-3/10*ln(2*x^(1/3)+x^(1/6)+5^(1/2)*x^(1/6)+2)-6/(10-2*5^(1/2))^(1/2)
*arctan((4*x^(1/6)+1+5^(1/2))/(10-2*5^(1/2))^(1/2))+6/5/(10-2*5^(1/2))^(1/2
)*5^(1/2)*arctan((4*x^(1/6)+1+5^(1/2))/(10-2*5^(1/2))^(1/2))+3/10*5^(1/2)*l
n(2*x^(1/3)+x^(1/6)-5^(1/2)*x^(1/6)+2)-3/10*ln(2*x^(1/3)+x^(1/6)-5^(1/2)*x^
(1/6)+2)-6/(10+2*5^(1/2))^(1/2)*arctan((4*x^(1/6)+1-5^(1/2))/(10+2*5^(1/2))
^(1/2))-6/5/(10+2*5^(1/2))^(1/2)*5^(1/2)*arctan((4*x^(1/6)+1-5^(1/2))/(10+2
*5^(1/2))^(1/2))
```

maxima [B] time = 1.52, size = 293, normalized size = 1.46

$$-\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}-1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}-\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}+1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}\cdots}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}\cdots}\right)}{5\sqrt{-2\sqrt{5}-10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) - 1)*log((sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*
sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) - (-1)^(
1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6)))/sqrt(2*sqrt(5) - 10) -
 3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) + 1)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*
sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sqrt(5) - 10
) - 6/5*(-1)^(1/5)*log((-1)^(1/5) + x^(1/6)) + x - 3/5*(sqrt(5) + 3)*log(-x
^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(
5)*(-1)^(4/5) + (-1)^(4/5)) - 3/5*(sqrt(5) - 3)*log(x^(1/6)*(sqrt(5)*(-1)^(
1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(4/5) - (-1)^(
4/5)) + 6*x^(1/6)
```

mupad [B] time = 0.06, size = 208, normalized size = 1.03

$$x+\frac{6\ln\left(1296x^{1/6}-1296\right)}{5}-\ln\left(270\sqrt{2}\sqrt{-\sqrt{5}-5}-270\sqrt{5}+1080x^{1/6}+270\right)\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] x + (6*log(1296*x^(1/6) - 1296))/5 - log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2)
- 270*5^(1/2) + 1080*x^(1/6) + 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 -
 (3*5^(1/2))/10 + 3/10) + log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2) + 270*5^(1/
2) - 1080*x^(1/6) - 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 + (3*5^(1/2)
)/10 - 3/10) + 6*x^(1/6) - log(270*5^(1/2) + 1080*x^(1/6) - 270*2^(1/2)*(5^
(1/2) - 5)^(1/2) + 270)*((3*5^(1/2))/10 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/1
0 + 3/10) - log(270*5^(1/2) + 1080*x^(1/6) + 270*2^(1/2)*(5^(1/2) - 5)^(1/2
) + 270)*((3*5^(1/2))/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)
```

sympy [A] time = 24.15, size = 311, normalized size = 1.55

$$6\sqrt[6]{x}+x+\frac{6\log\left(\sqrt[6]{x}-1\right)}{5}-\frac{3\sqrt{5}\log\left(8\sqrt[6]{x}+8\sqrt{5}\sqrt[6]{x}+16\sqrt[3]{x}+16\right)}{10}-\frac{3\log\left(8\sqrt[6]{x}+8\sqrt{5}\sqrt[6]{x}+16\sqrt[3]{x}+16\right)}{10}-\frac{3\log\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] 6*x**(1/6) + x + 6*log(x**(1/6) - 1)/5 - 3*sqrt(5)*log(8*x**(1/6) + 8*sqrt(
5)*x**(1/6) + 16*x**(1/3) + 16)/10 - 3*log(8*x**(1/6) + 8*sqrt(5)*x**(1/6)
+ 16*x**(1/3) + 16)/10 - 3*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/
3) + 16)/10 + 3*sqrt(5)*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/3)
+ 16)/10 - 3*sqrt(2)*sqrt(5 - sqrt(5))*atan(2*sqrt(2)*x**(1/6)/sqrt(5 - sqr
t(5)) + sqrt(2)/(2*sqrt(5 - sqrt(5))) + sqrt(10)/(2*sqrt(5 - sqrt(5))))/5 -
 3*sqrt(2)*sqrt(sqrt(5) + 5)*atan(2*sqrt(2)*x**(1/6)/sqrt(sqrt(5) + 5) - sq
rt(10)/(2*sqrt(sqrt(5) + 5)) + sqrt(2)/(2*sqrt(sqrt(5) + 5)))/5
```

3.581 $\int \frac{\sqrt{b-\frac{a}{x}}\, x^m}{\sqrt{a-bx}}\, dx$

Optimal. Leaf size=36

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

```
[Out] 2*x^(1+m)*(b-a/x)^(1/2)/(1+2*m)/(-b*x+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^(1 + m))/((1 + 2*m)*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x}}\, x^m}{\sqrt{a-bx}}\, dx &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{x^{-\frac{1}{2}+m}\sqrt{-a+bx}}{\sqrt{a-bx}}\, dx}{\sqrt{-a+bx}} \\
&= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int x^{-\frac{1}{2}+m}\, dx}{\sqrt{a-bx}} \\
&= \frac{2\sqrt{b-\frac{a}{x}}\, x^{1+m}}{(1+2m)\sqrt{a-bx}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.97

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x}}}{\left(m+\frac{1}{2}\right)\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[b - a/x]*x^(1 + m))/((1/2 + m)*Sqrt[a - b*x])
```

fricas [A] time = 0.43, size = 44, normalized size = 1.22

$$\frac{2\sqrt{-bx+a}\,xx^m\sqrt{\frac{bx-a}{x}}}{2\,am-(2\,bm+b)x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-b*x + a)*x*x^m*sqrt((b*x - a)/x)/(2*a*m - (2*b*m + b)*x + a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Undef/Unsigned Inf encou
ntered in limitLimit: Max order reached or unable to make series expansion
Error: Bad Argument Value
```

maple [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{2\sqrt{-\frac{-bx+a}{x}}\;x^{m+1}}{(2m+1)\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2*x^(m+1)/(1+2*m)*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

maxima [C] time = 0.71, size = 15, normalized size = 0.42

$$\frac{2\sqrt{x}\,x^m}{2i\,m+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*x^m/(2*I*m + I)
```

mupad [B] time = 3.26, size = 32, normalized size = 0.89

$$\frac{2\,x^{m+1}\,\sqrt{b-\frac{a}{x}}}{(2\,m+1)\,\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
[Out] (2*x^(m + 1)*(b - a/x)^(1/2))/((2*m + 1)*(a - b*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
[Out] Integral(x**m*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

3.582 $\int \frac{\sqrt{b-\frac{a}{x}}\, x^2}{\sqrt{a-bx}}\, dx$

Optimal. Leaf size=29

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

```
[Out] 2/5*x^3*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x}}\, x^2}{\sqrt{a-bx}}\, dx &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{x^{3/2}\sqrt{-a+bx}}{\sqrt{a-bx}}\, dx}{\sqrt{-a+bx}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int x^{3/2}\, dx}{\sqrt{a-bx}}\\
&= \frac{2\sqrt{b-\frac{a}{x}}\, x^3}{5\sqrt{a-bx}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

fricas [A] time = 0.41, size = 35, normalized size = 1.21

$$-\frac{2\sqrt{-bx+a}\,x^3\sqrt{\frac{bx-a}{x}}}{5\,(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(-b*x + a)*x^3*sqrt((b*x - a)/x)/(b*x - a)
```

giac [B] time = 0.38, size = 76, normalized size = 2.62

$$\frac{2\sqrt{-ab}\,a^2|b|\mathrm{sgn}(x)}{5\,b^4}-\frac{2\left(\sqrt{-ab}\,a^2-\frac{((bx-a)b+ab)^2\sqrt{-(bx-a)b-ab}}{b^2}\right)|b|\mathrm{sgn}(x)}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(-a*b)*a^2*abs(b)*sgn(x)/b^4 - 2/5*(sqrt(-a*b)*a^2 - ((b*x - a)*b +
 a*b)^2*sqrt(-(b*x - a)*b - a*b)/b^2)*abs(b)*sgn(x)/b^4
```

maple [A] time = 0.00, size = 27, normalized size = 0.93

$$\frac{2\sqrt{-\frac{-bx+a}{x}}\,x^3}{5\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2/5*x^3*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

maxima [C] time = 0.81, size = 5, normalized size = 0.17

$$-\frac{2}{5}i\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*I*x^(5/2)
```

mupad [B] time = 3.10, size = 23, normalized size = 0.79

$$\frac{2\,x^3\,\sqrt{b-\frac{a}{x}}}{5\,\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

```
[Out] (2*x^3*(b - a/x)^(1/2))/(5*(a - b*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

3.583 $\int \frac{\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

```
[Out] 2/3*x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {515, 23, 30}

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{\sqrt{x}\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \sqrt{x}\,dx}{\sqrt{a-bx}}\\
&= \frac{2\sqrt{b-\frac{a}{x}}\,x^2}{3\sqrt{a-bx}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

fricas [A] time = 0.40, size = 35, normalized size = 1.21

$$-\frac{2\sqrt{-bx+a}\,x^2\sqrt{\frac{bx-a}{x}}}{3\,(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(-b*x + a)*x^2*sqrt((b*x - a)/x)/(b*x - a)
```

giac [B] time = 0.49, size = 56, normalized size = 1.93

$$\frac{2\sqrt{-ab}\,a|b|\mathrm{sgn}(x)}{3\,b^3}-\frac{2\left(\sqrt{-ab}\,a+\frac{(-(bx-a)b-ab)^{\frac{3}{2}}}{b}\right)|b|\mathrm{sgn}(x)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(-a*b)*a*abs(b)*sgn(x)/b^3 - 2/3*(sqrt(-a*b)*a + (-(b*x - a)*b - a*
b)^(3/2)/b)*abs(b)*sgn(x)/b^3
```

maple [A] time = 0.00, size = 27, normalized size = 0.93

$$\frac{2\sqrt{-\frac{-bx+a}{x}}\,x^2}{3\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
[Out] 2/3*x^2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

maxima [C] time = 0.73, size = 5, normalized size = 0.17

$$-\frac{2}{3}i\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2/3*I*x^(3/2)
```

mupad [B] time = 3.06, size = 23, normalized size = 0.79

$$\frac{2\,x^2\,\sqrt{b-\frac{a}{x}}}{3\,\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

```
[Out] (2*x^2*(b - a/x)^(1/2))/(3*(a - b*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

3.584 $\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=25

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

[Out] 2*x*(b-a/x)^(1/2)/(-b*x+a)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {435, 23, 30}

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b - a/x]/Sqrt[a - b*x],x]

[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{\sqrt{x}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}}\,dx}{\sqrt{a-bx}}\\ &= \frac{2\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/Sqrt[a - b*x],x]
[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

fricas [A] time = 0.40, size = 33, normalized size = 1.32

$$-\frac{2\sqrt{-bx+a}\,x\sqrt{\frac{bx-a}{x}}}{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-b*x + a)*x*sqrt((b*x - a)/x)/(b*x - a)
```

giac [B] time = 0.40, size = 51, normalized size = 2.04

$$\frac{2\left(\sqrt{-(bx-a)b-ab}-\sqrt{-ab}\right)|b|\mathrm{sgn}(x)}{b^2}+\frac{2\sqrt{-ab}\,|b|\mathrm{sgn}(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*(sqrt(-(b*x - a)*b - a*b) - sqrt(-a*b))*abs(b)*sgn(x)/b^2 + 2*sqrt(-a*b)*
abs(b)*sgn(x)/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{2\sqrt{-\frac{-bx+a}{x}}\,x}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
[Out] 2*x*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

maxima [C] time = 0.67, size = 5, normalized size = 0.20

$$-2i\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2*I*sqrt(x)
```

mupad [B] time = 3.04, size = 21, normalized size = 0.84

$$\frac{2\,x\,\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x)^(1/2)/(a - b*x)^(1/2),x)
```

```
[Out] (2*x*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x} + b}}{\sqrt{a - bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/sqrt(a - b*x), x)
```

3.585 $\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

```
[Out] -2*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{x^{3/2}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{x^{3/2}}\,dx}{\sqrt{a-bx}}\\ &= -\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\end{aligned}$$

Mathematica [**A**]   time = 0.01, size = 24, normalized size = 1.00

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

fricas [**A**]   time = 0.40, size = 32, normalized size = 1.33

$$\frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x - a)
```

giac [**B**]   time = 0.43, size = 42, normalized size = 1.75

$$\frac{2\left(\frac{b^3}{\sqrt{-(bx-a)b-ab}}-\frac{b^3}{\sqrt{-ab}}\right)|b|\text{sgn}(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*(b^3/sqrt(-(b*x - a)*b - a*b) - b^3/sqrt(-a*b))*abs(b)*sgn(x)/b^3
```

maple [**A**]   time = 0.00, size = 24, normalized size = 1.00

$$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x)
[Out] -2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

maxima [**C**]   time = 0.86, size = 5, normalized size = 0.21

$$\frac{2i}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2*I/sqrt(x)
```

mupad [**B**]   time = 3.08, size = 20, normalized size = 0.83

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x)^(1/2)/(x*(a - b*x)^(1/2)),x)
```

```
[Out] -(2*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x}+b}}{x\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/x/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/(x*sqrt(a - b*x)), x)
```

3.586 $\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

```
[Out] -2/3*(b-a/x)^(1/2)/x/(-b*x+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 30}

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{x^{5/2}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{x^{5/2}}\,dx}{\sqrt{a-bx}}\\
&= -\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

fricas [A] time = 0.41, size = 35, normalized size = 1.21

$$\frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{3\left(bx^2-ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x^2 - a*x)
```

giac [B] time = 0.51, size = 60, normalized size = 2.07

$$\frac{2\left(\frac{b^5}{((bx-a)b+ab)\sqrt{-(bx-a)b-ab}}-\frac{b^4}{\sqrt{-ab}\,a}\right)|b|\mathrm{sgn}(x)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/3*(b^5/(((b*x - a)*b + a*b)*sqrt(-(b*x - a)*b - a*b)) - b^4/(sqrt(-a*b)*a
))*abs(b)*sgn(x)/b^3
```

maple [A] time = 0.00, size = 27, normalized size = 0.93

$$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{3\sqrt{-bx+a}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x)
[Out] -2/3*(-(-b*x+a)/x)^(1/2)/x/(-b*x+a)^(1/2)
```

maxima [C] time = 0.84, size = 5, normalized size = 0.17

$$\frac{2i}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*I/x^(3/2)
```

mupad [B] time = 3.10, size = 23, normalized size = 0.79

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3\,x\,\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x)^(1/2)/(x^2*(a - b*x)^(1/2)),x)
```

```
[Out] -(2*(b - a/x)^(1/2))/(3*x*(a - b*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x}+b}}{x^2\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/x**2/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/(x**2*sqrt(a - b*x)), x)
```

3.587 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^n \, dx$

Optimal. Leaf size=80

$$\frac{x\left(a+\frac{b}{x}\right)^m \left(\frac{ax}{b}+1\right)^{-m} (c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

```
[Out] (a+b/x)^m*x*(d*x+c)^n*AppellF1(1-m,-m,-n,2-m,-a*x/b,-d*x/c)/(1-m)/((1+a*x/b
)^m)/((1+d*x/c)^n)
```

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {435, 135, 133}

$$\frac{x\left(a+\frac{b}{x}\right)^m \left(\frac{ax}{b}+1\right)^{-m} (c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m*(c + d*x)^n,x]
```

```
[Out] ((a + b/x)^m*x*(c + d*x)^n*AppellF1[1 - m, -m, -n, 2 - m, -((a*x)/b), -((d*
x)/c)])/((1 - m)*(1 + (a*x)/b)^m*(1 + (d*x)/c)^n)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^m (c+dx)^n \, dx &= \left(\left(a+\frac{b}{x}\right)^m x^m (b+ax)^{-m}\right) \int x^{-m} (b+ax)^m (c+dx)^n \, dx \\
&= \left(\left(a+\frac{b}{x}\right)^m x^m \left(1+\frac{ax}{b}\right)^{-m}\right) \int x^{-m} \left(1+\frac{ax}{b}\right)^m (c+dx)^n \, dx \\
&= \left(\left(a+\frac{b}{x}\right)^m x^m \left(1+\frac{ax}{b}\right)^{-m} (c+dx)^n \left(1+\frac{dx}{c}\right)^{-n}\right) \int x^{-m} \left(1+\frac{ax}{b}\right)^m \left(1+\frac{dx}{c}\right)^n dx \\
&= \frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m} (c+dx)^n \left(1+\frac{dx}{c}\right)^{-n} F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}
\end{aligned}$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b/x)^m*(c + d*x)^n,x]
[Out] Integrate[(a + b/x)^m*(c + d*x)^n, x]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^n\left(\frac{ax+b}{x}\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="fricas")
[Out] integral((d*x + c)^n*((a*x + b)/x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^n\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="giac")
[Out] integrate((d*x + c)^n*(a + b/x)^m, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m*(d*x+c)^n,x)
[Out] int((a+b/x)^m*(d*x+c)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^n\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^n*(a + b/x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{x}\right)^m (c+d\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m*(c + d*x)^n,x)
```

```
[Out] int((a + b/x)^m*(c + d*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x}\right)^{m} (c + dx)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m*(d*x+c)**n,x)
```

```
[Out] Integral((a + b/x)**m*(c + d*x)**n, x)
```

3.588 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx$

Optimal. Leaf size=138

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(2-m))}{6a^2}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(1-m)+b^2d^2\left(m^2-3m+2\right)\right)\,{}_2F_1\left(2,m+1;m\right.}{6a^4(m+1)}$$

```
[Out] 1/6*d*(6*a*c-b*d*(2-m))*(a+b/x)^(1+m)*x^2/a^2+1/3*d^2*(a+b/x)^(1+m)*x^3/a-1
/6*b*(6*a^2*c^2-6*a*b*c*d*(1-m)+b^2*d^2*(m^2-3*m+2))*(a+b/x)^(1+m)*hypergeo
m([2, 1+m],[2+m],1+b/a/x)/a^4/(1+m)
```

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {434, 446, 89, 78, 65}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(1-m)+b^2d^2\left(m^2-3m+2\right)\right)\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{6a^4(m+1)}+\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6a}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m*(c + d*x)^2,x]
```

```
[Out] (d*(6*a*c - b*d*(2 - m))*(a + b/x)^(1 + m)*x^2)/(6*a^2) + (d^2*(a + b/x)^(1
 + m)*x^3)/(3*a) - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 - m) + b^2*d^2*(2 - 3*m + m
^2))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(6*
a^4*(1 + m))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
```

```
n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx = \int \left(a+\frac{b}{x}\right)^m \left(d+\frac{c}{x}\right)^2 x^2\,dx$$
$$= -\text{Subst}\left(\int \frac{(a+bx)^m(d+cx)^2}{x^4}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m} x^3}{3a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^m\left(d(6ac-bd(2-m))+3ac^2x\right)}{x^3}\,dx, x, \frac{1}{x}\right)}{3a}$$
$$= \frac{d(6ac-bd(2-m))\left(a+\frac{b}{x}\right)^{1+m} x^2}{6a^2} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+m} x^3}{3a} - \frac{1}{6}\left(6c^2 - \frac{bd(6ac-bd(2-m))}{a^2}\right.$$
$$= \frac{d(6ac-bd(2-m))\left(a+\frac{b}{x}\right)^{1+m} x^2}{6a^2} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+m} x^3}{3a} - \frac{b\left(6a^2c^2-6abcd(1-m)+b^2\right.}{}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.81

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^m \left(a^2d(m+1)x^2(2a(3c+dx)+bd(m-2)) - b\left(6a^2c^2+6abcd(m-1)+b^2d^2\left(m^2-3m+2\right)\right)\,{}_2F_1\right.}{6a^4(m+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m*(c + d*x)^2,x]
```

```
[Out] ((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2*(b*d*(-2 + m) + 2*a*(3*c + d*x))
- b*(6*a^2*c^2 + 6*a*b*c*d*(-1 + m) + b^2*d^2*(2 - 3*m + m^2))*Hypergeometr
ic2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(6*a^4*(1 + m)*x)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^2 + 2\,cdx + c^2\right)\left(\frac{ax+b}{x}\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*((a*x + b)/x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\left(a+\frac{b}{x}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(a + b/x)^m, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \left(a+\frac{b}{x}\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m*(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^m*(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \left(a+\frac{b}{x}\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*(a + b/x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{x}\right)^m (c+d\,x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m*(c + d*x)^2,x)
```

```
[Out] int((a + b/x)^m*(c + d*x)^2, x)
```

sympy [C] time = 6.14, size = 121, normalized size = 0.88

$$\frac{b^m c^2 x x^{-m} \Gamma(1-m)\, {}_2F_1\left(\begin{matrix} -m, 1-m \\ 2-m \end{matrix} \middle| \frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)} + \frac{2b^m c d x^2 x^{-m} \Gamma(2-m)\, {}_2F_1\left(\begin{matrix} -m, 2-m \\ 3-m \end{matrix} \middle| \frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)} + \frac{b^m d^2 x^3 x^{-m} \Gamma(3-m)\, {}_2}{\Gamma(4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m*(d*x+c)**2,x)
```

```
[Out] b**m*c**2*x*x**(-m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar
(I*pi)/b)/gamma(2 - m) + 2*b**m*c*d*x**2*x**(-m)*gamma(2 - m)*hyper((-m, 2
- m), (3 - m,), a*x*exp_polar(I*pi)/b)/gamma(3 - m) + b**m*d**2*x**3*x**(-m
)*gamma(3 - m)*hyper((-m, 3 - m), (4 - m,), a*x*exp_polar(I*pi)/b)/gamma(4
- m)
```

3.589 $\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx$

Optimal. Leaf size=79

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(1-m))\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{2a^3(m+1)}$$

```
[Out] 1/2*d*(a+b/x)^(1+m)*x^2/a-1/2*b*(2*a*c-b*d*(1-m))*(a+b/x)^(1+m)*hypergeom([
2, 1+m],[2+m],1+b/a/x)/a^3/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {434, 446, 78, 65}

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(1-m))\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{2a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m*(c + d*x),x]
```

```
[Out] (d*(a + b/x)^(1 + m)*x^2)/(2*a) - (b*(2*a*c - b*d*(1 - m))*(a + b/x)^(1 + m
)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(2*a^3*(1 + m))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx = \int \left(a+\frac{b}{x}\right)^m \left(d+\frac{c}{x}\right)x\,dx$$
$$= -\text{Subst}\left(\int \frac{(a+bx)^m(d+cx)}{x^3}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{d\left(a+\frac{b}{x}\right)^{1+m}x^2}{2a} - \frac{(2ac+bd(-1+m))\,\text{Subst}\left(\int \frac{(a+bx)^m}{x^2}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{d\left(a+\frac{b}{x}\right)^{1+m}x^2}{2a} - \frac{b(2ac-bd(1-m))\left(a+\frac{b}{x}\right)^{1+m}{}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)}{2a^3(1+m)}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 0.92

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^m\left(a^2d(m+1)x^2+b(-2ac-bd(m-1))\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)\right)}{2a^3(m+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m*(c + d*x),x]
```

```
[Out] ((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2 + b*(-2*a*c - b*d*(-1 + m))*Hyper
geometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(2*a^3*(1 + m)*x)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)\left(\frac{ax+b}{x}\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*((a*x + b)/x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(a + b/x)^m, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (dx+c)\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m*(d*x+c),x)
```

```
[Out] int((a+b/x)^m*(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\left(a + \frac{b}{x}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(a + b/x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{x}\right)^m (c + d\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m*(c + d*x),x)
```

```
[Out] int((a + b/x)^m*(c + d*x), x)
```

sympy [C] time = 4.10, size = 75, normalized size = 0.95

$$\frac{b^m c x x^{-m} \Gamma(1-m)\, {}_2F_1\left(\begin{matrix} -m, 1-m \\ 2-m \end{matrix}\middle| \frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)} + \frac{b^m d x^2 x^{-m} \Gamma(2-m)\, {}_2F_1\left(\begin{matrix} -m, 2-m \\ 3-m \end{matrix}\middle| \frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m*(d*x+c),x)
```

```
[Out] b**m*c*x*x**(-m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I*
pi)/b)/gamma(2 - m) + b**m*d*x**2*x**(-m)*gamma(2 - m)*hyper((-m, 2 - m), (
3 - m,), a*x*exp_polar(I*pi)/b)/gamma(3 - m)
```

3.590 $\int \left(a+\frac{b}{x}\right)^m\,dx$

Optimal. Leaf size=40

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{a^2(m+1)}$$

```
[Out] -b*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],1+b/a/x)/a^2/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {242, 65}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(a^
2*(1 + m)))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^m\,dx &= -\text{Subst}\left(\int \frac{(a+bx)^m}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= -\frac{b\left(a+\frac{b}{x}\right)^{1+m}\,{}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)}{a^2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.25

$$-\frac{x\left(a+\frac{b}{x}\right)^m\left(\frac{ax}{b}+1\right)^{-m}\,{}_2F_1\left(1-m,-m;2-m;-\frac{ax}{b}\right)}{m-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m,x]
```

```
[Out] -(((a + b/x)^m*x*Hypergeometric2F1[1 - m, -m, 2 - m, -((a*x)/b)])/((-1 + m)
*(1 + (a*x)/b)^m))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax+b}{x}\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m,x)
```

```
[Out] int((a+b/x)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m, x)
```

mupad [B] time = 3.13, size = 51, normalized size = 1.28

$$-\frac{x\left(a+\frac{b}{x}\right)^m\,{}_2\mathrm{F}_1\left(1-m,-m;\ 2-m;\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^m\,(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m,x)
```

```
[Out] -(x*(a + b/x)^m*hypergeom([1 - m, -m], 2 - m, -(a*x)/b))/(((a*x)/b + 1)^m*(
m - 1))
```

sympy [C] time = 1.47, size = 34, normalized size = 0.85

$$\frac{b^m x x^{-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix}-m,1-m\\2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m,x)
```

```
[Out] b**m*x*x**(-m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I*pi
)/b)/gamma(2 - m)
```

3.591 $\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx$

Optimal. Leaf size=101

$$\frac{\left(a+\frac{b}{x}\right)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{b}{ax}+1\right)}{ad(m+1)}-\frac{c\left(a+\frac{b}{x}\right)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(m+1)(ac-bd)}$$

```
[Out] -c*(a+b/x)^(1+m)*hypergeom([1, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/d/(a*c-b*d)/
(1+m)+(a+b/x)^(1+m)*hypergeom([1, 1+m],[2+m],1+b/a/x)/a/d/(1+m)
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {434, 446, 86, 65, 68}

$$\frac{\left(a+\frac{b}{x}\right)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{b}{ax}+1\right)}{ad(m+1)}-\frac{c\left(a+\frac{b}{x}\right)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(m+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x),x]
```

```
[Out] -((c*(a + b/x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/(d*(a*c - b*d)*(1 + m))) + ((a + b/x)^(1 + m)*Hypergeometric2F1[
1, 1 + m, 2 + m, 1 + b/(a*x)])/(a*d*(1 + m))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)x}\,dx\\
&= -\operatorname{Subst}\left(\int \frac{(a+bx)^m}{x(d+cx)}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\operatorname{Subst}\left(\int \frac{(a+bx)^m}{x}\,dx, x, \frac{1}{x}\right)}{d}+\frac{c\operatorname{Subst}\left(\int \frac{(a+bx)^m}{d+cx}\,dx, x, \frac{1}{x}\right)}{d}\\
&= -\frac{c\left(a+\frac{b}{x}\right)^{1+m}{}_2F_1\left(1,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+m)}+\frac{\left(a+\frac{b}{x}\right)^{1+m}{}_2F_1\left(1,1+m;2+m;1+\frac{b}{ax}\right)}{ad(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.96

$$\frac{(ax+b)\left(a+\frac{b}{x}\right)^m\left(ac\,{}_2F_1\left(1,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(bd-ac)\,{}_2F_1\left(1,m+1;m+2;\frac{b}{ax}+1\right)\right)}{ad(m+1)x(bd-ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x),x]
[Out] ((a + b/x)^m*(b + a*x)*(a*c*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x)
)/(a*c - b*d)] + (-(a*c) + b*d)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + b/(a
*x)]))/(a*d*(-(a*c) + b*d)*(1 + m)*x)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^m}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^m/(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^m/(d*x + c), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c),x)
```

```
[Out] int((a+b/x)^m/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m/(c + d*x),x)
```

```
[Out] int((a + b/x)^m/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c),x)
```

```
[Out] Integral((a + b/x)**m/(c + d*x), x)
```

3.592 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(m+1)(ac-bd)^2}$$

```
[Out] -b*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)^2/
(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {434, 444, 68}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(m+1)(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/((a*c - b*d)^2*(1 + m)))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^2 x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{(a+bx)^m}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{b\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+m)}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.02

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(2,m+1;m+2;-\frac{c\left(a+\frac{b}{x}\right)}{bd-ac}\right)}{(m+1)(bd-ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((c*(a + b/x))/(
-(a*c) + b*d))])/((-(a*c) + b*d)^2*(1 + m)))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^m}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(dx + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^m/(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d\, x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m/(c + d*x)^2,x)
[Out] int((a + b/x)^m/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + dx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c)**2,x)
[Out] Integral((a + b/x)**m/(c + d*x)**2, x)
```

3.593 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3} dx$

Optimal. Leaf size=112

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(m+1))\,{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(m+1)(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

```
[Out] -1/2*d*(a+b/x)^(1+m)/c/(a*c-b*d)/(d+c/x)^2-1/2*b*(2*a*c-b*d*(1+m))*(a+b/x)^
(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/c/(a*c-b*d)^3/(1+m)
```

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {434, 446, 78, 68}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(m+1))\,{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(m+1)(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x)^3,x]
```

```
[Out] -(d*(a + b/x)^(1 + m))/(2*c*(a*c - b*d)*(d + c/x)^2) - (b*(2*a*c - b*d*(1 +
 m))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/(2*c*(a*c - b*d)^3*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^3 x^3}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{x(a+bx)^m}{(d+cx)^3}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2c(ac-bd)\left(d+\frac{c}{x}\right)^2} - \frac{(2ac-bd(1+m))\operatorname{Subst}\left(\int \frac{(a+bx)^m}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{2c(ac-bd)} \\
&= -\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2c(ac-bd)\left(d+\frac{c}{x}\right)^2} - \frac{b(2ac-bd(1+m))\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(ac-bd)^3(1+m)}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 99, normalized size = 0.88

$$
\frac{\left(a+\frac{b}{x}\right)^{m+1}\left(\frac{b(bd(m+1)-2ac)\,{}_2F_1\left(2,m+1;m+2;\frac{bc+axc}{acx-bdx}\right)}{(m+1)(ac-bd)^2}-\frac{dx^2}{(c+dx)^2}\right)}{2c(ac-bd)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x)^3,x]
```

```
[Out] ((a + b/x)^(1 + m)*(-((d*x^2)/(c + d*x)^2) + (b*(-2*a*c + b*d*(1 + m))*Hype
rgeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x - b*d*x)])/((a*c - b*d)
^2*(1 + m))))/(2*c*(a*c - b*d))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^m}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^3, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c)^3,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{m}}{(dx + c)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x}\right)^{m}}{(c + d\, x)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m/(c + d*x)^3,x)
```

```
[Out] int((a + b/x)^m/(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{m}}{(c + dx)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c)**3,x)
```

```
[Out] Integral((a + b/x)**m/(c + d*x)**3, x)
```

3.594 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx$

Optimal. Leaf size=185

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(m+1)+b^2d^2\left(m^2+3m+2\right)\right){}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{6c^2(m+1)(ac-bd)^4}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-$$

```
[Out] 1/3*d^2*(a+b/x)^(1+m)/c^2/(a*c-b*d)/(d+c/x)^3-1/6*d*(6*a*c-b*d*(4+m))*(a+b/
x)^(1+m)/c^2/(a*c-b*d)^2/(d+c/x)^2-1/6*b*(6*a^2*c^2-6*a*b*c*d*(1+m)+b^2*d^2
*(m^2+3*m+2))*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/c
^2/(a*c-b*d)^4/(1+m)
```

Rubi [A] time = 0.18, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {434, 446, 89, 78, 68}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(m+1)+b^2d^2\left(m^2+3m+2\right)\right){}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{6c^2(m+1)(ac-bd)^4}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x)^4,x]
```

```
[Out] (d^2*(a + b/x)^(1 + m))/(3*c^2*(a*c - b*d)*(d + c/x)^3) - (d*(6*a*c - b*d*(
4 + m))*(a + b/x)^(1 + m))/(6*c^2*(a*c - b*d)^2*(d + c/x)^2) - (b*(6*a^2*c^
2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2 + 3*m + m^2))*(a + b/x)^(1 + m)*Hypergeo
metric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*c - b*d)])/(6*c^2*(a*c - b*d)^4
*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^4 x^4}\,dx$$

$$= -\operatorname{Subst}\left(\int \frac{x^2(a+bx)^m}{(d+cx)^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^m(-d(3ac-bd(1+m))+3c(ac-bd)x)}{(d+cx)^3}\,dx, x, \frac{1}{x}\right)}{3c^2(ac-bd)}$$

$$= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} - \frac{d(6ac-bd(4+m))\left(a+\frac{b}{x}\right)^{1+m}}{6c^2(ac-bd)^2\left(d+\frac{c}{x}\right)^2} - \frac{\left(6a^2c^2-6abcd(1+m)+b^2d^2\left(2+3\right.\right.}{6c^2(ac}$$

$$= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} - \frac{d(6ac-bd(4+m))\left(a+\frac{b}{x}\right)^{1+m}}{6c^2(ac-bd)^2\left(d+\frac{c}{x}\right)^2} - \frac{b\left(6a^2c^2-6abcd(1+m)+b^2d^2\left(2+\right.\right.}{6c}$$

Mathematica [A] time = 0.15, size = 155, normalized size = 0.84

$$\frac{\left(a+\frac{b}{x}\right)^{m+1}\left(-\frac{b\left(6a^2c^2-6abcd(m+1)+b^2d^2\left(m^2+3m+2\right)\right){}_2F_1\left(2,m+1;m+2;\frac{bc+axc}{acx-bdx}\right)}{(m+1)(ac-bd)^2}+\frac{2d^2x^3(ac-bd)}{(c+dx)^3}+\frac{dx^2(bd(m+4)-6ac)}{(c+dx)^2}\right)}{6c^2(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x)^4,x]
```

```
[Out] ((a + b/x)^(1 + m)*((2*d^2*(a*c - b*d)*x^3)/(c + d*x)^3 + (d*(-6*a*c + b*d*
(4 + m))*x^2)/(c + d*x)^2 - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2
+ 3*m + m^2))*Hypergeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x - b*d
*x)])/((a*c - b*d)^2*(1 + m))))/(6*c^2*(a*c - b*d)^2)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^m}{d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3dx+c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x
 + c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(dx + c)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^4, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(dx + c)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c)^4,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(dx + c)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d\,x)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m/(c + d*x)^4,x)
```

```
[Out] int((a + b/x)^m/(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + dx)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c)**4,x)
```

```
[Out] Integral((a + b/x)**m/(c + d*x)**4, x)
```

3.595 $\int \frac{\sqrt{b-\frac{a}{x^2}}\, x^m}{\sqrt{a-bx^2}}\, dx$

Optimal. Leaf size=33

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

[Out] x^(1+m)*(b-a/x^2)^(1/2)/m/(-b*x^2+a)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x^2}}\, x^m}{\sqrt{a-bx^2}}\, dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\, x\right)\int \frac{x^{-1+m}\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\, dx}{\sqrt{-a+bx^2}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x^2}}\, x\right)\int x^{-1+m}\, dx}{\sqrt{a-bx^2}}\\
&= \frac{\sqrt{b-\frac{a}{x^2}}\, x^{1+m}}{m\sqrt{a-bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

fricas [A] time = 0.42, size = 44, normalized size = 1.33

$$-\frac{\sqrt{-bx^2+a}\,xx^m\sqrt{\frac{bx^2-a}{x^2}}}{bmx^2-am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b*x^2 + a)*x*x^m*sqrt((b*x^2 - a)/x^2)/(b*m*x^2 - a*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b-\frac{a}{x^2}}\,x^m}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a), x)
```

maple [A] time = 0.00, size = 35, normalized size = 1.06

$$\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}\,x^{m+1}}{\sqrt{-b\,x^2+a}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
[Out] x^(m+1)/m*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

maxima [C] time = 1.07, size = 8, normalized size = 0.24

$$-\frac{i\,x^m}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -I*x^m/m
```

mupad [B] time = 3.36, size = 29, normalized size = 0.88

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\,\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

```
[Out] (x^(m + 1)*(b - a/x^2)^(1/2))/(m*(a - b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

3.596 $\int \frac{\sqrt{b-\frac{a}{x^2}}\, x^2}{\sqrt{a-bx^2}}\, dx$

Optimal. Leaf size=31

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

[Out] 1/2*x^3*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b-\frac{a}{x^2}}\, x^2}{\sqrt{a-bx^2}}\, dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\, x\right)\int \frac{x\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\, dx}{\sqrt{-a+bx^2}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x^2}}\, x\right)\int x\, dx}{\sqrt{a-bx^2}}\\
&= \frac{\sqrt{b-\frac{a}{x^2}}\, x^3}{2\sqrt{a-bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

fricas [A] time = 0.42, size = 41, normalized size = 1.32

$$-\frac{\sqrt{-bx^2+a}\,x^3\sqrt{\frac{bx^2-a}{x^2}}}{2\left(bx^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-b*x^2 + a)*x^3*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

giac [C] time = 0.33, size = 15, normalized size = 0.48

$$-\frac{i\,bx^2-i\,a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] -1/2*(I*b*x^2 - I*a)/b
```

maple [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}\,x^3}{2\sqrt{-b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
[Out] 1/2*x^3*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

maxima [C] time = 1.02, size = 5, normalized size = 0.16

$$-\frac{1}{2}i\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -1/2*I*x^2
```

mupad [B] time = 3.14, size = 25, normalized size = 0.81

$$\frac{x^3\,\sqrt{b-\frac{a}{x^2}}}{2\,\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

```
[Out] (x^3*(b - a/x^2)^(1/2))/(2*(a - b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

3.597 $\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

[Out] x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {515, 23, 8}

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int 1\,dx}{\sqrt{a-bx^2}}\\ &= \frac{\sqrt{b-\frac{a}{x^2}}\,x^2}{\sqrt{a-bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

fricas [A] time = 0.40, size = 41, normalized size = 1.46

$$-\frac{\sqrt{-bx^2+a}\,x^2\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b*x^2 + a)*x^2*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a), x)
```

maple [A] time = 0.01, size = 42, normalized size = 1.50

$$-\frac{\sqrt{\frac{b\,x^2-a}{x^2}}\,\sqrt{-b\,x^2+a}\,x^2}{b\,x^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
[Out] -((b*x^2-a)/x^2)^(1/2)*x^2/(b*x^2-a)*(-b*x^2+a)^(1/2)
```

maxima [C] time = 1.12, size = 7, normalized size = 0.25

$$-i\,\sqrt{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -I*sqrt(x^2)
```

mupad [B] time = 3.10, size = 27, normalized size = 0.96

$$\frac{\sqrt{b\,x^2-a}\,\sqrt{x^2}}{\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
[Out] ((b*x^2 - a)^(1/2)*(x^2)^(1/2))/(a - b*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

3.598 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}} \, dx$

Optimal. Leaf size=28

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

[Out] x*ln(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {435, 23, 29}

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}} \, dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{x\sqrt{a-bx^2}} \, dx}{\sqrt{-a+bx^2}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{1}{x} \, dx}{\sqrt{a-bx^2}}\\ &= \frac{\sqrt{b-\frac{a}{x^2}}\,x\log(x)}{\sqrt{a-bx^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

fricas [B] time = 0.43, size = 51, normalized size = 1.82

$$-\arctan\left(\frac{\sqrt{-bx^2+a}\left(x^3+x\right)\sqrt{\frac{bx^2-a}{x^2}}}{bx^4-(a+b)x^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-b*x^2 + a)*(x^3 + x)*sqrt((b*x^2 - a)/x^2)/(b*x^4 - (a + b)*x
^2 + a))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Sign error %%%{b,2%%%}Li
mit: Max order reached or unable to make series expansion Error: Bad Argume
nt Value
```

maple [A] time = 0.01, size = 42, normalized size = 1.50

$$-\frac{\sqrt{\frac{b\,x^2-a}{x^2}}\,\sqrt{-b\,x^2+a}\,x\ln(x)}{b\,x^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] -((b*x^2-a)/x^2)^(1/2)*x/(b*x^2-a)*(-b*x^2+a)^(1/2)*ln(x)
```

maxima [C] time = 1.04, size = 4, normalized size = 0.14

$$-i\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2),x)
```

```
[Out] int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2} + b}}{\sqrt{a - bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

3.599 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

[Out] -(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]

[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{x^2\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{1}{x^2}\,dx}{\sqrt{a-bx^2}}\\ &= -\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

fricas [A] time = 0.40, size = 41, normalized size = 1.58

$$-\frac{\sqrt{-bx^2+a}\,(x-1)\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b*x^2 + a)*(x - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{-bx^2+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x), x)
```

maple [A] time = 0.01, size = 28, normalized size = 1.08

$$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{\sqrt{-b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x)
[Out] -(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

maxima [C] time = 1.11, size = 7, normalized size = 0.27

$$\frac{i}{\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] I/sqrt(x^2)
```

mupad [B] time = 3.34, size = 22, normalized size = 0.85

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x^2)^(1/2)/(x*(a - b*x^2)^(1/2)),x)
```

```
[Out] -(b - a/x^2)^(1/2)/(a - b*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2}+b}}{x\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/x/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/(x*sqrt(a - b*x**2)), x)
```

3.600 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

```
[Out] -1/2*(b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {515, 23, 30}

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
```

```
[Out] -Sqrt[b - a/x^2]/(2*x*Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 515

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x
^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{x^3\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{1}{x^3}\,dx}{\sqrt{a-bx^2}}\\ &= -\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
[Out] -1/2*Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2])
```

fricas [A] time = 0.40, size = 44, normalized size = 1.42

$$-\frac{\sqrt{-bx^2+a}\left(x^2-1\right)\sqrt{\frac{bx^2-a}{x^2}}}{2\left(bx^3-ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-b*x^2 + a)*(x^2 - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^3 - a*x)
```

giac [C] time = 0.37, size = 5, normalized size = 0.16

$$\frac{i}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/2*I/x^2
```

maple [A] time = 0.00, size = 31, normalized size = 1.00

$$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{2\sqrt{-b\,x^2+a}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x)
[Out] -1/2*(-(-b*x^2+a)/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

maxima [C] time = 1.06, size = 5, normalized size = 0.16

$$\frac{i}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/2*I/x^2
```

mupad [B] time = 3.46, size = 25, normalized size = 0.81

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2\,x\,\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)),x)
```

```
[Out] -(b - a/x^2)^(1/2)/(2*x*(a - b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2} + b}}{x^2 \sqrt{a - bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/x**2/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/(x**2*sqrt(a - b*x**2)), x)
```

3.601 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}} dx$

Optimal. Leaf size=406

$$-\frac{2\sqrt{b}\,c\sqrt{\frac{ax^2}{b}+1}\left(ac^2+bd^2\right)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx}}+\frac{2\sqrt{b}\sqrt{\frac{ax^2}{b}+1}\sqrt{c+dx}\left(ac^2-3b\ldots\right.}{5(-a)^{3/2}dx\sqrt{a\ldots}}$$

```
[Out] 2/5*(d*x+c)^(3/2)*(a*x^2+b)/a/x/(a+b/x^2)^(1/2)+2/5*c*(a*x^2+b)*(d*x+c)^(1/
2)/a/x/(a+b/x^2)^(1/2)+2/5*(a*c^2-3*b*d^2)*EllipticE(1/2*(1-x*(-a)^(1/2)/b^
(1/2))^(1/2)*2^(1/2),(-2*d*(-a)^(1/2)*b^(1/2)/(a*c-d*(-a)^(1/2)*b^(1/2)))^(
1/2))*b^(1/2)*(d*x+c)^(1/2)*(1+a*x^2/b)^(1/2)/(-a)^(3/2)/d/x/(a+b/x^2)^(1/2
)/(a*(d*x+c)/(a*c-d*(-a)^(1/2)*b^(1/2)))^(1/2)-2/5*c*(a*c^2+b*d^2)*Elliptic
F(1/2*(1-x*(-a)^(1/2)/b^(1/2))^(1/2)*2^(1/2),(-2*d*(-a)^(1/2)*b^(1/2)/(a*c-
d*(-a)^(1/2)*b^(1/2)))^(1/2))*b^(1/2)*(1+a*x^2/b)^(1/2)*(a*(d*x+c)/(a*c-d*(
-a)^(1/2)*b^(1/2)))^(1/2)/(-a)^(3/2)/d/x/(a+b/x^2)^(1/2)/(d*x+c)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1450, 833, 844, 719, 424, 419}

$$-\frac{2\sqrt{b}\,c\sqrt{\frac{ax^2}{b}+1}\left(ac^2+bd^2\right)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx}}+\frac{2\sqrt{b}\sqrt{\frac{ax^2}{b}+1}\sqrt{c+dx}\left(ac^2-3b\ldots\right.}{5(-a)^{3/2}dx\sqrt{a\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

```
[Out] (2*c*Sqrt[c + d*x]*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*(c + d*x)^(3/2
)*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*Sqrt[b]*(a*c^2 - 3*b*d^2)*Sqrt[
c + d*x]*Sqrt[1 + (a*x^2)/b]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]
]/Sqrt[2]], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3
/2)*d*Sqrt[a + b/x^2]*x*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]) - (
2*Sqrt[b]*c*(a*c^2 + b*d^2)*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]*
Sqrt[1 + (a*x^2)/b]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]]/Sqrt[2]
], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3/2)*d*Sqr
t[a + b/x^2]*x*Sqrt[c + d*x])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1450

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Sy
mbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/(c + a*x^(2
*n))^FracPart[p], Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x], x] /
; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[mn2, -2*n] && !IntegerQ[p] && !I
ntegerQ[q] && PosQ[n]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{b+ax^2}\int \frac{x(c+dx)^{3/2}}{\sqrt{b+ax^2}}\,dx}{\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(2\sqrt{b+ax^2}\right)\int \frac{\left(-\frac{3bd}{2}+\frac{3acx}{2}\right)\sqrt{c+dx}}{\sqrt{b+ax^2}}\,dx}{5a\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(4\sqrt{b+ax^2}\right)\int \frac{-3abcd+\frac{3}{4}a\left(ac^2-3bd^2\right)x}{\sqrt{c+dx}\,\sqrt{b+ax^2}}\,dx}{15a^2\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(\left(ac^2-3bd^2\right)\sqrt{b+ax^2}\right)\int \frac{\sqrt{c+dx}}{\sqrt{b+ax^2}}\,dx}{5ad\sqrt{a+\frac{b}{x^2}}\,x} - \frac{\left(c\left(\right.\right.}{}$$

$$= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(2\sqrt{-a}\,\sqrt{b}\,\left(ac^2-3bd^2\right)\sqrt{c+dx}\,\sqrt{1+\frac{ax^2}{b}}\right)}{5a^2d\sqrt{a+\frac{b}{x^2}}\,x\sqrt{}}$$

$$= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2\sqrt{b}\,\left(ac^2-3bd^2\right)\sqrt{c+dx}\,\sqrt{1+\frac{ax^2}{b}}\,E\left(\sin^{-1}\right.}{5(-a)^{3/2}d\sqrt{a+\frac{b}{x^2}}\,x\sqrt{\frac{a}{ac-}}}$$

Mathematica [C] time = 3.03, size = 540, normalized size = 1.33

$$\frac{\sqrt{c+dx}\left(\frac{2\left(ax^2+b\right)(2c+dx)}{a} + \frac{2\left(\sqrt{a}\,(c+dx)^{3/2}\left(-ia^{3/2}c^3+a\sqrt{b}\,c^2d+3i\sqrt{a}\,bcd^2-3b^{3/2}d^3\right)\sqrt{\frac{d\left(x+\frac{i\sqrt{b}}{\sqrt{a}}\right)}{c+dx}}\,\sqrt{-\frac{-dx+\frac{i\sqrt{b}\,d}{\sqrt{a}}}{c+dx}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{i\sqrt{b}\,d}{\sqrt{a}}}}{\sqrt{c+dx}}\right)\middle|\frac{\sqrt{a}\,c-i}{\sqrt{a}\,c+i}\right.\right.}{}\right.}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

```
[Out] (Sqrt[c + d*x]*((2*(2*c + d*x)*(b + a*x^2))/a + (2*(d^2*Sqrt[-c - (I*Sqrt[b
]*d)/Sqrt[a]]*(-3*b^2*d^2 + a^2*c^2*x^2 + a*b*(c^2 - 3*d^2*x^2)) + Sqrt[a]*
((-I)*a^(3/2)*c^3 + a*Sqrt[b]*c^2*d + (3*I)*Sqrt[a]*b*c*d^2 - 3*b^(3/2)*d^3
)*Sqrt[(d*((I*Sqrt[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[
a] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - (I*Sqrt
[b]*d)/Sqrt[a]]/Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sq
rt[b]*d)] - Sqrt[a]*Sqrt[b]*d*(a*c^2 + (4*I)*Sqrt[a]*Sqrt[b]*c*d - 3*b*d^2)
*Sqrt[(d*((I*Sqrt[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[a
] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - (I*Sqrt[
b]*d)/Sqrt[a]]/Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sqr
t[b]*d)]))/(a^2*d^2*Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]*(c + d*x))))/(5*Sqrt[a
 + b/x^2]*x)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^3+cx^2\right)\sqrt{dx+c}\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c*x^2)*sqrt(d*x + c)*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)
```

maple [B] time = 0.12, size = 1145, normalized size = 2.82

$$\frac{2a^2d^4x^4}{5}+\frac{6a^2cd^3x^3}{5}+\frac{4a^2c^2d^2x^2}{5}+\frac{2abd^4x^2}{5}-\frac{2\sqrt{\frac{\left(-ax+\sqrt{-ab}\right)d}{ac+\sqrt{-ab}\,d}}\sqrt{\frac{\left(ax+\sqrt{-ab}\right)d}{-ac+\sqrt{-ab}\,d}}\sqrt{-\frac{(dx+c)a}{-ac+\sqrt{-ab}\,d}}\,a^2c^4\,\mathrm{EllipticE}\left(\sqrt{-\frac{(dx+c)a}{-ac+\sqrt{-ab}\,d}},\sqrt{-\frac{-ac+\sqrt{-}}{ac+\sqrt{-a}}}\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x)
```

```
[Out] 2/5*(((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))
*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticF((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^
(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*(-(d*x+c)*a/((-a*
b)^(1/2)*d-a*c))^(1/2)*(-a*b)^(1/2)*a*c^3*d+((-a*x+(-a*b)^(1/2))*d/((-a*b)^
(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*Ellip
ticF((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)
^(1/2)*d+a*c))^(1/2))*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*(-a*b)^(1/2)*
b*c*d^3-3*((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(
1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticF((-(d*x+c)*a/((-a*b)^(1/2)*d-a
*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*(-(d*x+c)*a/
((-a*b)^(1/2)*d-a*c))^(1/2)*a*b*c^2*d^2-3*((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1
/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*Ellipti
cF((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(
1/2)*d+a*c))^(1/2))*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*b^2*d^4-((-a*x+
(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(
1/2)*d-a*c))^(1/2)*EllipticE((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a
*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*(-(d*x+c)*a/((-a*b)^(1/2)*d-a
*c))^(1/2)*a^2*c^4+2*((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a
*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticE((-(d*x+c)*a/((-a*b
)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*(
-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*a*b*c^2*d^2+3*((-a*x+(-a*b)^(1/2))*d
/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1
/2)*EllipticE((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c
)/((-a*b)^(1/2)*d+a*c))^(1/2))*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*b^2*
```

```
d^4+x^4*a^2*d^4+3*x^3*a^2*c*d^3+2*x^2*a^2*c^2*d^2+x^2*a*b*d^4+3*x*a*b*c*d^3
+2*a*b*c^2*d^2)/(d*x+c)^(1/2)/d^2/a^2/x/((a*x^2+b)/x^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b/x^2)^(1/2),x)
[Out] int((c + d*x)^(3/2)/(a + b/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(a+b/x**2)**(1/2),x)
[Out] Integral((c + d*x)**(3/2)/sqrt(a + b/x**2), x)
```

3.602 $\int \frac{-1+x^3}{\left(-4x+x^4\right)^{2/3}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{4}\sqrt[3]{x^4-4x}$$

[Out] 3/4*(x^4-4*x)^(1/3)

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{3}{4}\sqrt[3]{x^4-4x}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^3)/(-4*x + x^4)^(2/3),x]

[Out] (3*(-4*x + x^4)^(1/3))/4

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+x^3}{\left(-4x+x^4\right)^{2/3}}\,dx=\frac{3}{4}\sqrt[3]{-4x+x^4}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 1.00

$$\frac{3}{4}\sqrt[3]{x\left(x^3-4\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^3)/(-4*x + x^4)^(2/3),x]

[Out] (3*(x*(-4 + x^3))^(1/3))/4

fricas [A] time = 0.38, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^4-4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="fricas")

[Out] 3/4*(x^4 - 4*x)^(1/3)

giac [A] time = 0.32, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^4-4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="giac")
```

```
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

maple [A] time = 0.01, size = 18, normalized size = 1.20

$$\frac{3\left(x^{3}-4\right)x}{4\left(x^{4}-4x\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^4-4*x)^(2/3),x)
```

```
[Out] 3/4*x*(x^3-4)/(x^4-4*x)^(2/3)
```

maxima [A] time = 0.89, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^{4}-4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

mupad [B] time = 3.52, size = 11, normalized size = 0.73

$$\frac{3\left(x^{4}-4\,x\right)^{1/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^4 - 4*x)^(2/3),x)
```

```
[Out] (3*(x^4 - 4*x)^(1/3))/4
```

sympy [A] time = 0.20, size = 12, normalized size = 0.80

$$\frac{3\sqrt[3]{x^{4}-4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**4-4*x)**(2/3),x)
```

```
[Out] 3*(x**4 - 4*x)**(1/3)/4
```

3.603 $\int (2-x^2)\sqrt[4]{6x-x^3}\,dx$

Optimal. Leaf size=17

$$\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

```
[Out] 4/15*(-x^3+6*x)^(5/4)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1588}

$$\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

```
[Out] (4*(6*x - x^3)^(5/4))/15
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2-x^2\right)\sqrt[4]{6x-x^3}\,dx=\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

Mathematica [C] time = 0.05, size = 72, normalized size = 4.24

$$-\frac{4\sqrt[4]{-x\left(x^2-6\right)}\left(5x^3\,{}_2F_1\left(-\frac{1}{4},\frac{13}{8};\frac{21}{8};\frac{x^2}{6}\right)-26x\,{}_2F_1\left(-\frac{1}{4},\frac{5}{8};\frac{13}{8};\frac{x^2}{6}\right)\right)}{65\sqrt[4]{1-\frac{x^2}{6}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

```
[Out] (-4*(-(x*(-6 + x^2)))^(1/4)*(-26*x*Hypergeometric2F1[-1/4, 5/8, 13/8, x^2/6
] + 5*x^3*Hypergeometric2F1[-1/4, 13/8, 21/8, x^2/6]))/(65*(1 - x^2/6)^(1/4
))
```

fricas [A] time = 0.39, size = 20, normalized size = 1.18

$$-\frac{4}{15}\left(x^3-6\,x\right)\left(-x^3+6\,x\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/15*(x^3 - 6*x)*(-x^3 + 6*x)^(1/4)
```

giac [A] time = 0.33, size = 13, normalized size = 0.76

$$\frac{4}{15}\left(-x^3+6\,x\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="giac")
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

maple [A] time = 0.00, size = 20, normalized size = 1.18

$$-\frac{4\left(-x^3+6x\right)^{\frac{1}{4}}\left(x^2-6\right)x}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2)*(-x^3+6*x)^(1/4),x)
[Out] -4/15*(-x^3+6*x)^(1/4)*x*(x^2-6)
```

maxima [A] time = 0.88, size = 13, normalized size = 0.76

$$\frac{4}{15}\left(-x^3+6\,x\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="maxima")
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

mupad [B] time = 3.16, size = 19, normalized size = 1.12

$$-\frac{4\,x\,\left(x^2-6\right)\,\left(6\,x-x^3\right)^{1/4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 2)*(6*x - x^3)^(1/4),x)
[Out] -(4*x*(x^2 - 6)*(6*x - x^3)^(1/4))/15
```

sympy [B] time = 0.27, size = 31, normalized size = 1.82

$$-\frac{4x^3\sqrt[4]{-x^3+6x}}{15}+\frac{8x\sqrt[4]{-x^3+6x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2)*(-x**3+6*x)**(1/4),x)
[Out] -4*x**3*(-x**3 + 6*x)**(1/4)/15 + 8*x*(-x**3 + 6*x)**(1/4)/5
```

3.604 $\int (1+x^4) \sqrt{5x+x^5}\, dx$

Optimal. Leaf size=15

$$\frac{2}{15}\left(x^5+5x\right)^{3/2}$$

[Out] 2/15*(x^5+5*x)^(3/2)

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2}{15}\left(x^5+5x\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)*Sqrt[5*x + x^5],x]

[Out] (2*(5*x + x^5)^(3/2))/15

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(1+x^4\right) \sqrt{5x+x^5}\, dx = \frac{2}{15}\left(5x+x^5\right)^{3/2}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 1.00

$$\frac{2}{15}\left(x\left(x^4+5\right)\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)*Sqrt[5*x + x^5],x]

[Out] (2*(x*(5 + x^4))^(3/2))/15

fricas [A] time = 0.38, size = 11, normalized size = 0.73

$$\frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="fricas")

[Out] 2/15*(x^5 + 5*x)^(3/2)

giac [A] time = 0.40, size = 11, normalized size = 0.73

$$\frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

maple [A] time = 0.01, size = 18, normalized size = 1.20

$$\frac{2\left(x^{4}+5\right)\sqrt{x^{5}+5x}\,x}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(x^5+5*x)^(1/2),x)
```

```
[Out] 2/15*x*(x^4+5)*(x^5+5*x)^(1/2)
```

maxima [A] time = 0.87, size = 11, normalized size = 0.73

$$\frac{2}{15}\left(x^{5}+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

mupad [B] time = 3.10, size = 11, normalized size = 0.73

$$\frac{2\left(x^{5}+5\,x\right)^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^5)^(1/2)*(x^4 + 1),x)
```

```
[Out] (2*(5*x + x^5)^(3/2))/15
```

sympy [B] time = 0.26, size = 31, normalized size = 2.07

$$\frac{2x^{5}\sqrt{x^{5}+5x}}{15}+\frac{2x\sqrt{x^{5}+5x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(x**5+5*x)**(1/2),x)
```

```
[Out] 2*x**5*sqrt(x**5 + 5*x)/15 + 2*x*sqrt(x**5 + 5*x)/3
```

3.605 $\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx$

Optimal. Leaf size=15

$$\frac{2}{3}\left(x^5+2x\right)^{3/2}$$

```
[Out] 2/3*(x^5+2*x)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$\frac{2}{3}\left(x^5+2x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
```

```
[Out] (2*(2*x + x^5)^(3/2))/3
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx=\frac{2}{3}\left(2x+x^5\right)^{3/2}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 1.00

$$\frac{2}{3}\left(x\left(x^4+2\right)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
```

```
[Out] (2*(x*(2 + x^4))^(3/2))/3
```

fricas [A] time = 0.39, size = 11, normalized size = 0.73

$$\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

giac [A] time = 0.40, size = 11, normalized size = 0.73

$$\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

maple [A] time = 0.00, size = 18, normalized size = 1.20

$$\frac{2\left(x^{4}+2\right)\sqrt{x^{5}+2x}\,x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4+2)*(x^5+2*x)^(1/2),x)
```

```
[Out] 2/3*x*(x^4+2)*(x^5+2*x)^(1/2)
```

maxima [A] time = 0.87, size = 11, normalized size = 0.73

$$\frac{2}{3}\left(x^{5}+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

mupad [B] time = 3.11, size = 11, normalized size = 0.73

$$\frac{2\left(x^{5}+2\,x\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^5)^(1/2)*(5*x^4 + 2),x)
```

```
[Out] (2*(2*x + x^5)^(3/2))/3
```

sympy [B] time = 0.26, size = 31, normalized size = 2.07

$$\frac{2x^{5}\sqrt{x^{5}+2x}}{3}+\frac{4x\sqrt{x^{5}+2x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4+2)*(x**5+2*x)**(1/2),x)
```

```
[Out] 2*x**5*sqrt(x**5 + 2*x)/3 + 4*x*sqrt(x**5 + 2*x)/3
```

3.606 $\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx$

Optimal. Leaf size=13

$$\sqrt{2x^3+x^2}$$

```
[Out] (2*x^3+x^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1588}

$$\sqrt{2x^3+x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
```

```
[Out] Sqrt[x^2 + 2*x^3]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx = \sqrt{x^2+2x^3}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\sqrt{x^2(2x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
```

```
[Out] Sqrt[x^2*(1 + 2*x)]
```

fricas [A] time = 0.40, size = 11, normalized size = 0.85

$$\sqrt{2\,x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2*x^3 + x^2)
```

giac [A] time = 0.36, size = 11, normalized size = 0.85

$$\sqrt{2\,x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="giac")
[Out] sqrt(2*x^3 + x^2)
```

maple [A] time = 0.00, size = 21, normalized size = 1.62

$$\frac{(2x+1)\,x^2}{\sqrt{2x^3+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+x)/(2*x^3+x^2)^(1/2),x)
[Out] x^2*(2*x+1)/(2*x^3+x^2)^(1/2)
```

maxima [A] time = 0.88, size = 11, normalized size = 0.85

$$\sqrt{2\,x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(2*x^3 + x^2)
```

mupad [B] time = 3.23, size = 10, normalized size = 0.77

$$|x|\,\sqrt{2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2)/(x^2 + 2*x^3)^(1/2),x)
[Out] abs(x)*(2*x + 1)^(1/2)
```

sympy [A] time = 0.16, size = 10, normalized size = 0.77

$$\sqrt{2x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+x)/(2*x**3+x**2)**(1/2),x)
[Out] sqrt(2*x**3 + x**2)
```

3.607 $\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx$

Optimal. Leaf size=44

$$x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

```
[Out] -9/5*(1-5*x)^(1/3)+3/10*(1-5*x)^(2/3)+x+27/5*ln(3+(1-5*x)^(1/3))
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {431, 376, 77}

$$x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 + (1 - 5*x)
^(1/3)])/5
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx &= -\left(\frac{1}{5}\operatorname{Subst}\left(\int \frac{2+\sqrt[3]{x}}{3+\sqrt[3]{x}}\,dx,x,1-5x\right)\right)\\
&= -\left(\frac{3}{5}\operatorname{Subst}\left(\int \frac{x^2(2+x)}{3+x}\,dx,x,\sqrt[3]{1-5x}\right)\right)\\
&= -\left(\frac{3}{5}\operatorname{Subst}\left(\int \left(3-x+x^2-\frac{9}{3+x}\right)dx,x,\sqrt[3]{1-5x}\right)\right)\\
&= -\frac{9}{5}\sqrt[3]{1-5x}+\frac{3}{10}(1-5x)^{2/3}+x+\frac{27}{5}\log\left(3+\sqrt[3]{1-5x}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 + (1 - 5*x)
^(1/3)])/5
```

fricas [A] time = 0.41, size = 32, normalized size = 0.73

$$x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="fricas")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3)
```

giac [A] time = 0.48, size = 33, normalized size = 0.75

$$x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)-\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="giac")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3) - 1/5
```

maple [A] time = 0.00, size = 34, normalized size = 0.77

$$x+\frac{27\ln\left(3+(-5x+1)^{\frac{1}{3}}\right)}{5}-\frac{1}{5}+\frac{3\,(-5x+1)^{\frac{2}{3}}}{10}-\frac{9\,(-5x+1)^{\frac{1}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x)
```

```
[Out] -1/5+x+3/10*(1-5*x)^(2/3)-9/5*(1-5*x)^(1/3)+27/5*ln(3+(1-5*x)^(1/3))
```

maxima [A] time = 0.84, size = 33, normalized size = 0.75

$$x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)-\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="maxima")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3) - 1/5
```

mupad [B] time = 0.11, size = 32, normalized size = 0.73

$$x+\frac{27\,\ln\left((1-5\,x)^{1/3}+3\right)}{5}-\frac{9\,(1-5\,x)^{1/3}}{5}+\frac{3\,(1-5\,x)^{2/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - 5*x)^(1/3) + 2)/((1 - 5*x)^(1/3) + 3),x)
```

```
[Out] x + (27*log((1 - 5*x)^(1/3) + 3))/5 - (9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^
(2/3))/10
```

sympy [A] time = 0.20, size = 39, normalized size = 0.89

$$x + \frac{3(1-5x)^{\frac{2}{3}}}{10} - \frac{9\sqrt[3]{1-5x}}{5} + \frac{27\log\left(\sqrt[3]{1-5x}+3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)**(1/3))/(3+(1-5*x)**(1/3)),x)
```

```
[Out] x + 3*(1 - 5*x)**(2/3)/10 - 9*(1 - 5*x)**(1/3)/5 + 27*log((1 - 5*x)**(1/3)
+ 3)/5
```

3.608 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$x+4\sqrt{x}+4\log\left(1-\sqrt{x}\right)$$

```
[Out] x+4*ln(1-x^(1/2))+4*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {376, 77}

$$x+4\sqrt{x}+4\log\left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x(1+x)}{-1+x}\,dx,x,\sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(2+\frac{2}{-1+x}+x\right)dx,x,\sqrt{x}\right)\\ &= 4\sqrt{x}+x+4\log\left(1-\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.95

$$x+4\left(\sqrt{x}+\log\left(1-\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] x + 4*(Sqrt[x] + Log[1 - Sqrt[x]])
```

fricas [A] time = 0.41, size = 15, normalized size = 0.71

$$x+4\sqrt{x}+4\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

giac [A] time = 0.31, size = 16, normalized size = 0.76

$$x + 4\sqrt{x} + 4\log\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="giac")
```

```
[Out] x + 4*sqrt(x) + 4*log(abs(sqrt(x) - 1))
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$x + 4\ln\left(\sqrt{x} - 1\right) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)/(x^(1/2)-1),x)
```

```
[Out] x+4*x^(1/2)+4*ln(x^(1/2)-1)
```

maxima [A] time = 0.88, size = 15, normalized size = 0.71

$$x + 4\sqrt{x} + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

mupad [B] time = 3.01, size = 15, normalized size = 0.71

$$x + 4\ln\left(\sqrt{x} - 1\right) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)/(x^(1/2) - 1),x)
```

```
[Out] x + 4*log(x^(1/2) - 1) + 4*x^(1/2)
```

sympy [A] time = 0.15, size = 17, normalized size = 0.81

$$4\sqrt{x} + x + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(-1+x**(1/2)),x)
```

```
[Out] 4*sqrt(x) + x + 4*log(sqrt(x) - 1)
```

3.609 $\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx$

Optimal. Leaf size=33

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

[Out] -x-4/3*ln(1+(2+3*x)^(1/2))+4/3*(2+3*x)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {431, 376, 77}

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]

[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1-\sqrt{x}}{1+\sqrt{x}}\,dx,x,2+3x\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{(1-x)x}{1+x}\,dx,x,\sqrt{2+3x}\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \left(2-x-\frac{2}{1+x}\right)dx,x,\sqrt{2+3x}\right)\\
&= -x+\frac{4}{3}\sqrt{2+3x}-\frac{4}{3}\log\left(1+\sqrt{2+3x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3
```

fricas [A] time = 0.40, size = 25, normalized size = 0.76

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="fricas")
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1)
```

giac [A] time = 0.33, size = 26, normalized size = 0.79

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="giac")
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

maple [A] time = 0.01, size = 27, normalized size = 0.82

$$-x-\frac{4\ln\left(1+\sqrt{3x+2}\right)}{3}+\frac{4\sqrt{3x+2}}{3}-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(3*x+2)^(1/2))/(1+(3*x+2)^(1/2)),x)
[Out] 4/3*(3*x+2)^(1/2)-x-2/3-4/3*ln(1+(3*x+2)^(1/2))
```

maxima [A] time = 0.88, size = 26, normalized size = 0.79

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="maxima")
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

mupad [B] time = 3.10, size = 25, normalized size = 0.76

$$\frac{4\sqrt{3x+2}}{3}-\frac{4\ln\left(\sqrt{3x+2}+1\right)}{3}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((3*x + 2)^(1/2) - 1)/((3*x + 2)^(1/2) + 1),x)
[Out] (4*(3*x + 2)^(1/2))/3 - (4*log((3*x + 2)^(1/2) + 1))/3 - x
```

sympy [A] time = 0.17, size = 27, normalized size = 0.82

$$-x+\frac{4\sqrt{3x+2}}{3}-\frac{4\log\left(\sqrt{3x+2}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)**(1/2))/(1+(2+3*x)**(1/2)),x)
```

```
[Out] -x + 4*sqrt(3*x + 2)/3 - 4*log(sqrt(3*x + 2) + 1)/3
```

3.610 $\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx$

Optimal. Leaf size=33

$$-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+x$$

[Out] x+4*ln(1+(b*x+a)^(1/2))/b-4*(b*x+a)^(1/2)/b

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {431, 376, 77}

$$-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+x$$

Antiderivative was successfully verified.

[In] Int[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]

[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx &= \frac{\text{Subst}\left(\int \frac{-1+\sqrt{x}}{1+\sqrt{x}}\,dx,x,a+bx\right)}{b}\\ &= \frac{2\,\text{Subst}\left(\int \frac{(-1+x)x}{1+x}\,dx,x,\sqrt{a+bx}\right)}{b}\\ &= \frac{2\,\text{Subst}\left(\int \left(-2+x+\frac{2}{1+x}\right)dx,x,\sqrt{a+bx}\right)}{b}\\ &= x-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(1+\sqrt{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b
```

fricas [A] time = 0.39, size = 29, normalized size = 0.88

$$\frac{bx-4\sqrt{bx+a}+4\log\left(\sqrt{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="fricas")
[Out] (b*x - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

giac [A] time = 0.36, size = 38, normalized size = 1.15

$$\frac{4\log\left(\sqrt{bx+a}+1\right)}{b}+\frac{(bx+a)b-4\sqrt{bx+a}\,b}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="giac")
[Out] 4*log(sqrt(b*x + a) + 1)/b + ((b*x + a)*b - 4*sqrt(b*x + a)*b)/b^2
```

maple [A] time = 0.00, size = 35, normalized size = 1.06

$$x+\frac{a}{b}+\frac{4\ln\left(1+\sqrt{bx+a}\right)}{b}-\frac{4\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x)
[Out] -4*(b*x+a)^(1/2)/b+x+a/b+4*ln(1+(b*x+a)^(1/2))/b
```

maxima [A] time = 1.05, size = 30, normalized size = 0.91

$$\frac{bx+a-4\sqrt{bx+a}+4\log\left(\sqrt{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="maxima")
[Out] (b*x + a - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

mupad [B] time = 3.02, size = 29, normalized size = 0.88

$$x+\frac{4\ln\left(\sqrt{a+b\,x}+1\right)}{b}-\frac{4\sqrt{a+b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^(1/2) - 1)/((a + b*x)^(1/2) + 1),x)
```

```
[Out] x + (4*log((a + b*x)^(1/2) + 1))/b - (4*(a + b*x)^(1/2))/b
```

sympy [A] time = 0.44, size = 42, normalized size = 1.27

$$\begin{cases} x - \frac{4\sqrt{a+bx}}{b} + \frac{4\log\left(\sqrt{a+bx}+1\right)}{b} & \text{for } b \neq 0 \\ \frac{x\left(\sqrt{a}-1\right)}{\sqrt{a}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)**(1/2))/(1+(b*x+a)**(1/2)),x)
```

```
[Out] Piecewise((x - 4*sqrt(a + b*x)/b + 4*log(sqrt(a + b*x) + 1)/b, Ne(b, 0)), (
x*(sqrt(a) - 1)/(sqrt(a) + 1), True))
```

3.611 $\int \frac{a+bnx^{-1+n}}{ax+bx^n} dx$

Optimal. Leaf size=10

$$\log(ax+bx^n)$$

[Out] ln(a*x+b*x^n)

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.70, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 514, 446, 72}

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]

[Out] n*Log[x] + Log[b + a*x^(1 - n)]

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx &= \int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}}\,dx\\
&= \int \frac{bn+ax^{1-n}}{x\left(b+ax^{1-n}\right)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{bn+ax}{x(b+ax)}\,dx,x,x^{1-n}\right)}{1-n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{n}{x}+\frac{a-an}{b+ax}\right)dx,x,x^{1-n}\right)}{1-n}\\
&= n\log(x)+\log\left(b+ax^{1-n}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 17, normalized size = 1.70

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

fricas [A] time = 0.42, size = 10, normalized size = 1.00

$$\log\left(ax+bx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="fricas")
[Out] log(a*x + b*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{bnx^{n-1}+a}{ax+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="giac")
[Out] integrate((b*n*x^(n - 1) + a)/(a*x + b*x^n), x)
```

maple [A] time = 0.02, size = 13, normalized size = 1.30

$$\ln\left(ax+b\,\mathrm{e}^{n\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*n*x^(n-1))/(a*x+b*x^n),x)
[Out] ln(a*x+b*exp(n*ln(x)))
```

maxima [A] time = 0.87, size = 10, normalized size = 1.00

$$\log\left(ax+bx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="maxima")
[Out] log(a*x + b*x^n)
```

mupad [B] time = 3.27, size = 10, normalized size = 1.00

$$\ln\left(b\,x^n+a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*n*x^(n - 1))/(b*x^n + a*x),x)
[Out] log(b*x^n + a*x)
```

sympy [A] time = 6.96, size = 32, normalized size = 3.20

$$\begin{cases}\log\left(x+\frac{bx^n}{a}\right) & \text{for } a\neq 0\\ n\left(\frac{n^2\log(x)}{n^2-n}-\frac{n\log(x)}{n^2-n}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x**(-1+n))/(a*x+b*x**n),x)
```

```
[Out] Piecewise((log(x + b*x**n/a), Ne(a, 0)), (n*(n**2*log(x)/(n**2 - n) - n*log
(x)/(n**2 - n)), True))
```

3.612 $\int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}}\,dx$

Optimal. Leaf size=17

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

[Out] n*ln(x)+ln(b+a*x^(1-n))

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {514, 446, 72}

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]

[Out] n*Log[x] + Log[b + a*x^(1 - n)]

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}}\,dx &= \int \frac{bn+ax^{1-n}}{x\left(b+ax^{1-n}\right)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{bn+ax}{x(b+ax)}\,dx,x,x^{1-n}\right)}{1-n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{n}{x}+\frac{a-an}{b+ax}\right)\,dx,x,x^{1-n}\right)}{1-n}\\
&= n\log(x)+\log\left(b+ax^{1-n}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

fricas [A] time = 0.41, size = 10, normalized size = 0.59

$$\log (ax + bx^n)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="fricas")
[Out] log(a*x + b*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bnx^{n-1} + a}{\left(ax^{-n+1} + b\right)x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="giac")
[Out] integrate((b*n*x^(n - 1) + a)/((a*x^(-n + 1) + b)*x^n), x)
```

maple [A] time = 0.03, size = 13, normalized size = 0.76

$$\ln\left(ax + b\,\mathrm{e}^{n\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*n*x^(n-1))/(x^n)/(b+a*x^(-n+1)),x)
[Out] ln(a*x+b*exp(n*ln(x)))
```

maxima [B] time = 0.95, size = 86, normalized size = 5.06

$$bn\left(\frac{\log(x)}{b} - \frac{n\log(x)}{b(n-1)} + \frac{\log\left(\frac{ax+bx^n}{b}\right)}{b(n-1)}\right) + a\left(\frac{n\log(x)}{a(n-1)} - \frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="maxima")
[Out] b*n*(log(x)/b - n*log(x)/(b*(n - 1)) + log((a*x + b*x^n)/b)/(b*(n - 1))) +
a*(n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1)))
```

mupad [B] time = 3.37, size = 39, normalized size = 2.29

$$-\frac{\ln\left(b + a\,x^{1-n}\right) - 2\,n\,\mathrm{atanh}\left(\frac{2\,a\,x^{1-n}}{b} + 1\right)}{n-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*n*x^(n - 1))/(x^n*(b + a*x^(1 - n))),x)
[Out] -(log(b + a*x^(1 - n)) - 2*n*atanh((2*a*x^(1 - n))/b + 1))/(n - 1)
```

sympy [A] time = 53.07, size = 8, normalized size = 0.47

$$\log (ax + bx^n)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x**(-1+n))/(x**n)/(b+a*x**(1-n)),x)
[Out] log(a*x + b*x**n)
```

3.613 $\int x\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3b\right.$

Optimal. Leaf size=37

$$x^2\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

```
[Out] x^2*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Rubi [A] time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 176, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {1590}

$$x^2\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d + 3*a
*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*e*n +
2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n +
 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 +
 c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+be\right.$$

Mathematica [A] time = 0.35, size = 34, normalized size = 0.92

$$x^2(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d
 + 3*a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*
e*n + 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b
*f*n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)
*x^4 + c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 38, normalized size = 1.03

$$x^{2} \left(c\,x^{2}+b x +a \right)^{m +1} \left(g\,x^{3}+f\,x^{2}+e x +d \right)^{n +1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e+3*b*d
)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*m+2*b*
f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+6*b
*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x)
```

```
[Out] x^2*(c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)
```

maxima [B] time = 1.76, size = 97, normalized size = 2.62

$$\left(cgx^{7} + \left(cf + bg\right)x^{6} + \left(ce + bf + ag\right)x^{5} + \left(cd + be + af\right)x^{4} + adx^{2} + \left(bd + ae\right)x^{3}\right)e^{\left(n \log\left(gx^{3} + fx^{2} + ex + d\right) + m \log\left(cx^{2} + bx + \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="maxima")
```

```
[Out] (c*g*x^7 + (c*f + b*g)*x^6 + (c*e + b*f + a*g)*x^5 + (c*d + b*e + a*f)*x^4
+ a*d*x^2 + (b*d + a*e)*x^3)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^
2 + b*x + a))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + x^4*(6*b*g +
 6*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(4*a*f + 4*b*e + 4*c*d
+ b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(3*a*e + 3*b*d + b*d*m + a*e*n) +
x^3*(5*a*g + 5*b*f + 5*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) +
 c*g*x^5*(2*m + 3*n + 7)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(2*a*d+(a*e*n+b*d*m+
3*a*e+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x**2+(3*a*g*
n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*
m+2*c*f*n+6*b*g+6*c*f)*x**4+c*g*(7+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

3.614 $\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+$

Optimal. Leaf size=35

$$x\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

```
[Out] x*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Rubi [A] time = 0.10, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 174, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {1590}

$$x\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2*a*e +
 b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*
f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a
*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g
*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(ad+(2bd+2ae+bdm+aen)x+(3cd+3be+3af+2cdm+bem+b\right.$$

Mathematica [A] time = 0.33, size = 32, normalized size = 0.91

$$x(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2
*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n
+ 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n
 + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4
 + c*g*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 36, normalized size = 1.03

$$x\left(c\,x^{2}+bx+a\right)^{m+1}\left(g\,x^{3}+f\,x^{2}+ex+d\right)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b*d)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+
2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5*b*g+5
*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x)
```

```
[Out] x*(c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)
```

maximea [B] time = 1.73, size = 95, normalized size = 2.71

$$\left(cgx^{6}+\left(cf+bg\right)x^{5}+\left(ce+bf+ag\right)x^{4}+\left(cd+be+af\right)x^{3}+adx+(bd+ae)x^{2}\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="maxima")
```

```
[Out] (c*g*x^6 + (c*f + b*g)*x^5 + (c*e + b*f + a*g)*x^4 + (c*d + b*e + a*f)*x^3
+ a*d*x + (b*d + a*e)*x^2)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2
+ b*x + a))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + x^4*(5*b*g + 5*c
*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*f + 3*b*e + 3*c*d + b*
e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(2*a*e + 2*b*d + b*d*m + a*e*n) + x^3*
(4*a*g + 4*b*f + 4*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g
*x^5*(2*m + 3*n + 6)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(a*d+(a*e*n+b*d*m+2*a*
e+2*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x**2+(3*a*g*n+b*
f*m+2*b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*
c*f*n+5*b*g+5*c*f)*x**4+c*g*(6+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

3.615 $\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(bd+ae+b\right.$

Optimal. Leaf size=34

$$\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Rubi [A] time = 0.12, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 164, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {1590}

$$\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*m + a*
e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x + (3*c*
e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2 + (4*c
*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m + 3*n)
*x^4),x]
```

```
[Out] (a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2a\right.$$

Mathematica [A] time = 0.31, size = 31, normalized size = 0.91

$$(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*
m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x +
 (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2
+ (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m
+ 3*n)*x^4),x]
```

```
[Out] (a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 35, normalized size = 1.03

$$\left(c\,x^{2}+bx+a\right)^{m+1}\left(g\,x^{3}+f\,x^{2}+ex+d\right)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f*n+b*e
*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+
3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^3+c*g*
(5+2*m+3*n)*x^4),x)
```

```
[Out] (c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)
```

maxima [B] time = 1.70, size = 92, normalized size = 2.71

$$\left(cgx^{5}+\left(cf+bg\right)x^{4}+\left(ce+bf+ag\right)x^{3}+\left(cd+be+af\right)x^{2}+ad+\left(bd+ae\right)x\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))
```

mupad [B] time = 9.78, size = 148, normalized size = 4.35

$$\left(g\,x^{3}+f\,x^{2}+e\,x+d\right)^{n}\,\left(x^{4}\,\left(b\,g+c\,f\right)\,\left(c\,x^{2}+b\,x+a\right)^{m}+x^{2}\,\left(c\,x^{2}+b\,x+a\right)^{m}\,\left(a\,f+b\,e+c\,d\right)+x^{3}\,\left(c\,x^{2}+b\,x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*e + b*d + x^3*(4*b*g
 + 4*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*g + 3*b*f + 3*c*
e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + x*(2*a*f + 2*b*e + 2*c*d
 + b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + b*d*m + a*e*n + c*g*x^4*(2*m + 3*n
+ 5)),x)
```

```
[Out] (d + e*x + f*x^2 + g*x^3)^n*(x^4*(b*g + c*f)*(a + b*x + c*x^2)^m + x^2*(a +
 b*x + c*x^2)^m*(a*f + b*e + c*d) + x^3*(a + b*x + c*x^2)^m*(a*g + b*f + c*
```

```
e) + a*d*(a + b*x + c*x^2)^m + x*(a*e + b*d)*(a + b*x + c*x^2)^m + c*g*x^5*
(a + b*x + c*x^2)^m)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(b*d+a*e+b*d*m+a*e*n+(
2*a*f*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c
*e*m+c*e*n+3*a*g+3*b*f+3*c*e)*x**2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c
*f)*x**3+c*g*(5+2*m+3*n)*x**4),x)
```

```
[Out] Timed out
```

3.616 $\int \frac{\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(-ad+(bdm+aen)x+(cd+be+af+2cdm+b\right.}{}$

Optimal. Leaf size=37

$$\frac{\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}}{x}$$

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x
```

Rubi [F] time = 3.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2\right.}{}$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m + a*e
*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (2*c*e
+ 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (3*c*f
 + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m + 3*n)*x
^5))/x^2,x]
```

```
[Out] (c*(d + 2*d*m) + b*e*(1 + m + n) + a*f*(1 + 2*n))*Defer[Int][(a + b*x + c*x
^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - a*d*Defer[Int][((a + b*x + c*x^2)^m
*(d + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (b*d*m + a*e*n)*Defer[Int][((a + b*
x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x, x] + (c*e*(2 + 2*m + n) + b*f*
(2 + m + 2*n) + a*g*(2 + 3*n))*Defer[Int][x*(a + b*x + c*x^2)^m*(d + e*x +
f*x^2 + g*x^3)^n, x] + (c*f*(3 + 2*m + 2*n) + b*g*(3 + m + 3*n))*Defer[Int]
[x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(4 + 2*m + 3
*n)*Defer[Int][x^3*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2\right.}{}$$

Mathematica [A] time = 0.81, size = 34, normalized size = 0.92

$$\frac{(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m
 + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (
2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 +
(3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m +
3*n)*x^5))/x^2,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.03, size = 38, normalized size = 1.03

$$\frac{\left(c\,x^{2}+bx+a\right)^{m+1}\left(g\,x^{3}+f\,x^{2}+ex+d\right)^{n+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*f*n+b*
e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+2*a
*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c*g*(4+
2*m+3*n)*x^5)/x^2,x)
```

```
[Out] (c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)/x
```

maxima [B] time = 1.76, size = 95, normalized size = 2.57

$$\frac{\left(cgx^{5}+\left(cf+bg\right)x^{4}+\left(ce+bf+ag\right)x^{3}+\left(cd+be+af\right)x^{2}+ad+\left(bd+ae\right)x\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+bx+\right.\right.}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))/x
```

mupad [B] time = 9.68, size = 37, normalized size = 1.00

$$\frac{\left(c\,x^{2}+b\,x+a\right)^{m+1}\left(g\,x^{3}+f\,x^{2}+e\,x+d\right)^{n+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(3*b*g + 3*c*f +
b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - a*d + x^2*(a*f + b*e + c*d + b*e*m +
 2*c*d*m + 2*a*f*n + b*e*n) + x*(b*d*m + a*e*n) + x^3*(2*a*g + 2*b*f + 2*c*
e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 4))
)/x^2,x)
```

```
[Out] ((a + b*x + c*x^2)^(m + 1)*(d + e*x + f*x^2 + g*x^3)^(n + 1))/x
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-a*d+(a*e*n+b*d*m)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e
*m+c*e*n+2*a*g+2*b*f+2*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f
)*x**4+c*g*(4+2*m+3*n)*x**5)/x**2,x)
```

```
[Out] Timed out
```

3.617 $\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+b}{}$

Optimal. Leaf size=37

$$\frac{(a+bx+cx^2)^{m+1}(d+ex+fx^2+gx^3)^{n+1}}{x^2}$$

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x^2
```

Rubi [F] time = 3.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2}{}$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*d) - a*
e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e + b*f
 + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f + 2*b*g
+ 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5))/x^3,
x]
```

```
[Out] (c*e*(1 + 2*m + n) + b*f*(1 + m + 2*n) + a*g*(1 + 3*n))*Defer[Int][(a + b*x
 + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - 2*a*d*Defer[Int][((a + b*x +
c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^3, x] - (b*d*(1 - m) + a*e*(1 - n))
*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (2*
c*d*m + 2*a*f*n + b*e*(m + n))*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f
*x^2 + g*x^3)^n)/x, x] + (2*c*f*(1 + m + n) + b*g*(2 + m + 3*n))*Defer[Int]
[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(3 + 2*m + 3*n
)*Defer[Int][x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2}{}$$

Mathematica [A] time = 1.15, size = 34, normalized size = 0.92

$$\frac{(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*d
) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e
 + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f +
2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5)
)/x^3,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 38, normalized size = 1.03

$$\frac{\left(c\,x^{2}+bx+a\right)^{m+1}\left(g\,x^{3}+f\,x^{2}+ex+d\right)^{n+1}}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*d)*x+(
2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+a*g+b
*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*m+3*n)
*x^5)/x^3,x)
```

```
[Out] (c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)/x^2
```

maxima [B] time = 1.81, size = 95, normalized size = 2.57

$$\frac{\left(cgx^{5}+\left(cf+bg\right)x^{4}+\left(ce+bf+ag\right)x^{3}+\left(cd+be+af\right)x^{2}+ad+\left(bd+ae\right)x\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+bx+a\right)\right)}}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
 + a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))/x^2
```

mupad [B] time = 9.22, size = 146, normalized size = 3.95

$$\left(c\,x^2+b\,x+a\right)^m\,\left(g\,x^3+f\,x^2+e\,x+d\right)^n\,\left(a\,f+b\,e+c\,d+c\,g\,x^3+a\,g\,x+b\,f\,x+c\,e\,x+b\,g\,x^2+c\,f\,x^2\right)+\frac{\left(a\,e+b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(2*b*g + 2*c*f +
b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - 2*a*d - x*(a*e + b*d - b*d*m - a*e*n
) + x^2*(b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x^3*(a*g + b*f + c*e + b*f*m
+ 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 3)))/x^3,x)
```

```
[Out] (a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*f + b*e + c*d + c*g*x^3
+ a*g*x + b*f*x + c*e*x + b*g*x^2 + c*f*x^2) + ((a*e + b*d)*(a + b*x + c*x^
2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x + (a*d*(a + b*x + c*x^2)^m*(d + e*x + f
*x^2 + g*x^3)^n)/x^2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-2*a*d+(a*e*n+b*d*m-a
*e-b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m
+c*e*n+a*g+b*f+c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x**4+c
*g*(3+2*m+3*n)*x**5)/x**3,x)
```

```
[Out] Timed out
```

3.618 $\int x^3 \left(a + b\sqrt{c + dx}\right)^2 \, dx$

Optimal. Leaf size=185

$$\frac{c^2\left(3a^2-b^2c\right)(c+dx)^2}{2d^4}+\frac{\left(a^2-3b^2c\right)(c+dx)^4}{4d^4}-\frac{c\left(a^2-b^2c\right)(c+dx)^3}{d^4}-\frac{a^2c^3x}{d^3}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}$$

```
[Out] -a^2*c^3*x/d^3-4/3*a*b*c^3*(d*x+c)^(3/2)/d^4+1/2*c^2*(-b^2*c+3*a^2)*(d*x+c)
^2/d^4+12/5*a*b*c^2*(d*x+c)^(5/2)/d^4-c*(-b^2*c+a^2)*(d*x+c)^3/d^4-12/7*a*b
*c*(d*x+c)^(7/2)/d^4+1/4*(-3*b^2*c+a^2)*(d*x+c)^4/d^4+4/9*a*b*(d*x+c)^(9/2)
/d^4+1/5*b^2*(d*x+c)^5/d^4
```

Rubi [A] time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{c^2\left(3a^2-b^2c\right)(c+dx)^2}{2d^4}+\frac{\left(a^2-3b^2c\right)(c+dx)^4}{4d^4}-\frac{c\left(a^2-b^2c\right)(c+dx)^3}{d^4}-\frac{a^2c^3x}{d^3}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c^3*x)/d^3) - (4*a*b*c^3*(c + d*x)^(3/2))/(3*d^4) + (c^2*(3*a^2 - b^
2*c)*(c + d*x)^2)/(2*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (c*(a^2
- b^2*c)*(c + d*x)^3)/d^4 - (12*a*b*c*(c + d*x)^(7/2))/(7*d^4) + ((a^2 - 3*
b^2*c)*(c + d*x)^4)/(4*d^4) + (4*a*b*(c + d*x)^(9/2))/(9*d^4) + (b^2*(c + d
*x)^5)/(5*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^2(-c+x)^3\,dx,x,c+dx\right)}{d^4}$$
$$=\frac{2\,\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)^3\,dx,x,\sqrt{c+dx}\right)}{d^4}$$
$$=\frac{2\,\text{Subst}\left(\int\left(-a^2c^3x-2abc^3x^2-c^2\left(-3a^2+b^2c\right)x^3+6abc^2x^4+3c\left(-a^2+b^2c\right)x^5\right.}{d^4}$$
$$=-\frac{a^2c^3x}{d^3}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{c^2\left(3a^2-b^2c\right)(c+dx)^2}{2d^4}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}-\frac{c\left(a^2\right.}{}$$

Mathematica [A] time = 0.31, size = 88, normalized size = 0.48

$$\frac{a^2x^4}{4}+\frac{4ab\sqrt{c+dx}\left(-16c^4+8c^3dx-6c^2d^2x^2+5cd^3x^3+35d^4x^4\right)}{315d^4}+\frac{1}{20}b^2x^4(5c+4dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*x^4)/4 + (b^2*x^4*(5*c + 4*d*x))/20 + (4*a*b*Sqrt[c + d*x]*(-16*c^4 +
8*c^3*d*x - 6*c^2*d^2*x^2 + 5*c*d^3*x^3 + 35*d^4*x^4))/(315*d^4)
```

fricas [A] time = 0.89, size = 94, normalized size = 0.51

$$\frac{252\,b^2d^5x^5+315\left(b^2c+a^2\right)d^4x^4+16\left(35\,abd^4x^4+5\,abcd^3x^3-6\,abc^2d^2x^2+8\,abc^3dx-16\,abc^4\right)\sqrt{dx+c}}{1260\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(252*b^2*d^5*x^5 + 315*(b^2*c + a^2)*d^4*x^4 + 16*(35*a*b*d^4*x^4 +
5*a*b*c*d^3*x^3 - 6*a*b*c^2*d^2*x^2 + 8*a*b*c^3*d*x - 16*a*b*c^4)*sqrt(d*x
+ c))/d^4
```

giac [A] time = 0.41, size = 151, normalized size = 0.82

$$\frac{252\,b^2d^2x^5+315\,b^2cdx^4+315\,a^2dx^4+\frac{144\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)abc}{d^3}+\frac{16\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c\right.}{}}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/1260*(252*b^2*d^2*x^5 + 315*b^2*c*d*x^4 + 315*a^2*d*x^4 + 144*(5*(d*x + c
)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*
c^3)*a*b*c/d^3 + 16*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x
+ c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b/d^3)/
d
```

maple [A] time = 0.00, size = 78, normalized size = 0.42

$$\frac{a^2x^4}{4}+\left(\frac{1}{5}d\,x^5+\frac{1}{4}c\,x^4\right)b^2+\frac{4\left(-\frac{(dx+c)^{\frac{3}{2}}c^3}{3}+\frac{3(dx+c)^{\frac{5}{2}}c^2}{5}-\frac{3(dx+c)^{\frac{7}{2}}c}{7}+\frac{(dx+c)^{\frac{9}{2}}}{9}\right)ab}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/5*d*x^5+1/4*c*x^4)+4*a*b/d^4*(1/9*(d*x+c)^(9/2)-3/7*c*(d*x+c)^(7/2)+
3/5*c^2*(d*x+c)^(5/2)-1/3*c^3*(d*x+c)^(3/2))+1/4*a^2*x^4
```

maxima [A] time = 0.88, size = 151, normalized size = 0.82

$$\frac{252\,(dx+c)^5b^2 + 560\,(dx+c)^{\frac{9}{2}}ab - 2160\,(dx+c)^{\frac{7}{2}}abc + 3024\,(dx+c)^{\frac{5}{2}}abc^2 - 1680\,(dx+c)^{\frac{3}{2}}abc^3 - 1260\,(dx+c}{1260\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(252*(d*x + c)^5*b^2 + 560*(d*x + c)^(9/2)*a*b - 2160*(d*x + c)^(7/2
)*a*b*c + 3024*(d*x + c)^(5/2)*a*b*c^2 - 1680*(d*x + c)^(3/2)*a*b*c^3 - 126
0*(d*x + c)*a^2*c^3 - 315*(3*b^2*c - a^2)*(d*x + c)^4 + 1260*(b^2*c^2 - a^2
*c)*(d*x + c)^3 - 630*(b^2*c^3 - 3*a^2*c^2)*(d*x + c)^2)/d^4
```

mupad [B] time = 3.37, size = 167, normalized size = 0.90

$$\frac{b^2\,(c+d\,x)^5}{5\,d^4} - \frac{\left(6\,b^2\,c-2\,a^2\right)\,(c+d\,x)^4}{8\,d^4} + \frac{\left(6\,a^2\,c^2-2\,b^2\,c^3\right)\,(c+d\,x)^2}{4\,d^4} - \frac{a^2\,c^3\,x}{d^3} + \frac{4\,a\,b\,(c+d\,x)^{9/2}}{9\,d^4} + \frac{c\,\left(b^2\,c-a^2\right)\,(c+}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^5)/(5*d^4) - ((6*b^2*c - 2*a^2)*(c + d*x)^4)/(8*d^4) + ((6*a
^2*c^2 - 2*b^2*c^3)*(c + d*x)^2)/(4*d^4) - (a^2*c^3*x)/d^3 + (4*a*b*(c + d*
x)^(9/2))/(9*d^4) + (c*(b^2*c - a^2)*(c + d*x)^3)/d^4 - (4*a*b*c^3*(c + d*x
)^(3/2))/(3*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (12*a*b*c*(c + d*
x)^(7/2))/(7*d^4)
```

sympy [A] time = 7.02, size = 139, normalized size = 0.75

$$\begin{cases} \frac{\frac{a^2dx^4}{4} + \frac{4ab\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3} + \frac{3c^2(c+dx)^{\frac{5}{2}}}{5} - \frac{3c(c+dx)^{\frac{7}{2}}}{7} + \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^3} + \frac{2b^2\left(-\frac{c^3(c+dx)^2}{4} + \frac{c^2(c+dx)^3}{2} - \frac{3c(c+dx)^4}{8} + \frac{(c+dx)^5}{10}\right)}{d^3}}{d} & \text{for}\ d \neq 0 \\ \frac{x^4\left(a+b\sqrt{c}\right)^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((a**2*d*x**4/4 + 4*a*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d
*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 2*b**2*(
-c**3*(c + d*x)**2/4 + c**2*(c + d*x)**3/2 - 3*c*(c + d*x)**4/8 + (c + d*x)
**5/10)/d**3)/d, Ne(d, 0)), (x**4*(a + b*sqrt(c))**2/4, True))
```

3.619 $\int x^2 \left(a+b\sqrt{c+dx}\right)^2 dx$

Optimal. Leaf size=138

$$\frac{\left(a^2-2b^2c\right)(c+dx)^3}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}+\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{b^2(c$$

```
[Out] a^2*c^2*x/d^2+4/3*a*b*c^2*(d*x+c)^(3/2)/d^3-1/2*c*(-b^2*c+2*a^2)*(d*x+c)^2/
d^3-8/5*a*b*c*(d*x+c)^(5/2)/d^3+1/3*(-2*b^2*c+a^2)*(d*x+c)^3/d^3+4/7*a*b*(d
*x+c)^(7/2)/d^3+1/4*b^2*(d*x+c)^4/d^3
```

Rubi [A] time = 0.17, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{\left(a^2-2b^2c\right)(c+dx)^3}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}+\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{b^2(c$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*c^2*x)/d^2 + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (c*(2*a^2 - b^2*c)*
(c + d*x)^2)/(2*d^3) - (8*a*b*c*(c + d*x)^(5/2))/(5*d^3) + ((a^2 - 2*b^2*c)
*(c + d*x)^3)/(3*d^3) + (4*a*b*(c + d*x)^(7/2))/(7*d^3) + (b^2*(c + d*x)^4)
/(4*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^2(-c+x)^2dx,x,c+dx\right)}{d^3}$$
$$=\frac{2\,\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)^2dx,x,\sqrt{c+dx}\right)}{d^3}$$
$$=\frac{2\,\text{Subst}\left(\int\left(a^2c^2x+2abc^2x^2+c\left(-2a^2+b^2c\right)x^3-4abcx^4+\left(a^2-2b^2c\right)x^5+2abx^6\right.\right.}{d^3}$$
$$=\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{\left(a^2-2b^2c\right)(}{3d^3}$$

Mathematica [A] time = 0.19, size = 77, normalized size = 0.56

$$\frac{a^2x^3}{3}+\frac{4ab\sqrt{c+dx}\left(8c^3-4c^2dx+3cd^2x^2+15d^3x^3\right)}{105d^3}+\frac{1}{12}b^2x^3(4c+3dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*x^3)/3 + (b^2*x^3*(4*c + 3*d*x))/12 + (4*a*b*Sqrt[c + d*x]*(8*c^3 - 4*
c^2*d*x + 3*c*d^2*x^2 + 15*d^3*x^3))/(105*d^3)
```

fricas [A] time = 0.85, size = 81, normalized size = 0.59

$$\frac{105\,b^2d^4x^4+140\left(b^2c+a^2\right)d^3x^3+16\left(15\,abd^3x^3+3\,abcd^2x^2-4\,abc^2dx+8\,abc^3\right)\sqrt{dx+c}}{420\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/420*(105*b^2*d^4*x^4 + 140*(b^2*c + a^2)*d^3*x^3 + 16*(15*a*b*d^3*x^3 + 3
*a*b*c*d^2*x^2 - 4*a*b*c^2*d*x + 8*a*b*c^3)*sqrt(d*x + c))/d^3
```

giac [A] time = 0.43, size = 127, normalized size = 0.92

$$\frac{105\,b^2d^2x^4+140\,b^2cdx^3+140\,a^2dx^3+\frac{112\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)abc}{d^2}+\frac{48\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-3\right.}{d^2}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/420*(105*b^2*d^2*x^4 + 140*b^2*c*d*x^3 + 140*a^2*d*x^3 + 112*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b*c/d^2 + 48*(5*(d*
x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x
+ c)*c^3)*a*b/d^2)/d
```

maple [A] time = 0.00, size = 66, normalized size = 0.48

$$\frac{a^2x^3}{3}+\left(\frac{1}{4}d\,x^4+\frac{1}{3}c\,x^3\right)b^2+\frac{4\left(\frac{(dx+c)^{\frac{3}{2}}c^2}{3}-\frac{2(dx+c)^{\frac{5}{2}}c}{5}+\frac{(dx+c)^{\frac{7}{2}}}{7}\right)ab}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/4*x^4*d+1/3*c*x^3)+4*a*b/d^3*(1/7*(d*x+c)^(7/2)-2/5*(d*x+c)^(5/2)*c+
1/3*c^2*(d*x+c)^(3/2))+1/3*a^2*x^3
```

maxima [A] time = 0.88, size = 112, normalized size = 0.81

$$\frac{105\,(dx+c)^4b^2+240\,(dx+c)^{\frac{7}{2}}ab-672\,(dx+c)^{\frac{5}{2}}abc+560\,(dx+c)^{\frac{3}{2}}abc^2+420\,(dx+c)a^2c^2-140\left(2\,b^2c-a^2\right.}{420\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/420*(105*(d*x + c)^4*b^2 + 240*(d*x + c)^(7/2)*a*b - 672*(d*x + c)^(5/2)*
a*b*c + 560*(d*x + c)^(3/2)*a*b*c^2 + 420*(d*x + c)*a^2*c^2 - 140*(2*b^2*c
- a^2)*(d*x + c)^3 + 210*(b^2*c^2 - 2*a^2*c)*(d*x + c)^2)/d^3
```

mupad [B] time = 0.06, size = 124, normalized size = 0.90

$$\frac{b^2\,(c+d\,x)^4}{4\,d^3}-\frac{\left(4\,a^2\,c-2\,b^2\,c^2\right)\,(c+d\,x)^2}{4\,d^3}-\frac{\left(4\,b^2\,c-2\,a^2\right)\,(c+d\,x)^3}{6\,d^3}+\frac{a^2\,c^2\,x}{d^2}+\frac{4\,a\,b\,(c+d\,x)^{7/2}}{7\,d^3}+\frac{4\,a\,b\,c^2\,(c+d\,x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^4)/(4*d^3) - ((4*a^2*c - 2*b^2*c^2)*(c + d*x)^2)/(4*d^3) - (
(4*b^2*c - 2*a^2)*(c + d*x)^3)/(6*d^3) + (a^2*c^2*x)/d^2 + (4*a*b*(c + d*x)
^(7/2))/(7*d^3) + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (8*a*b*c*(c + d*x)^
(5/2))/(5*d^3)
```

sympy [A] time = 5.97, size = 110, normalized size = 0.80

$$\begin{cases} \frac{\frac{a^2dx^3}{3}+\frac{4ab\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2}+\frac{2b^2\left(\frac{c^2(c+dx)^2}{4}-\frac{c(c+dx)^3}{3}+\frac{(c+dx)^4}{8}\right)}{d^2}}{d} & \text{for } d\neq 0 \\ \frac{x^3\left(a+b\sqrt{c}\right)^2}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((a**2*d*x**3/3 + 4*a*b*(c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)*
*(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 2*b**2*(c**2*(c + d*x)**2/4 - c*(c +
d*x)**3/3 + (c + d*x)**4/8)/d**2)/d, Ne(d, 0)), (x**3*(a + b*sqrt(c))**2/3,
 True))
```

3.620 $\int x \left(a+b\sqrt{c+dx}\right)^2 \, dx$

Optimal. Leaf size=89

$$\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}-\frac{a^2cx}{d}+\frac{4ab(c+dx)^{5/2}}{5d^2}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

```
[Out] -a^2*c*x/d-4/3*a*b*c*(d*x+c)^(3/2)/d^2+1/2*(-b^2*c+a^2)*(d*x+c)^2/d^2+4/5*a
*b*(d*x+c)^(5/2)/d^2+1/3*b^2*(d*x+c)^3/d^2
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {371, 1398, 772}

$$\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}-\frac{a^2cx}{d}+\frac{4ab(c+dx)^{5/2}}{5d^2}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c*x)/d) - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2) + ((a^2 - b^2*c)*(c + d*
x)^2)/(2*d^2) + (4*a*b*(c + d*x)^(5/2))/(5*d^2) + (b^2*(c + d*x)^3)/(3*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\sqrt{c+dx}\right)^2 \, dx &= \frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^2(-c+x)\,dx,x,c+dx\right)}{d^2}\\ &= \frac{2\,\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}\\ &= \frac{2\,\text{Subst}\left(\int\left(-a^2cx-2abcx^2+\left(a^2-b^2c\right)x^3+2abx^4+b^2x^5\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}\\ &= -\frac{a^2cx}{d}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}+\frac{4ab(c+dx)^{5/2}}{5d^2}+\frac{b^2(c+dx)^3}{3d^2}\end{aligned}$$

Mathematica [A] time = 0.10, size = 63, normalized size = 0.71

$$\frac{1}{30}\left(15a^2x^2+\frac{8ab\sqrt{c+dx}\left(-2c^2+cdx+3d^2x^2\right)}{d^2}+5b^2x^2(3c+2dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (15*a^2*x^2 + 5*b^2*x^2*(3*c + 2*d*x) + (8*a*b*Sqrt[c + d*x]*(-2*c^2 + c*d*
x + 3*d^2*x^2))/d^2)/30
```

fricas [A] time = 0.55, size = 67, normalized size = 0.75

$$\frac{10\,b^2d^3x^3+15\left(b^2c+a^2\right)d^2x^2+8\left(3\,abd^2x^2+abcdx-2\,abc^2\right)\sqrt{dx+c}}{30\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^2*d^3*x^3 + 15*(b^2*c + a^2)*d^2*x^2 + 8*(3*a*b*d^2*x^2 + a*b*c*
d*x - 2*a*b*c^2)*sqrt(d*x + c))/d^2
```

giac [A] time = 0.51, size = 131, normalized size = 1.47

$$\frac{10\,b^2d^2x^3+\frac{40\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)abc}{d}+\frac{15\left((dx+c)^2-2\,(dx+c)c\right)b^2c}{d}+\frac{15\left((dx+c)^2-2\,(dx+c)c\right)a^2}{d}+\frac{8\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)ab}{d}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/30*(10*b^2*d^2*x^3 + 40*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c/d + 1
5*((d*x + c)^2 - 2*(d*x + c)*c)*b^2*c/d + 15*((d*x + c)^2 - 2*(d*x + c)*c)*
a^2/d + 8*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)
*a*b/d)/d
```

maple [A] time = 0.00, size = 54, normalized size = 0.61

$$\frac{a^2x^2}{2}+\left(\frac{1}{3}d\,x^3+\frac{1}{2}c\,x^2\right)b^2+\frac{4\left(-\frac{(dx+c)^{\frac{3}{2}}c}{3}+\frac{(dx+c)^{\frac{5}{2}}}{5}\right)ab}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/3*d*x^3+1/2*c*x^2)+4*a*b/d^2*(1/5*(d*x+c)^(5/2)-1/3*(d*x+c)^(3/2)*c)
+1/2*a^2*x^2
```

maxima [A] time = 0.88, size = 72, normalized size = 0.81

$$\frac{10\,(dx+c)^3b^2+24\,(dx+c)^{\frac{5}{2}}ab-40\,(dx+c)^{\frac{3}{2}}abc-30\,(dx+c)a^2c-15\left(b^2c-a^2\right)(dx+c)^2}{30\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(10*(d*x + c)^3*b^2 + 24*(d*x + c)^(5/2)*a*b - 40*(d*x + c)^(3/2)*a*b*
c - 30*(d*x + c)*a^2*c - 15*(b^2*c - a^2)*(d*x + c)^2)/d^2
```

mupad [**B**] time = 0.03, size = 79, normalized size = 0.89

$$\frac{b^2\,(c+d\,x)^3}{3\,d^2}-\frac{\left(2\,b^2\,c-2\,a^2\right)\,(c+d\,x)^2}{4\,d^2}+\frac{4\,a\,b\,(c+d\,x)^{5/2}}{5\,d^2}-\frac{a^2\,c\,x}{d}-\frac{4\,a\,b\,c\,(c+d\,x)^{3/2}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^3)/(3*d^2) - ((2*b^2*c - 2*a^2)*(c + d*x)^2)/(4*d^2) + (4*a*
b*(c + d*x)^(5/2))/(5*d^2) - (a^2*c*x)/d - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2
)
```

sympy [**A**] time = 4.77, size = 94, normalized size = 1.06

$$\begin{cases} \frac{\frac{2a^2\left(-\frac{c(c+dx)}{2}+\frac{(c+dx)^2}{4}\right)}{d}+\frac{4ab\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{2b^2\left(-\frac{c(c+dx)^2}{4}+\frac{(c+dx)^3}{6}\right)}{d}}{d} & \text{for}\ d\neq 0 \\ \frac{x^2\left(a+b\sqrt{c}\right)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((2*a**2*(-c*(c + d*x)/2 + (c + d*x)**2/4)/d + 4*a*b*(-c*(c + d*x
)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*b**2*(-c*(c + d*x)**2/4 + (c + d*x)*
*3/6)/d)/d, Ne(d, 0)), (x**2*(a + b*sqrt(c))**2/2, True))
```

3.621 $\int \left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal. Leaf size=41

$$a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

```
[Out] a^2*x+4/3*a*b*(d*x+c)^(3/2)/d+1/2*b^2*(d*x+c)^2/d
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 190, 43}

$$a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] a^2*x + (4*a*b*(c + d*x)^(3/2))/(3*d) + (b^2*(c + d*x)^2)/(2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{c+dx}\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^2\,dx,x,c+dx\right)}{d}\\ &= \frac{2\,\text{Subst}\left(\int x(a+bx)^2\,dx,x,\sqrt{c+dx}\right)}{d}\\ &= \frac{2\,\text{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right)\,dx,x,\sqrt{c+dx}\right)}{d}\\ &= a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.98

$$\frac{6a^2dx+8ab(c+dx)^{3/2}+3b^2(c+dx)^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (6*a^2*d*x + 8*a*b*(c + d*x)^(3/2) + 3*b^2*(c + d*x)^2)/(6*d)
```

fricas [A] time = 0.52, size = 49, normalized size = 1.20

$$\frac{3\,b^2d^2x^2+6\left(b^2c+a^2\right)dx+8\,(abdx+abc)\sqrt{dx+c}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*d^2*x^2 + 6*(b^2*c + a^2)*d*x + 8*(a*b*d*x + a*b*c)*sqrt(d*x + c
))/d
```

giac [B] time = 0.36, size = 82, normalized size = 2.00

$$\frac{6\,(dx+c)b^2c+24\,\sqrt{dx+c}\,abc+6\,(dx+c)a^2+8\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)ab+3\left((dx+c)^2-2\,(dx+c)c\right)b^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*b^2*c + 24*sqrt(d*x + c)*a*b*c + 6*(d*x + c)*a^2 + 8*((d*x
 + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b + 3*((d*x + c)^2 - 2*(d*x + c)*c)*b^2)
/d
```

maple [A] time = 0.00, size = 35, normalized size = 0.85

$$a^2x+\left(\frac{1}{2}d\,x^2+cx\right)b^2+\frac{4\,(dx+c)^{\frac{3}{2}}\,ab}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/2*d*x^2+c*x)+4/3*a*b*(d*x+c)^(3/2)/d+a^2*x
```

maxima [A] time = 0.83, size = 35, normalized size = 0.85

$$\frac{1}{2}\left(dx^2+2\,cx\right)b^2+a^2x+\frac{4\,(dx+c)^{\frac{3}{2}}ab}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*b^2 + a^2*x + 4/3*(d*x + c)^(3/2)*a*b/d
```

mupad [B] time = 0.05, size = 36, normalized size = 0.88

$$\frac{3\,b^2\,(c+d\,x)^2+8\,a\,b\,(c+d\,x)^{3/2}+6\,a^2\,d\,x}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (3*b^2*(c + d*x)^2 + 8*a*b*(c + d*x)^(3/2) + 6*a^2*d*x)/(6*d)
```

sympy [A] time = 0.21, size = 68, normalized size = 1.66

$$\begin{cases} a^2x + \frac{4abc\sqrt{c+dx}}{3d} + \frac{4abx\sqrt{c+dx}}{3} + b^2cx + \frac{b^2dx^2}{2} & \text{for } d \neq 0 \\ x\left(a + b\sqrt{c}\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((a**2*x + 4*a*b*c*sqrt(c + d*x)/(3*d) + 4*a*b*x*sqrt(c + d*x)/3 +
 b**2*c*x + b**2*d*x**2/2, Ne(d, 0)), (x*(a + b*sqrt(c))**2, True))
```

3.622 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x} dx$

Optimal. Leaf size=57

$$\log(x)\left(a^2+b^2c\right)+4ab\sqrt{c+dx}-4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+b^2dx$$

```
[Out] b^2*d*x+(b^2*c+a^2)*ln(x)-4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a*
b*(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 801, 635, 207, 260}

$$\log(x)\left(a^2+b^2c\right)+4ab\sqrt{c+dx}-4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+b^2dx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b^2*d*x + 4*a*b*Sqrt[c + d*x] - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]
] + (a^2 + b^2*c)*Log[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx &= \text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{-c+x}\,dx, x, c+dx\right)\\
&= 2\,\text{Subst}\left(\int \frac{x(a+bx)^2}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= 2\,\text{Subst}\left(\int \left(2ab+b^2x+\frac{2abc+\left(a^2+b^2c\right)x}{-c+x^2}\right)dx, x, \sqrt{c+dx}\right)\\
&= b^2dx+4ab\sqrt{c+dx}+2\,\text{Subst}\left(\int \frac{2abc+\left(a^2+b^2c\right)x}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= b^2dx+4ab\sqrt{c+dx}+(4abc)\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)+\left(2\left(a^2+b^2c\right)\right)\text{Sul}\\
&= b^2dx+4ab\sqrt{c+dx}-4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\left(a^2+b^2c\right)\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.16, size = 79, normalized size = 1.39

$$
b\left(4a\sqrt{c+dx}+bdx\right)+\left(a-b\sqrt{c}\right)^2\log\left(\sqrt{c+dx}+\sqrt{c}\right)+\left(a+b\sqrt{c}\right)^2\log\left(\sqrt{c}-\sqrt{c+dx}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b*(b*d*x + 4*a*Sqrt[c + d*x]) + (a + b*Sqrt[c])^2*Log[Sqrt[c] - Sqrt[c + d*
x]] + (a - b*Sqrt[c])^2*Log[Sqrt[c] + Sqrt[c + d*x]]
```

fricas [A] time = 0.45, size = 118, normalized size = 2.07

$$
\left[b^2dx+2\,ab\sqrt{c}\,\log\left(\frac{dx-2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)+4\,\sqrt{dx+c}\,ab+\left(b^2c+a^2\right)\log(x), b^2dx+4\,ab\sqrt{-c}\,\arctan\left(\frac{\sqrt{}}{}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] [b^2*d*x + 2*a*b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 4*s
qrt(d*x + c)*a*b + (b^2*c + a^2)*log(x), b^2*d*x + 4*a*b*sqrt(-c)*arctan(sq
rt(d*x + c)*sqrt(-c)/c) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x)]
```

giac [A] time = 0.42, size = 59, normalized size = 1.04

$$
\frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+(dx+c)b^2+4\,\sqrt{dx+c}\,ab+\left(b^2c+a^2\right)\log\left(dx\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] 4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + (d*x + c)*b^2 + 4*sqrt(d*
x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

maple [A] time = 0.01, size = 51, normalized size = 0.89

$$b^2c\ln(x) + b^2dx - 4ab\sqrt{c}\ \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right) + a^2\ln(x) + 4\sqrt{dx+c}\ ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x,x)
```

```
[Out] ln(x)*b^2*c+b^2*d*x-4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a*b*(d*x
+c)^(1/2)+a^2*ln(x)
```

maxima [A] time = 1.93, size = 70, normalized size = 1.23

$$2\,ab\sqrt{c}\,\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right) + (dx+c)b^2 + 4\sqrt{dx+c}\,ab + \left(b^2c+a^2\right)\log(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] 2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + (d
*x + c)*b^2 + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

mupad [B] time = 0.09, size = 130, normalized size = 2.28

$$\ln\left(\left(2\,a^2+2\,c\,b^2\right)\,\sqrt{c+d\,x}-2\,\left(a+b\,\sqrt{c}\right)^2\,\sqrt{c+d\,x}+4\,a\,b\,c\right)\,\left(a+b\,\sqrt{c}\right)^2+\ln\left(\left(2\,a^2+2\,c\,b^2\right)\,\sqrt{c+d\,x}-2\,\left(a-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2/x,x)
```

```
[Out] log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(a + b*c^(1/2))^2*(c + d*x)^(1/2)
 + 4*a*b*c)*(a + b*c^(1/2))^2 + log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(
a - b*c^(1/2))^2*(c + d*x)^(1/2) + 4*a*b*c)*(a - b*c^(1/2))^2 + 4*a*b*(c +
d*x)^(1/2) + b^2*d*x
```

sympy [A] time = 27.29, size = 65, normalized size = 1.14

$$a^2\log(x) - 2ab\left(-\frac{2c\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} - 2\sqrt{c+dx}\right) + b^2c\log(x) + b^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x,x)
```

```
[Out] a**2*log(x) - 2*a*b*(-2*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) - 2*sqrt(c
+ d*x)) + b**2*c*log(x) + b**2*d*x
```

3.623 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2} dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(x)$$

```
[Out] b^2*d*ln(x)-2*a*b*d*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(1/2)-(a+b*(d*x+c)^(1/
2))^2/x
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 819, 635, 207, 260}

$$-\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] -((a + b*Sqrt[c + d*x])^2/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/Sqr
t[c] + b^2*d*Log[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
```

```
!ILtQ[m + 2*p + 3, 0])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = d\,\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{(-c+x)^2}\,dx, x, c+dx\right)$$
$$= (2d)\,\text{Subst}\left(\int \frac{x(a+bx)^2}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$
$$= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} - \frac{d\,\text{Subst}\left(\int \frac{-2abc-2b^2cx}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)}{c}$$
$$= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} + (2abd)\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right) + \left(2b^2d\right)\text{Subst}\left(\int \frac{}{-c}\right.$$
$$= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} - \frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2d\log(x)$$

Mathematica [B] time = 0.20, size = 161, normalized size = 2.98

$$\frac{\sqrt{c}\left(a^4+2a^3b\sqrt{c+dx}-2ab^3c\sqrt{c+dx}-b^4c(c+2dx)\right)+bdx\left(a+b\sqrt{c}\right)\left(a-b\sqrt{c}\right)^2\log\left(\sqrt{c+dx}+\sqrt{c}\right)+bdx}{\sqrt{c}\,x\left(b^2c-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] (Sqrt[c]*(a^4 + 2*a^3*b*Sqrt[c + d*x] - 2*a*b^3*c*Sqrt[c + d*x] - b^4*c*(c
+ 2*d*x)) + b*(-a + b*Sqrt[c])*(a + b*Sqrt[c])^2*d*x*Log[Sqrt[c] - Sqrt[c +
 d*x]] + b*(a - b*Sqrt[c])^2*(a + b*Sqrt[c])*d*x*Log[Sqrt[c] + Sqrt[c + d*x
]])/(Sqrt[c]*(-a^2 + b^2*c)*x)
```

fricas [A] time = 0.48, size = 147, normalized size = 2.72

$$\left[\frac{b^2cdx\log(x)+ab\sqrt{c}\,dx\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-b^2c^2-2\sqrt{dx+c}\,abc-a^2c}{cx}, \frac{b^2cdx\log(x)+2ab\sqrt{-c}\,dx\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="fricas")
```

```
[Out] [(b^2*c*d*x*log(x) + a*b*sqrt(c)*d*x*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2
*c)/x) - b^2*c^2 - 2*sqrt(d*x + c)*a*b*c - a^2*c)/(c*x), (b^2*c*d*x*log(x)
+ 2*a*b*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqrt(-c)/c) - b^2*c^2 - 2*sqrt(d*
x + c)*a*b*c - a^2*c)/(c*x)]
```

giac [A] time = 0.44, size = 80, normalized size = 1.48

$$\frac{b^2d^2\log(dx) + \frac{2\,abd^2\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{b^2cd^2+2\,\sqrt{dx+c}\,abd^2+a^2d^2}{dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*log(d*x) + 2*a*b*d^2*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) - (b^
2*c*d^2 + 2*sqrt(d*x + c)*a*b*d^2 + a^2*d^2)/(d*x))/d
```

maple [A] time = 0.02, size = 60, normalized size = 1.11

$$b^2d\ln(x) - \frac{2abd\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b^2c}{x} - \frac{a^2}{x} - \frac{2\sqrt{dx+c}\,ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x^2,x)
```

```
[Out] -b^2*c/x+b^2*d*ln(x)-2*a*b/x*(d*x+c)^(1/2)-2*a*b*d*arctanh((d*x+c)^(1/2)/c^
(1/2))/c^(1/2)-a^2/x
```

maxima [A] time = 1.97, size = 73, normalized size = 1.35

$$\left(b^2\log(dx) + \frac{ab\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\sqrt{c}} - \frac{b^2c + 2\,\sqrt{dx+c}\,ab + a^2}{dx}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="maxima")
```

```
[Out] (b^2*log(d*x) + a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))
)/sqrt(c) - (b^2*c + 2*sqrt(d*x + c)*a*b + a^2)/(d*x))*d
```

mupad [B] time = 0.12, size = 131, normalized size = 2.43

$$b\,d\,\ln\left(2\,b\,d\,\left(b+\frac{a}{\sqrt{c}}\right)\,\sqrt{c+d\,x}-2\,b^2\,d\,\sqrt{c+d\,x}-2\,a\,b\,d\right)\,\left(b+\frac{a}{\sqrt{c}}\right)-\frac{a^2\,d+b^2\,c\,d+2\,a\,b\,d\,\sqrt{c+d\,x}}{d\,x}+b\,d\,\mathrm{ln}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2/x^2,x)
```

```
[Out] b*d*log(2*b*d*(b + a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2) - 2
*a*b*d)*(b + a/c^(1/2)) - (a^2*d + b^2*c*d + 2*a*b*d*(c + d*x)^(1/2))/(d*x)
 + b*d*log(2*b*d*(b - a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2)
- 2*a*b*d)*(b - a/c^(1/2))
```

sympy [B] time = 87.77, size = 139, normalized size = 2.57

$$-\frac{a^2}{x}-abcd\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{c+dx}\right)+abcd\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{c+dx}\right)+\frac{4abd\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2ab\sqrt{c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x**2,x)
```

```
[Out] -a**2/x - a*b*c*d*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(c + d*x)) +
a*b*c*d*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(c + d*x)) + 4*a*b*d*ata
n(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) - 2*a*b*sqrt(c + d*x)/x - b**2*c/x + b**
2*d*log(x)
```

3.624 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3} \, dx$

Optimal. Leaf size=80

$$\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}-\frac{bd\left(a\sqrt{c+dx}+bc\right)}{2cx}$$

```
[Out] 1/2*a*b*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(3/2)-1/2*b*d*(b*c+a*(d*x+c)^(
1/2))/c/x-1/2*(a+b*(d*x+c)^(1/2))^2/x^2
```

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 821, 12, 639, 207}

$$\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}-\frac{bd\left(a\sqrt{c+dx}+bc\right)}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] -(b*d*(b*c + a*Sqrt[c + d*x]))/(2*c*x) - (a + b*Sqrt[c + d*x])^2/(2*x^2) +
(a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx &= d^2\,\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{(-c+x)^3}\,dx, x, c+dx\right)\\
&= \left(2d^2\right)\text{Subst}\left(\int \frac{x(a+bx)^2}{\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} - \frac{d^2\,\text{Subst}\left(\int -\frac{2bc(a+bx)}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c}\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} + \left(bd^2\right)\text{Subst}\left(\int \frac{a+bx}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx} - \frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} - \frac{\left(abd^2\right)\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)}{2c}\\
&= -\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx} - \frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} + \frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [B] time = 0.35, size = 221, normalized size = 2.76

$$\frac{-\frac{2\sqrt{c}\left(a^6c+a^5b\sqrt{c+dx}(2c+dx)+a^4b^2\left(-c^2+2cdx+3d^2x^2\right)-2a^3b^3c\sqrt{c+dx}(2c+dx)-a^2b^4c\left(c^2+4cdx+2d^2x^2\right)+ab^5c^2\sqrt{c+dx}(2c+dx)+b^6c^2(c+dx)^2\right)}{x^2\left(a^2-b^2c\right)^2} - ab\dots}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] ((-2*Sqrt[c]*(a^6*c + b^6*c^2*(c + d*x)^2 + a^5*b*Sqrt[c + d*x]*(2*c + d*x)
 - 2*a^3*b^3*c*Sqrt[c + d*x]*(2*c + d*x) + a*b^5*c^2*Sqrt[c + d*x]*(2*c + d
*x) - a^2*b^4*c*(c^2 + 4*c*d*x + 2*d^2*x^2) + a^4*b^2*(-c^2 + 2*c*d*x + 3*d
^2*x^2)))/((a^2 - b^2*c)^2*x^2) - a*b*d^2*Log[Sqrt[c] - Sqrt[c + d*x]] + a*
b*d^2*Log[Sqrt[c] + Sqrt[c + d*x]])/(4*c^(3/2))
```

fricas [A] time = 0.49, size = 181, normalized size = 2.26

$$\left[\frac{ab\sqrt{c}\,d^2x^2\log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c}+2c}{x}\right) - 4b^2c^2dx - 2b^2c^3 - 2a^2c^2 - 2\left(abcdx+2abc^2\right)\sqrt{dx+c}}{4c^2x^2}, -\frac{ab\sqrt{-c}\,d^2x^2\arctan\dots}{\dots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(a*b*sqrt(c)*d^2*x^2*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 4*
b^2*c^2*d*x - 2*b^2*c^3 - 2*a^2*c^2 - 2*(a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x +
```

```
c))/(c^2*x^2), -1/2*(a*b*sqrt(-c)*d^2*x^2*arctan(sqrt(d*x + c)*sqrt(-c)/c)
+ 2*b^2*c^2*d*x + b^2*c^3 + a^2*c^2 + (a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x + c)
)/(c^2*x^2)]
```

giac [A] time = 0.36, size = 105, normalized size = 1.31

$$-\frac{\frac{abd^3\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}+\frac{2\,(dx+c)b^2cd^3-b^2c^2d^3+(dx+c)^{\frac{3}{2}}abd^3+\sqrt{dx+c}\,abcd^3+a^2cd^3}{cd^2x^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d^3*arctan(sqrt(d*x + c)/sqrt(-c))/(sqrt(-c)*c) + (2*(d*x + c)*b^
2*c*d^3 - b^2*c^2*d^3 + (d*x + c)^(3/2)*a*b*d^3 + sqrt(d*x + c)*a*b*c*d^3 +
 a^2*c*d^3)/(c*d^2*x^2))/d
```

maple [A] time = 0.02, size = 81, normalized size = 1.01

$$4\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}+\frac{-\frac{(dx+c)^{\frac{3}{2}}}{8c}-\frac{\sqrt{dx+c}}{8}}{d^2x^2}\right)ab\,d^2+\left(-\frac{d}{x}-\frac{c}{2x^2}\right)b^2-\frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x^3,x)
```

```
[Out] b^2*(-d/x-1/2*c/x^2)+4*a*b*d^2*((-1/8/c*(d*x+c)^(3/2)-1/8*(d*x+c)^(1/2))/x^
2/d^2+1/8/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))-1/2*a^2/x^2
```

maxima [A] time = 1.96, size = 113, normalized size = 1.41

$$-\frac{1}{4}\left(\frac{ab\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(2\,(dx+c)b^2c-b^2c^2+(dx+c)^{\frac{3}{2}}ab+\sqrt{dx+c}\,abc+a^2c\right)}{(dx+c)^2c-2\,(dx+c)c^2+c^3}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/c^(3/2)
+ 2*(2*(d*x + c)*b^2*c - b^2*c^2 + (d*x + c)^(3/2)*a*b + sqrt(d*x + c)*a*b*
c + a^2*c)/((d*x + c)^2*c - 2*(d*x + c)*c^2 + c^3))*d^2
```

mupad [B] time = 3.40, size = 80, normalized size = 1.00

$$\frac{a\,b\,d^2\operatorname{atanh}\left(\frac{\sqrt{c+d\,x}}{\sqrt{c}}\right)}{2\,c^{3/2}}-\frac{b^2\,c}{2\,x^2}-\frac{b^2\,d}{x}-\frac{a\,b\,\sqrt{c+d\,x}}{2\,x^2}-\frac{a\,b\,{\left(c+d\,x\right)}^{3/2}}{2\,c\,x^2}-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2/x^3,x)
```

```
[Out] (a*b*d^2*atanh((c + d*x)^(1/2)/c^(1/2)))/(2*c^(3/2)) - (b^2*c)/(2*x^2) - (b
^2*d)/x - (a*b*(c + d*x)^(1/2))/(2*x^2) - (a*b*(c + d*x)^(3/2))/(2*c*x^2) -
 a^2/(2*x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x**3,x)
```

```
[Out] Timed out
```

3.625 $\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=326

$$\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{12a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c}\cdots\right.}{7b^8d^4}$$

```
[Out] -4/3*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(3/2)/b^8/d^4+4/5*(-b^2*c+a^2)^2*
(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^8/d^4-12/7*a*(-3*b^2*c+7*a^2)*(-
b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(7/2)/b^8/d^4+4/9*(3*b^4*c^2-30*a^2*b^2*c+35
*a^4)*(a+b*(d*x+c)^(1/2))^(9/2)/b^8/d^4-20/11*a*(-3*b^2*c+7*a^2)*(a+b*(d*x+
c)^(1/2))^(11/2)/b^8/d^4+12/13*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)/b^
8/d^4-28/15*a*(a+b*(d*x+c)^(1/2))^(15/2)/b^8/d^4+4/17*(a+b*(d*x+c)^(1/2))^(
17/2)/b^8/d^4
```

Rubi [A] time = 0.24, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-30a^2b^2c+35a^4+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}+\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\cdots\right.}{11b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) + (4*(a^2 -
b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) - (12*a*(
7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4) + (
4*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d
^4) - (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) +
(12*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^8*d^4) - (28*a*(a +
 b*Sqrt[c + d*x])^(15/2))/(15*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(17/2))/(
17*b^8*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\,(-c+x)^3\,dx, x, c+dx\right)}{d^4}$$
$$= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\,\left(-c+x^2\right)^3\,dx, x, \sqrt{c+dx}\right)}{d^4}$$
$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{a(a^2-b^2c)^3\sqrt{a+bx}}{b^7} - \frac{(-7a^2+b^2c)(-a^2+b^2c)^2(a+bx)^{3/2}}{b^7} - \frac{3(7a^5-10a^3b^2c+3ab^4c^2)(a+bx)^{5/2}}{b^7}\right.\right.}{}$$
$$= -\frac{4a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} + \frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4} - 12$$

Mathematica [A] time = 0.42, size = 232, normalized size = 0.71

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-14336a^7+21504a^6b\sqrt{c+dx}+3840a^5b^2(10c-7dx)-640a^4b^3(104c-49dx)\sqrt{c+dx}-4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-14336*a^7 + 3840*a^5*b^2*(10*c - 7*d*x) +
21504*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(104*c - 49*d*x)*Sqrt[c + d*x] - 48
*a^3*b^4*(616*c^2 - 1080*c*d*x + 735*d^2*x^2) + 24*a^2*b^5*Sqrt[c + d*x]*(2
960*c^2 - 2716*c*d*x + 1617*d^2*x^2) + 6*a*b^6*(320*c^3 - 3936*c^2*d*x + 57
54*c*d^2*x^2 - 7007*d^3*x^3) - 231*b^7*Sqrt[c + d*x]*(128*c^3 - 160*c^2*d*x
 + 180*c*d^2*x^2 - 195*d^3*x^3)))/(765765*b^8*d^4)
```

fricas [A] time = 0.54, size = 286, normalized size = 0.88

$$\frac{4\left(45045\,b^8d^4x^4-29568\,b^8c^4+72960\,a^2b^6c^3-96128\,a^4b^4c^2+59904\,a^6b^2c-14336\,a^8+231\left(15\,b^8c-14\,a^2b^6\right)d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/765765*(45045*b^8*d^4*x^4 - 29568*b^8*c^4 + 72960*a^2*b^6*c^3 - 96128*a^4
*b^4*c^2 + 59904*a^6*b^2*c - 14336*a^8 + 231*(15*b^8*c - 14*a^2*b^6)*d^3*x^
3 - 28*(165*b^8*c^2 - 291*a^2*b^6*c + 140*a^4*b^4)*d^2*x^2 + 32*(231*b^8*c^
3 - 555*a^2*b^6*c^2 + 520*a^4*b^4*c - 168*a^6*b^2)*d*x + (3003*a*b^7*d^3*x^
3 - 27648*a*b^7*c^3 + 41472*a^3*b^5*c^2 - 28160*a^5*b^3*c + 7168*a^7*b - 35
28*(2*a*b^7*c - a^3*b^5)*d^2*x^2 + 32*(417*a*b^7*c^2 - 417*a^3*b^5*c + 140*
a^5*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

giac [B] time = 0.63, size = 915, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/765765*(17*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(
d*x + c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 810
81*(sqrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/
```

```
2)*a^2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt
(d*x + c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c +
 193050*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b + a
)^(5/2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135*s
qrt(sqrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) + 24
255*(sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)*a^2
 + 175175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b + a)^(7
/2)*a^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x + c)*b
+ a)^(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)*a/(b^7*d^3) + (153153
*(sqrt(d*x + c)*b + a)^(5/2)*b^6*c^3 - 510510*(sqrt(d*x + c)*b + a)^(3/2)*a
*b^6*c^3 + 765765*sqrt(sqrt(d*x + c)*b + a)*a^2*b^6*c^3 - 255255*(sqrt(d*x
+ c)*b + a)^(9/2)*b^4*c^2 + 1312740*(sqrt(d*x + c)*b + a)^(7/2)*a*b^4*c^2 -
 2756754*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^4*c^2 + 3063060*(sqrt(d*x + c)*b
 + a)^(3/2)*a^3*b^4*c^2 - 2297295*sqrt(sqrt(d*x + c)*b + a)*a^4*b^4*c^2 + 1
76715*(sqrt(d*x + c)*b + a)^(13/2)*b^2*c - 1253070*(sqrt(d*x + c)*b + a)^(1
1/2)*a*b^2*c + 3828825*(sqrt(d*x + c)*b + a)^(9/2)*a^2*b^2*c - 6563700*(sqr
t(d*x + c)*b + a)^(7/2)*a^3*b^2*c + 6891885*(sqrt(d*x + c)*b + a)^(5/2)*a^4
*b^2*c - 4594590*(sqrt(d*x + c)*b + a)^(3/2)*a^5*b^2*c + 2297295*sqrt(sqrt(
d*x + c)*b + a)*a^6*b^2*c - 45045*(sqrt(d*x + c)*b + a)^(17/2) + 408408*(sq
rt(d*x + c)*b + a)^(15/2)*a - 1649340*(sqrt(d*x + c)*b + a)^(13/2)*a^2 + 38
98440*(sqrt(d*x + c)*b + a)^(11/2)*a^3 - 5955950*(sqrt(d*x + c)*b + a)^(9/2
)*a^4 + 6126120*(sqrt(d*x + c)*b + a)^(7/2)*a^5 - 4288284*(sqrt(d*x + c)*b
+ a)^(5/2)*a^6 + 2042040*(sqrt(d*x + c)*b + a)^(3/2)*a^7 - 765765*sqrt(sqrt
(d*x + c)*b + a)*a^8)/(b^7*d^3))/(b*d)
```

maple [A] time = 0.01, size = 383, normalized size = 1.17

$$-\frac{28\left(a+\sqrt{dx+c}\,b\right)^{\frac{15}{2}}a}{15}-\frac{4\left(-b^2c+a^2\right)^3\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}}a}{3}+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{17}{2}}}{17}+\frac{4\left(-3b^2c+21a^2\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{13}{2}}}{13}+\frac{4\left(-8\left(-b^2c+a^2\right)a-2\left(-2b^2c+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^4/b^8*(1/17*(a+b*(d*x+c)^(1/2))^(17/2)-7/15*a*(a+b*(d*x+c)^(1/2))^(15/2
)+1/13*(-3*b^2*c+21*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)+1/11*(-8*(-b^2*c+a^2)*a
-2*a*(-2*b^2*c+6*a^2)-(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9*(
(-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2-(-8*(-b^2*c
+a^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(-6*(-b^2*c+
a^2)^2*a-((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2)*
a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*((-b^2*c+a^2)^3+6*(-b^2*c+a^2)^2*a^2)*(a+b
*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(3/2))
```

maxima [A] time = 0.96, size = 268, normalized size = 0.82

$$4\left(45045\left(\sqrt{dx+c}\,b+a\right)^{\frac{17}{2}}-357357\left(\sqrt{dx+c}\,b+a\right)^{\frac{15}{2}}a-176715\left(b^2c-7a^2\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}+348075\left(3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/765765*(45045*(sqrt(d*x + c)*b + a)^(17/2) - 357357*(sqrt(d*x + c)*b + a)
^(15/2)*a - 176715*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(13/2) + 348075*(3
*a*b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(11/2) + 85085*(3*b^4*c^2 - 30*a^2*
b^2*c + 35*a^4)*(sqrt(d*x + c)*b + a)^(9/2) - 328185*(3*a*b^4*c^2 - 10*a^3*
b^2*c + 7*a^5)*(sqrt(d*x + c)*b + a)^(7/2) - 153153*(b^6*c^3 - 9*a^2*b^4*c^
2 + 15*a^4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(5/2) + 255255*(a*b^6*c^3 -
 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*(sqrt(d*x + c)*b + a)^(3/2))/(b^8*d^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sqrt{a + b \sqrt{c + d x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^3*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a + b\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*sqrt(c + d*x)), x)
```

3.626 $\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=224

$$\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{4a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4\left(5a^4\right.}{}$$

```
[Out] -4/3*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(3/2)/b^6/d^3+4/5*(b^4*c^2-6*a^2*
b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(5/2)/b^6/d^3-8/7*a*(-3*b^2*c+5*a^2)*(a+b*
(d*x+c)^(1/2))^(7/2)/b^6/d^3+8/9*(-b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)/b
^6/d^3-20/11*a*(a+b*(d*x+c)^(1/2))^(11/2)/b^6/d^3+4/13*(a+b*(d*x+c)^(1/2))^
(13/2)/b^6/d^3
```

Rubi [A] time = 0.16, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}+\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right.}{7b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) + (4*(5*a^4
 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) - (8*a*(5
*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) + (8*(5*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(
11/2))/(11*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^6*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int\sqrt{a+b\sqrt{x}}\,(-c+x)^2\,dx,x,c+dx\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\,\left(-c+x^2\right)^2\,dx,x,\sqrt{c+dx}\right)}{d^3}$$
$$= \frac{2\,\text{Subst}\left(\int\left(-\frac{a\left(a^2-b^2c\right)^2\sqrt{a+bx}}{b^5}+\frac{\left(5a^4-6a^2b^2c+b^4c^2\right)(a+bx)^{3/2}}{b^5}-\frac{2\left(5a^3-3ab^2c\right)(a+bx)^{5/2}}{b^5}-\frac{2\left(-5a^2+b^2\right.}{b}\right.\right.}{d^3}$$
$$= -\frac{4a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{8a}{}$$

Mathematica [A] time = 0.24, size = 147, normalized size = 0.66

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-1280a^5+1920a^4b\sqrt{c+dx}+32a^3b^2(68c-75dx)+16a^2b^3\sqrt{c+dx}\,(175dx-254c)-6ab^4\right.}{45045b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-1280*a^5 + 32*a^3*b^2*(68*c - 75*d*x) + 19
20*a^4*b*Sqrt[c + d*x] + 16*a^2*b^3*Sqrt[c + d*x]*(-254*c + 175*d*x) + 77*b
^5*Sqrt[c + d*x]*(32*c^2 - 40*c*d*x + 45*d^2*x^2) - 6*a*b^4*(96*c^2 - 380*c
*d*x + 525*d^2*x^2)))/(45045*b^6*d^3)
```

fricas [A] time = 0.52, size = 184, normalized size = 0.82

$$\frac{4\left(3465\,b^6d^3x^3+2464\,b^6c^3-4640\,a^2b^4c^2+4096\,a^4b^2c-1280\,a^6+35\left(11\,b^6c-10\,a^2b^4\right)d^2x^2-8\left(77\,b^6c^2-127\,a\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/45045*(3465*b^6*d^3*x^3 + 2464*b^6*c^3 - 4640*a^2*b^4*c^2 + 4096*a^4*b^2*
c - 1280*a^6 + 35*(11*b^6*c - 10*a^2*b^4)*d^2*x^2 - 8*(77*b^6*c^2 - 127*a^2
*b^4*c + 60*a^4*b^2)*d*x + (315*a*b^5*d^2*x^2 + 1888*a*b^5*c^2 - 1888*a^3*b
^3*c + 640*a^5*b - 400*(2*a*b^5*c - a^3*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(
d*x + c)*b + a)/(b^6*d^3)
```

giac [B] time = 0.46, size = 549, normalized size = 2.45

$$4\left(\frac{13\left(1155\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^4c^2-3465\sqrt{\sqrt{dx+c}\,b+a}\,ab^4c^2-990\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}b^2c+4158\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}ab^2c-6930\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^2b^2c+6930\sqrt{\sqrt{dx+}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/45045*(13*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x
+ c)*b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(
d*x + c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c
+ 6930*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/
```

```
2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)*
a^2 - 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3/
2)*a^4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)*a/(b^5*d^2) + (9009*(sqrt(d*x
+ c)*b + a)^(5/2)*b^4*c^2 - 30030*(sqrt(d*x + c)*b + a)^(3/2)*a*b^4*c^2 + 4
5045*sqrt(sqrt(d*x + c)*b + a)*a^2*b^4*c^2 - 10010*(sqrt(d*x + c)*b + a)^(9
/2)*b^2*c + 51480*(sqrt(d*x + c)*b + a)^(7/2)*a*b^2*c - 108108*(sqrt(d*x +
c)*b + a)^(5/2)*a^2*b^2*c + 120120*(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^2*c -
90090*sqrt(sqrt(d*x + c)*b + a)*a^4*b^2*c + 3465*(sqrt(d*x + c)*b + a)^(13/
2) - 24570*(sqrt(d*x + c)*b + a)^(11/2)*a + 75075*(sqrt(d*x + c)*b + a)^(9/
2)*a^2 - 128700*(sqrt(d*x + c)*b + a)^(7/2)*a^3 + 135135*(sqrt(d*x + c)*b +
 a)^(5/2)*a^4 - 90090*(sqrt(d*x + c)*b + a)^(3/2)*a^5 + 45045*sqrt(sqrt(d*x
 + c)*b + a)*a^6)/(b^5*d^2))/(b*d)
```

maple [A] time = 0.00, size = 183, normalized size = 0.82

$$\frac{-\frac{20\left(a+\sqrt{dx+c}\,b\right)^{\frac{11}{2}}a}{11}-\frac{4\left(-b^2c+a^2\right)^2\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}}a}{3}+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{13}{2}}}{13}+\frac{4\left(-2b^2c+10a^2\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{9}{2}}}{9}+\frac{4\left(-4\left(-b^2c+a^2\right)a-\left(-2b^2c+6a\right.\right.}{7}}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d^3/b^6*(1/13*(a+(d*x+c)^(1/2)*b)^(13/2)-5/11*a*(a+(d*x+c)^(1/2)*b)^(11/2
)+1/9*(-2*b^2*c+10*a^2)*(a+(d*x+c)^(1/2)*b)^(9/2)+1/7*(-4*(-b^2*c+a^2)*a-(-
2*b^2*c+6*a^2)*a)*(a+(d*x+c)^(1/2)*b)^(7/2)+1/5*((-b^2*c+a^2)^2+4*(-b^2*c+a
^2)*a^2)*(a+(d*x+c)^(1/2)*b)^(5/2)-1/3*(-b^2*c+a^2)^2*a*(a+(d*x+c)^(1/2)*b)
^(3/2))
```

maxima [A] time = 0.94, size = 167, normalized size = 0.75

$$\frac{4\left(3465\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}-20475\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}a-10010\left(b^2c-5a^2\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}+12870\left(3ab^2c-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/45045*(3465*(sqrt(d*x + c)*b + a)^(13/2) - 20475*(sqrt(d*x + c)*b + a)^(1
1/2)*a - 10010*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(9/2) + 12870*(3*a*b^2
*c - 5*a^3)*(sqrt(d*x + c)*b + a)^(7/2) + 9009*(b^4*c^2 - 6*a^2*b^2*c + 5*a
^4)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(sq
rt(d*x + c)*b + a)^(3/2))/(b^6*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{a+b\,\sqrt{c+d\,x}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^2*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*sqrt(c + d*x)), x)
```

3.627 $\int x\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=133

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{4a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

```
[Out] -4/3*a*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^4/d^2+4/5*(-b^2*c+3*a^2)*(a
+b*(d*x+c)^(1/2))^(5/2)/b^4/d^2-12/7*a*(a+b*(d*x+c)^(1/2))^(7/2)/b^4/d^2+4/
9*(a+b*(d*x+c)^(1/2))^(9/2)/b^4/d^2
```

Rubi [A] time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{4a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) + (4*(3*a^2 -
b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x
])^(7/2))/(7*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^4*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\,(-c+x)\,dx, x, c+dx\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\,\left(-c+x^2\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int\left(\frac{\left(-a^3+ab^2c\right)\sqrt{a+bx}}{b^3}+\frac{\left(3a^2-b^2c\right)(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx, x, \sqrt{c+dx}\right.}{d^2}$$
$$= -\frac{4a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{12a\left(a+b\sqrt{\right.}}{7b^4d}$$

Mathematica [A] time = 0.14, size = 84, normalized size = 0.63

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-16a^3+24a^2b\sqrt{c+dx}+6ab^2(2c-5dx)+7b^3\sqrt{c+dx}\,(5dx-4c)\right)}{315b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-16*a^3 + 6*a*b^2*(2*c - 5*d*x) + 24*a^2*b*
Sqrt[c + d*x] + 7*b^3*Sqrt[c + d*x]*(-4*c + 5*d*x)))/(315*b^4*d^2)
```

fricas [A] time = 0.56, size = 103, normalized size = 0.77

$$\frac{4\left(35\,b^4d^2x^2-28\,b^4c^2+36\,a^2b^2c-16\,a^4+\left(7\,b^4c-6\,a^2b^2\right)dx+\left(5\,ab^3dx-16\,ab^3c+8\,a^3b\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+}}}{315\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/315*(35*b^4*d^2*x^2 - 28*b^4*c^2 + 36*a^2*b^2*c - 16*a^4 + (7*b^4*c - 6*a
^2*b^2)*d*x + (5*a*b^3*d*x - 16*a*b^3*c + 8*a^3*b)*sqrt(d*x + c))*sqrt(sqrt
(d*x + c)*b + a)/(b^4*d^2)
```

giac [B] time = 0.37, size = 279, normalized size = 2.10

$$-\frac{4\left(\frac{3\left(35\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^2c-105\sqrt{\sqrt{dx+c}\,b+a}\,ab^2c-15\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}+63\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a-105\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^2+105\sqrt{\sqrt{dx+c}\,b+a}\,a^3\right)a}{b^3d}+\frac{6}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/315*(3*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b
+ a)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5
/2)*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)
*a^3)*a/(b^3*d) + (63*(sqrt(d*x + c)*b + a)^(5/2)*b^2*c - 210*(sqrt(d*x + c
)*b + a)^(3/2)*a*b^2*c + 315*sqrt(sqrt(d*x + c)*b + a)*a^2*b^2*c - 35*(sqrt
(d*x + c)*b + a)^(9/2) + 180*(sqrt(d*x + c)*b + a)^(7/2)*a - 378*(sqrt(d*x
+ c)*b + a)^(5/2)*a^2 + 420*(sqrt(d*x + c)*b + a)^(3/2)*a^3 - 315*sqrt(sqrt
(d*x + c)*b + a)*a^4)/(b^3*d))/(b*d)
```

maple [A] time = 0.00, size = 94, normalized size = 0.71

$$\frac{-\frac{12\left(a+\sqrt{dx+c}\,b\right)^{\frac{7}{2}}a}{7}-\frac{4\left(-b^{2}c+a^{2}\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}}a}{3}+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{9}{2}}}{9}+\frac{4\left(-b^{2}c+3a^{2}\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{5}{2}}}{5}}{b^{4}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d^2/b^4*(1/9*(a+(d*x+c)^(1/2)*b)^(9/2)-3/7*a*(a+(d*x+c)^(1/2)*b)^(7/2)+1/
5*(-b^2*c+3*a^2)*(a+(d*x+c)^(1/2)*b)^(5/2)-1/3*(-b^2*c+a^2)*a*(a+(d*x+c)^(1
/2)*b)^(3/2))
```

maxima [A] time = 0.94, size = 93, normalized size = 0.70

$$\frac{4\left(35\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}-135\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a-63\left(b^{2}c-3\,a^{2}\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}+105\left(ab^{2}c-a^{3}\right)\left(\sqrt{dx+c}\,b+\right.\right.}{315\,b^{4}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/315*(35*(sqrt(d*x + c)*b + a)^(9/2) - 135*(sqrt(d*x + c)*b + a)^(7/2)*a -
 63*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(5/2) + 105*(a*b^2*c - a^3)*(sqrt
(d*x + c)*b + a)^(3/2))/(b^4*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*sqrt(c + d*x)), x)
```

3.628 $\int \sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=56

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

```
[Out] -4/3*a*(a+b*(d*x+c)^(1/2))^(3/2)/b^2/d+4/5*(a+b*(d*x+c)^(1/2))^(5/2)/b^2/d
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {247, 190, 43}

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2*d) + (4*(a + b*Sqrt[c + d*x])^(5/
2))/(5*b^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\sqrt{c+dx}}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\,dx,x,c+dx\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int x\sqrt{a+bx}\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx,x,\sqrt{c+dx}\right)}{d}\\
&= -\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.77

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(3b\sqrt{c+dx}-2a\right)}{15b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-2*a + 3*b*Sqrt[c + d*x]))/(15*b^2*d)
```

fricas [A] time = 0.52, size = 50, normalized size = 0.89

$$\frac{4\left(3\,b^2dx+3\,b^2c+\sqrt{dx+c}\,ab-2\,a^2\right)\sqrt{\sqrt{dx+c}\,b+a}}{15\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/15*(3*b^2*d*x + 3*b^2*c + sqrt(d*x + c)*a*b - 2*a^2)*sqrt(sqrt(d*x + c)*b
 + a)/(b^2*d)
```

giac [B] time = 0.36, size = 99, normalized size = 1.77

$$\frac{4\left(\frac{5\left(\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}-3\,\sqrt{\sqrt{dx+c}\,b+a}\,a\right)a}{b}+\frac{3\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}-10\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a+15\,\sqrt{\sqrt{dx+c}\,b+a}\,a^2}{b}\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(5*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)*a/b +
 (3*(sqrt(d*x + c)*b + a)^(5/2) - 10*(sqrt(d*x + c)*b + a)^(3/2)*a + 15*sqr
t(sqrt(d*x + c)*b + a)*a^2)/b)/(b*d)
```

maple [A] time = 0.00, size = 41, normalized size = 0.73

$$\frac{-\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}}a}{3}+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{5}{2}}}{5}}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d/b^2*(1/5*(a+(d*x+c)^(1/2)*b)^(5/2)-1/3*a*(a+(d*x+c)^(1/2)*b)^(3/2))
```

maxima [A] time = 0.91, size = 43, normalized size = 0.77

$$\frac{4\left(\frac{3\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/15*(3*(sqrt(d*x + c)*b + a)^(5/2)/b^2 - 5*(sqrt(d*x + c)*b + a)^(3/2)*a/b
^2)/d
```

mupad [B] time = 3.39, size = 44, normalized size = 0.79

$$\frac{4\left(a+b\sqrt{c+d\,x}\right)^{5/2}}{5\,b^2\,d}-\frac{4\,a\left(a+b\sqrt{c+d\,x}\right)^{3/2}}{3\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] (4*(a + b*(c + d*x)^(1/2))^(5/2))/(5*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))^
(3/2))/(3*b^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x)), x)
```

3.629 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$

Optimal. Leaf size=116

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\ \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\ \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

```
[Out] -2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(a-b*c^(1/2))^(1/
2)-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))*(a+b*c^(1/2))^(
1/2)+4*(a+b*(d*x+c)^(1/2))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {371, 1398, 825, 827, 1166, 207}

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\ \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\ \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]*ArcTanh[Sqrt[a + b*
Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{-c+x}\,dx, x, c+dx\right)\\
&= 2\,\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= 4\sqrt{a+b\sqrt{c+dx}} + 2\,\text{Subst}\left(\int \frac{bc+ax}{\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)\\
&= 4\sqrt{a+b\sqrt{c+dx}} + 4\,\text{Subst}\left(\int \frac{-a^2+b^2c+ax^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&= 4\sqrt{a+b\sqrt{c+dx}} + \left(2\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&= 4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(
\end{aligned}$$

Mathematica [A] time = 0.13, size = 116, normalized size = 1.00

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]*ArcTanh[Sqrt[a + b*
Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

fricas [B] time = 0.54, size = 194, normalized size = 1.67

$$-\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}\,b+a}+2\sqrt{a+\sqrt{b^2c}}\right)+\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}\,b+a}-2\sqrt{a+\sqrt{b^2c}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*sqrt(a + sqrt(b^
2*c))) + sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) - 2*sqrt(a +
 sqrt(b^2*c))) - sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*
sqrt(a - sqrt(b^2*c))) + sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b +
 a) - 2*sqrt(a - sqrt(b^2*c))) + 4*sqrt(sqrt(d*x + c)*b + a)
```

giac [A] time = 0.49, size = 150, normalized size = 1.29

$$\frac{2\left(2\sqrt{\sqrt{dx+c}\,b+a}\,b-\frac{\left(b^3c-a^2b\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}+a\right)\sqrt{b\sqrt{c}-a}}+\frac{\left(b^3c-a^2b\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}-a\right)\sqrt{-b\sqrt{c}-a}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*(2*sqrt(sqrt(d*x + c)*b + a)*b - (b^3*c - a^2*b)*arctan(sqrt(sqrt(d*x + c
)*b + a)/sqrt(-a + sqrt(b^2*c)))/((b*sqrt(c) + a)*sqrt(b*sqrt(c) - a)) + (b
^3*c - a^2*b)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*
sqrt(c) - a)*sqrt(-b*sqrt(c) - a)))/b
```

maple [B] time = 0.04, size = 221, normalized size = 1.91

$$\frac{2b^2c\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\sqrt{b^2c}\sqrt{-a-\sqrt{b^2c}}}-\frac{2b^2c\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\sqrt{b^2c}\sqrt{-a+\sqrt{b^2c}}}+\frac{2a\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\sqrt{-a-\sqrt{b^2c}}}+\frac{2a\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\sqrt{-a+\sqrt{b^2c}}}+4\sqrt{a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^(1/2)*b)^(1/2)/x,x)
```

```
[Out] 4*(a+(d*x+c)^(1/2)*b)^(1/2)+2/(b^2*c)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan
((a+(d*x+c)^(1/2)*b)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))*b^2*c+2/(-(b^2*c)^(1/2
)-a)^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))*a-2/(
b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/((b^2
*c)^(1/2)-a)^(1/2))*b^2*c+2/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+(d*x+c)^(1/2)
*b)^(1/2)/((b^2*c)^(1/2)-a)^(1/2))*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sqrt{dx+c}\,b+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{a+b\sqrt{c+d\,x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x,x)
```

```
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x, x)
```

3.630 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$

Optimal. Leaf size=137

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}$$

```
[Out] 1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/c^(1/2)/(a-b
*c^(1/2))^(1/2)-1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/
2))/c^(1/2)/(a+b*c^(1/2))^(1/2)-(a+b*(d*x+c)^(1/2))^(1/2)/x
```

Rubi [A] time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {371, 1398, 821, 12, 708, 1093, 207}

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + b*Sqrt[c + d*x]]/x) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqr
t[a - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTanh[Sqrt[a +
b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]]*Sqrt[c])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
```

```
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx &= d\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{(-c+x)^2}\,dx, x, c+dx\right)\\
&= (2d)\,\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} - \frac{d\,\text{Subst}\left(\int -\frac{bc}{2\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{c}\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{1}{2}(bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \left(b^2d\right)\text{Subst}\left(\int \frac{1}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{(bd)\,\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{2\sqrt{c}} - \frac{(bd)\,\text{Subst}\left(\right.}{}\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}\sqrt{c}} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.18, size = 181, normalized size = 1.32

$$\frac{\left(a-b\sqrt{c}\right)\left(2\sqrt{c}\left(a+b\sqrt{c}\right)\sqrt{a+b\sqrt{c+dx}} + bdx\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)\right) - bdx\sqrt{a-b\sqrt{c}}\left(a+b\sqrt{c}\right.}{2\sqrt{c}\,x\left(b^2c-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] (-(b*Sqrt[a - b*Sqrt[c]]*(a + b*Sqrt[c])*d*x*ArcTanh[Sqrt[a + b*Sqrt[c + d*
x]]/Sqrt[a - b*Sqrt[c]]]) + (a - b*Sqrt[c])*(2*(a + b*Sqrt[c])*Sqrt[c]*Sqrt
[a + b*Sqrt[c + d*x]] + b*Sqrt[a + b*Sqrt[c]]*d*x*ArcTanh[Sqrt[a + b*Sqrt[c
 + d*x]]/Sqrt[a + b*Sqrt[c]]]))/(2*Sqrt[c]*(-a^2 + b^2*c)*x)
```

fricas [B] time = 0.60, size = 1003, normalized size = 7.32

$$-\frac{x\sqrt{-\frac{ab^2d^2+\sqrt{\frac{b^6d^4}{b^4c^3-2a^2b^2c^2+a^4c}}\left(b^2c^2-a^2c\right)}{b^2c^2-a^2c}}\log\left(\sqrt{\sqrt{dx+c}\,b+a}\,b^4d^3+\left(b^4cd^2-\sqrt{\frac{b^6d^4}{b^4c^3-2\,a^2b^2c^2+a^4c}}\left(ab^2c^2-a^3c\right)\right)\sqrt{-\frac{ab^2\cdots}{\cdots}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*
(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3
 + (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2
- a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)
)*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^
4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*
log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3
- 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*
d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c)
)) + x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(
b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3
+ (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 -
 a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))
*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4
/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*l
og(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 -
 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d
^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))
) + 4*sqrt(sqrt(d*x + c)*b + a))/x
```

giac [B] time = 0.66, size = 232, normalized size = 1.69

$$\frac{\frac{2\sqrt{\sqrt{dx+c}\,b+a}\,b^3d^2}{b^2c-\left(\sqrt{dx+c}\,b+a\right)^2+2\left(\sqrt{dx+c}\,b+a\right)a-a^2}-\frac{\left(b^3cd^2|b|+ab^3\sqrt{c}\,d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(bc^{\frac{3}{2}}+ac\right)\sqrt{b\sqrt{c}-a}\,|b|}+\frac{\left(b^3cd^2|b|-ab^3\sqrt{c}\,d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(bc^{\frac{3}{2}}-ac\right)\sqrt{-b\sqrt{c}-a}\,|b|}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(sqrt(d*x + c)*b + a)*b^3*d^2/(b^2*c - (sqrt(d*x + c)*b + a)^2 +
 2*(sqrt(d*x + c)*b + a)*a - a^2) - (b^3*c*d^2*abs(b) + a*b^3*sqrt(c)*d^2)*
arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqrt(b^2*c)))/((b*c^(3/2) + a*c)
*sqrt(b*sqrt(c) - a)*abs(b)) + (b^3*c*d^2*abs(b) - a*b^3*sqrt(c)*d^2)*arcta
n(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*c^(3/2) - a*c)*sqrt
(-b*sqrt(c) - a)*abs(b)))/(b*d)
```

maple [A] time = 0.03, size = 151, normalized size = 1.10

$$\frac{b^2d\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{2\sqrt{b^2c}\,\sqrt{-a-\sqrt{b^2c}}}-\frac{b^2d\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{2\sqrt{b^2c}\,\sqrt{-a+\sqrt{b^2c}}}-\frac{\sqrt{a+\sqrt{dx+c}\,b}\,b^2d}{-b^2c+(dx+c)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^(1/2)*b)^(1/2)/x^2,x)
```

```
[Out] -d*b^2*(a+(d*x+c)^(1/2)*b)^(1/2)/((d*x+c)*b^2-b^2*c)+1/2*d*b^2/(b^2*c)^(1/2
)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/
2))^(1/2))-1/2*d*b^2/(b^2*c)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+
c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{dx+c}\,b+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sqrt{c+d\,x}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**2, x)
```

3.631 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$

Optimal. Leaf size=224

$$\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8cx\left(a^2-b^2c\right)}-\frac{bd^2\left(2a-3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd^2\left(2a+3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{3/2}}$$

```
[Out] -1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(2*a-3*b
*c^(1/2))/c^(3/2)/(a-b*c^(1/2))^(3/2)+1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2)
)^(1/2)/(a+b*c^(1/2))^(1/2))*(2*a+3*b*c^(1/2))/c^(3/2)/(a+b*c^(1/2))^(3/2)-
1/2*(a+b*(d*x+c)^(1/2))^(1/2)/x^2+1/8*b*d*(b*c-a*(d*x+c)^(1/2))*(a+b*(d*x+c
)^(1/2))^(1/2)/c/(-b^2*c+a^2)/x
```

Rubi [A] time = 0.43, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {371, 1398, 821, 12, 741, 827, 1166, 207}

$$\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8cx\left(a^2-b^2c\right)}-\frac{bd^2\left(2a-3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd^2\left(2a+3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] -Sqrt[a + b*Sqrt[c + d*x]]/(2*x^2) + (b*d*(b*c - a*Sqrt[c + d*x])*Sqrt[a +
b*Sqrt[c + d*x]])/(8*c*(a^2 - b^2*c)*x) - (b*(2*a - 3*b*Sqrt[c])*d^2*ArcTan
h[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(3/2)
*c^(3/2)) + (b*(2*a + 3*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sq
rt[a + b*Sqrt[c]]])/(16*(a + b*Sqrt[c])^(3/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = d^2\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{(-c+x)^3}\,dx, x, c+dx\right)$$
$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$
$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} - \frac{d^2\,\text{Subst}\left(\int -\frac{bc}{2\sqrt{a+bx}\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c}$$
$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{1}{4}\left(bd^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$
$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(bd^2\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-2a^2+3b^2c\right)}{\sqrt{a+bx}\left(-c+\right.}\right.}{8c\left(a^2-b\right.}$$
$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(bd^2\right)\text{Subst}\left(\int \frac{\frac{a^2b}{2}+\frac{1}{2}b\left(-2a^2\right.}{a^2-b^2c-}\right.}{4c}$$
$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(b\left(2a-3b\sqrt{c}\right)d^2\right)\text{Subst}\left(\right.}{16\left(\right.}$$
$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} - \frac{b\left(2a-3b\sqrt{c}\right)d^2\tanh^{-1}\left(\right.}{16\left(a-b\sqrt{c}\right)^{3/2}c}$$

Mathematica [A] time = 0.42, size = 258, normalized size = 1.15

$$\frac{-\left(a-b\sqrt{c}\right)\left(2\sqrt{c}\left(a+b\sqrt{c}\right)\sqrt{a+b\sqrt{c+dx}}\left(4a^2c+abdx\sqrt{c+dx}-b^2c(4c+dx)\right)-bd^2x^2\sqrt{a+b\sqrt{c}}\left(2a^2+ab\right.\right.}{16c^{3/2}x^2\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] (-(b*(2*a - 3*b*Sqrt[c])*Sqrt[a - b*Sqrt[c]]*(a + b*Sqrt[c])^2*d^2*x^2*ArcT
anh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]]) - (a - b*Sqrt[c])*(2*(a
 + b*Sqrt[c])*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]]*(4*a^2*c + a*b*d*x*Sqrt[c +
 d*x] - b^2*c*(4*c + d*x)) - b*Sqrt[a + b*Sqrt[c]]*(2*a^2 + a*b*Sqrt[c] - 3
*b^2*c)*d^2*x^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]))/(1
6*c^(3/2)*(a^2 - b^2*c)^2*x^2)
```

fricas [B] time = 0.81, size = 2856, normalized size = 12.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/32*((b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2
)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c
^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b
^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^
6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2
*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2
*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*
c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b
^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*
a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^
4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*
sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10
*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 +
a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - (b^2*c^
2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*
c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b
^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a
^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2
*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a
^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*
a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b
^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^1
2*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 -
 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b
^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14
*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4
*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(
b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) + (b^2*c^2 - a^2*c)*x^
2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b
^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^
4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 +
15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*
a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*sqrt(
sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*
d^4 + 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*
c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2
*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c
^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^
6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2
*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20
*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a
^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - (b^2*c^2 - a^2*c)*x^2*sqrt(-((15*
a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^
4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(
b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^
5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 -
 a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)
*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 + 2*(2*a*
b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81
*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 1
5*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3
)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2
*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*
a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6
+ 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 +
3*a^4*b^2*c^4 - a^6*c^3))) - 4*(b^2*c*d*x - sqrt(d*x + c)*a*b*d*x + 4*b^2*c
^2 - 4*a^2*c)*sqrt(sqrt(d*x + c)*b + a))/((b^2*c^2 - a^2*c)*x^2)
```

giac [B] time = 1.10, size = 895, normalized size = 4.00

$$\frac{\left(\left(b^3c^2-a^2bc\right)^2ab^3\sqrt{c}\,d^3-\left(3\,b^7c^3-4\,a^2b^5c^2+a^4b^3c\right)d^3\left|b^3c^2-a^2bc\right|+\left(3\,ab^9c^{\frac{9}{2}}-8\,a^3b^7c^{\frac{7}{2}}+7\,a^5b^5c^{\frac{5}{2}}-2\,a^7b^3c^{\frac{3}{2}}\right)d^3\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b \cdots}}{\sqrt{-\frac{ab^2c^2-a^3c+\sqrt{\left(ab^2c^2-a^3c\right)^2+\left(b^4 \cdots\right.}}{b^2c^2- \cdots}}}\right.}{\left(b^5c^{\frac{9}{2}}-ab^4c^4-2\,a^2b^3c^{\frac{7}{2}}+2\,a^3b^2c^3+a^4bc^{\frac{5}{2}}-a^5c^2\right)\sqrt{-b\sqrt{c}-a}\,\left|b^3c^2-a^2bc\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(((b^3*c^2 - a^2*b*c)^2*a*b^3*sqrt(c)*d^3 - (3*b^7*c^3 - 4*a^2*b^5*c^2
 + a^4*b^3*c)*d^3*abs(b^3*c^2 - a^2*b*c) + (3*a*b^9*c^(9/2) - 8*a^3*b^7*c^(
7/2) + 7*a^5*b^5*c^(5/2) - 2*a^7*b^3*c^(3/2))*d^3)*arctan(sqrt(sqrt(d*x + c
)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c + sqrt((a*b^2*c^2 - a^3*c)^2 + (b^4*c^3 -
 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c)))/(b^2*c^2 - a^2*c)))/((b^5*c^(9/
2) - a*b^4*c^4 - 2*a^2*b^3*c^(7/2) + 2*a^3*b^2*c^3 + a^4*b*c^(5/2) - a^5*c^
2)*sqrt(-b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*c)) + ((b^3*c^2 - a^2*b*c)^2*a*
b^3*d^3 + (3*b^7*c^(5/2) - 4*a^2*b^5*c^(3/2) + a^4*b^3*sqrt(c))*d^3*abs(b^3
*c^2 - a^2*b*c) + (3*a*b^9*c^4 - 8*a^3*b^7*c^3 + 7*a^5*b^5*c^2 - 2*a^7*b^3*
c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c - sqrt((a
*b^2*c^2 - a^3*c)^2 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c)))
/(b^2*c^2 - a^2*c)))/((b^5*c^4 + a*b^4*c^(7/2) - 2*a^2*b^3*c^3 - 2*a^3*b^2*
c^(5/2) + a^4*b*c^2 + a^5*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*
c)) - 2*(3*sqrt(sqrt(d*x + c)*b + a)*b^7*c^2*d^3 + (sqrt(d*x + c)*b + a)^(5
/2)*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(3/2)*a*b^5*c*d^3 - 4*sqrt(sqrt(d*x +
 c)*b + a)*a^2*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a*b^3*d^3 + 3*(sqrt(
d*x + c)*b + a)^(5/2)*a^2*b^3*d^3 - 3*(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^3*d
^3 + sqrt(sqrt(d*x + c)*b + a)*a^4*b^3*d^3)/((b^2*c^2 - a^2*c)*(b^2*c - (sq
rt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^2))/(b*d)
```

maple [B] time = 0.04, size = 784, normalized size = 3.50

$$-\frac{\left(a+\sqrt{dx+c}\,b\right)^{\frac{3}{2}}a\,b^4d^2}{8\left(-b^2c+(dx+c)\,b^2\right)^2\left(-b^2c+a^2\right)}+\frac{3b^4d^2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{16\left(-b^2c+a^2\right)\sqrt{b^2c}\,\sqrt{-a-\sqrt{b^2c}}}-\frac{3b^4d^2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{16\left(-b^2c+a^2\right)\sqrt{b^2c}\,\sqrt{-a+\sqrt{b^2c}}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^(1/2)*b)^(1/2)/x^3,x)
```

```
[Out] -1/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2*a/c/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*b)^(7
/2)+1/8*b^4*d^2/(-b^2*c+(d*x+c)*b^2)^2/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*b)^(5/
2)+3/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2/c/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*b)^(5
/2)*a^2-1/8*b^4*d^2/(-b^2*c+(d*x+c)*b^2)^2*a/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*
b)^(3/2)-3/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2*a^3/c/(-b^2*c+a^2)*(a+(d*x+c)^(
1/2)*b)^(3/2)-3/8*b^4*d^2/(-b^2*c+(d*x+c)*b^2)^2*(a+(d*x+c)^(1/2)*b)^(1/2)+
1/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2/c*(a+(d*x+c)^(1/2)*b)^(1/2)*a^2+3/16*b^4
*d^2/(-b^2*c+a^2)/(b^2*c)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^
(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2))-1/16*b^2*d^2/c/(-b^2*c+a^2)/(-a-(b
^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2
))*a-1/8*b^2*d^2/c/(-b^2*c+a^2)/(b^2*c)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2)*arct
an((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2))*a^2-3/16*b^4*d^2/(-b
^2*c+a^2)/(b^2*c)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)
^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))-1/16*b^2*d^2/c/(-b^2*c+a^2)/(-a+(b^2*c)^(1
/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))*a+1/8
```

```
*b^2*d^2/c/(-b^2*c+a^2)/(b^2*c)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a+(d
*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))*a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{dx+c}\,b+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\sqrt{c+d\,x}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x^3,x)
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**3,x)
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**3, x)
```

3.632 $\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=230

$$-\frac{2a\left(a^2-b^2c\right)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}+\frac{2\left(a^2-b^2c\right)^3\sqrt{c+dx}}{b^7d^4}-\frac{a\left(a^2-3b^2c\right)(c+dx)^2}{2b^4d^4}+\frac{2\left(a^2-3b^2c\right)(c+dx)^{5/2}}{5b^3d^4}-\frac{a}{}$$

```
[Out] -a*(3*b^4*c^2-3*a^2*b^2*c+a^4)*x/b^6/d^3+2/3*(3*b^4*c^2-3*a^2*b^2*c+a^4)*(d
*x+c)^(3/2)/b^5/d^4-1/2*a*(-3*b^2*c+a^2)*(d*x+c)^2/b^4/d^4+2/5*(-3*b^2*c+a^
2)*(d*x+c)^(5/2)/b^3/d^4-1/3*a*(d*x+c)^3/b^2/d^4+2/7*(d*x+c)^(7/2)/b/d^4-2*
a*(-b^2*c+a^2)^3*ln(a+b*(d*x+c)^(1/2))/b^8/d^4+2*(-b^2*c+a^2)^3*(d*x+c)^(1/
2)/b^7/d^4
```

Rubi [A] time = 0.26, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(-3a^2b^2c+a^4+3b^4c^2\right)(c+dx)^{3/2}}{3b^5d^4}-\frac{ax\left(-3a^2b^2c+a^4+3b^4c^2\right)}{b^6d^3}+\frac{2\left(a^2-3b^2c\right)(c+dx)^{5/2}}{5b^3d^4}-\frac{a\left(a^2-3b^2c\right)(c+dx}{2b^4d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3)) + (2*(a^2 - b^2*c)^3*Sqr
t[c + d*x])/(b^7*d^4) + (2*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*(c + d*x)^(3/2))
/(3*b^5*d^4) - (a*(a^2 - 3*b^2*c)*(c + d*x)^2)/(2*b^4*d^4) + (2*(a^2 - 3*b^
2*c)*(c + d*x)^(5/2))/(5*b^3*d^4) - (a*(c + d*x)^3)/(3*b^2*d^4) + (2*(c + d
*x)^(7/2))/(7*b*d^4) - (2*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(b^8*
d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^3}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{\left(-a^2+b^2c\right)^3}{b^7} - \frac{a\left(a^4-3a^2b^2c+3b^4c^2\right)x}{b^6} + \frac{\left(a^4-3a^2b^2c+3b^4c^2\right)x^2}{b^5} - \frac{a\left(a^2-3b^2c\right)x^3}{b^4} - \frac{\left(-a^2+3b^2c\right.}{b^3}\right.\right.}{d^4}$$

$$= -\frac{a\left(a^4-3a^2b^2c+3b^4c^2\right)x}{b^6d^3} + \frac{2\left(a^2-b^2c\right)^3\sqrt{c+dx}}{b^7d^4} + \frac{2\left(a^4-3a^2b^2c+3b^4c^2\right)(c+dx)}{3b^5d^4}$$

Mathematica [A] time = 0.20, size = 213, normalized size = 0.93

$$\frac{b\left(420a^6\sqrt{c+dx} - 210a^5bdx - 140a^4b^2(8c-dx)\sqrt{c+dx} - 105a^3b^3dx(dx-4c) + 84a^2b^4\sqrt{c+dx}\left(11c^2-3cd\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-210*a^5*b*d*x - 105*a^3*b^3*d*x*(-4*c + d*x) + 420*a^6*Sqrt[c + d*x] -
 140*a^4*b^2*(8*c - d*x)*Sqrt[c + d*x] + 84*a^2*b^4*Sqrt[c + d*x]*(11*c^2 -
 3*c*d*x + d^2*x^2) - 35*a*b^5*d*x*(6*c^2 - 3*c*d*x + 2*d^2*x^2) + 12*b^6*S
qrt[c + d*x]*(-16*c^3 + 8*c^2*d*x - 6*c*d^2*x^2 + 5*d^3*x^3)) - 420*a*(a^2
- b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(210*b^8*d^4)
```

fricas [A] time = 0.45, size = 228, normalized size = 0.99

$$-\frac{70\,ab^6d^3x^3 - 105\left(ab^6c - a^3b^4\right)d^2x^2 + 210\left(ab^6c^2 - 2\,a^3b^4c + a^5b^2\right)dx - 420\left(ab^6c^3 - 3\,a^3b^4c^2 + 3\,a^5b^2c - a^7\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/210*(70*a*b^6*d^3*x^3 - 105*(a*b^6*c - a^3*b^4)*d^2*x^2 + 210*(a*b^6*c^2
 - 2*a^3*b^4*c + a^5*b^2)*d*x - 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*
c - a^7)*log(sqrt(d*x + c)*b + a) - 4*(15*b^7*d^3*x^3 - 48*b^7*c^3 + 231*a^
2*b^5*c^2 - 280*a^4*b^3*c + 105*a^6*b - 3*(6*b^7*c - 7*a^2*b^5)*d^2*x^2 + (
24*b^7*c^2 - 63*a^2*b^5*c + 35*a^4*b^3)*d*x)*sqrt(d*x + c))/(b^8*d^4)
```

giac [A] time = 0.37, size = 341, normalized size = 1.48

$$\frac{2\left(ab^6c^3 - 3\,a^3b^4c^2 + 3\,a^5b^2c - a^7\right)\log\left(\left|\sqrt{dx+c}\,b + a\right|\right)}{b^8d^4} + \frac{60\,(dx+c)^{\frac{7}{2}}b^6d^{24} - 252\,(dx+c)^{\frac{5}{2}}b^6cd^{24} + 420\,(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(abs(sqrt(d*x + c)*b +
 a))/(b^8*d^4) + 1/210*(60*(d*x + c)^(7/2)*b^6*d^24 - 252*(d*x + c)^(5/2)*b
^6*c*d^24 + 420*(d*x + c)^(3/2)*b^6*c^2*d^24 - 420*sqrt(d*x + c)*b^6*c^3*d^
24 - 70*(d*x + c)^3*a*b^5*d^24 + 315*(d*x + c)^2*a*b^5*c*d^24 - 630*(d*x +
c)*a*b^5*c^2*d^24 + 84*(d*x + c)^(5/2)*a^2*b^4*d^24 - 420*(d*x + c)^(3/2)*a
```

```
^2*b^4*c*d^24 + 1260*sqrt(d*x + c)*a^2*b^4*c^2*d^24 - 105*(d*x + c)^2*a^3*b
^3*d^24 + 630*(d*x + c)*a^3*b^3*c*d^24 + 140*(d*x + c)^(3/2)*a^4*b^2*d^24 -
 1260*sqrt(d*x + c)*a^4*b^2*c*d^24 - 210*(d*x + c)*a^5*b*d^24 + 420*sqrt(d*
x + c)*a^6*d^24)/(b^7*d^28)
```

maple [A] time = 0.01, size = 394, normalized size = 1.71

$$-\frac{a\,x^3}{3b^2d}+\frac{ac\,x^2}{2b^2d^2}-\frac{a^3x^2}{2b^4d^2}+\frac{2a\,c^3\ln\left(a+\sqrt{dx+c}\;b\right)}{b^2d^4}-\frac{a\,c^2x}{b^2d^3}-\frac{6a^3c^2\ln\left(a+\sqrt{dx+c}\;b\right)}{b^4d^4}+\frac{2a^3cx}{b^4d^3}-\frac{11a\,c^3}{6b^2d^4}-\frac{2\sqrt{dx+c}\;c^3}{b\,d^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] -11/6/d^4/b^2*a*c^3-6/5/d^4/b*(d*x+c)^(5/2)*c+2/5/d^4/b^3*(d*x+c)^(5/2)*a^2
+2/d^4/b*(d*x+c)^(3/2)*c^2-2/d^4/b*(d*x+c)^(1/2)*c^3+2/3/d^4/b^5*(d*x+c)^(3
/2)*a^4+2/7*(d*x+c)^(7/2)/b/d^4+2/d^4/b^7*(d*x+c)^(1/2)*a^6-1/d^3/b^6*x*a^5
-1/2/d^2/b^4*x^2*a^3+6/d^4/b^3*(d*x+c)^(1/2)*a^2*c^2-6/d^4/b^5*(d*x+c)^(1/2
)*a^4*c-1/3/d/b^2*x^3*a-2/d^4/b^3*(d*x+c)^(3/2)*a^2*c+1/2/d^2/b^2*x^2*a*c-1
/d^3/b^2*x*a*c^2+2/d^3/b^4*x*a^3*c-1/d^4/b^6*a^5*c+5/2/d^4/b^4*a^3*c^2+2/d^
4*a/b^2*ln(a+(d*x+c)^(1/2)*b)*c^3-6/d^4*a^3/b^4*ln(a+(d*x+c)^(1/2)*b)*c^2+6
/d^4*a^5/b^6*ln(a+(d*x+c)^(1/2)*b)*c-2/d^4*a^7/b^8*ln(a+(d*x+c)^(1/2)*b)
```

maxima [A] time = 0.90, size = 243, normalized size = 1.06

$$\frac{\frac{60\,(dx+c)^{\frac{7}{2}}b^6-70\,(dx+c)^3ab^5-84\left(3\,b^6c-a^2b^4\right)(dx+c)^{\frac{5}{2}}+105\left(3\,ab^5c-a^3b^3\right)(dx+c)^2+140\left(3\,b^6c^2-3\,a^2b^4c+a^4b^2\right)(dx+c)^{\frac{3}{2}}-210\left(3\,ab^5c^2-3\,a^3b^3c+a^5b\right)(dx}{b^7}}{210\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/210*((60*(d*x + c)^(7/2)*b^6 - 70*(d*x + c)^3*a*b^5 - 84*(3*b^6*c - a^2*b
^4)*(d*x + c)^(5/2) + 105*(3*a*b^5*c - a^3*b^3)*(d*x + c)^2 + 140*(3*b^6*c^
2 - 3*a^2*b^4*c + a^4*b^2)*(d*x + c)^(3/2) - 210*(3*a*b^5*c^2 - 3*a^3*b^3*c
 + a^5*b)*(d*x + c) - 420*(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqr
t(d*x + c))/b^7 + 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(s
qrt(d*x + c)*b + a)/b^8)/d^4
```

mupad [B] time = 0.07, size = 317, normalized size = 1.38

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}}{7\,b\,d^4}-\left(\frac{a^2\,\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{b^2}-\frac{6\,c^2}{b\,d^4}\right)}{b^2}+\frac{2\,c^3}{b\,d^4}\right)\,\sqrt{c+d\,x}-\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{3\,b^2}-\frac{2\,c^2}{b\,d^4}\right)\,{\left(c+d\,x\right)}^{3/2}-\left(\frac{6\,c}{5\,b\,d^4}-\frac{2\,a^2}{5\,b^3\,d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(7/2))/(7*b*d^4) - ((a^2*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d
^4)))/b^2 - (6*c^2)/(b*d^4)))/b^2 + (2*c^3)/(b*d^4))*(c + d*x)^(1/2) - ((a^
2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/(3*b^2) - (2*c^2)/(b*d^4))*(c + d*x)
^(3/2) - ((6*c)/(5*b*d^4) - (2*a^2)/(5*b^3*d^4))*(c + d*x)^(5/2) + (a*((6*c
)/(b*d^4) - (2*a^2)/(b^3*d^4))*(c + d*x)^2)/(4*b) - (a*(c + d*x)^3)/(3*b^2*
d^4) - (log(a + b*(c + d*x)^(1/2))*(2*a^7 - 6*a^5*b^2*c - 2*a*b^6*c^3 + 6*a
^3*b^4*c^2))/(b^8*d^4) + (a*d*x*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/
b^2 - (6*c^2)/(b*d^4)))/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(x**3/(a + b*sqrt(c + d*x)), x)
```

3.633 $\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=151

$$-\frac{2a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{2\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^5d^3}-\frac{ax\left(a^2-2b^2c\right)}{b^4d^2}+\frac{2\left(a^2-2b^2c\right)(c+dx)^{3/2}}{3b^3d^3}-\frac{a(c+dx)}{2b^2d^3}$$

```
[Out] -a*(-2*b^2*c+a^2)*x/b^4/d^2+2/3*(-2*b^2*c+a^2)*(d*x+c)^(3/2)/b^3/d^3-1/2*a*
(d*x+c)^2/b^2/d^3+2/5*(d*x+c)^(5/2)/b/d^3-2*a*(-b^2*c+a^2)^2*ln(a+b*(d*x+c)
^(1/2))/b^6/d^3+2*(-b^2*c+a^2)^2*(d*x+c)^(1/2)/b^5/d^3
```

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(a^2-2b^2c\right)(c+dx)^{3/2}}{3b^3d^3}+\frac{2\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^5d^3}-\frac{ax\left(a^2-2b^2c\right)}{b^4d^2}-\frac{2a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}-\frac{a(c+dx)^2}{2b^2d^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*(a^2 - 2*b^2*c)*x)/(b^4*d^2)) + (2*(a^2 - b^2*c)^2*Sqrt[c + d*x])/(b^5
*d^3) + (2*(a^2 - 2*b^2*c)*(c + d*x)^(3/2))/(3*b^3*d^3) - (a*(c + d*x)^2)/(
2*b^2*d^3) + (2*(c + d*x)^(5/2))/(5*b*d^3) - (2*a*(a^2 - b^2*c)^2*Log[a + b
*Sqrt[c + d*x]])/(b^6*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^2}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{\left(-a^2+b^2c\right)^2}{b^5} - \frac{a\left(a^2-2b^2c\right)x}{b^4} - \frac{\left(-a^2+2b^2c\right)x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a\left(a^2-b^2c\right)^2}{b^5(a+bx)}\right) dx, x, \sqrt{c+dx}}{d^3}$$

$$= -\frac{a\left(a^2-2b^2c\right)x}{b^4d^2} + \frac{2\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^5d^3} + \frac{2\left(a^2-2b^2c\right)(c+dx)^{3/2}}{3b^3d^3} - \frac{a(c+dx)^2}{2b^2d^3} + 2($$

Mathematica [A] time = 0.13, size = 138, normalized size = 0.91

$$\frac{b\left(60a^4\sqrt{c+dx} - 30a^3bdx - 20a^2b^2(5c-dx)\sqrt{c+dx} - 15ab^3dx(dx-2c) + 4b^4\sqrt{c+dx}\left(8c^2-4cdx+3d^2x^2\right)\right.}{30b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-30*a^3*b*d*x - 15*a*b^3*d*x*(-2*c + d*x) + 60*a^4*Sqrt[c + d*x] - 20*a
^2*b^2*(5*c - d*x)*Sqrt[c + d*x] + 4*b^4*Sqrt[c + d*x]*(8*c^2 - 4*c*d*x + 3
*d^2*x^2)) - 60*a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x]])/(30*b^6*d^3)
```

fricas [A] time = 0.44, size = 138, normalized size = 0.91

$$-\frac{15\,ab^4d^2x^2 - 30\left(ab^4c - a^3b^2\right)dx + 60\left(ab^4c^2 - 2\,a^3b^2c + a^5\right)\log\left(\sqrt{dx+c}\,b + a\right) - 4\left(3\,b^5d^2x^2 + 8\,b^5c^2 - 25\,a\right.}{30\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/30*(15*a*b^4*d^2*x^2 - 30*(a*b^4*c - a^3*b^2)*d*x + 60*(a*b^4*c^2 - 2*a^
3*b^2*c + a^5)*log(sqrt(d*x + c)*b + a) - 4*(3*b^5*d^2*x^2 + 8*b^5*c^2 - 25
*a^2*b^3*c + 15*a^4*b - (4*b^5*c - 5*a^2*b^3)*d*x)*sqrt(d*x + c))/(b^6*d^3)
```

giac [A] time = 0.41, size = 198, normalized size = 1.31

$$-\frac{2\left(ab^4c^2 - 2\,a^3b^2c + a^5\right)\log\left(\left|\sqrt{dx+c}\,b + a\right|\right)}{b^6d^3} + \frac{12\,(dx+c)^{\frac{5}{2}}b^4d^{12} - 40\,(dx+c)^{\frac{3}{2}}b^4cd^{12} + 60\,\sqrt{dx+c}\,b^4c^2d^{12}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3)
+ 1/30*(12*(d*x + c)^(5/2)*b^4*d^12 - 40*(d*x + c)^(3/2)*b^4*c*d^12 + 60*sq
rt(d*x + c)*b^4*c^2*d^12 - 15*(d*x + c)^2*a*b^3*d^12 + 60*(d*x + c)*a*b^3*c
*d^12 + 20*(d*x + c)^(3/2)*a^2*b^2*d^12 - 120*sqrt(d*x + c)*a^2*b^2*c*d^12
- 30*(d*x + c)*a^3*b*d^12 + 60*sqrt(d*x + c)*a^4*d^12)/(b^5*d^15)
```

maple [A] time = 0.01, size = 235, normalized size = 1.56

$$-\frac{a\,x^2}{2b^2d} - \frac{2a\,c^2\ln\left(a+\sqrt{dx+c}\;b\right)}{b^2d^3} + \frac{acx}{b^2d^2} + \frac{4a^3c\ln\left(a+\sqrt{dx+c}\;b\right)}{b^4d^3} - \frac{a^3x}{b^4d^2} + \frac{3a\,c^2}{2b^2d^3} + \frac{2\sqrt{dx+c}\;c^2}{b\,d^3} - \frac{2a^5\ln\left(a+\sqrt{d}\right.}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] 2/5*(d*x+c)^(5/2)/b/d^3-1/2/d/b^2*x^2*a+1/d^2/b^2*x*a*c+3/2/d^3/b^2*a*c^2-4
/3/d^3/b*(d*x+c)^(3/2)*c+2/3/d^3/b^3*(d*x+c)^(3/2)*a^2+2/d^3/b*(d*x+c)^(1/2
)*c^2-1/d^2/b^4*x*a^3-1/d^3/b^4*a^3*c-4/d^3/b^3*(d*x+c)^(1/2)*a^2*c+2/d^3/b
^5*(d*x+c)^(1/2)*a^4-2/d^3*a/b^2*ln(a+(d*x+c)^(1/2)*b)*c^2+4/d^3*a^3/b^4*ln
(a+(d*x+c)^(1/2)*b)*c-2/d^3*a^5/b^6*ln(a+(d*x+c)^(1/2)*b)
```

maxima [A] time = 0.88, size = 148, normalized size = 0.98

$$\frac{\frac{12 (dx+c)^{\frac{5}{2}} b^4 - 15 (dx+c)^2 ab^3 - 20 \left(2 b^4 c - a^2 b^2\right) (dx+c)^{\frac{3}{2}} + 30 \left(2 ab^3 c - a^3 b\right) (dx+c) + 60 \left(b^4 c^2 - 2 a^2 b^2 c + a^4\right) \sqrt{dx+c}}{b^5} - \frac{60 \left(ab^4 c^2 - 2 a^3 b^2 c + a^5\right) \log\left(\sqrt{dx+c}\, b + \cdots\right.}{b^6}}{30 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/30*((12*(d*x + c)^(5/2)*b^4 - 15*(d*x + c)^2*a*b^3 - 20*(2*b^4*c - a^2*b^
2)*(d*x + c)^(3/2) + 30*(2*a*b^3*c - a^3*b)*(d*x + c) + 60*(b^4*c^2 - 2*a^2
*b^2*c + a^4)*sqrt(d*x + c))/b^5 - 60*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(s
qrt(d*x + c)*b + a)/b^6)/d^3
```

mupad [B] time = 3.21, size = 184, normalized size = 1.22

$$\frac{2 (c + d x)^{5/2}}{5 b d^3} - \left(\frac{a^2 \left(\frac{4 c}{b d^3} - \frac{2 a^2}{b^3 d^3}\right)}{b^2} - \frac{2 c^2}{b d^3}\right) \sqrt{c + d x} - \left(\frac{4 c}{3 b d^3} - \frac{2 a^2}{3 b^3 d^3}\right) (c + d x)^{3/2} - \frac{\ln\left(a + b \sqrt{c + d x}\right) \left(2 a^5 - 4 a^{3} \cdots\right.}{b^6 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(5/2))/(5*b*d^3) - ((a^2*((4*c)/(b*d^3) - (2*a^2)/(b^3*d^3)))/
b^2 - (2*c^2)/(b*d^3))*(c + d*x)^(1/2) - ((4*c)/(3*b*d^3) - (2*a^2)/(3*b^3*
d^3))*(c + d*x)^(3/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^5 - 4*a^3*b^2*c +
2*a*b^4*c^2))/(b^6*d^3) - (a*(c + d*x)^2)/(2*b^2*d^3) + (a*d*x*((4*c)/(b*d^
3) - (2*a^2)/(b^3*d^3)))/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(x**2/(a + b*sqrt(c + d*x)), x)
```

3.634 $\int \frac{x}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=90

$$-\frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{2\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d^2}-\frac{ax}{b^2d}+\frac{2(c+dx)^{3/2}}{3bd^2}$$

```
[Out] -a*x/b^2/d+2/3*(d*x+c)^(3/2)/b/d^2-2*a*(-b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/b
^4/d^2+2*(-b^2*c+a^2)*(d*x+c)^(1/2)/b^3/d^2
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{2\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d^2}-\frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{ax}{b^2d}+\frac{2(c+dx)^{3/2}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*x)/(b^2*d)) + (2*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^3*d^2) + (2*(c + d*x)
^(3/2))/(3*b*d^2) - (2*a*(a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a+b\sqrt{c+dx}}\,dx &= \frac{\text{Subst}\left(\int \frac{-c+x}{a+b\sqrt{x}}\,dx,x,c+dx\right)}{d^2}\\
&= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)}{a+bx}\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\,\text{Subst}\left(\int \left(\frac{a^2-b^2c}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}+\frac{-a^3+ab^2c}{b^3(a+bx)}\right)dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= -\frac{ax}{b^2d}+\frac{2\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d^2}+\frac{2(c+dx)^{3/2}}{3bd^2}-\frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 82, normalized size = 0.91

$$\frac{b\left(6a^2\sqrt{c+dx}-3abdx+2b^2(dx-2c)\sqrt{c+dx}\right)-6\left(a^3-ab^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{3b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-3*a*b*d*x + 6*a^2*Sqrt[c + d*x] + 2*b^2*(-2*c + d*x)*Sqrt[c + d*x]) -
6*(a^3 - a*b^2*c)*Log[a + b*Sqrt[c + d*x]])/(3*b^4*d^2)
```

fricas [A] time = 0.46, size = 71, normalized size = 0.79

$$-\frac{3\,ab^2dx-6\left(ab^2c-a^3\right)\log\left(\sqrt{dx+c}\,b+a\right)-2\left(b^3dx-2\,b^3c+3\,a^2b\right)\sqrt{dx+c}}{3\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*b^2*d*x - 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a) - 2*(b^3*d*x
 - 2*b^3*c + 3*a^2*b)*sqrt(d*x + c))/(b^4*d^2)
```

giac [A] time = 0.33, size = 105, normalized size = 1.17

$$\frac{\frac{6\left(ab^2c-a^3\right)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^4d}+\frac{2\,(dx+c)^{\frac{3}{2}}b^2d^2-6\,\sqrt{dx+c}\,b^2cd^2-3\,(dx+c)abd^2+6\,\sqrt{dx+c}\,a^2d^2}{b^3d^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(6*(a*b^2*c - a^3)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) + (2*(d*x + c)
^(3/2)*b^2*d^2 - 6*sqrt(d*x + c)*b^2*c*d^2 - 3*(d*x + c)*a*b*d^2 + 6*sqrt(d
*x + c)*a^2*d^2)/(b^3*d^3))/d
```

maple [A] time = 0.00, size = 116, normalized size = 1.29

$$\frac{2ac\ln\left(a+\sqrt{dx+c}\,b\right)}{b^2d^2}-\frac{ax}{b^2d}-\frac{2a^3\ln\left(a+\sqrt{dx+c}\,b\right)}{b^4d^2}-\frac{ac}{b^2d^2}-\frac{2\sqrt{dx+c}\,c}{b\,d^2}+\frac{2\sqrt{dx+c}\,a^2}{b^3d^2}+\frac{2\,(dx+c)^{\frac{3}{2}}}{3b\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] 2/3*(d*x+c)^(3/2)/b/d^2-a*x/b^2/d-1/d^2/b^2*a*c-2/d^2/b*(d*x+c)^(1/2)*c+2/d
^2/b^3*(d*x+c)^(1/2)*a^2+2/d^2*a/b^2*ln(a+(d*x+c)^(1/2)*b)*c-2/d^2*a^3/b^4*
ln(a+(d*x+c)^(1/2)*b)
```

maxima [A] time = 0.88, size = 81, normalized size = 0.90

$$\frac{\frac{2\,(dx+c)^{\frac{3}{2}}b^2-3\,(dx+c)ab-6\left(b^2c-a^2\right)\sqrt{dx+c}}{b^3}+\frac{6\left(ab^2c-a^3\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^4}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*((2*(d*x + c)^(3/2)*b^2 - 3*(d*x + c)*a*b - 6*(b^2*c - a^2)*sqrt(d*x +
c))/b^3 + 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

mupad [B] time = 0.05, size = 89, normalized size = 0.99

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}}{3\,b\,d^2}-\left(\frac{2\,c}{b\,d^2}-\frac{2\,a^2}{b^3\,d^2}\right)\,\sqrt{c+d\,x}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^3-2\,a\,b^2\,c\right)}{b^4\,d^2}-\frac{a\,x}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(3/2))/(3*b*d^2) - ((2*c)/(b*d^2) - (2*a^2)/(b^3*d^2))*(c + d*
x)^(1/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^3 - 2*a*b^2*c))/(b^4*d^2) - (a*
x)/(b^2*d)
```

sympy [A] time = 4.72, size = 109, normalized size = 1.21

$$\begin{cases} \frac{2 \left(- \frac{a (c+dx)}{2b^2 d} - \frac{a\left(a^2-b^2c\right)\left(\begin{cases} \frac{\sqrt{c+dx}}{a} & \text{for}\: b = 0 \\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise} \end{cases}\right)}{b^3 d} + \frac{(c+dx)^{\frac{3}{2}}}{3bd} + \frac{\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3 d}\right)}{d} & \text{for}\: d \neq 0 \\ \frac{x^2}{2\left(a+b\sqrt{c}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*(-a*(c + d*x)/(2*b**2*d) - a*(a**2 - b**2*c)*Piecewise((sqrt(c
 + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(b**3*d) + (c + d
*x)**(3/2)/(3*b*d) + (a**2 - b**2*c)*sqrt(c + d*x)/(b**3*d))/d, Ne(d, 0)),
(x**2/(2*(a + b*sqrt(c))), True))
```

3.635 $\int \frac{1}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] -2*a*ln(a+b*(d*x+c)^(1/2))/b^2/d+2*(d*x+c)^(1/2)/b/d
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 190, 43}

$$\frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^(-1),x]
```

```
[Out] (2*Sqrt[c + d*x])/(b*d) - (2*a*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\sqrt{c+dx}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+b\sqrt{x}}\,dx,x,c+dx\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \frac{x}{a+bx}\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{c+dx}}{b}-\frac{a\log\left(a+b\sqrt{c+dx}\right)}{b^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^(-1),x]
[Out] (2*(Sqrt[c + d*x]/b - (a*Log[a + b*Sqrt[c + d*x]])/b^2))/d
```

fricas [A] time = 0.45, size = 33, normalized size = 0.80

$$-\frac{2\left(a\log\left(\sqrt{dx+c}\,b+a\right)-\sqrt{dx+c}\,b\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
[Out] -2*(a*log(sqrt(d*x + c)*b + a) - sqrt(d*x + c)*b)/(b^2*d)
```

giac [A] time = 0.35, size = 38, normalized size = 0.93

$$-\frac{2\,a\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^2d}+\frac{2\,\sqrt{dx+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
[Out] -2*a*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*sqrt(d*x + c)/(b*d)
```

maple [B] time = 0.01, size = 87, normalized size = 2.12

$$\frac{a\ln\left(-a+\sqrt{dx+c}\;b\right)}{b^2d}-\frac{a\ln\left(a+\sqrt{dx+c}\;b\right)}{b^2d}-\frac{a\ln\left(b^2dx+b^2c-a^2\right)}{b^2d}+\frac{2\sqrt{dx+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(d*x+c)^(1/2)*b),x)
[Out] 2*(d*x+c)^(1/2)/b/d+1/b^2/d*a*ln(-a+(d*x+c)^(1/2)*b)-a*ln(a+(d*x+c)^(1/2)*b
)/b^2/d-a*ln(b^2*d*x+b^2*c-a^2)/b^2/d
```

maxima [A] time = 0.89, size = 35, normalized size = 0.85

$$-\frac{2\left(\frac{a\log\left(\sqrt{dx+c}\,b+a\right)}{b^2}-\frac{\sqrt{dx+c}}{b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
[Out] -2*(a*log(sqrt(d*x + c)*b + a)/b^2 - sqrt(d*x + c)/b)/d
```

mupad [B] time = 0.05, size = 33, normalized size = 0.80

$$-\frac{2\,\left(a\,\ln\left(a+b\,\sqrt{c+d\,x}\right)-b\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^(1/2)),x)
[Out] -(2*(a*log(a + b*(c + d*x)^(1/2)) - b*(c + d*x)^(1/2)))/(b^2*d)
```

sympy [A] time = 0.55, size = 49, normalized size = 1.20

$$\begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ \frac{x}{a+b\sqrt{c}} & \text{for } d = 0 \\ -\frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{b^2d} + \frac{2\sqrt{c+dx}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x/a, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c)), Eq(d
, 0)), (-2*a*log(a/b + sqrt(c + d*x))/(b**2*d) + 2*sqrt(c + d*x)/(b*d), Tru
e))
```

3.636 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=82

$$-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}$$

```
[Out] a*ln(x)/(-b^2*c+a^2)-2*a*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)+2*b*arctanh((d*
x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 801, 635, 206, 260}

$$-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c) + (a*Log[x])/(a^
2 - b^2*c) - (2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
```

```
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)}\,dx, x, c+dx\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)\\
&= 2\,\text{Subst}\left(\int \left(-\frac{ab}{\left(a^2-b^2c\right)(a+bx)}+\frac{bc-ax}{\left(a^2-b^2c\right)\left(c-x^2\right)}\right)dx, x, \sqrt{c+dx}\right)\\
&= -\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2\,\text{Subst}\left(\int \frac{bc-ax}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{a^2-b^2c}\\
&= -\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{(2a)\,\text{Subst}\left(\int \frac{x}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{a^2-b^2c}+\frac{(2bc)\,\text{Subst}\left(\int \frac{1}{c-x^2}\,d\right.}{a^2-b^2c}\\
&= \frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}\end{aligned}$$

Mathematica [A] time = 0.09, size = 61, normalized size = 0.74

$$\frac{-2a\log\left(a+b\sqrt{c+dx}\right)+a\log(dx)+2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + a*Log[d*x] - 2*a*Log[a + b*Sq
rt[c + d*x]])/(a^2 - b^2*c)
```

fricas [A] time = 0.52, size = 125, normalized size = 1.52

$$\left[\frac{b\sqrt{c}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2a\log\left(\sqrt{dx+c}\,b+a\right)-a\log(x)}{b^2c-a^2}, \frac{2b\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+2a\log\left(\sqrt{dx+c}\right.}{b^2c-a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [(b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*a*log(sqrt(d*x
 + c)*b + a) - a*log(x))/(b^2*c - a^2), (2*b*sqrt(-c)*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + 2*a*log(sqrt(d*x + c)*b + a) - a*log(x))/(b^2*c - a^2)]
```

giac [A] time = 0.34, size = 88, normalized size = 1.07

$$\frac{2ab\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^3c-a^2b}+\frac{2bc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^2c-a^2\right)\sqrt{-c}}-\frac{a\log(dx)}{b^2c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*b*log(abs(sqrt(d*x + c)*b + a))/(b^3*c - a^2*b) + 2*b*c*arctan(sqrt(d*x
 + c)/sqrt(-c))/((b^2*c - a^2)*sqrt(-c)) - a*log(d*x)/(b^2*c - a^2)
```

maple [A] time = 0.01, size = 77, normalized size = 0.94

$$\frac{2b\sqrt{c}\ \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{-b^{2}c+a^{2}}+\frac{a\ln\left(dx\right)}{-b^{2}c+a^{2}}-\frac{2a\ln\left(a+\sqrt{dx+c}\ b\right)}{-b^{2}c+a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] 1/(-b^2*c+a^2)*a*ln(d*x)+2*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c
+a^2)-2*a*ln(a+(d*x+c)^(1/2)*b)/(-b^2*c+a^2)
```

maxima [A] time = 2.03, size = 95, normalized size = 1.16

$$\frac{b\sqrt{c}\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^{2}c-a^{2}}-\frac{a\log\left(dx\right)}{b^{2}c-a^{2}}+\frac{2\,a\log\left(\sqrt{dx+c}\,b+a\right)}{b^{2}c-a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^2*c -
 a^2) - a*log(d*x)/(b^2*c - a^2) + 2*a*log(sqrt(d*x + c)*b + a)/(b^2*c - a^
2)
```

mupad [B] time = 3.28, size = 181, normalized size = 2.21

$$\frac{\ln\left(2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c+6\,a\,b^2\,\sqrt{c}\,\sqrt{c+d\,x}\right)}{a+b\,\sqrt{c}}+\frac{\ln\left(-2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c-6\,a\,b^2 \ldots}{a-b\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] log(2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(1/2) - 6*a*b^2*c + 6*a*b^2*c^(1/2)*(
c + d*x)^(1/2))/(a + b*c^(1/2)) + log(- 2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(
1/2) - 6*a*b^2*c - 6*a*b^2*c^(1/2)*(c + d*x)^(1/2))/(a - b*c^(1/2)) + (2*a*
log(4*b^5*c^2*(c + d*x)^(1/2) - 36*a^3*b^2*c + 4*a*b^4*c^2 - 36*a^2*b^3*c*(
c + d*x)^(1/2)))/(b^2*c - a^2)
```

sympy [A] time = 13.01, size = 85, normalized size = 1.04

$$-\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for}\: b=0\\\frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{a^{2}-b^{2}c}-\frac{2\left(-\frac{a\log\left(-dx\right)}{2}+\frac{bc\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}\right)}{a^{2}-b^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] -2*a*b*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b,
True))/(a**2 - b**2*c) - 2*(-a*log(-d*x)/2 + b*c*atan(sqrt(c + d*x)/sqrt(-c
))/sqrt(-c))/(a**2 - b**2*c)
```

3.637 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=130

$$-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{bd\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}$$

```
[Out] a*b^2*d*ln(x)/(-b^2*c+a^2)^2-2*a*b^2*d*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^2
+b*(b^2*c+a^2)*d*arctanh((d*x+c)^(1/2)/c^(1/2))/(-b^2*c+a^2)^2/c^(1/2)+(-a+
b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x
```

Rubi [A] time = 0.18, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{bd\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -((a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x)) + (b*(a^2 + b^2*c)*d*ArcTanh[Sqr
t[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) + (a*b^2*d*Log[x])/(a^2 - b^
2*c)^2 - (2*a*b^2*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx &= d\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)^2}\,dx, x, c+dx\right)\\
&= (2d)\,\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x}+\frac{d\,\text{Subst}\left(\int \frac{-abc+b^2cx}{(a+bx)\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}\\
&= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x}+\frac{d\,\text{Subst}\left(\int \left(-\frac{2ab^3c}{\left(a^2-b^2c\right)(a+bx)}-\frac{bc\left(a^2+b^2c-2abx\right)}{\left(-a^2+b^2c\right)\left(c-x^2\right)}\right)dx, x, \sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}\\
&= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{(bd)\,\text{Subst}\left(\int \frac{a^2+b^2c-2abx}{c-x^2}\,dx, x, \sqrt{c
}{\left(a^2-b^2c\right)^2}\\
&= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}-\frac{\left(2ab^2d\right)\text{Subst}\left(\int \frac{x}{c-x^2}\,dx, x, \sqrt{c+
}{\left(a^2-b^2c\right)^2}\\
&= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x}+\frac{b\left(a^2+b^2c\right)d\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b
}{\left(a^2-b^2
\end{aligned}$$

Mathematica [A] time = 0.19, size = 144, normalized size = 1.11

$$\frac{\sqrt{c}\left(-\left(a^2-b^2c\right)\left(a-b\sqrt{c+dx}\right)-ab^2dx\log\left(a^2-b^2(c+dx)\right)+ab^2dx\log(x)\right)+bdx\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+d
}{\sqrt{c}x\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (-2*a*b^2*Sqrt[c]*d*x*ArcTanh[(b*Sqrt[c + d*x])/a] + b*(a^2 + b^2*c)*d*x*Ar
cTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*(-((a^2 - b^2*c)*(a - b*Sqrt[c + d*x
```

```
])) + a*b^2*d*x*Log[x] - a*b^2*d*x*Log[a^2 - b^2*(c + d*x)]))/(Sqrt[c]*(a^2
 - b^2*c)^2*x)
```

fricas [A] time = 0.61, size = 283, normalized size = 2.18

$$\left[-\frac{4\,ab^2cdx\log\left(\sqrt{dx+c}\,b+a\right)-2\,ab^2cdx\log(x)-2\,ab^2c^2-\left(b^3c+a^2b\right)\sqrt{c}\,dx\log\left(\frac{dx+2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)+2\,a^3c+2\right.}{2\left(b^4c^3-2\,a^2b^2c^2+a^4c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - 2*a*b^2*c*d*x*log(x) - 2*a*
b^2*c^2 - (b^3*c + a^2*b)*sqrt(c)*d*x*log((d*x + 2*sqrt(d*x + c)*sqrt(c) +
2*c)/x) + 2*a^3*c + 2*(b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*
b^2*c^2 + a^4*c)*x), -(2*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - a*b^2*c*d*x
*log(x) - a*b^2*c^2 + (b^3*c + a^2*b)*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqr
t(-c)/c) + a^3*c + (b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*b^2
*c^2 + a^4*c)*x)]
```

giac [A] time = 0.46, size = 191, normalized size = 1.47

$$-\frac{2\,ab^3d\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b}+\frac{ab^2d\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{\left(b^3cd+a^2bd\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{-c}}+\frac{ab^2cd-a^3d-\left(b^3cd-a^2bd\right)}{\left(b^2c-a^2\right)^2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*a*b^3*d*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) +
a*b^2*d*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c*d + a^2*b*d)*arcta
n(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(-c)) + (a*b^2
*c*d - a^3*d - (b^3*c*d - a^2*b*d)*sqrt(d*x + c))/((b^2*c - a^2)^2*d*x)
```

maple [A] time = 0.02, size = 216, normalized size = 1.66

$$\frac{b^3\sqrt{c}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\left(-b^2c+a^2\right)^2}+\frac{a\,b^2d\ln\left(dx\right)}{\left(-b^2c+a^2\right)^2}-\frac{2a\,b^2d\ln\left(a+\sqrt{dx+c}\,b\right)}{\left(-b^2c+a^2\right)^2}+\frac{a^2bd\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\left(-b^2c+a^2\right)^2\sqrt{c}}+\frac{a\,b^2c}{\left(-b^2c+a^2\right)^2x}-\frac{\sqrt{}}{(-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] -1/(-b^2*c+a^2)^2/x*(d*x+c)^(1/2)*b^3*c+1/(-b^2*c+a^2)^2/x*(d*x+c)^(1/2)*b*
a^2+1/(-b^2*c+a^2)^2/x*a*b^2*c-1/(-b^2*c+a^2)^2/x*a^3+d/(-b^2*c+a^2)^2*a*b^
2*ln(d*x)+d/(-b^2*c+a^2)^2*c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*b^3+d/(-b
^2*c+a^2)^2*b/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^2-2*a*b^2*d*ln(a+(d*
x+c)^(1/2)*b)/(-b^2*c+a^2)^2
```

maxima [A] time = 2.04, size = 191, normalized size = 1.47

$$\frac{1}{2}\left(\frac{2\,ab^2\log\left(dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{4\,ab^2\log\left(\sqrt{dx+c}\,b+a\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{\left(b^3c+a^2b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{c}}+\frac{2\left(\sqrt{dx+c}\,b-a\right)}{b^2c^2-a^2c-\left(b^2c-a^2\right)(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b^2*log(d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 4*a*b^2*log(sqrt(d*x
+ c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c + a^2*b)*log((sqrt(d*x +
 c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sq
rt(c)) + 2*(sqrt(d*x + c)*b - a)/(b^2*c^2 - a^2*c - (b^2*c - a^2)*(d*x + c)
))*d
```

mupad [B] time = 3.59, size = 220, normalized size = 1.69

$$\frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)\,\left(4\,a\,b^2\,c\,d-b\,\sqrt{c}\,d\,\left(2\,a^2+2\,c\,b^2\right)\right)}{4\,a^4\,c-8\,a^2\,b^2\,c^2+4\,b^4\,c^3}+\frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)\,\left(4\,a\,b^2\,c\,d+b\,\sqrt{c}\,d\,\left(2\,a^2+2\,c\,b^2\right.\right.}{4\,a^4\,c-8\,a^2\,b^2\,c^2+4\,b^4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] (log((c + d*x)^(1/2) - c^(1/2))*(4*a*b^2*c*d - b*c^(1/2)*d*(2*b^2*c + 2*a^2
)))/(4*a^4*c + 4*b^4*c^3 - 8*a^2*b^2*c^2) + (log((c + d*x)^(1/2) + c^(1/2))
*(4*a*b^2*c*d + b*c^(1/2)*d*(2*b^2*c + 2*a^2)))/(4*a^4*c + 4*b^4*c^3 - 8*a^
2*b^2*c^2) + ((a*d)/(b^2*c - a^2) - (b*d*(c + d*x)^(1/2))/(b^2*c - a^2))/(d
*x) - (2*a*b^2*d*log(a + b*(c + d*x)^(1/2)))/(b^2*c - a^2)^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(a + b*sqrt(c + d*x))), x)
```

3.638 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=204

$$-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4cx\left(a^2-b^2c\right)^2}+\frac{ab^4d^2\log(x)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}-\frac{bd^2\left(a^4-6a^2b^2c\cdots\right.}{4c^{3/2}\cdots}$$

```
[Out] -1/4*b*(-3*b^4*c^2-6*a^2*b^2*c+a^4)*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(3
/2)/(-b^2*c+a^2)^3+a*b^4*d^2*ln(x)/(-b^2*c+a^2)^3-2*a*b^4*d^2*ln(a+b*(d*x+c
)^(1/2))/(-b^2*c+a^2)^3+1/2*(-a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x^2-1/4*b*d*(
4*a*b*c-(3*b^2*c+a^2)*(d*x+c)^(1/2))/c/(-b^2*c+a^2)^2/x
```

Rubi [A] time = 0.28, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$-\frac{bd^2\left(-6a^2b^2c+a^4-3b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\left(a^2-b^2c\right)^3}+\frac{ab^4d^2\log(x)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\left(4a\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2) - (b*d*(4*a*b*c - (a^2 + 3*b^2
*c)*Sqrt[c + d*x]))/(4*c*(a^2 - b^2*c)^2*x) - (b*(a^4 - 6*a^2*b^2*c - 3*b^4
*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(4*c^(3/2)*(a^2 - b^2*c)^3) + (a*
b^4*d^2*Log[x])/(a^2 - b^2*c)^3 - (2*a*b^4*d^2*Log[a + b*Sqrt[c + d*x]])/(a
^2 - b^2*c)^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = d^2\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} + \frac{d^2\,\text{Subst}\left(\int \frac{-abc+3b^2cx}{(a+bx)\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} + \frac{d^2\,\text{Subst}\left(\int \frac{abc\left(a^2-5b^2c\right)+b^2c(\ldots}{(a+bx)\left(-c+\ldots\right.}\ldots\right)}{4c^2\left(a^2-\ldots\right.}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} + \frac{d^2\,\text{Subst}\left(\int\left(-\frac{8ab^5c^2}{\left(a^2-b^2c\right)(a+bx)}\ldots\right.\right.}{\ldots}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}\ldots$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}\ldots$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{b\left(a^4-6a^2b^2c-3b^4c^2\right)d^2\,\text{t}\ldots}{4c^{3/2}\left(a^2-b^2c\ldots\right.}$$

Mathematica [A] time = 0.43, size = 228, normalized size = 1.12

$$\frac{bd^2x^2\left(a^4-6a^2b^2c-3b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\sqrt{c}\left(4ab^4cd^2x^2\log\left(a^2-b^2(c+dx)\right)+\left(a^2-b^2c\right)\left(2a^3c-a^2b\sqrt{\ldots}\right.\right.}{4c^{3/2}x^2\left(\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (8*a*b^4*c^(3/2)*d^2*x^2*ArcTanh[(b*Sqrt[c + d*x])/a] + b*(a^4 - 6*a^2*b^2*
c - 3*b^4*c^2)*d^2*x^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*((a^2 - b^2
*c)*(2*a^3*c - 2*a*b^2*c*(c - 2*d*x) + b^3*c*(2*c - 3*d*x)*Sqrt[c + d*x] -
a^2*b*Sqrt[c + d*x]*(2*c + d*x)) - 4*a*b^4*c*d^2*x^2*Log[x] + 4*a*b^4*c*d^2
*x^2*Log[a^2 - b^2*(c + d*x)]))/(4*c^(3/2)*(-a^2 + b^2*c)^3*x^2)
```

fricas [A] time = 1.12, size = 534, normalized size = 2.62

$$\left[\frac{16\,ab^4c^2d^2x^2\log\left(\sqrt{dx+c}\,b+a\right)-8\,ab^4c^2d^2x^2\log(x)+4\,ab^4c^4-8\,a^3b^2c^3+4\,a^5c^2+\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\sqrt{c}}{8\left(b^6c^5-3\,a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/8*(16*a*b^4*c^2*d^2*x^2*log(sqrt(d*x + c)*b + a) - 8*a*b^4*c^2*d^2*x^2*l
og(x) + 4*a*b^4*c^4 - 8*a^3*b^2*c^3 + 4*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c
- a^4*b)*sqrt(c)*d^2*x^2*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 8*(
a*b^4*c^3 - a^3*b^2*c^2)*d*x - 2*(2*b^5*c^4 - 4*a^2*b^3*c^3 + 2*a^4*b*c^2 -
 (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqrt(d*x + c))/((b^6*c^5 - 3*a^
2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2), 1/4*(8*a*b^4*c^2*d^2*x^2*log(sqr
t(d*x + c)*b + a) - 4*a*b^4*c^2*d^2*x^2*log(x) + 2*a*b^4*c^4 - 4*a^3*b^2*c^
3 + 2*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*sqrt(-c)*d^2*x^2*arctan(s
qrt(d*x + c)*sqrt(-c)/c) - 4*(a*b^4*c^3 - a^3*b^2*c^2)*d*x - (2*b^5*c^4 - 4
*a^2*b^3*c^3 + 2*a^4*b*c^2 - (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqr
t(d*x + c))/((b^6*c^5 - 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2)]
```

giac [A] time = 0.46, size = 375, normalized size = 1.84

$$\frac{2\,ab^5d^2\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^7c^3-3\,a^2b^5c^2+3\,a^4b^3c-a^6b}-\frac{ab^4d^2\log(dx)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}+\frac{\left(3\,b^5c^2d^2+6\,a^2b^3cd^2-a^4bd^2\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{4\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c\right)\sqrt{-c}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*b^5*d^2*log(abs(sqrt(d*x + c)*b + a))/(b^7*c^3 - 3*a^2*b^5*c^2 + 3*a^4*
b^3*c - a^6*b) - a*b^4*d^2*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c
- a^6) + 1/4*(3*b^5*c^2*d^2 + 6*a^2*b^3*c*d^2 - a^4*b*d^2)*arctan(sqrt(d*x
+ c)/sqrt(-c))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(-c))
 + 1/4*(6*a*b^4*c^3*d^2 - 8*a^3*b^2*c^2*d^2 + 2*a^5*c*d^2 + (3*b^5*c^2*d^2
- 2*a^2*b^3*c*d^2 - a^4*b*d^2)*(d*x + c)^(3/2) - 4*(a*b^4*c^2*d^2 - a^3*b^2
*c*d^2)*(d*x + c) - (5*b^5*c^3*d^2 - 6*a^2*b^3*c^2*d^2 + a^4*b*c*d^2)*sqrt(
d*x + c))/((b^2*c - a^2)^3*c*d^2*x^2)
```

maple [B] time = 0.02, size = 459, normalized size = 2.25

$$\frac{3b^5\sqrt{c}\,d^2\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{4\left(-b^2c+a^2\right)^3}+\frac{a\,b^4d^2\ln(dx)}{\left(-b^2c+a^2\right)^3}-\frac{2a\,b^4d^2\ln\left(a+\sqrt{dx+c}\;b\right)}{\left(-b^2c+a^2\right)^3}+\frac{3a^2b^3d^2\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{2\left(-b^2c+a^2\right)^3\sqrt{c}}-\frac{a^4b\,d^2\operatorname{arctan}}{4\left(-b^2c+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+(d*x+c)^(1/2)*b),x)
```

```
[Out] -3/4/(-b^2*c+a^2)^3/x^2*b^5*c*(d*x+c)^(3/2)+1/2/(-b^2*c+a^2)^3/x^2*b^3*(d*x
+c)^(3/2)*a^2+1/4/(-b^2*c+a^2)^3/x^2*b/c*(d*x+c)^(3/2)*a^4+d/(-b^2*c+a^2)^3
/x*a*b^4*c-1/2/(-b^2*c+a^2)^3/x^2*b^4*a*c^2-d/(-b^2*c+a^2)^3/x*a^3*b^2+1/(-
b^2*c+a^2)^3/x^2*b^2*a^3*c-3/2/(-b^2*c+a^2)^3/x^2*(d*x+c)^(1/2)*a^2*b^3*c+1
/4/(-b^2*c+a^2)^3/x^2*(d*x+c)^(1/2)*b*a^4+5/4/(-b^2*c+a^2)^3/x^2*(d*x+c)^(1
/2)*c^2*b^5-1/2/(-b^2*c+a^2)^3/x^2*a^5+d^2/(-b^2*c+a^2)^3*b^4*a*ln(d*x)+3/4
*d^2/(-b^2*c+a^2)^3*b^5*c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))+3/2*d^2/(-b^
2*c+a^2)^3*b^3/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^2-1/4*d^2/(-b^2*c+a
^2)^3*b/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^4-2*a*b^4*d^2*ln(a+(d*x+c)
^(1/2)*b)/(-b^2*c+a^2)^3
```

maxima [A] time = 2.08, size = 367, normalized size = 1.80

$$-\frac{1}{8}\left(\frac{8\,ab^4\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{16\,ab^4\log\left(\sqrt{dx+c}\,b+a\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c\right)\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a*b^4*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) - 16*a
*b^4*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)
 - (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*
x + c) + sqrt(c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(
c)) + 2*(4*(d*x + c)*a*b^2*c - 6*a*b^2*c^2 + 2*a^3*c - (3*b^3*c + a^2*b)*(d
*x + c)^(3/2) + (5*b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/(b^4*c^5 - 2*a^2*b^2*c
^4 + a^4*c^3 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(d*x + c)^2 - 2*(b^4*c^4 -
 2*a^2*b^2*c^3 + a^4*c^2)*(d*x + c)))*d^2
```

mupad [B] time = 5.01, size = 1094, normalized size = 5.36

$$\frac{\ln\left(\frac{b^5\,d^4\,{\left(a^2+3\,c\,b^2\right)}^2\,\sqrt{c+d\,x}}{16\,c^2\,{\left(b^2\,c-a^2\right)}^4}-\frac{a\,b^4\,d^4\,\left(-a^4+2\,a^2\,b^2\,c+15\,b^4\,c^2\right)}{16\,c^2\,{\left(b^2\,c-a^2\right)}^4}-\frac{b\,d^2\,\sqrt{c^3}\,\left(\frac{b^2\,d^2\,\left(3\,b^2\,c-a^2\right)}{4\,c\,\left(b^2\,c-a^2\right)}+\frac{b^2\,d^2\,\sqrt{c^3}\,\left(a^2\,\sqrt{c+d\,x}+4\,a\,b\,c+3\,b^2\,c\,\sqrt{c+d\,x}\right)\,\left(3\,b^4\,c^2-a^4\right.}{4\,c^3\,{\left(b^2\,c-a^2\right)}^3}\right.}{8\,c^3\,(b}\right.}{8\,\left(a^6\,c^3-3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] (log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^(1/2))/(16*c^2*(b^2*c - a^2)^4) -
 (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b^2*c - a^2)^4) - (b
*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c - a^2)) + (b^2*d^2*
(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(3*b^
4*c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)^3) -
 (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^(1/2))/(2*c*(b^2*c - a^2)^2))*(3*b^4*
c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(8*c^3*(b^2*c - a^2)^3))*(8
*a*b^4*c^3*d^2 - a^4*b*d^2*(c^3)^(1/2) + 3*b^5*c^2*d^2*(c^3)^(1/2) + 6*a^2*
b^3*c*d^2*(c^3)^(1/2)))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^2*c^4 + 3*a^2*b^4*c
^5)) - ((a^3*d^2 - 3*a*b^2*c*d^2)/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) - ((a^2
*b*d^2 + 3*b^3*c*d^2)*(c + d*x)^(3/2))/(4*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c))
+ (b*d^2*(5*b^2*c - a^2)*(c + d*x)^(1/2))/(4*(a^4 + b^4*c^2 - 2*a^2*b^2*c))
 + (a*b^2*d^2*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/((c + d*x)^2 - 2*c*
(c + d*x) + c^2) + (log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^(1/2))/(16*c^2
*(b^2*c - a^2)^4) - (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b
^2*c - a^2)^4) - (b*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c
- a^2)) + (b^2*d^2*(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c
+ d*x)^(1/2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(4*c^3
```

```
*(b^2*c - a^2)^3) - (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^(1/2))/(2*c*(b^2*c
 - a^2)^2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(8*c^3*(
b^2*c - a^2)^3))*(8*a*b^4*c^3*d^2 + a^4*b*d^2*(c^3)^(1/2) - 3*b^5*c^2*d^2*(
c^3)^(1/2) - 6*a^2*b^3*c*d^2*(c^3)^(1/2)))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^
2*c^4 + 3*a^2*b^4*c^5)) + (2*a*b^4*d^2*log(a + b*(c + d*x)^(1/2)))/(b^2*c -
 a^2)^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Timed out
```

3.639 $\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=240

$$\frac{2a\left(a^2-b^2c\right)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(7a^2-b^2c\right)\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}-\frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4}-\frac{4a\left(2a^2-3b^2c\right.}{3b^5d}$$

```
[Out] (3*b^4*c^2-9*a^2*b^2*c+5*a^4)*x/b^6/d^3-4/3*a*(-3*b^2*c+2*a^2)*(d*x+c)^(3/2
)/b^5/d^4+3/2*(-b^2*c+a^2)*(d*x+c)^2/b^4/d^4-4/5*a*(d*x+c)^(5/2)/b^3/d^4+1/
3*(d*x+c)^3/b^2/d^4+2*(-b^2*c+a^2)^2*(-b^2*c+7*a^2)*ln(a+b*(d*x+c)^(1/2))/b
^8/d^4-12*a*(-b^2*c+a^2)^2*(d*x+c)^(1/2)/b^7/d^4+2*a*(-b^2*c+a^2)^3/b^8/d^4
/(a+b*(d*x+c)^(1/2))
```

Rubi [A] time = 0.28, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{x\left(-9a^2b^2c+5a^4+3b^4c^2\right)}{b^6d^3}+\frac{2a\left(a^2-b^2c\right)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}-\frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4}+\frac{3\left(a^2-b^2c\right)(c+dx)^2}{2b^4d^4}-\frac{4a\left(2a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((5*a^4 - 9*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3) - (12*a*(a^2 - b^2*c)^2*Sqr
t[c + d*x])/(b^7*d^4) - (4*a*(2*a^2 - 3*b^2*c)*(c + d*x)^(3/2))/(3*b^5*d^4)
 + (3*(a^2 - b^2*c)*(c + d*x)^2)/(2*b^4*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3
*d^4) + (c + d*x)^3/(3*b^2*d^4) + (2*a*(a^2 - b^2*c)^3)/(b^8*d^4*(a + b*Sqr
t[c + d*x])) + (2*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])
/(b^8*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^3}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{6a\left(a^2-b^2c\right)^2}{b^7} + \frac{\left(5a^4-9a^2b^2c+3b^4c^2\right)x}{b^6} - \frac{2a\left(2a^2-3b^2c\right)x^2}{b^5} - \frac{3\left(-a^2+b^2c\right)x^3}{b^4} - \frac{2ax^4}{b^3} + \frac{x^5}{b^2} - \right.\right.}{d^4}$$

$$= \frac{\left(5a^4-9a^2b^2c+3b^4c^2\right)x}{b^6d^3} - \frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4} - \frac{4a\left(2a^2-3b^2c\right)(c+dx)^{3/2}}{3b^5d^4} + \frac{3\left(\right.}{}$$

Mathematica [A] time = 0.27, size = 273, normalized size = 1.14

$$\frac{96a^7 - 324a^6b\sqrt{c+dx} - 6a^5b^2(102c+35dx) + 2a^4b^3\sqrt{c+dx}\,(284c+35dx) + a^3b^4\left(856c^2+380cdx-35d^2x^2\right) -}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (96*a^7 - 324*a^6*b*Sqrt[c + d*x] - 6*a^5*b^2*(102*c + 35*d*x) + 2*a^4*b^3*
Sqrt[c + d*x]*(284*c + 35*d*x) + a^3*b^4*(856*c^2 + 380*c*d*x - 35*d^2*x^2)
 - 3*a^2*b^5*Sqrt[c + d*x]*(76*c^2 + 36*c*d*x - 7*d^2*x^2) + 5*b^7*d*x*Sqrt
[c + d*x]*(6*c^2 - 3*c*d*x + 2*d^2*x^2) - a*b^6*(324*c^3 + 162*c^2*d*x - 33
*c*d^2*x^2 + 14*d^3*x^3) + 60*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c
 + d*x])*Log[a + b*Sqrt[c + d*x]])/(30*b^8*d^4*(a + b*Sqrt[c + d*x]))
```

fricas [A] time = 0.46, size = 392, normalized size = 1.63

$$\frac{10\,b^8d^4x^4 + 55\,b^8c^4 - 220\,a^2b^6c^3 + 195\,a^4b^4c^2 + 30\,a^6b^2c - 60\,a^8 - 5\left(b^8c - 7\,a^2b^6\right)d^3x^3 + 15\left(b^8c^2 - 8\,a^2b^6c + 7\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^8*d^4*x^4 + 55*b^8*c^4 - 220*a^2*b^6*c^3 + 195*a^4*b^4*c^2 + 30*
a^6*b^2*c - 60*a^8 - 5*(b^8*c - 7*a^2*b^6)*d^3*x^3 + 15*(b^8*c^2 - 8*a^2*b^
6*c + 7*a^4*b^4)*d^2*x^2 + 5*(17*b^8*c^3 - 87*a^2*b^6*c^2 + 96*a^4*b^4*c -
30*a^6*b^2)*d*x - 60*(b^8*c^4 - 10*a^2*b^6*c^3 + 24*a^4*b^4*c^2 - 22*a^6*b^
2*c + 7*a^8 + (b^8*c^3 - 9*a^2*b^6*c^2 + 15*a^4*b^4*c - 7*a^6*b^2)*d*x)*log
(sqrt(d*x + c)*b + a) - 4*(6*a*b^7*d^3*x^3 + 81*a*b^7*c^3 - 271*a^3*b^5*c^2
 + 295*a^5*b^3*c - 105*a^7*b - 2*(6*a*b^7*c - 7*a^3*b^5)*d^2*x^2 + 2*(24*a*
b^7*c^2 - 61*a^3*b^5*c + 35*a^5*b^3)*d*x)*sqrt(d*x + c))/(b^10*d^5*x + (b^1
0*c - a^2*b^8)*d^4)
```

giac [A] time = 0.48, size = 324, normalized size = 1.35

$$-\frac{2\left(b^6c^3 - 9\,a^2b^4c^2 + 15\,a^4b^2c - 7\,a^6\right)\log\left(\left|\sqrt{dx+c}\,b + a\right|\right)}{b^8d^4} - \frac{2\left(ab^6c^3 - 3\,a^3b^4c^2 + 3\,a^5b^2c - a^7\right)}{\left(\sqrt{dx+c}\,b + a\right)b^8d^4} + \frac{10\,(dx+c)^3b^{10}\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -2*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6)*log(abs(sqrt(d*x + c)*b
 + a))/(b^8*d^4) - 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/((sqrt
(d*x + c)*b + a)*b^8*d^4) + 1/30*(10*(d*x + c)^3*b^10*d^20 - 45*(d*x + c)^2
*b^10*c*d^20 + 90*(d*x + c)*b^10*c^2*d^20 - 24*(d*x + c)^(5/2)*a*b^9*d^20 +
 120*(d*x + c)^(3/2)*a*b^9*c*d^20 - 360*sqrt(d*x + c)*a*b^9*c^2*d^20 + 45*(
d*x + c)^2*a^2*b^8*d^20 - 270*(d*x + c)*a^2*b^8*c*d^20 - 80*(d*x + c)^(3/2)
*a^3*b^7*d^20 + 720*sqrt(d*x + c)*a^3*b^7*c*d^20 + 150*(d*x + c)*a^4*b^6*d^
20 - 360*sqrt(d*x + c)*a^5*b^5*d^20)/(b^12*d^24)
```

maple [A] time = 0.01, size = 416, normalized size = 1.73

$$\frac{x^3}{3b^2d}-\frac{c\,x^2}{2b^2d^2}+\frac{3a^2x^2}{2b^4d^2}-\frac{2a\,c^3}{\left(a+\sqrt{dx+c}\;b\right)b^2d^4}-\frac{2c^3\ln\left(a+\sqrt{dx+c}\;b\right)}{b^2d^4}+\frac{c^2x}{b^2d^3}+\frac{6a^3c^2}{\left(a+\sqrt{dx+c}\;b\right)b^4d^4}+\frac{18a^2c^2\ln\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] 1/3/d/b^2*x^3-1/2/d^2/b^2*x^2*c+1/d^3/b^2*x*c^2+11/6/d^4/b^2*c^3-4/5*a*(d*x
+c)^(5/2)/b^3/d^4+3/2/d^2/b^4*x^2*a^2-6/d^3/b^4*x*a^2*c-15/2/d^4/b^4*a^2*c^
2+4/d^4/b^3*(d*x+c)^(3/2)*a*c-8/3/d^4/b^5*(d*x+c)^(3/2)*a^3-12/d^4/b^3*a*c^
2*(d*x+c)^(1/2)+5/d^3/b^6*x*a^4+5/d^4/b^6*a^4*c+24/d^4/b^5*a^3*c*(d*x+c)^(1
/2)-12/d^4/b^7*a^5*(d*x+c)^(1/2)-2/d^4*a/b^2/(a+(d*x+c)^(1/2)*b)*c^3+6/d^4*
a^3/b^4/(a+(d*x+c)^(1/2)*b)*c^2-6/d^4*a^5/b^6/(a+(d*x+c)^(1/2)*b)*c+2/d^4*a
^7/b^8/(a+(d*x+c)^(1/2)*b)-2/d^4/b^2*ln(a+(d*x+c)^(1/2)*b)*c^3+18/d^4/b^4*l
n(a+(d*x+c)^(1/2)*b)*a^2*c^2-30/d^4/b^6*ln(a+(d*x+c)^(1/2)*b)*a^4*c+14/d^4/
b^8*ln(a+(d*x+c)^(1/2)*b)*a^6
```

maxima [A] time = 0.93, size = 251, normalized size = 1.05

$$-\frac{\frac{60\left(ab^6c^3-3a^3b^4c^2+3a^5b^2c-a^7\right)}{\sqrt{dx+c}\,b^9+ab^8}-\frac{10\,(dx+c)^3b^5-24\,(dx+c)^{\frac{5}{2}}ab^4-45\left(b^5c-a^2b^3\right)(dx+c)^2+40\left(3\,ab^4c-2\,a^3b^2\right)(dx+c)^{\frac{3}{2}}+30\left(3\,b^5c^2-9\,a^2b^3c+5\,a^4b\cdots\right.}{b^7}\cdots}{30\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(60*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/(sqrt(d*x + c)*b^
9 + a*b^8) - (10*(d*x + c)^3*b^5 - 24*(d*x + c)^(5/2)*a*b^4 - 45*(b^5*c - a
^2*b^3)*(d*x + c)^2 + 40*(3*a*b^4*c - 2*a^3*b^2)*(d*x + c)^(3/2) + 30*(3*b^
5*c^2 - 9*a^2*b^3*c + 5*a^4*b)*(d*x + c) - 360*(a*b^4*c^2 - 2*a^3*b^2*c + a
^5)*sqrt(d*x + c))/b^7 + 60*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6
)*log(sqrt(d*x + c)*b + a)/b^8)/d^4
```

mupad [B] time = 0.09, size = 461, normalized size = 1.92

$$\left(\frac{4\,a^3}{3\,b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{3\,b}\right)(c+d\,x)^{3/2}-\left(\frac{3\,c}{2\,b^2\,d^4}-\frac{3\,a^2}{2\,b^4\,d^4}\right)(c+d\,x)^2-\left(\frac{2\,a\left(\frac{a^2\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b^2}-\frac{2\,a\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\left(\frac{6\,c}{b^2\,d^4}\cdots\right.}{b}\cdots\right)}{b}\cdots\right)}{b}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] ((4*a^3)/(3*b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(3*b))*(
c + d*x)^(3/2) - ((3*c)/(2*b^2*d^4) - (3*a^2)/(2*b^4*d^4))*(c + d*x)^2 - ((
2*a*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b^2 - (2*a*((4*a^3)/(b^5*d
^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (6*c^2)/(b^2*d^4)
))/b + (a^2*((4*a^3)/(b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4))
)/b))/b^2)*(c + d*x)^(1/2) + (c + d*x)^3/(3*b^2*d^4) + (2*(a^7 - 3*a^5*b^2*
c - a*b^6*c^3 + 3*a^3*b^4*c^2))/(b*(b^8*d^4*(c + d*x)^(1/2) + a*b^7*d^4)) +
 d*x*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(2*b^2) - (a*((4*a^3)/(b^
5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (3*c^2)/(b^2*d
^4)) + (log(a + b*(c + d*x)^(1/2))*(14*a^6 - 2*b^6*c^3 - 30*a^4*b^2*c + 18*
a^2*b^4*c^2))/(b^8*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3*d^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a + b\sqrt{c + dx}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(a + b*sqrt(c + d*x))**2, x)
```

3.640 $\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=166

$$\frac{2a\left(a^2-b^2c\right)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}-\frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3}+\frac{x\left(3a^2-2b^2c\right)}{b^4d^2}+\frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}-4$$

```
[Out] (-2*b^2*c+3*a^2)*x/b^4/d^2-4/3*a*(d*x+c)^(3/2)/b^3/d^3+1/2*(d*x+c)^2/b^2/d^
3+2*(b^4*c^2-6*a^2*b^2*c+5*a^4)*ln(a+b*(d*x+c)^(1/2))/b^6/d^3-8*a*(-b^2*c+a
^2)*(d*x+c)^(1/2)/b^5/d^3+2*a*(-b^2*c+a^2)^2/b^6/d^3/(a+b*(d*x+c)^(1/2))
```

Rubi [A] time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{2\left(-6a^2b^2c+5a^4+b^4c^2\right)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{2a\left(a^2-b^2c\right)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}-\frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3}+\frac{x\left(3a^2-2b^2c\right)}{b^4d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - 2*b^2*c)*x)/(b^4*d^2) - (8*a*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^5*d^
3) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (c + d*x)^2/(2*b^2*d^3) + (2*a*(a^
2 - b^2*c)^2)/(b^6*d^3*(a + b*Sqrt[c + d*x])) + (2*(5*a^4 - 6*a^2*b^2*c + b
^4*c^2)*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^2}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{4a\left(a^2-b^2c\right)}{b^5}-\frac{\left(-3a^2+2b^2c\right)x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a\left(a^2-b^2c\right)^2}{b^5(a+bx)^2}+\frac{5a^4-6a^2b^2c+b^4c^2}{b^5(a+bx)}\right)dx, x, \sqrt{c\right.}}{d^3}$$

$$= \frac{\left(3a^2-2b^2c\right)x}{b^4d^2}-\frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3}-\frac{4a(c+dx)^{3/2}}{3b^3d^3}+\frac{(c+dx)^2}{2b^2d^3}+\frac{2a\left(a^2-b^2c\right)}{b^6d^3\left(a+b\sqrt{c+\right.}}$$

Mathematica [A] time = 0.17, size = 185, normalized size = 1.11

$$\frac{16a^5-44a^4b\sqrt{c+dx}-2a^3b^2(38c+15dx)+2a^2b^3\sqrt{c+dx}\,(18c+5dx)+12\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right.}{6b^6d^3\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (16*a^5 - 44*a^4*b*Sqrt[c + d*x] + 3*b^5*d*x*(-2*c + d*x)*Sqrt[c + d*x] + 2
*a^2*b^3*Sqrt[c + d*x]*(18*c + 5*d*x) - 2*a^3*b^2*(38*c + 15*d*x) + a*b^4*(
52*c^2 + 26*c*d*x - 5*d^2*x^2) + 12*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*
Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(6*b^6*d^3*(a + b*Sqrt[c + d*x]))
```

fricas [A] time = 0.47, size = 269, normalized size = 1.62

$$\frac{3\,b^6d^3x^3-9\,b^6c^3+15\,a^2b^4c^2+6\,a^4b^2c-12\,a^6-3\left(b^6c-5\,a^2b^4\right)d^2x^2-3\left(5\,b^6c^2-14\,a^2b^4c+6\,a^4b^2\right)dx+12\left(b^6c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^6*d^3*x^3 - 9*b^6*c^3 + 15*a^2*b^4*c^2 + 6*a^4*b^2*c - 12*a^6 - 3*
(b^6*c - 5*a^2*b^4)*d^2*x^2 - 3*(5*b^6*c^2 - 14*a^2*b^4*c + 6*a^4*b^2)*d*x
+ 12*(b^6*c^3 - 7*a^2*b^4*c^2 + 11*a^4*b^2*c - 5*a^6 + (b^6*c^2 - 6*a^2*b^4
*c + 5*a^4*b^2)*d*x)*log(sqrt(d*x + c)*b + a) - 4*(2*a*b^5*d^2*x^2 - 13*a*b
^5*c^2 + 28*a^3*b^3*c - 15*a^5*b - 2*(4*a*b^5*c - 5*a^3*b^3)*d*x)*sqrt(d*x
+ c))/(b^8*d^4*x + (b^8*c - a^2*b^6)*d^3)
```

giac [A] time = 0.37, size = 191, normalized size = 1.15

$$\frac{2\left(b^4c^2-6\,a^2b^2c+5\,a^4\right)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^6d^3}+\frac{2\left(ab^4c^2-2\,a^3b^2c+a^5\right)}{\left(\sqrt{dx+c}\,b+a\right)b^6d^3}+\frac{3\,(dx+c)^2b^6d^9-12\,(dx+c)b^6cd^9-8\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3) +
 2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/((sqrt(d*x + c)*b + a)*b^6*d^3) + 1/6*(3
```

```
*(d*x + c)^2*b^6*d^9 - 12*(d*x + c)*b^6*c*d^9 - 8*(d*x + c)^(3/2)*a*b^5*d^9
 + 48*sqrt(d*x + c)*a*b^5*c*d^9 + 18*(d*x + c)*a^2*b^4*d^9 - 48*sqrt(d*x +
c)*a^3*b^3*d^9)/(b^8*d^12)
```

maple [A] time = 0.01, size = 253, normalized size = 1.52

$$\frac{x^2}{2b^2d}+\frac{2a\,c^2}{\left(a+\sqrt{dx+c}\;b\right)b^2d^3}+\frac{2c^2\ln\left(a+\sqrt{dx+c}\;b\right)}{b^2d^3}-\frac{cx}{b^2d^2}-\frac{4a^3c}{\left(a+\sqrt{dx+c}\;b\right)b^4d^3}-\frac{12a^2c\ln\left(a+\sqrt{dx+c}\;b\right)}{b^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] 1/2/d/b^2*x^2-1/d^2/b^2*x*c-3/2/d^3/b^2*c^2-4/3*a*(d*x+c)^(3/2)/b^3/d^3+3/d
^2/b^4*x*a^2+3/d^3/b^4*a^2*c+8/d^3/b^3*a*c*(d*x+c)^(1/2)-8/d^3/b^5*a^3*(d*x
+c)^(1/2)+2/d^3*a/b^2/(a+(d*x+c)^(1/2)*b)*c^2-4/d^3*a^3/b^4/(a+(d*x+c)^(1/2
)*b)*c+2/d^3*a^5/b^6/(a+(d*x+c)^(1/2)*b)+2/d^3/b^2*ln(a+(d*x+c)^(1/2)*b)*c^
2-12/d^3/b^4*ln(a+(d*x+c)^(1/2)*b)*a^2*c+10/d^3/b^6*ln(a+(d*x+c)^(1/2)*b)*a
^4
```

maxima [A] time = 0.93, size = 158, normalized size = 0.95

$$\frac{\frac{12\left(ab^4c^2-2a^3b^2c+a^5\right)}{\sqrt{dx+c}\,b^7+ab^6}+\frac{3(dx+c)^2b^3-8(dx+c)^{\frac{3}{2}}ab^2-6\left(2b^3c-3a^2b\right)(dx+c)+48\left(ab^2c-a^3\right)\sqrt{dx+c}}{b^5}+\frac{12\left(b^4c^2-6a^2b^2c+5a^4\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^6}}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(12*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/(sqrt(d*x + c)*b^7 + a*b^6) + (3*(d
*x + c)^2*b^3 - 8*(d*x + c)^(3/2)*a*b^2 - 6*(2*b^3*c - 3*a^2*b)*(d*x + c) +
 48*(a*b^2*c - a^3)*sqrt(d*x + c))/b^5 + 12*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)
*log(sqrt(d*x + c)*b + a)/b^6)/d^3
```

mupad [B] time = 3.20, size = 197, normalized size = 1.19

$$\left(\frac{4\,a^3}{b^5\,d^3}+\frac{2\,a\,\left(\frac{4\,c}{b^2\,d^3}-\frac{6\,a^2}{b^4\,d^3}\right)}{b}\right)\sqrt{c+d\,x}+\frac{2\,\left(a^5-2\,a^3\,b^2\,c+a\,b^4\,c^2\right)}{b\,\left(b^6\,d^3\,\sqrt{c+d\,x}+a\,b^5\,d^3\right)}+\frac{(c+d\,x)^2}{2\,b^2\,d^3}-d\,x\,\left(\frac{2\,c}{b^2\,d^3}-\frac{3\,a^2}{b^4\,d^3}\right)-\frac{4\,a\,(c+d\,x)^{3/2}}{3\,b^3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] ((4*a^3)/(b^5*d^3) + (2*a*((4*c)/(b^2*d^3) - (6*a^2)/(b^4*d^3)))/b)*(c + d*
x)^(1/2) + (2*(a^5 - 2*a^3*b^2*c + a*b^4*c^2))/(b*(b^6*d^3*(c + d*x)^(1/2)
+ a*b^5*d^3)) + (c + d*x)^2/(2*b^2*d^3) - d*x*((2*c)/(b^2*d^3) - (3*a^2)/(b
^4*d^3)) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (log(a + b*(c + d*x)^(1/2))*
(10*a^4 + 2*b^4*c^2 - 12*a^2*b^2*c))/(b^6*d^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(a + b*sqrt(c + d*x))**2, x)
```

3.641 $\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{x}{b^2d}$$

```
[Out] x/b^2/d+2*(-b^2*c+3*a^2)*ln(a+b*(d*x+c)^(1/2))/b^4/d^2-4*a*(d*x+c)^(1/2)/b^
3/d^2+2*a*(-b^2*c+a^2)/b^4/d^2/(a+b*(d*x+c)^(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {371, 1398, 772}

$$\frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{x}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] x/(b^2*d) - (4*a*Sqrt[c + d*x])/(b^3*d^2) + (2*a*(a^2 - b^2*c))/(b^4*d^2*(a
 + b*Sqrt[c + d*x])) + (2*(3*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^
2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x(-c+x^2)}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}+\frac{-a^3+ab^2c}{b^3(a+bx)^2}+\frac{3a^2-b^2c}{b^3(a+bx)}\right)dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= \frac{x}{b^2d}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Mathematica [A] time = 0.09, size = 112, normalized size = 1.18

$$\frac{2a^3+2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)\log\left(a+b\sqrt{c+dx}\right)-4a^2b\sqrt{c+dx}-3ab^2(2c+dx)+b^3dx\sqrt{c+dx}}{b^4d^2\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (2*a^3 - 4*a^2*b*Sqrt[c + d*x] + b^3*d*x*Sqrt[c + d*x] - 3*a*b^2*(2*c + d*x
) + 2*(3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(b^4*
d^2*(a + b*Sqrt[c + d*x]))
```

fricas [A] time = 0.47, size = 163, normalized size = 1.72

$$\frac{b^4d^2x^2+b^4c^2+a^2b^2c-2\,a^4+\left(2\,b^4c-a^2b^2\right)dx-2\left(b^4c^2-4\,a^2b^2c+3\,a^4+\left(b^4c-3\,a^2b^2\right)dx\right)\log\left(\sqrt{dx+c}\,b\right.}{b^6d^3x+\left(b^6c-a^2b^4\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] (b^4*d^2*x^2 + b^4*c^2 + a^2*b^2*c - 2*a^4 + (2*b^4*c - a^2*b^2)*d*x - 2*(b
^4*c^2 - 4*a^2*b^2*c + 3*a^4 + (b^4*c - 3*a^2*b^2)*d*x)*log(sqrt(d*x + c)*b
 + a) - 2*(2*a*b^3*d*x + 3*a*b^3*c - 3*a^3*b)*sqrt(d*x + c))/(b^6*d^3*x + (
b^6*c - a^2*b^4)*d^2)
```

giac [A] time = 0.41, size = 102, normalized size = 1.07

$$-\frac{\frac{2\left(b^2c-3\,a^2\right)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^4d}-\frac{(dx+c)b^2d-4\,\sqrt{dx+c}\,abd}{b^4d^2}+\frac{2\left(ab^2c-a^3\right)}{\left(\sqrt{dx+c}\,b+a\right)b^4d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -(2*(b^2*c - 3*a^2)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) - ((d*x + c)*b^2*
d - 4*sqrt(d*x + c)*a*b*d)/(b^4*d^2) + 2*(a*b^2*c - a^3)/((sqrt(d*x + c)*b
+ a)*b^4*d))/d
```

maple [A] time = 0.01, size = 125, normalized size = 1.32

$$-\frac{2ac}{\left(a+\sqrt{dx+c}\;b\right)b^2d^2}-\frac{2c\ln\left(a+\sqrt{dx+c}\;b\right)}{b^2d^2}+\frac{x}{b^2d}+\frac{2a^3}{\left(a+\sqrt{dx+c}\;b\right)b^4d^2}+\frac{6a^2\ln\left(a+\sqrt{dx+c}\;b\right)}{b^4d^2}+\frac{c}{b^2d^2}-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] x/b^2/d+1/d^2/b^2*c-4*a*(d*x+c)^(1/2)/b^3/d^2-2/d^2*a/b^2/(a+(d*x+c)^(1/2)*
b)*c+2/d^2*a^3/b^4/(a+(d*x+c)^(1/2)*b)-2/d^2/b^2*ln(a+(d*x+c)^(1/2)*b)*c+6/
d^2/b^4*ln(a+(d*x+c)^(1/2)*b)*a^2
```

maxima [A] time = 1.05, size = 90, normalized size = 0.95

$$-\frac{\frac{2\left(ab^2c-a^3\right)}{\sqrt{dx+c}\,b^5+ab^4}-\frac{(dx+c)b-4\,\sqrt{dx+c}\,a}{b^3}+\frac{2\left(b^2c-3\,a^2\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^4}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b^2*c - a^3)/(sqrt(d*x + c)*b^5 + a*b^4) - ((d*x + c)*b - 4*sqrt(d*x
 + c)*a)/b^3 + 2*(b^2*c - 3*a^2)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

mupad [B] time = 0.06, size = 98, normalized size = 1.03

$$\frac{x}{b^2\,d}+\frac{2\,\left(a^3-a\,b^2\,c\right)}{b\,\left(b^4\,d^2\,\sqrt{c+d\,x}+a\,b^3\,d^2\right)}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,b^2\,c-6\,a^2\right)}{b^4\,d^2}-\frac{4\,a\,\sqrt{c+d\,x}}{b^3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] x/(b^2*d) + (2*(a^3 - a*b^2*c))/(b*(b^4*d^2*(c + d*x)^(1/2) + a*b^3*d^2)) -
 (log(a + b*(c + d*x)^(1/2))*(2*b^2*c - 6*a^2))/(b^4*d^2) - (4*a*(c + d*x)^
(1/2))/(b^3*d^2)
```

sympy [A] time = 42.38, size = 131, normalized size = 1.38

$$\begin{cases} \frac{2\left(-\frac{a\left(a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for}\ b=0\\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{b^3d}-\frac{2a\sqrt{c+dx}}{b^3d}+\frac{c+dx}{2b^2d}+\frac{\left(3a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^3d}\right)}{d} & \text{for}\ d\neq 0\\ \frac{x^2}{2\left(a+b\sqrt{c}\right)^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*(-a*(a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)),
(-1/(b*(a + b*sqrt(c + d*x))), True))/(b**3*d) - 2*a*sqrt(c + d*x)/(b**3*d)
 + (c + d*x)/(2*b**2*d) + (3*a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(
b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(b**3*d))/d, Ne(d, 0)), (x**2/(
2*(a + b*sqrt(c))**2), True))
```

3.642 $\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=47

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] 2*ln(a+b*(d*x+c)^(1/2))/b^2/d+2*a/b^2/d/(a+b*(d*x+c)^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 190, 43}

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^(-2),x]
```

```
[Out] (2*a)/(b^2*d*(a + b*Sqrt[c + d*x])) + (2*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.85

$$\frac{2\left(\frac{a}{a+b\sqrt{c+dx}}+\log\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^(-2),x]
[Out] (2*(a/(a + b*Sqrt[c + d*x]) + Log[a + b*Sqrt[c + d*x]]))/(b^2*d)
```

fricas [A] time = 0.46, size = 75, normalized size = 1.60

$$\frac{2\left(\sqrt{dx+c}\,ab-a^2+\left(b^2dx+b^2c-a^2\right)\log\left(\sqrt{dx+c}\,b+a\right)\right)}{b^4d^2x+\left(b^4c-a^2b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
[Out] 2*(sqrt(d*x + c)*a*b - a^2 + (b^2*d*x + b^2*c - a^2)*log(sqrt(d*x + c)*b +
a))/(b^4*d^2*x + (b^4*c - a^2*b^2)*d)
```

giac [A] time = 0.34, size = 44, normalized size = 0.94

$$\frac{2\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^2d}+\frac{2\,a}{\left(\sqrt{dx+c}\,b+a\right)b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
[Out] 2*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*a/((sqrt(d*x + c)*b + a)*b^2*d)
```

maple [B] time = 0.02, size = 142, normalized size = 3.02

$$-\frac{2a^2}{\left(b^2dx+b^2c-a^2\right)b^2d}+\frac{a}{\left(-a+\sqrt{dx+c}\;b\right)b^2d}+\frac{a}{\left(a+\sqrt{dx+c}\;b\right)b^2d}-\frac{\ln\left(-a+\sqrt{dx+c}\;b\right)}{b^2d}+\frac{\ln\left(a+\sqrt{dx+c}\;b\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(d*x+c)^(1/2)*b)^2,x)
[Out] -2*a^2/(b^2*d*x+b^2*c-a^2)/b^2/d+ln(b^2*d*x+b^2*c-a^2)/b^2/d+a/b^2/d/(-a+(d
*x+c)^(1/2)*b)-1/b^2/d*ln(-a+(d*x+c)^(1/2)*b)+a/b^2/d/(a+(d*x+c)^(1/2)*b)+l
n(a+(d*x+c)^(1/2)*b)/b^2/d
```

maxima [A] time = 0.87, size = 43, normalized size = 0.91

$$\frac{2\left(\frac{a}{\sqrt{dx+c}\,b^3+ab^2}+\frac{\log\left(\sqrt{dx+c}\,b+a\right)}{b^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
[Out] 2*(a/(sqrt(d*x + c)*b^3 + a*b^2) + log(sqrt(d*x + c)*b + a)/b^2)/d
```

mupad [B] time = 0.05, size = 43, normalized size = 0.91

$$\frac{2\ln\left(a+b\sqrt{c+dx}\right)}{b^2 d}+\frac{2a}{b^2\left(ad+bd\sqrt{c+dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (2*log(a + b*(c + d*x)^(1/2)))/(b^2*d) + (2*a)/(b^2*(a*d + b*d*(c + d*x)^(1
/2)))
```

sympy [A] time = 1.09, size = 124, normalized size = 2.64

$$\begin{cases} \frac{x}{a^2} & \text{for } b=0 \wedge (b=0 \vee d=0) \\ \frac{x}{\left(a+b\sqrt{c}\right)^2} & \text{for } d=0 \\ \frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2a}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((x/a**2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c))**2
, Eq(d, 0)), (2*a*log(a/b + sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)
) + 2*a/(a*b**2*d + b**3*d*sqrt(c + d*x)) + 2*b*sqrt(c + d*x)*log(a/b + sqr
t(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)), True))
```

3.643 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=129

$$\frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\left(a^2-b^2c\right)^2}+\frac{\log(x)\left(a^2+b^2c\right)}{\left(a^2-b^2c\right)^2}$$

```
[Out] (b^2*c+a^2)*ln(x)/(-b^2*c+a^2)^2-2*(b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/(-b^2*
c+a^2)^2+4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)^2+2*a/(-
b^2*c+a^2)/(a+b*(d*x+c)^(1/2))
```

Rubi [A] time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 801, 635, 206, 260}

$$\frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\left(a^2-b^2c\right)^2}+\frac{\log(x)\left(a^2+b^2c\right)}{\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (2*a)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (4*a*b*Sqrt[c]*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])/(a^2 - b^2*c)^2 + ((a^2 + b^2*c)*Log[x])/(a^2 - b^2*c)^2
- (2*(a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)}\,dx, x, c+dx\right)$$

$$= 2\,\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{ab}{\left(a^2-b^2c\right)(a+bx)^2} - \frac{b\left(a^2+b^2c\right)}{\left(a^2-b^2c\right)^2(a+bx)} + \frac{2abc-\left(a^2+b^2c\right)x}{\left(a^2-b^2c\right)^2\left(c-x^2\right)}\right) \ldots$$

$$= \frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)} - \frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2} + \frac{2\,\text{Subst}\left(\int \frac{2abc-(a \ldots}{c- \ldots} \ldots\right)}{(a^2 \ldots}$$

$$= \frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)} - \frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2} + \frac{(4abc)\,\text{Subst}\left(\int \ldots\right)}{(a^2 - \ldots}$$

$$= \frac{2a}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)} + \frac{4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\left(a^2-b^2c\right)^2} + \frac{\left(a^2+b^2c\right)\log(x)}{\left(a^2-b^2c\right)^2} - \frac{2\left(a^2 \ldots\right.}{\ldots}$$

Mathematica [A] time = 0.28, size = 164, normalized size = 1.27

$$\frac{2a^3-2\left(a^2+b^2c\right)\left(a+b\sqrt{c+dx}\right)\log\left(a+b\sqrt{c+dx}\right)-2ab^2c+\left(a-b\sqrt{c}\right)^2\log\left(\sqrt{c}-\sqrt{c+dx}\right)\left(a+b\sqrt{c+\ldots}\right.}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (2*a^3 - 2*a*b^2*c + (a - b*Sqrt[c])^2*(a + b*Sqrt[c + d*x])*Log[Sqrt[c] -
Sqrt[c + d*x]] + (a + b*Sqrt[c])^2*(a + b*Sqrt[c + d*x])*Log[Sqrt[c] + Sqrt
[c + d*x]] - 2*(a^2 + b^2*c)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]]
)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x]))
```

fricas [A] time = 0.55, size = 444, normalized size = 3.44

$$\left[\frac{2\,a^2b^2c-2\,a^4+2\left(ab^3dx+ab^3c-a^3b\right)\sqrt{c}\,\log\left(\frac{dx+2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)-2\left(b^4c^2-a^4+\left(b^4c+a^2b^2\right)dx\right)\log\left(\sqrt{dx\ldots}\right.}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6+\left(b^6c^2-2\,a^2b^4c\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a^2*b^2*c - 2*a^4 + 2*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(c)*log((d*x +
2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*
```

```
x)*log(sqrt(d*x + c)*b + a) + (b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x
) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2
*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c + a^4*b^2)*d*x), (2*a^2*b^2*c - 2*a^4 - 4
*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) -
2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(sqrt(d*x + c)*b + a) + (b^4*c
^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c
))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c +
a^4*b^2)*d*x)]
```

giac [A] time = 0.41, size = 174, normalized size = 1.35

$$-\frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{-c}}+\frac{\left(b^2c+a^2\right)\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{2\left(b^3c+a^2b\right)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b}-\frac{2\left(ab^2c-a^3\right)}{\left(b^2c-a^2\right)^2\left(\sqrt{dx+c}\,b+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt
(-c)) + (b^2*c + a^2)*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 2*(b^3*c +
a^2*b)*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) - 2*(a
*b^2*c - a^3)/((b^2*c - a^2)^2*(sqrt(d*x + c)*b + a))
```

maple [A] time = 0.01, size = 161, normalized size = 1.25

$$\frac{b^2c\ln\left(dx\right)}{\left(-b^2c+a^2\right)^2}-\frac{2b^2c\ln\left(a+\sqrt{dx+c}\;b\right)}{\left(-b^2c+a^2\right)^2}+\frac{4ab\sqrt{c}\;\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\left(-b^2c+a^2\right)^2}+\frac{a^2\ln\left(dx\right)}{\left(-b^2c+a^2\right)^2}-\frac{2a^2\ln\left(a+\sqrt{dx+c}\;b\right)}{\left(-b^2c+a^2\right)^2}+\frac{}{\left(-b^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] 1/(-b^2*c+a^2)^2*ln(d*x)*b^2*c+1/(-b^2*c+a^2)^2*ln(d*x)*a^2+4*a*b*arctanh((
d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)^2+2*a/(-b^2*c+a^2)/(a+(d*x+c)^(1
/2)*b)-2/(-b^2*c+a^2)^2*ln(a+(d*x+c)^(1/2)*b)*b^2*c-2/(-b^2*c+a^2)^2*ln(a+(
d*x+c)^(1/2)*b)*a^2
```

maxima [A] time = 1.99, size = 176, normalized size = 1.36

$$-\frac{2\,ab\sqrt{c}\,\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^4c^2-2\,a^2b^2c+a^4}+\frac{\left(b^2c+a^2\right)\log\left(dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{2\left(b^2c+a^2\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{2\,a}{ab^2c-a^3+\left(b^3c-a^2b\right)\sqrt{dx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^
4*c^2 - 2*a^2*b^2*c + a^4) + (b^2*c + a^2)*log(d*x)/(b^4*c^2 - 2*a^2*b^2*c
+ a^4) - 2*(b^2*c + a^2)*log(sqrt(d*x + c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c +
a^4) - 2*a/(a*b^2*c - a^3 + (b^3*c - a^2*b)*sqrt(d*x + c))
```

mupad [B] time = 3.51, size = 125, normalized size = 0.97

$$\frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{{\left(a+b\,\sqrt{c}\right)}^2}+\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(\frac{2}{b^2\,c-a^2}-\frac{4\,b^2\,c}{{\left(b^2\,c-a^2\right)}^2}\right)+\frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{{\left(a-b\,\sqrt{c}\right)}^2}-\frac{2\,a}{\left(b^2\,c-a^2\right)\,\left(a+b\,\sqrt{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] log((c + d*x)^(1/2) - c^(1/2))/(a + b*c^(1/2))^2 + log(a + b*(c + d*x)^(1/2
))*(2/(b^2*c - a^2) - (4*b^2*c)/(b^2*c - a^2)^2) + log((c + d*x)^(1/2) + c^
(1/2))/(a - b*c^(1/2))^2 - (2*a)/((b^2*c - a^2)*(a + b*(c + d*x)^(1/2)))
```

sympy [A] time = 50.51, size = 153, normalized size = 1.19

$$-\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0\\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{a^2-b^2c}-\frac{2b\left(a^2+b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{\left(a^2-b^2c\right)^2}-\frac{2\left(\frac{2abc\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\left(-\frac{a^2}{2}\right.\right.}{\left(a^2-b^2c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] -2*a*b*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*sqrt(c + d*x
))), True))/(a**2 - b**2*c) - 2*b*(a**2 + b**2*c)*Piecewise((sqrt(c + d*x)/
a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(a**2 - b**2*c)**2 - 2*(2
*a*b*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + (-a**2/2 - b**2*c/2)*log(-d*
x))/(a**2 - b**2*c)**2
```

3.644 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=202

$$\frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{b^2d\log(x)\left(3a^2+b^2c\right)}{\left(a^2-b^2c\right)^3}-\frac{2b^2d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c\right.}}{\left(a^2-b^2c\right)^3}$$

```
[Out] b^2*(b^2*c+3*a^2)*d*ln(x)/(-b^2*c+a^2)^3-2*b^2*(b^2*c+3*a^2)*d*ln(a+b*(d*x+
c)^(1/2))/(-b^2*c+a^2)^3+2*a*b*(3*b^2*c+a^2)*d*arctanh((d*x+c)^(1/2)/c^(1/2
))/(-b^2*c+a^2)^3/c^(1/2)+4*a*b^2*d/(-b^2*c+a^2)^2/(a+b*(d*x+c)^(1/2))+(-a+
b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x/(a+b*(d*x+c)^(1/2))
```

Rubi [A] time = 0.25, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$\frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{b^2d\log(x)\left(3a^2+b^2c\right)}{\left(a^2-b^2c\right)^3}-\frac{2b^2d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c\right.}}{\left(a^2-b^2c\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (4*a*b^2*d)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])) - (a - b*Sqrt[c + d*x])
/((a^2 - b^2*c)*x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh
[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^3) + (b^2*(3*a^2 + b^2*c)*d
*Log[x])/(a^2 - b^2*c)^3 - (2*b^2*(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]
])/(a^2 - b^2*c)^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = d\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (2d)\,\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} + \frac{d\,\text{Subst}\left(\int \frac{-2abc+2b^2cx}{(a+bx)^2\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} + \frac{d\,\text{Subst}\left(\int\left(-\frac{4ab^3c}{\left(a^2-b^2c\right)(a+bx)^2} - \frac{2b^3c\left(3a^2+b^2c\right)}{\left(-a^2+b^2c\right)^2(a+bx)} + \frac{2b\cdots}{\cdots}\right.\right.}{c\left(a^2-b^2c\cdots\right.}$$

$$= \frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} - \frac{2b^2\left(3a^2+b^2c\right)d\log\cdots}{\left(a^2-b\cdots\right.}$$

$$= \frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} - \frac{2b^2\left(3a^2+b^2c\right)d\log\cdots}{\left(a^2-b\cdots\right.}$$

$$= \frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{\left(a^2-b^2c\right)x\left(a+b\sqrt{c+dx}\right)} + \frac{2ab\left(a^2+3b^2c\right)d\tan\cdots}{\sqrt{c}\left(a^2-b\cdots\right.}$$

Mathematica [A] time = 0.76, size = 230, normalized size = 1.14

$$\frac{\frac{\sqrt{c}\left(2b^2\sqrt{c}\,d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)+\frac{\sqrt{c}\left(a^2-b^2c\right)\left(a^3-a^2b\sqrt{c+dx}-ab^2(c+4dx)+b^3c\sqrt{c+dx}\right)}{x\left(a+b\sqrt{c+dx}\right)}-bd\left(a+b\sqrt{c}\right)^3\log\left(\sqrt{c+dx}+\sqrt{c}\right)\right)}{\left(a^2-b^2c\right)^2}+\frac{\left(ab\sqrt{c}\,d-b^2cd\right)\log\left(\cdots\right.}{\left(a+b\sqrt{c}\right)\cdots}}{c\left(b^2c-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (((a*b*Sqrt[c]*d - b^2*c*d)*Log[Sqrt[c] - Sqrt[c + d*x]])/(a + b*Sqrt[c])^2
 + (Sqrt[c]*((Sqrt[c]*(a^2 - b^2*c)*(a^3 - a^2*b*Sqrt[c + d*x] + b^3*c*Sqrt
[c + d*x] - a*b^2*(c + 4*d*x))))/(x*(a + b*Sqrt[c + d*x])) - b*(a + b*Sqrt[c
])^3*d*Log[Sqrt[c] + Sqrt[c + d*x]] + 2*b^2*Sqrt[c]*(3*a^2 + b^2*c)*d*Log[a
 + b*Sqrt[c + d*x]]))/(a^2 - b^2*c)^2)/(c*(-a^2 + b^2*c))
```

fricas [B] time = 0.86, size = 854, normalized size = 4.23

$$\left[-\frac{b^6c^4 - a^2b^4c^3 - a^4b^2c^2 + a^6c + \left(b^6c^3 + 2\,a^2b^4c^2 - 3\,a^4b^2c\right)dx - \left(\left(3\,ab^5c + a^3b^3\right)d^2x^2 + \left(3\,ab^5c^2 - 2\,a^3b^3c - a^5b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [-(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 -
 3*a^4*b^2*c)*d*x - ((3*a*b^5*c + a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b
^3*c - a^5*b)*d*x)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2
*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)
*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^
3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2
 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)*sqrt(d*x + c))/((b^8*c^4 - 3*a^
2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6
*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x), -(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2
*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x - 2*((3*a*b^5*c
+ a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b^3*c - a^5*b)*d*x)*sqrt(-c)*arct
an(sqrt(d*x + c)*sqrt(-c)/c) - 2*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^
3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2
+ 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x
) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)
*sqrt(d*x + c))/((b^8*c^4 - 3*a^2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^
2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x)]
```

giac [A] time = 0.41, size = 311, normalized size = 1.54

$$-\frac{\left(b^4cd + 3\,a^2b^2d\right)\log\left(-dx\right)}{b^6c^3 - 3\,a^2b^4c^2 + 3\,a^4b^2c - a^6} + \frac{2\left(b^5cd + 3\,a^2b^3d\right)\log\left(\left|-\sqrt{dx+c}\,b - a\right|\right)}{b^7c^3 - 3\,a^2b^5c^2 + 3\,a^4b^3c - a^6b} + \frac{2\left(3\,ab^3cd + a^3bd\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^6c^3 - 3\,a^2b^4c^2 + 3\,a^4b^2c - a^6\right)\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -(b^4*c*d + 3*a^2*b^2*d)*log(-d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c -
 a^6) + 2*(b^5*c*d + 3*a^2*b^3*d)*log(abs(-sqrt(d*x + c)*b - a))/(b^7*c^3 -
 3*a^2*b^5*c^2 + 3*a^4*b^3*c - a^6*b) + 2*(3*a*b^3*c*d + a^3*b*d)*arctan(sq
rt(d*x + c)/sqrt(-c))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqrt(-
c)) - (sqrt(d*x + c)*b^3*c*d - 4*(d*x + c)*a*b^2*d + 3*a*b^2*c*d - sqrt(d*x
 + c)*a^2*b*d + a^3*d)/((b^4*c^2 - 2*a^2*b^2*c + a^4)*((d*x + c)^(3/2)*b -
sqrt(d*x + c)*b*c + (d*x + c)*a - a*c))
```

maple [A] time = 0.02, size = 312, normalized size = 1.54

$$\frac{b^4cd\ln\left(dx\right)}{\left(-b^2c+a^2\right)^3} - \frac{2b^4cd\ln\left(a+\sqrt{dx+c}\,b\right)}{\left(-b^2c+a^2\right)^3} + \frac{6a\,b^3\sqrt{c}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\left(-b^2c+a^2\right)^3} + \frac{3a^2b^2d\ln\left(dx\right)}{\left(-b^2c+a^2\right)^3} - \frac{6a^2b^2d\ln\left(a+\sqrt{dx+c}\,b\right.}{\left(-b^2c+a^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] -2/(-b^2*c+a^2)^3/x*(d*x+c)^(1/2)*a*b^3*c+2/(-b^2*c+a^2)^3/x*(d*x+c)^(1/2)*
a^3*b+1/(-b^2*c+a^2)^3/x*b^4*c^2-1/(-b^2*c+a^2)^3/x*a^4+d/(-b^2*c+a^2)^3*ln
(d*x)*b^4*c+3*d/(-b^2*c+a^2)^3*ln(d*x)*b^2*a^2+6*d/(-b^2*c+a^2)^3*c^(1/2)*a
rctanh((d*x+c)^(1/2)/c^(1/2))*a*b^3+2*d/(-b^2*c+a^2)^3*b/c^(1/2)*arctanh((d
*x+c)^(1/2)/c^(1/2))*a^3+2*a*b^2*d/(-b^2*c+a^2)^2/(a+(d*x+c)^(1/2)*b)-2*d*b
^4/(-b^2*c+a^2)^3*ln(a+(d*x+c)^(1/2)*b)*c-6*d*b^2/(-b^2*c+a^2)^3*ln(a+(d*x+
c)^(1/2)*b)*a^2
```

maxima [A] time = 2.01, size = 367, normalized size = 1.82

$$-d\left(\frac{\left(b^4c+3\,a^2b^2\right)\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{2\left(b^4c+3\,a^2b^2\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{\left(3\,ab^3c+a^3b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6\right)\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -d*((b^4*c + 3*a^2*b^2)*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a
^6) - 2*(b^4*c + 3*a^2*b^2)*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c
^2 + 3*a^4*b^2*c - a^6) - (3*a*b^3*c + a^3*b)*log((sqrt(d*x + c) - sqrt(c))
/(sqrt(d*x + c) + sqrt(c)))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*
sqrt(c)) + (4*(d*x + c)*a*b^2 - 3*a*b^2*c - a^3 - (b^3*c - a^2*b)*sqrt(d*x
+ c))/(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c - (b^5*c^2 - 2*a^2*b^3*c + a^4*b)*
(d*x + c)^(3/2) - (a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(d*x + c) + (b^5*c^3 - 2*
a^2*b^3*c^2 + a^4*b*c)*sqrt(d*x + c)))
```

mupad [B] time = 0.73, size = 275, normalized size = 1.36

$$\frac{b\,d\,\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{a^3\,\sqrt{c}-b^3\,c^2+3\,a\,b^2\,c^{3/2}-3\,a^2\,b\,c}-\frac{\frac{a\,d\,\left(a^2+3\,c\,b^2\right)}{{\left(b^2\,c-a^2\right)}^2}+\frac{b\,d\,\sqrt{c+d\,x}}{b^2\,c-a^2}-\frac{4\,a\,b^2\,d\,\left(c+d\,x\right)}{a^4-2\,a^2\,b^2\,c+b^4\,c^2}}{b\,{\left(c+d\,x\right)}^{3/2}-a\,c+a\,\left(c+d\,x\right)-b\,c\,\sqrt{c+d\,x}}-\frac{b\,d\,\ln\left(\sqrt{c+d\,x}-\right.}{a^3\,\sqrt{c}+b^3\,c^2+3\,a\,b^2\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] (b*d*log((c + d*x)^(1/2) + c^(1/2)))/(a^3*c^(1/2) - b^3*c^2 + 3*a*b^2*c^(3/
2) - 3*a^2*b*c) - ((a*d*(3*b^2*c + a^2))/(b^2*c - a^2)^2 + (b*d*(c + d*x)^(
1/2))/(b^2*c - a^2) - (4*a*b^2*d*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/
(b*(c + d*x)^(3/2) - a*c + a*(c + d*x) - b*c*(c + d*x)^(1/2)) - (b*d*log((c
 + d*x)^(1/2) - c^(1/2)))/(a^3*c^(1/2) + b^3*c^2 + 3*a*b^2*c^(3/2) + 3*a^2*
b*c) - log(a + b*(c + d*x)^(1/2))*((6*b^2*d)/(b^2*c - a^2)^2 - (8*b^4*c*d)/
(b^2*c - a^2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Timed out
```

3.645 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=306

$$\frac{ab^2d^2\left(a^2+11b^2c\right)}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2cx\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}+\frac{b^4d^2\log(x)\left(5a^2+b^2c\right)}{\left(a^2-b^2c\right)^4}$$

```
[Out] -1/2*a*b*(-15*b^4*c^2-10*a^2*b^2*c+a^4)*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/
c^(3/2)/(-b^2*c+a^2)^4+b^4*(b^2*c+5*a^2)*d^2*ln(x)/(-b^2*c+a^2)^4-2*b^4*(b^
2*c+5*a^2)*d^2*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^4+1/2*a*b^2*(11*b^2*c+a^2
)*d^2/c/(-b^2*c+a^2)^3/(a+b*(d*x+c)^(1/2))+1/2*(-a+b*(d*x+c)^(1/2))/(-b^2*c
+a^2)/x^2/(a+b*(d*x+c)^(1/2))-1/2*b*d*(3*a*b*c-(2*b^2*c+a^2)*(d*x+c)^(1/2))
/c/(-b^2*c+a^2)^2/x/(a+b*(d*x+c)^(1/2))
```

Rubi [A] time = 0.40, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$-\frac{abd^2\left(-10a^2b^2c+a^4-15b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\left(a^2-b^2c\right)^4}+\frac{ab^2d^2\left(a^2+11b^2c\right)}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)}+\frac{b^4d^2\log(x)\left(5a^2+b^2c\right)}{\left(a^2-b^2c\right)^4}-\frac{2b^4d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (a*b^2*(a^2 + 11*b^2*c)*d^2)/(2*c*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])) -
(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2*(a + b*Sqrt[c + d*x])) - (b*d*(3
*a*b*c - (a^2 + 2*b^2*c)*Sqrt[c + d*x]))/(2*c*(a^2 - b^2*c)^2*x*(a + b*Sqrt
[c + d*x])) - (a*b*(a^4 - 10*a^2*b^2*c - 15*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d
*x]/Sqrt[c]])/(2*c^(3/2)*(a^2 - b^2*c)^4) + (b^4*(5*a^2 + b^2*c)*d^2*Log[x]
)/(a^2 - b^2*c)^4 - (2*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d*x]])/(a
^2 - b^2*c)^4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = d^2\,\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} + \frac{d^2\,\text{Subst}\left(\int \frac{-2abc+4b^2cx}{(a+bx)^2\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} + \frac{d^2\,\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} + \frac{d^2\,\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2\ldots\right.\right.}{2c\left(a^2-b^2c\right)^2\ldots}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2\ldots\right.\right.}{2c\left(a^2-b^2c\right)^2\ldots}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2\ldots\right.\right.}{2c\left(a^2-b^2c\right)^2\ldots}$$

Mathematica [A] time = 0.83, size = 401, normalized size = 1.31

$$\frac{\frac{d^2\left(\frac{2b\sqrt{c}\left(a^2+2b^2c\right)\left(\left(b\sqrt{c}-a\right)\log\left(\sqrt{c}-\sqrt{c+dx}\right)+\left(a+b\sqrt{c}\right)\log\left(\sqrt{c+dx}+\sqrt{c}\right)-2b\sqrt{c}\log\left(a+b\sqrt{c+dx}\right)\right)}{b^2c-a^2}-abc\left(a^2+11b^2c\right)\left(\frac{2b\left(\frac{b^2c-a^2}{a+b\sqrt{c+dx}}+2a\log\left(a+b\sqrt{c+dx}\right)\right)}{\left(a^2-b^2c\right)^2}+\frac{\log\left(\sqrt{c}-\sqrt{c+\cdots}\right.}{\sqrt{c}\left(a+b\sqrt{c}\right)\cdots}\right.\right.}{2c\left(a^2-b^2c\right)}}{2c\left(a^2-b^2c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (-((c*(a - b*Sqrt[c + d*x]))/(x^2*(a + b*Sqrt[c + d*x]))) + (b*d*(-3*a*b*c
+ a^2*Sqrt[c + d*x] + 2*b^2*c*Sqrt[c + d*x]))/((a^2 - b^2*c)*x*(a + b*Sqrt[
c + d*x])) + (d^2*((2*b*Sqrt[c]*(a^2 + 2*b^2*c)*((-a + b*Sqrt[c])*Log[Sqrt[
c] - Sqrt[c + d*x]] + (a + b*Sqrt[c])*Log[Sqrt[c] + Sqrt[c + d*x]] - 2*b*Sq
rt[c]*Log[a + b*Sqrt[c + d*x]]))/(-a^2 + b^2*c) - a*b*c*(a^2 + 11*b^2*c)*(L
og[Sqrt[c] - Sqrt[c + d*x]]/((a + b*Sqrt[c])^2*Sqrt[c]) - Log[Sqrt[c] + Sqr
t[c + d*x]]/((a - b*Sqrt[c])^2*Sqrt[c]) + (2*b*((-a^2 + b^2*c)/(a + b*Sqrt[
c + d*x]) + 2*a*Log[a + b*Sqrt[c + d*x]]))/(a^2 - b^2*c)^2)))/(2*c*(a^2 - b
^2*c)))/(2*c*(a^2 - b^2*c))
```

fricas [B] time = 2.02, size = 1252, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b^8*c^6 - 4*a^2*b^6*c^5 + 4*a^6*b^2*c^3 - 2*a^8*c^2 - 4*(b^8*c^4 +
 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - 2*(b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^
4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x - ((15*a*b^7*c^2 + 10*a^3*b^5*c - a^5*b^3)*d
^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a^5*b^3*c + a^7*b)*d^2*x^2)*sqr
t(c)*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 8*((b^8*c^3 + 5*a^2*b^6
*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(sqrt
(d*x + c)*b + a) - 4*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*
b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - 2*(2*a*b^7*c^5 - 6*a^3*b^5*c^4 +
 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*c^2 - a^5*b^3*c)*
d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a^7*b*c)*d*x)*sqrt
(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*a^6*b^4*c^3 + a^8
*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c^5 - 10*a^6*b^4*c
^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2), -1/2*(b^8*c^6 - 2*a^2*b^6*c^5 + 2*a^6*
b^2*c^3 - a^8*c^2 - 2*(b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - (
b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x + ((15*a*b^7*c
^2 + 10*a^3*b^5*c - a^5*b^3)*d^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a
^5*b^3*c + a^7*b)*d^2*x^2)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) + 4*((
b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2
)*d^2*x^2)*log(sqrt(d*x + c)*b + a) - 2*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3
+ (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - (2*a*b^7*c^5
- 6*a^3*b^5*c^4 + 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*
c^2 - a^5*b^3*c)*d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a
^7*b*c)*d*x)*sqrt(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*
a^6*b^4*c^3 + a^8*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c
^5 - 10*a^6*b^4*c^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2)]
```

giac [A] time = 0.50, size = 521, normalized size = 1.70

$$\frac{\left(b^6cd^2+5\,a^2b^4d^2\right)\log\left(-dx\right)}{b^8c^4-4\,a^2b^6c^3+6\,a^4b^4c^2-4\,a^6b^2c+a^8}-\frac{2\left(b^7cd^2+5\,a^2b^5d^2\right)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^9c^4-4\,a^2b^7c^3+6\,a^4b^5c^2-4\,a^6b^3c+a^8b}-\frac{\left(15\,ab^5c^2d^2+10\,a^3b^3cd^2-\cdots\right.}{2\left(b^8c^5-4\,a^2b^6c^4+6\,a^4b^4\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] (b^6*c*d^2 + 5*a^2*b^4*d^2)*log(-d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*
c^2 - 4*a^6*b^2*c + a^8) - 2*(b^7*c*d^2 + 5*a^2*b^5*d^2)*log(abs(sqrt(d*x +
 c)*b + a))/(b^9*c^4 - 4*a^2*b^7*c^3 + 6*a^4*b^5*c^2 - 4*a^6*b^3*c + a^8*b)
 - 1/2*(15*a*b^5*c^2*d^2 + 10*a^3*b^3*c*d^2 - a^5*b*d^2)*arctan(sqrt(d*x +
c)/sqrt(-c))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^
8*c)*sqrt(-c)) - 1/2*(7*a*b^6*c^4*d^2 - a^3*b^4*c^3*d^2 - 7*a^5*b^2*c^2*d^2
 + a^7*c*d^2 + (11*a*b^6*c^2*d^2 - 10*a^3*b^4*c*d^2 - a^5*b^2*d^2)*(d*x + c
)^2 - (2*b^7*c^3*d^2 - 3*a^2*b^5*c^2*d^2 + a^6*b*d^2)*(d*x + c)^(3/2) - (19
*a*b^6*c^3*d^2 - 14*a^3*b^4*c^2*d^2 - 5*a^5*b^2*c*d^2)*(d*x + c) + 3*(b^7*c
^4*d^2 - 2*a^2*b^5*c^3*d^2 + a^4*b^3*c^2*d^2)*sqrt(d*x + c))/((b^2*c - a^2)
^4*(sqrt(d*x + c)*b + a)*c*d^2*x^2)
```

maple [B] time = 0.02, size = 610, normalized size = 1.99

$$\frac{b^{6} c\, d^{2} \ln\left(dx\right)}{\left(-b^{2} c+a^{2}\right)^{4}}-\frac{2 b^{6} c\, d^{2} \ln\left(a+\sqrt{dx+c}\, b\right)}{\left(-b^{2} c+a^{2}\right)^{4}}+\frac{15 a\, b^{5} \sqrt{c}\, d^{2} \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{2 \left(-b^{2} c+a^{2}\right)^{4}}+\frac{5 a^{2} b^{4} d^{2} \ln\left(dx\right)}{\left(-b^{2} c+a^{2}\right)^{4}}-\frac{10 a^{2} b^{4} d^{2} \ln\left(a+\cdots\right.}{\left(-b^{2} c+a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] -7/2/(-b^2*c+a^2)^4/x^2*a*b^5*c*(d*x+c)^(3/2)+3/(-b^2*c+a^2)^4/x^2*a^3*b^3*
(d*x+c)^(3/2)+1/2/(-b^2*c+a^2)^4/x^2*a^5*b/c*(d*x+c)^(3/2)+d/(-b^2*c+a^2)^4
/x*b^6*c^2-1/2/(-b^2*c+a^2)^4/x^2*c^3*b^6+2*d/(-b^2*c+a^2)^4/x*a^2*b^4*c+1/
2/(-b^2*c+a^2)^4/x^2*b^4*a^2*c^2-3*d/(-b^2*c+a^2)^4/x*a^4*b^2+1/2/(-b^2*c+a
^2)^4/x^2*b^2*a^4*c+9/2/(-b^2*c+a^2)^4/x^2*(d*x+c)^(1/2)*a*b^5*c^2-5/(-b^2*
c+a^2)^4/x^2*(d*x+c)^(1/2)*b^3*a^3*c+1/2/(-b^2*c+a^2)^4/x^2*(d*x+c)^(1/2)*b
*a^5-1/2/(-b^2*c+a^2)^4/x^2*a^6+d^2/(-b^2*c+a^2)^4*b^6*c*ln(d*x)+5*d^2/(-b^
2*c+a^2)^4*b^4*ln(d*x)*a^2+15/2*d^2/(-b^2*c+a^2)^4*b^5*c^(1/2)*arctanh((d*x
+c)^(1/2)/c^(1/2))*a+5*d^2/(-b^2*c+a^2)^4*b^3/c^(1/2)*arctanh((d*x+c)^(1/2)
/c^(1/2))*a^3-1/2*d^2/(-b^2*c+a^2)^4*b/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2
))*a^5+2*d^2*b^4/(-b^2*c+a^2)^3*a/(a+(d*x+c)^(1/2)*b)-2*d^2*b^6/(-b^2*c+a^2
)^4*ln(a+(d*x+c)^(1/2)*b)*c-10*d^2*b^4/(-b^2*c+a^2)^4*ln(a+(d*x+c)^(1/2)*b)
*a^2
```

maxima [B] time = 2.09, size = 659, normalized size = 2.15

$$\frac{1}{4} d^{2} \left(\frac{4 \left(b^{6} c+5 a^{2} b^{4}\right) \log\left(dx\right)}{b^{8} c^{4}-4 a^{2} b^{6} c^{3}+6 a^{4} b^{4} c^{2}-4 a^{6} b^{2} c+a^{8}}-\frac{8 \left(b^{6} c+5 a^{2} b^{4}\right) \log\left(\sqrt{dx+c}\, b+a\right)}{b^{8} c^{4}-4 a^{2} b^{6} c^{3}+6 a^{4} b^{4} c^{2}-4 a^{6} b^{2} c+a^{8}}-\frac{\left(15 a b^{5} c^{2}+10\cdots\right.}{\left(b^{8} c^{5}-4 a^{2} b^{6} c^{4}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*d^2*(4*(b^6*c + 5*a^2*b^4)*log(d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^
4*c^2 - 4*a^6*b^2*c + a^8) - 8*(b^6*c + 5*a^2*b^4)*log(sqrt(d*x + c)*b + a)
/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*c^2 - 4*a^6*b^2*c + a^8) - (15*a*b^5*
c^2 + 10*a^3*b^3*c - a^5*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) +
sqrt(c)))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c
)*sqrt(c)) - 2*(7*a*b^4*c^3 + 6*a^3*b^2*c^2 - a^5*c + (11*a*b^4*c + a^3*b^2
)*(d*x + c)^2 - (2*b^5*c^2 - a^2*b^3*c - a^4*b)*(d*x + c)^(3/2) - (19*a*b^4
*c^2 + 5*a^3*b^2*c)*(d*x + c) + 3*(b^5*c^3 - a^2*b^3*c^2)*sqrt(d*x + c))/(a
*b^6*c^6 - 3*a^3*b^4*c^5 + 3*a^5*b^2*c^4 - a^7*c^3 + (b^7*c^4 - 3*a^2*b^5*c
^3 + 3*a^4*b^3*c^2 - a^6*b*c)*(d*x + c)^(5/2) + (a*b^6*c^4 - 3*a^3*b^4*c^3
+ 3*a^5*b^2*c^2 - a^7*c)*(d*x + c)^2 - 2*(b^7*c^5 - 3*a^2*b^5*c^4 + 3*a^4*b
```

```
^3*c^3 - a^6*b*c^2)*(d*x + c)^(3/2) - 2*(a*b^6*c^5 - 3*a^3*b^4*c^4 + 3*a^5*
b^2*c^3 - a^7*c^2)*(d*x + c) + (b^7*c^6 - 3*a^2*b^5*c^5 + 3*a^4*b^3*c^4 - a
^6*b*c^3)*sqrt(d*x + c)))
```

mupad [B] time = 5.96, size = 1441, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] (((5*a^3*b^2*d^2 + 19*a*b^4*c*d^2)*(c + d*x))/(2*(b^2*c - a^2)*(a^4 + b^4*c
^2 - 2*a^2*b^2*c)) + ((a^3*b^2*d^2 + 11*a*b^4*c*d^2)*(c + d*x)^2)/(2*c*(a^6
 - b^6*c^3 - 3*a^4*b^2*c + 3*a^2*b^4*c^2)) - (a*(7*b^4*c^2*d^2 - a^4*d^2 +
6*a^2*b^2*c*d^2))/(2*(b^2*c - a^2)*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) + (b*(a^2
*d^2 + 2*b^2*c*d^2)*(c + d*x)^(3/2))/(2*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) -
(3*b^3*c*d^2*(c + d*x)^(1/2))/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)))/(a*(c + d*
x)^2 + b*(c + d*x)^(5/2) + a*c^2 - 2*a*c*(c + d*x) - 2*b*c*(c + d*x)^(3/2)
+ b*c^2*(c + d*x)^(1/2)) + log(a + b*(c + d*x)^(1/2))*((10*b^4*d^2)/(b^2*c
- a^2)^3 - (12*b^6*c*d^2)/(b^2*c - a^2)^4) + (log((a*b^4*d^4*(a^6 - 44*b^6*
c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (b*d^2*((b^
2*d^2*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(a^5*(c^3)^
(1/2) + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*a^3*b^2*c*(c^3)^(1/2) - 15*a*b^4*c^
2*(c^3)^(1/2)))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^(1/2)*(6*b^4*c
^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7*b^2*c - a^2))
/(2*c*(b^2*c - a^2)^2))*(a^5*(c^3)^(1/2) + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*
a^3*b^2*c*(c^3)^(1/2) - 15*a*b^4*c^2*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)^4)
+ (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^(1/2))/(4*c^2*(b^2*c - a^2)^6))
*(4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 + a^5*b*d^2*(c^3)^(1/2) - 10*a^3*b^3*c
*d^2*(c^3)^(1/2) - 15*a*b^5*c^2*d^2*(c^3)^(1/2)))/(4*(a^8*c^3 + b^8*c^7 - 4
*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6)) + (log((a*b^4*d^4*(a^6 - 44*
b^6*c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (b*d^2*
((b^2*d^2*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(4*b^5*
c^4 - a^5*(c^3)^(1/2) + 20*a^2*b^3*c^3 + 10*a^3*b^2*c*(c^3)^(1/2) + 15*a*b^
4*c^2*(c^3)^(1/2)))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^(1/2)*(6*b
^4*c^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7*b^2*c - a
^2))/(2*c*(b^2*c - a^2)^2))*(4*b^5*c^4 - a^5*(c^3)^(1/2) + 20*a^2*b^3*c^3 +
 10*a^3*b^2*c*(c^3)^(1/2) + 15*a*b^4*c^2*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)
^4) + (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^(1/2))/(4*c^2*(b^2*c - a^2)
^6))*(4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 - a^5*b*d^2*(c^3)^(1/2) + 10*a^3*b
^3*c*d^2*(c^3)^(1/2) + 15*a*b^5*c^2*d^2*(c^3)^(1/2)))/(4*(a^8*c^3 + b^8*c^7
 - 4*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Timed out
```

3.646 $\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}} \, dx$

Optimal. Leaf size=324

$$\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}-\frac{12a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c\right.}}{5b^8d^4}$$

```
[Out] 4/3*(-b^2*c+a^2)^2*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^8/d^4-12/5*a*
(-3*b^2*c+7*a^2)*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^8/d^4+4/7*(3*b^4*
c^2-30*a^2*b^2*c+35*a^4)*(a+b*(d*x+c)^(1/2))^(7/2)/b^8/d^4-20/9*a*(-3*b^2*c
+7*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)/b^8/d^4+12/11*(-b^2*c+7*a^2)*(a+b*(d*x+c)
^(1/2))^(11/2)/b^8/d^4-28/13*a*(a+b*(d*x+c)^(1/2))^(13/2)/b^8/d^4+4/15*(a+b
*(d*x+c)^(1/2))^(15/2)/b^8/d^4-4*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(1/2)
/b^8/d^4
```

Rubi [A] time = 0.23, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-30a^2b^2c+35a^4+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}+\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{\right.}}{9b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]])/(b^8*d^4) + (4*(a^2 - b^2*
c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) - (12*a*(7*a^
2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) + (4*(3
5*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4)
- (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d^4) + (12*(7
*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) - (28*a*(a + b*Sqr
t[c + d*x])^(13/2))/(13*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(15/2))/(15*b^8
*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^3}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\left(a^2-b^2c\right)^3}{b^7\sqrt{a+bx}} - \frac{\left(-7a^2+b^2c\right)\left(-a^2+b^2c\right)^2\sqrt{a+bx}}{b^7} - \frac{3\left(7a^5-10a^3b^2c+3ab^4c^2\right)(a+bx)^{3/2}}{b^7} + \frac{\left(35a^4-3\right.}{}\right.\right.}{}$$

$$= -\frac{4a\left(a^2-b^2c\right)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4} + \frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} - \frac{12a\left(\right.}{}$$

Mathematica [A] time = 0.35, size = 232, normalized size = 0.72

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-14336a^7+7168a^6b\sqrt{c+dx}+768a^5b^2(58c-7dx)-640a^4b^3(32c-7dx)\sqrt{c+dx}-16a^3b^4\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-14336*a^7 + 768*a^5*b^2*(58*c - 7*d*x) + 716
8*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(32*c - 7*d*x)*Sqrt[c + d*x] + 24*a^2*b
^5*Sqrt[c + d*x]*(784*c^2 - 356*c*d*x + 147*d^2*x^2) - 16*a^3*b^4*(2936*c^2
 - 680*c*d*x + 245*d^2*x^2) + 6*a*b^6*(2880*c^3 - 928*c^2*d*x + 658*c*d^2*x
^2 - 539*d^3*x^3) - 39*b^7*Sqrt[c + d*x]*(128*c^3 - 96*c^2*d*x + 84*c*d^2*x
^2 - 77*d^3*x^3)))/(45045*b^8*d^4)
```

fricas [A] time = 0.54, size = 231, normalized size = 0.71

$$-\frac{4\left(3234\,ab^6d^3x^3-17280\,ab^6c^3+46976\,a^3b^4c^2-44544\,a^5b^2c+14336\,a^7-28\left(141\,ab^6c-140\,a^3b^4\right)d^2x^2+64\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/45045*(3234*a*b^6*d^3*x^3 - 17280*a*b^6*c^3 + 46976*a^3*b^4*c^2 - 44544*
a^5*b^2*c + 14336*a^7 - 28*(141*a*b^6*c - 140*a^3*b^4)*d^2*x^2 + 64*(87*a*b
^6*c^2 - 170*a^3*b^4*c + 84*a^5*b^2)*d*x - (3003*b^7*d^3*x^3 - 4992*b^7*c^3
 + 18816*a^2*b^5*c^2 - 20480*a^4*b^3*c + 7168*a^6*b - 252*(13*b^7*c - 14*a^
2*b^5)*d^2*x^2 + 32*(117*b^7*c^2 - 267*a^2*b^5*c + 140*a^4*b^3)*d*x)*sqrt(d
*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

giac [A] time = 0.42, size = 409, normalized size = 1.26

$$-\frac{4\left(15015\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^6c^3-45045\sqrt{\sqrt{dx+c}\,b+a}\,ab^6c^3-19305\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}b^4c^2+81081\left(\sqrt{dx+c}\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/45045*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(d*x +
 c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 81081*(s
qrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/2)*a^
2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt(d*x
+ c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c + 1930
50*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b + a)^(5/
2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135*sqrt(s
qrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) + 24255*(
sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)*a^2 + 17
5175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b + a)^(7/2)*a
^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x + c)*b + a)^
(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)/(b^8*d^4)
```

maple [A] time = 0.00, size = 383, normalized size = 1.18

$$-\frac{28\left(a+\sqrt{dx+c}\,b\right)^{\frac{13}{2}}a}{13}-4\left(-b^2c+a^2\right)^3\sqrt{a+\sqrt{dx+c}\,b}\,a+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{15}{2}}}{15}+\frac{4\left(-3b^2c+21a^2\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{11}{2}}}{11}+\frac{4\left(-8\left(-b^2c+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d^4/b^8*(1/15*(a+(d*x+c)^(1/2)*b)^(15/2)-7/13*a*(a+(d*x+c)^(1/2)*b)^(13/2
)+1/11*(-3*b^2*c+21*a^2)*(a+(d*x+c)^(1/2)*b)^(11/2)+1/9*(-8*(-b^2*c+a^2)*a-
2*(-2*b^2*c+6*a^2)*a-(-3*b^2*c+15*a^2)*a)*(a+(d*x+c)^(1/2)*b)^(9/2)+1/7*(8*
(-b^2*c+a^2)*a^2-(-8*(-b^2*c+a^2)*a-2*(-2*b^2*c+6*a^2)*a)*a+(-b^2*c+a^2)*(-
2*b^2*c+6*a^2)+(-b^2*c+a^2)^2)*(a+(d*x+c)^(1/2)*b)^(7/2)+1/5*(-6*(-b^2*c+a^
2)^2*a-(8*(-b^2*c+a^2)*a^2+(-b^2*c+a^2)*(-2*b^2*c+6*a^2)+(-b^2*c+a^2)^2)*a)
*(a+(d*x+c)^(1/2)*b)^(5/2)+1/3*(6*(-b^2*c+a^2)^2*a^2+(-b^2*c+a^2)^3)*(a+(d*
x+c)^(1/2)*b)^(3/2)-(-b^2*c+a^2)^3*a*(a+(d*x+c)^(1/2)*b)^(1/2))
```

maxima [A] time = 0.94, size = 268, normalized size = 0.83

$$\frac{4\left(3003\left(\sqrt{dx+c}\,b+a\right)^{\frac{15}{2}}-24255\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}a-12285\left(b^2c-7a^2\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}+25025\left(3ab^2c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/45045*(3003*(sqrt(d*x + c)*b + a)^(15/2) - 24255*(sqrt(d*x + c)*b + a)^(1
3/2)*a - 12285*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(11/2) + 25025*(3*a*b^
2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(9/2) + 6435*(3*b^4*c^2 - 30*a^2*b^2*c +
 35*a^4)*(sqrt(d*x + c)*b + a)^(7/2) - 27027*(3*a*b^4*c^2 - 10*a^3*b^2*c +
7*a^5)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^
4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(3/2) + 45045*(a*b^6*c^3 - 3*a^3*b^4
*c^2 + 3*a^5*b^2*c - a^7)*sqrt(sqrt(d*x + c)*b + a))/(b^8*d^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^3/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*sqrt(c + d*x)), x)
```

3.647 $\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=222

$$\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{4a\left(a^2-b^2c\right)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3}+\frac{4\left(5a^4-\right.}{}$$

```
[Out] 4/3*(b^4*c^2-6*a^2*b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(3/2)/b^6/d^3-8/5*a*(-3
*b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^6/d^3+8/7*(-b^2*c+5*a^2)*(a+b*(d*
x+c)^(1/2))^(7/2)/b^6/d^3-20/9*a*(a+b*(d*x+c)^(1/2))^(9/2)/b^6/d^3+4/11*(a+
b*(d*x+c)^(1/2))^(11/2)/b^6/d^3-4*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(1/2
)/b^6/d^3
```

Rubi [A] time = 0.16, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$\frac{4\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{8\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{8a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right.}{5b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^2*Sqrt[a + b*Sqrt[c + d*x]])/(b^6*d^3) + (4*(5*a^4 - 6*
a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) - (8*a*(5*a^2
 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) + (8*(5*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(9/2)
)/(9*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^6*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^2}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\left(a^2-b^2c\right)^2}{b^5\sqrt{a+bx}} + \frac{\left(5a^4-6a^2b^2c+b^4c^2\right)\sqrt{a+bx}}{b^5} - \frac{2\left(5a^3-3ab^2c\right)(a+bx)^{3/2}}{b^5} - \frac{2\left(-5a^2+b^2c\right)(a+bx)^{5/2}}{b^5}\right.\right.}{d^3}$$

$$= -\frac{4a\left(a^2-b^2c\right)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3} + \frac{4\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} - \frac{8a\left(5a\right.}{}$$

Mathematica [A] time = 0.17, size = 147, normalized size = 0.66

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-1280a^5+640a^4b\sqrt{c+dx}+96a^3b^2(28c-5dx)-16a^2b^3(74c-25dx)\sqrt{c+dx}-2ab^4\left(736c^2\right.\right.}{3465b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-1280*a^5 + 96*a^3*b^2*(28*c - 5*d*x) + 640*a
^4*b*Sqrt[c + d*x] - 16*a^2*b^3*(74*c - 25*d*x)*Sqrt[c + d*x] + 15*b^5*Sqrt
[c + d*x]*(32*c^2 - 24*c*d*x + 21*d^2*x^2) - 2*a*b^4*(736*c^2 - 244*c*d*x +
 175*d^2*x^2)))/(3465*b^6*d^3)
```

fricas [A] time = 0.55, size = 140, normalized size = 0.63

$$-\frac{4\left(350\,ab^4d^2x^2+1472\,ab^4c^2-2688\,a^3b^2c+1280\,a^5-8\left(61\,ab^4c-60\,a^3b^2\right)dx-\left(315\,b^5d^2x^2+480\,b^5c^2-1184\right.\right.}{3465\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/3465*(350*a*b^4*d^2*x^2 + 1472*a*b^4*c^2 - 2688*a^3*b^2*c + 1280*a^5 - 8
*(61*a*b^4*c - 60*a^3*b^2)*d*x - (315*b^5*d^2*x^2 + 480*b^5*c^2 - 1184*a^2*
b^3*c + 640*a^4*b - 40*(9*b^5*c - 10*a^2*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt
(d*x + c)*b + a)/(b^6*d^3)
```

giac [A] time = 0.33, size = 238, normalized size = 1.07

$$\frac{4\left(1155\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^4c^2-3465\sqrt{\sqrt{dx+c}\,b+a}\,ab^4c^2-990\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}b^2c+4158\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3465*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x + c)*
b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(d*x +
 c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c + 693
0*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/2) -
```

```
1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)*a^2 -
 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3/2)*a^
4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)/(b^6*d^3)
```

maple [A] time = 0.00, size = 183, normalized size = 0.82

$$\frac{-\frac{20\left(a+\sqrt{dx+c}\,b\right)^{\frac{9}{2}}a}{9}-4\left(-b^2c+a^2\right)^2\sqrt{a+\sqrt{dx+c}\,b}\,a+\frac{4\left(a+\sqrt{dx+c}\,b\right)^{\frac{11}{2}}}{11}+\frac{4\left(-2b^2c+10a^2\right)\left(a+\sqrt{dx+c}\,b\right)^{\frac{7}{2}}}{7}+\frac{4\left(-4\left(-b^2c+a^2\right)\right.}{}}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d^3/b^6*(1/11*(a+(d*x+c)^(1/2)*b)^(11/2)-5/9*a*(a+(d*x+c)^(1/2)*b)^(9/2)+
1/7*(-2*b^2*c+10*a^2)*(a+(d*x+c)^(1/2)*b)^(7/2)+1/5*(-4*(-b^2*c+a^2)*a-(-2*
b^2*c+6*a^2)*a)*(a+(d*x+c)^(1/2)*b)^(5/2)+1/3*(4*(-b^2*c+a^2)*a^2+(-b^2*c+a
^2)^2)*(a+(d*x+c)^(1/2)*b)^(3/2)-(-b^2*c+a^2)^2*a*(a+(d*x+c)^(1/2)*b)^(1/2)
)
```

maxima [A] time = 0.92, size = 167, normalized size = 0.75

$$\frac{4\left(315\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}-1925\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}a-990\left(b^2c-5\,a^2\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}+1386\left(3\,ab^2c-5\,a^3\right)\left(\right.\right.}{346}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3465*(315*(sqrt(d*x + c)*b + a)^(11/2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)
*a - 990*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(7/2) + 1386*(3*a*b^2*c - 5*
a^3)*(sqrt(d*x + c)*b + a)^(5/2) + 1155*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*(sq
rt(d*x + c)*b + a)^(3/2) - 3465*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*sqrt(sqrt(d
*x + c)*b + a))/(b^6*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*sqrt(c + d*x)), x)
```

3.648 $\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=131

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}-\frac{4a\left(a^2-b^2c\right)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}$$

```
[Out] 4/3*(-b^2*c+3*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^4/d^2-12/5*a*(a+b*(d*x+c)^(1
/2))^(5/2)/b^4/d^2+4/7*(a+b*(d*x+c)^(1/2))^(7/2)/b^4/d^2-4*a*(-b^2*c+a^2)*(
a+b*(d*x+c)^(1/2))^(1/2)/b^4/d^2
```

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}-\frac{4a\left(a^2-b^2c\right)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*Sqrt[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (4*(3*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x])^(
5/2))/(5*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4*d^2)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x(-c+x^2)}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{-a^3+ab^2c}{b^3\sqrt{a+bx}} + \frac{(3a^2-b^2c)\sqrt{a+bx}}{b^3} - \frac{3a(a+bx)^{3/2}}{b^3} + \frac{(a+bx)^{5/2}}{b^3}\right) dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= -\frac{4a\left(a^2-b^2c\right)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2} + \frac{4\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2} - \frac{12a\left(a+b\sqrt{c+\right.}}{5b^4d^2}$$

Mathematica [A] time = 0.10, size = 84, normalized size = 0.64

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-48a^3+24a^2b\sqrt{c+dx}+2ab^2(26c-9dx)+5b^3\sqrt{c+dx}\,(3dx-4c)\right)}{105b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-48*a^3 + 2*a*b^2*(26*c - 9*d*x) + 24*a^2*b*S
qrt[c + d*x] + 5*b^3*Sqrt[c + d*x]*(-4*c + 3*d*x)))/(105*b^4*d^2)
```

fricas [A] time = 0.58, size = 71, normalized size = 0.54

$$-\frac{4\left(18\,ab^2dx - 52\,ab^2c + 48\,a^3 - \left(15\,b^3dx - 20\,b^3c + 24\,a^2b\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}\,b+a}}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/105*(18*a*b^2*d*x - 52*a*b^2*c + 48*a^3 - (15*b^3*d*x - 20*b^3*c + 24*a^
2*b)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^4*d^2)
```

giac [A] time = 0.37, size = 115, normalized size = 0.88

$$-\frac{4\left(35\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^2c - 105\sqrt{\sqrt{dx+c}\,b+a}\,ab^2c - 15\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}} + 63\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a - 105\left(\right.\right.}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/105*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b + a
)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5/2)
*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)*a^
3)/(b^4*d^2)
```

maple [A] time = 0.00, size = 94, normalized size = 0.72

$$\frac{-\frac{12\left(a+\sqrt{dx+c}\;b\right)^{\frac{5}{2}}a}{5} - 4\left(-b^2c+a^2\right)\sqrt{a+\sqrt{dx+c}\;b}\;a + \frac{4\left(a+\sqrt{dx+c}\;b\right)^{\frac{7}{2}}}{7} + \frac{4\left(-b^2c+3a^2\right)\left(a+\sqrt{dx+c}\;b\right)^{\frac{3}{2}}}{3}}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d^2/b^4*(1/7*(a+(d*x+c)^(1/2)*b)^(7/2)-3/5*(a+(d*x+c)^(1/2)*b)^(5/2)*a+1/
3*(-b^2*c+3*a^2)*(a+(d*x+c)^(1/2)*b)^(3/2)-(-b^2*c+a^2)*a*(a+(d*x+c)^(1/2)*
b)^(1/2))
```

maxima [A] time = 0.93, size = 93, normalized size = 0.71

$$\frac{4\left(15\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}-63\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a-35\left(b^2c-3\,a^2\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}+105\left(ab^2c-a^3\right)\sqrt{\sqrt{dx+c}\,b+}\right.}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/105*(15*(sqrt(d*x + c)*b + a)^(7/2) - 63*(sqrt(d*x + c)*b + a)^(5/2)*a -
35*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(3/2) + 105*(a*b^2*c - a^3)*sqrt(s
qrt(d*x + c)*b + a))/(b^4*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*sqrt(c + d*x)), x)
```

3.649 $\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=54

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

```
[Out] 4/3*(a+b*(d*x+c)^(1/2))^(3/2)/b^2/d-4*a*(a+b*(d*x+c)^(1/2))^(1/2)/b^2/d
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {247, 190, 43}

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*Sqrt[a + b*Sqrt[c + d*x]])/(b^2*d) + (4*(a + b*Sqrt[c + d*x])^(3/2))/
(3*b^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx,x,c+dx\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx,x,\sqrt{c+dx}\right)}{d}\\
&= -\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.78

$$\frac{4\left(b\sqrt{c+dx}-2a\right)\sqrt{a+b\sqrt{c+dx}}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sqrt[c + d*x]],x]
[Out] (4*(-2*a + b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(3*b^2*d)
```

fricas [A] time = 0.54, size = 34, normalized size = 0.63

$$\frac{4\sqrt{\sqrt{dx+c}\,b+a}\left(\sqrt{dx+c}\,b-2\,a\right)}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 4/3*sqrt(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - 2*a)/(b^2*d)
```

giac [A] time = 0.44, size = 38, normalized size = 0.70

$$\frac{4\left(\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}-3\sqrt{\sqrt{dx+c}\,b+a}\,a\right)}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)/(b^2*d)
```

maple [A] time = 0.01, size = 41, normalized size = 0.76

$$\frac{-4\sqrt{a+\sqrt{dx+c}\;b}\;a+\frac{4\left(a+\sqrt{dx+c}\;b\right)^{\frac{3}{2}}}{3}}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(d*x+c)^(1/2)*b)^(1/2),x)
[Out] 4/d/b^2*(1/3*(a+(d*x+c)^(1/2)*b)^(3/2)-a*(a+(d*x+c)^(1/2)*b)^(1/2))
```

maxima [A] time = 0.86, size = 42, normalized size = 0.78

$$\frac{4\left(\frac{\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}}{b^2}-\frac{3\sqrt{\sqrt{dx+c}\,b+a}\,a}{b^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2)/b^2 - 3*sqrt(sqrt(d*x + c)*b + a)*a/b^2)/d
```

mupad [B] time = 3.26, size = 44, normalized size = 0.81

$$\frac{4\left(a+b\sqrt{c+d\,x}\right)^{3/2}}{3\,b^2\,d}-\frac{4\,a\sqrt{a+b\sqrt{c+d\,x}}}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] (4*(a + b*(c + d*x)^(1/2))^(3/2))/(3*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))^
(1/2))/(b^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sqrt{c + dx}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sqrt(c + d*x)), x)
```

3.650 $\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=97

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

```
[Out] -2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/(a-b*c^(1/2))^(1/
2)-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))/(a+b*c^(1/2))^(
1/2)
```

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = $\{371, 1398, 827, 1166, 207\}$

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt
[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/Sqrt[a +
b*Sqrt[c]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}\,(-c+x)}\,dx, x, c+dx\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)\\
&= 4\,\text{Subst}\left(\int \frac{-a+x^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)+2\,\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,d\right.\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 97, normalized size = 1.00

$$
-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt
[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/Sqrt[a +
b*Sqrt[c]]
```

fricas [B] time = 0.56, size = 743, normalized size = 7.66

$$
\sqrt{-\frac{\left(b^2c-a^2\right)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a}{b^2c-a^2}}\,\log\left(4\left(\left(b^2c-a^2\right)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a\right)\sqrt{-\frac{\left(b^2c-a^2\right)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}}{b^2c-a^2}}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
- a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)
*sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
 - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) - sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(
b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2))*log(-4*((b^2*c - a^2)*sqr
t(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)*sqrt(-((b^2*c - a^2)*sqrt(b^2*c
```

```
/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*
b + a)) - sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a
)/(b^2*c - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a
^4)) + a)*sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a
)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) + sqrt(((b^2*c - a^2)*sqrt(
b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2))*log(-4*((b^2*c - a
^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)*sqrt(((b^2*c - a^2)*sqrt
(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x
 + c)*b + a))
```

giac [A] time = 0.50, size = 140, normalized size = 1.44

$$\frac{2\left(\frac{\left(b^2\sqrt{c}|b|+ab^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}+a\right)\sqrt{b\sqrt{c}-a}}+\frac{\left(b^2\sqrt{c}|b|-ab^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}-a\right)\sqrt{-b\sqrt{c}-a}}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*((b^2*sqrt(c)*abs(b) + a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a +
sqrt(b^2*c)))/((b*sqrt(c) + a)*sqrt(b*sqrt(c) - a)) + (b^2*sqrt(c)*abs(b) -
 a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*sqrt(c
) - a)*sqrt(-b*sqrt(c) - a)))/b^2
```

maple [A] time = 0.02, size = 92, normalized size = 0.95

$$\frac{2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\sqrt{-a-\sqrt{b^2c}}}+\frac{2\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\sqrt{-a+\sqrt{b^2c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 2/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/
2))^(1/2))+2/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a+
(b^2*c)^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{dx+c}\,b+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*sqrt(c + d*x))), x)
```

3.651 $\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=163

$$-\frac{\sqrt{a+b\sqrt{c+dx}}\,\left(a-b\sqrt{c+dx}\right)}{x\left(a^2-b^2c\right)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{3/2}}$$

```
[Out] -1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/c^(1/2)/(a-
b*c^(1/2))^(3/2)+1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1
/2))/c^(1/2)/(a+b*c^(1/2))^(3/2)-(a-b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1
/2)/(-b^2*c+a^2)/x
```

Rubi [A] time = 0.20, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {371, 1398, 823, 827, 1166, 207}

$$-\frac{\sqrt{a+b\sqrt{c+dx}}\,\left(a-b\sqrt{c+dx}\right)}{x\left(a^2-b^2c\right)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -(((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x)) - (b
*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*(a - b*Sqrt[c
])^(3/2)*Sqrt[c]) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[
c]]])/(2*(a + b*Sqrt[c])^(3/2)*Sqrt[c])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = d\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}\,(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (2d)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x} + \frac{d\,\text{Subst}\left(\int \frac{-\frac{1}{2}abc+\frac{1}{2}b^2cx}{\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{c\left(a^2-b^2c\right)}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x} + \frac{(2d)\,\text{Subst}\left(\int \frac{-ab^2c+\frac{1}{2}b^2cx^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c}}\right.}{c\left(a^2-b^2c\right)}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x} + \frac{(bd)\,\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+d}}\right.}{2\left(a-b\sqrt{c}\right)\sqrt{c}}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{\left(a^2-b^2c\right)x} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\left(a-b\sqrt{c}\right)^{3/2}\sqrt{c}} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c}}}{\sqrt{a+b}}\right.}{2\left(a+b\sqrt{c}\right)^{3/2}}$$

Mathematica [A] time = 0.25, size = 216, normalized size = 1.33

$$\frac{\sqrt{a-b\sqrt{c}}\left(bdx\left(a-b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)-2\sqrt{c}\sqrt{a+b\sqrt{c}}\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}\right)-bdx\left(a\right.}{2\sqrt{c}\,x\sqrt{a-b\sqrt{c}}\sqrt{a+b\sqrt{c}}\left(a^2-b^2c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-(b*(a + b*Sqrt[c])^(3/2)*d*x*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b
*Sqrt[c]]]) + Sqrt[a - b*Sqrt[c]]*(-2*Sqrt[a + b*Sqrt[c]]*Sqrt[c]*(a - b*Sq
rt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]] + b*(a - b*Sqrt[c])*d*x*ArcTanh[Sqrt
[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]))/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[a
+ b*Sqrt[c]]*Sqrt[c]*(a^2 - b^2*c)*x)
```

fricas [B] time = 0.64, size = 2493, normalized size = 15.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^
4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^
4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4
*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2
- a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6
*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)
*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 +
 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c
)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2
*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a
^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2
*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2
*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3
*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c
^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*
a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))
*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*
a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^
10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b
^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(
-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^
6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c
^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^
12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) + (b^2*c - a^2)
*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*
c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^
2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*
c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6
*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6*c^2 + 3*a^3*b^4*c
)*d^2 + (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 +
6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 -
20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^
4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt
((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a
^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(
b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2*c - a^2)*x*sqrt(-
((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6
*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^
6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^1
2*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6*c + 3*a^
2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 + (
b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8
*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^
6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3
*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^
2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^
5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 -
```

```
3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - 4*sqrt(sqrt(d*x + c)*b + a)*(sq
rt(d*x + c)*b - a))/((b^2*c - a^2)*x)
```

giac [B] time = 0.76, size = 654, normalized size = 4.01

$$\frac{\left(\left(b^3c-a^2b\right)^2 b^4 c^{\frac{3}{2}} d^2 - 2\left(ab^6c^2-a^3b^4c\right)d^2\left|-b^3c+a^2b\right| + \left(a^2b^8c^{\frac{5}{2}} - 2\,a^4b^6c^{\frac{3}{2}} + a^6b^4\sqrt{c}\right)d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-\frac{ab^2c-a^3+\sqrt{\left(ab^2c-a^3\right)^2+\left(b^4c^2-2\,a^2b^2c+a^4\right)\left(b^2c-a^2\right)}}{b^2c-a^2}}}\right)}{\left(b^5c^{\frac{7}{2}}+ab^4c^3-2\,a^2b^3c^{\frac{5}{2}}-2\,a^3b^2c^2+a^4bc^{\frac{3}{2}}+a^5c\right)\sqrt{b\sqrt{c}-a}\left|-b^3c+a^2b\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(((b^3*c - a^2*b)^2*b^4*c^(3/2)*d^2 - 2*(a*b^6*c^2 - a^3*b^4*c)*d^2*abs
(-b^3*c + a^2*b) + (a^2*b^8*c^(5/2) - 2*a^4*b^6*c^(3/2) + a^6*b^4*sqrt(c))*
d^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c - a^3 + sqrt((a*b^2*c
- a^3)^2 + (b^4*c^2 - 2*a^2*b^2*c + a^4)*(b^2*c - a^2)))/(b^2*c - a^2)))/((
b^5*c^(7/2) + a*b^4*c^3 - 2*a^2*b^3*c^(5/2) - 2*a^3*b^2*c^2 + a^4*b*c^(3/2)
 + a^5*c)*sqrt(b*sqrt(c) - a)*abs(-b^3*c + a^2*b)) + ((b^3*c - a^2*b)^2*b^4
*c^(3/2)*d^2 + 2*(a*b^6*c^2 - a^3*b^4*c)*d^2*abs(-b^3*c + a^2*b) + (a^2*b^8
*c^(5/2) - 2*a^4*b^6*c^(3/2) + a^6*b^4*sqrt(c))*d^2)*arctan(sqrt(sqrt(d*x +
 c)*b + a)/sqrt(-(a*b^2*c - a^3 - sqrt((a*b^2*c - a^3)^2 + (b^4*c^2 - 2*a^2
*b^2*c + a^4)*(b^2*c - a^2)))/(b^2*c - a^2)))/((b^5*c^(7/2) - a*b^4*c^3 - 2
*a^2*b^3*c^(5/2) + 2*a^3*b^2*c^2 + a^4*b*c^(3/2) - a^5*c)*sqrt(-b*sqrt(c) -
 a)*abs(-b^3*c + a^2*b)) + 2*((sqrt(d*x + c)*b + a)^(3/2)*b^4*d^2 - 2*sqrt(
sqrt(d*x + c)*b + a)*a*b^4*d^2)/((b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*(sqrt
(d*x + c)*b + a)*a - a^2)*(b^2*c - a^2)))/(b^2*d)
```

maple [B] time = 0.03, size = 265, normalized size = 1.63

$$\frac{2\sqrt{b^2c}\,d\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(-4a-4\sqrt{b^2c}\right)\sqrt{-a-\sqrt{b^2c}}\;c}-\frac{2\sqrt{b^2c}\,d\arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(-4a+4\sqrt{b^2c}\right)\sqrt{-a+\sqrt{b^2c}}\;c}-\frac{2\sqrt{b^2c}\,\sqrt{a+\sqrt{dx+c}\,b}\,d}{\left(-4a-4\sqrt{b^2c}\right)\left(-\sqrt{dx+c}\,b+\sqrt{b^2c}\right)c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] -2*d*(b^2*c)^(1/2)/c*(a+(d*x+c)^(1/2)*b)^(1/2)/(-4*(b^2*c)^(1/2)-4*a)/(-(d*
x+c)^(1/2)*b+(b^2*c)^(1/2))+2*d*(b^2*c)^(1/2)/c/(-4*(b^2*c)^(1/2)-4*a)/(-a-
(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1
/2))-2*d*(b^2*c)^(1/2)/c*(a+(d*x+c)^(1/2)*b)^(1/2)/(4*(b^2*c)^(1/2)-4*a)/((
d*x+c)^(1/2)*b+(b^2*c)^(1/2))-2*d*(b^2*c)^(1/2)/c/(4*(b^2*c)^(1/2)-4*a)/(-a
+(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{dx+c}\,b+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*sqrt(c + d*x))), x)
```

3.652 $\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=261

$$-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8cx\left(a^2-b^2c\right)^2}+\frac{bd^2\left(2a-5b\sqrt{c}\right)\tanh^{-1}}{16c^{3/2}\left(a-b\sqrt{\vphantom{c}}\right.}$$

```
[Out] 1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(2*a-5*b*
c^(1/2))/c^(3/2)/(a-b*c^(1/2))^(5/2)-1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2))
^(1/2)/(a+b*c^(1/2))^(1/2))*(2*a+5*b*c^(1/2))/c^(3/2)/(a+b*c^(1/2))^(5/2)-1
/2*(a-b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1/2)/(-b^2*c+a^2)/x^2-1/8*b*d*(
6*a*b*c-(5*b^2*c+a^2)*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1/2)/c/(-b^2*c+a^
2)^2/x
```

Rubi [A] time = 0.48, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {371, 1398, 823, 827, 1166, 207}

$$-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8cx\left(a^2-b^2c\right)^2}+\frac{bd^2\left(2a-5b\sqrt{c}\right)\tanh^{-1}}{16c^{3/2}\left(a-b\sqrt{\vphantom{c}}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(2*(a^2 - b^2*c)*x^2) -
(b*d*Sqrt[a + b*Sqrt[c + d*x]]*(6*a*b*c - (a^2 + 5*b^2*c)*Sqrt[c + d*x]))/(
8*c*(a^2 - b^2*c)^2*x) + (b*(2*a - 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(5/2)*c^(3/2)) - (b*(2
*a + 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]
])/(16*(a + b*Sqrt[c])^(5/2)*c^(3/2))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}\,(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} + \frac{d^2\,\text{Subst}\left(\int \frac{-\frac{1}{2}abc+\frac{5}{2}b^2cx}{\sqrt{a+bx}\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right.}{8c\left(a^2-b^2c\right)^2x}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right.}{8c\left(a^2-b^2c\right)^2x}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right.}{8c\left(a^2-b^2c\right)^2x}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\,\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right.}{8c\left(a^2-b^2c\right)^2x}$$

Mathematica [A] time = 0.75, size = 281, normalized size = 1.08

$$-\frac{\frac{8(a^2-b^2c)\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{x^2}-\frac{2bd\sqrt{a+b\sqrt{c+dx}}\left(a^2\sqrt{c+dx}-6abc+5b^2c\sqrt{c+dx}\right)}{cx}+\frac{bd^2\left(\left(a-b\sqrt{c}\right)^{5/2}\left(2a+5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right.\right.}{c^{3/2}\sqrt{a-b\sqrt{c}}}}{16\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -1/16*((8*(a^2 - b^2*c)*(a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/x^
2 - (2*b*d*Sqrt[a + b*Sqrt[c + d*x]]*(-6*a*b*c + a^2*Sqrt[c + d*x] + 5*b^2*
c*Sqrt[c + d*x]))/(c*x) + (b*d^2*(-((2*a - 5*b*Sqrt[c])*(a + b*Sqrt[c])^(5/
2)*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]]) + (a - b*Sqrt[c]
)^(5/2)*(2*a + 5*b*Sqrt[c])*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sq
rt[c]]]))/(Sqrt[a - b*Sqrt[c]]*Sqrt[a + b*Sqrt[c]]*c^(3/2)))/(a^2 - b^2*c)^
2
```

fricas [B] time = 1.28, size = 4390, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3
*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a
^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4
+ 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10
)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10
+ 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^
6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c
^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*
b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x
+ c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 +
35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*
a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c
^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6
*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^
11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^
8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*
sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b
^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 -
 a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10
780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*
b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*
a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*
c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*
b^2*c^4 - a^10*c^3))) - (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a
*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a
^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sq
rt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*
c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 1
20*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7
- 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*
c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^1
0*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^
6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 6
```

09*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 + (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) - (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 + (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2

```
*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) +
4*(6*a*b^2*c*d*x - 4*a*b^2*c^2 + 4*a^3*c + (4*b^3*c^2 - 4*a^2*b*c - (5*b^3*
c + a^2*b)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a))/((b^4*c^3 - 2*a^2
*b^2*c^2 + a^4*c)*x^2)
```

giac [B] time = 1.27, size = 1303, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*b^4)*d^3 - (13*
a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a^7*b^4*sqrt(c))
*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*c^6 - 17*a^4*b^
12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 - a^12*b^4*c)*d^
3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*
c + sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 - 3*a^2*b^4*c^3 +
 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/(b^4*c^3 - 2*a^
2*b^2*c^2 + a^4*c)))/((b^9*c^6 - a*b^8*c^(11/2) - 4*a^2*b^7*c^5 + 4*a^3*b^6
*c^(9/2) + 6*a^4*b^5*c^4 - 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3 + 4*a^7*b^2*c^
(5/2) + a^8*b*c^2 - a^9*c^(3/2))*sqrt(-b*sqrt(c) - a)*abs(b^5*c^3 - 2*a^2*b
^3*c^2 + a^4*b*c)) + ((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*
b^4)*d^3 + (13*a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a
^7*b^4*sqrt(c))*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*
c^6 - 17*a^4*b^12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 -
 a^12*b^4*c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3
*b^2*c^2 + a^5*c - sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 -
3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/
(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/((b^9*c^6 + a*b^8*c^(11/2) - 4*a^2*b^7*
c^5 - 4*a^3*b^6*c^(9/2) + 6*a^4*b^5*c^4 + 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3
 - 4*a^7*b^2*c^(5/2) + a^8*b*c^2 + a^9*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^5
*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)) - 2*(9*(sqrt(d*x + c)*b + a)^(3/2)*b^8*c^2
*d^3 - 19*sqrt(sqrt(d*x + c)*b + a)*a*b^8*c^2*d^3 - 5*(sqrt(d*x + c)*b + a)
^(7/2)*b^6*c*d^3 + 21*(sqrt(d*x + c)*b + a)^(5/2)*a*b^6*c*d^3 - 30*(sqrt(d*
x + c)*b + a)^(3/2)*a^2*b^6*c*d^3 + 18*sqrt(sqrt(d*x + c)*b + a)*a^3*b^6*c*
d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a^2*b^4*d^3 + 3*(sqrt(d*x + c)*b + a)^(5/
2)*a^3*b^4*d^3 - 3*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^4*d^3 + sqrt(sqrt(d*x
+ c)*b + a)*a^5*b^4*d^3)/((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c - (sqrt(
d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^2))/(b^2*d)
```

maple [B] time = 0.10, size = 840, normalized size = 3.22

$$\frac{a b^2 d^2 \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{8\sqrt{b^2c}\,\left(b^2c+a^2+2\sqrt{b^2c}\,a\right)\sqrt{-a-\sqrt{b^2c}}\;c}+\frac{a b^2 d^2 \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}\,b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{8\sqrt{b^2c}\,\left(-b^2c-a^2+2\sqrt{b^2c}\,a\right)\sqrt{-a+\sqrt{b^2c}}\;c}+\frac{(}{8\sqrt{b^2c}\,\left(\sqrt{dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 5/16*b^2*d^2/c/((d*x+c)^(1/2)*b-(b^2*c)^(1/2))^2/(b^2*c+2*a*(b^2*c)^(1/2)+a
^2)*(a+(d*x+c)^(1/2)*b)^(3/2)+1/8*b^2*d^2/c/(b^2*c)^(1/2)/((d*x+c)^(1/2)*b-
(b^2*c)^(1/2))^2/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)*(a+(d*x+c)^(1/2)*b)^(3/2)*a-
7/16*b^2*d^2/c/((d*x+c)^(1/2)*b-(b^2*c)^(1/2))^2/((b^2*c)^(1/2)+a)*(a+(d*x+
c)^(1/2)*b)^(1/2)-1/8*b^2*d^2/c/(b^2*c)^(1/2)/((d*x+c)^(1/2)*b-(b^2*c)^(1/2
))^2/((b^2*c)^(1/2)+a)*(a+(d*x+c)^(1/2)*b)^(1/2)*a+5/16*b^2*d^2/c/(b^2*c+2*
a*(b^2*c)^(1/2)+a^2)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1
/2)/(-a-(b^2*c)^(1/2))^(1/2))+1/8*b^2*d^2/c/(b^2*c)^(1/2)/(b^2*c+2*a*(b^2*c
```

```
)^(1/2)+a^2)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-
(b^2*c)^(1/2))^(1/2))*a+5/16*b^2*d^2/c/((d*x+c)^(1/2)*b+(b^2*c)^(1/2))^2/(b
^2*c-2*a*(b^2*c)^(1/2)+a^2)*(a+(d*x+c)^(1/2)*b)^(3/2)-1/8*b^2*d^2/c/(b^2*c)
^(1/2)/((d*x+c)^(1/2)*b+(b^2*c)^(1/2))^2/(b^2*c-2*a*(b^2*c)^(1/2)+a^2)*(a+(
d*x+c)^(1/2)*b)^(3/2)*a+1/8*b^2*d^2/c/(b^2*c)^(1/2)/((d*x+c)^(1/2)*b+(b^2*c
)^(1/2))^2/(-(b^2*c)^(1/2)+a)*(a+(d*x+c)^(1/2)*b)^(1/2)*a-7/16*b^2*d^2/c/((
d*x+c)^(1/2)*b+(b^2*c)^(1/2))^2/(-(b^2*c)^(1/2)+a)*(a+(d*x+c)^(1/2)*b)^(1/2
)-5/16*b^2*d^2/c/(-b^2*c+2*a*(b^2*c)^(1/2)-a^2)/(-a+(b^2*c)^(1/2))^(1/2)*ar
ctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))+1/8*b^2*d^2/c/(b^2
*c)^(1/2)/(-b^2*c+2*a*(b^2*c)^(1/2)-a^2)/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a
+(d*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{dx+c}\,b+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)),x)
[Out] int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + b*sqrt(c + d*x))), x)
```

3.653 $\int x^3 \left(a+b\sqrt{c+dx}\right)^p dx$

Optimal. Leaf size=350

$$-\frac{2a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^8d^4(p+1)}+\frac{2\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^8d^4(p+2)}-\frac{6a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(\right.}{b^8d^4(p+3}$$

```
[Out] -2*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(1+p)/b^8/d^4/(1+p)+2*(-b^2*c+a^2)^
2*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(2+p)/b^8/d^4/(2+p)-6*a*(-3*b^2*c+7*a^
2)*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3+p)/b^8/d^4/(3+p)+2*(3*b^4*c^2-30*a^2
*b^2*c+35*a^4)*(a+b*(d*x+c)^(1/2))^(4+p)/b^8/d^4/(4+p)-10*a*(-3*b^2*c+7*a^2
)*(a+b*(d*x+c)^(1/2))^(5+p)/b^8/d^4/(5+p)+6*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/
2))^(6+p)/b^8/d^4/(6+p)-14*a*(a+b*(d*x+c)^(1/2))^(7+p)/b^8/d^4/(7+p)+2*(a+b
*(d*x+c)^(1/2))^(8+p)/b^8/d^4/(8+p)
```

Rubi [A] time = 0.28, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{2\left(-30a^2b^2c+35a^4+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^8d^4(p+4)}-\frac{2a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^8d^4(p+1)}+\frac{2\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)}{b^8d^4(p+}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(1 + p))/(b^8*d^4*(1 + p)) + (2
*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^8*d^4*(2
 + p)) - (6*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p)
)/(b^8*d^4*(3 + p)) + (2*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c
+ d*x])^(4 + p))/(b^8*d^4*(4 + p)) - (10*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c
+ d*x])^(5 + p))/(b^8*d^4*(5 + p)) + (6*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x
])^(6 + p))/(b^8*d^4*(6 + p)) - (14*a*(a + b*Sqrt[c + d*x])^(7 + p))/(b^8*d
^4*(7 + p)) + (2*(a + b*Sqrt[c + d*x])^(8 + p))/(b^8*d^4*(8 + p))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^p(-c+x)^3\,dx,x,c+dx\right)}{d^4}$$
$$=\frac{2\,\text{Subst}\left(\int x(a+bx)^p\left(-c+x^2\right)^3\,dx,x,\sqrt{c+dx}\right)}{d^4}$$
$$=\frac{2\,\text{Subst}\left(\int\left(-\frac{a\left(a^2-b^2c\right)^3(a+bx)^p}{b^7}-\frac{\left(-7a^2+b^2c\right)\left(-a^2+b^2c\right)^2(a+bx)^{1+p}}{b^7}-\frac{3\left(7a^5-10a^3b^2c+3ab^4c^2\right)(a+bx)^2}{b^7}\right.\right.}{}$$
$$=-\frac{2a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^8d^4(1+p)}+\frac{2\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^8d^4(2+p)}-$$

Mathematica [**A**] time = 0.85, size = 555, normalized size = 1.59

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(5040a^7-5040a^6b(p+1)\sqrt{c+dx}+360a^5b^2\left(6c\left(p^2+p-7\right)+7d\left(p^2+3p+2\right)x\right)-120a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(5040*a^7 - 5040*a^6*b*(1 + p)*Sqrt[c + d
*x] + 360*a^5*b^2*(6*c*(-7 + p + p^2) + 7*d*(2 + 3*p + p^2)*x) - 120*a^4*b^
3*(1 + p)*Sqrt[c + d*x]*(2*c*(-63 - 5*p + 2*p^2) + 7*d*(6 + 5*p + p^2)*x) +
 6*a^3*b^4*(8*c^2*(315 - 124*p - 139*p^2 - 14*p^3 + p^4) + 40*c*d*(-42 - 61
*p - 16*p^2 + 4*p^3 + p^4)*x + 35*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)*x
^2) - 6*a^2*b^5*(1 + p)*Sqrt[c + d*x]*(-24*c^2*(-105 - 24*p + 5*p^2 + p^3)
+ 4*c*d*(-420 - 386*p - 94*p^2 - p^3 + p^4)*x + 7*d^2*(120 + 154*p + 71*p^2
 + 14*p^3 + p^4)*x^2) - b^7*(105 + 176*p + 86*p^2 + 16*p^3 + p^4)*Sqrt[c +
d*x]*(-48*c^3 + 24*c^2*d*(2 + p)*x - 6*c*d^2*(8 + 6*p + p^2)*x^2 + d^3*(48
+ 44*p + 12*p^2 + p^3)*x^3) + a*b^6*(48*c^3*(-105 + 103*p + 138*p^2 + 38*p^
3 + 3*p^4) - 24*c^2*d*(-210 - 283*p - 21*p^2 + 74*p^3 + 24*p^4 + 2*p^5)*x +
 6*c*d^2*(-840 - 1726*p - 1151*p^2 - 265*p^3 + 10*p^4 + 11*p^5 + p^6)*x^2 +
 7*d^3*(720 + 1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6)*x^3)))/
(b^8*d^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p)*(7 + p)*(8 + p))
```

fricas [**B**] time = 0.67, size = 1416, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(5040*b^8*c^4 - 20160*a^2*b^6*c^3 + 30240*a^4*b^4*c^2 - 20160*a^6*b^2*c
+ 5040*a^8 + 48*(b^8*c^4 + 6*a^2*b^6*c^3 + a^4*b^4*c^2)*p^4 - (b^8*d^4*p^7
+ 28*b^8*d^4*p^6 + 322*b^8*d^4*p^5 + 1960*b^8*d^4*p^4 + 6769*b^8*d^4*p^3 +
13132*b^8*d^4*p^2 + 13068*b^8*d^4*p + 5040*b^8*d^4)*x^4 + 384*(2*b^8*c^4 +
7*a^2*b^6*c^3 - 3*a^4*b^4*c^2)*p^3 - (b^8*c*d^3*p^7 + (22*b^8*c - 7*a^2*b^6
)*d^3*p^6 + 5*(38*b^8*c - 21*a^2*b^6)*d^3*p^5 + 5*(164*b^8*c - 119*a^2*b^6)
*d^3*p^4 + (1849*b^8*c - 1575*a^2*b^6)*d^3*p^3 + 2*(1019*b^8*c - 959*a^2*b^
6)*d^3*p^2 + 840*(b^8*c - a^2*b^6)*d^3*p)*x^3 + 48*(86*b^8*c^4 + 81*a^2*b^6
*c^3 - 124*a^4*b^4*c^2 + 45*a^6*b^2*c)*p^2 + 6*(18*b^8*c^2*d^2*p^5 + (b^8*c
^2 + a^2*b^6*c)*d^2*p^6 + (118*b^8*c^2 - 95*a^2*b^6*c + 35*a^4*b^4)*d^2*p^4
 + 6*(58*b^8*c^2 - 80*a^2*b^6*c + 35*a^4*b^4)*d^2*p^3 + (457*b^8*c^2 - 806*
a^2*b^6*c + 385*a^4*b^4)*d^2*p^2 + 210*(b^8*c^2 - 2*a^2*b^6*c + a^4*b^4)*d^
```

```
2*p)*x^2 + 192*(44*b^8*c^4 - 71*a^2*b^6*c^3 + 54*a^4*b^4*c^2 - 15*a^6*b^2*c
)*p - 24*((b^8*c^3 + 3*a^2*b^6*c^2)*d*p^5 + 2*(8*b^8*c^3 + 9*a^2*b^6*c^2 -
5*a^4*b^4*c)*d*p^4 + (86*b^8*c^3 - 57*a^2*b^6*c^2 + 15*a^4*b^4*c)*d*p^3 + (
176*b^8*c^3 - 387*a^2*b^6*c^2 + 340*a^4*b^4*c - 105*a^6*b^2)*d*p^2 + 105*(b
^8*c^3 - 3*a^2*b^6*c^2 + 3*a^4*b^4*c - a^6*b^2)*d*p)*x + (192*(a*b^7*c^3 +
a^3*b^5*c^2)*p^4 + 96*(27*a*b^7*c^3 + 2*a^3*b^5*c^2 - 5*a^5*b^3*c)*p^3 - (a
*b^7*d^3*p^7 + 21*a*b^7*d^3*p^6 + 175*a*b^7*d^3*p^5 + 735*a*b^7*d^3*p^4 + 1
624*a*b^7*d^3*p^3 + 1764*a*b^7*d^3*p^2 + 720*a*b^7*d^3*p)*x^3 + 192*(56*a*b
^7*c^3 - 49*a^3*b^5*c^2 + 15*a^5*b^3*c)*p^2 + 6*(2*a*b^7*c*d^2*p^6 + (33*a*
b^7*c - 7*a^3*b^5)*d^2*p^5 + 10*(20*a*b^7*c - 7*a^3*b^5)*d^2*p^4 + 5*(111*a
*b^7*c - 49*a^3*b^5)*d^2*p^3 + 2*(349*a*b^7*c - 175*a^3*b^5)*d^2*p^2 + 24*(
13*a*b^7*c - 7*a^3*b^5)*d^2*p)*x^2 + 48*(279*a*b^7*c^3 - 511*a^3*b^5*c^2 +
385*a^5*b^3*c - 105*a^7*b)*p - 24*((3*a*b^7*c^2 + a^3*b^5*c)*d*p^5 + 2*(21*
a*b^7*c^2 - 5*a^3*b^5*c)*d*p^4 + (192*a*b^7*c^2 - 135*a^3*b^5*c + 35*a^5*b^
3)*d*p^3 + (327*a*b^7*c^2 - 320*a^3*b^5*c + 105*a^5*b^3)*d*p^2 + 2*(87*a*b^
7*c^2 - 98*a^3*b^5*c + 35*a^5*b^3)*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b
+ a)^p/(b^8*d^4*p^8 + 36*b^8*d^4*p^7 + 546*b^8*d^4*p^6 + 4536*b^8*d^4*p^5 +
 22449*b^8*d^4*p^4 + 67284*b^8*d^4*p^3 + 118124*b^8*d^4*p^2 + 109584*b^8*d^
4*p + 40320*b^8*d^4)
```

giac [B] time = 0.97, size = 5699, normalized size = 16.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] -2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^7 - (sqrt(d*x
 + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^7 + 34*(sqrt(d*x + c)*b +
a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^6 - 35*(sqrt(d*x + c)*b + a)*(sqrt(d
*x + c)*b + a)^p*a*b^6*c^3*p^6 - 3*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b
 + a)^p*b^4*c^2*p^7 + 9*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b
^4*c^2*p^7 - 9*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*
p^7 + 3*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^7 + 478
*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^5 - 511*(sqrt(d*
x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^5 - 96*(sqrt(d*x + c)*b +
 a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^6 + 297*(sqrt(d*x + c)*b + a)^3*(sq
rt(d*x + c)*b + a)^p*a*b^4*c^2*p^6 - 306*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x
+ c)*b + a)^p*a^2*b^4*c^2*p^6 + 105*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b
+ a)^p*a^3*b^4*c^2*p^6 + 3*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*
b^2*c*p^7 - 15*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^7
+ 30*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^7 - 30*(sq
rt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^7 + 15*(sqrt(d*x +
 c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^7 - 3*(sqrt(d*x + c)*b + a
)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^7 + 3580*(sqrt(d*x + c)*b + a)^2*(sqr
t(d*x + c)*b + a)^p*b^6*c^3*p^4 - 4025*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)
*b + a)^p*a*b^6*c^3*p^4 - 1254*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a
)^p*b^4*c^2*p^5 + 4023*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^
4*c^2*p^5 - 4302*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^
2*p^5 + 1533*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^5
+ 90*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^6 - 465*(sqrt(
d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^6 + 960*(sqrt(d*x + c)*
b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^6 - 990*(sqrt(d*x + c)*b + a)^
3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^6 + 510*(sqrt(d*x + c)*b + a)^2*(sqrt
(d*x + c)*b + a)^p*a^4*b^2*c*p^6 - 105*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)
*b + a)^p*a^5*b^2*c*p^6 - (sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p
^7 + 7*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^7 - 21*(sqrt(d*x
 + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^7 + 35*(sqrt(d*x + c)*b + a)^5
*(sqrt(d*x + c)*b + a)^p*a^3*p^7 - 35*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c
)*b + a)^p*a^4*p^7 + 21*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5
```

p^7 - 7(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6*p^7 + (sqrt(d*
x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^7 + 15289*(sqrt(d*x + c)*b + a)
^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^3 - 18424*(sqrt(d*x + c)*b + a)*(sqrt(
d*x + c)*b + a)^p*a*b^6*c^3*p^3 - 8592*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x +
c)*b + a)^p*b^4*c^2*p^4 + 28755*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
a)^p*a*b^4*c^2*p^4 - 32220*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*
a^2*b^4*c^2*p^4 + 12075*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b
^4*c^2*p^4 + 1098*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^5
 - 5865*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^5 + 12540
*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^5 - 13410*(sqr
t(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^5 + 7170*(sqrt(d*x
+ c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^5 - 1533*(sqrt(d*x + c)*b
 + a)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^5 - 28*(sqrt(d*x + c)*b + a)^8*(s
qrt(d*x + c)*b + a)^p*p^6 + 203*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b +
a)^p*a*p^6 - 630*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^6 +
1085*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p^6 - 1120*(sqrt(d
*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4*p^6 + 693*(sqrt(d*x + c)*b + a
)^3*(sqrt(d*x + c)*b + a)^p*a^5*p^6 - 238*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x
 + c)*b + a)^p*a^6*p^6 + 35*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a
^7*p^6 + 36706*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^2
- 48860*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^2 - 32979
*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^3 + 115776*(sqrt
(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^3 - 137601*(sqrt(d*x
 + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*p^3 + 55272*(sqrt(d*x +
c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^3 + 7020*(sqrt(d*x + c)*b +
 a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^4 - 38715*(sqrt(d*x + c)*b + a)^5*(sq
rt(d*x + c)*b + a)^p*a*b^2*c*p^4 + 85920*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x
+ c)*b + a)^p*a^2*b^2*c*p^4 - 95850*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*
b + a)^p*a^3*b^2*c*p^4 + 53700*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a
)^p*a^4*b^2*c*p^4 - 12075*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5
*b^2*c*p^4 - 322*(sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^5 + 2401
*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^5 - 7686*(sqrt(d*x + c
)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^5 + 13685*(sqrt(d*x + c)*b + a)^5*
(sqrt(d*x + c)*b + a)^p*a^3*p^5 - 14630*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x +
 c)*b + a)^p*a^4*p^5 + 9387*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p
*a^5*p^5 - 3346*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6*p^5 + 5
11*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^5 + 44712*(sqrt(d*x
+ c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p - 69264*(sqrt(d*x + c)*b +
a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p - 69936*(sqrt(d*x + c)*b + a)^4*(sqr
t(d*x + c)*b + a)^p*b^4*c^2*p^2 + 258228*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x
+ c)*b + a)^p*a*b^4*c^2*p^2 - 330354*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)
*b + a)^p*a^2*b^4*c^2*p^2 + 146580*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b +
 a)^p*a^3*b^4*c^2*p^2 + 25227*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)
^p*b^2*c*p^3 - 143160*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2
*c*p^3 + 329790*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p
^3 - 385920*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^3 +
 229335*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^3 - 552
72*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^3 - 1960*(sqrt
(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^4 + 14945*(sqrt(d*x + c)*b + a
)^7*(sqrt(d*x + c)*b + a)^p*a*p^4 - 49140*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x
 + c)*b + a)^p*a^2*p^4 + 90335*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a
)^p*a^3*p^4 - 100240*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4*p^
4 + 67095*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5*p^4 - 25060*(
sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6*p^4 + 4025*(sqrt(d*x + c
)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^4 + 20160*(sqrt(d*x + c)*b + a)^2*(s
qrt(d*x + c)*b + a)^p*b^6*c^3 - 40320*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*
b + a)^p*a*b^6*c^3 - 74628*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*
b^4*c^2*p + 288432*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^
2*p - 402408*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*p

```
+ 207792*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p + 5049
0*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^2 - 293460*(sqrt(
d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^2 + 699360*(sqrt(d*x +
c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^2 - 860760*(sqrt(d*x + c)*b
 + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^2 + 550590*(sqrt(d*x + c)*b + a
)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^2 - 146580*(sqrt(d*x + c)*b + a)*(s
qrt(d*x + c)*b + a)^p*a^5*b^2*c*p^2 - 6769*(sqrt(d*x + c)*b + a)^8*(sqrt(d*
x + c)*b + a)^p*p^3 + 52528*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p
*a*p^3 - 176589*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^3 + 3
34040*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p^3 - 384755*(sqr
t(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4*p^3 + 270144*(sqrt(d*x + c)
*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5*p^3 - 107023*(sqrt(d*x + c)*b + a)^2*
(sqrt(d*x + c)*b + a)^p*a^6*p^3 + 18424*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c
)*b + a)^p*a^7*p^3 - 30240*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*
b^4*c^2 + 120960*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2
- 181440*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2 + 1209
60*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2 + 51432*(sqrt(
d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p - 304560*(sqrt(d*x + c)*b
 + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p + 746280*(sqrt(d*x + c)*b + a)^4*
(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p - 961440*(sqrt(d*x + c)*b + a)^3*(sqrt(
d*x + c)*b + a)^p*a^3*b^2*c*p + 670680*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x +
c)*b + a)^p*a^4*b^2*c*p - 207792*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a
)^p*a^5*b^2*c*p - 13132*(sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^2
 + 103292*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^2 - 353430*(s
qrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^2 + 684740*(sqrt(d*x +
c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p^2 - 815920*(sqrt(d*x + c)*b + a)^
4*(sqrt(d*x + c)*b + a)^p*a^4*p^2 + 602532*(sqrt(d*x + c)*b + a)^3*(sqrt(d*
x + c)*b + a)^p*a^5*p^2 - 256942*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b +
 a)^p*a^6*p^2 + 48860*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^2
 + 20160*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c - 120960*(sq
rt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c + 302400*(sqrt(d*x + c
)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c - 403200*(sqrt(d*x + c)*b + a)
^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c + 302400*(sqrt(d*x + c)*b + a)^2*(sqrt
(d*x + c)*b + a)^p*a^4*b^2*c - 120960*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*
b + a)^p*a^5*b^2*c - 13068*(sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*
p + 103824*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p - 360024*(sq
rt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p + 710640*(sqrt(d*x + c)*
b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p - 870660*(sqrt(d*x + c)*b + a)^4*(sq
rt(d*x + c)*b + a)^p*a^4*p + 673008*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*
b + a)^p*a^5*p - 312984*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6
*p + 69264*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p - 5040*(sqrt
(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p + 40320*(sqrt(d*x + c)*b + a)^7*
(sqrt(d*x + c)*b + a)^p*a - 141120*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b
 + a)^p*a^2 + 282240*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3 -
352800*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4 + 282240*(sqrt(d
*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5 - 141120*(sqrt(d*x + c)*b + a)
^2*(sqrt(d*x + c)*b + a)^p*a^6 + 40320*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)
*b + a)^p*a^7)/((b^6*d^3*p^8 + 36*b^6*d^3*p^7 + 546*b^6*d^3*p^6 + 4536*b^6*
d^3*p^5 + 22449*b^6*d^3*p^4 + 67284*b^6*d^3*p^3 + 118124*b^6*d^3*p^2 + 1095
84*b^6*d^3*p + 40320*b^6*d^3)*b^2*d)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x^3 \left(a + \sqrt{dx + c}\, b\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+(d*x+c)^(1/2)*b)^p,x)
```

```
[Out] int(x^3*(a+(d*x+c)^(1/2)*b)^p,x)
```

maxima [B] time = 1.13, size = 728, normalized size = 2.08

$$-\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p c^3}{(p^2+3p+2)b^2}-\frac{3\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)a^2b^2+6\sqrt{dx+c}\right.}{(p^4+10p^3+35p^2+50p+24)b^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
a)^p*c^3/((p^2 + 3*p + 2)*b^2) - 3*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^
4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b
^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c^2/((p^4 + 1
0*p^3 + 35*p^2 + 50*p + 24)*b^4) + 3*((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 27
4*p + 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x +
 c)^(5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(
p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2
 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p*c/((p^6 + 2
1*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6) - ((p^7 + 28*p^6
+ 322*p^5 + 1960*p^4 + 6769*p^3 + 13132*p^2 + 13068*p + 5040)*(d*x + c)^4*b
^8 + (p^7 + 21*p^6 + 175*p^5 + 735*p^4 + 1624*p^3 + 1764*p^2 + 720*p)*(d*x
+ c)^(7/2)*a*b^7 - 7*(p^6 + 15*p^5 + 85*p^4 + 225*p^3 + 274*p^2 + 120*p)*(d
*x + c)^3*a^2*b^6 + 42*(p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(5
/2)*a^3*b^5 - 210*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^4*b^4 + 840*(p
^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^5*b^3 - 2520*(p^2 + p)*(d*x + c)*a^6*b^
2 + 5040*sqrt(d*x + c)*a^7*b*p - 5040*a^8)*(sqrt(d*x + c)*b + a)^p/((p^8 +
36*p^7 + 546*p^6 + 4536*p^5 + 22449*p^4 + 67284*p^3 + 118124*p^2 + 109584*p
 + 40320)*b^8))/d^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\left(a+b\sqrt{c+d\,x}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] int(x^3*(a + b*(c + d*x)^(1/2))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral(x**3*(a + b*sqrt(c + d*x))**p, x)
```

3.654 $\int x^2 \left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal. Leaf size=242

$$-\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^6d^3(p+1)}-\frac{4a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^6d^3(p+3)}+\frac{4\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^6d^3(p+4)}+\frac{2}{}$$

```
[Out] -2*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(1+p)/b^6/d^3/(1+p)+2*(b^4*c^2-6*a^
2*b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(2+p)/b^6/d^3/(2+p)-4*a*(-3*b^2*c+5*a^2)
*(a+b*(d*x+c)^(1/2))^(3+p)/b^6/d^3/(3+p)+4*(-b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2
))^(4+p)/b^6/d^3/(4+p)-10*a*(a+b*(d*x+c)^(1/2))^(5+p)/b^6/d^3/(5+p)+2*(a+b*
(d*x+c)^(1/2))^(6+p)/b^6/d^3/(6+p)
```

Rubi [A] time = 0.18, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{2\left(-6a^2b^2c+5a^4+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^6d^3(p+2)}-\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^6d^3(p+1)}-\frac{4a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+}\right.}{b^6d^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(1 + p))/(b^6*d^3*(1 + p)) + (2
*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^6*d^3*(2
 + p)) - (4*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p))/(b^6*d^3*(3
+ p)) + (4*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(4 + p))/(b^6*d^3*(4 + p))
 - (10*a*(a + b*Sqrt[c + d*x])^(5 + p))/(b^6*d^3*(5 + p)) + (2*(a + b*Sqrt[
c + d*x])^(6 + p))/(b^6*d^3*(6 + p))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^2 \left(a+b\sqrt{c+dx}\right)^p dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p (-c+x)^2 dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int x(a+bx)^p \left(-c+x^2\right)^2 dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a\left(a^2-b^2c\right)^2(a+bx)^p}{b^5} + \frac{\left(5a^4-6a^2b^2c+b^4c^2\right)(a+bx)^{1+p}}{b^5} - \frac{2\left(5a^3-3ab^2c\right)(a+bx)^{2+p}}{b^5} - \frac{2\left(-5a^2\right.}{}\right.\right.}{d^3}$$

$$= -\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^6d^3(1+p)} + \frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^6d^3(2+p)} -$$

Mathematica [A] time = 0.37, size = 284, normalized size = 1.17

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(120a^5-120a^4b(p+1)\sqrt{c+dx}+12a^3b^2\left(4c\left(p^2+p-5\right)+5d\left(p^2+3p+2\right)x\right)-4a^2b^3(p\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(120*a^5 - 120*a^4*b*(1 + p)*Sqrt[c + d*x
] + 12*a^3*b^2*(4*c*(-5 + p + p^2) + 5*d*(2 + 3*p + p^2)*x) - 4*a^2*b^3*(1
+ p)*Sqrt[c + d*x]*(2*c*(-30 - 4*p + p^2) + 5*d*(6 + 5*p + p^2)*x) - b^5*(1
5 + 23*p + 9*p^2 + p^3)*Sqrt[c + d*x]*(8*c^2 - 4*c*d*(2 + p)*x + d^2*(8 + 6
*p + p^2)*x^2) + a*b^4*(-8*c^2*(-15 + 10*p + 12*p^2 + 2*p^3) + 4*c*d*(-30 -
 43*p - 10*p^2 + 4*p^3 + p^4)*x + 5*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)
*x^2)))/(b^6*d^3*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p))
```

fricas [B] time = 0.57, size = 712, normalized size = 2.94

$$\frac{2\left(120\,b^6c^3-360\,a^2b^4c^2+360\,a^4b^2c-120\,a^6+8\left(b^6c^3+3\,a^2b^4c^2\right)p^3+\left(b^6d^3p^5+15\,b^6d^3p^4+85\,b^6d^3p^3+225\,b^6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(120*b^6*c^3 - 360*a^2*b^4*c^2 + 360*a^4*b^2*c - 120*a^6 + 8*(b^6*c^3 + 3
*a^2*b^4*c^2)*p^3 + (b^6*d^3*p^5 + 15*b^6*d^3*p^4 + 85*b^6*d^3*p^3 + 225*b^
6*d^3*p^2 + 274*b^6*d^3*p + 120*b^6*d^3)*x^3 + 24*(3*b^6*c^3 + 3*a^2*b^4*c^
2 - 2*a^4*b^2*c)*p^2 + (b^6*c*d^2*p^5 + (11*b^6*c - 5*a^2*b^4)*d^2*p^4 + (4
1*b^6*c - 30*a^2*b^4)*d^2*p^3 + (61*b^6*c - 55*a^2*b^4)*d^2*p^2 + 30*(b^6*c
 - a^2*b^4)*d^2*p)*x^2 + 8*(23*b^6*c^3 - 24*a^2*b^4*c^2 + 9*a^4*b^2*c)*p -
4*((b^6*c^2 + a^2*b^4*c)*d*p^4 + 3*(3*b^6*c^2 - a^2*b^4*c)*d*p^3 + (23*b^6*
c^2 - 34*a^2*b^4*c + 15*a^4*b^2)*d*p^2 + 15*(b^6*c^2 - 2*a^2*b^4*c + a^4*b^
2)*d*p)*x + (8*(3*a*b^5*c^2 + a^3*b^3*c)*p^3 + 24*(7*a*b^5*c^2 - 3*a^3*b^3*
c)*p^2 + (a*b^5*d^2*p^5 + 10*a*b^5*d^2*p^4 + 35*a*b^5*d^2*p^3 + 50*a*b^5*d^
2*p^2 + 24*a*b^5*d^2*p)*x^2 + 8*(33*a*b^5*c^2 - 40*a^3*b^3*c + 15*a^5*b)*p
- 4*(2*a*b^5*c*d*p^4 + 5*(3*a*b^5*c - a^3*b^3)*d*p^3 + (31*a*b^5*c - 15*a^3
*b^3)*d*p^2 + 2*(9*a*b^5*c - 5*a^3*b^3)*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x +
c)*b + a)^p/(b^6*d^3*p^6 + 21*b^6*d^3*p^5 + 175*b^6*d^3*p^4 + 735*b^6*d^3*p
^3 + 1624*b^6*d^3*p^2 + 1764*b^6*d^3*p + 720*b^6*d^3)
```

giac [B] time = 0.59, size = 2511, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^5 - (sqrt(d*x
+ c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^5 + 19*(sqrt(d*x + c)*b + a
)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^4 - 20*(sqrt(d*x + c)*b + a)*(sqrt(d*
x + c)*b + a)^p*a*b^4*c^2*p^4 - 2*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b
+ a)^p*b^2*c*p^5 + 6*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*
c*p^5 - 6*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^5 + 2
*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^5 + 137*(sqrt(d*
x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^3 - 155*(sqrt(d*x + c)*b
+ a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^3 - 34*(sqrt(d*x + c)*b + a)^4*(sq
rt(d*x + c)*b + a)^p*b^2*c*p^4 + 108*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)
*b + a)^p*a*b^2*c*p^4 - 114*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p
*a^2*b^2*c*p^4 + 40*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c
*p^4 + (sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^5 - 5*(sqrt(d*x +
c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^5 + 10*(sqrt(d*x + c)*b + a)^4*(sqr
t(d*x + c)*b + a)^p*a^2*p^5 - 10*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
 a)^p*a^3*p^5 + 5*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p^5 -
 (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^5 + 461*(sqrt(d*x + c)
*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^2 - 580*(sqrt(d*x + c)*b + a)*(
sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^2 - 214*(sqrt(d*x + c)*b + a)^4*(sqrt(d*
x + c)*b + a)^p*b^2*c*p^3 + 726*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
a)^p*a*b^2*c*p^3 - 822*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*
b^2*c*p^3 + 310*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^3
 + 15*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^4 - 80*(sqrt(d*x +
c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^4 + 170*(sqrt(d*x + c)*b + a)^4*(sq
rt(d*x + c)*b + a)^p*a^2*p^4 - 180*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b
 + a)^p*a^3*p^4 + 95*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p^
4 - 20*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^4 + 702*(sqrt(d*
x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p - 1044*(sqrt(d*x + c)*b +
 a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p - 614*(sqrt(d*x + c)*b + a)^4*(sqrt
(d*x + c)*b + a)^p*b^2*c*p^2 + 2232*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*
b + a)^p*a*b^2*c*p^2 - 2766*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p
*a^2*b^2*c*p^2 + 1160*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2
*c*p^2 + 85*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^3 - 475*(sqrt
(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^3 + 1070*(sqrt(d*x + c)*b +
a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p^3 - 1210*(sqrt(d*x + c)*b + a)^3*(sqrt(d
*x + c)*b + a)^p*a^3*p^3 + 685*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a
)^p*a^4*p^3 - 155*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^3 + 3
60*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2 - 720*(sqrt(d*x
+ c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2 - 792*(sqrt(d*x + c)*b + a)^4
*(sqrt(d*x + c)*b + a)^p*b^2*c*p + 3048*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x +
 c)*b + a)^p*a*b^2*c*p - 4212*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)
^p*a^2*b^2*c*p + 2088*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2
*c*p + 225*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^2 - 1300*(sqrt
(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^2 + 3070*(sqrt(d*x + c)*b +
a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p^2 - 3720*(sqrt(d*x + c)*b + a)^3*(sqrt(d
*x + c)*b + a)^p*a^3*p^2 + 2305*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b +
a)^p*a^4*p^2 - 580*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^2 -
360*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c + 1440*(sqrt(d*x
+ c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*c - 2160*(sqrt(d*x + c)*b + a)^
2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c + 1440*(sqrt(d*x + c)*b + a)*(sqrt(d*x
+ c)*b + a)^p*a^3*b^2*c + 274*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)
^p*p - 1620*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p + 3960*(sqr
t(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p - 5080*(sqrt(d*x + c)*b +
 a)^3*(sqrt(d*x + c)*b + a)^p*a^3*p + 3510*(sqrt(d*x + c)*b + a)^2*(sqrt(d*
x + c)*b + a)^p*a^4*p - 1044*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*
a^5*p + 120*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p - 720*(sqrt(d*x
```

```
 + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a + 1800*(sqrt(d*x + c)*b + a)^4*(sq
rt(d*x + c)*b + a)^p*a^2 - 2400*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
a)^p*a^3 + 1800*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4 - 720*(
sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5)/((b^4*d^2*p^6 + 21*b^4*d^
2*p^5 + 175*b^4*d^2*p^4 + 735*b^4*d^2*p^3 + 1624*b^4*d^2*p^2 + 1764*b^4*d^2
*p + 720*b^4*d^2)*b^2*d)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + \sqrt{dx + c}\, b\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+(d*x+c)^(1/2)*b)^p,x)
```

```
[Out] int(x^2*(a+(d*x+c)^(1/2)*b)^p,x)
```

maxima [A] time = 1.06, size = 402, normalized size = 1.66

$$2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p c^2}{\left(p^2+3\,p+2\right)b^2} - \frac{2\left(\left(p^3+6\,p^2+11\,p+6\right)(dx+c)^2 b^4+\left(p^3+3\,p^2+2\,p\right)(dx+c)^{\frac{3}{2}}ab^3-3\left(p^2+p\right)(dx+c)a^2b^2+6\,\sqrt{dx+c}\,a^3\right.}{\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a
)^p*c^2/((p^2 + 3*p + 2)*b^2) - 2*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4
 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^
2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c/((p^4 + 10*p
^3 + 35*p^2 + 50*p + 24)*b^4) + ((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p +
 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(
5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(p^3 +
 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2 + 12
0*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p/((p^6 + 21*p^5 +
 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6))/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b\,\sqrt{c + d\,x}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] int(x^2*(a + b*(c + d*x)^(1/2))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b\sqrt{c + dx}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral(x**2*(a + b*sqrt(c + d*x))**p, x)
```

3.655 $\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal. Leaf size=145

$$-\frac{2a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^4d^2(p+1)}+\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^4d^2(p+2)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4d^2(p+3)}+\frac{2\left(a+b\sqrt{c+dx}\right.}{b^4d^2(p+4)}$$

```
[Out] -2*a*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(1+p)/b^4/d^2/(1+p)+2*(-b^2*c+3*a^2)*
(a+b*(d*x+c)^(1/2))^(2+p)/b^4/d^2/(2+p)-6*a*(a+b*(d*x+c)^(1/2))^(3+p)/b^4/d
^2/(3+p)+2*(a+b*(d*x+c)^(1/2))^(4+p)/b^4/d^2/(4+p)
```

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {371, 1398, 772}

$$-\frac{2a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^4d^2(p+1)}+\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^4d^2(p+2)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4d^2(p+3)}+\frac{2\left(a+b\sqrt{c+dx}\right.}{b^4d^2(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(1 + p))/(b^4*d^2*(1 + p)) + (2*(
3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^4*d^2*(2 + p)) - (6*a*(a +
 b*Sqrt[c + d*x])^(3 + p))/(b^4*d^2*(3 + p)) + (2*(a + b*Sqrt[c + d*x])^(4
+ p))/(b^4*d^2*(4 + p))
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^p(-c+x)\,dx,x,c+dx\right)}{d^2}$$
$$=\frac{2\,\text{Subst}\left(\int x(a+bx)^p\left(-c+x^2\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}$$
$$=\frac{2\,\text{Subst}\left(\int\left(\frac{(-a^3+ab^2c)(a+bx)^p}{b^3}+\frac{(3a^2-b^2c)(a+bx)^{1+p}}{b^3}-\frac{3a(a+bx)^{2+p}}{b^3}+\frac{(a+bx)^{3+p}}{b^3}\right)dx,x,\sqrt{c+dx}\right.}{d^2}$$
$$=-\frac{2a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^4d^2(1+p)}+\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^4d^2(2+p)}-\frac{6a\left(a+b\sqrt{c\right.}}{b^4d^2(3}$$

Mathematica [A] time = 0.16, size = 128, normalized size = 0.88

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(6a^3-6a^2b(p+1)\sqrt{c+dx}+ab^2\left(2c\left(p^2+p-3\right)+3d\left(p^2+3p+2\right)x\right)-b^3\left(p^2+4p+3\right)\right.}{b^4d^2(p+1)(p+2)(p+3)(p+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(6*a^3 - 6*a^2*b*(1 + p)*Sqrt[c + d*x] -
b^3*(3 + 4*p + p^2)*Sqrt[c + d*x]*(-2*c + d*(2 + p)*x) + a*b^2*(2*c*(-3 + p
 + p^2) + 3*d*(2 + 3*p + p^2)*x)))/(b^4*d^2*(1 + p)*(2 + p)*(3 + p)*(4 + p)
)
```

fricas [B] time = 0.54, size = 294, normalized size = 2.03

$$-\frac{2\left(6\,b^4c^2-12\,a^2b^2c+6\,a^4+2\left(b^4c^2+a^2b^2c\right)p^2-\left(b^4d^2p^3+6\,b^4d^2p^2+11\,b^4d^2p+6\,b^4d^2\right)x^2+4\left(2\,b^4c^2-a^2b^2c\right.\right.}{b^4d^2p^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(6*b^4*c^2 - 12*a^2*b^2*c + 6*a^4 + 2*(b^4*c^2 + a^2*b^2*c)*p^2 - (b^4*d
^2*p^3 + 6*b^4*d^2*p^2 + 11*b^4*d^2*p + 6*b^4*d^2)*x^2 + 4*(2*b^4*c^2 - a^2
*b^2*c)*p - (b^4*c*d*p^3 + (4*b^4*c - 3*a^2*b^2)*d*p^2 + 3*(b^4*c - a^2*b^2
)*d*p)*x + (4*a*b^3*c*p^2 + 2*(5*a*b^3*c - 3*a^3*b)*p - (a*b^3*d*p^3 + 3*a*
b^3*d*p^2 + 2*a*b^3*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b + a)^p/(b^4*d^2
*p^4 + 10*b^4*d^2*p^3 + 35*b^4*d^2*p^2 + 50*b^4*d^2*p + 24*b^4*d^2)
```

giac [B] time = 0.44, size = 806, normalized size = 5.56

$$-\frac{2\left(\left(\sqrt{dx+c}\,b+a\right)^2\left(\sqrt{dx+c}\,b+a\right)^pb^2cp^3-\left(\sqrt{dx+c}\,b+a\right)\left(\sqrt{dx+c}\,b+a\right)^pab^2cp^3+8\left(\sqrt{dx+c}\,b+a\right)^2\left(\sqrt{dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] -2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c*p^3 - (sqrt(d*x +
 c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^3 + 8*(sqrt(d*x + c)*b + a)^2*
(sqrt(d*x + c)*b + a)^p*b^2*c*p^2 - 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*
b + a)^p*a*b^2*c*p^2 - (sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^3
+ 3*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*p^3 - 3*(sqrt(d*x + c
```

```
)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p^3 + (sqrt(d*x + c)*b + a)*(sqrt(d*
x + c)*b + a)^p*a^3*p^3 + 19*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^
p*b^2*c*p - 26*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p - 6*
(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^2 + 21*(sqrt(d*x + c)*b +
 a)^3*(sqrt(d*x + c)*b + a)^p*a*p^2 - 24*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x
+ c)*b + a)^p*a^2*p^2 + 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3
*p^2 + 12*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c - 24*(sqrt(
d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c - 11*(sqrt(d*x + c)*b + a)^
4*(sqrt(d*x + c)*b + a)^p*p + 42*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
 a)^p*a*p - 57*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p + 26*(
sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*p - 6*(sqrt(d*x + c)*b + a
)^4*(sqrt(d*x + c)*b + a)^p + 24*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
 a)^p*a - 36*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2 + 24*(sqrt
(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3)/((b^2*p^4 + 10*b^2*p^3 + 35*b
^2*p^2 + 50*b^2*p + 24*b^2)*b^2*d^2)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+\sqrt{dx+c}\,b\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+(d*x+c)^(1/2)*b)^p,x)
```

```
[Out] int(x*(a+(d*x+c)^(1/2)*b)^p,x)
```

maxima [A] time = 0.99, size = 187, normalized size = 1.29

$$-\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p c}{(p^2+3p+2)b^2}-\frac{\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)a^2b^2+6\sqrt{dx+}\right.}{(p^4+10p^3+35p^2+50p+24)b^4}\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
a)^p*c/((p^2 + 3*p + 2)*b^2) - ((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4 +
(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^2 +
 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p/((p^4 + 10*p^3 +
35*p^2 + 50*p + 24)*b^4))/d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\sqrt{c+d\,x}\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] int(x*(a + b*(c + d*x)^(1/2))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\sqrt{c+dx}\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral(x*(a + b*sqrt(c + d*x))**p, x)
```

3.656 $\int \left(a+b\sqrt{c+dx}\right)^p \, dx$

Optimal. Leaf size=62

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2d(p+2)}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2d(p+1)}$$

```
[Out] -2*a*(a+b*(d*x+c)^(1/2))^(1+p)/b^2/d/(1+p)+2*(a+b*(d*x+c)^(1/2))^(2+p)/b^2/
d/(2+p)
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 190, 43}

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2d(p+2)}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a + b*Sqrt[c + d*x])^(1 + p))/(b^2*d*(1 + p)) + (2*(a + b*Sqrt[c + d
*x])^(2 + p))/(b^2*d*(2 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sqrt{c+dx}\right)^p \, dx &= \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p \, dx,x,c+dx\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int x(a+bx)^p \, dx,x,\sqrt{c+dx}\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right) dx,x,\sqrt{c+dx}\right)}{d}\\
&= -\frac{2a\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^2d(1+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^2d(2+p)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.85

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(b(p+1)\sqrt{c+dx}-a\right)}{b^2d(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (2*(a + b*Sqrt[c + d*x])^(1 + p)*(-a + b*(1 + p)*Sqrt[c + d*x]))/(b^2*d*(1
+ p)*(2 + p))
```

fricas [A] time = 0.53, size = 81, normalized size = 1.31

$$\frac{2\left(b^2cp+\sqrt{dx+c}\,abp+b^2c-a^2+\left(b^2dp+b^2d\right)x\right)\left(\sqrt{dx+c}\,b+a\right)^p}{b^2dp^2+3\,b^2dp+2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(b^2*c*p + sqrt(d*x + c)*a*b*p + b^2*c - a^2 + (b^2*d*p + b^2*d)*x)*(sqrt
(d*x + c)*b + a)^p/(b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)
```

giac [B] time = 0.44, size = 129, normalized size = 2.08

$$\frac{2\left(\left(\sqrt{dx+c}\,b+a\right)^2\left(\sqrt{dx+c}\,b+a\right)^pp-\left(\sqrt{dx+c}\,b+a\right)\left(\sqrt{dx+c}\,b+a\right)^pap+\left(\sqrt{dx+c}\,b+a\right)^2\left(\sqrt{dx+c}\,b+a\right.}{\left(p^2+3\,p+2\right)b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*p - (sqrt(d*x + c)*b + a
)*(sqrt(d*x + c)*b + a)^p*a*p + (sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b +
a)^p - 2*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a)/((p^2 + 3*p + 2)*
b^2*d)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int\left(a+\sqrt{dx+c}\,b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^(1/2)*b)^p,x)
```

```
[Out] int((a+(d*x+c)^(1/2)*b)^p,x)
```

maxima [A] time = 0.93, size = 60, normalized size = 0.97

$$\frac{2\left((dx+c)b^2\left(p+1\right)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p}{\left(p^2+3\,p+2\right)b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a)
^p/((p^2 + 3*p + 2)*b^2*d)
```

mupad [B] time = 3.58, size = 146, normalized size = 2.35

$$\begin{cases} -\frac{2\,a\,\ln\left(a+b\,\sqrt{c+d\,x}\right)-2\,b\,\sqrt{c+d\,x}}{b^2\,d} & \text{if } p=-1 \\ \frac{2\left(\ln\left(a+b\,\sqrt{c+d\,x}\right)+\frac{a}{a+b\,\sqrt{c+d\,x}}\right)}{b^2\,d} & \text{if } p=-2 \\ \frac{4\left(a+b\,\sqrt{c+d\,x}\right)^{p+2}}{b^2\,d\,\left(2\,p+4\right)}-\frac{4\,a\,\left(a+b\,\sqrt{c+d\,x}\right)^{p+1}}{b^2\,d\,\left(2\,p+2\right)} & \text{if } p\neq -1\wedge p\neq -2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] piecewise(p == -1, -(2*a*log(a + b*(c + d*x)^(1/2)) - 2*b*(c + d*x)^(1/2))/
(b^2*d), p == -2, (2*(log(a + b*(c + d*x)^(1/2)) + a/(a + b*(c + d*x)^(1/2)
)))/(b^2*d), p ~= -1 & p ~= -2, (4*(a + b*(c + d*x)^(1/2))^(p + 2))/(b^2*d*
(2*p + 4)) - (4*a*(a + b*(c + d*x)^(1/2))^(p + 1))/(b^2*d*(2*p + 2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sqrt{c + dx}\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral((a + b*sqrt(c + d*x))**p, x)
```

3.657 $\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx$

Optimal. Leaf size=139

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{(p+1)\left(a-b\sqrt{c}\right)}-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{(p+1)\left(a+b\sqrt{c}\right)}$$

```
[Out] -hypergeom([1, 1+p],[2+p],(a+b*(d*x+c)^(1/2))/(a-b*c^(1/2)))*(a+b*(d*x+c)^(
1/2))^(1+p)/(1+p)/(a-b*c^(1/2))-hypergeom([1, 1+p],[2+p],(a+b*(d*x+c)^(1/2)
)/(a+b*c^(1/2)))*(a+b*(d*x+c)^(1/2))^(1+p)/(1+p)/(a+b*c^(1/2))
```

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {371, 1398, 831, 68}

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{(p+1)\left(a-b\sqrt{c}\right)}-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{(p+1)\left(a+b\sqrt{c}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^p/x,x]
```

```
[Out] -(((a + b*Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*
Sqrt[c + d*x])/(a - b*Sqrt[c])])/((a - b*Sqrt[c])*(1 + p))) - ((a + b*Sqrt[
c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x])/
(a + b*Sqrt[c])])/((a + b*Sqrt[c])*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx = \text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^p}{-c+x}\,dx, x, c+dx\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x(a+bx)^p}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)$$
$$= 2\,\text{Subst}\left(\int \left(-\frac{(a+bx)^p}{2\left(\sqrt{c}-x\right)}+\frac{(a+bx)^p}{2\left(\sqrt{c}+x\right)}\right)dx, x, \sqrt{c+dx}\right)$$
$$= -\text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)+\text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)$$
$$= -\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{\left(a-b\sqrt{c}\right)(1+p)} - \frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\,{}_2F_1\left(1,1+p;\right.}{\left(a+b\sqrt{c}\right)(1+p}$$

Mathematica [A] time = 0.11, size = 136, normalized size = 0.98

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\left(\left(a+b\sqrt{c}\right)\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)+\left(a-b\sqrt{c}\right)\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)\right)}{(p+1)\left(a-b\sqrt{c}\right)\left(a+b\sqrt{c}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^p/x,x]
```

```
[Out] -(((a + b*Sqrt[c + d*x])^(1 + p)*((a + b*Sqrt[c])*Hypergeometric2F1[1, 1 +
p, 2 + p, (a + b*Sqrt[c + d*x])/(a - b*Sqrt[c])] + (a - b*Sqrt[c])*Hypergeo
metric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x])/(a + b*Sqrt[c])]))/((a - b
*Sqrt[c])*(a + b*Sqrt[c])*(1 + p)))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\sqrt{dx+c}\,b+a\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((sqrt(d*x + c)*b + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{dx+c}\,b+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\sqrt{dx+c}\,b\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^(1/2)*b)^p/x,x)
```

```
[Out] int((a+(d*x+c)^(1/2)*b)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{dx+c}\,b+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sqrt{c+d\,x}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^p/x,x)
```

```
[Out] int((a + b*(c + d*x)^(1/2))^p/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**p/x,x)
```

```
[Out] Integral((a + b*sqrt(c + d*x))**p/x, x)
```

3.658 $\int \frac{(a+b(cx)^n)^{5/2}}{x} \, dx$

Optimal. Leaf size=93

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{a+b(cx)^n}}{n}+\frac{2a\,(a+b(cx)^n)^{3/2}}{3n}+\frac{2\,(a+b(cx)^n)^{5/2}}{5n}$$

```
[Out] 2/3*a*(a+b*(c*x)^n)^(3/2)/n+2/5*(a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctanh((a+
b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 50, 63, 208}

$$\frac{2a^2\sqrt{a+b(cx)^n}}{n}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\,(a+b(cx)^n)^{3/2}}{3n}+\frac{2\,(a+b(cx)^n)^{5/2}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[a + b*(c*x)^n])/n + (2*a*(a + b*(c*x)^n)^(3/2))/(3*n) + (2*(a +
 b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])
/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{5/2}}{x}\,dx, x, cx\right)}{c}$$
$$= \text{Subst}\left(\int \frac{(a+bx^n)^{5/2}}{x}\,dx, x, cx\right)$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, (cx)^n\right)}{n}$$
$$= \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}$$
$$= \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}$$
$$= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (c\right.}{n}$$
$$= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{bn}$$
$$= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.83

$$\frac{2\sqrt{a+b(cx)^n}\left(23a^2+11ab(cx)^n+3b^2(cx)^{2n}\right)-30a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(23*a^2 + 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*a
^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

fricas [A] time = 0.48, size = 164, normalized size = 1.76

$$\left[\frac{15\,a^{\frac{5}{2}}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\,\sqrt{a}+2\,a}{(cx)^n}\right)+2\left(11\,(cx)^n ab+3\,(cx)^{2n}b^2+23\,a^2\right)\sqrt{(cx)^n b+a}}{15\,n}, \frac{2\left(15\sqrt{-a}\,a^2\arctan\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^(5/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*
x)^n) + 2*(11*(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a)
```

```
)/n, 2/15*(15*sqrt(-a)*a^2*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (11*(c*
x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n b + a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

maple [A] time = 0.01, size = 70, normalized size = 0.75

$$\frac{-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right)+2\sqrt{b\,(cx)^n+a}\;a^2+\frac{2\left(b(cx)^n+a\right)^{\frac{3}{2}}a}{3}+\frac{2\left(b(cx)^n+a\right)^{\frac{5}{2}}}{5}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*x)^n)^(5/2)/x,x)
```

```
[Out] 1/n*(2/5*(a+b*(c*x)^n)^(5/2)+2/3*a*(a+b*(c*x)^n)^(3/2)+2*(a+b*(c*x)^n)^(1/2
)*a^2-2*a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n b + a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,{\left(c\,x\right)}^n\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^(5/2)/x,x)
```

```
[Out] int((a + b*(c*x)^n)^(5/2)/x, x)
```

sympy [A] time = 92.66, size = 122, normalized size = 1.31

$$\begin{cases} \frac{\frac{2a^3\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2a^2\sqrt{a+b(cx)^n}+\frac{2a\left(a+b(cx)^n\right)^{\frac{3}{2}}}{3}+\frac{2\left(a+b(cx)^n\right)^{\frac{5}{2}}}{5}}{n} & \text{for}\ n \neq 0 \\ -\left(-a^2\sqrt{a+b}-2ab\sqrt{a+b}-b^2\sqrt{a+b}\right)\log(cx) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] Piecewise(((2*a**3*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) + 2*a**2*sq
rt(a + b*(c*x)**n) + 2*a*(a + b*(c*x)**n)**(3/2)/3 + 2*(a + b*(c*x)**n)**(5
/2)/5)/n, Ne(n, 0)), (-(-a**2*sqrt(a + b) - 2*a*b*sqrt(a + b) - b**2*sqrt(a
 + b))*log(c*x), True))
```

3.659 $\int \frac{(a+b(cx)^n)^{3/2}}{x} dx$

Optimal. Leaf size=70

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+b(cx)^n}}{n}+\frac{2\left(a+b(cx)^n\right)^{3/2}}{3n}$$

```
[Out] 2/3*(a+b*(c*x)^n)^(3/2)/n-2*a^(3/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n+
2*a*(a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 50, 63, 208}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+b(cx)^n}}{n}+\frac{2\left(a+b(cx)^n\right)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*a*Sqrt[a + b*(c*x)^n])/n + (2*(a + b*(c*x)^n)^(3/2))/(3*n) - (2*a^(3/2)*
ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{3/2}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{(a+bx^n)^{3/2}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.87

$$\frac{2\sqrt{a+b(cx)^n}\,(4a+b(cx)^n) - 6a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(4*a + b*(c*x)^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*(c*
x)^n]/Sqrt[a]])/(3*n)
```

fricas [A] time = 0.50, size = 130, normalized size = 1.86

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\,\sqrt{a}+2\,a}{(cx)^n}\right)+2\left((cx)^n\,b+4\,a\right)\sqrt{(cx)^n\,b+a}}{3\,n}, \frac{2\left(3\sqrt{-a}\,a\arctan\left(\frac{\sqrt{(cx)^n b+a}\,\sqrt{-a}}{a}\right)+\left((cx)^n\right.\right.}{3\,n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)
^n) + 2*((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n, 2/3*(3*sqrt(-a)*a*arctan(
sqrt((c*x)^n*b + a)*sqrt(-a)/a) + ((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n\,b+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

maple [A] time = 0.00, size = 54, normalized size = 0.77

$$\frac{-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b(cx)^{n}+a}}{\sqrt{a}}\right)+2\sqrt{b\,(cx)^{n}+a}\;a+\frac{2\left(b(cx)^{n}+a\right)^{\frac{3}{2}}}{3}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n+a)^(3/2)/x,x)
```

```
[Out] 1/n*(2/3*(b*(c*x)^n+a)^(3/2)+2*a*(b*(c*x)^n+a)^(1/2)-2*a^(3/2)*arctanh((b*(
c*x)^n+a)^(1/2)/a^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((cx)^{n}\,b+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,(c\,x)^{n}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^(3/2)/x,x)
```

```
[Out] int((a + b*(c*x)^n)^(3/2)/x, x)
```

sympy [A] time = 66.15, size = 102, normalized size = 1.46

$$\begin{cases}\frac{-a\left(-\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2\sqrt{a+b(cx)^{n}}\right)-b\left(\begin{cases}-\sqrt{a}\;(cx)^{n} & \text{for}\ b=0\\-\frac{2\left(a+b(cx)^{n}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{n} & \text{for}\ n\neq 0\\\left(a\sqrt{a+b}\;+b\sqrt{a+b}\right)\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise(((-a*(-2*a*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) - 2*sqrt(
a + b*(c*x)**n)) - b*Piecewise((-sqrt(a)*(c*x)**n, Eq(b, 0)), (-2*(a + b*(c
*x)**n)**(3/2)/(3*b), True)))/n, Ne(n, 0)), ((a*sqrt(a + b) + b*sqrt(a + b)
)*log(x), True))
```

3.660 $\int \frac{\sqrt{a+b(cx)^n}}{x} \, dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 50, 63, 208}

$$\frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]
)/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c\sqrt{a+bx^n}}{x}\,dx,x,cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx,x,cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,(cx)^n\right)}{n}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n}+\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,(cx)^n\right)}{n}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b(cx)^n}\right)}{bn}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.96

$$\frac{2\sqrt{a+b(cx)^n}-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

fricas [A] time = 0.50, size = 103, normalized size = 2.10

$$\left[\frac{\sqrt{a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\,\sqrt{a}+2a}{(cx)^n}\right)+2\sqrt{(cx)^n b+a}}{n},\ \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^n b+a}\,\sqrt{-a}}{a}\right)+\sqrt{(cx)^n b+a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) +
2*sqrt((c*x)^n*b + a))/n, 2*(sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a
) + sqrt((c*x)^n*b + a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(cx)^n b+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt((c*x)^n*b + a)/x, x)
```

maple [A] time = 0.00, size = 40, normalized size = 0.82

$$\frac{-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right)+2\sqrt{b\,(cx)^n+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n+a)^(1/2)/x,x)
[Out] 1/n*(2*(b*(c*x)^n+a)^(1/2)-2*a^(1/2)*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(cx)^n\,b+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt((c*x)^n*b + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,{(c\,x)}^n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^(1/2)/x,x)
[Out] int((a + b*(c*x)^n)^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,(cx)^n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(1/2)/x,x)
[Out] Integral(sqrt(a + b*(c*x)**n)/x, x)
```

3.661 $\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*(c*x)^n]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}}\,dx, x, cx\right)}{c}$$
$$= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, cx\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*x)^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

fricas [A] time = 0.48, size = 78, normalized size = 2.60

$$\left[\frac{\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\,\sqrt{a}+2a}{(cx)^n}\right)}{\sqrt{a}\,n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^n b+a}\,\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
[Out] [log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n)/(sqrt(a)*n)
, 2*sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)/(a*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n b+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x)^n+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2))/n/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n b + a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,x\right)}^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^n)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*x)^n)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + b\,(cx)^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*x)**n)), x)
```

3.662 $\int \frac{1}{x(a+b(cx)^n)^{3/2}} dx$

Optimal. Leaf size=52

$$\frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

```
[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)/n+2/a/n/(a+b*(c*x)^n)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {367, 12, 266, 51, 63, 208}

$$\frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(
3/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(a+bx^n)^{3/2}}\,dx,x,cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx,x,cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,(cx)^n\right)}{n}\\
&= \frac{2}{an\sqrt{a+b(cx)^n}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,(cx)^n\right)}{an}\\
&= \frac{2}{an\sqrt{a+b(cx)^n}}+\frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b(cx)^n}\right)}{abn}\\
&= \frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 0.79

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b(cx)^n}{a}+1\right)}{an\sqrt{a+b(cx)^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*(c*x)^n)/a])/(a*n*Sqrt[a + b*(c*x
)^n])
```

fricas [A] time = 0.49, size = 164, normalized size = 3.15

$$\left[\frac{\left((cx)^n\sqrt{a}\,b+a^{\frac{3}{2}}\right)\log\left(\frac{(cx)^nb-2\sqrt{(cx)^nb+a}\,\sqrt{a}+2\,a}{(cx)^n}\right)+2\sqrt{(cx)^n\,b+a}\,a}{(cx)^n\,a^2bn+a^3n},\ \frac{2\left(\left((cx)^n\sqrt{-a}\,b+\sqrt{-a}\,a\right)\arctan\left(\frac{\sqrt{(cx)^nb+a}\,\sqrt{-}}{a}\right.\right.}{(cx)^n\,a^2bn+a^3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [(((c*x)^n*sqrt(a)*b + a^(3/2))*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt
(a) + 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n), 2
*(((c*x)^n*sqrt(-a)*b + sqrt(-a)*a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)
+ sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^n\,b+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

maple [A] time = 0.01, size = 43, normalized size = 0.83

$$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b(cx)^{n}+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{b(cx)^{n}+a}\;a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x)^n+a)^(3/2),x)
```

```
[Out] 1/n*(2/a/(b*(c*x)^n+a)^(1/2)-2/a^(3/2)*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2))
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^{n}\, b + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\left(a+b\,{\left(c\,x\right)}^n\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^n)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*x)^n)^(3/2)), x)
```

sympy [A] time = 11.43, size = 48, normalized size = 0.92

$$\frac{2}{an\sqrt{a+b\,(cx)^{n}}}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{-a}}\right)}{an\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(3/2),x)
```

```
[Out] 2/(a*n*sqrt(a + b*(c*x)**n)) + 2*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a*n*s
qrt(-a))
```

3.663 $\int \frac{1}{x(a+b(cx)^n)^{5/2}} \, dx$

Optimal. Leaf size=75

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}+\frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}$$

```
[Out] 2/3/a/n/(a+b*(c*x)^n)^(3/2)-2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(5/2)/
n+2/a^2/n/(a+b*(c*x)^n)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {367, 12, 266, 51, 63, 208}

$$\frac{2}{a^2n\sqrt{a+b(cx)^n}}-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] 2/(3*a*n*(a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTan
h[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(a+bx^n)^{5/2}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(a+bx^n\right)^{5/2}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{an}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{a^2n}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{a^2bn}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 43, normalized size = 0.57

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b(cx)^n}{a}+1\right)}{3an\left(a+b(cx)^n\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*(c*x)^n)/a])/(3*a*n*(a + b*(c*x)
^n)^(3/2))
```

fricas [B] time = 0.49, size = 262, normalized size = 3.49

$$\left[\frac{3\left(2\,(cx)^n a^{\frac{3}{2}}b+(cx)^{2n}\sqrt{a}\,b^2+a^{\frac{5}{2}}\right)\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\,\sqrt{a}+2a}{(cx)^n}\right)+2\left(3\,(cx)^n ab+4a^2\right)\sqrt{(cx)^n b+a}}{3\left(2\,(cx)^n a^4bn+(cx)^{2n}a^3b^2n+a^5n\right)}, \frac{2\left(3\left(2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(2*(c*x)^n*a^(3/2)*b + (c*x)^(2*n)*sqrt(a)*b^2 + a^(5/2))*log(((c*x
)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) + 2*(3*(c*x)^n*a*b +
4*a^2)*sqrt((c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^
5*n), 2/3*(3*(2*(c*x)^n*sqrt(-a)*a*b + (c*x)^(2*n)*sqrt(-a)*b^2 + sqrt(-a)*
```

```
a^2)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (3*(c*x)^n*a*b + 4*a^2)*sqrt(
(c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^5*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^n b + a\right)^{\frac{5}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

maple [A] time = 0.01, size = 59, normalized size = 0.79

$$\frac{\frac{2}{3\left(b(cx)^n+a\right)^{\frac{3}{2}} a} - \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2}{\sqrt{b(cx)^n+a}\; a^2}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x)^n+a)^(5/2),x)
```

```
[Out] 1/n*(-2/a^(5/2)*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2))+2/a^2/(b*(c*x)^n+a)^(1
/2)+2/3/a/(b*(c*x)^n+a)^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^n b + a\right)^{\frac{5}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a + b\,(c\,x)^n\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^n)^(5/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*x)^n)^(5/2)), x)
```

sympy [A] time = 16.79, size = 70, normalized size = 0.93

$$\frac{2}{3an\left(a + b\,(cx)^n\right)^{\frac{3}{2}}} + \frac{2}{a^2 n\sqrt{a + b\,(cx)^n}} + \frac{2 \operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{a^2 n\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(5/2),x)
```

```
[Out] 2/(3*a*n*(a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(a + b*(c*x)**n)) + 2*ata
n(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a**2*n*sqrt(-a))
```

3.664 $\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx$

Optimal. Leaf size=101

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{b(cx)^n-a}}{n}-\frac{2a\,(b(cx)^n-a)^{3/2}}{3n}+\frac{2\,(b(cx)^n-a)^{5/2}}{5n}$$

```
[Out] -2/3*a*(-a+b*(c*x)^n)^(3/2)/n+2/5*(-a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctan((
-a+b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(-a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2a^2\sqrt{b(cx)^n-a}}{n}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\,(b(cx)^n-a)^{3/2}}{3n}+\frac{2\,(b(cx)^n-a)^{5/2}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[-a + b*(c*x)^n])/n - (2*a*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*(-
a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a
]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{5/2}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{(-a+bx^n)^{5/2}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{5/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{a\,\text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= -\frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx,\right.}{n}\\
&= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{bn}\\
&= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 81, normalized size = 0.80

$$\frac{2\sqrt{b(cx)^n-a}\,\left(23a^2-11ab(cx)^n+3b^2(cx)^{2n}\right)-30a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n]*(23*a^2 - 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*
a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

fricas [A] time = 0.48, size = 169, normalized size = 1.67

$$\left[\frac{15\sqrt{-a}\,a^2\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)-2\left(11(cx)^n ab-3(cx)^{2n}b^2-23a^2\right)\sqrt{(cx)^n b-a}}{15n}, -\frac{2\left(15a^{\frac{5}{2}}\arctan\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*sqrt(-a)*a^2*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*
a)/(c*x)^n) - 2*(11*(c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*
```

```
b - a))/n, -2/15*(15*a^(5/2)*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) + (11*(c*x
)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*b - a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n b - a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

maple [A] time = 0.01, size = 86, normalized size = 0.85

$$-\frac{2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{b\,(cx)^n-a}\,a^2}{n}-\frac{2\left(b\,(cx)^n-a\right)^{\frac{3}{2}}a}{3n}+\frac{2\left(b\,(cx)^n-a\right)^{\frac{5}{2}}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*(c*x)^n)^(5/2)/x,x)
[Out] -2/3*a*(-a+b*(c*x)^n)^(3/2)/n+2/5*(-a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctan((
-a+b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(-a+b*(c*x)^n)^(1/2)/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n b - a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,{\left(c\,x\right)}^n-a\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n - a)^(5/2)/x,x)
[Out] int((b*(c*x)^n - a)^(5/2)/x, x)
```

sympy [A] time = 85.59, size = 114, normalized size = 1.13

$$\begin{cases} \frac{-2a^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)+2a^2\sqrt{-a+b(cx)^n}-\frac{2a\left(-a+b(cx)^n\right)^{\frac{3}{2}}}{3}+\frac{2\left(-a+b(cx)^n\right)^{\frac{5}{2}}}{5}}{n} & \text{for } n \neq 0 \\ -\left(-a^2\sqrt{-a+b}+2ab\sqrt{-a+b}-b^2\sqrt{-a+b}\right)\log(cx) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(5/2)/x,x)
[Out] Piecewise(((-2*a**(5/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) + 2*a**2*sqrt(-
a + b*(c*x)**n) - 2*a*(-a + b*(c*x)**n)**(3/2)/3 + 2*(-a + b*(c*x)**n)**(5/
2)/5)/n, Ne(n, 0)), (-(-a**2*sqrt(-a + b) + 2*a*b*sqrt(-a + b) - b**2*sqrt(
-a + b))*log(c*x), True))
```

3.665 $\int \frac{(-a+b(cx)^n)^{3/2}}{x} dx$

Optimal. Leaf size=76

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\sqrt{b(cx)^n-a}}{n}+\frac{2\left(b(cx)^n-a\right)^{3/2}}{3n}$$

```
[Out] 2/3*(-a+b*(c*x)^n)^(3/2)/n+2*a^(3/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n
-2*a*(-a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\sqrt{b(cx)^n-a}}{n}+\frac{2\left(b(cx)^n-a\right)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (-2*a*Sqrt[-a + b*(c*x)^n])/n + (2*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*a^(3/
2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{3/2}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{(-a+bx^n)^{3/2}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(-a+b(cx)^n)^{3/2}}{3n} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}\\
&= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.87

$$\frac{6a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right) - 2(4a-b(cx)^n)\sqrt{b(cx)^n-a}}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*(c*x)^n)^(3/2)/x,x]
[Out] (-2*(4*a - b*(c*x)^n)*Sqrt[-a + b*(c*x)^n] + 6*a^(3/2)*ArcTan[Sqrt[-a + b*(
c*x)^n]/Sqrt[a]])/(3*n)
```

fricas [A] time = 0.46, size = 135, normalized size = 1.78

$$\left[\frac{3\sqrt{-a}\,a\log\left(\frac{(cx)^n b+2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right) + 2\sqrt{(cx)^n b-a}\left((cx)^n b-4a\right)}{3n}, \frac{2\left(3a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right) + \sqrt{(cx)^n b-\right.}{3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
[Out] [1/3*(3*sqrt(-a)*a*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(
c*x)^n) + 2*sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n, 2/3*(3*a^(3/2)*arctan
(sqrt((c*x)^n*b - a)/sqrt(a)) + sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n b-a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

maple [A] time = 0.01, size = 65, normalized size = 0.86

$$\frac{2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2\sqrt{b\,(cx)^n-a}\;a}{n}+\frac{2\left(b\,(cx)^n-a\right)^{\frac{3}{2}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n-a)^(3/2)/x,x)
```

```
[Out] 2/3*(b*(c*x)^n-a)^(3/2)/n+2*a^(3/2)*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))/n-2
*a*(b*(c*x)^n-a)^(1/2)/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((cx)^n\,b-a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,(c\,x)^n-a\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n - a)^(3/2)/x,x)
```

```
[Out] int((b*(c*x)^n - a)^(3/2)/x, x)
```

sympy [A] time = 70.06, size = 95, normalized size = 1.25

$$\begin{cases}\frac{a\left(2\sqrt{a}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)-2\sqrt{-a+b(cx)^n}\right)-b\left(\begin{cases}-\sqrt{-a}\,(cx)^n & \text{for}\ b=0\\-\frac{2\left(-a+b(cx)^n\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{n} & \text{for}\ n\neq 0\\\left(-a\sqrt{-a+b}+b\sqrt{-a+b}\right)\log\left(x\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise(((a*(2*sqrt(a)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) - 2*sqrt(-a +
b*(c*x)**n)) - b*Piecewise((-sqrt(-a)*(c*x)**n, Eq(b, 0)), (-2*(-a + b*(c*x
)**n)**(3/2)/(3*b), True)))/n, Ne(n, 0)), ((-a*sqrt(-a + b) + b*sqrt(-a + b
))*log(x), True))
```

3.666 $\int \frac{\sqrt{-a+b(cx)^n}}{x} \, dx$

Optimal. Leaf size=53

$$\frac{2\sqrt{b(cx)^n-a}}{n}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(-a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2\sqrt{b(cx)^n-a}}{n}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]
])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c\sqrt{-a+bx^n}}{x}\,dx, x, cx\right)}{c} \\
&= \text{Subst}\left(\int \frac{\sqrt{-a+bx^n}}{x}\,dx, x, cx\right) \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n} \\
&= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n} \\
&= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn} \\
&= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.96

$$\frac{2\sqrt{b(cx)^n-a}-2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*(c*x)^n]/x,x]
[Out] (2*Sqrt[-a + b*(c*x)^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

fricas [A] time = 0.48, size = 110, normalized size = 2.08

$$\left[\frac{\sqrt{-a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}}{n}, -\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)-\sqrt{(cx)^n b-a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
[Out] [(sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)
+ 2*sqrt((c*x)^n*b - a))/n, -2*(sqrt(a)*arctan(sqrt((c*x)^n*b - a)/sqrt(a))
 - sqrt((c*x)^n*b - a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(cx)^n b-a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt((c*x)^n*b - a)/x, x)
```

maple [A] time = 0.00, size = 46, normalized size = 0.87

$$-\frac{2\sqrt{a}\ \arctan\left(\frac{\sqrt{b(cx)^{n}-a}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{b\,(cx)^{n}-a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n-a)^(1/2)/x,x)
[Out] -2*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))*a^(1/2)/n+2*(b*(c*x)^n-a)^(1/2)/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(cx)^{n}\,b-a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt((c*x)^n*b - a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,(c\,x)^{n}-a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n - a)^(1/2)/x,x)
[Out] int((b*(c*x)^n - a)^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a+b\,(cx)^{n}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(1/2)/x,x)
[Out] Integral(sqrt(-a + b*(c*x)**n)/x, x)
```

3.667 $\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

```
[Out] 2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {367, 12, 266, 63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{-a+bx^n}}\,dx,x,cx\right)}{c}$$
$$= \text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx^n}}\,dx,x,cx\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx,x,(cx)^n\right)}{n}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+b(cx)^n}\right)}{bn}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

fricas [A] time = 0.50, size = 80, normalized size = 2.50

$$\left[-\frac{\sqrt{-a}\,\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n b-a}\,\sqrt{-a}-2\,a}{(cx)^n}\right)}{an}, \frac{2\,\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)/
(a*n), 2*arctan(sqrt((c*x)^n*b - a)/sqrt(a))/(sqrt(a)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n\,b-a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

maple [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x)^n-a)^(1/2),x)
```

```
[Out] 2*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))/n/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n b - a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{b\,(c\,x)^n - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*(c*x)^n - a)^(1/2)),x)
```

```
[Out] int(1/(x*(b*(c*x)^n - a)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-a + b\,(cx)^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-a + b*(c*x)**n)), x)
```

3.668 $\int \frac{1}{x(-a+b(cx)^n)^{3/2}} dx$

Optimal. Leaf size=56

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

```
[Out] -2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)/n-2/a/n/(-a+b*(c*x)^n)^(1/2
)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 51, 63, 205}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a
^(3/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(-a+bx^n)^{3/2}}\,dx,x,cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(-a+bx^n\right)^{3/2}}\,dx,x,cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}}\,dx,x,(cx)^n\right)}{n}\\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx,x,(cx)^n\right)}{an}\\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+b(cx)^n}\right)}{abn}\\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 44, normalized size = 0.79

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{b(cx)^n}{a}\right)}{an\sqrt{b(cx)^n-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (b*(c*x)^n)/a])/(a*n*Sqrt[-a + b*(c
*x)^n])
```

fricas [A] time = 0.53, size = 175, normalized size = 3.12

$$\left[-\frac{\left((cx)^n\sqrt{-a}\,b-\sqrt{-a}\,a\right)\log\left(\frac{(cx)^n b+2\sqrt{(cx)^n b-a}\,\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}\,a}{(cx)^n a^2bn-a^3n},-\frac{2\left(\left((cx)^n\sqrt{a}\,b-a^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)\right.}{(cx)^n a^2bn-a^3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(((c*x)^n*sqrt(-a)*b - sqrt(-a)*a)*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)
*sqrt(-a) - 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3
*n), -2*(((c*x)^n*sqrt(a)*b - a^(3/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a))
+ sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^n b-a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="giac")

[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)

maple [A] time = 0.01, size = 49, normalized size = 0.88

$$-\frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}-\frac{2}{\sqrt{b\,(cx)^n-a}\;an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*(c*x)^n-a)^(3/2),x)

[Out] -2*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))/a^(3/2)/n-2/a/n/(b*(c*x)^n-a)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^n\,b-a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="maxima")

[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\left(b\,(c\,x)^n-a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(b*(c*x)^n - a)^(3/2)),x)

[Out] int(1/(x*(b*(c*x)^n - a)^(3/2)), x)

sympy [A] time = 15.94, size = 44, normalized size = 0.79

$$-\frac{2}{an\sqrt{-a+b\,(cx)^n}}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)**n)**(3/2),x)

[Out] -2/(a*n*sqrt(-a + b*(c*x)**n)) - 2*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**(3/2)*n)

3.669 $\int \frac{1}{x(-a+b(cx)^n)^{5/2}}\,dx$

Optimal. Leaf size=81

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{b(cx)^n-a}}-\frac{2}{3an\,(b(cx)^n-a)^{3/2}}$$

[Out] -2/3/a/n/(-a+b*(c*x)^n)^(3/2)+2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(5/2)/n+2/a^2/n/(-a+b*(c*x)^n)^(1/2)

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {367, 12, 266, 51, 63, 205}

$$\frac{2}{a^2n\sqrt{b(cx)^n-a}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{5/2}n}-\frac{2}{3an\,(b(cx)^n-a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(-a + b*(c*x)^n)^(5/2)),x]

[Out] -2/(3*a*n*(-a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[-a + b*(c*x)^n]) + (2*ArcTan[Sqrt[-a + b*(c*x)^n/Sqrt[a]])/(a^(5/2)*n)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x(-a+b(cx)^n)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{c}{x(-a+bx^n)^{5/2}}\,dx, x, cx\right)}{c}$$
$$= \text{Subst}\left(\int \frac{1}{x(-a+bx^n)^{5/2}}\,dx, x, cx\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{5/2}}\,dx, x, (cx)^n\right)}{n}$$
$$= -\frac{2}{3an(-a+b(cx)^n)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{an}$$
$$= -\frac{2}{3an(-a+b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{a^2n}$$
$$= -\frac{2}{3an(-a+b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{a^2bn}$$
$$= -\frac{2}{3an(-a+b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

Mathematica [C] time = 0.04, size = 46, normalized size = 0.57

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{b(cx)^n}{a}\right)}{3an(b(cx)^n-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, 1 - (b*(c*x)^n)/a])/(3*a*n*(-a + b*(c*
x)^n)^(3/2))
```

fricas [A] time = 0.51, size = 277, normalized size = 3.42

$$\left[-\frac{3\left(2(cx)^n\sqrt{-a}\,ab-(cx)^{2n}\sqrt{-a}\,b^2-\sqrt{-a}\,a^2\right)\log\left(\frac{(cx)^nb-2\sqrt{(cx)^nb-a}\,\sqrt{-a}-2a}{(cx)^n}\right)+2\left(3(cx)^nab-4a^2\right)\sqrt{(cx)^nb-}}{3\left(2(cx)^na^4bn-(cx)^{2n}a^3b^2n-a^5n\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(2*(c*x)^n*sqrt(-a)*a*b - (c*x)^(2*n)*sqrt(-a)*b^2 - sqrt(-a)*a^2)
*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n) + 2*(3*(c*
x)^n*a*b - 4*a^2)*sqrt((c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3
*b^2*n - a^5*n), 2/3*(3*(2*(c*x)^n*a^(3/2)*b - (c*x)^(2*n)*sqrt(a)*b^2 - a^
```

```
(5/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) - (3*(c*x)^n*a*b - 4*a^2)*sqrt((
c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3*b^2*n - a^5*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^n b - a\right)^{\frac{5}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)
```

maple [A] time = 0.01, size = 70, normalized size = 0.86

$$-\frac{2}{3\left(b\,(cx)^n - a\right)^{\frac{3}{2}} an} + \frac{2\arctan\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}} n} + \frac{2}{\sqrt{b\,(cx)^n - a}\; a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x)^n-a)^(5/2),x)
```

```
[Out] -2/3/a/n/(b*(c*x)^n-a)^(3/2)+2*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))/a^(5/2)/
n+2/a^2/n/(b*(c*x)^n-a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx)^n b - a\right)^{\frac{5}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(b\,(c\,x)^n - a\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*(c*x)^n - a)^(5/2)),x)
```

```
[Out] int(1/(x*(b*(c*x)^n - a)^(5/2)), x)
```

sympy [A] time = 15.63, size = 63, normalized size = 0.78

$$-\frac{2}{3an\left(-a + b\,(cx)^n\right)^{\frac{3}{2}}} + \frac{2}{a^2 n\sqrt{-a + b\,(cx)^n}} + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{-a + b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(5/2),x)
```

```
[Out] -2/(3*a*n*(-a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(-a + b*(c*x)**n)) + 2*
atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**(5/2)*n)
```

3.670 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 0.48, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

giac [A] time = 0.41, size = 21, normalized size = 0.91

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [A] time = 1.97, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 3.11, size = 17, normalized size = 0.74

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
```

```
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

sympy [A] time = 1.01, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

3.671 $\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}$$

```
[Out] -2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*x)^m]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^m}}\,dx, x, cx\right)}{c}$$
$$= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^m}}\,dx, x, cx\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^m\right)}{m}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^m}\right)}{bm}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*x)^m]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

fricas [A] time = 0.50, size = 78, normalized size = 2.60

$$\left[\frac{\log\left(\frac{(cx)^m b-2\sqrt{(cx)^m b+a}\,\sqrt{a}+2\,a}{(cx)^m}\right)}{\sqrt{a}\,m}, \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{(cx)^m b+a}\,\sqrt{-a}}{a}\right)}{am}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="fricas")
[Out] [log(((c*x)^m*b - 2*sqrt((c*x)^m*b + a)*sqrt(a) + 2*a)/(c*x)^m)/(sqrt(a)*m)
, 2*sqrt(-a)*arctan(sqrt((c*x)^m*b + a)*sqrt(-a)/a)/(a*m)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^m b+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{b(cx)^m+a}}{\sqrt{a}}\right)}{\sqrt{a}\;m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*x)^m)^(1/2),x)
[Out] -2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^m b + a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{a+b\,{(c\,x)}^m}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^m)^(1/2)),x)
[Out] int(1/(x*(a + b*(c*x)^m)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + b\,(cx)^m}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**m)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*(c*x)**m)), x)
```

3.672 $\int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$$

```
[Out] -2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx, x, (dx)^m\right)}{m}$$
$$= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}}\,dx, x, c(dx)^m\right)}{cm}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, c(dx)^m\right)}{m}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(c(dx)^m\right)^n\right)}{mn}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c(dx)^m\right)^n}\right)}{bmn}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$$

Mathematica [A] time = 0.07, size = 37, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

fricas [A] time = 0.49, size = 116, normalized size = 3.14

$$\left[\frac{\log\left(\left(be^{(mn\log(dx)+n\log(c))} - 2\sqrt{be^{(mn\log(dx)+n\log(c))}+a}\,\sqrt{a} + 2a\right)e^{(-mn\log(dx)-n\log(c))}\right)}{\sqrt{a}\,mn}, \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{be^{(mn\log(d}}}{}\right.}{amn}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*e^(m*n*log(d*x) + n*log(c)) - 2*sqrt(b*e^(m*n*log(d*x) + n*log(c))
+ a)*sqrt(a) + 2*a)*e^(-m*n*log(d*x) - n*log(c)))/(sqrt(a)*m*n), 2*sqrt(-a)
*arctan(sqrt(b*e^(m*n*log(d*x) + n*log(c)) + a)*sqrt(-a)/a)/(a*m*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left((dx)^m c\right)^n b + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="giac")
```

[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)

maple [A] time = 0.01, size = 32, normalized size = 0.86

$$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b\left(c(dx)^{m}\right)^{n}+a}}{\sqrt{a}}\right)}{\sqrt{a}\, mn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x)

[Out] -2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left((dx)^{m} c\right)^{n} b + a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,{\left(d\,x\right)}^m\right)}^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)),x)

[Out] int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + b \left(c (dx)^{m}\right)^{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*x)**m)**n)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*(c*(d*x)**m)**n)), x)

3.673 $\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}$$

```
[Out] -2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(c(dx)^n)^p}}\,dx, x, (ex)^m\right)}{m}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^p}}\,dx, x, (d(ex)^m)^n\right)}{mn}$$

$$= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^p}}\,dx, x, c\,(d(ex)^m)^n\right)}{cmn}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^p}}\,dx, x, c\,(d(ex)^m)^n\right)}{mn}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(c\,(d(ex)^m)^n\right)^p\right)}{mnp}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c\,(d(ex)^m)^n\right)^p}\right)}{bmnp}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c(d(ex)^m)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}$$

Mathematica [A] time = 0.15, size = 44, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c(d(ex)^m)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

fricas [A] time = 0.48, size = 147, normalized size = 3.34

$$\left[\frac{\log\left(\left(be^{\left(mnp\log(ex)+np\log(d)+p\log(c)\right)} - 2\sqrt{be^{\left(mnp\log(ex)+np\log(d)+p\log(c)\right)}+a}\,\sqrt{a} + 2\,a\right)e^{\left(-mnp\log(ex)-np\log(d)-p\log(c\right.}}{\sqrt{a}\,mnp}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) - 2*sqrt(b*e^(m*n*p*log(
e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(a) + 2*a)*e^(-m*n*p*log(e*x) - n*p*
log(d) - p*log(c)))/(sqrt(a)*m*n*p), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*log(
e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

maple [A] time = 0.02, size = 39, normalized size = 0.89

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\left(c\left(d(ex)^{m}\right)^{n}\right)^{p}+a}}{\sqrt{a}}\right)}{\sqrt{a}\;mnp}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x)
[Out] -2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(\left((ex)^{m}\,d\right)^{n}c\right)^{p}b+a}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a+b\left(c\left(d\,(e\,x)^{m}\right)^{n}\right)^{p}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)),x)
[Out] int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\,(ex)^{m}\right)^{n}\right)^{p}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)**m)**n)**p)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*x)**m)**n)**p)), x)
```

3.674 $\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

```
[Out] -2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q/a^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {367, 12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p*
q)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^n\right)^p\right)^q}}\,dx, x, (fx)^m\right)}{m}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(c(dx)^p)^q}}\,dx, x, \left(e(fx)^m\right)^n\right)}{mn}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^q}}\,dx, x, \left(d\left(e(fx)^m\right)^n\right)^p\right)}{mnp}\\
&= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^q}}\,dx, x, c\left(d\left(e(fx)^m\right)^n\right)^p\right)}{cmnp}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^q}}\,dx, x, c\left(d\left(e(fx)^m\right)^n\right)^p\right)}{mnp}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q\right)}{mnpq}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}\right)}{bmnpq}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}
\end{aligned}
$$

Mathematica [A] time = 0.26, size = 51, normalized size = 1.00

$$
-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q/Sqrt[a]])/(Sqrt[a]*m*n*p*
q)
```

fricas [A] time = 0.48, size = 182, normalized size = 3.57

$$
\left[\frac{\log\left(\left(\left(be^{\left(mnpq\log\left(fx\right)+npq\log(e)+pq\log(d)+q\log(c)\right)} - 2\sqrt{be^{\left(mnpq\log\left(fx\right)+npq\log(e)+pq\log(d)+q\log(c)\right)}+a}\,\sqrt{a} + 2a\right)e^{\left(-mnpq\log\right.}\right.\right.}{\sqrt{a}\,mnpq}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) - 2*sqr
t(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) + a)*sqrt(a
) + 2*a)*e^(-m*n*p*q*log(f*x) - n*p*q*log(e) - p*q*log(d) - q*log(c)))/(sqr
t(a)*m*n*p*q), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e)
+ p*q*log(d) + q*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p*q)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(\left(\left(\left(fx\right)^{m} e\right)^{n} d\right)^{p} c\right)^{q} b + a x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x), x)
```

maple [A] time = 0.02, size = 46, normalized size = 0.90

$$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b\left(c\left(d\left(e\left(fx\right)^{m}\right)^{n}\right)^{p}\right)^{q}+a}}{\sqrt{a}}\right)}{\sqrt{a}\, mnpq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q/a^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a+b\left(c\left(d\left(e\left(f\,x\right)^{m}\right)^{n}\right)^{p}\right)^{q}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + b\left(c\left(d\left(e\left(fx\right)^{m}\right)^{n}\right)^{p}\right)^{q}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)**m)**n)**p)**q)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*(f*x)**m)**n)**p)**q)), x)
```

3.675 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x}\,dx$

Optimal. Leaf size=76

$$-\frac{35}{48}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{35}{16}\sqrt{\frac{1}{x^2}-1}-\frac{35}{16}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)-\frac{1}{6}\left(\frac{1}{x^2}-1\right)^{7/2}x^6-\frac{7}{24}\left(\frac{1}{x^2}-1\right)^{5/2}x^4$$

```
[Out] -35/48*(-1+1/x^2)^(3/2)*x^2-7/24*(-1+1/x^2)^(5/2)*x^4-1/6*(-1+1/x^2)^(7/2)*
x^6-35/16*arctan((-1+1/x^2)^(1/2))+35/16*(-1+1/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {25, 266, 47, 50, 63, 203}

$$-\frac{1}{6}\left(\frac{1}{x^2}-1\right)^{7/2}x^6-\frac{7}{24}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{35}{48}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{35}{16}\sqrt{\frac{1}{x^2}-1}-\frac{35}{16}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (35*Sqrt[-1 + x^(-2)])/16 - (35*(-1 + x^(-2))^(3/2)*x^2)/48 - (7*(-1 + x^(-
2))^(5/2)*x^4)/24 - ((-1 + x^(-2))^(7/2)*x^6)/6 - (35*ArcTan[Sqrt[-1 + x^(-
2)]])/16
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x}\,dx = -\int \left(-1+\frac{1}{x^2}\right)^{7/2} x^5\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^{7/2}}{x^4}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 + \frac{7}{12}\text{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 + \frac{35}{48}\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 + \frac{35}{32}\text{Subst}\left(\int \frac{\sqrt{-}}{}\right.$$

$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 - \frac{35}{32}$$

$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 - \frac{35}{16}$$

$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 - \frac{35}{16}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.45

$$\frac{\sqrt{\frac{1}{x^2}-1}\,{}_2F_1\left(-\frac{7}{2},-\frac{1}{2};\frac{1}{2};x^2\right)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*Hypergeometric2F1[-7/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2]
```

fricas [A] time = 0.48, size = 55, normalized size = 0.72

$$\frac{1}{48}\left(8\,x^6-38\,x^4+87\,x^2+48\right)\sqrt{-\frac{x^2-1}{x^2}} - \frac{35}{8}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^6 - 38*x^4 + 87*x^2 + 48)*sqrt(-(x^2 - 1)/x^2) - 35/8*arctan((x*s
qrt(-(x^2 - 1)/x^2) - 1)/x)
```

giac [A] time = 0.39, size = 77, normalized size = 1.01

$$\frac{1}{48}\left(2\left(4x^2\mathrm{sgn}(x)-19\,\mathrm{sgn}(x)\right)x^2+87\,\mathrm{sgn}(x)\right)\sqrt{-x^2+1}\,x+\frac{35}{16}\arcsin(x)\mathrm{sgn}(x)-\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2*sgn(x) - 19*sgn(x))*x^2 + 87*sgn(x))*sqrt(-x^2 + 1)*x + 35/1
6*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1
) - 1)*sgn(x)/x
```

maple [A] time = 0.02, size = 83, normalized size = 1.09

$$\frac{\sqrt{-\frac{x^2-1}{x^2}}\left(-8\left(-x^2+1\right)^{\frac{3}{2}}x^4+30\left(-x^2+1\right)^{\frac{3}{2}}x^2+105\sqrt{-x^2+1}\,x^2+105x\arcsin(x)+48\left(-x^2+1\right)^{\frac{3}{2}}\right)}{48\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x)
```

```
[Out] 1/48*(-(x^2-1)/x^2)^(1/2)*(-8*x^4*(-x^2+1)^(3/2)+30*x^2*(-x^2+1)^(3/2)+48*(
-x^2+1)^(3/2)+105*x^2*(-x^2+1)^(1/2)+105*arcsin(x)*x)/(-x^2+1)^(1/2)
```

maxima [B] time = 2.07, size = 120, normalized size = 1.58

$$\frac{3}{2}x^2\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}-1}-\frac{3\left(\frac{1}{x^2}-1\right)^{\frac{5}{2}}+8\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-3\sqrt{\frac{1}{x^2}-1}}{48\left(\left(\frac{1}{x^2}-1\right)^3+3\left(\frac{1}{x^2}-1\right)^2+\frac{3}{x^2}-2\right)}+\frac{3\left(\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^2}-1}\right)}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)}-\frac{35}{16}\arctan\left(\sqrt{\frac{1}{x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 3/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 1/48*(3*(1/x^2 - 1)^(5/2) + 8*(
1/x^2 - 1)^(3/2) - 3*sqrt(1/x^2 - 1))/((1/x^2 - 1)^3 + 3*(1/x^2 - 1)^2 + 3/
x^2 - 2) + 3/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^2 - 1))/((1/x^2 - 1)^2 + 2/x^2
 - 1) - 35/16*arctan(sqrt(1/x^2 - 1))
```

mupad [B] time = 3.50, size = 54, normalized size = 0.71

$$\sqrt{\frac{1}{x^2}-1}-\frac{35\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{16}+\frac{19\,x^6\,\sqrt{\frac{1}{x^2}-1}}{16}+\frac{17\,x^6\,{\left(\frac{1}{x^2}-1\right)}^{3/2}}{6}+\frac{29\,x^6\,{\left(\frac{1}{x^2}-1\right)}^{5/2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^3)/x,x)
```

```
[Out] (1/x^2 - 1)^(1/2) - (35*atan((1/x^2 - 1)^(1/2)))/16 + (19*x^6*(1/x^2 - 1)^(
1/2))/16 + (17*x^6*(1/x^2 - 1)^(3/2))/6 + (29*x^6*(1/x^2 - 1)^(5/2))/16
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**3*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

3.676 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x} dx$

Optimal. Leaf size=60

$$\frac{5}{8}\left(\frac{1}{x^2}-1\right)^{3/2}x^2-\frac{15}{8}\sqrt{\frac{1}{x^2}-1}+\frac{15}{8}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)+\frac{1}{4}\left(\frac{1}{x^2}-1\right)^{5/2}x^4$$

```
[Out] 5/8*(-1+1/x^2)^(3/2)*x^2+1/4*(-1+1/x^2)^(5/2)*x^4+15/8*arctan((-1+1/x^2)^(1
/2))-15/8*(-1+1/x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {25, 266, 47, 50, 63, 203}

$$\frac{1}{4}\left(\frac{1}{x^2}-1\right)^{5/2}x^4+\frac{5}{8}\left(\frac{1}{x^2}-1\right)^{3/2}x^2-\frac{15}{8}\sqrt{\frac{1}{x^2}-1}+\frac{15}{8}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] (-15*Sqrt[-1 + x^(-2)])/8 + (5*(-1 + x^(-2))^(3/2)*x^2)/8 + ((-1 + x^(-2))^
(5/2)*x^4)/4 + (15*ArcTan[Sqrt[-1 + x^(-2)]])/8
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x}\,dx &= \int \left(-1+\frac{1}{x^2}\right)^{5/2} x^3\,dx \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{5}{8}\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{15}{16}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{16}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\ldots}\right. \\
&= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\ldots\right. \\
&= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{8}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.58

$$
-\frac{\sqrt{\frac{1}{x^2}-1}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};x^2\right)}{\sqrt{1-x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] -((Sqrt[-1 + x^(-2)]*Hypergeometric2F1[-5/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2]
)
```

fricas [A] time = 0.45, size = 50, normalized size = 0.83

$$
\frac{1}{8}\left(2\,x^4-9\,x^2-8\right)\sqrt{-\frac{x^2-1}{x^2}} + \frac{15}{4}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^4 - 9*x^2 - 8)*sqrt(-(x^2 - 1)/x^2) + 15/4*arctan((x*sqrt(-(x^2 -
1)/x^2) - 1)/x)
```

giac [A] time = 0.43, size = 67, normalized size = 1.12

$$\frac{1}{8}\left(2\,x^2\mathrm{sgn}(x)-9\,\mathrm{sgn}(x)\right)\sqrt{-x^2+1}\,x-\frac{15}{8}\,\arcsin(x)\mathrm{sgn}(x)+\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*x^2*sgn(x) - 9*sgn(x))*sqrt(-x^2 + 1)*x - 15/8*arcsin(x)*sgn(x) + 1/
2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

maple [A] time = 0.01, size = 69, normalized size = 1.15

$$-\frac{\sqrt{-\frac{x^2-1}{x^2}}\,\left(2\left(-x^2+1\right)^{\frac{3}{2}}x^2+15\sqrt{-x^2+1}\,x^2+15x\arcsin(x)+8\left(-x^2+1\right)^{\frac{3}{2}}\right)}{8\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x)
```

```
[Out] -1/8*(-(x^2-1)/x^2)^(1/2)*(2*(-x^2+1)^(3/2)*x^2+8*(-x^2+1)^(3/2)+15*(-x^2+1
)^(1/2)*x^2+15*x*arcsin(x))/(-x^2+1)^(1/2)
```

maxima [A] time = 2.00, size = 67, normalized size = 1.12

$$-x^2\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}-1}-\frac{\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^2}-1}}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)}+\frac{15}{8}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -x^2*sqrt(1/x^2 - 1) - sqrt(1/x^2 - 1) - 1/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^
2 - 1))/((1/x^2 - 1)^2 + 2/x^2 - 1) + 15/8*arctan(sqrt(1/x^2 - 1))
```

mupad [B] time = 3.36, size = 44, normalized size = 0.73

$$\frac{15\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{8}-\sqrt{\frac{1}{x^2}-1}-\frac{7\,x^4\,\sqrt{\frac{1}{x^2}-1}}{8}-\frac{9\,x^4\,{\left(\frac{1}{x^2}-1\right)}^{3/2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^2)/x,x)
```

```
[Out] (15*atan((1/x^2 - 1)^(1/2)))/8 - (1/x^2 - 1)^(1/2) - (7*x^4*(1/x^2 - 1)^(1/
2))/8 - (9*x^4*(1/x^2 - 1)^(3/2))/8
```

sympy [A] time = 118.56, size = 60, normalized size = 1.00

$$\frac{x^4\sqrt{-1+\frac{1}{x^2}}\left(2-\frac{1}{x^2}\right)}{8}-x^2\sqrt{-1+\frac{1}{x^2}}-\sqrt{-1+\frac{1}{x^2}}+\frac{15\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**2*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/8 - x**2*sqrt(-1 + x**(-2)) - sqrt(-1
+ x**(-2)) + 15*atan(sqrt(-1 + x**(-2)))/8
```

3.677 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)}{x} dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{3}{2}\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

```
[Out] -1/2*(-1+1/x^2)^(3/2)*x^2-3/2*arctan((-1+1/x^2)^(1/2))+3/2*(-1+1/x^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {25, 266, 47, 50, 63, 203}

$$-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{3}{2}\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

```
[Out] (3*Sqrt[-1 + x^(-2)])/2 - ((-1 + x^(-2))^(3/2)*x^2)/2 - (3*ArcTan[Sqrt[-1 +
 x^(-2)]])/2
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)}{x}\,dx &= -\int \left(-1+\frac{1}{x^2}\right)^{3/2} x\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2+\frac{3}{4}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x^2}}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{2}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.77

$$\frac{\sqrt{\frac{1}{x^2}-1}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};x^2\right)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
[Out] (Sqrt[-1 + x^(-2)]*Hypergeometric2F1[-3/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2]
```

fricas [A] time = 0.45, size = 43, normalized size = 0.98

$$\frac{1}{2}\left(x^2+2\right)\sqrt{-\frac{x^2-1}{x^2}}-3\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
[Out] 1/2*(x^2 + 2)*sqrt(-(x^2 - 1)/x^2) - 3*arctan((x*sqrt(-(x^2 - 1)/x^2) - 1)/
x)
```

giac [A] time = 0.38, size = 57, normalized size = 1.30

$$\frac{1}{2}\sqrt{-x^2+1}\,x\,\text{sgn}(x)+\frac{3}{2}\arcsin(x)\text{sgn}(x)-\frac{x\,\text{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\text{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*sgn(x) + 3/2*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^
2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

maple [A] time = 0.01, size = 55, normalized size = 1.25

$$\frac{\sqrt{-\frac{x^2-1}{x^2}}\left(3\sqrt{-x^2+1}\,x^2+3x\arcsin(x)+2\left(-x^2+1\right)^{\frac{3}{2}}\right)}{2\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(-1+1/x^2)^(1/2)/x,x)
```

```
[Out] 1/2*(-(x^2-1)/x^2)^(1/2)*(2*(-x^2+1)^(3/2)+3*(-x^2+1)^(1/2)*x^2+3*x*arcsin(
x))/(-x^2+1)^(1/2)
```

maxima [A] time = 1.99, size = 30, normalized size = 0.68

$$\frac{1}{2}\,x^2\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\,\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 3/2*arctan(sqrt(1/x^2 - 1))
```

mupad [B] time = 3.57, size = 30, normalized size = 0.68

$$\sqrt{\frac{1}{x^2}-1}-\frac{3\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{2}+\frac{x^2\,\sqrt{\frac{1}{x^2}-1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1))/x,x)
```

```
[Out] (1/x^2 - 1)^(1/2) - (3*atan((1/x^2 - 1)^(1/2)))/2 + (x^2*(1/x^2 - 1)^(1/2))
/2
```

sympy [A] time = 43.65, size = 39, normalized size = 0.89

$$\frac{x^2\sqrt{-1+\frac{1}{x^2}}}{2}+\sqrt{-1+\frac{1}{x^2}}-\frac{3\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 3*atan(sqrt(-1 + x**(-2)))
/2
```

3.678 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)}\,dx$

Optimal. Leaf size=9

$$\sqrt{\frac{1}{x^2}-1}$$

[Out] (-1+1/x^2)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {25, 261}

$$\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]

[Out] Sqrt[-1 + x^(-2)]

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx &= -\int \frac{1}{\sqrt{-1+\frac{1}{x^2}}\,x^3}\,dx\\ &= \sqrt{-1+\frac{1}{x^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]

[Out] Sqrt[-1 + x^(-2)]

fricas [A] time = 0.45, size = 12, normalized size = 1.33

$$\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="fricas")
```

```
[Out] sqrt(-(x^2 - 1)/x^2)
```

giac [B] time = 0.40, size = 37, normalized size = 4.11

$$-\frac{x\operatorname{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\operatorname{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

maple [A] time = 0.00, size = 13, normalized size = 1.44

$$\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1),x)
```

```
[Out] (-(x^2-1)/x^2)^(1/2)
```

maxima [B] time = 0.58, size = 16, normalized size = 1.78

$$\frac{\sqrt{x+1}\,\sqrt{-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="maxima")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1)/x
```

mupad [B] time = 3.18, size = 14, normalized size = 1.56

$$\frac{\sqrt{1-x^2}}{|x|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)),x)
```

```
[Out] (1 - x^2)^(1/2)/abs(x)
```

sympy [A] time = 2.25, size = 8, normalized size = 0.89

$$\sqrt{-1+\frac{1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1),x)
```

```
[Out] sqrt(-1 + x**(-2))
```

3.679 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=21

$$\frac{1}{\sqrt{\frac{1}{x^2}-1}}-\sqrt{\frac{1}{x^2}-1}$$

[Out] 1/(-1+1/x^2)^(1/2)-(-1+1/x^2)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {25, 266, 43}

$$\frac{1}{\sqrt{\frac{1}{x^2}-1}}-\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]

[Out] 1/Sqrt[-1 + x^(-2)] - Sqrt[-1 + x^(-2)]

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = \int \frac{1}{\left(-1+\frac{1}{x^2}\right)^{3/2}x^5}\,dx$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(-1+x)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^{3/2}} + \frac{1}{\sqrt{-1+x}}\right)dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{1}{\sqrt{-1+\frac{1}{x^2}}} - \sqrt{-1+\frac{1}{x^2}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.14

$$\frac{\sqrt{\frac{1}{x^2}-1}\,(1-2x^2)}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(1 - 2*x^2))/(-1 + x^2)
```

fricas [A] time = 0.45, size = 28, normalized size = 1.33

$$-\frac{\left(2\,x^2-1\right)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -(2*x^2 - 1)*sqrt(-(x^2 - 1)/x^2)/(x^2 - 1)
```

giac [B] time = 0.38, size = 58, normalized size = 2.76

$$-\frac{\sqrt{-x^2+1}\,x\operatorname{sgn}(x)}{x^2-1} + \frac{x\operatorname{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} - \frac{\left(\sqrt{-x^2+1}-1\right)\operatorname{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)*x*sgn(x)/(x^2 - 1) + 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/
2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

maple [A] time = 0.01, size = 29, normalized size = 1.38

$$-\frac{\left(2x^2-1\right)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x)
```

```
[Out] -(2*x^2-1)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)
```

maxima [A] time = 0.73, size = 30, normalized size = 1.43

$$-\frac{\left(2\,x^2-1\right)\sqrt{x+1}\,\sqrt{-x+1}}{x^3-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="maxima")
[Out] -(2*x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x^3 - x)
```

mupad [B] time = 3.11, size = 25, normalized size = 1.19

$$\frac{x\,\sqrt{\frac{1}{x^2}-1}\,\left(2\,x^2-1\right)}{x-x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^2),x)
[Out] (x*(1/x^2 - 1)^(1/2)*(2*x^2 - 1))/(x - x^3)
```

sympy [A] time = 3.85, size = 20, normalized size = 0.95

$$-\sqrt{-1+\frac{1}{x^2}}+\frac{1}{\sqrt{-1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**2,x)
[Out] -sqrt(-1 + x**(-2)) + 1/sqrt(-1 + x**(-2))
```

3.680 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx$

Optimal. Leaf size=34

$$\sqrt{\frac{1}{x^2}-1}-\frac{2}{\sqrt{\frac{1}{x^2}-1}}-\frac{1}{3\left(\frac{1}{x^2}-1\right)^{3/2}}$$

```
[Out] -1/3/(-1+1/x^2)^(3/2)-2/(-1+1/x^2)^(1/2)+(-1+1/x^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {25, 266, 43}

$$\sqrt{\frac{1}{x^2}-1}-\frac{2}{\sqrt{\frac{1}{x^2}-1}}-\frac{1}{3\left(\frac{1}{x^2}-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

```
[Out] -1/(3*(-1 + x^(-2))^(3/2)) - 2/Sqrt[-1 + x^(-2)] + Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx = -\int \frac{1}{\left(-1+\frac{1}{x^2}\right)^{5/2}x^7}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(-1+x)^{5/2}}\,dx, x, \frac{1}{x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^{5/2}} + \frac{2}{(-1+x)^{3/2}} + \frac{1}{\sqrt{-1+x}}\right)dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{3/2}} - \frac{2}{\sqrt{-1+\frac{1}{x^2}}} + \sqrt{-1+\frac{1}{x^2}}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.94

$$\frac{\sqrt{\frac{1}{x^2}-1}\left(8x^4-12x^2+3\right)}{3\left(x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
[Out] (Sqrt[-1 + x^(-2)]*(3 - 12*x^2 + 8*x^4))/(3*(-1 + x^2)^2)
```

fricas [A] time = 0.46, size = 38, normalized size = 1.12

$$\frac{\left(8\,x^4-12\,x^2+3\right)\sqrt{-\frac{x^2-1}{x^2}}}{3\left(x^4-2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="fricas")
[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(-(x^2 - 1)/x^2)/(x^4 - 2*x^2 + 1)
```

giac [B] time = 0.44, size = 68, normalized size = 2.00

$$-\frac{x\text{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\text{sgn}(x)}{2\,x} - \frac{\left(5\,x^2\text{sgn}(x)-6\,\text{sgn}(x)\right)x}{3\left(x^2-1\right)\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="giac")
[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x - 1/
3*(5*x^2*sgn(x) - 6*sgn(x))*x/((x^2 - 1)*sqrt(-x^2 + 1))
```

maple [A] time = 0.00, size = 34, normalized size = 1.00

$$\frac{\left(8x^4-12x^2+3\right)\sqrt{-\frac{x^2-1}{x^2}}}{3\left(x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x)
```

```
[Out] 1/3*(8*x^4-12*x^2+3)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)^2
```

maxima [A] time = 0.64, size = 38, normalized size = 1.12

$$\frac{\left(8\,x^4-12\,x^2+3\right)\sqrt{x+1}\,\sqrt{-x+1}}{3\left(x^5-2\,x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(x + 1)*sqrt(-x + 1)/(x^5 - 2*x^3 + x)
```

mupad [B] time = 3.09, size = 28, normalized size = 0.82

$$\frac{\sqrt{\frac{1}{x^2}-1}\,\left(8\,x^4-12\,x^2+3\right)}{3\left(x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^3),x)
```

```
[Out] ((1/x^2 - 1)^(1/2)*(8*x^4 - 12*x^2 + 3))/(3*(x^2 - 1)^2)
```

sympy [A] time = 5.12, size = 34, normalized size = 1.00

$$\sqrt{-1+\frac{1}{x^2}}-\frac{2}{\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**3,x)
```

```
[Out] sqrt(-1 + x**(-2)) - 2/sqrt(-1 + x**(-2)) - 1/(3*(-1 + x**(-2))**(3/2))
```

3.681 $\int \frac{\sqrt{1+\frac{1}{x^2}}\, x}{(1+x^2)^2}\, dx$

Optimal. Leaf size=9

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

```
[Out] 1/(1+1/x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {25, 261}

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+\frac{1}{x^2}}\, x}{(1+x^2)^2}\, dx &= \int \frac{1}{\left(1+\frac{1}{x^2}\right)^{3/2} x^3}\, dx\\ &= \frac{1}{\sqrt{1+\frac{1}{x^2}}}\end{aligned}$$

Mathematica [B] time = 0.01, size = 20, normalized size = 2.22

$$\frac{\sqrt{\frac{1}{x^2}+1}\, x^2}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

```
[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

fricas [**B**] time = 0.46, size = 28, normalized size = 3.11

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

giac [**A**] time = 0.34, size = 11, normalized size = 1.22

$$\frac{x\mathrm{sgn}(x)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="giac")
[Out] x*sgn(x)/sqrt(x^2 + 1)
```

maple [**B**] time = 0.01, size = 23, normalized size = 2.56

$$\frac{\sqrt{\frac{x^2+1}{x^2}}\,x^2}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x)
[Out] 1/(x^2+1)*x^2*((x^2+1)/x^2)^(1/2)
```

maxima [**A**] time = 1.40, size = 11, normalized size = 1.22

$$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/sqrt((x^2 + 1)/x^2)
```

mupad [**B**] time = 3.11, size = 18, normalized size = 2.00

$$\frac{x^2\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1/x^2 + 1)^(1/2))/(x^2 + 1)^2,x)
[Out] (x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

sympy [**A**] time = 2.83, size = 8, normalized size = 0.89

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x**2)**(1/2)/(x**2+1)**2,x)
[Out] x/sqrt(x**2 + 1)
```

3.682 $\int \frac{1}{\sqrt{1+\frac{1}{x^2}}\, x(1+x^2)}\, dx$

Optimal. Leaf size=9

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

```
[Out] 1/(1+1/x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {25, 261}

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
```

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1+\frac{1}{x^2}}\, x\left(1+x^2\right)}\, dx &= \int \frac{1}{\left(1+\frac{1}{x^2}\right)^{3/2} x^3}\, dx \\ &= \frac{1}{\sqrt{1+\frac{1}{x^2}}} \end{aligned}$$

Mathematica [B] time = 0.00, size = 20, normalized size = 2.22

$$\frac{\sqrt{\frac{1}{x^2}+1}\, x^2}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
```

```
[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

fricas [**B**] time = 0.44, size = 28, normalized size = 3.11

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="fricas")
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

giac [**B**] time = 0.33, size = 20, normalized size = 2.22

$$\frac{1}{x^2-\sqrt{x^4+x^2}+1}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="giac")
[Out] 1/(x^2 - sqrt(x^4 + x^2) + 1) - 1
```

maple [**A**] time = 0.00, size = 12, normalized size = 1.33

$$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1)/(1+1/x^2)^(1/2),x)
[Out] 1/((x^2+1)/x^2)^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(x^2+1)x\sqrt{\frac{1}{x^2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)), x)
```

mupad [**B**] time = 3.10, size = 18, normalized size = 2.00

$$\frac{x^2\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/x^2 + 1)^(1/2)*(x^2 + 1)),x)
[Out] (x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

sympy [**A**] time = 3.17, size = 10, normalized size = 1.11

$$\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)/(1+1/x**2)**(1/2),x)
[Out] 1/sqrt(1 + x**(-2))
```

3.683 $\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

```
[Out] ln(1+(b*x^2+a)^(1/2))/b
```

Rubi [A] time = 0.07, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2155, 31}

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

```
[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a+bx+\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\ &= \frac{\log\left(1+\sqrt{a+bx^2}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

```
[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

fricas [B] time = 0.47, size = 67, normalized size = 3.72

$$\frac{2 \log \left(bx^2+a-1\right)+\log \left(\frac{bx^2+a+2 \sqrt{bx^2+a}+1}{x^2}\right)-\log \left(\frac{bx^2+a-2 \sqrt{bx^2+a}+1}{x^2}\right)}{4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(b*x^2 + a - 1) + log((b*x^2 + a + 2*sqrt(b*x^2 + a) + 1)/x^2) -
log((b*x^2 + a - 2*sqrt(b*x^2 + a) + 1)/x^2))/b
```

giac [A] time = 0.34, size = 16, normalized size = 0.89

$$\frac{\log \left(\sqrt{bx^2+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(sqrt(b*x^2 + a) + 1)/b
```

maple [B] time = 0.06, size = 1059, normalized size = 58.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^2+(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*((x+(
-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2/((-(a-1)*b)
^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(-a*b)^(1/2)*ln(((x+(
-a*b)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b))^(1/2))/b^(1/2)-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(
-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b))^(1/2)-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2
)+(-a*b)^(1/2))*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1/2)
+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))/b^(1/2)+
1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(b*(x+
(-(a-1)*b)^(1/2)/b)^2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2)-1/
2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(-(a-1)*
b)^(1/2)*ln((b*(x+(-(a-1)*b)^(1/2)/b)-(-(a-1)*b)^(1/2))/b^(1/2)+(b*(x+(-(a-
1)*b)^(1/2)/b)^2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2
)-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*arct
anh(1/2*(2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b))/(b*(x+(-(a-1)*b)^(1/2
)/b)^2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2))+1/2/((-(a-1)*b)^
(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(b*(x-(-(a-1)*b)^(1/2)
/b)^2+2*(-(a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2)+1/2/((-(a-1)*b)^(1
/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(-(a-1)*b)^(1/2)*ln((b*(
x-(-(a-1)*b)^(1/2)/b)+(-(a-1)*b)^(1/2))/b^(1/2)+(b*(x-(-(a-1)*b)^(1/2)/b)^2
+2*(-(a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2)-1/2/((-(a-1)*b
)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*arctanh(1/2*(2+2*(-(
a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b))/(b*(x-(-(a-1)*b)^(1/2)/b)^2+2*(-(a-1)
*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2))+1/2/b*ln(b*x^2+a-1)
```

maxima [A] time = 0.57, size = 16, normalized size = 0.89

$$\frac{\log \left(\sqrt{bx^2+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] log(sqrt(b*x^2 + a) + 1)/b
```

mupad [B] time = 3.38, size = 26, normalized size = 1.44

$$\frac{\operatorname{atanh}\left(\sqrt{b\,x^2+a}\right)+\frac{\ln\left(b\,x^2+a-1\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + (a + b*x^2)^(1/2)),x)
[Out] (atanh((a + b*x^2)^(1/2)) + log(a + b*x^2 - 1)/2)/b
```

sympy [A] time = 3.39, size = 14, normalized size = 0.78

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**2+(b*x**2+a)**(1/2)),x)
[Out] log(sqrt(a + b*x**2) + 1)/b
```

3.684 $\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx$

Optimal. Leaf size=16

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

```
[Out] 3/4*ln(1-(x^2)^(2/3))
```

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6715, 1593, 260}

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x^2 - (x^2)^(1/3)),x]
```

```
[Out] (3*Log[1 - (x^2)^(2/3)])/4
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\sqrt[3]{x}+x}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-1+x^{2/3}\right)\sqrt[3]{x}}\,dx,x,x^2\right)\\&=\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x^2 - (x^2)^(1/3)),x]
```

```
[Out] (3*Log[1 - (x^2)^(2/3)])/4
```

fricas [B] time = 0.45, size = 32, normalized size = 2.00

$$-3 \log\left(\frac{(x^2)^{\frac{1}{3}}}{x}\right) + \frac{3}{4} \log\left(-\frac{x^2 - (x^2)^{\frac{1}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*log((x^2)^(1/3)/x) + 3/4*log(-(x^2 - (x^2)^(1/3))/x^2)
```

giac [A] time = 0.47, size = 16, normalized size = 1.00

$$\frac{3}{4} \log\left(\left|(x\mathrm{sgn}(x))^{\frac{1}{3}} x\mathrm{sgn}(x) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/4*log(abs((x*sgn(x))^(1/3)*x*sgn(x) - 1))
```

maple [B] time = 0.05, size = 70, normalized size = 4.38

$$\frac{\ln\left((x^2)^{\frac{1}{3}} - 1\right)}{2} + \frac{\ln\left((x^2)^{\frac{1}{3}} + 1\right)}{2} - \frac{\ln\left((x^2)^{\frac{2}{3}} - (x^2)^{\frac{1}{3}} + 1\right)}{4} - \frac{\ln\left((x^2)^{\frac{2}{3}} + (x^2)^{\frac{1}{3}} + 1\right)}{4} + \frac{\ln\left(x^2 - 1\right)}{4} + \frac{\ln\left(x^2 + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-(x^2)^(1/3)),x)
```

```
[Out] 1/4*ln(x^2-1)+1/4*ln(x^2+1)+1/2*ln((x^2)^(1/3)-1)-1/4*ln((x^2)^(2/3)+(x^2)^
(1/3)+1)-1/4*ln((x^2)^(2/3)-(x^2)^(1/3)+1)+1/2*ln((x^2)^(1/3)+1)
```

maxima [A] time = 0.64, size = 21, normalized size = 1.31

$$\frac{3}{4} \log\left((x^2)^{\frac{1}{3}} + 1\right) + \frac{3}{4} \log\left((x^2)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/4*log((x^2)^(1/3) + 1) + 3/4*log((x^2)^(1/3) - 1)
```

mupad [B] time = 3.34, size = 10, normalized size = 0.62

$$\frac{3 \ln\left((x^2)^{2/3} - 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^2)^(1/3) - x^2),x)
```

```
[Out] (3*log((x^2)^(2/3) - 1))/4
```

sympy [A] time = 0.22, size = 19, normalized size = 1.19

$$-\frac{\log(x)}{2} + \frac{3 \log\left(x^2 - \sqrt[3]{x^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-(x**2)**(1/3)),x)
```

```
[Out] -log(x)/2 + 3*log(x**2 - (x**2)**(1/3))/4
```

3.685 $\int x\left(1+x^2\right)^3\sqrt{2+2x^2+x^4}\,dx$

Optimal. Leaf size=44

$$\frac{1}{10}\left(x^2+1\right)^2\left(x^4+2x^2+2\right)^{3/2}-\frac{1}{15}\left(x^4+2x^2+2\right)^{3/2}$$

```
[Out] -1/15*(x^4+2*x^2+2)^(3/2)+1/10*(x^2+1)^2*(x^4+2*x^2+2)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1247, 692, 629}

$$\frac{1}{10}\left(x^2+1\right)^2\left(x^4+2x^2+2\right)^{3/2}-\frac{1}{15}\left(x^4+2x^2+2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
```

```
[Out] -(2 + 2*x^2 + x^4)^(3/2)/15 + ((1 + x^2)^2*(2 + 2*x^2 + x^4)^(3/2))/10
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(1+x^2\right)^3\sqrt{2+2x^2+x^4}\,dx&=\frac{1}{2}\text{Subst}\left(\int(1+x)^3\sqrt{2+2x+x^2}\,dx,x,x^2\right)\\&=\frac{1}{10}\left(1+x^2\right)^2\left(2+2x^2+x^4\right)^{3/2}-\frac{1}{5}\text{Subst}\left(\int(1+x)\sqrt{2+2x+x^2}\,dx,x,x^2\right)\\&=-\frac{1}{15}\left(2+2x^2+x^4\right)^{3/2}+\frac{1}{10}\left(1+x^2\right)^2\left(2+2x^2+x^4\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.68

$$\frac{1}{30}\left(x^4+2x^2+2\right)^{3/2}\left(3x^4+6x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
[Out] ((2 + 2*x^2 + x^4)^(3/2)*(1 + 6*x^2 + 3*x^4))/30
```

fricas [A] time = 0.43, size = 36, normalized size = 0.82

$$\frac{1}{30}\left(3\,x^8+12\,x^6+19\,x^4+14\,x^2+2\right)\sqrt{x^4+2\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/30*(3*x^8 + 12*x^6 + 19*x^4 + 14*x^2 + 2)*sqrt(x^4 + 2*x^2 + 2)
```

giac [A] time = 0.35, size = 29, normalized size = 0.66

$$\frac{1}{10}\left(x^4+2\,x^2+2\right)^{\frac{5}{2}}-\frac{1}{6}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="giac")
[Out] 1/10*(x^4 + 2*x^2 + 2)^(5/2) - 1/6*(x^4 + 2*x^2 + 2)^(3/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.61

$$\frac{\left(x^4+2x^2+2\right)^{\frac{3}{2}}\left(3x^4+6x^2+1\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x)
[Out] 1/30*(x^4+2*x^2+2)^(3/2)*(3*x^4+6*x^2+1)
```

maxima [A] time = 1.47, size = 49, normalized size = 1.11

$$\frac{1}{10}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}x^4+\frac{1}{5}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}x^2+\frac{1}{30}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
[Out] 1/10*(x^4 + 2*x^2 + 2)^(3/2)*x^4 + 1/5*(x^4 + 2*x^2 + 2)^(3/2)*x^2 + 1/30*(
x^4 + 2*x^2 + 2)^(3/2)
```

mupad [B] time = 0.09, size = 26, normalized size = 0.59

$$\frac{\left(x^4+2\,x^2+2\right)^{3/2}\left(3\,x^4+6\,x^2+1\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^3*(2*x^2 + x^4 + 2)^(1/2),x)
[Out] ((2*x^2 + x^4 + 2)^(3/2)*(6*x^2 + 3*x^4 + 1))/30
```

sympy [B] time = 0.65, size = 94, normalized size = 2.14

$$\frac{x^8\sqrt{x^4+2x^2+2}}{10}+\frac{2x^6\sqrt{x^4+2x^2+2}}{5}+\frac{19x^4\sqrt{x^4+2x^2+2}}{30}+\frac{7x^2\sqrt{x^4+2x^2+2}}{15}+\frac{\sqrt{x^4+2x^2+2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**3*(x**4+2*x**2+2)**(1/2),x)
```

```
[Out] x**8*sqrt(x**4 + 2*x**2 + 2)/10 + 2*x**6*sqrt(x**4 + 2*x**2 + 2)/5 + 19*x**
4*sqrt(x**4 + 2*x**2 + 2)/30 + 7*x**2*sqrt(x**4 + 2*x**2 + 2)/15 + sqrt(x**
4 + 2*x**2 + 2)/15
```

3.686 $\int x^5\sqrt{1-x^3}\,\left(1+x^9\right)^2\,dx$

Optimal. Leaf size=121

$$\frac{2}{51}\left(1-x^3\right)^{17/2}-\frac{14}{45}\left(1-x^3\right)^{15/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{8}{9}$$

```
[Out] -8/9*(-x^3+1)^(3/2)+32/15*(-x^3+1)^(5/2)-22/7*(-x^3+1)^(7/2)+86/27*(-x^3+1)
^(9/2)-74/33*(-x^3+1)^(11/2)+14/13*(-x^3+1)^(13/2)-14/45*(-x^3+1)^(15/2)+2/
51*(-x^3+1)^(17/2)
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1821, 1620}

$$\frac{2}{51}\left(1-x^3\right)^{17/2}-\frac{14}{45}\left(1-x^3\right)^{15/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{8}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (-8*(1 - x^3)^(3/2))/9 + (32*(1 - x^3)^(5/2))/15 - (22*(1 - x^3)^(7/2))/7 +
 (86*(1 - x^3)^(9/2))/27 - (74*(1 - x^3)^(11/2))/33 + (14*(1 - x^3)^(13/2))
/13 - (14*(1 - x^3)^(15/2))/45 + (2*(1 - x^3)^(17/2))/51
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{1-x^3}\,\left(1+x^9\right)^2\,dx &= \frac{1}{3}\text{Subst}\left(\int \sqrt{1-x}\,x\left(1+x^3\right)^2\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(4\sqrt{1-x}\,-16(1-x)^{3/2}+33(1-x)^{5/2}-43(1-x)^{7/2}+37(1-x)^{9/2}-2\right.\right.\\ &= -\frac{8}{9}\left(1-x^3\right)^{3/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.47

$$\frac{2\sqrt{1-x^3}\,\left(45045x^{24}-3003x^{21}-3234x^{18}+135702x^{15}-19390x^{12}-22160x^9+126561x^6-86507x^3-173014\right)}{2297295}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (2*Sqrt[1 - x^3]*(-173014 - 86507*x^3 + 126561*x^6 - 22160*x^9 - 19390*x^12
 + 135702*x^15 - 3234*x^18 - 3003*x^21 + 45045*x^24))/2297295
```

fricas [A] time = 0.45, size = 53, normalized size = 0.44

$$\frac{2}{2297295}\left(45045\,x^{24}-3003\,x^{21}-3234\,x^{18}+135702\,x^{15}-19390\,x^{12}-22160\,x^{9}+126561\,x^{6}-86507\,x^{3}-173\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/2297295*(45045*x^24 - 3003*x^21 - 3234*x^18 + 135702*x^15 - 19390*x^12 -
22160*x^9 + 126561*x^6 - 86507*x^3 - 173014)*sqrt(-x^3 + 1)
```

giac [A] time = 0.37, size = 138, normalized size = 1.14

$$\frac{2}{51}\left(x^{3}-1\right)^{8}\sqrt{-x^{3}+1}+\frac{14}{45}\left(x^{3}-1\right)^{7}\sqrt{-x^{3}+1}+\frac{14}{13}\left(x^{3}-1\right)^{6}\sqrt{-x^{3}+1}+\frac{74}{33}\left(x^{3}-1\right)^{5}\sqrt{-x^{3}+1}+\frac{86}{27}\left(x^{3}-1\right)^{4}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/51*(x^3 - 1)^8*sqrt(-x^3 + 1) + 14/45*(x^3 - 1)^7*sqrt(-x^3 + 1) + 14/13*
(x^3 - 1)^6*sqrt(-x^3 + 1) + 74/33*(x^3 - 1)^5*sqrt(-x^3 + 1) + 86/27*(x^3
- 1)^4*sqrt(-x^3 + 1) + 22/7*(x^3 - 1)^3*sqrt(-x^3 + 1) + 32/15*(x^3 - 1)^2
*sqrt(-x^3 + 1) - 8/9*(-x^3 + 1)^(3/2)
```

maple [A] time = 0.01, size = 58, normalized size = 0.48

$$\frac{2\sqrt{-x^{3}+1}\;\left(45045x^{21}+42042x^{18}+38808x^{15}+174510x^{12}+155120x^{9}+132960x^{6}+259521x^{3}+173014\right)(}{2297295}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x)
```

```
[Out] 2/2297295*(-x^3+1)^(1/2)*(45045*x^21+42042*x^18+38808*x^15+174510*x^12+1551
20*x^9+132960*x^6+259521*x^3+173014)*(x-1)*(x^2+x+1)
```

maxima [A] time = 1.54, size = 89, normalized size = 0.74

$$\frac{2}{51}\left(-x^{3}+1\right)^{\frac{17}{2}}-\frac{14}{45}\left(-x^{3}+1\right)^{\frac{15}{2}}+\frac{14}{13}\left(-x^{3}+1\right)^{\frac{13}{2}}-\frac{74}{33}\left(-x^{3}+1\right)^{\frac{11}{2}}+\frac{86}{27}\left(-x^{3}+1\right)^{\frac{9}{2}}-\frac{22}{7}\left(-x^{3}+1\right)^{\frac{7}{2}}+\frac{32}{15}\left(-x^{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/51*(-x^3 + 1)^(17/2) - 14/45*(-x^3 + 1)^(15/2) + 14/13*(-x^3 + 1)^(13/2)
- 74/33*(-x^3 + 1)^(11/2) + 86/27*(-x^3 + 1)^(9/2) - 22/7*(-x^3 + 1)^(7/2)
+ 32/15*(-x^3 + 1)^(5/2) - 8/9*(-x^3 + 1)^(3/2)
```

mupad [B] time = 3.27, size = 124, normalized size = 1.02

$$\frac{84374\,x^{6}\,\sqrt{1-x^{3}}}{765765}-\frac{173014\,x^{3}\,\sqrt{1-x^{3}}}{2297295}-\frac{8864\,x^{9}\,\sqrt{1-x^{3}}}{459459}-\frac{1108\,x^{12}\,\sqrt{1-x^{3}}}{65637}+\frac{1436\,x^{15}\,\sqrt{1-x^{3}}}{12155}-\frac{28\,x^{18}\,\sqrt{1-}}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(1 - x^3)^(1/2)*(x^9 + 1)^2,x)
```

```
[Out] (84374*x^6*(1 - x^3)^(1/2))/765765 - (173014*x^3*(1 - x^3)^(1/2))/2297295 -
 (8864*x^9*(1 - x^3)^(1/2))/459459 - (1108*x^12*(1 - x^3)^(1/2))/65637 + (1
436*x^15*(1 - x^3)^(1/2))/12155 - (28*x^18*(1 - x^3)^(1/2))/9945 - (2*x^21*
(1 - x^3)^(1/2))/765 + (2*x^24*(1 - x^3)^(1/2))/51 - (346028*(1 - x^3)^(1/2
))/2297295
```

sympy [**A**] time = 12.71, size = 133, normalized size = 1.10

$$\frac{2x^{24}\sqrt{1-x^3}}{51}-\frac{2x^{21}\sqrt{1-x^3}}{765}-\frac{28x^{18}\sqrt{1-x^3}}{9945}+\frac{1436x^{15}\sqrt{1-x^3}}{12155}-\frac{1108x^{12}\sqrt{1-x^3}}{65637}-\frac{8864x^9\sqrt{1-x^3}}{459459}+\frac{84374x^6\sqrt{1-}}{765765}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**9+1)**2*(-x**3+1)**(1/2),x)
```

```
[Out] 2*x**24*sqrt(1 - x**3)/51 - 2*x**21*sqrt(1 - x**3)/765 - 28*x**18*sqrt(1 -
x**3)/9945 + 1436*x**15*sqrt(1 - x**3)/12155 - 1108*x**12*sqrt(1 - x**3)/65
637 - 8864*x**9*sqrt(1 - x**3)/459459 + 84374*x**6*sqrt(1 - x**3)/765765 -
173014*x**3*sqrt(1 - x**3)/2297295 - 346028*sqrt(1 - x**3)/2297295
```

3.687 $\int \left(\frac{x}{(a+bx^2)^{3/2}} + \frac{x}{(1+x^2)\sqrt{a+bx^2}} \right) dx$

Optimal. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {261, 444, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx = \int \frac{x}{(a+bx^2)^{3/2}}\,dx+\int \frac{x}{(1+x^2)\sqrt{a+bx^2}}\,dx$$
$$= -\frac{1}{b\sqrt{a+bx^2}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx,x,x^2\right)$$
$$= -\frac{1}{b\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}$$
$$= -\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.42

$$\frac{b\sqrt{a-b}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)+a-b}{b(b-a)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (a - b + Sqrt[a - b]*b*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]
)/(b*(-a + b)*Sqrt[a + b*x^2])
```

fricas [B] time = 0.49, size = 268, normalized size = 5.36

$$\left[\frac{(b^2x^2+ab)\sqrt{a-b}\log\left(\frac{b^2x^4+2(4ab-3b^2)x^2-4(bx^2+2a-b)\sqrt{bx^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{x^4+2x^2+1}\right)-4\sqrt{bx^2+a}(a-b)}{4(a^2b-ab^2+(ab^2-b^3)x^2)}, -\frac{(b^2x^2+ab)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*
(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 +
 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^
2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b
*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 + a)*(
a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

giac [A] time = 0.35, size = 41, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

maple [A] time = 0.03, size = 42, normalized size = 0.84

$$\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{b\,x^{2}+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/b/(b*x^2+a)^(1/2)+1/(b-a)^(1/2)*arctan((b*x^2+a)^(1/2)/(b-a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 3.76, size = 42, normalized size = 0.84

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(3/2) + x/((x^2 + 1)*(a + b*x^2)^(1/2)),x)
```

```
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^(
1/2))
```

sympy [A] time = 3.71, size = 49, normalized size = 0.98

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \frac{\mathrm{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) +
 atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b)
```

3.688 $\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {6, 571, 78, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\int \frac{x\left(1+a+x^2+bx^2\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{x\left(1+a+(1+b)x^2\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx$$
$$=\frac{1}{2}\operatorname{Subst}\left(\int \frac{1+a+(1+b)x}{(1+x)(a+bx)^{3/2}}\,dx,x,x^2\right)$$
$$=-\frac{1}{b\sqrt{a+bx^2}}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx,x,x^2\right)$$
$$=-\frac{1}{b\sqrt{a+bx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}$$
$$=-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A] time = 0.01, size = 71, normalized size = 1.42

$$\frac{b\sqrt{a-b}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)+a-b}{b(b-a)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (a - b + Sqrt[a - b]*b*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]
)/(b*(-a + b)*Sqrt[a + b*x^2])
```

fricas [B] time = 0.48, size = 268, normalized size = 5.36

$$\left[\frac{\left(b^2x^2+ab\right)\sqrt{a-b}\,\log\left(\frac{b^2x^4+2\left(4\,ab-3\,b^2\right)x^2-4\left(bx^2+2\,a-b\right)\sqrt{bx^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-4\,\sqrt{bx^2+a}\,(a-b)}{4\left(a^2b-ab^2+\left(ab^2-b^3\right)x^2\right)},-\frac{\left(b^2x^2+a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*
(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 +
 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^
2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b
*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 + a)*(
a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

giac [A] time = 0.38, size = 41, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

maple [B] time = 0.02, size = 133, normalized size = 2.66

$$\frac{a \arctan\left(\frac{\sqrt{b x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}-\frac{b \arctan\left(\frac{\sqrt{b x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}+\frac{a}{(a-b)\sqrt{b x^2+a}}-\frac{b}{(a-b)\sqrt{b x^2+a}}-\frac{1}{\sqrt{b x^2+a}\, b}-\frac{1}{\sqrt{b x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/(b*x^2+a)^(1/2)-1/(b*x^2+a)^(1/2)/b-b/(-b+a)/(b*x^2+a)^(1/2)-b/(-b+a)/(-
a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))+a/(-b+a)/(b*x^2+a)^(1/2)+a/
(-b+a)/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 4.52, size = 96, normalized size = 1.92

$$\frac{1}{\sqrt{b x^2+a}\,(a-b)}-\frac{a}{\sqrt{b x^2+a}\,\left(a b-b^2\right)}-\frac{a \operatorname{atanh}\left(\frac{\sqrt{b x^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}+\frac{b \operatorname{atanh}\left(\frac{\sqrt{b x^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + x^2 + 1))/((x^2 + 1)*(a + b*x^2)^(3/2)),x)
```

```
[Out] 1/((a + b*x^2)^(1/2)*(a - b)) - a/((a + b*x^2)^(1/2)*(a*b - b^2)) - (a*atan
h((a + b*x^2)^(1/2)/(a - b)^(1/2)))/(a - b)^(3/2) + (b*atanh((a + b*x^2)^(1
/2)/(a - b)^(1/2)))/(a - b)^(3/2)
```

sympy [A] time = 79.83, size = 37, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+x**2+a+1)/(x**2+1)/(b*x**2+a)**(3/2),x)
```

```
[Out] atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b) - 1/(b*sqrt(a + b*x**2))
```

3.689 $\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx$

Optimal. Leaf size=68

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b
/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.085, Rules used = {261, 444, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2
]),x]
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \left(\frac{x}{\left(a+bx^2\right)^{5/2}}+\frac{x}{\left(a+bx^2\right)^{3/2}}+\frac{x}{\left(1+x^2\right)\sqrt{a+bx^2}}\right)dx = \int \frac{x}{\left(a+bx^2\right)^{5/2}}\,dx+\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx+\int \frac{}{\left(1+x^2\right.}$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{}{(1+x}\right.$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx\right.}{b}$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A] time = 0.19, size = 63, normalized size = 0.93

$$\frac{-3a-3bx^2-1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a +
 b*x^2]),x]
```

```
[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a - b]]/Sqrt[a - b]
```

fricas [B] time = 0.49, size = 382, normalized size = 5.62

$$\left[\frac{3\left(b^3x^4+2\,ab^2x^2+a^2b\right)\sqrt{a-b}\,\log\left(\frac{b^2x^4+2\left(4\,ab-3\,b^2\right)x^2-4\left(bx^2+2\,a-b\right)\sqrt{bx^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-4\left(3\left(ab-b^2\right)x^2+3\,a\right.}{12\left(\left(ab^3-b^4\right)x^4+a^3b-a^2b^2+2\left(a^2b^2-ab^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*
b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 -
8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*
b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 -
 a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arctan(-
1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 -
 a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a))/(
(a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

giac [A] time = 0.35, size = 55, normalized size = 0.81

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}\,b}-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b) -
 1/3/((b*x^2 + a)^(3/2)*b)
```

maple [A] time = 0.02, size = 56, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{3\left(b\,x^2+a\right)^{\frac{3}{2}}b}-\frac{1}{\sqrt{b\,x^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-1/(b*x^2+a)^(1/2)/b+1/(-a+b)^(1/2)*arctan((b*x^2+a)^
(1/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 3.69, size = 56, normalized size = 0.82

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\,\sqrt{b\,x^2+a}}-\frac{1}{3\,b\,\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(3/2) + x/(a + b*x^2)^(5/2) + x/((x^2 + 1)*(a + b*x^2)^(1
/2)),x)
```

```
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^(
1/2)) - 1/(3*b*(a + b*x^2)^(3/2))
```

sympy [A] time = 4.39, size = 97, normalized size = 1.43

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\: b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for}\: b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases} + \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2)+x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1
/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) +
 Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(
b, 0)), (x**2/(2*a**(5/2)), True)) + atan(sqrt(a + b*x**2)/sqrt(-a + b))/sq
rt(-a + b)
```

3.690 $\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b
/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {6, 6715, 897, 1261, 207}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x^4))/(
(1 + x^2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{x\left(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{x\left(1+a+a^2+(1+a)x^2+bx^2+2abx^2+bx^4+\right.}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}$$

$$= \int \frac{x\left(1+a+a^2+2abx^2+(1+a+b)x^2+bx^4+b\right.}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}$$

$$= \int \frac{x\left(1+a+a^2+(1+a+b+2ab)x^2+bx^4+b^2x\right.}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}$$

$$= \int \frac{x\left(1+a+a^2+(1+a+b+2ab)x^2+\left(b+b^2\right)x\right.}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}$$

$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1+a+a^2+(1+a+b+2ab)x+\left(b+\right.}{(1+x)(a+bx)^{5/2}}\right.$$

$$= \frac{\mathrm{Subst}\left(\int \frac{\frac{\left(1+a+a^2\right)b^2-ab(1+a+b+2ab)+a^2\left(b+b^2\right)}{b^2}-\frac{\left(-b(1+a+b+2ab)+2a\right.}{b^2}}{x^4\left(\frac{-a+b}{b}+\frac{x^2}{b}\right)}\right.}{b}$$

$$= \frac{\mathrm{Subst}\left(\int\left(\frac{1}{x^4}+\frac{1}{x^2}+\frac{b}{-a+b+x^2}\right)dx, x, \sqrt{a+bx^2}\right)}{b}$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}+\mathrm{Subst}\left(\int \frac{1}{-a+b}\right.$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.93

$$\frac{-3a-3bx^2-1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x
^4))/((1 + x^2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a - b]]/Sqrt[a - b]
```

fricas [B] time = 0.50, size = 382, normalized size = 5.62

$$\left[\frac{3\left(b^3x^4+2\,ab^2x^2+a^2b\right)\sqrt{a-b}\,\log\left(\frac{b^2x^4+2\left(4\,ab-3\,b^2\right)x^2-4\left(bx^2+2\,a-b\right)\sqrt{bx^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-4\left(3\left(ab-b^2\right)x^2+3\,a\right.}{12\left(\left(ab^3-b^4\right)x^4+a^3b-a^2b^2+2\left(a^2b^2-ab^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*
b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 -
8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*
b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 -
 a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arctan(-
1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 -
 a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a))/(
(a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

giac [A] time = 0.46, size = 52, normalized size = 0.76

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{3\,bx^2+3\,a+1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/3*(3*b*x^2 + 3*a + 1)
/((b*x^2 + a)^(3/2)*b)
```

maple [B] time = 0.03, size = 314, normalized size = 4.62

$$\frac{a^2\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}} - \frac{2ab\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}} + \frac{b^2\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}} - \frac{b\,x^2}{\left(b\,x^2+a\right)^{\frac{3}{2}}} + \frac{a^2}{(a-b)^2\sqrt{b\,x^2+a}} + \frac{a}{3\,(a-b)\,(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(
5/2),x)
```

```
[Out] -x^2*b/(b*x^2+a)^(3/2)-x^2/(b*x^2+a)^(3/2)-4/3*a/(b*x^2+a)^(3/2)-a/b/(b*x^2
+a)^(3/2)+1/3*b/(b*x^2+a)^(3/2)-1/3/(b*x^2+a)^(3/2)/b+1/(a-b)^2/(b*x^2+a)^(
1/2)*a^2-2/(a-b)^2/(b*x^2+a)^(1/2)*a*b+1/(a-b)^2/(b*x^2+a)^(1/2)*b^2+1/(a-b
)^2/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))*a^2-2/(a-b)^2/(-a+b)^
(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))*a*b+1/(a-b)^2/(-a+b)^(1/2)*arcta
n((b*x^2+a)^(1/2)/(-a+b)^(1/2))*b^2+1/3/(a-b)/(b*x^2+a)^(3/2)*a^2-2/3/(a-b)
/(b*x^2+a)^(3/2)*a*b+1/3/(a-b)/(b*x^2+a)^(3/2)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 3.90, size = 50, normalized size = 0.74

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{b\,x^2+a+\frac{1}{3}}{b\,\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + a*x^2 + b*x^2 + b*x^4 + a^2 + x^2 + b^2*x^4 + 2*a*b*x^2 + 1))/(
(x^2 + 1)*(a + b*x^2)^(5/2)),x)
```

```
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - (a + b*x^2 + 1/3)/
(b*(a + b*x^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+b*x**4+2*a*b*x**2+a*x**2+b*x**2+a**2+x**2+a+1)/(x**2
+1)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

3.691 $\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx$

Optimal. Leaf size=34

$$2\sqrt{x+\sqrt{x}}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

[Out] -2*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))+2*(x+x^(1/2))^(1/2)

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2010, 2013, 620, 206}

$$2\sqrt{x+\sqrt{x}}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Sqrt[x] + x],x]

[Out] 2*Sqrt[Sqrt[x] + x] - 2*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2010

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt
[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n -
 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = 2\sqrt{\sqrt{x}+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+x}}\,dx$$
$$= 2\sqrt{\sqrt{x}+x} - \text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{\sqrt{x}+x} - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$
$$= 2\sqrt{\sqrt{x}+x} - 2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.15

$$2\sqrt{x+\sqrt{x}}\left(1-\frac{\sinh^{-1}\left(\sqrt[4]{x}\right)}{\sqrt{\sqrt{x}+1}\,\sqrt[4]{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sqrt[x] + x],x]
[Out] 2*Sqrt[Sqrt[x] + x]*(1 - ArcSinh[x^(1/4)]/(Sqrt[1 + Sqrt[x]]*x^(1/4)))
```

fricas [A] time = 0.81, size = 39, normalized size = 1.15

$$2\sqrt{x+\sqrt{x}} + \frac{1}{2}\log\left(4\sqrt{x+\sqrt{x}}\left(2\sqrt{x}+1\right) - 8x - 8\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x + sqrt(x)) + 1/2*log(4*sqrt(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8
*sqrt(x) - 1)
```

giac [A] time = 0.37, size = 27, normalized size = 0.79

$$2\sqrt{x+\sqrt{x}} + \log\left(-2\sqrt{x+\sqrt{x}} + 2\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x + sqrt(x)) + log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)
```

maple [A] time = 0.01, size = 44, normalized size = 1.29

$$-\frac{\sqrt{x+\sqrt{x}}\left(\ln\left(\sqrt{x}+\frac{1}{2}+\sqrt{x+\sqrt{x}}\right)-2\sqrt{x+\sqrt{x}}\right)}{\sqrt{\left(\sqrt{x}+1\right)\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x^(1/2))^(1/2),x)
```

```
[Out] -(x+x^(1/2))^(1/2)*(-2*(x+x^(1/2))^(1/2)+ln(x^(1/2)+1/2+(x+x^(1/2))^(1/2)))
/(x^(1/2)*(x^(1/2)+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(x)), x)
```

mupad [B] time = 3.33, size = 39, normalized size = 1.15

$$\frac{2\sqrt{x}\,\left(\sqrt{x}+1\right)+x^{1/4}\,\mathrm{asin}\left(x^{1/4}\,1\mathrm{i}\right)\,\sqrt{\sqrt{x}+1}\,2\mathrm{i}}{\sqrt{x+\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^(1/2))^(1/2),x)
```

```
[Out] (2*x^(1/2)*(x^(1/2) + 1) + x^(1/4)*asin(x^(1/4)*1i)*(x^(1/2) + 1)^(1/2)*2i)
/(x + x^(1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(x) + x), x)
```

3.692 $\int \sqrt{\sqrt{x}+x}\,dx$

Optimal. Leaf size=74

$$\frac{2}{3}\sqrt{x+\sqrt{x}}\,x+\frac{1}{6}\sqrt{x+\sqrt{x}}\,\sqrt{x}-\frac{\sqrt{x+\sqrt{x}}}{4}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

```
[Out] 1/4*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))-1/4*(x+x^(1/2))^(1/2)+2/3*x*(x+x^(1/
2))^(1/2)+1/6*x^(1/2)*(x+x^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2004, 2018, 670, 640, 620, 206}

$$\frac{2}{3}\sqrt{x+\sqrt{x}}\,x+\frac{1}{6}\sqrt{x+\sqrt{x}}\,\sqrt{x}-\frac{\sqrt{x+\sqrt{x}}}{4}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[x] + x],x]
```

```
[Out] -Sqrt[Sqrt[x] + x]/4 + (Sqrt[x]*Sqrt[Sqrt[x] + x])/6 + (2*x*Sqrt[Sqrt[x] +
x])/3 + ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n
```

```
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sqrt{x}+x}\,dx &= \frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{6}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\,dx\\
&= \frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)\\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.69

$$\frac{1}{12}\sqrt{x+\sqrt{x}}\left(8x+2\sqrt{x}+\frac{3\sinh^{-1}\left(\sqrt[4]{x}\right)}{\sqrt{\sqrt{x}+1}\sqrt[4]{x}}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[x] + x],x]
[Out] (Sqrt[Sqrt[x] + x]*(-3 + 2*Sqrt[x] + 8*x + (3*ArcSinh[x^(1/4)])/(Sqrt[1 + S
qrt[x]]*x^(1/4))))/12
```

fricas [A] time = 0.86, size = 49, normalized size = 0.66

$$\frac{1}{12}\left(8x+2\sqrt{x}-3\right)\sqrt{x+\sqrt{x}}+\frac{1}{16}\log\left(4\sqrt{x+\sqrt{x}}\left(2\sqrt{x}+1\right)+8x+8\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/12*(8*x + 2*sqrt(x) - 3)*sqrt(x + sqrt(x)) + 1/16*log(4*sqrt(x + sqrt(x))
*(2*sqrt(x) + 1) + 8*x + 8*sqrt(x) + 1)
```

giac [A] time = 0.45, size = 43, normalized size = 0.58

$$\frac{1}{12}\left(2\sqrt{x}\left(4\sqrt{x}+1\right)-3\right)\sqrt{x+\sqrt{x}}-\frac{1}{8}\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 3)*sqrt(x + sqrt(x)) - 1/8*log(-2*sqrt(x
+ sqrt(x)) + 2*sqrt(x) + 1)
```

maple [A] time = 0.00, size = 42, normalized size = 0.57

$$\frac{\ln\left(\sqrt{x}+\frac{1}{2}+\sqrt{x+\sqrt{x}}\right)}{8}+\frac{2\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^(1/2))^(1/2),x)
```

```
[Out] 2/3*(x+x^(1/2))^(3/2)-1/4*(1+2*x^(1/2))*(x+x^(1/2))^(1/2)+1/8*ln(x^(1/2)+1/
2+(x+x^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x)), x)
```

mupad [B] time = 3.16, size = 27, normalized size = 0.36

$$\frac{4\,x\,\sqrt{x+\sqrt{x}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{5}{2};\,\frac{7}{2};\,-\sqrt{x}\right)}{5\,\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(1/2))^(1/2),x)
```

```
[Out] (4*x*(x + x^(1/2))^(1/2)*hypergeom([-1/2, 5/2], 7/2, -x^(1/2)))/(5*(x^(1/2)
 + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sqrt{x}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) + x), x)
```

3.693 $\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx$

Optimal. Leaf size=19

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

```
[Out] 2/5*(-x)^(5/2)-1/2*x^2
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x]*(Sqrt[-x] + x),x]
```

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx &= \int \left(-(-x)^{3/2}-x\right)\,dx\\ &= \frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x]*(Sqrt[-x] + x),x]
```

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

fricas [A] time = 0.44, size = 16, normalized size = 0.84

$$\frac{2}{5}\sqrt{-x}\,x^2-\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

giac [A] time = 0.31, size = 16, normalized size = 0.84

$$\frac{2}{5}\sqrt{-x}\,x^2-\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{x^2}{2}+\frac{2\left(-x\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^(1/2)*(x+(-x)^(1/2)),x)
```

```
[Out] 2/5*(-x)^(5/2)-1/2*x^2
```

maxima [A] time = 0.67, size = 13, normalized size = 0.68

$$\frac{2}{5}\left(-x\right)^{\frac{5}{2}}-\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*(-x)^(5/2) - 1/2*x^2
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{2\left(-x\right)^{5/2}}{5}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^(1/2)*(x + (-x)^(1/2)),x)
```

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

sympy [C] time = 0.19, size = 14, normalized size = 0.74

$$\frac{2ix^{\frac{5}{2}}}{5}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**(1/2)*(x+(-x)**(1/2)),x)
```

```
[Out] 2*I*x**(5/2)/5 - x**2/2
```

3.694 $\int \frac{5+\sqrt[4]{x}}{-6+x} \, dx$

Optimal. Leaf size=54

$$4\sqrt[4]{x}+5\log(6-x)-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

```
[Out] 4*x^(1/4)-2*6^(1/4)*arctan(1/6*x^(1/4)*6^(3/4))-2*6^(1/4)*arctanh(1/6*x^(1/
4)*6^(3/4))+5*ln(6-x)
```

Rubi [A] time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1831, 260, 321, 212, 206, 203}

$$4\sqrt[4]{x}+5\log(6-x)-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^(1/4))/(-6 + x),x]
```

```
[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/6
^(1/4)] + 5*Log[6 - x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
```

```
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx &= 4\,\text{Subst}\left(\int \frac{x^3(5+x)}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\,\text{Subst}\left(\int \left(\frac{5x^3}{-6+x^4}+\frac{x^4}{-6+x^4}\right)dx, x, \sqrt[4]{x}\right)\\
&= 4\,\text{Subst}\left(\int \frac{x^4}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)+20\,\text{Subst}\left(\int \frac{x^3}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\sqrt[4]{x}+5\log(6-x)+24\,\text{Subst}\left(\int \frac{1}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\sqrt[4]{x}+5\log(6-x)-\left(2\sqrt{6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{6}-x^2}\,dx, x, \sqrt[4]{x}\right)-\left(2\sqrt{6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{6}+x^2}\,dx, x,\right.\\
&= 4\sqrt[4]{x}-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)+5\log(6-x)
\end{aligned}$$

Mathematica [C] time = 0.08, size = 107, normalized size = 1.98

$$4\sqrt[4]{x}+\left(5+\sqrt[4]{6}\right)\log\left(\sqrt[4]{6}-\sqrt[4]{x}\right)+\left(5-i\sqrt[4]{6}\right)\log\left(\sqrt[4]{6}-i\sqrt[4]{x}\right)+\left(5+i\sqrt[4]{6}\right)\log\left(\sqrt[4]{6}+i\sqrt[4]{x}\right)-\left(\sqrt[4]{6}-5\right)\log\left(\sqrt[4]{x}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^(1/4))/(-6 + x),x]
```

```
[Out] 4*x^(1/4) + (5 + 6^(1/4))*Log[6^(1/4) - x^(1/4)] + (5 - I*6^(1/4))*Log[6^(1
/4) - I*x^(1/4)] + (5 + I*6^(1/4))*Log[6^(1/4) + I*x^(1/4)] - (-5 + 6^(1/4)
)*Log[6^(1/4) + x^(1/4)]
```

fricas [B] time = 0.47, size = 86, normalized size = 1.59

$$-\left(6^{\frac{1}{4}}-5\right)\log\left(2\cdot 6^{\frac{1}{4}}+2\,x^{\frac{1}{4}}\right)+\left(6^{\frac{1}{4}}+5\right)\log\left(-2\cdot 6^{\frac{1}{4}}+2\,x^{\frac{1}{4}}\right)+4\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}\sqrt{\sqrt{6}+\sqrt{x}}-\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)+4\,x^{\frac{1}{4}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="fricas")
```

```
[Out] -(6^(1/4) - 5)*log(2*6^(1/4) + 2*x^(1/4)) + (6^(1/4) + 5)*log(-2*6^(1/4) +
2*x^(1/4)) + 4*6^(1/4)*arctan(1/6*6^(3/4)*sqrt(sqrt(6) + sqrt(x)) - 1/6*6^(
3/4)*x^(1/4)) + 4*x^(1/4) + 5*log(4*sqrt(6) + 4*sqrt(x))
```

giac [A] time = 0.49, size = 55, normalized size = 1.02

$$-2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)-6^{\frac{1}{4}}\log\left(6^{\frac{1}{4}}+x^{\frac{1}{4}}\right)+6^{\frac{1}{4}}\log\left(\left|-6^{\frac{1}{4}}+x^{\frac{1}{4}}\right|\right)+4\,x^{\frac{1}{4}}+5\,\log\left(|x-6|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="giac")
```

```
[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) - 6^(1/4)*log(6^(1/4) + x^(1/4)) + 6
^(1/4)*log(abs(-6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(abs(x - 6))
```

maple [A] time = 0.00, size = 52, normalized size = 0.96

$$-2\,6^{\frac{1}{4}}\arctan\left(\frac{6^{\frac{3}{4}}x^{\frac{1}{4}}}{6}\right)-6^{\frac{1}{4}}\ln\left(\frac{x^{\frac{1}{4}}+6^{\frac{1}{4}}}{x^{\frac{1}{4}}-6^{\frac{1}{4}}}\right)+5\ln\left(x-6\right)+4x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+x^(1/4))/(-6+x),x)
```

```
[Out] 4*x^(1/4)-2*6^(1/4)*arctan(1/6*x^(1/4)*6^(3/4))-6^(1/4)*ln((x^(1/4)+6^(1/4)
)/(x^(1/4)-6^(1/4)))+5*ln(-6+x)
```

maxima [A] time = 1.33, size = 67, normalized size = 1.24

$$-2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)+6^{\frac{1}{4}}\log\left(-\frac{6^{\frac{1}{4}}-x^{\frac{1}{4}}}{6^{\frac{1}{4}}+x^{\frac{1}{4}}}\right)+4\,x^{\frac{1}{4}}+5\,\log\left(\sqrt{6}+\sqrt{x}\right)+5\,\log\left(-\sqrt{6}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="maxima")
```

```
[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) + 6^(1/4)*log(-(6^(1/4) - x^(1/4))/(
6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(sqrt(6) + sqrt(x)) + 5*log(-sqrt(6)
 + sqrt(x))
```

mupad [B] time = 0.09, size = 162, normalized size = 3.00

$$\ln\left(11520\,x^{1/4}-\left(6^{1/4}+5\right)\,\left(2304\,x^{1/4}-2304\,6^{1/4}+11520\right)+57600\right)\,\left(6^{1/4}+5\right)-\ln\left(\left(6^{1/4}-5\right)\,\left(2304\,6^{1/4}+23\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/4) + 5)/(x - 6),x)
```

```
[Out] log(11520*x^(1/4) - (6^(1/4) + 5)*(2304*x^(1/4) - 2304*6^(1/4) + 11520) + 5
7600)*(6^(1/4) + 5) - log((6^(1/4) - 5)*(2304*6^(1/4) + 2304*x^(1/4) + 1152
0) + 11520*x^(1/4) + 57600)*(6^(1/4) - 5) - log(11520*x^(1/4) + ((-6^(1/2))
^(1/2) - 5)*(2304*(-6^(1/2))^(1/2) + 2304*x^(1/4) + 11520) + 57600)*((-6^(1
/2))^(1/2) - 5) + log(11520*x^(1/4) - ((-6^(1/2))^(1/2) + 5)*(2304*x^(1/4)
- 2304*(-6^(1/2))^(1/2) + 11520) + 57600)*((-6^(1/2))^(1/2) + 5) + 4*x^(1/4
)
```

sympy [A] time = 1.45, size = 100, normalized size = 1.85

$$4\sqrt[4]{x}+\sqrt[4]{6}\log\left(\sqrt[4]{x}-\sqrt[4]{6}\right)+5\log\left(\sqrt[4]{x}-\sqrt[4]{6}\right)-\sqrt[4]{6}\log\left(\sqrt[4]{x}+\sqrt[4]{6}\right)+5\log\left(\sqrt[4]{x}+\sqrt[4]{6}\right)+5\log\left(\sqrt{x}+\sqrt{6}\right)-2\sqrt[4]{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x**(1/4))/(-6+x),x)
```

```
[Out] 4*x**(1/4) + 6**(1/4)*log(x**(1/4) - 6**(1/4)) + 5*log(x**(1/4) - 6**(1/4))
 - 6**(1/4)*log(x**(1/4) + 6**(1/4)) + 5*log(x**(1/4) + 6**(1/4)) + 5*log(s
qrt(x) + sqrt(6)) - 2*6**(1/4)*atan(6**(3/4)*x**(1/4)/6)
```

3.695 $\int \frac{1}{4+\sqrt{4-x}-x}\,dx$

Optimal. Leaf size=14

$$-2\log\left(\sqrt{4-x}+1\right)$$

```
[Out] -2*ln(1+(4-x)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {31}

$$-2\log\left(\sqrt{4-x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + Sqrt[4 - x] - x)^(-1),x]
```

```
[Out] -2*Log[1 + Sqrt[4 - x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4+\sqrt{4-x}-x}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{4-x}\right)\right)\\ &= -2\log\left(1+\sqrt{4-x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$-2\log\left(\sqrt{4-x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + Sqrt[4 - x] - x)^(-1),x]
```

```
[Out] -2*Log[1 + Sqrt[4 - x]]
```

fricas [A] time = 0.44, size = 12, normalized size = 0.86

$$-2\,\log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*log(sqrt(-x + 4) + 1)
```

giac [A] time = 0.31, size = 12, normalized size = 0.86

$$-2\,\log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="giac")
[Out] -2*log(sqrt(-x + 4) + 1)
```

maple [A] time = 0.01, size = 18, normalized size = 1.29

$$-2\operatorname{arctanh}\left(\sqrt{-x+4}\right)-\ln\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-x+(4-x)^(1/2)),x)
[Out] -ln(x-3)-2*arctanh((4-x)^(1/2))
```

maxima [A] time = 0.66, size = 12, normalized size = 0.86

$$-2\log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="maxima")
[Out] -2*log(sqrt(-x + 4) + 1)
```

mupad [B] time = 0.19, size = 12, normalized size = 0.86

$$-2\ln\left(\sqrt{4-x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4 - x)^(1/2) - x + 4),x)
[Out] -2*log((4 - x)^(1/2) + 1)
```

sympy [B] time = 4.23, size = 32, normalized size = 2.29

$$\log\left(2\sqrt{4-x}\right)-\log\left(2\sqrt{4-x}+2\right)-\log\left(x-\sqrt{4-x}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)**(1/2)),x)
[Out] log(2*sqrt(4 - x)) - log(2*sqrt(4 - x) + 2) - log(x - sqrt(4 - x) - 4)
```

3.696 $\int \frac{1}{1+x-\sqrt{2+x}}\,dx$

Optimal. Leaf size=61

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)$$

```
[Out] 1/5*ln(1-5^(1/2)-2*(2+x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)-2*(2+x)^(1/2))
*(5+5^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {632, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[2 + x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x-\sqrt{2+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{-1-x+x^2}\,dx, x, \sqrt{2+x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, \sqrt{2+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\right.\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{2+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{2+x}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.95

$$\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]]) + (5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[2 + x]])/5
```

fricas [A] time = 0.44, size = 63, normalized size = 1.03

$$\frac{1}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(x+3)-\left(\sqrt{5}(2x+1)-5\right)\sqrt{x+2}+7x+3}{x^2+x-1}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 3) - (sqrt(5)*(2*x + 1) - 5)*sqrt(x +
 2) + 7*x + 3)/(x^2 + x - 1)) + log(x - sqrt(x + 2) + 1)
```

giac [A] time = 0.33, size = 50, normalized size = 0.82

$$\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{x+2}-1\right|}{\left|\sqrt{5}+2\sqrt{x+2}-1\right|}\right)+\log\left(\left|x-\sqrt{x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 2) - 1)/abs(sqrt(5) + 2*sqrt(x +
2) - 1)) + log(abs(x - sqrt(x + 2) + 1))
```

maple [A] time = 0.01, size = 91, normalized size = 1.49

$$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{x+2}-1\right)\sqrt{5}}{5}\right)}{5}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{x+2}+1\right)\sqrt{5}}{5}\right)}{5}+\frac{\ln\left(x+1-\sqrt{x+2}\right)}{2}-\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x-(x+2)^(1/2)),x)
```

```
[Out] -1/5*5^(1/2)*arctanh(1/5*(2*x+1)*5^(1/2))+1/2*ln(x^2+x-1)-1/2*ln(1+x+(x+2)^
(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(x+2)^(1/2)+1)*5^(1/2))+1/2*ln(1+x-(x+2)^
(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(x+2)^(1/2)-1)*5^(1/2))
```

maxima [A] time = 1.48, size = 46, normalized size = 0.75

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+2}+1}{\sqrt{5}+2\sqrt{x+2}-1}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 2) + 1)/(sqrt(5) + 2*sqrt(x + 2) - 1
)) + log(x - sqrt(x + 2) + 1)
```

mupad [B] time = 3.24, size = 71, normalized size = 1.16

$$\ln\left(2\sqrt{x+2}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{x+2}-1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right)-\ln\left(2\sqrt{x+2}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{x+2}-1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x + 2)^(1/2) + 1),x)
```

```
[Out] log(2*(x + 2)^(1/2) - (5^(1/2)/5 + 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)/5 + 1
) - log(2*(x + 2)^(1/2) + (5^(1/2)/5 - 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)/5
 - 1)
```

sympy [A] time = 2.55, size = 94, normalized size = 1.54

$$4\left(\begin{cases}-\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{x+2}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for } \left(\sqrt{x+2}-\frac{1}{2}\right)^2>\frac{5}{4}\\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{x+2}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for } \left(\sqrt{x+2}-\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)**(1/2)),x)
```

```
[Out] 4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(x + 2) - 1/2)/5)/10, (sqrt(x +
2) - 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(x + 2) - 1/2)/5)/10, (
sqrt(x + 2) - 1/2)**2 < 5/4)) + log(x - sqrt(x + 2) + 1)
```

3.697 $\int \frac{1}{4+x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=37

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

```
[Out] ln(4+x+(1+x)^(1/2))-2/11*arctan(1/11*(1+2*(1+x)^(1/2))*11^(1/2))*11^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {634, 618, 204, 628}

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= -\text{Subst}\left(\int \frac{1}{3+x+x^2}\,dx, x, \sqrt{1+x}\right) + \text{Subst}\left(\int \frac{1+2x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= \log\left(4+x+\sqrt{1+x}\right) + 2\,\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x, 1+2\sqrt{1+x}\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}} + \log\left(4+x+\sqrt{1+x}\right)$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\log\left(x+\sqrt{x+1}+4\right) - \frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

fricas [A] time = 0.48, size = 32, normalized size = 0.86

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{2}{11}\sqrt{11}\sqrt{x+1} + \frac{1}{11}\sqrt{11}\right) + \log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/11*sqrt(11)*arctan(2/11*sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)) + log(x +
sqrt(x + 1) + 4)
```

giac [A] time = 0.37, size = 30, normalized size = 0.81

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right) + \log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

maple [B] time = 0.01, size = 93, normalized size = 2.51

$$-\frac{\sqrt{11}\arctan\left(\frac{\left(1+2\sqrt{x+1}\right)\sqrt{11}}{11}\right)}{11} + \frac{\sqrt{11}\arctan\left(\frac{(2x+7)\sqrt{11}}{11}\right)}{11} - \frac{\sqrt{11}\arctan\left(\frac{\left(2\sqrt{x+1}-1\right)\sqrt{11}}{11}\right)}{11} + \frac{\ln\left(x+4+\sqrt{x+1}\right)}{2} - \frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+x+(x+1)^(1/2)),x)
```

```
[Out] -1/2*ln(x+4-(x+1)^(1/2))-1/11*11^(1/2)*arctan(1/11*(2*(x+1)^(1/2)-1)*11^(1/
2))+1/2*ln(4+x+(x+1)^(1/2))-1/11*arctan(1/11*(1+2*(x+1)^(1/2))*11^(1/2))*11
^(1/2)+1/11*11^(1/2)*arctan(1/11*(2*x+7)*11^(1/2))+1/2*ln(x^2+7*x+15)
```

maxima [A] time = 1.73, size = 30, normalized size = 0.81

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

mupad [B] time = 0.07, size = 32, normalized size = 0.86

$$\ln\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{\sqrt{11}}{11}+\frac{2\sqrt{11}\sqrt{x+1}}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x + 1)^(1/2) + 4),x)
```

```
[Out] log(x + (x + 1)^(1/2) + 4) - (2*11^(1/2)*atan(11^(1/2)/11 + (2*11^(1/2)*(x
+ 1)^(1/2))/11))/11
```

sympy [A] time = 2.31, size = 39, normalized size = 1.05

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}\left(\sqrt{x+1}+\frac{1}{2}\right)}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)**(1/2)),x)
```

```
[Out] log(x + sqrt(x + 1) + 4) - 2*sqrt(11)*atan(2*sqrt(11)*(sqrt(x + 1) + 1/2)/1
1)/11
```

3.698 $\int \frac{1}{x-\sqrt{1+x}}\,dx$

Optimal. Leaf size=61

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)$$

```
[Out] 1/5*ln(1-5^(1/2)-2*(1+x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)-2*(1+x)^(1/2))
*(5+5^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {632, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[1 + x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x-\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{-1-x+x^2}\,dx,x,\sqrt{1+x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,\sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx\right.\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 0.95

$$\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]]) + (5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[1 + x]])/5
```

fricas [A] time = 0.48, size = 64, normalized size = 1.05

$$\frac{1}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(x+2)-\left(\sqrt{5}(2x-1)-5\right)\sqrt{x+1}+3x-2}{x^2-x-1}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 2) - (sqrt(5)*(2*x - 1) - 5)*sqrt(x +
 1) + 3*x - 2)/(x^2 - x - 1)) + log(x - sqrt(x + 1))
```

giac [A] time = 0.36, size = 49, normalized size = 0.80

$$\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{x+1}-1\right|}{\left|\sqrt{5}+2\sqrt{x+1}-1\right|}\right)+\log\left(\left|x-\sqrt{x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 1) - 1)/abs(sqrt(5) + 2*sqrt(x +
1) - 1)) + log(abs(x - sqrt(x + 1)))
```

maple [A] time = 0.01, size = 91, normalized size = 1.49

$$-\frac{\sqrt{5}\ \mathrm{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{5}-\frac{\sqrt{5}\ \mathrm{arctanh}\left(\frac{\left(1+2\sqrt{x+1}\right)\sqrt{5}}{5}\right)}{5}-\frac{\sqrt{5}\ \mathrm{arctanh}\left(\frac{\left(2\sqrt{x+1}-1\right)\sqrt{5}}{5}\right)}{5}+\frac{\ln\left(x-\sqrt{x+1}\right)}{2}-\frac{\ln\left(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(x+1)^(1/2)),x)
```

```
[Out] 1/2*ln(x^2-x-1)-1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))-1/2*ln(x+(x+1)^(1/
2))-1/5*5^(1/2)*arctanh(1/5*(1+2*(x+1)^(1/2))*5^(1/2))+1/2*ln(x-(x+1)^(1/2)
)-1/5*5^(1/2)*arctanh(1/5*(2*(x+1)^(1/2)-1)*5^(1/2))
```

maxima [A] time = 1.51, size = 45, normalized size = 0.74

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+1}+1}{\sqrt{5}+2\sqrt{x+1}-1}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 1) + 1)/(sqrt(5) + 2*sqrt(x + 1) - 1
)) + log(x - sqrt(x + 1))
```

mupad [B] time = 0.12, size = 71, normalized size = 1.16

$$\ln\left(2\sqrt{x+1}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{x+1}-1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right)-\ln\left(2\sqrt{x+1}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{x+1}-1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x + 1)^(1/2)),x)
```

```
[Out] log(2*(x + 1)^(1/2) - (5^(1/2)/5 + 1)*(2*(x + 1)^(1/2) - 1))*(5^(1/2)/5 + 1
) - log(2*(x + 1)^(1/2) + (5^(1/2)/5 - 1)*(2*(x + 1)^(1/2) - 1))*(5^(1/2)/5
 - 1)
```

sympy [A] time = 2.21, size = 92, normalized size = 1.51

$$4\left(\begin{cases} -\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{x+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+1}-\frac{1}{2}\right)^2>\frac{5}{4} \\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{x+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+1}-\frac{1}{2}\right)^2<\frac{5}{4} \end{cases}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)**(1/2)),x)
```

```
[Out] 4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(x + 1) - 1/2)/5)/10, (sqrt(x +
1) - 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(x + 1) - 1/2)/5)/10, (
sqrt(x + 1) - 1/2)**2 < 5/4)) + log(x - sqrt(x + 1))
```

3.699 $\int \frac{1}{x-\sqrt{2+x}}\,dx$

Optimal. Leaf size=31

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

```
[Out] 4/3*ln(2-(2+x)^(1/2))+2/3*ln(1+(2+x)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {632, 31}

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x-\sqrt{2+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{-2-x+x^2}\,dx, x, \sqrt{2+x}\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{2+x}\right)+\frac{4}{3}\text{Subst}\left(\int \frac{1}{-2+x}\,dx, x, \sqrt{2+x}\right)\\ &= \frac{4}{3}\log\left(2-\sqrt{2+x}\right)+\frac{2}{3}\log\left(1+\sqrt{2+x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

fricas [A] time = 0.45, size = 21, normalized size = 0.68

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

giac [A] time = 0.39, size = 22, normalized size = 0.71

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(abs(sqrt(x + 2) - 2))
```

maple [B] time = 0.02, size = 54, normalized size = 1.74

$$\frac{2\ln(x-2)}{3}+\frac{2\ln\left(-2+\sqrt{x+2}\right)}{3}+\frac{\ln(x+1)}{3}+\frac{\ln\left(1+\sqrt{x+2}\right)}{3}-\frac{\ln\left(\sqrt{x+2}-1\right)}{3}-\frac{2\ln\left(\sqrt{x+2}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(x+2)^(1/2)),x)
```

```
[Out] 2/3*ln(x-2)+1/3*ln(x+1)-1/3*ln((x+2)^(1/2)-1)+2/3*ln(-2+(x+2)^(1/2))-2/3*ln
((x+2)^(1/2)+2)+1/3*ln(1+(x+2)^(1/2))
```

maxima [A] time = 0.59, size = 21, normalized size = 0.68

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

mupad [B] time = 3.08, size = 25, normalized size = 0.81

$$\frac{2\ln\left(\frac{2\sqrt{x+2}}{3}+\frac{2}{3}\right)}{3}+\frac{4\ln\left(\frac{4}{3}-\frac{2\sqrt{x+2}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x + 2)^(1/2)),x)
```

```
[Out] (2*log((2*(x + 2)^(1/2))/3 + 2/3))/3 + (4*log(4/3 - (2*(x + 2)^(1/2))/3))/3
```

sympy [A] time = 2.43, size = 36, normalized size = 1.16

$$\log\left(x-\sqrt{x+2}\right)+\frac{\log\left(2\sqrt{x+2}-4\right)}{3}-\frac{\log\left(2\sqrt{x+2}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)**(1/2)),x)
```

```
[Out] log(x - sqrt(x + 2)) + log(2*sqrt(x + 2) - 4)/3 - log(2*sqrt(x + 2) + 2)/3
```

3.700 $\int \frac{1}{-\sqrt{1-x}+x} dx$

Optimal. Leaf size=65

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)$$

```
[Out] 1/5*ln(1-5^(1/2)+2*(1-x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)+2*(1-x)^(1/2))
*(5+5^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {632, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 2*Sqrt[1 - x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\sqrt{1-x}+x}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{-1+x+x^2}\,dx,x,\sqrt{1-x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx,x,\sqrt{1-x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,d\right.\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2\sqrt{1-x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2\sqrt{1-x}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.95

$$\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]]) + (5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 2*Sqrt[1 - x]])/5
```

fricas [A] time = 0.43, size = 65, normalized size = 1.00

$$\frac{1}{5}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(x-2)-\left(\sqrt{5}(2x+1)+5\right)\sqrt{-x+1}-3x-2}{x^2+x-1}\right)+\log\left(-x+\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 + sqrt(5)*(x - 2) - (sqrt(5)*(2*x + 1) + 5)*sqrt(-x
+ 1) - 3*x - 2)/(x^2 + x - 1)) + log(-x + sqrt(-x + 1))
```

giac [A] time = 0.35, size = 54, normalized size = 0.83

$$-\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{-x+1}+1\right|}{\sqrt{5}+2\sqrt{-x+1}+1}\right)+\log\left(\left|-x+\sqrt{-x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(-x + 1) + 1)/(sqrt(5) + 2*sqrt(-x +
1) + 1)) + log(abs(-x + sqrt(-x + 1)))
```

maple [B] time = 0.01, size = 101, normalized size = 1.55

$$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{-x+1}-1\right)\sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{-x+1}+1\right)\sqrt{5}}{5}\right)}{5}-\frac{\ln\left(-x-\sqrt{-x+1}\right)}{2}+\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(-x+1)^(1/2)),x)
```

```
[Out] 1/2*ln(x^2+x-1)+1/5*5^(1/2)*arctanh(1/5*(2*x+1)*5^(1/2))+1/2*ln(-x+(-x+1)^(
1/2))+1/5*5^(1/2)*arctanh(1/5*(2*(-x+1)^(1/2)+1)*5^(1/2))-1/2*ln(-x-(-x+1)^
(1/2))+1/5*5^(1/2)*arctanh(1/5*(2*(-x+1)^(1/2)-1)*5^(1/2))
```

maxima [A] time = 1.32, size = 51, normalized size = 0.78

$$-\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{-x+1}-1}{\sqrt{5}+2\sqrt{-x+1}+1}\right)+\log\left(-x+\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(-x + 1) - 1)/(sqrt(5) + 2*sqrt(-x + 1)
+ 1)) + log(-x + sqrt(-x + 1))
```

mupad [B] time = 3.16, size = 79, normalized size = 1.22

$$\ln\left(2\sqrt{1-x}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{1-x}+1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right)-\ln\left(2\sqrt{1-x}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{1-x}+1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (1 - x)^(1/2)),x)
```

```
[Out] log(2*(1 - x)^(1/2) - (5^(1/2)/5 + 1)*(2*(1 - x)^(1/2) + 1))*(5^(1/2)/5 + 1
) - log(2*(1 - x)^(1/2) + (5^(1/2)/5 - 1)*(2*(1 - x)^(1/2) + 1))*(5^(1/2)/5
 - 1)
```

sympy [**A**] time = 2.31, size = 92, normalized size = 1.42

$$-4\left(\begin{cases} -\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{1-x}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for } \left(\sqrt{1-x}+\frac{1}{2}\right)^{2} > \frac{5}{4} \\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{1-x}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for } \left(\sqrt{1-x}+\frac{1}{2}\right)^{2} < \frac{5}{4} \end{cases}\right)+\log\left(x-\sqrt{1-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)**(1/2)),x)
```

```
[Out] -4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(1 - x) + 1/2)/5)/10, (sqrt(1 -
 x) + 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(1 - x) + 1/2)/5)/10,
(sqrt(1 - x) + 1/2)**2 < 5/4)) + log(x - sqrt(1 - x))
```

3.701 $\int \sqrt{1+\sqrt{x}+x}\,dx$

Optimal. Leaf size=62

$$\frac{2}{3}\left(x+\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}+1}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)$$

```
[Out] -3/8*arcsinh(1/3*(1+2*x^(1/2))*3^(1/2))+2/3*(1+x+x^(1/2))^(3/2)-1/4*(1+2*x^
(1/2))*(1+x+x^(1/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1341, 640, 612, 619, 215}

$$\frac{2}{3}\left(x+\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}+1}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[x] + x],x]
```

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 + Sqrt[x] + x])/4 + (2*(1 + Sqrt[x] + x)^(3/2))/3
- (3*ArcSinh[(1 + 2*Sqrt[x])/Sqrt[3]])/8
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \sqrt{1+\sqrt{x}+x}\,dx = 2\,\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx,x,\sqrt{x}\right)$$
$$= \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1+x+x^2}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x} + \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x} + \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \frac{1}{8}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+\right.$$
$$= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x} + \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \frac{3}{8}\sinh^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.79

$$\frac{1}{24}\left(2\sqrt{x+\sqrt{x}+1}\left(8x+2\sqrt{x}+5\right)-9\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[x] + x],x]
[Out] (2*Sqrt[1 + Sqrt[x] + x]*(5 + 2*Sqrt[x] + 8*x) - 9*ArcSinh[(1 + 2*Sqrt[x])/
Sqrt[3]])/24
```

fricas [A] time = 0.90, size = 51, normalized size = 0.82

$$\frac{1}{12}\left(8x+2\sqrt{x}+5\right)\sqrt{x+\sqrt{x}+1}+\frac{3}{16}\log\left(4\sqrt{x+\sqrt{x}+1}\left(2\sqrt{x}+1\right)-8x-8\sqrt{x}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/12*(8*x + 2*sqrt(x) + 5)*sqrt(x + sqrt(x) + 1) + 3/16*log(4*sqrt(x + sqrt
(x) + 1)*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 5)
```

giac [A] time = 0.40, size = 45, normalized size = 0.73

$$\frac{1}{12}\left(2\sqrt{x}\left(4\sqrt{x}+1\right)+5\right)\sqrt{x+\sqrt{x}+1}+\frac{3}{8}\log\left(2\sqrt{x+\sqrt{x}+1}-2\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) + 5)*sqrt(x + sqrt(x) + 1) + 3/8*log(2*sqrt
(x + sqrt(x) + 1) - 2*sqrt(x) - 1)
```

maple [A] time = 0.01, size = 42, normalized size = 0.68

$$-\frac{3\,\text{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x}+\frac{1}{2}\right)}{3}\right)}{8}+\frac{2\left(x+\sqrt{x}+1\right)^{\frac{3}{2}}}{3}-\frac{\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+x^(1/2))^(1/2),x)
```

```
[Out] 2/3*(1+x+x^(1/2))^(3/2)-1/4*(2*x^(1/2)+1)*(1+x+x^(1/2))^(1/2)-3/8*arcsinh(2
/3*3^(1/2)*(x^(1/2)+1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x+\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(1/2) + 1)^(1/2),x)
[Out] int((x + x^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x}+x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x**(1/2))**(1/2),x)
[Out] Integral(sqrt(sqrt(x) + x + 1), x)
```

3.702 $\int \sqrt{1+x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=75

$$\frac{2}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)$$

```
[Out] 1/4*arctanh((1+x)^(1/2)/(1+x+(1+x)^(1/2))^(1/2))+2/3*(1+x+(1+x)^(1/2))^(3/2
)-1/4*(1+2*(1+x)^(1/2))*(1+x+(1+x)^(1/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1980, 640, 612, 620, 206}

$$\frac{2}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (2*(1 + x + Sqrt[1 + x])^(3/2))/3 - (Sqrt[1 + x + Sqrt[1 + x]]*(1 + 2*Sqrt[
1 + x]))/4 + ArcTanh[Sqrt[1 + x]/Sqrt[1 + x + Sqrt[1 + x]]]/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = 2\,\text{Subst}\left(\int x\sqrt{x(1+x)}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\,\text{Subst}\left(\int x\sqrt{x+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \text{Subst}\left(\int \sqrt{x+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx\right.$$
$$= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x\right.$$
$$= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1+x+\sqrt{1}}}\right.$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.83

$$\frac{1}{12}\sqrt{x+\sqrt{x+1}+1}\left(8x+2\sqrt{x+1}+\frac{3\sinh^{-1}\left(\sqrt[4]{x+1}\right)}{\sqrt[4]{x+1}\sqrt{\sqrt{x+1}+1}}+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (Sqrt[1 + x + Sqrt[1 + x]]*(5 + 8*x + 2*Sqrt[1 + x] + (3*ArcSinh[(1 + x)^(1
/4)])/((1 + x)^(1/4)*Sqrt[1 + Sqrt[1 + x]])))/12
```

fricas [A] time = 0.87, size = 61, normalized size = 0.81

$$\frac{1}{12}\left(8x+2\sqrt{x+1}+5\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{16}\log\left(-4\sqrt{x+\sqrt{x+1}+1}\left(2\sqrt{x+1}+1\right)-8x-8\sqrt{x+1}-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x + 1) + 5)*sqrt(x + sqrt(x + 1) + 1) + 1/16*log(-4*sqrt
(x + sqrt(x + 1) + 1)*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 9)
```

giac [A] time = 0.41, size = 55, normalized size = 0.73

$$\frac{1}{12}\left(2\sqrt{x+1}\left(4\sqrt{x+1}+1\right)-3\right)\sqrt{x+\sqrt{x+1}+1}-\frac{1}{8}\log\left(-2\sqrt{x+\sqrt{x+1}+1}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x + 1)*(4*sqrt(x + 1) + 1) - 3)*sqrt(x + sqrt(x + 1) + 1) - 1/
8*log(-2*sqrt(x + sqrt(x + 1) + 1) + 2*sqrt(x + 1) + 1)
```

maple [A] time = 0.01, size = 55, normalized size = 0.73

$$\frac{\ln\left(\sqrt{x+1}+\frac{1}{2}+\sqrt{x+1+\sqrt{x+1}}\right)}{8}+\frac{2\left(x+1+\sqrt{x+1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x+1}\right)\sqrt{x+1+\sqrt{x+1}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+(x+1)^(1/2))^(1/2),x)
```

```
[Out] 2/3*(1+x+(x+1)^(1/2))^(3/2)-1/4*(1+2*(x+1)^(1/2))*(1+x+(x+1)^(1/2))^(1/2)+1
/8*ln((x+1)^(1/2)+1/2+(1+x+(x+1)^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + \sqrt{x+1} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x + \sqrt{x+1} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int((x + (x + 1)^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + \sqrt{x+1} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1) + 1), x)
```

3.703 $\int \sqrt{\sqrt{-1+x}+x}\,dx$

Optimal. Leaf size=68

$$\frac{2}{3}\left(x+\sqrt{x-1}\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x-1}+1\right)\sqrt{x+\sqrt{x-1}}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)$$

```
[Out] -3/8*arcsinh(1/3*(1+2*(-1+x)^(1/2))*3^(1/2))+2/3*(x+(-1+x)^(1/2))^(3/2)-1/4
*(1+2*(-1+x)^(1/2))*(x+(-1+x)^(1/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {640, 612, 619, 215}

$$\frac{2}{3}\left(x+\sqrt{x-1}\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x-1}+1\right)\sqrt{x+\sqrt{x-1}}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[-1 + x] + x],x]
```

```
[Out] -((1 + 2*Sqrt[-1 + x])*Sqrt[Sqrt[-1 + x] + x])/4 + (2*(Sqrt[-1 + x] + x)^(3
/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/8
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = 2\,\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{-1+x}\right)$$
$$= \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{-1+x}\right)$$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\right.$$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{1}{8}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\right.$$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{3}{8}\sinh^{-1}\left(\frac{1+2\sqrt{-1+x}}{\sqrt{3}}\right.$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.79

$$\frac{1}{24}\left(2\sqrt{x+\sqrt{x-1}}\left(8x+2\sqrt{x-1}-3\right)-9\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[-1 + x] + x],x]
```

```
[Out] (2*Sqrt[Sqrt[-1 + x] + x]*(-3 + 2*Sqrt[-1 + x] + 8*x) - 9*ArcSinh[(1 + 2*Sq
rt[-1 + x])/Sqrt[3]])/24
```

fricas [A] time = 0.93, size = 59, normalized size = 0.87

$$\frac{1}{12}\left(8\,x+2\,\sqrt{x-1}-3\right)\sqrt{x+\sqrt{x-1}}+\frac{3}{16}\,\log\left(-4\,\sqrt{x+\sqrt{x-1}}\left(2\,\sqrt{x-1}+1\right)+8\,x+8\,\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x - 1) - 3)*sqrt(x + sqrt(x - 1)) + 3/16*log(-4*sqrt(x +
 sqrt(x - 1))*(2*sqrt(x - 1) + 1) + 8*x + 8*sqrt(x - 1) - 3)
```

giac [A] time = 0.40, size = 53, normalized size = 0.78

$$\frac{1}{12}\left(2\,\sqrt{x-1}\left(4\,\sqrt{x-1}+1\right)+5\right)\sqrt{x+\sqrt{x-1}}+\frac{3}{8}\,\log\left(2\,\sqrt{x+\sqrt{x-1}}-2\,\sqrt{x-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x - 1)*(4*sqrt(x - 1) + 1) + 5)*sqrt(x + sqrt(x - 1)) + 3/8*lo
g(2*sqrt(x + sqrt(x - 1)) - 2*sqrt(x - 1) - 1)
```

maple [A] time = 0.01, size = 48, normalized size = 0.71

$$-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}+\frac{1}{2}\right)}{3}\right)}{8}+\frac{2\left(x+\sqrt{x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x-1}\right)\sqrt{x+\sqrt{x-1}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x-1)^(1/2))^(1/2),x)
```

```
[Out] 2/3*(x+(x-1)^(1/2))^(3/2)-1/4*(1+2*(x-1)^(1/2))*(x+(x-1)^(1/2))^(1/2)-3/8*a
rcsinh(2/3*3^(1/2)*((x-1)^(1/2)+1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x+\sqrt{x-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x - 1)^(1/2))^(1/2),x)
```

```
[Out] int((x + (x - 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x - 1)), x)
```

3.704 $\int \sqrt{2x+\sqrt{-1+2x}}\,dx$

Optimal. Leaf size=80

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}-\frac{1}{8}\left(2\sqrt{2x-1}+1\right)\sqrt{2x+\sqrt{2x-1}}-\frac{3}{16}\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)$$

```
[Out] -3/16*arcsinh(1/3*(1+2*(-1+2*x)^(1/2))*3^(1/2))+1/3*(2*x+(-1+2*x)^(1/2))^(3
/2)-1/8*(1+2*(-1+2*x)^(1/2))*(2*x+(-1+2*x)^(1/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {640, 612, 619, 215}

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}-\frac{1}{8}\left(2\sqrt{2x-1}+1\right)\sqrt{2x+\sqrt{2x-1}}-\frac{3}{16}\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
```

```
[Out] (2*x + Sqrt[-1 + 2*x])^(3/2)/3 - (Sqrt[2*x + Sqrt[-1 + 2*x]]*(1 + 2*Sqrt[-1
 + 2*x]))/8 - (3*ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/16
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx,x,\sqrt{-1+2x}\right)$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{2}\text{Subst}\left(\int \sqrt{1+x+x^2}\,dx,x,\sqrt{-1+2x}\right)$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1+x}}\right.$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{1}{16}\sqrt{3}\,\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{3}{16}\sinh^{-1}\left(\frac{1+2\sqrt{-1}}{\sqrt{3}}\right.$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.78

$$\frac{1}{48}\left(2\sqrt{2x+\sqrt{2x-1}}\left(16x+2\sqrt{2x-1}-3\right)-9\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
```

```
[Out] (2*Sqrt[2*x + Sqrt[-1 + 2*x]]*(-3 + 16*x + 2*Sqrt[-1 + 2*x]) - 9*ArcSinh[(1
 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/48
```

fricas [A] time = 0.93, size = 73, normalized size = 0.91

$$\frac{1}{24}\left(16\,x+2\,\sqrt{2\,x-1}-3\right)\sqrt{2\,x+\sqrt{2\,x-1}}+\frac{3}{32}\,\log\left(-4\,\sqrt{2\,x+\sqrt{2\,x-1}}\left(2\,\sqrt{2\,x-1}+1\right)+16\,x+8\,\sqrt{2\,x-1}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(16*x + 2*sqrt(2*x - 1) - 3)*sqrt(2*x + sqrt(2*x - 1)) + 3/32*log(-4*s
qrt(2*x + sqrt(2*x - 1))*(2*sqrt(2*x - 1) + 1) + 16*x + 8*sqrt(2*x - 1) - 3
)
```

giac [A] time = 0.45, size = 67, normalized size = 0.84

$$\frac{1}{24}\left(2\,\sqrt{2\,x-1}\left(4\,\sqrt{2\,x-1}+1\right)+5\right)\sqrt{2\,x+\sqrt{2\,x-1}}+\frac{3}{16}\,\log\left(2\,\sqrt{2\,x+\sqrt{2\,x-1}}-2\,\sqrt{2\,x-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(2*sqrt(2*x - 1)*(4*sqrt(2*x - 1) + 1) + 5)*sqrt(2*x + sqrt(2*x - 1))
+ 3/16*log(2*sqrt(2*x + sqrt(2*x - 1)) - 2*sqrt(2*x - 1) - 1)
```

maple [A] time = 0.01, size = 60, normalized size = 0.75

$$-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{2x-1}+\frac{1}{2}\right)}{3}\right)}{16}+\frac{\left(2x+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{2x-1}\right)\sqrt{2x+\sqrt{2x-1}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+(2*x-1)^(1/2))^(1/2),x)
```

```
[Out] 1/3*(2*x+(2*x-1)^(1/2))^(3/2)-1/8*(1+2*(2*x-1)^(1/2))*(2*x+(2*x-1)^(1/2))^(
1/2)-3/16*arcsinh(2/3*3^(1/2)*((2*x-1)^(1/2)+1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2\,x+\sqrt{2\,x-1}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x + sqrt(2*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2\,x+\sqrt{2\,x-1}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + (2*x - 1)^(1/2))^(1/2),x)
```

```
[Out] int((2*x + (2*x - 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x+\sqrt{2x-1}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(2*x + sqrt(2*x - 1)), x)
```

3.705 $\int \sqrt{3x+\sqrt{-7+8x}}\,dx$

Optimal. Leaf size=109

$$\frac{\left(-3(7-8x)+8\sqrt{8x-7}+21\right)^{3/2}}{72\sqrt{2}}-\frac{\left(3\sqrt{8x-7}+4\right)\sqrt{-3(7-8x)+8\sqrt{8x-7}+21}}{36\sqrt{2}}-\frac{47\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

```
[Out] -47/216*arcsinh(1/47*(4+3*(-7+8*x)^(1/2))*47^(1/2))*6^(1/2)+1/144*(24*x+8*(
-7+8*x)^(1/2))^(3/2)*2^(1/2)-1/36*(4+3*(-7+8*x)^(1/2))*(6*x+2*(-7+8*x)^(1/2
))^(1/2)*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {640, 612, 619, 215}

$$\frac{\left(-3(7-8x)+8\sqrt{8x-7}+21\right)^{3/2}}{72\sqrt{2}}-\frac{\left(3\sqrt{8x-7}+4\right)\sqrt{-3(7-8x)+8\sqrt{8x-7}+21}}{36\sqrt{2}}-\frac{47\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

```
[Out] -((4 + 3*Sqrt[-7 + 8*x])*Sqrt[21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x]])/(36*Sqr
t[2]) + (21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x])^(3/2)/(72*Sqrt[2]) - (47*ArcS
inh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/(36*Sqrt[6])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx = \frac{1}{4}\text{Subst}\left(\int x\sqrt{\frac{21}{8}+x+\frac{3x^2}{8}}\,dx, x, \sqrt{-7+8x}\right)$$

$$= \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}} - \frac{1}{3}\text{Subst}\left(\int \sqrt{\frac{21}{8}+x+\frac{3x^2}{8}}\,dx, x, \sqrt{-7+8x}\right)$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right.}{72\sqrt{2}}$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right.}{72\sqrt{2}}$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right.}{72\sqrt{2}}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 0.60

$$\frac{1}{216}\left(12\sqrt{3x+\sqrt{8x-7}}\left(12x+\sqrt{8x-7}-4\right)-47\sqrt{6}\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

```
[Out] (12*Sqrt[3*x + Sqrt[-7 + 8*x]]*(-4 + 12*x + Sqrt[-7 + 8*x]) - 47*Sqrt[6]*Ar
cSinh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/216
```

fricas [A] time = 1.79, size = 101, normalized size = 0.93

$$\frac{1}{18}\left(12x+\sqrt{8x-7}-4\right)\sqrt{3x+\sqrt{8x-7}}+\frac{47}{864}\sqrt{6}\log\left(-41472x^2-192(144x-47)\sqrt{8x-7}+8\left(3\sqrt{6}(14\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(12*x + sqrt(8*x - 7) - 4)*sqrt(3*x + sqrt(8*x - 7)) + 47/864*sqrt(6)*
log(-41472*x^2 - 192*(144*x - 47)*sqrt(8*x - 7) + 8*(3*sqrt(6)*(144*x + 17)
*sqrt(8*x - 7) + 4*sqrt(6)*(432*x - 299))*sqrt(3*x + sqrt(8*x - 7)) - 9792*
x + 30047)
```

giac [A] time = 0.47, size = 88, normalized size = 0.81

$$\frac{1}{216}\sqrt{2}\left(3\sqrt{2}\left(\sqrt{8x-7}\left(3\sqrt{8x-7}+2\right)+13\right)\sqrt{3x+\sqrt{8x-7}}+47\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}\sqrt{8x-7}-2\sqrt{2}\sqrt{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*sqrt(2)*(3*sqrt(2)*(sqrt(8*x - 7)*(3*sqrt(8*x - 7) + 2) + 13)*sqrt(3*
x + sqrt(8*x - 7)) + 47*sqrt(3)*log(-sqrt(3)*(sqrt(3)*sqrt(8*x - 7) - 2*sqr
t(2)*sqrt(3*x + sqrt(8*x - 7))) - 4))
```

maple [A] time = 0.01, size = 67, normalized size = 0.61

$$-\frac{47\sqrt{6}\,\operatorname{arcsinh}\left(\frac{3\sqrt{47}\left(\sqrt{8x-7}+\frac{4}{3}\right)}{47}\right)}{216}+\frac{\left(48x+16\sqrt{8x-7}\right)^{\frac{3}{2}}}{288}-\frac{\left(12\sqrt{8x-7}+16\right)\sqrt{48x+16\sqrt{8x-7}}}{288}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+(-7+8*x)^(1/2))^(1/2),x)
```

```
[Out] 1/288*(48*x+16*(-7+8*x)^(1/2))^(3/2)-1/288*(12*(-7+8*x)^(1/2)+16)*(48*x+16*
(-7+8*x)^(1/2))^(1/2)-47/216*6^(1/2)*arcsinh(3/47*47^(1/2)*((-7+8*x)^(1/2)+
4/3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x+\sqrt{8x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x + sqrt(8*x - 7)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{3x+\sqrt{8x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + (8*x - 7)^(1/2))^(1/2),x)
```

```
[Out] int((3*x + (8*x - 7)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x+\sqrt{8x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(3*x + sqrt(8*x - 7)), x)
```

3.706 $\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=47

$$2\sqrt{x+\sqrt{x+1}}-\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

```
[Out] -arctanh(1/2*(1+2*(1+x)^(1/2))/(x+(1+x)^(1/2))^(1/2))+2*(x+(1+x)^(1/2))^(1/
2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {640, 621, 206}

$$2\sqrt{x+\sqrt{x+1}}-\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
x]])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\sqrt{x+\sqrt{1+x}} - \mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\sqrt{x+\sqrt{1+x}} - 2\,\mathrm{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+2\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)$$
$$= 2\sqrt{x+\sqrt{1+x}} - \tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.00

$$2\sqrt{x+\sqrt{x+1}} - \tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

fricas [A] time = 0.81, size = 47, normalized size = 1.00

$$2\sqrt{x+\sqrt{x+1}} + \frac{1}{2}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right) - 8\,x - 8\sqrt{x+1} - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(x + 1)) + 1/2*log(4*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 5)
```

giac [A] time = 0.43, size = 33, normalized size = 0.70

$$2\sqrt{x+\sqrt{x+1}} + \log\left(-2\sqrt{x+\sqrt{x+1}} + 2\sqrt{x+1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + sqrt(x + 1)) + log(-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1)
```

maple [A] time = 0.01, size = 32, normalized size = 0.68

$$-\ln\left(\sqrt{x+1} + \frac{1}{2} + \sqrt{x+\sqrt{x+1}}\right) + 2\sqrt{x+\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x+1)^(1/2))^(1/2),x)
```

```
[Out] 2*(x+(x+1)^(1/2))^(1/2)-ln((x+1)^(1/2)+1/2+(x+(x+1)^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x + sqrt(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x + 1)^(1/2))^(1/2),x)
[Out] int(1/(x + (x + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(x + sqrt(x + 1)), x)
```

3.707 $\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx$

Optimal. Leaf size=67

$$x-2\sqrt{3}\,\sqrt{2x-3}+3\log\left(x+\sqrt{3}\,\sqrt{2x-3}+4\right)+4\sqrt{6}\,\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

```
[Out] x+3*ln(4+x+(-3+2*x)^(1/2)*3^(1/2))+4*arctan(1/12*(3+(-9+6*x)^(1/2))*6^(1/2)
)*6^(1/2)-2*(-3+2*x)^(1/2)*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1628, 634, 618, 204, 628}

$$x-2\sqrt{3}\,\sqrt{2x-3}+3\log\left(x+\sqrt{3}\,\sqrt{2x-3}+4\right)+4\sqrt{6}\,\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] x - 2*Sqrt[3]*Sqrt[-3 + 2*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqr
t[6])] + 3*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(15+x^2)}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-6+x+\frac{18(11+x)}{33+6x+x^2}\right)dx, x, \sqrt{-9+6x}\right)$$
$$= x-2\sqrt{3}\sqrt{-3+2x}+6\,\text{Subst}\left(\int \frac{11+x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)$$
$$= x-2\sqrt{3}\sqrt{-3+2x}+3\,\text{Subst}\left(\int \frac{6+2x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)+48\,\text{Subst}\left(\int \frac{}{3}\right.$$
$$= x-2\sqrt{3}\sqrt{-3+2x}+3\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)-96\,\text{Subst}\left(\int \frac{1}{-96-x^2}\,dx, x\right.$$
$$= x-2\sqrt{3}\sqrt{-3+2x}+4\sqrt{6}\tan^{-1}\left(\frac{3+\sqrt{3}\sqrt{-3+2x}}{2\sqrt{6}}\right)+3\log\left(4+x+\sqrt{3}\sqrt{-3+\right.$$

Mathematica [A] time = 0.07, size = 56, normalized size = 0.84

$$x-2\sqrt{6x-9}+3\log\left(x+\sqrt{6x-9}+4\right)+4\sqrt{6}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] x - 2*Sqrt[-9 + 6*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqrt[6])] +
 3*Log[4 + x + Sqrt[-9 + 6*x]]
```

fricas [A] time = 0.47, size = 48, normalized size = 0.72

$$4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\sqrt{6x-9}+\frac{1}{4}\sqrt{6}\right)+x-2\sqrt{6x-9}+3\log\left(x+\sqrt{6x-9}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(6*x - 9) + 1/4*sqrt(6)) + x - 2*sqrt(6*x
 - 9) + 3*log(x + sqrt(6*x - 9) + 4)
```

giac [A] time = 0.38, size = 49, normalized size = 0.73

$$4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)+x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6*x - 9) +
3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2
```

maple [A] time = 0.01, size = 52, normalized size = 0.78

$$x+4\sqrt{6}\arctan\left(\frac{\left(2\sqrt{6x-9}+6\right)\sqrt{6}}{24}\right)+3\ln\left(6x+24+6\sqrt{6x-9}\right)-2\sqrt{6x-9}-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(4+x+(-9+6*x)^(1/2)),x)
```

```
[Out] -2*(-9+6*x)^(1/2)-3/2+x+3*ln(24+6*x+6*(-9+6*x)^(1/2))+4*6^(1/2)*arctan(1/24
*(2*(-9+6*x)^(1/2)+6)*6^(1/2))
```

maxima [A] time = 2.24, size = 49, normalized size = 0.73

$$4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)+x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6*x - 9) +
3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2
```

mupad [B] time = 3.09, size = 102, normalized size = 1.52

$$x+3\ln\left(\left(6\sqrt{6x-9}+\left(-3+\sqrt{6}\,2\mathrm{i}\right)\left(2\sqrt{6x-9}+6\right)+66\right)\left(6\sqrt{6x-9}-\left(3+\sqrt{6}\,2\mathrm{i}\right)\left(2\sqrt{6x-9}+6\right)+66\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x + (6*x - 9)^(1/2) + 4),x)
```

```
[Out] x + 3*log((6*(6*x - 9)^(1/2) + (6^(1/2)*2i - 3)*(2*(6*x - 9)^(1/2) + 6) + 6
6)*(6*(6*x - 9)^(1/2) - (6^(1/2)*2i + 3)*(2*(6*x - 9)^(1/2) + 6) + 66)) + 4
*6^(1/2)*atan((6^(1/2)*(6*x - 9)^(1/2))/12 + 6^(1/2)/4) - 2*(6*x - 9)^(1/2)
```

sympy [A] time = 38.42, size = 58, normalized size = 0.87

$$x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)+4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\left(\sqrt{6x-9}+3\right)}{12}\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] x - 2*sqrt(6*x - 9) + 3*log(6*x + 6*sqrt(6*x - 9) + 24) + 4*sqrt(6)*atan(sq
rt(6)*(sqrt(6*x - 9) + 3)/12) - 3/2
```

3.708 $\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx$

Optimal. Leaf size=71

$$-x+2\sqrt{3}\sqrt{2x-3}+10\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

```
[Out] -x+10*ln(4+x+(-3+2*x)^(1/2)*3^(1/2))-21/2*arctan(1/12*(3+(-9+6*x)^(1/2))*6^
(1/2))*6^(1/2)+2*(-3+2*x)^(1/2)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1628, 634, 618, 204, 628}

$$-x+2\sqrt{3}\sqrt{2x-3}+10\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -x + 2*Sqrt[3]*Sqrt[-3 + 2*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2
*Sqrt[6])] + 10*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -\left(\frac{1}{3}\text{Subst}\left(\int \frac{x\left(-63+x^2\right)}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\right)$$
$$= -\left(\frac{1}{3}\text{Subst}\left(\int \left(-6+x+\frac{6(33-10x)}{33+6x+x^2}\right)dx, x, \sqrt{-9+6x}\right)\right)$$
$$= -x+2\sqrt{3}\sqrt{-3+2x} - 2\,\text{Subst}\left(\int \frac{33-10x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)$$
$$= -x+2\sqrt{3}\sqrt{-3+2x} + 10\,\text{Subst}\left(\int \frac{6+2x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right) - 126\,\text{Subst}\left(\int$$
$$= -x+2\sqrt{3}\sqrt{-3+2x} + 10\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right) + 252\,\text{Subst}\left(\int \frac{1}{-96-x^2}\,dx,$$
$$= -x+2\sqrt{3}\sqrt{-3+2x} - 21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{3+\sqrt{3}\sqrt{-3+2x}}{2\sqrt{6}}\right) + 10\log\left(4+x+\sqrt{3}\sqrt{-3\right.$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.85

$$-x+2\sqrt{6x-9}+10\log\left(x+\sqrt{6x-9}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -x + 2*Sqrt[-9 + 6*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqrt[6]
)] + 10*Log[4 + x + Sqrt[-9 + 6*x]]
```

fricas [A] time = 0.45, size = 59, normalized size = 0.83

$$-\frac{21}{2}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{12}\sqrt{3}\sqrt{2}\sqrt{6x-9}+\frac{1}{4}\sqrt{3}\sqrt{2}\right)-x+2\sqrt{6x-9}+10\log\left(x+\sqrt{6x-9}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -21/2*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(6*x - 9) + 1/4*sqrt(
3)*sqrt(2)) - x + 2*sqrt(6*x - 9) + 10*log(x + sqrt(6*x - 9) + 4)
```

giac [A] time = 0.33, size = 51, normalized size = 0.72

$$-\frac{21}{2}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)-x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -21/2*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) - x + 2*sqrt(6*x - 9
) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) + 3/2
```

maple [A] time = 0.00, size = 54, normalized size = 0.76

$$-x-\frac{21\sqrt{6}\arctan\left(\frac{\left(2\sqrt{6x-9}+6\right)\sqrt{6}}{24}\right)}{2}+10\ln\left(6x+24+6\sqrt{6x-9}\right)+2\sqrt{6x-9}+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12-x)/(4+x+(6*x-9)^(1/2)),x)
```

```
[Out] 2*(6*x-9)^(1/2)+3/2-x+10*ln(6*x+24+6*(6*x-9)^(1/2))-21/2*6^(1/2)*arctan(1/2
4*(2*(6*x-9)^(1/2)+6)*6^(1/2))
```

maxima [A] time = 2.08, size = 51, normalized size = 0.72

$$-\frac{21}{2}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)-x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -21/2*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) - x + 2*sqrt(6*x - 9
) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) + 3/2
```

mupad [B] time = 0.03, size = 118, normalized size = 1.66

$$2\sqrt{6x-9}+10\ln\left(\left(\left(2\sqrt{6x-9}+6\right)\left(-10+\frac{\sqrt{2}\sqrt{3}\,21\mathrm{i}}{4}\right)+20\sqrt{6x-9}-66\right)\left(\left(2\sqrt{6x-9}+6\right)\left(10+\frac{\sqrt{2}\,\sqrt{}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 12)/(x + (6*x - 9)^(1/2) + 4),x)
```

```
[Out] 10*log(((2*(6*x - 9)^(1/2) + 6)*((2^(1/2)*3^(1/2)*21i)/4 - 10) + 20*(6*x -
9)^(1/2) - 66)*((2*(6*x - 9)^(1/2) + 6)*((2^(1/2)*3^(1/2)*21i)/4 + 10) - 20
*(6*x - 9)^(1/2) + 66)) - x + 2*(6*x - 9)^(1/2) - (21*2^(1/2)*3^(1/2)*atan(
(2^(1/2)*3^(1/2))/4 + (2^(1/2)*3^(1/2)*(6*x - 9)^(1/2))/12))/2
```

sympy [A] time = 73.42, size = 60, normalized size = 0.85

$$-x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{21\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}\left(\sqrt{6x-9}+3\right)}{12}\right)}{2}+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] -x + 2*sqrt(6*x - 9) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) - 21*sqrt(6)*atan
(sqrt(6)*(sqrt(6*x - 9) + 3)/12)/2 + 3/2
```

3.709 $\int \frac{-1+x^3}{\sqrt{x}\left(1+x^2\right)} dx$

Optimal. Leaf size=52

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

```
[Out] 2/3*x^(3/2)-arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-arctan(1+2^(1/2)*x^(1/2))*2^
(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1802, 827, 1162, 617, 204}

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*ArcTan[1 + Sq
rt[2]*Sqrt[x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^3}{\sqrt{x}\left(1+x^2\right)}\,dx &= \int \left(\sqrt{x}-\frac{1+x}{\sqrt{x}\left(1+x^2\right)}\right)dx \\
&= \frac{2x^{3/2}}{3}-\int \frac{1+x}{\sqrt{x}\left(1+x^2\right)}\,dx \\
&= \frac{2x^{3/2}}{3}-2\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3}-\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3}-\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)+\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\right. \\
&= \frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.00

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*ArcTan[1 + Sq
rt[2]*Sqrt[x]]
```

fricas [A] time = 0.45, size = 23, normalized size = 0.44

$$\frac{2}{3}x^{\frac{3}{2}}-\sqrt{2}\arctan\left(\frac{\sqrt{2}(x-1)}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(x - 1)/sqrt(x))
```

giac [A] time = 0.31, size = 46, normalized size = 0.88

$$\frac{2}{3}x^{\frac{3}{2}}-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

maple [B] time = 0.01, size = 97, normalized size = 1.87

$$\frac{2x^{\frac{3}{2}}}{3}-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)-\frac{\sqrt{2}\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)}{4}-\frac{\sqrt{2}\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^2+1)/x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)-arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-arctan(-1+2^(1/2)*x^(1/2))*2^
(1/2)-1/4*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))-1/4*2^(1/
2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))
```

maxima [A] time = 1.81, size = 46, normalized size = 0.88

$$\frac{2}{3}x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

mupad [B] time = 3.17, size = 43, normalized size = 0.83

$$\frac{2x^{3/2}}{3} - \frac{\sqrt{2}\left(2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{x}}{2}+\frac{\sqrt{2}x^{3/2}}{2}\right)+2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{x}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^(1/2)*(x^2 + 1)),x)
```

```
[Out] (2*x^(3/2))/3 - (2^(1/2)*(2*atan((2^(1/2)*x^(1/2))/2 + (2^(1/2)*x^(3/2))/2)
 + 2*atan((2^(1/2)*x^(1/2))/2)))/2
```

sympy [A] time = 0.77, size = 44, normalized size = 0.85

$$\frac{2x^{\frac{3}{2}}}{3} - \sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right) - \sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**2+1)/x**(1/2),x)
```

```
[Out] 2*x**(3/2)/3 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1) - sqrt(2)*atan(sqrt(2)*sqr
t(x) + 1)
```

3.710 $\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx$

Optimal. Leaf size=20

$$-\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

```
[Out] -arcsinh(1/3*(1-2*(-1+x)^(1/2))*3^(1/2))
```

Rubi [A] time = 0.10, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {12, 619, 215}

$$-\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] -ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{1-x+x^2}}\,dx, x, \sqrt{-1+x}\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, -1+2\sqrt{-1+x}\right)}{\sqrt{3}}\\
&= -\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 0.90

$$\sinh^{-1}\left(\frac{2\sqrt{x-1}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]
```

fricas [B] time = 0.80, size = 37, normalized size = 1.85

$$\frac{1}{2}\,\log\left(4\,\sqrt{x-\sqrt{x-1}}\,\left(2\,\sqrt{x-1}-1\right)+8\,x-8\,\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) -
 3)
```

giac [A] time = 0.37, size = 25, normalized size = 1.25

$$-\log\left(2\,\sqrt{x-\sqrt{x-1}}-2\,\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

maple [A] time = 0.01, size = 14, normalized size = 0.70

$$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x)
```

```
[Out] arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\int\frac{1}{\sqrt{x-\sqrt{x-1}}\,\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int\frac{1}{2\,\sqrt{x-\sqrt{x-1}}\,\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)),x)
```

```
[Out] int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{x-1}\sqrt{x-\sqrt{x-1}}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)/2
```

3.711 $\int \frac{1+x^{7/2}}{1-x^2} \, dx$

Optimal. Leaf size=43

$$-\frac{2x^{5/2}}{5} - 2\sqrt{x} - \log\left(1-\sqrt{x}\right) + \frac{1}{2}\log(x+1) + \tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -2/5*x^(5/2)+arctan(x^(1/2))+1/2*ln(1+x)-ln(1-x^(1/2))-2*x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 31, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1833, 275, 206, 302, 212, 203}

$$-\frac{2x^{5/2}}{5} - 2\sqrt{x} + \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right) + \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(7/2))/(1 - x^2),x]
```

```
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]] + ArcTanh[x
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
```

```
] && !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^{7/2}}{1-x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x\left(1+x^7\right)}{1-x^4}\,dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{x}{1-x^4}+\frac{x^8}{1-x^4}\right)dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{1-x^4}\,dx,x,\sqrt{x}\right)+2\,\text{Subst}\left(\int \frac{x^8}{1-x^4}\,dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-1-x^4+\frac{1}{1-x^4}\right)dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,x\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tanh^{-1}(x)+2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tanh^{-1}(x)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)+\tanh^{-1}(x)
\end{aligned}$$

Mathematica [C] time = 0.03, size = 67, normalized size = 1.56

$$-\frac{2x^{5/2}}{5}-2\sqrt{x}+\left(\frac{1}{2}-\frac{i}{2}\right)\log\left(-\sqrt{x}+i\right)-\log\left(1-\sqrt{x}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\log\left(\sqrt{x}+i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(7/2))/(1 - x^2),x]
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + (1/2 - I/2)*Log[I - Sqrt[x]] - Log[1 - Sqrt[x]
] + (1/2 + I/2)*Log[I + Sqrt[x]]
```

fricas [A] time = 0.46, size = 29, normalized size = 0.67

$$-\frac{2}{5}\left(x^2+5\right)\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="fricas")
[Out] -2/5*(x^2 + 5)*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) - 1
)
```

giac [A] time = 0.34, size = 30, normalized size = 0.70

$$-\frac{2}{5}x^{\frac{5}{2}}-2\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="giac")
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(abs(sqrt(
x) - 1))
```

maple [A] time = 0.01, size = 34, normalized size = 0.79

$$-\frac{2x^{\frac{5}{2}}}{5} + \operatorname{arctanh}(x) + \arctan\left(\sqrt{x}\right) - \frac{\ln\left(\sqrt{x}-1\right)}{2} + \frac{\ln\left(\sqrt{x}+1\right)}{2} - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(7/2))/(-x^2+1),x)
```

```
[Out] -2/5*x^(5/2)-2*x^(1/2)-1/2*ln(x^(1/2)-1)+1/2*ln(x^(1/2)+1)+arctan(x^(1/2))+
arctanh(x)
```

maxima [A] time = 1.80, size = 29, normalized size = 0.67

$$-\frac{2}{5}x^{\frac{5}{2}} - 2\sqrt{x} + \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(x+1\right) - \log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) -
 1)
```

mupad [B] time = 3.11, size = 53, normalized size = 1.23

$$-\ln\left(10\sqrt{x}-10\right) - 2\sqrt{x} - \frac{2x^{5/2}}{5} + \ln\left(1+\sqrt{x}\,\left(-3-\mathrm{i}\right)-3\mathrm{i}\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \ln\left(1+\sqrt{x}\,\left(-3+1\mathrm{i}\right)+3\mathrm{i}\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(7/2) + 1)/(x^2 - 1),x)
```

```
[Out] log((1 - 3i) - x^(1/2)*(3 + 1i))*(1/2 + 1i/2) - log(10*x^(1/2) - 10) + log(
(1 + 3i) - x^(1/2)*(3 - 1i))*(1/2 - 1i/2) - 2*x^(1/2) - (2*x^(5/2))/5
```

sympy [A] time = 2.43, size = 36, normalized size = 0.84

$$-\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \log\left(\sqrt{x}-1\right) + \frac{\log\left(x+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(7/2))/(-x**2+1),x)
```

```
[Out] -2*x**(5/2)/5 - 2*sqrt(x) - log(sqrt(x) - 1) + log(x + 1)/2 + atan(sqrt(x))
```

3.712 $\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx$

Optimal. Leaf size=116

$$\frac{1}{3}(2x-1)^{3/2}-\frac{3}{8}(2x-1)^{4/3}+\frac{3}{7}(2x-1)^{7/6}+\frac{3}{5}(2x-1)^{5/6}-\frac{3}{4}(2x-1)^{2/3}+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-x-18\log\left(\sqrt[6]{\right.$$

```
[Out] -x+18*(-1+2*x)^(1/6)-9*(-1+2*x)^(1/3)-3/4*(-1+2*x)^(2/3)+3/5*(-1+2*x)^(5/6)
+3/7*(-1+2*x)^(7/6)-3/8*(-1+2*x)^(4/3)+1/3*(-1+2*x)^(3/2)-18*ln(1+(-1+2*x)^
(1/6))+6*(-1+2*x)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1620}

$$\frac{1}{3}(2x-1)^{3/2}-\frac{3}{8}(2x-1)^{4/3}+\frac{3}{7}(2x-1)^{7/6}+\frac{3}{5}(2x-1)^{5/6}-\frac{3}{4}(2x-1)^{2/3}+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-x-18\log\left(\sqrt[6]{\right.$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] -x + 18*(-1 + 2*x)^(1/6) - 9*(-1 + 2*x)^(1/3) + 6*Sqrt[-1 + 2*x] - (3*(-1 +
 2*x)^(2/3))/4 + (3*(-1 + 2*x)^(5/6))/5 + (3*(-1 + 2*x)^(7/6))/7 - (3*(-1 +
 2*x)^(4/3))/8 + (-1 + 2*x)^(3/2)/3 - 18*Log[1 + (-1 + 2*x)^(1/6)]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^3\left(5+x^6\right)}{1+x}\,dx, x, \sqrt[6]{-1+2x}\right)\\ &= 3\,\text{Subst}\left(\int \left(6-6x+6x^2-x^3+x^4-x^5+x^6-x^7+x^8-\frac{6}{1+x}\right)dx, x, \sqrt[6]{-1\right.\\ &= -x+18\sqrt[6]{-1+2x}-9\sqrt[3]{-1+2x}+6\sqrt{-1+2x}-\frac{3}{4}(-1+2x)^{2/3}+\frac{3}{5}(-1+2x)^{\frac{}{}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 127, normalized size = 1.09

$$2\left(x\left(\frac{1}{3}\sqrt{2x-1}-\frac{3}{8}\sqrt[3]{2x-1}+\frac{3}{7}\sqrt[6]{2x-1}-\frac{1}{2}\right)+\frac{3}{10}(2x-1)^{5/6}-\frac{3}{8}(2x-1)^{2/3}+\frac{17}{6}\sqrt{2x-1}-\frac{69}{16}\sqrt[3]{2x-1}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] 2*((123*(-1 + 2*x)^(1/6))/14 - (69*(-1 + 2*x)^(1/3))/16 + (17*Sqrt[-1 + 2*x
])/6 - (3*(-1 + 2*x)^(2/3))/8 + (3*(-1 + 2*x)^(5/6))/10 + x*(-1/2 + (3*(-1
+ 2*x)^(1/6))/7 - (3*(-1 + 2*x)^(1/3))/8 + Sqrt[-1 + 2*x]/3) - 9*Log[1 + (-
1 + 2*x)^(1/6)])
```

fricas [A] time = 0.46, size = 76, normalized size = 0.66

$$\frac{1}{3}\left(2x+17\right)\sqrt{2x-1}-\frac{3}{8}\left(2x+23\right)\left(2x-1\right)^{\frac{1}{3}}+\frac{3}{7}\left(2x+41\right)\left(2x-1\right)^{\frac{1}{6}}-x+\frac{3}{5}\left(2x-1\right)^{\frac{5}{6}}-\frac{3}{4}\left(2x-1\right)^{\frac{2}{3}}-18\log\left((2x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x + 17)*sqrt(2*x - 1) - 3/8*(2*x + 23)*(2*x - 1)^(1/3) + 3/7*(2*x +
41)*(2*x - 1)^(1/6) - x + 3/5*(2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) - 18*lo
g((2*x - 1)^(1/6) + 1)
```

giac [A] time = 0.39, size = 89, normalized size = 0.77

$$\frac{1}{3}\left(2x-1\right)^{\frac{3}{2}}-\frac{3}{8}\left(2x-1\right)^{\frac{4}{3}}+\frac{3}{7}\left(2x-1\right)^{\frac{7}{6}}-x+\frac{3}{5}\left(2x-1\right)^{\frac{5}{6}}-\frac{3}{4}\left(2x-1\right)^{\frac{2}{3}}+6\sqrt{2x-1}-9\left(2x-1\right)^{\frac{1}{3}}+18\left(2x-1\right)^{\frac{1}{6}}-18\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*(
2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3)
+ 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

maple [A] time = 0.00, size = 90, normalized size = 0.78

$$-x-18\ln\left(1+\left(2x-1\right)^{\frac{1}{6}}\right)+\frac{\left(2x-1\right)^{\frac{3}{2}}}{3}-\frac{3\left(2x-1\right)^{\frac{4}{3}}}{8}+\frac{3\left(2x-1\right)^{\frac{7}{6}}}{7}+\frac{1}{2}+\frac{3\left(2x-1\right)^{\frac{5}{6}}}{5}-\frac{3\left(2x-1\right)^{\frac{2}{3}}}{4}+6\sqrt{2x-1}-9\left(2x-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+2*x)/((2*x-1)^(1/3)+(2*x-1)^(1/2)),x)
```

```
[Out] 1/3*(2*x-1)^(3/2)-3/8*(2*x-1)^(4/3)+3/7*(2*x-1)^(7/6)-x+1/2+3/5*(2*x-1)^(5/
6)-3/4*(2*x-1)^(2/3)+6*(2*x-1)^(1/2)-9*(2*x-1)^(1/3)+18*(2*x-1)^(1/6)-18*ln
(1+(2*x-1)^(1/6))
```

maxima [A] time = 0.90, size = 89, normalized size = 0.77

$$\frac{1}{3}\left(2x-1\right)^{\frac{3}{2}}-\frac{3}{8}\left(2x-1\right)^{\frac{4}{3}}+\frac{3}{7}\left(2x-1\right)^{\frac{7}{6}}-x+\frac{3}{5}\left(2x-1\right)^{\frac{5}{6}}-\frac{3}{4}\left(2x-1\right)^{\frac{2}{3}}+6\sqrt{2x-1}-9\left(2x-1\right)^{\frac{1}{3}}+18\left(2x-1\right)^{\frac{1}{6}}-18\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*(
2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3)
+ 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

mupad [B] time = 0.13, size = 88, normalized size = 0.76

$$6\sqrt{2x-1}-18\ln\left(\left(2x-1\right)^{1/6}+1\right)-x-9\left(2x-1\right)^{1/3}-\frac{3\left(2x-1\right)^{2/3}}{4}+\frac{\left(2x-1\right)^{3/2}}{3}+18\left(2x-1\right)^{1/6}-\frac{3\left(2x-1\right)^{4/3}}{8}+\frac{3\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 4)/((2*x - 1)^(1/2) + (2*x - 1)^(1/3)),x)
```

```
[Out] 6*(2*x - 1)^(1/2) - 18*log((2*x - 1)^(1/6) + 1) - x - 9*(2*x - 1)^(1/3) - (
3*(2*x - 1)^(2/3))/4 + (2*x - 1)^(3/2)/3 + 18*(2*x - 1)^(1/6) - (3*(2*x - 1
)^(4/3))/8 + (3*(2*x - 1)^(5/6))/5 + (3*(2*x - 1)^(7/6))/7
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2\left(\int \frac{x}{\sqrt[3]{2x-1}+\sqrt{2x-1}}\,dx+\int \frac{2}{\sqrt[3]{2x-1}+\sqrt{2x-1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)**(1/3)+(-1+2*x)**(1/2)),x)
```

```
[Out] 2*(Integral(x/((2*x - 1)**(1/3) + sqrt(2*x - 1)), x) + Integral(2/((2*x - 1
)**(1/3) + sqrt(2*x - 1)), x))
```

3.713 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal. Leaf size=83

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Rubi **[A]** time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{\sqrt{2+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \frac{-1+x}{\sqrt{2+\sqrt{x}}}\,dx, x, 1+\sqrt{x}\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x\left(-1+x^2\right)}{\sqrt{2+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= 4\,\text{Subst}\left(\int \left(-\frac{6}{\sqrt{2+x}} + 11\sqrt{2+x} - 6(2+x)^{3/2} + (2+x)^{5/2}\right)dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= -48\sqrt{2+\sqrt{1+\sqrt{x}}} + \frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2} - \frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2} + \frac{8}{7}\left(2+\sqrt{1\right.$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.70

$$\frac{8}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\left(3\sqrt{x}\left(5\sqrt{\sqrt{x}+1}-12\right)+76\sqrt{\sqrt{x}+1}-280\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sq
rt[1 + Sqrt[x]])*Sqrt[x]))/105
```

fricas [A] time = 0.46, size = 35, normalized size = 0.42

$$\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sq
rt(x) + 1) + 2)
```

giac [A] time = 3.46, size = 82, normalized size = 0.99

$$\frac{8\left(15\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-126\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+385\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-630\sqrt{\sqrt{\sqrt{x}+1}+2}\right)}{105\,\text{sgn}\left(4\left(\sqrt{x}+1\right)^2-8\sqrt{x}-7\right)\text{sgn}\left(4x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(15*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 126*(sqrt(sqrt(x) + 1) + 2)^(5/2)
 + 385*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 630*sqrt(sqrt(sqrt(x) + 1) + 2))/(sg
n(4*(sqrt(x) + 1)^2 - 8*sqrt(x) - 7)*sgn(4*x - 3))
```

maple [A] time = 0.01, size = 54, normalized size = 0.65

$$\frac{88\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+(x^(1/2)+1)^(1/2))^(1/2),x)
```

```
[Out] 88/3*(2+(x^(1/2)+1)^(1/2))^(3/2)-48/5*(2+(x^(1/2)+1)^(1/2))^(5/2)+8/7*(2+(x
^(1/2)+1)^(1/2))^(7/2)-48*(2+(x^(1/2)+1)^(1/2))^(1/2)
```

maxima [A] time = 0.90, size = 53, normalized size = 0.64

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 88
/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)
```

```
[Out] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)
```

3.714 $\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$

Optimal. Leaf size=64

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{9/2}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{7/2}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}$$

```
[Out] 64/5*(2+(4+x^(1/2))^(1/2))^(5/2)-48/7*(2+(4+x^(1/2))^(1/2))^(7/2)+8/9*(2+(4
+x^(1/2))^(1/2))^(9/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{9/2}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{7/2}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

```
[Out] (64*(2 + Sqrt[4 + Sqrt[x]])^(5/2))/5 - (48*(2 + Sqrt[4 + Sqrt[x]])^(7/2))/7
 + (8*(2 + Sqrt[4 + Sqrt[x]])^(9/2))/9
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = 2\,\text{Subst}\left(\int x\sqrt{2+\sqrt{4+x}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \sqrt{2+\sqrt{x}}\,(-4+x)\,dx, x, 4+\sqrt{x}\right)$$
$$= 4\,\text{Subst}\left(\int x\sqrt{2+x}\,\left(-4+x^2\right)dx, x, \sqrt{4+\sqrt{x}}\right)$$
$$= 4\,\text{Subst}\left(\int \left(8(2+x)^{3/2}-6(2+x)^{5/2}+(2+x)^{7/2}\right)dx, x, \sqrt{4+\sqrt{x}}\right)$$
$$= \frac{64}{5}\left(2+\sqrt{4+\sqrt{x}}\right)^{5/2} - \frac{48}{7}\left(2+\sqrt{4+\sqrt{x}}\right)^{7/2} + \frac{8}{9}\left(2+\sqrt{4+\sqrt{x}}\right)^{9/2}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.67

$$-\frac{8}{315}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}\left(130\sqrt{\sqrt{x}+4}-35\sqrt{x}-244\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

```
[Out] (-8*(2 + Sqrt[4 + Sqrt[x]])^(5/2)*(-244 + 130*Sqrt[4 + Sqrt[x]] - 35*Sqrt[x
]))/315
```

fricas [A] time = 0.46, size = 39, normalized size = 0.61

$$\frac{8}{315}\left(2\left(5\sqrt{x}-32\right)\sqrt{\sqrt{x}+4}+35\,x+4\sqrt{x}-128\right)\sqrt{\sqrt{\sqrt{x}+4}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/315*(2*(5*sqrt(x) - 32)*sqrt(sqrt(x) + 4) + 35*x + 4*sqrt(x) - 128)*sqrt(
sqrt(sqrt(x) + 4) + 2)
```

giac [B] time = 7.65, size = 268, normalized size = 4.19

$$\frac{8}{315}\left(\left(35\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}}-360\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}}+1512\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}-3360\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{3}{2}}+5040\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/315*((35*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 360*(sqrt(sqrt(x) + 4) + 2)^(7/2
) + 1512*(sqrt(sqrt(x) + 4) + 2)^(5/2) - 3360*(sqrt(sqrt(x) + 4) + 2)^(3/2)
 + 5040*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 7
9) + 18*(5*(sqrt(sqrt(x) + 4) + 2)^(7/2) - 42*(sqrt(sqrt(x) + 4) + 2)^(5/2)
 + 140*(sqrt(sqrt(x) + 4) + 2)^(3/2) - 280*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn
(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 84*(3*(sqrt(sqrt(x) + 4) + 2)^(5/2)
 - 20*(sqrt(sqrt(x) + 4) + 2)^(3/2) + 60*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4
*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 840*((sqrt(sqrt(x) + 4) + 2)^(3/2) -
6*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79))*sg
n(4*x - 15)
```

maple [A] time = 0.01, size = 41, normalized size = 0.64

$$\frac{64\left(2+\sqrt{\sqrt{x}+4}\right)^{\frac{5}{2}}}{5}-\frac{48\left(2+\sqrt{\sqrt{x}+4}\right)^{\frac{7}{2}}}{7}+\frac{8\left(2+\sqrt{\sqrt{x}+4}\right)^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(x^(1/2)+4)^(1/2))^(1/2),x)
```

```
[Out] 64/5*(2+(x^(1/2)+4)^(1/2))^(5/2)-48/7*(2+(x^(1/2)+4)^(1/2))^(7/2)+8/9*(2+(x
^(1/2)+4)^(1/2))^(9/2)
```

maxima [A] time = 0.88, size = 40, normalized size = 0.62

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 48/7*(sqrt(sqrt(x) + 4) + 2)^(7/2) + 64
/5*(sqrt(sqrt(x) + 4) + 2)^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{\sqrt{x}+4}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) + 4)^(1/2) + 2)^(1/2),x)
```

```
[Out] int(((x^(1/2) + 4)^(1/2) + 2)^(1/2), x)
```

sympy [B] time = 2.51, size = 216, normalized size = 3.38

$$-\frac{2\sqrt{2}\sqrt{x}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{63\pi}-\frac{4\sqrt{2}\sqrt{x}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}-\frac{\sqrt{2}x\sqrt{\sqrt{\sqrt{x}+4}}}{9\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] -2*sqrt(2)*sqrt(x)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/4
)*gamma(1/4)/(63*pi) - 4*sqrt(2)*sqrt(x)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(
-1/4)*gamma(1/4)/(315*pi) - sqrt(2)*x*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/
4)*gamma(1/4)/(9*pi) + 64*sqrt(2)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4)
+ 2)*gamma(-1/4)*gamma(1/4)/(315*pi) + 128*sqrt(2)*sqrt(sqrt(sqrt(x) + 4) +
 2)*gamma(-1/4)*gamma(1/4)/(315*pi)
```

3.715 $\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx$

Optimal. Leaf size=82

$$\frac{8}{45}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{9/2}-\frac{48}{35}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{7/2}+\frac{64}{25}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}$$

```
[Out] 64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2))
^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {371, 1398, 772}

$$\frac{8}{45}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{9/2}-\frac{48}{35}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{7/2}+\frac{64}{25}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (64*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2))/25 - (48*(2 - Sqrt[4 + Sqrt[-9 +
5*x]])^(7/2))/35 + (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(9/2))/45
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx &= \frac{2}{5}\operatorname{Subst}\left(\int x\sqrt{2-\sqrt{4+x}}\,dx, x, \sqrt{-9+5x}\right)\\ &= \frac{2}{5}\operatorname{Subst}\left(\int \sqrt{2-\sqrt{x}}\,(-4+x)\,dx, x, 4+\sqrt{-9+5x}\right)\\ &= \frac{4}{5}\operatorname{Subst}\left(\int \sqrt{2-x}\,x\left(-4+x^2\right)\,dx, x, \sqrt{4+\sqrt{-9+5x}}\right)\\ &= \frac{4}{5}\operatorname{Subst}\left(\int \left(-8(2-x)^{3/2}+6(2-x)^{5/2}-(2-x)^{7/2}\right)dx, x, \sqrt{4+\sqrt{-9+5x}}\right)\\ &= \frac{64}{25}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{5/2}-\frac{48}{35}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{7/2}+\frac{8}{45}\left(2-\sqrt{4+\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.70

$$\frac{8\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}\left(35\sqrt{5x-9}+130\sqrt{\sqrt{5x-9}+4}+244\right)}{1575}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2)*(244 + 35*Sqrt[-9 + 5*x] + 130*Sqrt
[4 + Sqrt[-9 + 5*x]]))/1575
```

fricas [A] time = 0.47, size = 57, normalized size = 0.70

$$-\frac{8}{1575}\left(2\left(5\sqrt{5x-9}-32\right)\sqrt{\sqrt{5x-9}+4}-175x-4\sqrt{5x-9}+443\right)\sqrt{-\sqrt{\sqrt{5x-9}+4}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/1575*(2*(5*sqrt(5*x - 9) - 32)*sqrt(sqrt(5*x - 9) + 4) - 175*x - 4*sqrt(
5*x - 9) + 443)*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2)
```

giac [B] time = 7.17, size = 474, normalized size = 5.78

$$-\frac{8}{1575}\left(\left(\left(35\left(\sqrt{\sqrt{5x-9}+4}-2\right)^4\sqrt{-\sqrt{\sqrt{5x-9}+4}+2}+360\left(\sqrt{\sqrt{5x-9}+4}-2\right)^3\sqrt{-\sqrt{\sqrt{5x-9}+4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -8/1575*((35*(sqrt(sqrt(5*x - 9) + 4) - 2)^4*sqrt(-sqrt(sqrt(5*x - 9) + 4)
+ 2) + 360*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) +
2) + 1512*(sqrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2
) - 3360*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 5040*sqrt(-sqrt(sqrt(5*x -
9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 18*(5
*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) + 42*(s
qrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 140*(-sq
rt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 280*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*
sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 84*(3*(sqrt(sqrt(5*
```

```
x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 20*(-sqrt(sqrt(5*x
- 9) + 4) + 2)^(3/2) + 60*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(
5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 840*((-sqrt(sqrt(5*x - 9) + 4) +
 2)^(3/2) - 6*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4
)^2 + 32*sqrt(5*x - 9) + 79))*sgn(20*x - 51)
```

maple [A] time = 0.01, size = 59, normalized size = 0.72

$$\frac{64\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{5}{2}}}{25}-\frac{48\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{7}{2}}}{35}+\frac{8\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{9}{2}}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2))
^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

maxima [A] time = 0.87, size = 58, normalized size = 0.71

$$\frac{8}{45}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{9}{2}}-\frac{48}{35}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{7}{2}}+\frac{64}{25}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/45*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(9/2) - 48/35*(-sqrt(sqrt(5*x - 9) + 4)
 + 2)^(7/2) + 64/25*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2-\sqrt{\sqrt{5\,x-9}+4}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2),x)
```

```
[Out] int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2-\sqrt{\sqrt{5x-9}+4}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(2 - sqrt(sqrt(5*x - 9) + 4)), x)
```

3.716 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal. Leaf size=83

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {371, 1398, 772}

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{\sqrt{2+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \frac{-1+x}{\sqrt{2+\sqrt{x}}}\,dx, x, 1+\sqrt{x}\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x(-1+x^2)}{\sqrt{2+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= 4\,\text{Subst}\left(\int \left(-\frac{6}{\sqrt{2+x}} + 11\sqrt{2+x} - 6(2+x)^{3/2} + (2+x)^{5/2}\right)dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= -48\sqrt{2+\sqrt{1+\sqrt{x}}} + \frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2} - \frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2} + \frac{8}{7}\left(2+\sqrt{1+\right.$$

Mathematica [A] time = 0.02, size = 58, normalized size = 0.70

$$\frac{8}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\left(3\sqrt{x}\left(5\sqrt{\sqrt{x}+1}-12\right)+76\sqrt{\sqrt{x}+1}-280\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sq
rt[1 + Sqrt[x]])*Sqrt[x]))/105
```

fricas [A] time = 0.45, size = 35, normalized size = 0.42

$$\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sq
rt(x) + 1) + 2)
```

giac [A] time = 3.18, size = 82, normalized size = 0.99

$$\frac{8\left(15\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-126\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+385\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-630\sqrt{\sqrt{\sqrt{x}+1}+2}\right)}{105\,\text{sgn}\left(4\left(\sqrt{x}+1\right)^2-8\sqrt{x}-7\right)\text{sgn}\left(4x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(15*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 126*(sqrt(sqrt(x) + 1) + 2)^(5/2)
 + 385*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 630*sqrt(sqrt(sqrt(x) + 1) + 2))/(sg
n(4*(sqrt(x) + 1)^2 - 8*sqrt(x) - 7)*sgn(4*x - 3))
```

maple [A] time = 0.01, size = 54, normalized size = 0.65

$$\frac{88\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{\sqrt{x}+1}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+(x^(1/2)+1)^(1/2))^(1/2),x)
```

```
[Out] 88/3*(2+(x^(1/2)+1)^(1/2))^(3/2)-48/5*(2+(x^(1/2)+1)^(1/2))^(5/2)+8/7*(2+(x
^(1/2)+1)^(1/2))^(7/2)-48*(2+(x^(1/2)+1)^(1/2))^(1/2)
```

maxima [A] time = 0.88, size = 53, normalized size = 0.64

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 88
/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)
```

```
[Out] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)
```

3.717 $\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal. Leaf size=190

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}\right.$$

```
[Out] -32/5*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(1+x^(1/2))^(1/2))^(
1/2))^(7/2)+112/9*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(1+x^(
1/2))^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(13/2)-11
2/15*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(1+x^(1/2))^(1/2))^
(1/2))^(17/2)
```

Rubi [A] time = 0.37, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1618, 1620}

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] (-32*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2))/5 + (48*(1 + Sqrt[1 + Sqrt[1
+ Sqrt[x]]])^(7/2))/7 + (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(9/2))/9 - (
320*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(11/2))/11 + (288*(1 + Sqrt[1 + Sqrt[
1 + Sqrt[x]]])^(13/2))/13 - (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(15/2))/
15 + (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(17/2))/17
```

Rule 1618

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
 /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder
[Px, a + b*x, x], 0]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = 2\,\text{Subst}\left(\int x\sqrt{1+\sqrt{1+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)$$
$$= 4\,\text{Subst}\left(\int x\left(-1+x^2\right)\sqrt{1+\sqrt{1+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= 8\,\text{Subst}\left(\int x^3\sqrt{1+x}\left(-2+x^2\right)\left(-1+x^2\right)dx, x, \sqrt{1+\sqrt{1+\sqrt{x}}}\right)$$
$$= 8\,\text{Subst}\left(\int x^3(1+x)^{3/2}\left(2-2x-x^2+x^3\right)dx, x, \sqrt{1+\sqrt{1+\sqrt{x}}}\right)$$
$$= 8\,\text{Subst}\left(\int \left(-2(1+x)^{3/2}+3(1+x)^{5/2}+7(1+x)^{7/2}-20(1+x)^{9/2}+18(1+x\right.\right.$$
$$= -\frac{32}{5}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{5/2}+\frac{48}{7}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{7/2}+\frac{112}{9}\left(1+\sqrt{1\right.$$

Mathematica [A] time = 0.10, size = 135, normalized size = 0.71

$$\frac{16\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{5/2}\left(231\sqrt{x}\left(-377\sqrt{\sqrt{\sqrt{x}+1}+1}+195\sqrt{\sqrt{x}+1}+365\right)+8\left(252\sqrt{\sqrt{x}+1}\sqrt{\sqrt{\right.\right.}}{765765}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2)*(8*(-8221 + 8642*Sqrt[1 + Sqrt[
1 + Sqrt[x]]] - 4865*Sqrt[1 + Sqrt[x]] + 252*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sq
rt[1 + Sqrt[x]]) + 231*(365 - 377*Sqrt[1 + Sqrt[1 + Sqrt[x]]] + 195*Sqrt[1
+ Sqrt[x]])*Sqrt[x]))/765765
```

fricas [A] time = 0.47, size = 76, normalized size = 0.40

$$\frac{16}{765765}\left(\left(231\sqrt{x}-1304\right)\sqrt{\sqrt{x}+1}+\left(\left(3003\sqrt{x}-4672\right)\sqrt{\sqrt{x}+1}-3528\sqrt{x}+8752\right)\sqrt{\sqrt{\sqrt{x}+1}+1}+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 16/765765*((231*sqrt(x) - 1304)*sqrt(sqrt(x) + 1) + ((3003*sqrt(x) - 4672)*
sqrt(sqrt(x) + 1) - 3528*sqrt(x) + 8752)*sqrt(sqrt(sqrt(x) + 1) + 1) + 4504
5*x + 4613*sqrt(x) - 28152)*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 121, normalized size = 0.64

$$-\frac{32\left(1+\sqrt{1+\sqrt{\sqrt{x}+1}}\right)^{\frac{5}{2}}}{5}+\frac{48\left(1+\sqrt{1+\sqrt{\sqrt{x}+1}}\right)^{\frac{7}{2}}}{7}+\frac{112\left(1+\sqrt{1+\sqrt{\sqrt{x}+1}}\right)^{\frac{9}{2}}}{9}-\frac{320\left(1+\sqrt{1+\sqrt{\sqrt{x}+\right.}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1+(x^(1/2)+1)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] -32/5*(1+(1+(x^(1/2)+1)^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(x^(1/2)+1)^(1/2))^(
1/2))^(7/2)+112/9*(1+(1+(x^(1/2)+1)^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(x^(1/
2)+1)^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(x^(1/2)+1)^(1/2))^(1/2))^(13/2)-11
2/15*(1+(1+(x^(1/2)+1)^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(x^(1/2)+1)^(1/2))^
(1/2))^(17/2)
```

maxima [A] time = 0.92, size = 120, normalized size = 0.63

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 16/17*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 112/15*(sqrt(sqrt(sqrt(x)
+ 1) + 1) + 1)^(15/2) + 288/13*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(13/2) - 3
20/11*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 112/9*(sqrt(sqrt(sqrt(x) +
 1) + 1) + 1)^(9/2) + 48/7*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(7/2) - 32/5*(
sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1), x)
```

3.718 $\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$

Optimal. Leaf size=233

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-\frac{760}{11}\left(\sqrt{\sqrt{2}\right.$$

```
[Out] -16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^(
1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(2+
(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))^(1
/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(3+(-
1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(
17/2)
```

Rubi [A] time = 0.38, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1620}

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-\frac{760}{11}\left(\sqrt{\sqrt{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] (-16*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2))/3 + (136*(2 + Sqrt[3 + Sqr
t[-1 + 2*Sqrt[x]]])^(5/2))/5 - (480*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(7
/2))/7 + (304*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(9/2))/3 - (760*(2 + Sqr
t[3 + Sqrt[-1 + 2*Sqrt[x]]])^(11/2))/11 + (300*(2 + Sqrt[3 + Sqrt[-1 + 2*Sq
rt[x]]])^(13/2))/13 - (56*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(15/2))/15 +
 (4*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(17/2))/17
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = 2\,\text{Subst}\left(\int x\sqrt{2+\sqrt{3+\sqrt{-1+2x}}}\,dx, x, \sqrt{x}\right)$$
$$= \text{Subst}\left(\int x\left(1+x^2\right)\sqrt{2+\sqrt{3+x}}\,dx, x, \sqrt{-1+2\sqrt{x}}\right)$$
$$= 2\,\text{Subst}\left(\int x\sqrt{2+x}\left(-3+x^2\right)\left(1+\left(-3+x^2\right)^2\right)dx, x, \sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)$$
$$= 2\,\text{Subst}\left(\int \left(-4\sqrt{2+x}+34(2+x)^{3/2}-120(2+x)^{5/2}+228(2+x)^{7/2}-190(2$$
$$= -\frac{16}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{3/2}+\frac{136}{5}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{5/2}-\frac{480}{7}\left(2\right.$$

Mathematica [A] time = 0.13, size = 183, normalized size = 0.79

$$\frac{8\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{3/2}\left(7\sqrt{x}\left(2145\sqrt{2\sqrt{x}-1}\sqrt{\sqrt{2\sqrt{x}-1}+3}+1452\sqrt{\sqrt{2\sqrt{x}-1}+3}-4004\sqrt{2\sqrt{x}-\right.\right.}{255255}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] (8*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2)*(4*(-9786 - 2535*Sqrt[3 + Sqr
t[-1 + 2*Sqrt[x]]] - 4286*Sqrt[-1 + 2*Sqrt[x]] + 3843*Sqrt[3 + Sqrt[-1 + 2*
Sqrt[x]]]*Sqrt[-1 + 2*Sqrt[x]]) + 7*(-3576 + 1452*Sqrt[3 + Sqrt[-1 + 2*Sqrt
[x]]] - 4004*Sqrt[-1 + 2*Sqrt[x]] + 2145*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]*Sqr
t[-1 + 2*Sqrt[x]])*Sqrt[x]))/255255
```

fricas [A] time = 0.46, size = 85, normalized size = 0.36

$$-\frac{8}{255255}\left(\left(847\sqrt{x}-1688\right)\sqrt{2\sqrt{x}-1}-2\left(\left(1001\sqrt{x}+6800\right)\sqrt{2\sqrt{x}-1}-2352\sqrt{x}-29712\right)\sqrt{\sqrt{2\sqrt{x}-1}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/255255*((847*sqrt(x) - 1688)*sqrt(2*sqrt(x) - 1) - 2*((1001*sqrt(x) + 68
00)*sqrt(2*sqrt(x) - 1) - 2352*sqrt(x) - 29712)*sqrt(sqrt(2*sqrt(x) - 1) +
3) - 30030*x + 3843*sqrt(x) + 124080)*sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) +
2)
```

giac [A] time = 51.25, size = 271, normalized size = 1.16

$$\frac{4}{255255}\left(15015\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}-238238\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}+1472625\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/255255*(15015*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 238238*(sqrt(s
qrt(2*sqrt(x) - 1) + 3) + 2)^(15/2) + 1472625*(sqrt(sqrt(2*sqrt(x) - 1) + 3
) + 2)^(13/2) - 4408950*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 646646
0*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(9/2) - 4375800*(sqrt(sqrt(2*sqrt(x)
- 1) + 3) + 2)^(7/2) + 1735734*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/2) -
340340*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(3/2))*sgn(131072*x^23 + 6029312
*x^22 + 131596288*x^21 + 1823539200*x^20 + 18092523520*x^19 + 137313009664*
x^18 + 830934196224*x^17 + 4121209913344*x^16 + 17059018985472*x^15 + 59571
270234112*x^14 + 176317166240256*x^13 + 442104199109632*x^12 + 934792487842
816*x^11 + 1653389259996160*x^10 + 2419262240692992*x^9 + 2886578907966976*
x^8 + 2756595188687360*x^7 + 2055315711024768*x^6 + 1156127428771360*x^5 +
466803251648192*x^4 + 125285938081152*x^3 + 19649836876032*x^2 + 1399854182
400*x + 14929920000)
```

maple [A] time = 0.02, size = 154, normalized size = 0.66

$$-\frac{16\left(2+\sqrt{3+\sqrt{2\sqrt{x}-1}}\right)^{\frac{3}{2}}}{3}+\frac{136\left(2+\sqrt{3+\sqrt{2\sqrt{x}-1}}\right)^{\frac{5}{2}}}{5}-\frac{480\left(2+\sqrt{3+\sqrt{2\sqrt{x}-1}}\right)^{\frac{7}{2}}}{7}+\frac{304\left(2+\sqrt{3-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x)
```

```
[Out] -16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^(
1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(2+
(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))^(1
/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(3+(-
1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(
17/2)
```

maxima [A] time = 0.93, size = 153, normalized size = 0.66

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/17*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 56/15*(sqrt(sqrt(2*sqrt(x
) - 1) + 3) + 2)^(15/2) + 300/13*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(13/2)
 - 760/11*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 304/3*(sqrt(sqrt(2*s
qrt(x) - 1) + 3) + 2)^(9/2) - 480/7*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(7/
2) + 136/5*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/2) - 16/3*(sqrt(sqrt(2*sq
rt(x) - 1) + 3) + 2)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2),x)
```

```
[Out] int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x**(1/2))**(1/2))**(1/2))**(1/2),x)
[Out] Integral(sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2), x)
```

3.719 $\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}\, x\, dx$

Optimal. Leaf size=160

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)$$

```
[Out] 16/5*(1+(1+(-1+x)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(-1+x)^(1/2))^(1/2))^(7/2)
+8*(1+(1+(-1+x)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(-1+x)^(1/2))^(1/2))^(11/2
)+144/13*(1+(1+(-1+x)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(-1+x)^(1/2))^(1/2))
^(15/2)+8/17*(1+(1+(-1+x)^(1/2))^(1/2))^(17/2)
```

Rubi [A] time = 0.28, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1618, 1620}

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (16*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2))/5 - (24*(1 + Sqrt[1 + Sqrt[-1 + x]]
)^(7/2))/7 + 8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(9/2) - (160*(1 + Sqrt[1 + Sqrt
[-1 + x]])^(11/2))/11 + (144*(1 + Sqrt[1 + Sqrt[-1 + x]])^(13/2))/13 - (56*
(1 + Sqrt[1 + Sqrt[-1 + x]])^(15/2))/15 + (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(
17/2))/17
```

Rule 1618

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
 /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder
[Px, a + b*x, x], 0]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}\, x\, dx &= 2\,\text{Subst}\left(\int x\left(1+x^2\right)\sqrt{1+\sqrt{1+x}}\, dx, x, \sqrt{-1+x}\right)\\
&= 4\,\text{Subst}\left(\int x\sqrt{1+x}\left(-1+x^2\right)\left(1+\left(-1+x^2\right)^2\right) dx, x, \sqrt{1+\sqrt{-1+x}}\right)\\
&= 4\,\text{Subst}\left(\int x(1+x)^{3/2}\left(-2+2x+2x^2-2x^3-x^4+x^5\right) dx, x, \sqrt{1+\sqrt{-1+x}}\right.\\
&= 4\,\text{Subst}\left(\int \left(2(1+x)^{3/2}-3(1+x)^{5/2}+9(1+x)^{7/2}-20(1+x)^{9/2}+18(1+x)^{1}\right.\right.\\
&= \frac{16}{5}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{5/2}-\frac{24}{7}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{7/2}+8\left(1+\sqrt{1+\sqrt{-1}}\right.
\end{aligned}$$

Mathematica [A] time = 0.09, size = 103, normalized size = 0.64

$$\frac{8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{5/2}\left(8\left(84\sqrt{x-1}\sqrt{\sqrt{x-1}+1}-3030\sqrt{\sqrt{x-1}+1}+1715\sqrt{x-1}+2591\right)+77\left(-377\sqrt{\sqrt{}}\right.\right.}{255255}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2)*(8*(2591 - 3030*Sqrt[1 + Sqrt[-1 + x]
] + 1715*Sqrt[-1 + x] + 84*Sqrt[1 + Sqrt[-1 + x]]*Sqrt[-1 + x]) + 77*(365 -
 377*Sqrt[1 + Sqrt[-1 + x]] + 195*Sqrt[-1 + x])*x))/255255
```

fricas [A] time = 0.47, size = 62, normalized size = 0.39

$$\frac{8}{255255}\left(15015\,x^2+(77\,x+1032)\sqrt{x-1}+\left((1001\,x+4544)\sqrt{x-1}-1176\,x-7696\right)\sqrt{\sqrt{x-1}+1}-1799\,x-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/255255*(15015*x^2 + (77*x + 1032)*sqrt(x - 1) + ((1001*x + 4544)*sqrt(x -
 1) - 1176*x - 7696)*sqrt(sqrt(x - 1) + 1) - 1799*x - 22088)*sqrt(sqrt(sqrt
(x - 1) + 1) + 1)
```

giac [B] time = 15.54, size = 859, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/765765*(7*(6435*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 58344*(sqrt(sqrt(x -
 1) + 1) + 1)^(15/2) + 235620*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 556920*(
sqrt(sqrt(x - 1) + 1) + 1)^(11/2) + 850850*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2
) - 875160*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 612612*(sqrt(sqrt(x - 1) + 1
) + 1)^(5/2) - 291720*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 109395*sqrt(sqrt(
sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 119
*(429*(sqrt(sqrt(x - 1) + 1) + 1)^(15/2) - 3465*(sqrt(sqrt(x - 1) + 1) + 1)
^(13/2) + 12285*(sqrt(sqrt(x - 1) + 1) + 1)^(11/2) - 25025*(sqrt(sqrt(x - 1
) + 1) + 1)^(9/2) + 32175*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 27027*(sqrt(s
qrt(x - 1) + 1) + 1)^(5/2) + 15015*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 6435
*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1)
 - 7) - 765*(231*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 1638*(sqrt(sqrt(x - 1
) + 1) + 1)^(11/2) + 5005*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) - 8580*(sqrt(sq
rt(x - 1) + 1) + 1)^(7/2) + 9009*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) - 6006*(
sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 3003*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sg
n(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 3315*(63*(sqrt(sqrt(x - 1) +
 1) + 1)^(11/2) - 385*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) + 990*(sqrt(sqrt(x
- 1) + 1) + 1)^(7/2) - 1386*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) + 1155*(sqrt(
sqrt(x - 1) + 1) + 1)^(3/2) - 693*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(s
qrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 9724*(35*(sqrt(sqrt(x - 1) + 1) +
1)^(9/2) - 180*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 378*(sqrt(sqrt(x - 1) +
1) + 1)^(5/2) - 420*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 315*sqrt(sqrt(sqrt(
x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 87516*(5
*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 21*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) +
 35*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 35*sqrt(sqrt(sqrt(x - 1) + 1) + 1))
*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 102102*(3*(sqrt(sqrt(x -
```

```
1) + 1) + 1)^(5/2) - 10*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 15*sqrt(sqrt(sq
rt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 51051
0*((sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 3*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*s
gn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7))*sgn(4*x - 7)
```

maple [A] time = 0.01, size = 107, normalized size = 0.67

$$\frac{16\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{5}{2}}}{5}-\frac{24\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{7}{2}}}{7}+8\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{9}{2}}-\frac{160\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{11}{2}}}{11}+\frac{144\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(1+(x-1)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 16/5*(1+(1+(x-1)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(x-1)^(1/2))^(1/2))^(7/2)+8
*(1+(1+(x-1)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(x-1)^(1/2))^(1/2))^(11/2)+14
4/13*(1+(1+(x-1)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(x-1)^(1/2))^(1/2))^(15/2
)+8/17*(1+(1+(x-1)^(1/2))^(1/2))^(17/2)
```

maxima [A] time = 0.89, size = 106, normalized size = 0.66

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/17*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 56/15*(sqrt(sqrt(x - 1) + 1) + 1)
^(15/2) + 144/13*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 160/11*(sqrt(sqrt(x -
 1) + 1) + 1)^(11/2) + 8*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) - 24/7*(sqrt(sqr
t(x - 1) + 1) + 1)^(7/2) + 16/5*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{\sqrt{\sqrt{x-1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sqrt{\sqrt{x-1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(sqrt(sqrt(x - 1) + 1) + 1), x)
```

3.720 $\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx$

Optimal. Leaf size=20

$$-2\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

[Out] -2*arcsinh(1/3*(1-2*(-1+x)^(1/2))*3^(1/2))

Rubi [A] time = 0.08, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{619, 215\}$

$$-2\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] -2*ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x+x^2}}\,dx, x, \sqrt{-1+x}\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, -1+2\sqrt{-1+x}\right)}{\sqrt{3}}\\
&= -2\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$2\sinh^{-1}\left(\frac{2\sqrt{x-1}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] 2*ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]

fricas [B] time = 0.80, size = 35, normalized size = 1.75

$$\log\left(4\sqrt{x-\sqrt{x-1}}\left(2\sqrt{x-1}-1\right)+8x-8\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) - 3)
```

giac [A] time = 0.32, size = 25, normalized size = 1.25

$$-2\log\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] -2*log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

maple [A] time = 0.00, size = 16, normalized size = 0.80

$$2\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x)
[Out] 2*arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-\sqrt{x-1}}\,\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{x-\sqrt{x-1}}\,\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)),x)
[Out] int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1}\,\sqrt{x-\sqrt{x-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)
```

3.721 $\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx$

Optimal. Leaf size=44

$$2\sqrt{x+\sqrt{2x-1}+1}-\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

```
[Out] -arcsinh(1/2*(1+(-1+2*x)^(1/2))*2^(1/2))*2^(1/2)+2*(1+x+(-1+2*x)^(1/2))^(1/
2)
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 619, 215}

$$\sqrt{2}\sqrt{2x+2\sqrt{2x-1}+2}-\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

```
[Out] Sqrt[2]*Sqrt[2 + 2*x + 2*Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2
*x])/Sqrt[2]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{\frac{3}{2}+x+\frac{x^2}{2}}}\,dx,x,\sqrt{-1+2x}\right)\\ &= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}}-\text{Subst}\left(\int \frac{1}{\sqrt{\frac{3}{2}+x+\frac{x^2}{2}}}\,dx,x,\sqrt{-1+2x}\right)\\ &= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}}-\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}}\,dx,x,1+\sqrt{-1+2x}\right)\\ &= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}}-\sqrt{2}\sinh^{-1}\left(\frac{1+\sqrt{-1+2x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$2\sqrt{x+\sqrt{2x-1}+1}-\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

```
[Out] 2*Sqrt[1 + x + Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2*x])/Sqrt[
2]]
```

fricas [B] time = 1.08, size = 85, normalized size = 1.93

$$\frac{1}{4}\sqrt{2}\log\left(-8x^2-8(2x+1)\sqrt{2x-1}+2\left(\sqrt{2}(2x+3)\sqrt{2x-1}+\sqrt{2}(6x-1)\right)\sqrt{x+\sqrt{2x-1}+1}-24x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-8*x^2 - 8*(2*x + 1)*sqrt(2*x - 1) + 2*(sqrt(2)*(2*x + 3)*s
qrt(2*x - 1) + sqrt(2)*(6*x - 1))*sqrt(x + sqrt(2*x - 1) + 1) - 24*x + 7) +
 2*sqrt(x + sqrt(2*x - 1) + 1)
```

giac [A] time = 0.30, size = 49, normalized size = 1.11

$$\sqrt{2}\left(\sqrt{2x+2\sqrt{2x-1}+2}+\log\left(\sqrt{2x+2\sqrt{2x-1}+2}-\sqrt{2x-1}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(2*x + 2*sqrt(2*x - 1) + 2) + log(sqrt(2*x + 2*sqrt(2*x - 1) +
 2) - sqrt(2*x - 1) - 1))
```

maple [A] time = 0.01, size = 38, normalized size = 0.86

$$-\sqrt{2}\ \operatorname{arcsinh}\left(\frac{\left(1+\sqrt{2x-1}\right)\sqrt{2}}{2}\right)+\sqrt{4x+4+4\sqrt{2x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x+(2*x-1)^(1/2))^(1/2),x)
```

```
[Out] (4*x+4+4*(2*x-1)^(1/2))^(1/2)-arcsinh(1/2*(1+(2*x-1)^(1/2))*2^(1/2))*2^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x+\sqrt{2x-1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x+\sqrt{2\,x-1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{2x-1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

3.722 $\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)} dx$

Optimal. Leaf size=54

$$-\frac{2\left(-aq+bp+f^2(-p)\right)\log\left(\sqrt{ax+b}+f\right)}{a^2}-\frac{2fp\sqrt{ax+b}}{a^2}+\frac{px}{a}$$

```
[Out] p*x/a-2*(-f^2*p-a*q+b*p)*ln(f+(a*x+b)^(1/2))/a^2-2*f*p*(a*x+b)^(1/2)/a^2
```

Rubi [A] time = 0.38, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {697}

$$-\frac{2\left(-aq+bp+f^2(-p)\right)\log\left(\sqrt{ax+b}+f\right)}{a^2}-\frac{2fp\sqrt{ax+b}}{a^2}+\frac{px}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

```
[Out] (p*x)/a - (2*f*p*Sqrt[b + a*x])/a^2 - (2*(b*p - f^2*p - a*q)*Log[f + Sqrt[b
 + a*x]])/a^2
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)} dx &= \frac{2\,\text{Subst}\left(\int \frac{-bp+aq+px^2}{f+x} dx, x, \sqrt{b+ax}\right)}{a^2}\\
&= \frac{2\,\text{Subst}\left(\int \left(-fp+px+\frac{-bp+f^2p+aq}{f+x}\right) dx, x, \sqrt{b+ax}\right)}{a^2}\\
&= \frac{px}{a}-\frac{2fp\sqrt{b+ax}}{a^2}-\frac{2\left(bp-f^2p-aq\right)\log\left(f+\sqrt{b+ax}\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 50, normalized size = 0.93

$$\frac{2\left(aq-bp+f^2p\right)\log\left(\sqrt{ax+b}+f\right)+p\left(ax-2f\sqrt{ax+b}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

```
[Out] (p*(a*x - 2*f*Sqrt[b + a*x]) + 2*(-(b*p) + f^2*p + a*q)*Log[f + Sqrt[b + a*
x]])/a^2
```

fricas [A] time = 0.47, size = 45, normalized size = 0.83

$$\frac{apx-2\sqrt{ax+b}\,fp+2\left(\left(f^2-b\right)p+aq\right)\log\left(f+\sqrt{ax+b}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="fricas")
```

```
[Out] (a*p*x - 2*sqrt(a*x + b)*f*p + 2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))
)/a^2
```

giac [A] time = 0.32, size = 61, normalized size = 1.13

$$\frac{2\left(f^{2}p - bp + aq\right)\log\left(\left|f + \sqrt{ax + b}\right|\right)}{a^{2}} - \frac{2\sqrt{ax + b}\,a^{2}fp - (ax + b)a^{2}p}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(f^2*p - b*p + a*q)*log(abs(f + sqrt(a*x + b)))/a^2 - (2*sqrt(a*x + b)*a^
2*f*p - (a*x + b)*a^2*p)/a^4
```

maple [A] time = 0.00, size = 80, normalized size = 1.48

$$\frac{2f^{2}p\ln\left(f + \sqrt{ax + b}\right)}{a^{2}} + \frac{px}{a} + \frac{2q\ln\left(f + \sqrt{ax + b}\right)}{a} - \frac{2bp\ln\left(f + \sqrt{ax + b}\right)}{a^{2}} + \frac{bp}{a^{2}} - \frac{2\sqrt{ax + b}\,fp}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x)
```

```
[Out] p*x/a+1/a^2*b*p-2*f*p*(a*x+b)^(1/2)/a^2+2/a^2*ln(f+(a*x+b)^(1/2))*f^2*p+2/a
*ln(f+(a*x+b)^(1/2))*q-2/a^2*ln(f+(a*x+b)^(1/2))*b*p
```

maxima [A] time = 0.88, size = 58, normalized size = 1.07

$$\frac{\frac{2\left(\left(f^{2}-b\right)p+aq\right)\log\left(f+\sqrt{ax+b}\right)}{a} - \frac{2\sqrt{ax+b}\,fp-(ax+b)p}{a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="maxima")
```

```
[Out] (2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))/a - (2*sqrt(a*x + b)*f*p - (a
*x + b)*p)/a)/a
```

mupad [B] time = 3.10, size = 50, normalized size = 0.93

$$\frac{\ln\left(f + \sqrt{b + a\,x}\right)\left(2\,p\,f^{2} + 2\,a\,q - 2\,b\,p\right)}{a^{2}} + \frac{p\,x}{a} - \frac{2\,f\,p\,\sqrt{b + a\,x}}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((q + p*x)/((f + (b + a*x)^(1/2))*(b + a*x)^(1/2)),x)
```

```
[Out] (log(f + (b + a*x)^(1/2))*(2*a*q - 2*b*p + 2*f^2*p))/a^2 + (p*x)/a - (2*f*p
*(b + a*x)^(1/2))/a^2
```

sympy [A] time = 32.90, size = 99, normalized size = 1.83

$$-\frac{2fp\sqrt{ax+b}}{a^{2}} - \frac{2f\left(-aq + bp - f^{2}p\right)\left(\begin{cases}\frac{1}{\sqrt{ax+b}} & \text{for}\ f = 0\\ \frac{\log\left(\frac{f}{\sqrt{ax+b}}+1\right)}{f} & \text{otherwise}\end{cases}\right)}{a^{2}} + \frac{p\,(ax+b)}{a^{2}} + \frac{2\left(-aq + bp - f^{2}p\right)\log\left(\frac{1}{\sqrt{ax+b}}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)**(1/2)/(f+(a*x+b)**(1/2)),x)
```

```
[Out] -2*f*p*sqrt(a*x + b)/a**2 - 2*f*(-a*q + b*p - f**2*p)*Piecewise((1/sqrt(a*x
 + b), Eq(f, 0)), (log(f/sqrt(a*x + b) + 1)/f, True))/a**2 + p*(a*x + b)/a*
*2 + 2*(-a*q + b*p - f**2*p)*log(1/sqrt(a*x + b))/a**2
```

3.723 $\int \sqrt{1-\sqrt{x}-x}\,dx$

Optimal. Leaf size=70

$$-\frac{2}{3}\left(-x-\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

```
[Out] -5/8*arcsin(1/5*(1+2*x^(1/2))*5^(1/2))-2/3*(1-x-x^(1/2))^(3/2)-1/4*(1+2*x^(
1/2))*(1-x-x^(1/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1341, 640, 612, 619, 216}

$$-\frac{2}{3}\left(-x-\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/4 - (2*(1 - Sqrt[x] - x)^(3/2))/3
- (5*ArcSin[(1 + 2*Sqrt[x])/Sqrt[5]])/8
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-\sqrt{x}-x}\,dx &= 2\,\text{Subst}\left(\int x\sqrt{1-x-x^2}\,dx,x,\sqrt{x}\right)\\ &= -\frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2}-\text{Subst}\left(\int \sqrt{1-x-x^2}\,dx,x,\sqrt{x}\right)\\ &= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}-\frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{5}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x-x^2}}\,dx,x,\sqrt{x}\right)\\ &= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}-\frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2}+\frac{1}{8}\sqrt{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{5}}}\,dx,x,-1-\right.\\ &= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}-\frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{5}{8}\sin^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.76

$$\frac{1}{12}\sqrt{-x-\sqrt{x}+1}\left(8x+2\sqrt{x}-11\right)+\frac{5}{8}\sin^{-1}\left(\frac{-2\sqrt{x}-1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(-11 + 2*Sqrt[x] + 8*x))/12 + (5*ArcSin[(-1 - 2*Sqrt
[x])/Sqrt[5]])/8
```

fricas [A] time = 1.35, size = 84, normalized size = 1.20

$$\frac{1}{12}\left(8\,x+2\,\sqrt{x}-11\right)\sqrt{-x-\sqrt{x}+1}+\frac{5}{16}\arctan\left(-\frac{\left(8\,x^2-\left(16\,x^2-38\,x+11\right)\sqrt{x}-9\,x+3\right)\sqrt{-x-\sqrt{x}+1}}{4\left(4\,x^3-13\,x^2+7\,x-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x) - 11)*sqrt(-x - sqrt(x) + 1) + 5/16*arctan(-1/4*(8*x^
2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x) + 1)/(4*x^3 -
 13*x^2 + 7*x - 1))
```

giac [A] time = 0.35, size = 44, normalized size = 0.63

$$\frac{1}{12}\left(2\,\sqrt{x}\left(4\,\sqrt{x}+1\right)-11\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\arcsin\left(\frac{1}{5}\,\sqrt{5}\left(2\,\sqrt{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 11)*sqrt(-x - sqrt(x) + 1) - 5/8*arcsin(1
/5*sqrt(5)*(2*sqrt(x) + 1))
```

maple [A] time = 0.01, size = 50, normalized size = 0.71

$$-\frac{5\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8}-\frac{2\left(-x-\sqrt{x}+1\right)^{\frac{3}{2}}}{3}+\frac{\left(-2\sqrt{x}-1\right)\sqrt{-x-\sqrt{x}+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-x^(1/2))^(1/2),x)
```

```
[Out] -2/3*(1-x-x^(1/2))^(3/2)+1/4*(-2*x^(1/2)-1)*(1-x-x^(1/2))^(1/2)-5/8*arcsin(
2/5*5^(1/2)*(x^(1/2)+1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x-\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x - sqrt(x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{1-\sqrt{x}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^(1/2) - x)^(1/2),x)
```

```
[Out] int((1 - x^(1/2) - x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sqrt{x}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(-sqrt(x) - x + 1), x)
```

3.724 $\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx$

Optimal. Leaf size=19

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

[Out] x+2*ln(4+x^(1/2))+4*x^(1/2)

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {28, 1397, 771}

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

[In] Int[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]

[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1397

```
Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g
 - 1)*(d + e*x^(g*n))^q*(a + b*x^(g*n) + c*x^(2*g*n))^p, x], x, x^(1/g)], x
]] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx &= \int \frac{\left(3+\sqrt{x}\right)^2}{4\sqrt{x}+x}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{x(3+x)^2}{4x+x^2}\,dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(2+x+\frac{1}{4+x}\right)dx,x,\sqrt{x}\right)\\
&= 4\sqrt{x}+x+2\log\left(4+\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

fricas [A] time = 0.45, size = 15, normalized size = 0.79

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="fricas")
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

giac [A] time = 0.39, size = 15, normalized size = 0.79

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="giac")
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$x + 2\ln\left(\sqrt{x} + 4\right) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9+x+6*x^(1/2))/(x+4*x^(1/2)),x)
[Out] x+2*ln(x^(1/2)+4)+4*x^(1/2)
```

maxima [A] time = 0.88, size = 15, normalized size = 0.79

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="maxima")
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

mupad [B] time = 3.04, size = 15, normalized size = 0.79

$$x + 2\ln\left(\sqrt{x} + 4\right) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 6*x^(1/2) + 9)/(x + 4*x^(1/2)),x)
[Out] x + 2*log(x^(1/2) + 4) + 4*x^(1/2)
```

sympy [A] time = 0.17, size = 17, normalized size = 0.89

$$4\sqrt{x} + x + 2\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x**(1/2))/(x+4*x**(1/2)),x)
[Out] 4*sqrt(x) + x + 2*log(sqrt(x) + 4)
```

3.725 $\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx$

Optimal. Leaf size=77

$$\frac{80x^{7/2}}{567}+\frac{400x^{5/2}}{6561}+\frac{50000x^{3/2}}{1594323}+\frac{2x^4}{9}+\frac{200x^3}{2187}+\frac{2500x^2}{59049}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

```
[Out] 125000/4782969*x+50000/1594323*x^(3/2)+2500/59049*x^2+400/6561*x^(5/2)+200/
2187*x^3+80/567*x^(7/2)+2/9*x^4-280728140/387420489*ln(5-9*x^(1/2))-5614562
8/43046721*x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1893, 190, 43, 266}

$$\frac{2x^4}{9}+\frac{80x^{7/2}}{567}+\frac{200x^3}{2187}+\frac{400x^{5/2}}{6561}+\frac{2500x^2}{59049}+\frac{50000x^{3/2}}{1594323}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

Antiderivative was successfully verified.

```
[In] Int[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (-56145628*Sqrt[x])/43046721 + (125000*x)/4782969 + (50000*x^(3/2))/1594323
 + (2500*x^2)/59049 + (400*x^(5/2))/6561 + (200*x^3)/2187 + (80*x^(7/2))/56
7 + (2*x^4)/9 - (280728140*Log[5 - 9*Sqrt[x]])/387420489
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \int \left(-\frac{6}{-5+9\sqrt{x}} + \frac{8x^{7/2}}{-5+9\sqrt{x}}\right) dx$$
$$= -\left(6\int \frac{1}{-5+9\sqrt{x}}\,dx\right) + 8\int \frac{x^{7/2}}{-5+9\sqrt{x}}\,dx$$
$$= -\left(12\,\text{Subst}\left(\int \frac{x}{-5+9x}\,dx, x, \sqrt{x}\right)\right) + 16\,\text{Subst}\left(\int \frac{x^8}{-5+9x}\,dx, x, \sqrt{x}\right)$$
$$= -\left(12\,\text{Subst}\left(\int \left(\frac{1}{9} + \frac{5}{9(-5+9x)}\right) dx, x, \sqrt{x}\right)\right) + 16\,\text{Subst}\left(\int \left(\frac{78125}{43046721} + \frac{15625x}{4782969} + \frac{3125x}{53144}\right.\right.$$
$$= -\frac{56145628\sqrt{x}}{43046721} + \frac{125000x}{4782969} + \frac{50000x^{3/2}}{1594323} + \frac{2500x^2}{59049} + \frac{400x^{5/2}}{6561} + \frac{200x^3}{2187} + \frac{80x^{7/2}}{567} + \frac{2x^4}{9} - \frac{2807}{}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.86

$$\frac{2\left(9\left(21257640x^{7/2} + 9185400x^{5/2} + 4725000x^{3/2} + 33480783x^4 + 13778100x^3 + 6378750x^2 + 3937500x - 1965\right.\right.}{2711943423}$$

Antiderivative was successfully verified.

```
[In] Integrate[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (2*(9*(-196509698*Sqrt[x] + 3937500*x + 4725000*x^(3/2) + 6378750*x^2 + 918
5400*x^(5/2) + 13778100*x^3 + 21257640*x^(7/2) + 33480783*x^4) - 982548490*
Log[5 - 9*Sqrt[x]]))/2711943423
```

fricas [A] time = 0.47, size = 49, normalized size = 0.64

$$\frac{2}{9}x^4 + \frac{200}{2187}x^3 + \frac{2500}{59049}x^2 + \frac{4}{301327047}\left(10628820x^3 + 4592700x^2 + 2362500x - 98254849\right)\sqrt{x} + \frac{125000}{4782969}x - \frac{28}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/9*x^4 + 200/2187*x^3 + 2500/59049*x^2 + 4/301327047*(10628820*x^3 + 45927
00*x^2 + 2362500*x - 98254849)*sqrt(x) + 125000/4782969*x - 280728140/38742
0489*log(9*sqrt(x) - 5)
```

giac [A] time = 0.35, size = 50, normalized size = 0.65

$$\frac{2}{9}x^4 + \frac{80}{567}x^{\frac{7}{2}} + \frac{200}{2187}x^3 + \frac{400}{6561}x^{\frac{5}{2}} + \frac{2500}{59049}x^2 + \frac{50000}{1594323}x^{\frac{3}{2}} + \frac{125000}{4782969}x - \frac{56145628}{43046721}\sqrt{x} - \frac{280728140}{387420489}\log\left(\left|9\sqrt{x} - \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2
 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 2
80728140/387420489*log(abs(9*sqrt(x) - 5))
```

maple [A] time = 0.00, size = 50, normalized size = 0.65

$$\frac{2x^4}{9} + \frac{80x^{\frac{7}{2}}}{567} + \frac{200x^3}{2187} + \frac{400x^{\frac{5}{2}}}{6561} + \frac{2500x^2}{59049} + \frac{50000x^{\frac{3}{2}}}{1594323} + \frac{125000x}{4782969} - \frac{280728140\ln\left(9\sqrt{x}-5\right)}{387420489} - \frac{56145628\sqrt{x}}{43046721}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6-8*x^(7/2))/(5-9*x^(1/2)),x)
```

```
[Out] 2/9*x^4+80/567*x^(7/2)+200/2187*x^3+400/6561*x^(5/2)+2500/59049*x^2+50000/1
594323*x^(3/2)+125000/4782969*x-56145628/43046721*x^(1/2)-280728140/3874204
89*ln(-5+9*x^(1/2))
```

maxima [A] time = 0.88, size = 49, normalized size = 0.64

$$\frac{2}{9}x^4+\frac{80}{567}x^{\frac{7}{2}}+\frac{200}{2187}x^3+\frac{400}{6561}x^{\frac{5}{2}}+\frac{2500}{59049}x^2+\frac{50000}{1594323}x^{\frac{3}{2}}+\frac{125000}{4782969}x-\frac{56145628}{43046721}\sqrt{x}-\frac{280728140}{387420489}\log\left(9\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2
 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 2
80728140/387420489*log(9*sqrt(x) - 5)
```

mupad [B] time = 0.05, size = 47, normalized size = 0.61

$$\frac{125000\,x}{4782969}-\frac{280728140\,\ln\left(\sqrt{x}-\frac{5}{9}\right)}{387420489}+\frac{2500\,x^2}{59049}-\frac{56145628\,\sqrt{x}}{43046721}+\frac{200\,x^3}{2187}+\frac{2\,x^4}{9}+\frac{50000\,x^{3/2}}{1594323}+\frac{400\,x^{5/2}}{6561}+\frac{80\,x^{7/2}}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^(7/2) - 6)/(9*x^(1/2) - 5),x)
```

```
[Out] (125000*x)/4782969 - (280728140*log(x^(1/2) - 5/9))/387420489 + (2500*x^2)/
59049 - (56145628*x^(1/2))/43046721 + (200*x^3)/2187 + (2*x^4)/9 + (50000*x
^(3/2))/1594323 + (400*x^(5/2))/6561 + (80*x^(7/2))/567
```

sympy [A] time = 2.32, size = 71, normalized size = 0.92

$$\frac{80x^{\frac{7}{2}}}{567}+\frac{400x^{\frac{5}{2}}}{6561}+\frac{50000x^{\frac{3}{2}}}{1594323}-\frac{56145628\sqrt{x}}{43046721}+\frac{2x^4}{9}+\frac{200x^3}{2187}+\frac{2500x^2}{59049}+\frac{125000x}{4782969}-\frac{280728140\log\left(\sqrt{x}-\frac{5}{9}\right)}{387420489}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x**(7/2))/(5-9*x**(1/2)),x)
```

```
[Out] 80*x**(7/2)/567 + 400*x**(5/2)/6561 + 50000*x**(3/2)/1594323 - 56145628*sqr
t(x)/43046721 + 2*x**4/9 + 200*x**3/2187 + 2500*x**2/59049 + 125000*x/47829
69 - 280728140*log(sqrt(x) - 5/9)/387420489
```

3.726 $\int \frac{\sqrt{1+x}\left(1+x^3\right)}{1+x^2}\,dx$

Optimal. Leaf size=80

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}+(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)$$

```
[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)+(1-I)^(3/2)*arctanh((1+x)^(1/2)/(1-I)^(1/2
))+(1+I)^(3/2)*arctanh((1+x)^(1/2)/(1+I)^(1/2))-2*(1+x)^(1/2)
```

Rubi [B] time = 0.29, antiderivative size = 224, normalized size of antiderivative = 2.80, number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1625, 1629, 825, 12, 708, 1094, 634, 618, 204, 628}

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}-\frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}\right.}{2\sqrt{1+\sqrt{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5 - Sqrt[1 + Sqrt[
2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]]
+ Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/Sqrt[2*(
-1 + Sqrt[2])]] - Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/
(2*Sqrt[1 + Sqrt[2]]) + Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1
+ x]]/(2*Sqrt[1 + Sqrt[2]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1625

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*(a + c*x^2)^p,
x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemaind
er[Pq, d + e*x, x], 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\sqrt{1+x}\,(1+x^3)}{1+x^2}\,dx = \int \frac{(1+x)^{3/2}(1-x+x^2)}{1+x^2}\,dx$$
$$= \int \left((1+x)^{3/2} - \frac{x(1+x)^{3/2}}{1+x^2}\right)dx$$
$$= \frac{2}{5}(1+x)^{5/2} - \int \frac{x(1+x)^{3/2}}{1+x^2}\,dx$$
$$= -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \int \frac{(-1+x)\sqrt{1+x}}{1+x^2}\,dx$$
$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \int -\frac{2}{\sqrt{1+x}\,(1+x^2)}\,dx$$
$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + 2\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx$$
$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + 4\,\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)$$
$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2(1+\sqrt{2})}-x}{\sqrt{2}-\sqrt{2(1+\sqrt{2})}\,x+x^2}\,dx, x, \sqrt{1+x}\right)}{\sqrt{1+\sqrt{2}}} + \text{Su}$$
$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2(1+\sqrt{2})}\,x+x^2}\,dx, x, \sqrt{1+x}\right)}{\sqrt{2}} + \text{Su}$$
$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \frac{\log\left(1+\sqrt{2}+x-\sqrt{2(1+\sqrt{2})}\sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}} + \text{log}$$
$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \frac{\tan^{-1}\left(\frac{\sqrt{2(1+\sqrt{2})}-2\sqrt{1+x}}{\sqrt{2(-1+\sqrt{2})}}\right)}{\sqrt{-1+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2(1+\sqrt{2})}+2}{\sqrt{2(-1+\sqrt{2}}}\right)}{\sqrt{-1+\sqrt{2}}}$$

Mathematica [A] time = 0.10, size = 68, normalized size = 0.85

$$\frac{2}{15}\sqrt{x+1}\,(3x^2+x-17) + (1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right) + (1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] (2*Sqrt[1 + x]*(-17 + x + 3*x^2))/15 + (1 - I)^(3/2)*ArcTanh[Sqrt[1 + x]/Sq
rt[1 - I]] + (1 + I)^(3/2)*ArcTanh[Sqrt[1 + x]/Sqrt[1 + I]]
```

fricas [B] time = 0.52, size = 302, normalized size = 3.78

$$-\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(2\cdot 8^{\frac{1}{4}}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+4x+4\sqrt{2}+4\right)+\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2*8^(1/4)*sqrt(x + 1)*sq
rt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) + 1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*
(sqrt(2) - 2)*log(-2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt
(2) + 4) - 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/16*8^(3/4)*sqrt
(2)*sqrt(2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*s
qrt(2*sqrt(2) + 4) - 1/8*8^(3/4)*sqrt(2)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/16*8^(3/4)*
sqrt(2)*sqrt(-2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) +
 4)*sqrt(2*sqrt(2) + 4) - 1/8*8^(3/4)*sqrt(2)*sqrt(x + 1)*sqrt(2*sqrt(2) +
4) + sqrt(2) + 1) + 2/15*(3*x^2 + x - 17)*sqrt(x + 1)
```

giac [B] time = 1.71, size = 171, normalized size = 2.14

$$\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{2}{3}(x+1)^{\frac{3}{2}}+\sqrt{\sqrt{2}+1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)+\sqrt{\sqrt{2}+1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+\right.}{2\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2) + sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4
)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) + sqrt(sq
rt(2) + 1)*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/
sqrt(-sqrt(2) + 2)) + 1/2*sqrt(sqrt(2) - 1)*log(2^(1/4)*sqrt(x + 1)*sqrt(sq
rt(2) + 2) + x + sqrt(2) + 1) - 1/2*sqrt(sqrt(2) - 1)*log(-2^(1/4)*sqrt(x +
 1)*sqrt(sqrt(2) + 2) + x + sqrt(2) + 1) - 2*sqrt(x + 1)
```

maple [B] time = 0.05, size = 443, normalized size = 5.54

$$\frac{\left(2+2\sqrt{2}\right)\sqrt{2}\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{2\sqrt{-2+2\sqrt{2}}}-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*(x+1)^(1/2)/(x^2+1),x)
```

```
[Out] 2/5*(x+1)^(5/2)-2/3*(x+1)^(3/2)-2*(x+1)^(1/2)-1/4*ln(1+x+2^(1/2)+(x+1)^(1/2
)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)*2^(1/2)+1/2*ln(1+x+2^(1/2)+(x+1)
^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2/(-2+2*2^(1/2))^(1/2)*ar
ctan((2*(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2)
)*2^(1/2)-1/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)
^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2
)-(x+1)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)*2^(1/2)-1/2*ln(1+x+2
^(1/2)-(x+1)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2/(-2+2*2^(1/
2))^(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*
(2+2*2^(1/2))*2^(1/2)-1/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(
1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arc
tan((2*(x+1)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+1\right)\sqrt{x+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1), x)
```

mupad [B] time = 0.10, size = 255, normalized size = 3.19

$$\frac{2\,(x+1)^{5/2}}{5}-\frac{2\,(x+1)^{3/2}}{3}-2\,\sqrt{x+1}-\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}-\frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}\right)\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)*(x + 1)^(1/2))/(x^2 + 1),x)
```

```
[Out] (2*(x + 1)^(5/2))/5 - (2*(x + 1)^(3/2))/3 - 2*(x + 1)^(1/2) - atan((2^(1/2)
*(- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*
(- 2^(1/2)/4 - 1/4)^(1/2) - 64) - (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^
(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64))*((
- 2^(1/2)/4 - 1/4)^(1/2)*2i + (2^(1/2)/4 - 1/4)^(1/2)*2i) + atan((2^(1/2)*(
- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(-
 2^(1/2)/4 - 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1
/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 64))*((-
2^(1/2)/4 - 1/4)^(1/2)*2i - (2^(1/2)/4 - 1/4)^(1/2)*2i)
```

sympy [A] time = 12.15, size = 56, normalized size = 0.70

$$\frac{2\,(x+1)^{\frac{5}{2}}}{5}-\frac{2\,(x+1)^{\frac{3}{2}}}{3}-2\sqrt{x+1}+4\,\mathrm{RootSum}\left(512t^4+32t^2+1,\left(t\mapsto t\log\left(-128t^3+\sqrt{x+1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*(1+x)**(1/2)/(x**2+1),x)
```

```
[Out] 2*(x + 1)**(5/2)/5 - 2*(x + 1)**(3/2)/3 - 2*sqrt(x + 1) + 4*RootSum(512*_t*
*4 + 32*_t**2 + 1, Lambda(_t, _t*log(-128*_t**3 + sqrt(x + 1))))
```

3.727 $\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx$

Optimal. Leaf size=89

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

```
[Out] arctan(1/2*(3-x^(1/2))/(-1+x-x^(1/2))^(1/2))-2*arctanh(1/2*(1-2*x^(1/2))/(-
1+x-x^(1/2))^(1/2))-arctanh(1/2*(1+3*x^(1/2))/(-1+x-x^(1/2))^(1/2))
```

Rubi [A] time = 0.26, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {990, 621, 206, 1033, 724, 204}

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```

```
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 - 2*Sqrt[x]
)/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x
] + x])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{-1-x+x^2}}{-1+x^2}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right) - 2\,\text{Subst}\left(\int \frac{x}{\left(-1+x^2\right)\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+2\sqrt{x}}{\sqrt{-1-\sqrt{x}+x}}\right) - \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right)$$

$$= -2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) + 2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3+\sqrt{x}}{\sqrt{-1-\sqrt{x}+x}}\right) + 2\,\text{Subst}\left(\right.$$

$$= \tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - 2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - \tanh^{-1}\left(\frac{1+3\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right)$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.00

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right) - 2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right) - \tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 - 2*Sqrt[x]
)/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x
] + x])]
```

fricas [A] time = 4.14, size = 87, normalized size = 0.98

$$-\arctan\left(\frac{\left((x-4)\sqrt{x}-2x+3\right)\sqrt{x-\sqrt{x}-1}}{2\left(x^2-3x+1\right)}\right) + \log\left(-\frac{8x^2+2\left((4x-5)\sqrt{x}+2x-1\right)\sqrt{x-\sqrt{x}-1}-17x-2}{x-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*((x - 4)*sqrt(x) - 2*x + 3)*sqrt(x - sqrt(x) - 1)/(x^2 - 3*x +
1)) + log(-(8*x^2 + 2*((4*x - 5)*sqrt(x) + 2*x - 1)*sqrt(x - sqrt(x) - 1) -
 17*x - 2*sqrt(x) + 11)/(x - 1))
```

giac [A] time = 1.24, size = 81, normalized size = 0.91

$$-2\arctan\left(\sqrt{x-\sqrt{x}-1}-\sqrt{x}+1\right) - \log\left(-\sqrt{x-\sqrt{x}-1}+\sqrt{x}+2\right) + \log\left(-\sqrt{x-\sqrt{x}-1}+\sqrt{x}\right) - 2\log\left(\left|2\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(x - sqrt(x) - 1) - sqrt(x) + 1) - log(-sqrt(x - sqrt(x) - 1)
 + sqrt(x) + 2) + log(-sqrt(x - sqrt(x) - 1) + sqrt(x)) - 2*log(abs(2*sqrt(
x - sqrt(x) - 1) - 2*sqrt(x) + 1))
```

maple [A] time = 0.02, size = 130, normalized size = 1.46

$$\operatorname{arctanh}\left(\frac{-3\sqrt{x}-1}{2\sqrt{-3\sqrt{x}+\left(\sqrt{x}+1\right)^{2}-2}}\right)-\arctan\left(\frac{\sqrt{x}-3}{2\sqrt{\sqrt{x}+\left(\sqrt{x}-1\right)^{2}-2}}\right)+\frac{3\ln\left(\sqrt{x}-\frac{1}{2}+\sqrt{-3\sqrt{x}+\left(\sqrt{x}+\right.}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x-x^(1/2))^(1/2)/(x-1)/x^(1/2),x)
```

```
[Out] ((x^(1/2)-1)^2+x^(1/2)-2)^(1/2)+1/2*ln(x^(1/2)-1/2+((x^(1/2)-1)^2+x^(1/2)-2
)^(1/2))-arctan(1/2*(x^(1/2)-3)/((x^(1/2)-1)^2+x^(1/2)-2)^(1/2))-((x^(1/2)+
1)^2-3*x^(1/2)-2)^(1/2)+3/2*ln(-1/2+x^(1/2)+((x^(1/2)+1)^2-3*x^(1/2)-2)^(1/
2))+arctanh(1/2*(-1-3*x^(1/2))/((x^(1/2)+1)^2-3*x^(1/2)-2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x-\sqrt{x}-1}}{(x-1)\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x - sqrt(x) - 1)/((x - 1)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x-\sqrt{x}-1}}{\sqrt{x}\,\left(x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)),x)
```

```
[Out] int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\sqrt{x}+x-1}}{\sqrt{x}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x**(1/2))**(1/2)/(-1+x)/x**(1/2),x)
```

```
[Out] Integral(sqrt(-sqrt(x) + x - 1)/(sqrt(x)*(x - 1)), x)
```

3.728 $\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=61

$$3\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)$$

```
[Out] -arctan(1/2*(3+(1+x)^(1/2))/(x+(1+x)^(1/2))^(1/2))+3*arctanh(1/2*(1-3*(1+x)
^(1/2))/(x+(1+x)^(1/2))^(1/2))
```

Rubi [A] time = 0.51, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1033, 724, 206, 204}

$$3\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 3*ArcTanh[(1 - 3*Sqr
t[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = 2\,\text{Subst}\left(\int \frac{1+2x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$

$$= 3\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right) + \text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+x}\cdots}\right.$$

$$= -\left(2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)\right) - 6\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1\cdots}{\sqrt{x\cdots}}\right.$$

$$= -\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right) + 3\tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.00

$$\tan^{-1}\left(\frac{-\sqrt{x+1}-3}{2\sqrt{x+\sqrt{x+1}}}\right) - 3\tanh^{-1}\left(\frac{3\sqrt{x+1}-1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] ArcTan[(-3 - Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] - 3*ArcTanh[(-1 + 3*Sq
rt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

fricas [A] time = 2.40, size = 62, normalized size = 1.02

$$\arctan\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}-3\right)}{x-8}\right) + 3\log\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}+1\right) - 3x - 2\sqrt{x+1} - 2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="fricas")
```

```
[Out] arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) + 3*log((2*sqrt(x
 + sqrt(x + 1))*(sqrt(x + 1) + 1) - 3*x - 2*sqrt(x + 1) - 2)/x)
```

giac [A] time = 1.16, size = 65, normalized size = 1.07

$$2\arctan\left(\sqrt{x+\sqrt{x+1}} - \sqrt{x+1} - 1\right) - 3\log\left(\left|\sqrt{x+\sqrt{x+1}} - \sqrt{x+1} + 2\right|\right) + 3\log\left(\left|\sqrt{x+\sqrt{x+1}} - \sqrt{x+\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="giac")
```

```
[Out] 2*arctan(sqrt(x + sqrt(x + 1)) - sqrt(x + 1) - 1) - 3*log(abs(sqrt(x + sqrt
(x + 1)) - sqrt(x + 1) + 2)) + 3*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1
)))
```

maple [A] time = 0.02, size = 68, normalized size = 1.11

$$-3\,\text{arctanh}\left(\frac{3\sqrt{x+1}-1}{2\sqrt{\left(\sqrt{x+1}-1\right)^2+3\sqrt{x+1}-2}}\right) + \arctan\left(\frac{-\sqrt{x+1}-3}{2\sqrt{\left(1+\sqrt{x+1}\right)^2-\sqrt{x+1}-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*(x+1)^(1/2))/x/(x+1)^(1/2)/(x+(x+1)^(1/2))^(1/2),x)
```

```
[Out] -3*arctanh(1/2*(3*(x+1)^(1/2)-1)/(((x+1)^(1/2)-1)^2+3*(x+1)^(1/2)-2)^(1/2))
+arctan(1/2*(-(x+1)^(1/2)-3)/((1+(x+1)^(1/2))^2-(x+1)^(1/2)-2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\sqrt{x+1}+1}{\sqrt{x+\sqrt{x+1}}\,\sqrt{x+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((2*sqrt(x + 1) + 1)/(sqrt(x + sqrt(x + 1))*sqrt(x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2\sqrt{x+1}+1}{x\sqrt{x+\sqrt{x+1}}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\sqrt{x+1}+1}{x\sqrt{x+1}\,\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)**(1/2))/x/(1+x)**(1/2)/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral((2*sqrt(x + 1) + 1)/(x*sqrt(x + 1)*sqrt(x + sqrt(x + 1))), x)
```

3.729 $\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$

Optimal. Leaf size=8

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*arcsinh(x^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {54, 215}

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)\\ &= 2\sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

fricas [B] time = 0.44, size = 18, normalized size = 2.25

$$-\log\left(2\sqrt{x+1}\sqrt{x} - 2x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

giac [B] time = 0.25, size = 14, normalized size = 1.75

$$-2\log\left(\sqrt{x+1} - \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(x + 1) - sqrt(x))
```

maple [B] time = 0.00, size = 28, normalized size = 3.50

$$\frac{\sqrt{(x+1)\,x}\;\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{x+1}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x+1)^(1/2),x)
[Out] (x*(x+1))^(1/2)/x^(1/2)/(x+1)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

maxima [B] time = 0.87, size = 27, normalized size = 3.38

$$\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)-\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] log(sqrt(x + 1)/sqrt(x) + 1) - log(sqrt(x + 1)/sqrt(x) - 1)
```

mupad [B] time = 0.16, size = 14, normalized size = 1.75

$$4\,\mathrm{atanh}\left(\frac{\sqrt{x+1}\,-1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x + 1)^(1/2)),x)
[Out] 4*atanh(((x + 1)^(1/2) - 1)/x^(1/2))
```

sympy [A] time = 0.95, size = 26, normalized size = 3.25

$$\begin{cases} 2\,\mathrm{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1|>1 \\ -2i\,\mathrm{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x)**(1/2),x)
[Out] Piecewise((2*acosh(sqrt(x + 1)), Abs(x + 1) > 1), (-2*I*asin(sqrt(x + 1)),
True))
```

3.730 $\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx$

Optimal. Leaf size=8

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*arcsinh(x^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1958, 54, 215}

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)]/x,x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx &= \int \frac{1}{\sqrt{x}\sqrt{1+x}} \, dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} \, dx, x, \sqrt{x}\right) \\
&= 2\sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)]/x,x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

fricas [B] time = 0.43, size = 27, normalized size = 3.38

$$\log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="fricas")
[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

giac [B] time = 0.40, size = 22, normalized size = 2.75

$$-\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="giac")
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1)
```

maple [B] time = 0.02, size = 32, normalized size = 4.00

$$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{(x+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(x+1))^(1/2)/x,x)
[Out] (x/(x+1))^(1/2)*(x+1)/((x+1)*x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

maxima [B] time = 0.88, size = 27, normalized size = 3.38

$$\log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="maxima")
[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

mupad [B] time = 0.06, size = 12, normalized size = 1.50

$$2\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(x + 1))^(1/2)/x,x)
[Out] 2*atanh((x/(x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{x}{x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2)/x,x)
[Out] Integral(sqrt(x/(x + 1))/x, x)
```

3.731 $\int \frac{\sqrt{x}}{\sqrt{1+x}} dx$

Optimal. Leaf size=22

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 54, 215}

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx &= \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx \\
&= \sqrt{x}\sqrt{1+x} - \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right) \\
&= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1) - \sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]
```

fricas [A] time = 0.44, size = 28, normalized size = 1.27

$$\sqrt{x+1}\,\sqrt{x} + \frac{1}{2}\,\log\left(2\,\sqrt{x+1}\,\sqrt{x} - 2\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(x) + 1/2*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

giac [A] time = 0.39, size = 22, normalized size = 1.00

$$\sqrt{x+1}\,\sqrt{x} + \log\left(\sqrt{x+1} - \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) - sqrt(x))
```

maple [B] time = 0.00, size = 39, normalized size = 1.77

$$\sqrt{x+1}\,\sqrt{x} - \frac{\sqrt{(x+1)\,x}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2\sqrt{x+1}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x+1)^(1/2),x)
```

```
[Out] x^(1/2)*(x+1)^(1/2)-1/2*((x+1)*x)^(1/2)/(x+1)^(1/2)/x^(1/2)*ln(x+1/2+(x^2+x
)^(1/2))
```

maxima [B] time = 0.87, size = 49, normalized size = 2.23

$$\frac{\sqrt{x+1}}{\sqrt{x}\left(\frac{x+1}{x}-1\right)} - \frac{1}{2}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right) + \frac{1}{2}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x + 1)/(sqrt(x)*((x + 1)/x - 1)) - 1/2*log(sqrt(x + 1)/sqrt(x) + 1) +
1/2*log(sqrt(x + 1)/sqrt(x) - 1)
```

mupad [B] time = 3.72, size = 26, normalized size = 1.18

$$\sqrt{x}\,\sqrt{x+1} - 2\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + 1)^(1/2),x)
```

```
[Out] x^(1/2)*(x + 1)^(1/2) - 2*atanh(x^(1/2)/((x + 1)^(1/2) - 1))
```

sympy [A] time = 1.48, size = 60, normalized size = 2.73

$$\begin{cases} -\operatorname{acosh}\left(\sqrt{x+1}\right) + \frac{(x+1)^{\frac{3}{2}}}{\sqrt{x}} - \frac{\sqrt{x+1}}{\sqrt{x}} & \text{for } |x+1| > 1 \\ i\sqrt{-x}\,\sqrt{x+1} + i\operatorname{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-acosh(sqrt(x + 1)) + (x + 1)**(3/2)/sqrt(x) - sqrt(x + 1)/sqrt(
x), Abs(x + 1) > 1), (I*sqrt(-x)*sqrt(x + 1) + I*asin(sqrt(x + 1)), True))
```

3.732 $\int \sqrt{\frac{x}{1+x}}\, dx$

Optimal. Leaf size=22

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1958, 50, 54, 215}

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{x}{1+x}}\, dx &= \int \frac{\sqrt{x}}{\sqrt{1+x}}\, dx \\
&= \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\, dx \\
&= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\, dx, x, \sqrt{x}\right) \\
&= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1)-\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]
```

fricas [B] time = 0.42, size = 42, normalized size = 1.91

$$(x+1)\sqrt{\frac{x}{x+1}}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
+ 1)) - 1)
```

giac [B] time = 0.44, size = 35, normalized size = 1.59

$$\frac{1}{2}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)+\sqrt{x^2+x}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x +
 1)
```

maple [B] time = 0.01, size = 45, normalized size = 2.05

$$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\left(-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)+2\sqrt{x^2+x}\right)}{2\sqrt{(x+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x+1)*x)^(1/2),x)
```

```
[Out] 1/2*(1/(x+1)*x)^(1/2)*(x+1)*(2*(x^2+x)^(1/2)-ln(x+1/2+(x^2+x)^(1/2)))/((x+1
)*x)^(1/2)
```

maxima [B] time = 0.82, size = 51, normalized size = 2.32

$$-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(s
qrt(x/(x + 1)) - 1)
```

mupad [B] time = 3.12, size = 35, normalized size = 1.59

$$-\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right)-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(x + 1))^(1/2),x)
[Out] - atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2),x)
[Out] Integral(sqrt(x/(x + 1)), x)
```

3.733 $\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=36

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[Out] arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {94, 92, 203}

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\sqrt{1+x}\right)\\ &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+\tan^{-1}\left(\sqrt{-1+x}\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.39

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(-x^2+\sqrt{x^2-1}\,x\tan^{-1}\left(\sqrt{x^2-1}\right)+1\right)}{(x-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(1 - x^2 + x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])
)/((-1 + x)*x)
```

fricas [A] time = 0.42, size = 39, normalized size = 1.08

$$\frac{2\,x\arctan\left(\sqrt{x+1}\,\sqrt{x-1}-x\right)-\sqrt{x+1}\,\sqrt{x-1}-x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

giac [A] time = 0.33, size = 42, normalized size = 1.17

$$-\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4}-2\,\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -8/((sqrt(x + 1) - sqrt(x - 1))^4 + 4) - 2*arctan(1/2*(sqrt(x + 1) - sqrt(x
 - 1))^2)
```

maple [A] time = 0.02, size = 43, normalized size = 1.19

$$\frac{\left(-x\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{x-1}\,\sqrt{x+1}}{\sqrt{x^2-1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/2)/x^2/(x+1)^(1/2),x)
```

```
[Out] (-arctan(1/(x^2-1)^(1/2))*x-(x^2-1)^(1/2))*(x-1)^(1/2)*(x+1)^(1/2)/x/(x^2-1
)^(1/2)
```

maxima [A] time = 1.98, size = 20, normalized size = 0.56

$$-\frac{\sqrt{x^2-1}}{x}-\arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x^2 - 1)/x - arcsin(1/abs(x))
```

mupad [B] time = 5.09, size = 138, normalized size = 3.83

$$-\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)1\mathrm{i}+\ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)1\mathrm{i}-\frac{\sqrt{x-1}-\mathrm{i}}{4\left(\sqrt{x+1}-1\right)}-\frac{\frac{5\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1}{\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)}{\sqrt{x+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x^2*(x + 1)^(1/2)),x)
```

```
[Out] log(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))*1i - log(((x - 1)^(1/2) - 1i)
^2/((x + 1)^(1/2) - 1)^2 + 1)*1i - ((x - 1)^(1/2) - 1i)/(4*((x + 1)^(1/2) -
 1)) - ((5*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 + 1)/((4*((x - 1)^
(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (4*((x - 1)^(1/2) - 1i))/((x + 1)^(1
/2) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x-1}}{x^2\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/x**2/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x - 1)/(x**2*sqrt(x + 1)), x)
```

3.734 $\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} \, dx$

Optimal. Leaf size=36

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[Out] arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1958, 94, 92, 203}

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx &= \int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + \int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\,\sqrt{1+x}\right)\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + \tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 50, normalized size = 1.39

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(-x^2+\sqrt{x^2-1}\,x\tan^{-1}\left(\sqrt{x^2-1}\right)+1\right)}{(x-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(1 - x^2 + x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])
)/((-1 + x)*x)
```

fricas [A] time = 0.44, size = 36, normalized size = 1.00

$$\frac{2\,x\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)-(x+1)\sqrt{\frac{x-1}{x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*x*arctan(sqrt((x - 1)/(x + 1))) - (x + 1)*sqrt((x - 1)/(x + 1)))/x
```

giac [A] time = 0.41, size = 51, normalized size = 1.42

$$-\frac{1}{2}(\pi-2)\text{sgn}\,(x+1)+2\,\arctan\left(-x+\sqrt{x^2-1}\right)\text{sgn}\,(x+1)-\frac{2\,\text{sgn}\,(x+1)}{\left(x-\sqrt{x^2-1}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(pi - 2)*sgn(x + 1) + 2*arctan(-x + sqrt(x^2 - 1))*sgn(x + 1) - 2*sgn(
x + 1)/((x - sqrt(x^2 - 1))^2 + 1)
```

maple [B] time = 0.02, size = 59, normalized size = 1.64

$$\frac{\sqrt{\frac{x-1}{x+1}}\,(x+1)\left(-\sqrt{x^2-1}\,x^2-x\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)+\left(x^2-1\right)^{\frac{3}{2}}\right)}{\sqrt{(x-1)\,(x+1)}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-1)/(x+1))^(1/2)/x^2,x)
```

```
[Out] ((x-1)/(x+1))^(1/2)*(x+1)*((x^2-1)^(3/2)-x^2*(x^2-1)^(1/2)-x*arctan(1/(x^2-
1)^(1/2)))/((x-1)*(x+1))^(1/2)/x
```

maxima [A] time = 1.95, size = 41, normalized size = 1.14

$$-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}+2\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) + 1) + 2*arctan(sqrt((x - 1)/(x +
 1)))
```

mupad [B] time = 0.06, size = 41, normalized size = 1.14

$$2\,\mathrm{atan}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)/(x + 1))^(1/2)/x^2,x)
```

```
[Out] 2*atan(((x - 1)/(x + 1))^(1/2)) - (2*((x - 1)/(x + 1))^(1/2))/((x - 1)/(x +
 1) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{x-1}{x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((x - 1)/(x + 1))/x**2, x)
```

3.735 $\int \frac{\sqrt{-1+x}\, x^3}{\sqrt{1+x}}\, dx$

Optimal. Leaf size=69

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}\,x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\,\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

```
[Out] 3/8*arccosh(x)+1/24*(7-2*x)*(-1+x)^(3/2)*(1+x)^(1/2)+1/4*(-1+x)^(3/2)*x^2*(
1+x)^(1/2)-3/8*(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {100, 147, 50, 52}

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}\,x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\,\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int \frac{\sqrt{-1+x}\,x^3}{\sqrt{1+x}}\,dx = \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{1}{4}\int \frac{(2-x)\sqrt{-1+x}\,x}{\sqrt{1+x}}\,dx$$
$$= \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} - \frac{3}{8}\int \frac{\sqrt{-1+x}}{\sqrt{1+x}}\,dx$$
$$= -\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x} + \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{3}{8}\int \frac{}{\sqrt{-1+}}$$
$$= -\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x} + \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{3}{8}\cosh^{-1}(x)$$

Mathematica [A] time = 0.07, size = 76, normalized size = 1.10

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(6x^5-8x^4+3x^3-8x^2-18\sqrt{1-x^2}\,\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-9x+16\right)}{24(x-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(16 - 9*x - 8*x^2 + 3*x^3 - 8*x^4 + 6*x^5 - 18*Sqrt
[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/(24*(-1 + x))
```

fricas [A] time = 0.42, size = 46, normalized size = 0.67

$$\frac{1}{24}\left(6x^3-8x^2+9x-16\right)\sqrt{x+1}\,\sqrt{x-1} - \frac{3}{8}\log\left(\sqrt{x+1}\,\sqrt{x-1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 - 8*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(x - 1) - 3/8*log(sqrt(x +
1)*sqrt(x - 1) - x)
```

giac [A] time = 0.33, size = 47, normalized size = 0.68

$$\frac{1}{24}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\,\sqrt{x-1} - \frac{3}{4}\log\left(\sqrt{x+1}-\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(x - 1) - 3
/4*log(sqrt(x + 1) - sqrt(x - 1))
```

maple [A] time = 0.01, size = 76, normalized size = 1.10

$$\frac{\sqrt{x-1}\,\sqrt{x+1}\,\left(6\sqrt{x^2-1}\,x^3-8\sqrt{x^2-1}\,x^2+9\sqrt{x^2-1}\,x+9\ln\left(x+\sqrt{x^2-1}\right)-16\sqrt{x^2-1}\right)}{24\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x-1)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] 1/24*(x-1)^(1/2)*(x+1)^(1/2)*(6*x^3*(x^2-1)^(1/2)-8*(x^2-1)^(1/2)*x^2+9*x*(
x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2))-16*(x^2-1)^(1/2))/(x^2-1)^(1/2)
```

maxima [A] time = 0.88, size = 55, normalized size = 0.80

$$\frac{1}{4}\left(x^2-1\right)^{\frac{3}{2}}x-\frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{x^2-1}\,x-\sqrt{x^2-1}+\frac{3}{8}\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 - 1)^(3/2)*x - 1/3*(x^2 - 1)^(3/2) + 5/8*sqrt(x^2 - 1)*x - sqrt(x^
2 - 1) + 3/8*log(2*x + 2*sqrt(x^2 - 1))
```

mupad [B] time = 12.86, size = 473, normalized size = 6.86

$$\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)}{2}+\frac{\frac{23\left(\sqrt{x-1}-\mathrm{i}\right)^3}{2\left(\sqrt{x+1}-1\right)^3}-\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^4 64\mathrm{i}}{\left(\sqrt{x+1}-1\right)^4}+\frac{333\left(\sqrt{x-1}-\mathrm{i}\right)^5}{2\left(\sqrt{x+1}-1\right)^5}+\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^6 256\mathrm{i}}{3\left(\sqrt{x+1}-1\right)^6}+\frac{671\left(\sqrt{x-1}-\mathrm{i}\right)^7}{2\left(\sqrt{x+1}-1\right)^7}-\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^8 128\mathrm{i}}{3\left(\sqrt{x+1}-1\right)^8}+\cdots}{1+\frac{28\left(\sqrt{x-1}-\mathrm{i}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{56\left(\sqrt{x-1}-\mathrm{i}\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{x-1}-\mathrm{i}\right)^8}{\left(\sqrt{x+1}-1\right)^8}-\frac{56\left(\sqrt{x\cdots}\right.}{\left(\sqrt{x+\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x - 1)^(1/2))/(x + 1)^(1/2),x)
```

```
[Out] (3*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)))/2 + ((23*((x - 1)^(1/2)
 - 1i)^3)/(2*((x + 1)^(1/2) - 1)^3) - (((x - 1)^(1/2) - 1i)^4*64i)/((x + 1)
^(1/2) - 1)^4 + (333*((x - 1)^(1/2) - 1i)^5)/(2*((x + 1)^(1/2) - 1)^5) + ((
(x - 1)^(1/2) - 1i)^6*256i)/(3*((x + 1)^(1/2) - 1)^6) + (671*((x - 1)^(1/2)
 - 1i)^7)/(2*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^8*128i)/(3*((x
+ 1)^(1/2) - 1)^8) + (671*((x - 1)^(1/2) - 1i)^9)/(2*((x + 1)^(1/2) - 1)^9)
 + (((x - 1)^(1/2) - 1i)^10*256i)/(3*((x + 1)^(1/2) - 1)^10) + (333*((x - 1
)^(1/2) - 1i)^11)/(2*((x + 1)^(1/2) - 1)^11) - (((x - 1)^(1/2) - 1i)^12*64i
)/((x + 1)^(1/2) - 1)^12 + (23*((x - 1)^(1/2) - 1i)^13)/(2*((x + 1)^(1/2) -
 1)^13) - (3*((x - 1)^(1/2) - 1i)^15)/(2*((x + 1)^(1/2) - 1)^15) - (3*((x -
 1)^(1/2) - 1i))/(2*((x + 1)^(1/2) - 1)))/((28*((x - 1)^(1/2) - 1i)^4)/((x
+ 1)^(1/2) - 1)^4 - (8*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - (56*
((x - 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + (70*((x - 1)^(1/2) - 1i)^8)
/((x + 1)^(1/2) - 1)^8 - (56*((x - 1)^(1/2) - 1i)^10)/((x + 1)^(1/2) - 1)^1
0 + (28*((x - 1)^(1/2) - 1i)^12)/((x + 1)^(1/2) - 1)^12 - (8*((x - 1)^(1/2)
 - 1i)^14)/((x + 1)^(1/2) - 1)^14 + ((x - 1)^(1/2) - 1i)^16/((x + 1)^(1/2)
- 1)^16 + 1)
```

sympy [A] time = 13.85, size = 83, normalized size = 1.20

$$\frac{(x-1)^{\frac{7}{2}}\sqrt{x+1}}{4}+\frac{5\,(x-1)^{\frac{5}{2}}\sqrt{x+1}}{12}+\frac{11\,(x-1)^{\frac{3}{2}}\sqrt{x+1}}{24}-\frac{3\sqrt{x-1}\,\sqrt{x+1}}{8}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{x-1}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] (x - 1)**(7/2)*sqrt(x + 1)/4 + 5*(x - 1)**(5/2)*sqrt(x + 1)/12 + 11*(x - 1)
**(3/2)*sqrt(x + 1)/24 - 3*sqrt(x - 1)*sqrt(x + 1)/8 + 3*asinh(sqrt(2)*sqrt
(x - 1)/2)/4
```

3.736 $\int x^3 \sqrt{\frac{-1+x}{1+x}}\, dx$

Optimal. Leaf size=69

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}\,x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\,\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

```
[Out] 3/8*arccosh(x)+1/24*(7-2*x)*(-1+x)^(3/2)*(1+x)^(1/2)+1/4*(-1+x)^(3/2)*x^2*(
1+x)^(1/2)-3/8*(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1958, 100, 147, 50, 52}

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}\,x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\,\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx &= \int \frac{\sqrt{-1+x}\,x^3}{\sqrt{1+x}}\,dx\\ &= \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{1}{4}\int \frac{(2-x)\sqrt{-1+x}\,x}{\sqrt{1+x}}\,dx\\ &= \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}-\frac{3}{8}\int \frac{\sqrt{-1+x}}{\sqrt{1+x}}\,dx\\ &= -\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\int \frac{}{\sqrt{-1}}\\ &= -\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\cosh^{-1}\end{aligned}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 1.10

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(6x^5-8x^4+3x^3-8x^2-18\sqrt{1-x^2}\,\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-9x+16\right)}{24(x-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(16 - 9*x - 8*x^2 + 3*x^3 - 8*x^4 + 6*x^5 - 18*Sqrt
[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/(24*(-1 + x))
```

fricas [A] time = 0.43, size = 64, normalized size = 0.93

$$\frac{1}{24}\left(6x^4-2x^3+x^2-7x-16\right)\sqrt{\frac{x-1}{x+1}}+\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^4 - 2*x^3 + x^2 - 7*x - 16)*sqrt((x - 1)/(x + 1)) + 3/8*log(sqrt(
(x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

giac [A] time = 0.34, size = 62, normalized size = 0.90

$$-\frac{3}{8}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\mathrm{sgn}(x+1)+\frac{1}{24}\left(\left(2\left(3x\mathrm{sgn}(x+1)-4\,\mathrm{sgn}(x+1)\right)x+9\,\mathrm{sgn}(x+1)\right)x-16\,\mathrm{sgn}(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] -3/8*log(abs(-x + sqrt(x^2 - 1)))*sgn(x + 1) + 1/24*((2*(3*x*sgn(x + 1) - 4
*sgn(x + 1))*x + 9*sgn(x + 1))*x - 16*sgn(x + 1))*sqrt(x^2 - 1)
```

maple [A] time = 0.01, size = 79, normalized size = 1.14

$$\frac{\sqrt{\frac{x-1}{x+1}}\,(x+1)\left(6\left(x^{2}-1\right)^{\frac{3}{2}}x+15\sqrt{x^{2}-1}\,x+9\ln\left(x+\sqrt{x^{2}-1}\right)-8\left((x-1)(x+1)\right)^{\frac{3}{2}}-24\sqrt{x^{2}-1}\right)}{24\sqrt{(x-1)(x+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((x-1)/(x+1))^(1/2),x)
```

```
[Out] 1/24*((x-1)/(x+1))^(1/2)*(x+1)*(6*x*(x^2-1)^(3/2)-8*((x-1)*(x+1))^(3/2)+15*
(x^2-1)^(1/2)*x-24*(x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2)))/((x-1)*(x+1))^(1/2)
```

maxima [B] time = 0.85, size = 138, normalized size = 2.00

$$-\frac{39\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}-31\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+49\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-9\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^{2}}{(x+1)^{2}}+\frac{4(x-1)^{3}}{(x+1)^{3}}-\frac{(x-1)^{4}}{(x+1)^{4}}-1\right)}+\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(39*((x - 1)/(x + 1))^(7/2) - 31*((x - 1)/(x + 1))^(5/2) + 49*((x - 1
)/(x + 1))^(3/2) - 9*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^
2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 3/8*log(sq
rt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

mupad [B] time = 0.05, size = 119, normalized size = 1.72

$$\frac{3\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}-\frac{\frac{3\sqrt{\frac{x-1}{x+1}}}{4}-\frac{49\left(\frac{x-1}{x+1}\right)^{3/2}}{12}+\frac{31\left(\frac{x-1}{x+1}\right)^{5/2}}{12}-\frac{13\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6(x-1)^{2}}{(x+1)^{2}}-\frac{4(x-1)}{x+1}-\frac{4(x-1)^{3}}{(x+1)^{3}}+\frac{(x-1)^{4}}{(x+1)^{4}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] (3*atanh(((x - 1)/(x + 1))^(1/2)))/4 - ((3*((x - 1)/(x + 1))^(1/2))/4 - (49
*((x - 1)/(x + 1))^(3/2))/12 + (31*((x - 1)/(x + 1))^(5/2))/12 - (13*((x -
1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (4*(
x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{\frac{x-1}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((-1+x)/(1+x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((x - 1)/(x + 1)), x)
```

3.737 $\int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx$

Optimal. Leaf size=15

$$2\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[Out] 2*arctan((-x/(1+x))^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1960, 204}

$$2\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(x/(1 + x))]/x,x]
```

```
[Out] 2*ArcTan[Sqrt[-(x/(1 + x))]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, \sqrt{-\frac{x}{1+x}}\right)\right)\\ &= 2\tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)\end{aligned}$$

Mathematica [B] time = 0.02, size = 32, normalized size = 2.13

$$\frac{2\sqrt{-\frac{x}{x+1}}\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(x/(1 + x))]/x,x]
```

```
[Out] (2*Sqrt[-(x/(1 + x))]*Sqrt[1 + x]*ArcSinh[Sqrt[x]])/Sqrt[x]
```

fricas [A] time = 0.44, size = 13, normalized size = 0.87

$$2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="fricas")
[Out] 2*arctan(sqrt(-x/(x + 1)))
```

giac [A] time = 0.30, size = 20, normalized size = 1.33

$$-\frac{1}{2}\pi\operatorname{sgn}(x+1) - \arcsin(2x+1)\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="giac")
[Out] -1/2*pi*sgn(x + 1) - arcsin(2*x + 1)*sgn(x + 1)
```

maple [B] time = 0.02, size = 33, normalized size = 2.20

$$\frac{\sqrt{-\frac{x}{x+1}}\,(x+1)\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{(x+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/(x+1)*x)^(1/2)/x,x)
[Out] (-1/(x+1)*x)^(1/2)*(x+1)/((x+1)*x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

maxima [A] time = 1.95, size = 13, normalized size = 0.87

$$2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="maxima")
[Out] 2*arctan(sqrt(-x/(x + 1)))
```

mupad [B] time = 0.18, size = 13, normalized size = 0.87

$$2\,\mathrm{atan}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(x + 1))^(1/2)/x,x)
[Out] 2*atan((-x/(x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\frac{x}{x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))**(1/2)/x,x)
[Out] Integral(sqrt(-x/(x + 1))/x, x)
```

3.738 $\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx$

Optimal. Leaf size=18

$$2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

```
[Out] 2*arctan(((1-x)/(1+x))^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1961, 204}

$$2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
```

```
[Out] 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-2-2x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\ &= 2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.89

$$\frac{\sqrt{\frac{1-x}{x+1}}\sqrt{1-x^2}\,\sin^{-1}(x)}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 - x^2]*ArcSin[x])/(-1 + x)
```

fricas [A] time = 0.41, size = 15, normalized size = 0.83

$$2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="fricas")
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

giac [A] time = 0.31, size = 16, normalized size = 0.89

$$-\frac{1}{2}\pi\operatorname{sgn}(x+1)-\arcsin(x)\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="giac")
[Out] -1/2*pi*sgn(x + 1) - arcsin(x)*sgn(x + 1)
```

maple [A] time = 0.02, size = 30, normalized size = 1.67

$$-\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\arcsin(x)}{\sqrt{-(x-1)(x+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)/(x+1))^(1/2)/(x-1),x)
[Out] -(-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*arcsin(x)
```

maxima [A] time = 2.54, size = 15, normalized size = 0.83

$$2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="maxima")
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

mupad [B] time = 3.14, size = 15, normalized size = 0.83

$$2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/(x + 1))^(1/2)/(x - 1),x)
[Out] 2*atan((-(x - 1)/(x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\frac{x-1}{x+1}}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))**(1/2)/(-1+x),x)
[Out] Integral(sqrt(-(x - 1)/(x + 1))/(x - 1), x)
```

3.739 $\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} dx$

Optimal. Leaf size=24

$$\frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

```
[Out] 2*arctan(((b*x+a)/(-b*x+c))^(1/2))/b
```

Rubi [A] time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1961, 12, 203}

$$\frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

```
[Out] (2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} dx &= (2b(a+c))\,\text{Subst}\left(\int \frac{1}{b^2(a+c)\left(1+x^2\right)} dx, x, \sqrt{\frac{a+bx}{c-bx}}\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{a+bx}{c-bx}}\right)}{b}\\
&= \frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}
\end{aligned}$$

Mathematica [B] time = 0.23, size = 93, normalized size = 3.88

$$\frac{2b\sqrt{c-bx}\sqrt{\frac{a+bx}{c-bx}}\sin^{-1}\left(\frac{b\sqrt{c-bx}}{\sqrt{-b}\sqrt{-b(a+c)}}\right)}{(-b)^{3/2}\sqrt{-b(a+c)}\sqrt{\frac{a+bx}{a+c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

```
[Out] (2*b*Sqrt[c - b*x]*Sqrt[(a + b*x)/(c - b*x)]*ArcSin[(b*Sqrt[c - b*x])/(Sqrt
[-b]*Sqrt[-(b*(a + c))])])/((-b)^(3/2)*Sqrt[-(b*(a + c))]*Sqrt[(a + b*x)/(a
 + c)])
```

fricas [A] time = 0.42, size = 24, normalized size = 1.00

$$\frac{2\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

giac [A] time = 0.50, size = 41, normalized size = 1.71

$$-\frac{\arcsin\left(-\frac{2bx+a-c}{a+c}\right)\operatorname{sgn}(-ab-bc)\operatorname{sgn}(bx-c)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -arcsin(-(2*b*x + a - c)/(a + c))*sgn(-a*b - b*c)*sgn(b*x - c)/abs(b)
```

maple [B] time = 0.03, size = 85, normalized size = 3.54

$$-\frac{(bx-c)\sqrt{-\frac{bx+a}{bx-c}}\arctan\left(\frac{\sqrt{b^2}\,(2bx+a-c)}{2\sqrt{-(bx+a)(bx-c)}\,b}\right)}{\sqrt{b^2}\sqrt{-(bx+a)(bx-c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x)
```

```
[Out] -arctan(1/2*(b^2)^(1/2)/b*(2*b*x+a-c)/(-(b*x+a)*(b*x-c))^(1/2))*(b*x-c)*(-(
b*x+a)/(b*x-c))^(1/2)/(b^2)^(1/2)/(-(b*x+a)*(b*x-c))^(1/2)
```

maxima [A] time = 1.94, size = 24, normalized size = 1.00

$$\frac{2\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

mupad [B] time = 0.18, size = 36, normalized size = 1.50

$$-\frac{2\sqrt{-b}\,\operatorname{atanh}\left(\frac{\sqrt{-b}\,\sqrt{\frac{a+b\,x}{c-b\,x}}}{\sqrt{b}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/(c - b*x))^(1/2)/(a + b*x),x)
```

```
[Out] -(2*(-b)^(1/2)*atanh(((-b)^(1/2)*((a + b*x)/(c - b*x))^(1/2))/b^(1/2)))/b^(
3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a+bx}{-bx+c}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt((a + b*x)/(-b*x + c))/(a + b*x), x)
```

3.740 $\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} \, dx$

Optimal. Leaf size=41

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] 2*arctanh(d^(1/2)*((b*x+a)/(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = $\{1961, 12, 208\}$

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} \, dx &= (2(bc-ad))\,\text{Subst}\left(\int \frac{1}{(bc-ad)\left(b-dx^2\right)} \, dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{b-dx^2} \, dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [B] time = 0.08, size = 97, normalized size = 2.37

$$\frac{2\sqrt{bc-ad}\sqrt{\frac{a+bx}{c+dx}}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b\sqrt{d}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

```
[Out] (2*Sqrt[b*c - a*d]*Sqrt[(a + b*x)/(c + d*x)]*Sqrt[(b*(c + d*x))/(b*c - a*d)
]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*Sqrt[d]*Sqrt[a + b*x
])
```

fricas [A] time = 0.42, size = 105, normalized size = 2.56

$$\left[\frac{\sqrt{bd}\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}\,(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)}{bd}, -\frac{2\,\sqrt{-bd}\,\arctan\left(\frac{\sqrt{-bd}\,(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c)*sqrt((b*x + a)/(
d*x + c)))/(b*d), -2*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x + a)/
(d*x + c))/(b*d*x + a*d))/(b*d)]
```

giac [B] time = 0.49, size = 74, normalized size = 1.80

$$-\frac{\sqrt{bd}\log\left(\left|-2\left(\sqrt{bd}\,x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)\operatorname{sgn}(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))*b
*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))*sgn(d*x + c)/(b*d)
```

maple [B] time = 0.03, size = 80, normalized size = 1.95

$$\frac{(dx+c)\sqrt{\frac{bx+a}{dx+c}}\,\ln\left(\frac{2bdx+ad+bc+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}{2\sqrt{bd}}\right)}{\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x)
```

```
[Out] ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/((b*x+a)*(d*x+c))^(1/2)/(b*d)^(1/2)
```

maxima [A] time = 1.73, size = 59, normalized size = 1.44

$$-\frac{\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x + a)/(d*x + c))
 + sqrt(b*d)))/sqrt(b*d)
```

mupad [B] time = 0.20, size = 31, normalized size = 0.76

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/(c + d*x))^(1/2)/(a + b*x),x)
```

```
[Out] (2*atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2)))/(b^(1/2)*d^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt((a + b*x)/(c + d*x))/(a + b*x), x)
```

3.741 $\int \sqrt{-\frac{x}{1+x}}\,dx$

Optimal. Leaf size=32

$$\sqrt{-\frac{x}{x+1}}\,(x+1)-\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[Out] -arctan((-x/(1+x))^(1/2))+(1+x)*(-x/(1+x))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1959, 288, 204}

$$\sqrt{-\frac{x}{x+1}}\,(x+1)-\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(x/(1 + x))],x]
```

```
[Out] Sqrt[-(x/(1 + x))]*(1 + x) - ArcTan[Sqrt[-(x/(1 + x))]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-\frac{x}{1+x}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx,x,\sqrt{-\frac{x}{1+x}}\right)\right)\\ &= \sqrt{-\frac{x}{1+x}}\,(1+x)+\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{-\frac{x}{1+x}}\right)\\ &= \sqrt{-\frac{x}{1+x}}\,(1+x)-\tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.34

$$\frac{\sqrt{-\frac{x}{x+1}}\left(\sqrt{x}(x+1)-\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(x/(1 + x))],x]
```

```
[Out] (Sqrt[-(x/(1 + x))]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[
x]
```

fricas [A] time = 0.41, size = 28, normalized size = 0.88

$$(x+1)\sqrt{-\frac{x}{x+1}}-\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(-x/(x + 1)) - arctan(sqrt(-x/(x + 1)))
```

giac [A] time = 0.35, size = 36, normalized size = 1.12

$$\frac{1}{4}\pi\mathrm{sgn}\left(x+1\right)+\frac{1}{2}\arcsin\left(2x+1\right)\mathrm{sgn}\left(x+1\right)+\sqrt{-x^{2}-x}\,\mathrm{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x + 1) + 1/2*arcsin(2*x + 1)*sgn(x + 1) + sqrt(-x^2 - x)*sgn(x +
 1)
```

maple [A] time = 0.01, size = 46, normalized size = 1.44

$$\frac{\sqrt{-\frac{x}{x+1}}\,(x+1)\left(-\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)+2\sqrt{x^{2}+x}\right)}{2\sqrt{(x+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/(x+1)*x)^(1/2),x)
```

```
[Out] 1/2*(-1/(x+1)*x)^(1/2)*(x+1)*(-ln(x+1/2+(x^2+x)^(1/2))+2*(x^2+x)^(1/2))/((x
+1)*x)^(1/2)
```

maxima [A] time = 1.80, size = 37, normalized size = 1.16

$$-\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x/(x + 1))/(x/(x + 1) - 1) - arctan(sqrt(-x/(x + 1)))
```

mupad [B] time = 3.13, size = 37, normalized size = 1.16

$$-\mathrm{atan}\left(\sqrt{-\frac{x}{x+1}}\right)-\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(x + 1))^(1/2),x)
```

```
[Out] - atan((-x/(x + 1))^(1/2)) - (-x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\frac{x}{x+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt(-x/(x + 1)), x)
```

3.742 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal. Leaf size=38

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

```
[Out] -2*arctan(((1-x)/(1+x))^(1/2))+(1+x)*((1-x)/(1+x))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1959, 288, 204}

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1-x}{1+x}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\
&= \sqrt{\frac{1-x}{1+x}}(1+x)+2\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\\
&= \sqrt{\frac{1-x}{1+x}}(1+x)-2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 1.76

$$\frac{\sqrt{\frac{1-x}{x+1}}\sqrt{x+1}\left(\sqrt{x+1}(x-1)+2\sqrt{1-x}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)],x]
[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*((-1 + x)*Sqrt[1 + x] + 2*Sqrt[1 - x]*Ar
cSin[Sqrt[1 - x]/Sqrt[2]]))/(-1 + x)
```

fricas [A] time = 0.40, size = 32, normalized size = 0.84

$$(x+1)\sqrt{-\frac{x-1}{x+1}}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

giac [A] time = 0.40, size = 29, normalized size = 0.76

$$\frac{1}{2}\pi\mathrm{sgn}\left(x+1\right)+\arcsin(x)\mathrm{sgn}\left(x+1\right)+\sqrt{-x^{2}+1}\,\mathrm{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
[Out] 1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

maple [A] time = 0.01, size = 39, normalized size = 1.03

$$\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\left(\arcsin(x)+\sqrt{-x^{2}+1}\right)}{\sqrt{-(x-1)(x+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)/(x+1))^(1/2),x)
[Out] (-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*((-x^2+1)^(1/2)+arcsin(x))
```

maxima [A] time = 1.85, size = 43, normalized size = 1.13

$$-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(x
 + 1)))
```

mupad [B] time = 0.03, size = 43, normalized size = 1.13

$$-2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right)-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/(x + 1))^(1/2),x)
```

```
[Out] - 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)/
(x + 1) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1-x}{x+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt((1 - x)/(x + 1)), x)
```

3.743 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

Optimal. Leaf size=42

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

```
[Out] 2*a*arctan(((a+x)/(a-x))^(1/2))-(a-x)*((a+x)/(a-x))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1959, 288, 203}

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{a+x}{a-x}}\,dx &= (4a)\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx,x,\sqrt{\frac{a+x}{a-x}}\right)\\
&= -(a-x)\sqrt{\frac{a+x}{a-x}}+(2a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{\frac{a+x}{a-x}}\right)\\
&= -(a-x)\sqrt{\frac{a+x}{a-x}}+2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 83, normalized size = 1.98

$$\frac{\sqrt{x-a}\sqrt{\frac{a+x}{a-x}}\left(2a^{3/2}\sqrt{\frac{a+x}{a}}\sinh^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{2}\sqrt{a}}\right)+\sqrt{x-a}\,(a+x)\right)}{a+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + x)/(a - x)],x]
[Out] (Sqrt[-a + x]*Sqrt[(a + x)/(a - x)]*(Sqrt[-a + x]*(a + x) + 2*a^(3/2)*Sqrt[
(a + x)/a]*ArcSinh[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/(a + x)
```

fricas [A] time = 0.41, size = 38, normalized size = 0.90

$$2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

giac [A] time = 0.45, size = 36, normalized size = 0.86

$$a\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a-x)\operatorname{sgn}(a)-\sqrt{a^2-x^2}\operatorname{sgn}(a-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
[Out] a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

maple [A] time = 0.02, size = 64, normalized size = 1.52

$$-\frac{\sqrt{-\frac{a+x}{-a+x}}\,(-a+x)\left(a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)-\sqrt{a^2-x^2}\right)}{\sqrt{-(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a+x)/(a-x))^(1/2),x)
[Out] -(-(a+x)/(-a+x))^(1/2)*(-a+x)*(a*arctan(x/(a^2-x^2)^(1/2))-(a^2-x^2)^(1/2))
/(-(a+x)*(-a+x))^(1/2)
```

maxima [A] time = 2.01, size = 49, normalized size = 1.17

$$-2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}-\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
[Out] -2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
- x))))
```

mupad [B] time = 3.17, size = 49, normalized size = 1.17

$$2\,a\,\mathrm{atan}\left(\sqrt{\frac{a+x}{a-x}}\right)-\frac{2\,a\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + x)/(a - x))^(1/2),x)
```

```
[Out] 2*a*atan(((a + x)/(a - x))^(1/2)) - (2*a*((a + x)/(a - x))^(1/2))/((a + x)/
(a - x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a+x}{a-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))**(1/2),x)
```

```
[Out] Integral(sqrt((a + x)/(a - x)), x)
```

3.744 $\int \sqrt{\frac{-a+x}{a+x}}\,dx$

Optimal. Leaf size=41

$$\sqrt{-\frac{a-x}{a+x}}\,(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[Out] -2*a*arctanh(((-a+x)/(a+x))^(1/2))+(a+x)*((-a+x)/(a+x))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1959, 288, 206}

$$\sqrt{-\frac{a-x}{a+x}}\,(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + x)/(a + x)],x]
```

```
[Out] Sqrt[-((a - x)/(a + x))]*(a + x) - 2*a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-a+x}{a+x}}\,dx &= (4a)\,\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2}\,dx,x,\sqrt{\frac{-a+x}{a+x}}\right)\\ &= \sqrt{-\frac{a-x}{a+x}}\,(a+x)-(2a)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{\frac{-a+x}{a+x}}\right)\\ &= \sqrt{-\frac{a-x}{a+x}}\,(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)\end{aligned}$$

Mathematica [A] time = 0.07, size = 78, normalized size = 1.90

$$\frac{\sqrt{\frac{x-a}{a+x}}\left(\sqrt{x-a}\,(a+x)-2a^{3/2}\sqrt{\frac{a+x}{a}}\,\sinh^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{2}\sqrt{a}}\right)\right)}{\sqrt{x-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + x)/(a + x)],x]
```

```
[Out] (Sqrt[(-a + x)/(a + x)]*(Sqrt[-a + x]*(a + x) - 2*a^(3/2)*Sqrt[(a + x)/a]*A
rcSinh[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/Sqrt[-a + x]
```

fricas [A] time = 0.40, size = 58, normalized size = 1.41

$$-a\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+a\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)+(a+x)\sqrt{-\frac{a-x}{a+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -a*log(sqrt(-(a - x)/(a + x)) + 1) + a*log(sqrt(-(a - x)/(a + x)) - 1) + (a
 + x)*sqrt(-(a - x)/(a + x))
```

giac [A] time = 0.31, size = 40, normalized size = 0.98

$$a\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\operatorname{sgn}(a+x)+\sqrt{-a^2+x^2}\operatorname{sgn}(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + sqrt(-a^2 + x^2)*sgn(a + x)
```

maple [A] time = 0.01, size = 60, normalized size = 1.46

$$-\frac{\sqrt{\frac{-a+x}{a+x}}\,(a+x)\left(a\ln\left(x+\sqrt{-a^2+x^2}\right)-\sqrt{-a^2+x^2}\right)}{\sqrt{(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-a+x)/(a+x))^(1/2),x)
```

```
[Out] -((-a+x)/(a+x))^(1/2)*(a+x)*(a*ln(x+(-a^2+x^2)^(1/2))-(-a^2+x^2)^(1/2))/((a
+x)*(-a+x))^(1/2)
```

maxima [A] time = 0.91, size = 70, normalized size = 1.71

$$a\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1}-\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] a*(2*sqrt(-(a - x)/(a + x))/((a - x)/(a + x) + 1) - log(sqrt(-(a - x)/(a +
x)) + 1) + log(sqrt(-(a - x)/(a + x)) - 1))
```

mupad [B] time = 0.05, size = 51, normalized size = 1.24

$$\frac{2\,a\,\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1}-2\,a\,\mathrm{atanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - x)/(a + x))^(1/2),x)
```

```
[Out] (2*a*(-(a - x)/(a + x))^(1/2))/((a - x)/(a + x) + 1) - 2*a*atanh((-(a - x)/
(a + x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{-a+x}{a+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)/(a + x)), x)
```

3.745 $\int \sqrt{\frac{a+bx}{c+dx}}\, dx$

Optimal. Leaf size=76

$$\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\,d^{3/2}}$$

```
[Out] -(-a*d+b*c)*arctanh(d^(1/2)*((b*x+a)/(d*x+c))^(1/2)/b^(1/2))/d^(3/2)/b^(1/2
)+(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1959, 288, 208}

$$\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqr
t[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*d^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = (2(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{\left(b-dx^2\right)^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)$$

$$= \frac{\sqrt{\frac{a+bx}{c+dx}}\,(c+dx)}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{b-dx^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)}{d}$$

$$= \frac{\sqrt{\frac{a+bx}{c+dx}}\,(c+dx)}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\,d^{3/2}}$$

Mathematica [A] time = 0.27, size = 123, normalized size = 1.62

$$\frac{\sqrt{\frac{a+bx}{c+dx}}\left(b\sqrt{d}\,(a+bx)(c+dx)-\sqrt{a+bx}\,(bc-ad)^{3/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{bd^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(b*Sqrt[d]*(a + b*x)*(c + d*x) - (b*c - a*d)^(3/
2)*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a +
b*x])/Sqrt[b*c - a*d]]))/(b*d^(3/2)*(a + b*x))
```

fricas [A] time = 0.41, size = 180, normalized size = 2.37

$$\left[-\frac{(bc-ad)\sqrt{bd}\,\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}\,(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)-2\left(bd^2x+bcd\right)\sqrt{\frac{bx+a}{dx+c}}}{2\,bd^2}, \frac{(bc-ad)\sqrt{-bd}\,\arctan\left(\cdots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c - a*d)*sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c
)*sqrt((b*x + a)/(d*x + c))) - 2*(b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c)
))/(b*d^2), ((b*c - a*d)*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x +
 a)/(d*x + c))/(b*d*x + a*d)) + (b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c))
)/(b*d^2)]
```

giac [A] time = 0.57, size = 119, normalized size = 1.57

$$\frac{\sqrt{bdx^2+bcx+adx+ac}\,\text{sgn}\,(dx+c)}{d}+\frac{\left(bc\text{sgn}\,(dx+c)-ad\text{sgn}\,(dx+c)\right)\sqrt{bd}\,\log\left(\left|-2\left(\sqrt{bd}\,x-\sqrt{bdx^2+bcx+a\cdots}\right.\right.\right.}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b*d*x^2 + b*c*x + a*d*x + a*c)*sgn(d*x + c)/d + 1/2*(b*c*sgn(d*x + c)
- a*d*sgn(d*x + c))*sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*
x + a*d*x + a*c))*b*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d^2)
```

maple [B] time = 0.01, size = 152, normalized size = 2.00

$$\frac{\sqrt{\frac{bx+a}{dx+c}}\,(dx+c)\left(ad\ln\left(\frac{2bdx+ad+bc+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}{2\sqrt{bd}}\right)-bc\ln\left(\frac{2bdx+ad+bc+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}{2\sqrt{bd}}\right)+2\sqrt{(bx+a)\,(dx+c)}\cdots\right.}{2\sqrt{(bx+a)\,(dx+c)}\,\sqrt{bd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(d*x+c))^(1/2),x)
```

```
[Out] 1/2*((b*x+a)/(d*x+c))^(1/2)*(d*x+c)*(ln(1/2*(2*b*d*x+a*d+b*c+2*((b*x+a)*(d*
x+c))^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a*d-ln(1/2*(2*b*d*x+a*d+b*c+2*((b*x+a
)*(d*x+c))^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*b*c+2*((b*x+a)*(d*x+c))^(1/2)*(b
*d)^(1/2))/((b*x+a)*(d*x+c))^(1/2)/d/(b*d)^(1/2)
```

maxima [A] time = 1.23, size = 118, normalized size = 1.55

$$\frac{(bc-ad)\sqrt{\frac{bx+a}{dx+c}}}{bd-\frac{(bx+a)d^2}{dx+c}}+\frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{2\sqrt{bd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] (b*c - a*d)*sqrt((b*x + a)/(d*x + c))/(b*d - (b*x + a)*d^2/(d*x + c)) + 1/2
*(b*c - a*d)*log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x + a
)/(d*x + c)) + sqrt(b*d)))/(sqrt(b*d)*d)
```

mupad [B] time = 0.27, size = 90, normalized size = 1.18

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)\,(a\,d-b\,c)}{\sqrt{b}\,d^{3/2}}+\frac{(a\,d-b\,c)\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{b\,d\,\left(\frac{d\,(a+b\,x)}{b\,(c+d\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/(c + d*x))^(1/2),x)
```

```
[Out] (atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2))*(a*d - b*c))/(b^(1/2)
*d^(3/2)) + ((a*d - b*c)*((a + b*x)/(c + d*x))^(1/2))/(b*d*((d*(a + b*x))/(
b*(c + d*x)) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{a+bx}{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x)/(c + d*x)), x)
```

3.746 $\int \sqrt{\frac{-1+x}{5+3x}}\,dx$

Optimal. Leaf size=49

$$\frac{1}{3}\sqrt{x-1}\,\sqrt{3x+5}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\,\sqrt{x-1}\right)}{3\sqrt{3}}$$

```
[Out] -8/9*arcsinh(1/4*6^(1/2)*(-1+x)^(1/2))*3^(1/2)+1/3*(-1+x)^(1/2)*(5+3*x)^(1/
2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1958, 50, 54, 215}

$$\frac{1}{3}\sqrt{x-1}\,\sqrt{3x+5}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\,\sqrt{x-1}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (Sqrt[-1 + x]*Sqrt[5 + 3*x])/3 - (8*ArcSinh[(Sqrt[3/2]*Sqrt[-1 + x])/2])/(3
*Sqrt[3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-1+x}{5+3x}}\,dx &= \int \frac{\sqrt{-1+x}}{\sqrt{5+3x}}\,dx\\ &= \frac{1}{3}\sqrt{-1+x}\,\sqrt{5+3x}-\frac{4}{3}\int \frac{1}{\sqrt{-1+x}\,\sqrt{5+3x}}\,dx\\ &= \frac{1}{3}\sqrt{-1+x}\,\sqrt{5+3x}-\frac{8}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{8+3x^2}}\,dx,x,\sqrt{-1+x}\right)\\ &= \frac{1}{3}\sqrt{-1+x}\,\sqrt{5+3x}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\,\sqrt{-1+x}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [**A**] time = 0.04, size = 76, normalized size = 1.55

$$\frac{3(x-1)\sqrt{3x+5}-8\sqrt{3}\,\sqrt{x-1}\,\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\,\sqrt{x-1}\right)}{9\sqrt{\frac{x-1}{3x+5}}\,\sqrt{3x+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (3*(-1 + x)*Sqrt[5 + 3*x] - 8*Sqrt[3]*Sqrt[-1 + x]*ArcSinh[(Sqrt[3/2]*Sqrt[
-1 + x])/2])/(9*Sqrt[(-1 + x)/(5 + 3*x)]*Sqrt[5 + 3*x])
```

fricas [**A**] time = 0.41, size = 54, normalized size = 1.10

$$\frac{1}{3}\,(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}}+\frac{4}{9}\,\sqrt{3}\,\log\left(\sqrt{3}\,(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}}-3\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x + 5)*sqrt((x - 1)/(3*x + 5)) + 4/9*sqrt(3)*log(sqrt(3)*(3*x + 5)*s
qrt((x - 1)/(3*x + 5)) - 3*x - 1)
```

giac [**B**] time = 0.40, size = 74, normalized size = 1.51

$$-\frac{8}{9}\,\sqrt{3}\,\log(2)\text{sgn}\,(3\,x+5)+\frac{4}{9}\,\sqrt{3}\,\log\left(\left|-\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2+2\,x-5}\right)-1\right|\right)\text{sgn}\,(3\,x+5)+\frac{1}{3}\,\sqrt{3\,x^2+2\,x-5}\,\text{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="giac")
```

```
[Out] -8/9*sqrt(3)*log(2)*sgn(3*x + 5) + 4/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x
- sqrt(3*x^2 + 2*x - 5)) - 1))*sgn(3*x + 5) + 1/3*sqrt(3*x^2 + 2*x - 5)*sgn
(3*x + 5)
```

maple [**B**] time = 0.01, size = 76, normalized size = 1.55

$$-\frac{\sqrt{\frac{x-1}{3x+5}}\,(3x+5)\left(4\sqrt{3}\,\ln\left(\sqrt{3}\,x+\frac{\sqrt{3}}{3}+\sqrt{3x^2+2x-5}\right)-3\sqrt{3x^2+2x-5}\right)}{9\sqrt{(3x+5)\,(x-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-1)/(5+3*x))^(1/2),x)
```

```
[Out] -1/9*((x-1)/(5+3*x))^(1/2)*(5+3*x)*(4*ln(3^(1/2)*x+1/3*3^(1/2)+(3*x^2+2*x-5
)^(1/2))*3^(1/2)-3*(3*x^2+2*x-5)^(1/2))/((5+3*x)*(x-1))^(1/2)
```

maxima [B] time = 1.39, size = 80, normalized size = 1.63

$$\frac{4}{9}\sqrt{3}\log\left(-\frac{\sqrt{3}-3\sqrt{\frac{x-1}{3x+5}}}{\sqrt{3}+3\sqrt{\frac{x-1}{3x+5}}}\right)-\frac{8\sqrt{\frac{x-1}{3x+5}}}{3\left(\frac{3(x-1)}{3x+5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/9*sqrt(3)*log(-(sqrt(3) - 3*sqrt((x - 1)/(3*x + 5)))/(sqrt(3) + 3*sqrt((x
 - 1)/(3*x + 5)))) - 8/3*sqrt((x - 1)/(3*x + 5))/(3*(x - 1)/(3*x + 5) - 1)
```

mupad [B] time = 3.16, size = 57, normalized size = 1.16

$$-\frac{8\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\sqrt{\frac{x-1}{3x+5}}\right)}{9}-\frac{8\sqrt{\frac{x-1}{3x+5}}}{3\left(\frac{3x-3}{3x+5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)/(3*x + 5))^(1/2),x)
```

```
[Out] - (8*3^(1/2)*atanh(3^(1/2)*((x - 1)/(3*x + 5))^(1/2)))/9 - (8*((x - 1)/(3*x
 + 5))^(1/2))/(3*((3*x - 3)/(3*x + 5) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x-1}{3x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)/(3*x + 5)), x)
```

3.747 $\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2} dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}-12\tan^{-1}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)$$

```
[Out] -12*arctan((1+7*x)^(1/2)/(-1+5*x)^(1/2))-(-1+5*x)^(1/2)*(1+7*x)^(1/2)/x
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1958, 94, 93, 204}

$$-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}-12\tan^{-1}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])/x) - 12*ArcTan[Sqrt[1 + 7*x]/Sqrt[-1 + 5*x
]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = \int \frac{\sqrt{-1+5x}}{x^2\sqrt{1+7x}}\,dx$$
$$= -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x} + 6\int \frac{1}{x\sqrt{-1+5x}\sqrt{1+7x}}\,dx$$
$$= -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x} + 12\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)$$
$$= -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x} - 12\tan^{-1}\left(\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)$$

Mathematica [A] time = 0.03, size = 79, normalized size = 1.72

$$\frac{\sqrt{\frac{5x-1}{7x+1}}\left(12x\sqrt{7x+1}\tan^{-1}\left(\frac{\sqrt{5x-1}}{\sqrt{7x+1}}\right) - \sqrt{5x-1}\,(7x+1)\right)}{x\sqrt{5x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] (Sqrt[(-1 + 5*x)/(1 + 7*x)]*(-(Sqrt[-1 + 5*x]*(1 + 7*x)) + 12*x*Sqrt[1 + 7*
x]*ArcTan[Sqrt[-1 + 5*x]/Sqrt[1 + 7*x]]))/(x*Sqrt[-1 + 5*x])
```

fricas [A] time = 0.40, size = 46, normalized size = 1.00

$$\frac{12\,x\arctan\left(\sqrt{\frac{5\,x-1}{7\,x+1}}\right) - (7\,x+1)\sqrt{\frac{5\,x-1}{7\,x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (12*x*arctan(sqrt((5*x - 1)/(7*x + 1))) - (7*x + 1)*sqrt((5*x - 1)/(7*x + 1
)))/x
```

giac [B] time = 0.40, size = 114, normalized size = 2.48

$$\left(\sqrt{35} - 12\,\arctan\left(\frac{1}{7}\sqrt{35}\right)\right)\text{sgn}(7\,x+1) + 12\,\arctan\left(-\sqrt{35}\,x + \sqrt{35\,x^2-2\,x-1}\right)\text{sgn}(7\,x+1) - \frac{2\left(\left(\sqrt{35}\,x - \sqrt{\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (sqrt(35) - 12*arctan(1/7*sqrt(35)))*sgn(7*x + 1) + 12*arctan(-sqrt(35)*x +
 sqrt(35*x^2 - 2*x - 1))*sgn(7*x + 1) - 2*((sqrt(35)*x - sqrt(35*x^2 - 2*x
- 1))*sgn(7*x + 1) + sqrt(35)*sgn(7*x + 1))/((sqrt(35)*x - sqrt(35*x^2 - 2*
x - 1))^2 + 1)
```

maple [B] time = 0.02, size = 103, normalized size = 2.24

$$\frac{\sqrt{\frac{5x-1}{7x+1}}\,(7x+1)\left(-35\sqrt{35x^2-2x-1}\,x^2 - 6x\arctan\left(\frac{x+1}{\sqrt{35x^2-2x-1}}\right) + 2\sqrt{35x^2-2x-1}\,x + \left(35x^2-2x-1\right)^{\frac{3}{2}}\right)}{\sqrt{(5x-1)(7x+1)}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+5*x)/(1+7*x))^(1/2)/x^2,x)
```

```
[Out] ((-1+5*x)/(1+7*x))^(1/2)*(1+7*x)*((35*x^2-2*x-1)^(3/2)-35*(35*x^2-2*x-1)^(1
/2)*x^2+2*(35*x^2-2*x-1)^(1/2)*x-6*arctan((x+1)/(35*x^2-2*x-1)^(1/2))*x)/((
-1+5*x)*(1+7*x))^(1/2)/x
```

maxima [A] time = 1.43, size = 53, normalized size = 1.15

$$-\frac{12\sqrt{\frac{5x-1}{7x+1}}}{\frac{5x-1}{7x+1}+1}+12\arctan\left(\sqrt{\frac{5x-1}{7x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -12*sqrt((5*x - 1)/(7*x + 1))/((5*x - 1)/(7*x + 1) + 1) + 12*arctan(sqrt((5
*x - 1)/(7*x + 1)))
```

mupad [B] time = 3.24, size = 74, normalized size = 1.61

$$12\operatorname{atan}\left(\frac{\sqrt{5}\sqrt{7}\sqrt{35}\sqrt{\frac{5x-1}{7x+1}}}{35}\right)-\frac{12\sqrt{5}\sqrt{7}\sqrt{35}\sqrt{\frac{5x-1}{7x+1}}}{25\left(\frac{7x-\frac{7}{5}}{7x+1}+\frac{7}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x - 1)/(7*x + 1))^(1/2)/x^2,x)
```

```
[Out] 12*atan((5^(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/35) - (12*5^
(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/(25*((7*x - 7/5)/(7*x +
 1) + 7/5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{5x-1}{7x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((5*x - 1)/(7*x + 1))/x**2, x)
```

3.748 $\int \frac{x}{\sqrt{\frac{1-x}{1+x}}\,(1+x)}\,dx$

Optimal. Leaf size=20

$$-\sqrt{\frac{1-x}{x+1}}\,(x+1)$$

[Out] -(1+x)*((1-x)/(1+x))^(1/2)

Rubi [A] time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1962, 12, 383}

$$-\sqrt{\frac{1-x}{x+1}}\,(x+1)$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]

[Out] -(Sqrt[(1 - x)/(1 + x)]*(1 + x))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 1962

```
Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)
*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c
- a*d))/n, Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^
((m + 1)/n - 1)*(u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(
b*e - d*x^q)^((m + 1)/n + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q
)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] &
& IntegerQ[1/n] && IntegersQ[m, r]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{\frac{1-x}{1+x}}\,(1+x)}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1-x^2}{2\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\
&= -\sqrt{\frac{1-x}{1+x}}\,(1+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.95

$$\frac{x-1}{\sqrt{\frac{1-x}{x+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
[Out] (-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

fricas [A] time = 0.40, size = 17, normalized size = 0.85

$$-(x+1)\sqrt{-\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="fricas")
[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

giac [A] time = 0.42, size = 29, normalized size = 1.45

$$-\frac{2}{\sqrt{-\frac{x-1}{x+1}}+\frac{1}{\sqrt{-\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="giac")
[Out] -2/(sqrt(-(x - 1)/(x + 1)) + 1/sqrt(-(x - 1)/(x + 1)))
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/((-x+1)/(x+1))^(1/2),x)
[Out] (x-1)/(-(x-1)/(x+1))^(1/2)
```

maxima [A] time = 0.61, size = 27, normalized size = 1.35

$$\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

mupad [B] time = 0.06, size = 17, normalized size = 0.85

$$-\sqrt{-\frac{x-1}{x+1}}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((-(x - 1)/(x + 1))^(1/2)*(x + 1)),x)
[Out] -(-(x - 1)/(x + 1))^(1/2)*(x + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

3.749 $\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx$

Optimal. Leaf size=18

$$-\left((x+1)\sqrt{\frac{2}{x+1}-1}\right)$$

```
[Out] -(1+x)*(-1+2/(1+x))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {512, 514, 375, 74}

$$-(x+1)\sqrt{\frac{2}{x+1}-1}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
```

```
[Out] -((1 + x)*Sqrt[-1 + 2/(1 + x)])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 512

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIn
tegrand[(x - Coefficient[v, x, 0])^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x], x
, v], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[v, x] && IntegerQ[m]
 && NeQ[v, x]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx &= \text{Subst}\left(\int \frac{-1+x}{\sqrt{-1+\frac{2}{x}}\,x}\,dx,x,1+x\right)\\ &= \text{Subst}\left(\int \frac{1-\frac{1}{x}}{\sqrt{-1+\frac{2}{x}}}\,dx,x,1+x\right)\\ &= -\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{-1+2x}}\,dx,x,\frac{1}{1+x}\right)\\ &= -(1+x)\sqrt{-1+\frac{2}{1+x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.94

$$\frac{x-1}{\sqrt{\frac{2}{x+1}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
[Out] (-1 + x)/Sqrt[-1 + 2/(1 + x)]
```

fricas [A] time = 0.38, size = 17, normalized size = 0.94

$$-(x+1)\sqrt{-\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="fricas")
[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

giac [A] time = 0.26, size = 29, normalized size = 1.61

$$-\frac{2}{\sqrt{-\frac{x-1}{x+1}}+\frac{1}{\sqrt{-\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="giac")
[Out] -2/(sqrt(-(x - 1)/(x + 1)) + 1/sqrt(-(x - 1)/(x + 1)))
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/(-1+2/(x+1))^(1/2),x)
[Out] (x-1)/(-(x-1)/(x+1))^(1/2)
```

maxima [A] time = 0.61, size = 16, normalized size = 0.89

$$\frac{\sqrt{x+1}\,(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x + 1)*(x - 1)/sqrt(-x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x}{(x+1)\,\sqrt{\frac{2}{x+1}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)),x)
```

```
[Out] int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

3.750 $\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx$

Optimal. Leaf size=54

$$\sqrt{x+2}\,\sqrt{x+3}-\sinh^{-1}\left(\sqrt{x+2}\right)+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{\sqrt{x+3}}\right)$$

```
[Out] -arcsinh((2+x)^(1/2))+2*arctanh(2^(1/2)*(2+x)^(1/2)/(3+x)^(1/2))*2^(1/2)+(2
+x)^(1/2)*(3+x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1958, 154, 157, 54, 215, 93, 207}

$$\sqrt{x+2}\,\sqrt{x+3}-\sinh^{-1}\left(\sqrt{x+2}\right)+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{\sqrt{x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] Sqrt[2 + x]*Sqrt[3 + x] - ArcSinh[Sqrt[2 + x]] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[2 + x])/Sqrt[3 + x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx &= \int \frac{x\sqrt{3+x}}{(1+x)\sqrt{2+x}}\,dx \\
&= \sqrt{2+x}\,\sqrt{3+x} + \int \frac{-\frac{5}{2}-\frac{x}{2}}{(1+x)\sqrt{2+x}\,\sqrt{3+x}}\,dx \\
&= \sqrt{2+x}\,\sqrt{3+x} - \frac{1}{2}\int \frac{1}{\sqrt{2+x}\,\sqrt{3+x}}\,dx - 2\int \frac{1}{(1+x)\sqrt{2+x}\,\sqrt{3+x}}\,dx \\
&= \sqrt{2+x}\,\sqrt{3+x} - 4\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \frac{\sqrt{2+x}}{\sqrt{3+x}}\right) - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+\right. \\
&= \sqrt{2+x}\,\sqrt{3+x} - \sinh^{-1}\left(\sqrt{2+x}\right) + 2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{2+x}}{\sqrt{3+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 106, normalized size = 1.96

$$\frac{\sqrt{x+3}\left(x^2+5x+6\right)+2\sqrt{2}\,\sqrt{x+2}\,\sqrt{-(x+3)^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{\sqrt{-x-3}}\right)-\sqrt{x+2}\,(x+3)\sinh^{-1}\left(\sqrt{x+2}\right)}{\sqrt{\frac{x+2}{x+3}}\,(x+3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] (Sqrt[3 + x]*(6 + 5*x + x^2) - Sqrt[2 + x]*(3 + x)*ArcSinh[Sqrt[2 + x]] + 2
*Sqrt[2]*Sqrt[2 + x]*Sqrt[-(3 + x)^2]*ArcTan[(Sqrt[2]*Sqrt[2 + x])/Sqrt[-3
- x]])/(Sqrt[(2 + x)/(3 + x)]*(3 + x)^(3/2))
```

fricas [B] time = 0.41, size = 83, normalized size = 1.54

$$(x+3)\sqrt{\frac{x+2}{x+3}}+\sqrt{2}\,\log\left(\frac{2\,\sqrt{2}\,(x+3)\sqrt{\frac{x+2}{x+3}}+3\,x+7}{x+1}\right)-\frac{1}{2}\,\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\,\log\left(\sqrt{\frac{x+2}{x+3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 3)*sqrt((x + 2)/(x + 3)) + sqrt(2)*log((2*sqrt(2)*(x + 3)*sqrt((x + 2)
/(x + 3)) + 3*x + 7)/(x + 1)) - 1/2*log(sqrt((x + 2)/(x + 3)) + 1) + 1/2*lo
g(sqrt((x + 2)/(x + 3)) - 1)
```

giac [B] time = 0.30, size = 107, normalized size = 1.98

$$-\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sqrt{\frac{x+2}{x+3}}\right|}{2\left(\sqrt{2}+2\sqrt{\frac{x+2}{x+3}}\right)}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\left|\sqrt{\frac{x+2}{x+3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log(1/2*abs(-2*sqrt(2) + 4*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqr
t((x + 2)/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*lo
g(sqrt((x + 2)/(x + 3)) + 1) + 1/2*log(abs(sqrt((x + 2)/(x + 3)) - 1))
```

maple [A] time = 0.02, size = 81, normalized size = 1.50

$$\frac{(x+2)\left(2\sqrt{2}\ \operatorname{arctanh}\left(\frac{(3x+7)\sqrt{2}}{4\sqrt{x^2+5x+6}}\right)-\ln\left(x+\frac{5}{2}+\sqrt{x^2+5x+6}\right)+2\sqrt{x^2+5x+6}\right)}{2\sqrt{\frac{x+2}{x+3}}\ \sqrt{(x+3)(x+2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/((x+2)/(x+3))^(1/2),x)
```

```
[Out] 1/2*(x+2)*(2*2^(1/2)*arctanh(1/4*(7+3*x)*2^(1/2)/(x^2+5*x+6)^(1/2))+2*(x^2+
5*x+6)^(1/2)-ln(x+5/2+(x^2+5*x+6)^(1/2)))/((x+2)/(x+3))^(1/2)/((x+3)*(x+2))
^(1/2)
```

maxima [B] time = 1.22, size = 103, normalized size = 1.91

$$-\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{\frac{x+2}{x+3}}}{\sqrt{2}+2\sqrt{\frac{x+2}{x+3}}}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqrt((x + 2)
/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*log(sqrt((x
 + 2)/(x + 3)) + 1) + 1/2*log(sqrt((x + 2)/(x + 3)) - 1)
```

mupad [B] time = 0.09, size = 62, normalized size = 1.15

$$2\sqrt{2}\ \operatorname{atanh}\left(\sqrt{2}\ \sqrt{\frac{x+2}{x+3}}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\operatorname{atanh}\left(\sqrt{\frac{x+2}{x+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x + 2)/(x + 3))^(1/2)*(x + 1)),x)
```

```
[Out] 2*2^(1/2)*atanh(2^(1/2)*((x + 2)/(x + 3))^(1/2)) - ((x + 2)/(x + 3))^(1/2)/
((x + 2)/(x + 3) - 1) - atanh(((x + 2)/(x + 3))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{\frac{x+2}{x+3}}\ (x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 2)/(x + 3))*(x + 1)), x)
```

3.751 $\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} dx$

Optimal. Leaf size=11

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

[Out] 2/(1+1/x)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {25, 261}

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^(-1)]/(1 + x)^2,x]

[Out] 2/Sqrt[1 + x^(-1)]

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} dx &= \int \frac{1}{\left(1+\frac{1}{x}\right)^{3/2} x^2} dx\\ &= \frac{2}{\sqrt{1+\frac{1}{x}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^(-1)]/(1 + x)^2,x]

[Out] 2/Sqrt[1 + x^(-1)]

fricas [A] time = 0.39, size = 17, normalized size = 1.55

$$\frac{2\,x\sqrt{\frac{x+1}{x}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="fricas")
[Out] 2*x*sqrt((x + 1)/x)/(x + 1)
```

giac [B] time = 0.29, size = 23, normalized size = 2.09

$$\frac{2\,\mathrm{sgn}(x)}{x-\sqrt{x^2+x}+1}-2\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="giac")
[Out] 2*sgn(x)/(x - sqrt(x^2 + x) + 1) - 2*sgn(x)
```

maple [A] time = 0.01, size = 18, normalized size = 1.64

$$\frac{2\sqrt{\frac{x+1}{x}}\,x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2)/(x+1)^2,x)
[Out] 2/(x+1)*x*((x+1)/x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{x}+1}}{(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="maxima")
[Out] integrate(sqrt(1/x + 1)/(x + 1)^2, x)
```

mupad [B] time = 3.13, size = 15, normalized size = 1.36

$$\frac{2\,x\,\sqrt{\frac{1}{x}+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x + 1)^(1/2)/(x + 1)^2,x)
[Out] (2*x*(1/x + 1)^(1/2))/(x + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(1+x)**2,x)
[Out] Integral(sqrt(1 + 1/x)/(x + 1)**2, x)
```

3.752 $\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{\frac{1}{x}+1}\,\sqrt{x}\,\sin^{-1}(1-2x)}{\sqrt{x+1}}$$

```
[Out] arcsin(-1+2*x)*(1+1/x)^(1/2)*x^(1/2)/(1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1448, 26, 53, 619, 216}

$$-\frac{\sqrt{\frac{1}{x}+1}\,\sqrt{x}\,\sin^{-1}(1-2x)}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

```
[Out] -((Sqrt[1 + x^(-1)]*Sqrt[x]*ArcSin[1 - 2*x])/Sqrt[1 + x])
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1448

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*FracPart[q])*(1
 + d/(x^mn*e))^FracPart[q]), Int[x^(mn*q)*(1 + d/(x^mn*e))^q*(a + c*x^n2)^p
, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&  !Integer
Q[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{\sqrt{1+x}}{\sqrt{x}\sqrt{1-x^2}}\,dx}{\sqrt{1+x}}$$
$$= \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx}{\sqrt{1+x}}$$
$$= \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{x-x^2}}\,dx}{\sqrt{1+x}}$$
$$= -\frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right)}{\sqrt{1+x}}$$
$$= -\frac{\sqrt{1+\frac{1}{x}}\sqrt{x}\sin^{-1}(1-2x)}{\sqrt{1+x}}$$

Mathematica [A] time = 0.23, size = 41, normalized size = 1.41

$$-\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{x}}(2x-1)\sqrt{1-x^2}}{2(x^2-1)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

```
[Out] -ArcTan[(Sqrt[(1 + x)/x]*(-1 + 2*x)*Sqrt[1 - x^2])/(2*(-1 + x^2))]
```

fricas [A] time = 0.43, size = 34, normalized size = 1.17

$$-\arctan\left(\frac{2\sqrt{-x^2+1}\,x\sqrt{\frac{x+1}{x}}}{2x^2+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(2*sqrt(-x^2 + 1)*x*sqrt((x + 1)/x)/(2*x^2 + x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

maple [A] time = 0.02, size = 40, normalized size = 1.38

$$\frac{\sqrt{\frac{x+1}{x}}\sqrt{-x^2+1}\,x\arcsin(2x-1)}{(x+1)\sqrt{-(x-1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] ((x+1)/x)^(1/2)*x*(-x^2+1)^(1/2)/(x+1)/(-(x-1)*x)^(1/2)*arcsin(2*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x + 1)^(1/2)/(1 - x^2)^(1/2),x)
```

```
[Out] int((1/x + 1)^(1/2)/(1 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 + 1/x)/sqrt(-(x - 1)*(x + 1)), x)
```

3.753 $\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx$

Optimal. Leaf size=180

$$-\frac{1}{2}\log\left(-\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-x+3}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{7}+\sqrt{3}+1\right)+\frac{1}{1}$$

```
[Out] arctan((3^(1/2)-(-x^2-2*x+3)^(1/2))/x)-1/2*ln((-3+x+3^(1/2)*(-x^2-2*x+3)^(1
/2))/x^2)+1/14*ln(1+3^(1/2)+7^(1/2)-3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x)
*(7-7^(1/2))+1/14*ln(1+3^(1/2)-7^(1/2)-3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))
/x)*(7+7^(1/2))
```

Rubi [A] time = 0.20, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1074, 632, 31, 635, 203, 260}

$$-\frac{1}{2}\log\left(-\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-x+3}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{7}+\sqrt{3}+1\right)+\frac{1}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

```
[Out] ArcTan[(Sqrt[3] - Sqrt[3 - 2*x - x^2])/x] - Log[-((3 - x - Sqrt[3]*Sqrt[3 -
 2*x - x^2])/x^2)]/2 + ((7 + Sqrt[7])*Log[1 + Sqrt[3] - Sqrt[7] - (Sqrt[3]*
(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14 + ((7 - Sqrt[7])*Log[1 + Sqrt[3] +
Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1074

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d
*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c
*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x],
x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*
c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[
{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{3}-2x-\sqrt{3}\,x^2}{\left(1+x^2\right)\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}\,x^2\right)}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= \frac{1}{16}\,\text{Subst}\Bigg(\int \frac{-6+2\sqrt{3}\left(2-\sqrt{3}\right)-4\left(1+\sqrt{3}\right)-\left(-2\sqrt{3}+2\left(2-\sqrt{3}\right)+4\sqrt{3}\,(1+\sqrt{}\right.}{1+x^2}$$

$$= -\left(\frac{1}{2}\left(\sqrt{\frac{3}{7}}\left(1-\sqrt{7}\right)\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{3}+\sqrt{7}+\sqrt{3}\,x}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)\right) -$$

$$= -\tan^{-1}\left(\frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)-\frac{1}{2}\log\left(\frac{-3+x+\sqrt{3}\sqrt{3-2x-x^2}}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)$$

Mathematica [A] time = 0.38, size = 197, normalized size = 1.09

$$\frac{1}{28}\left(-\sqrt{14\left(4+\sqrt{7}\right)}\,\tanh^{-1}\left(\frac{\left(\sqrt{7}-1\right)x+\sqrt{7}+7}{\sqrt{2\left(4+\sqrt{7}\right)}\sqrt{-x^2-2x+3}}\right)-\sqrt{56-14\sqrt{7}}\,\tanh^{-1}\left(\frac{\sqrt{7}\,x+x+\sqrt{7}-7}{\sqrt{2}\sqrt{\left(\sqrt{7}-4\right)\left(x^2+2x-\right.}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

```
[Out] (14*ArcSin[(1 + x)/2] - Sqrt[14*(4 + Sqrt[7])]*ArcTanh[(7 + Sqrt[7] + (-1 +
 Sqrt[7])*x)/(Sqrt[2*(4 + Sqrt[7])]*Sqrt[3 - 2*x - x^2])] - Sqrt[56 - 14*Sq
rt[7]]*ArcTanh[(-7 + Sqrt[7] + x + Sqrt[7]*x)/(Sqrt[2]*Sqrt[(-4 + Sqrt[7])*
(-3 + 2*x + x^2)])] + 7*Log[1 - Sqrt[7] + 2*x] - Sqrt[7]*Log[1 - Sqrt[7] +
2*x] + 7*Log[1 + Sqrt[7] + 2*x] + Sqrt[7]*Log[1 + Sqrt[7] + 2*x])/28
```

fricas [B] time = 0.46, size = 372, normalized size = 2.07

$$\frac{1}{56}\sqrt{7}\log\left(\frac{24\,x^4+62\,x^3-153\,x^2+2\sqrt{7}\left(3\,x^4+x^3-45\,x^2+45\,x\right)-\left(14\,x^3-84\,x^2+\sqrt{7}\left(8\,x^3-30\,x^2+27\,x-\right.\right.}{4\,x^4+8\,x^3-8\,x^2-12\,x+9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 + 2*sqrt(7)*(3*x^4 + x^3 - 45*x
^2 + 45*x) - (14*x^3 - 84*x^2 + sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 126*
x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9))
+ 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 - 2*sqrt(7)*(3*x^4 + x^3 - 45
*x^2 + 45*x) + (14*x^3 - 84*x^2 - sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 12
6*x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
) + 1/28*sqrt(7)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*x - 3
```

```
)) - 1/2*arctan(sqrt(-x^2 - 2*x + 3)*(x + 1)/(x^2 + 2*x - 3)) + 1/4*log(2*x
^2 + 2*x - 3) - 1/8*log((2*sqrt(-x^2 - 2*x + 3)*x + 2*x - 3)/x^2) + 1/8*log
(-(2*sqrt(-x^2 - 2*x + 3)*x - 2*x + 3)/x^2)
```

giac [B] time = 0.52, size = 287, normalized size = 1.59

$$-\frac{1}{28}\sqrt{7}\log\left(\frac{\left|4x-2\sqrt{7}+2\right|}{\left|4x+2\sqrt{7}+2\right|}\right)+\frac{1}{28}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)-\frac{1}{28}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}\right.}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/28*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 1/28
*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/abs
(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 1/28*sqrt(7)*log(
abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) +
2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) + 1/2*arcsin(1/2*x + 1/2) + 1/4*
log(abs(2*x^2 + 2*x - 3)) + 1/4*log(abs(4*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1
) + 3*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 1)) - 1/4*log(abs(-4*(sqrt(-
x^2 - 2*x + 3) - 2)/(x + 1) + (sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 3))
```

maple [B] time = 0.07, size = 551, normalized size = 3.06

$$\frac{\sqrt{7}\,\operatorname{arctanh}\left(\frac{4-\sqrt{7}+\left(-1-\sqrt{7}\right)\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)}{\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)\sqrt{-4\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)^{2}+4\left(-1-\sqrt{7}\right)\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)+8-2\sqrt{7}}}\right)}{-\frac{7}{2}+\frac{7\sqrt{7}}{2}}-\frac{\operatorname{arctanh}\left(\frac{4-\sqrt{7}+\left(-1-\sqrt{7}\right)\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)}{\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)\sqrt{-4\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)^{2}+4\left(-1-\sqrt{7}\right)\left(x+\ldots\right.}}\right.}{4\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2)),x)
```

```
[Out] 1/28*7^(1/2)*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8
+2*7^(1/2))^(1/2)-1/28*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(x
+1))*7^(1/2)+1/4*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(x+1))-1
/7/(1/2+1/2*7^(1/2))*arctanh((4+7^(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(
1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2
))+8+2*7^(1/2))^(1/2))*7^(1/2)-1/4/(1/2+1/2*7^(1/2))*arctanh((4+7^(1/2)+(-1
+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^(1/2))^2+
4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2))-1/28*7^(1/2)*(-4*(x+
1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)+1/
28*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1/2)*(x+1))*7^(1/2)+1/4*arc
sin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1/2)*(x+1))+1/7/(-1/2+1/2*7^(1/2)
)*arctanh((4-7^(1/2)+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(
-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1
/2))*7^(1/2)-1/4/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7^(1/2))*(x+1/2-
1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(
x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2))+1/4*ln(2*x^2+2*x-3)+1/14*7^(1/2)*arc
tanh(1/14*(4*x+2)*7^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\sqrt{-x^2-2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(-x^2 - 2*x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x+\sqrt{-x^2-2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2)),x)
```

```
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\sqrt{-x^2-2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(-x**2 - 2*x + 3)), x)
```

3.754 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4\right)}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{8\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

```
[Out] 8/49*arctanh(1/7*(3-x-x*3^(1/2)-3^(1/2)*(-x^2-2*x+3)^(1/2))/x*7^(1/2))*7^(1
/2)+2/7*(4-3^(1/2)+3*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x)/(2-3^(1/2)-2*(1+3^(1/2
))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x+3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))^2/x^
2)
```

Rubi [A] time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1660, 12, 618, 206}

$$\frac{2\left(\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4\right)}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{8\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

```
[Out] (2*(4 - Sqrt[3] + (3*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(7*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (8*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sqr
t[3 - 2*x - x^2])/(Sqrt[7]*x)])/(7*Sqrt[7])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
```

```
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{-\sqrt{3}+2x+\sqrt{3}x^2}{\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2\right)^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} - \frac{1}{14}\,\text{Subst}\left(\int -\frac{}{2-\sqrt{3}\ldots}\right.$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{8}{7}\,\text{Subst}\left(\int \frac{}{2-\sqrt{3}+2\ldots}\right.$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} - \frac{16}{7}\,\text{Subst}\left(\int \frac{1}{28-x^2}\,dx\ldots\right.$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{8\tanh^{-1}\left(\frac{3-x-\sqrt{3}x-\sqrt{3}\ldots}{\sqrt{7}x}\right.}{7\sqrt{7}}$$

Mathematica [**A**] time = 0.45, size = 306, normalized size = 1.78

$$\frac{1}{98}\left(\frac{7(3-8x)}{2x^2+2x-3}-\frac{14(x-3)\sqrt{-x^2-2x+3}}{2x^2+2x-3}-2\left(1+\sqrt{7}\right)\sqrt{\frac{14}{4+\sqrt{7}}}\log\left(\sqrt{14\left(4+\sqrt{7}\right)}\sqrt{-x^2-2x+3}-\sqrt{7}\ldots\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

```
[Out] ((7*(3 - 8*x))/(-3 + 2*x + 2*x^2) - (14*(-3 + x)*Sqrt[3 - 2*x - x^2])/(-3 +
 2*x + 2*x^2) - 4*Sqrt[7]*Log[-1 + Sqrt[7] - 2*x] + (2*(-1 + Sqrt[7])*Sqrt[
14*(4 + Sqrt[7])]*Log[1 - Sqrt[7] + 2*x])/3 + 4*Sqrt[7]*Log[1 + Sqrt[7] + 2
*x] + 2*(1 + Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[1 + Sqrt[7] + 2*x] - 2*(1
+ Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[7 + 7*Sqrt[7] + 7*x - Sqrt[7]*x + Sqr
t[14*(4 + Sqrt[7])]*Sqrt[3 - 2*x - x^2]] - (2*(-1 + Sqrt[7])*Sqrt[14*(4 + S
qrt[7])]*Log[7 - 7*Sqrt[7] + (7 + Sqrt[7])*x - Sqrt[14]*Sqrt[(-4 + Sqrt[7])
*(-3 + 2*x + x^2)]])/3)/98
```

fricas [**A**] time = 0.45, size = 171, normalized size = 0.99

$$\frac{2\sqrt{7}\left(2x^2+2x-3\right)\log\left(\frac{x^4+44x^3-\sqrt{7}\left(3x^3+x^2-45x+45\right)\sqrt{-x^2-2x+3}+26x^2-276x+207}{4x^4+8x^3-8x^2-12x+9}\right)+4\sqrt{7}\left(2x^2+2x-3\right)\log\left(\frac{2x^2+\sqrt{7}\ldots}{2x\ldots}\right.}{98\left(2x^2+2x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/98*(2*sqrt(7)*(2*x^2 + 2*x - 3)*log((x^4 + 44*x^3 - sqrt(7)*(3*x^3 + x^2
- 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 - 276*x + 207)/(4*x^4 + 8*x^3 -
8*x^2 - 12*x + 9)) + 4*sqrt(7)*(2*x^2 + 2*x - 3)*log((2*x^2 + sqrt(7)*(2*x
+ 1) + 2*x + 4)/(2*x^2 + 2*x - 3)) - 14*sqrt(-x^2 - 2*x + 3)*(x - 3) - 56*x
 + 21)/(2*x^2 + 2*x - 3)
```

giac [B] time = 0.48, size = 350, normalized size = 2.03

$$-\frac{2}{49}\sqrt{7}\log\left(\frac{\left|4x-2\sqrt{7}+2\right|}{\left|4x+2\sqrt{7}+2\right|}\right)+\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)-\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}\right.}{x+}\right.}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2}\right.}{x+1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -2/49*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 2/49
*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/abs
(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 2/49*sqrt(7)*log(
abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) +
2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/14*(8*x - 3)/(2*x^2 + 2*x -
3) - 8/21*(5*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3)
- 2)^2/(x + 1)^2 + 11*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 6)/(8*(sqrt(
-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 +
8*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(
x + 1)^4 - 3)
```

maple [B] time = 0.05, size = 1066, normalized size = 6.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2))^2,x)
```

```
[Out] -3/28*(4*x+2)/(2*x^2+2*x-3)+4/49*7^(1/2)*arctanh(1/14*(4*x+2)*7^(1/2))+1/14
*(-2*x+6)/(2*x^2+2*x-3)-2*(-1/14-1/14*7^(1/2))*(-1/4/(2+1/2*7^(1/2))/(x+1/2
+1/2*7^(1/2))*(-(x+1/2+1/2*7^(1/2))^2+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+2+1/
2*7^(1/2))^(3/2)+1/8*(-1+7^(1/2))/(2+1/2*7^(1/2))*(1/2*(-4*(x+1/2+1/2*7^(1/
2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)+1/2*(-1+7^(1/2)
)*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(x+1))-(2+1/2*7^(1/2))/
(1/2+1/2*7^(1/2))*arctanh((4+7^(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2
+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+
8+2*7^(1/2))^(1/2)))-1/2/(2+1/2*7^(1/2))*(-1/4*(-2*x-2)*(-(x+1/2+1/2*7^(1/2
))^2+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+2+1/2*7^(1/2))^(1/2)-1/8*(-8-2*7^(1/2
)-(-1+7^(1/2))^2)*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(x+1)))
)+1/49*7^(1/2)*(1/4*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(
1/2))+8+2*7^(1/2))^(1/2)+1/4*(-1+7^(1/2))*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7
^(1/2))^2)^(1/2)*(x+1))-1/2*(2+1/2*7^(1/2))/(1/2+1/2*7^(1/2))*arctanh((4+7^
(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^
(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)))-1/49*7^(1/
2)*(1/4*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^
(1/2))^(1/2)+1/4*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(
1/2)*(x+1))-1/2*(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7
```

```
^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2+4
*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)))-2*(-1/14+1/14*7^(1/2
))*(-1/4/(2-1/2*7^(1/2))/(x+1/2-1/2*7^(1/2))*(-(x+1/2-1/2*7^(1/2))^2+(-1-7^
(1/2))*(x+1/2-1/2*7^(1/2))+2-1/2*7^(1/2))^(3/2)+1/8*(-1-7^(1/2))/(2-1/2*7^(
1/2))*(1/2*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2
*7^(1/2))^(1/2)+1/2*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2
)^(1/2)*(x+1))-(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7^
(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2+4*
(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)))-1/2/(2-1/2*7^(1/2))*(
-1/4*(-2*x-2)*(-(x+1/2-1/2*7^(1/2))^2+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+2-1/
2*7^(1/2))^(1/2)-1/8*(-8+2*7^(1/2)-(-1-7^(1/2))^2)*arcsin(1/(2-1/2*7^(1/2)+
1/4*(-1-7^(1/2))^2)^(1/2)*(x+1))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="maxima")
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2))^2,x)
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**2,x)
[Out] Integral((x + sqrt(-x**2 - 2*x + 3))**(-2), x)
```

3.755 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3} \, dx$

Optimal. Leaf size=307

$$-\frac{4\left(\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-5\sqrt{3}+9\right)}{21\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}+\frac{2\left(-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}\right.}{147\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2\right.}{x}\right.}$$

```
[Out] 12/343*arctanh(1/7*(3-x-x*3^(1/2)-3^(1/2)*(-x^2-2*x+3)^(1/2))/x*7^(1/2))*7^
(1/2)-4/21*(9-5*3^(1/2)+(21+5*3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x)/(2-3
^(1/2)-2*(1+3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x+3^(1/2)*(3^(1/2)-(-x^2-
2*x+3)^(1/2))^2/x^2)^2+2/147*(18-43*3^(1/2)-(18+49*3^(1/2))*(3^(1/2)-(-x^2-
2*x+3)^(1/2))/x)/(2-3^(1/2)-2*(1+3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x+3^
(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))^2/x^2)
```

Rubi [A] time = 0.25, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1660, 12, 618, 206}

$$-\frac{4\left(\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-5\sqrt{3}+9\right)}{21\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}+\frac{2\left(-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}\right.}{147\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2\right.}{x}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] (-4*(9 - 5*Sqrt[3] + ((21 + 5*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))
/(21*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (
Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)^2) + (2*(18 - 43*Sqrt[3] -
((18 + 49*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(147*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (12*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sq
rt[3 - 2*x - x^2])/(Sqrt[7]*x)])/(49*Sqrt[7])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{3}-2x-2x^3-\sqrt{3}\,x^4}{\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}\,x^2\right)^3}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} - \frac{1}{28}\,\text{Subst}\left(\int \frac{-\frac{8}{3}\left(\cdots\right.}{\left(2-\sqrt{\cdots}\right.}\right.$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\cdots\right.}{147\left(2-\sqrt{3}-\frac{2(1+\cdots}{\cdots}\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\cdots\right.}{147\left(2-\sqrt{3}-\frac{2(1+\cdots}{\cdots}\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\cdots\right.}{147\left(2-\sqrt{3}-\frac{2(1+\cdots}{\cdots}\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\cdots\right.}{147\left(2-\sqrt{3}-\frac{2(1+\cdots}{\cdots}\right.}$$

Mathematica [A] time = 1.11, size = 333, normalized size = 1.08

$$\frac{\frac{7(37-24x)}{2x^2+2x-3}+\frac{98(11x-12)}{\left(2x^2+2x-3\right)^2}-6\left(1+\sqrt{7}\right)\sqrt{\frac{14}{4+\sqrt{7}}}\,\log\left(\sqrt{14\left(4+\sqrt{7}\right)}\,\sqrt{-x^2-2x+3}-\sqrt{7}\,x+7x+7\sqrt{7}+7\right)-2\left(\sqrt{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] ((98*(-12 + 11*x))/(-3 + 2*x + 2*x^2)^2 + (7*(37 - 24*x))/(-3 + 2*x + 2*x^2
) - (14*Sqrt[3 - 2*x - x^2]*(-15 - 83*x + 58*x^2 + 34*x^3))/(-3 + 2*x + 2*x
^2)^2 - 12*Sqrt[7]*Log[-1 + Sqrt[7] - 2*x] + 2*(-1 + Sqrt[7])*Sqrt[14*(4 +
Sqrt[7])]*Log[1 - Sqrt[7] + 2*x] + 12*Sqrt[7]*Log[1 + Sqrt[7] + 2*x] + 6*(1
 + Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[1 + Sqrt[7] + 2*x] - 6*(1 + Sqrt[7])
*Sqrt[14/(4 + Sqrt[7])]*Log[7 + 7*Sqrt[7] + 7*x - Sqrt[7]*x + Sqrt[14*(4 +
Sqrt[7])]*Sqrt[3 - 2*x - x^2]] - 2*(-1 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7])]*Lo
g[7 - 7*Sqrt[7] + (7 + Sqrt[7])*x - Sqrt[14]*Sqrt[(-4 + Sqrt[7])*(-3 + 2*x
+ x^2)]])/1372
```

fricas [A] time = 0.43, size = 223, normalized size = 0.73

$$-\frac{336\,x^3 - 6\,\sqrt{7}\left(4\,x^4 + 8\,x^3 - 8\,x^2 - 12\,x + 9\right)\log\left(\frac{x^4+44\,x^3-\sqrt{7}\left(3\,x^3+x^2-45\,x+45\right)\sqrt{-x^2-2\,x+3}+26\,x^2-276\,x+207}{4\,x^4+8\,x^3-8\,x^2-12\,x+9}\right) - 12\,\sqrt{}}{1372\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/1372*(336*x^3 - 6*sqrt(7)*(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)*log((x^4 +
44*x^3 - sqrt(7)*(3*x^3 + x^2 - 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 -
276*x + 207)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)) - 12*sqrt(7)*(4*x^4 + 8*x^
3 - 8*x^2 - 12*x + 9)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*
x - 3)) - 182*x^2 + 14*(34*x^3 + 58*x^2 - 83*x - 15)*sqrt(-x^2 - 2*x + 3) -
 2100*x + 1953)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
```

giac [A] time = 0.56, size = 452, normalized size = 1.47

$$-\frac{3}{343}\,\sqrt{7}\,\log\left(\frac{\left|4\,x - 2\,\sqrt{7} + 2\right|}{\left|4\,x + 2\,\sqrt{7} + 2\right|}\right) + \frac{3}{343}\,\sqrt{7}\,\log\left(\frac{\left|-2\,\sqrt{7} + \frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1} + 4\right|}{\left|2\,\sqrt{7} + \frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1} + 4\right|}\right) - \frac{3}{343}\,\sqrt{7}\,\log\left(\frac{\left|-2\,\sqrt{7} + \frac{2\left(\sqrt{-}\right.}{}\right.}{\left|2\,\sqrt{7} + \frac{2\left(\sqrt{-}\right.}{}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/343*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 3/3
43*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/a
bs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 3/343*sqrt(7)*l
og(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7)
 + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/196*(48*x^3 - 26*x^2 - 30
0*x + 279)/(2*x^2 + 2*x - 3)^2 + 4/441*(231*(sqrt(-x^2 - 2*x + 3) - 2)/(x +
 1) + 3286*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 4441*(sqrt(-x^2 - 2*x +
 3) - 2)^3/(x + 1)^3 - 18906*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x + 1)^4 - 12487
*(sqrt(-x^2 - 2*x + 3) - 2)^5/(x + 1)^5 + 946*(sqrt(-x^2 - 2*x + 3) - 2)^6/
(x + 1)^6 + 1977*(sqrt(-x^2 - 2*x + 3) - 2)^7/(x + 1)^7 - 414)/(8*(sqrt(-x^
2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 + 8*(
sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x +
 1)^4 - 3)^2
```

maple [B] time = 0.07, size = 5984, normalized size = 19.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 2\,x + 3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 2\,x + 3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2))^3,x)
```

```
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 2x + 3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**3,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 2*x + 3))**(-3), x)
```

3.756 $\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+2\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{2}\log\left(\sqrt{x^2-2x-3}+x\right)$$

```
[Out] 2*ln(1-x-(x^2-2*x-3)^(1/2))-3/2*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)^
(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+2\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{2}\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 2*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - (
3*Log[x + Sqrt[-3 - 2*x + x^2]])/2
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{-3-2x+x^2}{x(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)\\
&= 2\,\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}-\frac{3}{4x}\right)dx, x, x+\sqrt{-3-2x+x^2}\right)\\
&= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+2\log\left(1-x-\sqrt{-3-2x+x^2}\right)-\frac{3}{2}\log\left(x+\sqrt{-3-2x+x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.91

$$2\left(\frac{1}{\sqrt{x^2-2x-3}+x-1}+\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{4}\log\left(\sqrt{x^2-2x-3}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

```
[Out] 2*((-1 + x + Sqrt[-3 - 2*x + x^2])^(-1) + Log[1 - x - Sqrt[-3 - 2*x + x^2]]
 - (3*Log[x + Sqrt[-3 - 2*x + x^2]])/4)
```

fricas [A] time = 0.43, size = 77, normalized size = 1.18

$$\frac{1}{2}x-\frac{1}{2}\sqrt{x^2-2x-3}-\frac{3}{4}\log(2x+3)-\frac{5}{4}\log\left(-x+\sqrt{x^2-2x-3}+1\right)+\frac{3}{4}\log\left(-x+\sqrt{x^2-2x-3}\right)-\frac{3}{4}\log\left(-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(2*x + 3) - 5/4*log(-x + sqrt(x^2
- 2*x - 3) + 1) + 3/4*log(-x + sqrt(x^2 - 2*x - 3)) - 3/4*log(-x + sqrt(x^2
 - 2*x - 3) - 3)
```

giac [A] time = 0.37, size = 81, normalized size = 1.25

$$\frac{1}{2}x-\frac{1}{2}\sqrt{x^2-2x-3}-\frac{3}{4}\log(|2x+3|)-\frac{5}{4}\log\left(\left|-x+\sqrt{x^2-2x-3}+1\right|\right)+\frac{3}{4}\log\left(\left|-x+\sqrt{x^2-2x-3}\right|\right)-\frac{3}{4}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(abs(2*x + 3)) - 5/4*log(abs(-x +
sqrt(x^2 - 2*x - 3) + 1)) + 3/4*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3/4*lo
g(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

maple [A] time = 0.01, size = 71, normalized size = 1.09

$$\frac{x}{2}+\frac{3\,\mathrm{arctanh}\left(\frac{-\frac{10x}{3}-2}{\sqrt{-20x+4\left(x+\frac{3}{2}\right)^2-21}}\right)}{4}-\frac{3\ln(2x+3)}{4}+\frac{5\ln\left(x-1+\sqrt{-5x+\left(x+\frac{3}{2}\right)^2-\frac{21}{4}}\right)}{4}-\frac{\sqrt{-20x+4\left(x+\frac{3}{2}\right)^2-}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2)),x)
```

```
[Out] -1/4*(4*(x+3/2)^2-20*x-21)^(1/2)+5/4*ln(-1+x+((x+3/2)^2-5*x-21/4)^(1/2))+3/
4*arctanh(2/3*(-3-5*x)/(4*(x+3/2)^2-20*x-21)^(1/2))+1/2*x-3/4*ln(2*x+3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x+\sqrt{x^2-2x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(x^2 - 2*x - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\frac{x}{2}-\frac{3\ln\left(x+\frac{3}{2}\right)}{4}-\int\frac{\sqrt{x^2-2x-3}}{2x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2)),x)
```

```
[Out] x/2 - (3*log(x + 3/2))/4 - int((x^2 - 2*x - 3)^(1/2)/(2*x + 3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\sqrt{x^2-2x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2)),x)
[Out] Integral(1/(x + sqrt(x**2 - 2*x - 3)), x)
```

3.757 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2} dx$

Optimal. Leaf size=83

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

```
[Out] 4*ln(1-x-(x^2-2*x-3)^(1/2))-4*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)^(1
/2))+3/2/(x+(x^2-2*x-3)^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])) + 4*Lo
g[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2*x + x^2]]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2} dx &= 2\,\text{Subst}\left(\int \frac{-3-2x+x^2}{x^2(-2+2x)^2} dx, x, x+\sqrt{-3-2x+x^2}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{2}{-1+x}-\frac{3}{4x^2}-\frac{2}{x}\right) dx, x, x+\sqrt{-3-2x+x^2}\right)\\ &= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{2\left(x+\sqrt{-3-2x+x^2}\right)}+4\log\left(1-x-\sqrt{-3-2x+x^2}\right.\end{aligned}$$

Mathematica [A] time = 0.03, size = 79, normalized size = 0.95

$$\frac{2}{\sqrt{x^2-2x-3}+x-1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

```
[Out] 2/(-1 + x + Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])) + 4*Lo
g[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2*x + x^2]]
```

fricas [A] time = 0.40, size = 97, normalized size = 1.17

$$\frac{4x^2 - 8(2x+3)\log\left(x^2 - \sqrt{x^2-2x-3}(x+1) - 3\right) - 8(2x+3)\log(2x+3) + 8(2x+3)\log\left(-x + \sqrt{x^2-2\right.}}{4(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^2 - 8*(2*x + 3)*log(x^2 - sqrt(x^2 - 2*x - 3)*(x + 1) - 3) - 8*(2*
x + 3)*log(2*x + 3) + 8*(2*x + 3)*log(-x + sqrt(x^2 - 2*x - 3)) - 4*sqrt(x^
2 - 2*x - 3)*(x + 3) + 2*x - 15)/(2*x + 3)
```

giac [B] time = 0.51, size = 143, normalized size = 1.72

$$\frac{1}{2}x - \frac{1}{2}\sqrt{x^2-2x-3} - \frac{3\left(5x - 5\sqrt{x^2-2x-3} + 3\right)}{4\left(\left(x-\sqrt{x^2-2x-3}\right)^2 + 3x - 3\sqrt{x^2-2x-3}\right)} - \frac{9}{4(2x+3)} - 2\log(|2x+3|) - 2\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*(5*x - 5*sqrt(x^2 - 2*x - 3) + 3)/((x
 - sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3)) - 9/4/(2*x + 3) -
2*log(abs(2*x + 3)) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)) + 2*log(abs(
-x + sqrt(x^2 - 2*x - 3))) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

maple [A] time = 0.02, size = 118, normalized size = 1.42

$$\frac{x}{2} + 2\,\mathrm{arctanh}\left(\frac{-\frac{10x}{3} - 2}{\sqrt{-20x + 4\left(x+\frac{3}{2}\right)^2 - 21}}\right) - 2\ln(2x+3) + 2\ln\left(x - 1 + \sqrt{-5x + \left(x+\frac{3}{2}\right)^2 - \frac{21}{4}}\right) - \frac{9}{4(2x+3)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2))^2,x)
```

```
[Out] -2*ln(2*x+3)+1/2*x-9/4/(2*x+3)-2/3*(-20*x+4*(x+3/2)^2-21)^(1/2)+2*ln(x-1+(-
5*x+(x+3/2)^2-21/4)^(1/2))+2*arctanh(2/3*(-5*x-3)/(-20*x+4*(x+3/2)^2-21)^(1
/2))-1/3/(x+3/2)*(-5*x+(x+3/2)^2-21/4)^(3/2)+1/6*(2*x-2)*(-5*x+(x+3/2)^2-21
/4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x + \sqrt{x^2-2x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2))^2,x)
[Out] int(1/(x + (x^2 - 2*x - 3)^(1/2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**2,x)
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-2), x)
```

3.758 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\right.$$

```
[Out] 6*ln(1-x-(x^2-2*x-3)^(1/2))-6*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)^(1
/2))+3/4/(x+(x^2-2*x-3)^(1/2))^2+4/(x+(x^2-2*x-3)^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2])^2) + 4/
(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 6*Log[x
+ Sqrt[-3 - 2*x + x^2]]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx &= 2\,\text{Subst}\left(\int \frac{-3-2x+x^2}{x^3(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{3}{-1+x}-\frac{3}{4x^3}-\frac{2}{x^2}-\frac{3}{x}\right)dx, x, x+\sqrt{-3-2x+x^2}\right.\\ &= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{4\left(x+\sqrt{-3-2x+x^2}\right)^2}+\frac{4}{x+\sqrt{-3-2x+x^2}}+6\end{aligned}$$

Mathematica [A] time = 0.05, size = 97, normalized size = 0.96

$$\frac{2}{\sqrt{x^2-2x-3}+x-1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\sqrt{x^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] 2/(-1 + x + Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2])^2) + 4/
(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 6*Log[x
+ Sqrt[-3 - 2*x + x^2]]
```

fricas [A] time = 0.40, size = 129, normalized size = 1.28

$$\frac{8\,x^3-10\,x^2-12\left(4\,x^2+12\,x+9\right)\log\left(x^2-\sqrt{x^2-2\,x-3}\,(x+1)-3\right)-12\left(4\,x^2+12\,x+9\right)\log\left(2\,x+3\right)+12\left(4\right.}{4\left(4\,x^2+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*x^3 - 10*x^2 - 12*(4*x^2 + 12*x + 9)*log(x^2 - sqrt(x^2 - 2*x - 3)*(
x + 1) - 3) - 12*(4*x^2 + 12*x + 9)*log(2*x + 3) + 12*(4*x^2 + 12*x + 9)*lo
g(-x + sqrt(x^2 - 2*x - 3)) - 2*(4*x^2 + 31*x + 33)*sqrt(x^2 - 2*x - 3) - 1
56*x - 171)/(4*x^2 + 12*x + 9)
```

giac [B] time = 0.42, size = 184, normalized size = 1.82

$$\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-2\,x-3}-\frac{104\left(x-\sqrt{x^2-2\,x-3}\right)^3+315\left(x-\sqrt{x^2-2\,x-3}\right)^2+162\,x-162\,\sqrt{x^2-2\,x-3}+27}{8\left(\left(x-\sqrt{x^2-2\,x-3}\right)^2+3\,x-3\,\sqrt{x^2-2\,x-3}\right)^2}-\frac{9}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 1/8*(104*(x - sqrt(x^2 - 2*x - 3))^3 + 31
5*(x - sqrt(x^2 - 2*x - 3))^2 + 162*x - 162*sqrt(x^2 - 2*x - 3) + 27)/((x -
 sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3))^2 - 9/8*(16*x + 21)/
(2*x + 3)^2 - 3*log(abs(2*x + 3)) - 3*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)
) + 3*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3*log(abs(-x + sqrt(x^2 - 2*x -
3) - 3))
```

maple [A] time = 0.03, size = 146, normalized size = 1.45

$$\frac{x}{2}+3\,\mathrm{arctanh}\left(\frac{-\frac{10x}{3}-2}{\sqrt{-20x+4\left(x+\frac{3}{2}\right)^2-21}}\right)-3\ln\left(2x+3\right)+3\ln\left(x-1+\sqrt{-5x+\left(x+\frac{3}{2}\right)^2-\frac{21}{4}}\right)-\frac{9}{2x+3}+\frac{27}{8\left(2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2))^3,x)
```

```
[Out] -9/(2*x+3)-3*ln(2*x+3)+1/2*x+27/8/(2*x+3)^2-1/2/(x+3/2)*(-5*x+(x+3/2)^2-21/
4)^(3/2)-(-20*x+4*(x+3/2)^2-21)^(1/2)+3*arctanh(2/3*(-5*x-3)/(-20*x+4*(x+3/
2)^2-21)^(1/2))+1/4*(2*x-2)*(-5*x+(x+3/2)^2-21/4)^(1/2)+3*ln(x-1+(-5*x+(x+3
/2)^2-21/4)^(1/2))+1/4/(x+3/2)^2*(-5*x+(x+3/2)^2-21/4)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2))^3,x)
```

```
[Out] int(1/(x + (x^2 - 2*x - 3)^(1/2))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**3,x)
```

```
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-3), x)
```

3.759 $\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx$

Optimal. Leaf size=108

$$\frac{1}{2}\log(x+3)+\frac{1}{2}\log\left(\frac{\sqrt{-x-1}\,x+\sqrt{x+3}\,x+3\sqrt{-x-1}}{(x+3)^{3/2}}\right)-\tan^{-1}\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\sqrt{2}\,\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)$$

```
[Out] -arctan((-1-x)^(1/2)/(3+x)^(1/2))+1/2*ln(3+x)+1/2*ln((3*(-1-x)^(1/2)+x*(-1-
x)^(1/2)+x*(3+x)^(1/2))/(3+x)^(3/2))-arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/
2))*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {12, 1023, 634, 618, 204, 628, 635, 203, 260}

$$\frac{1}{2}\log(x+3)+\frac{1}{2}\log\left(\frac{\sqrt{-x-1}\,x+\sqrt{x+3}\,x+3\sqrt{-x-1}}{(x+3)^{3/2}}\right)-\tan^{-1}\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\sqrt{2}\,\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

```
[Out] -ArcTan[Sqrt[-1 - x]/Sqrt[3 + x]] - Sqrt[2]*ArcTan[(1 - (3*Sqrt[-1 - x])/Sq
rt[3 + x])/Sqrt[2]] + Log[3 + x]/2 + Log[(3*Sqrt[-1 - x] + Sqrt[-1 - x]*x +
 x*Sqrt[3 + x])/(3 + x)^(3/2)]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1023

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rubi steps

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx = 2\,\text{Subst}\left(\int \frac{2x}{(1+x^2)(1-2x+3x^2)}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x}{(1+x^2)(1-2x+3x^2)}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{-2-2x}{1+x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{2+6x}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right.$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{-2+6x}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1}}{\sqrt{3}}\right.$$
$$= -\tan^{-1}\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + \frac{1}{2}\log(3+x) + \frac{1}{2}\log\left(\frac{3\sqrt{-1-x}+\sqrt{-1-x}\,x+x\sqrt{3+x}}{(3+x)^{3/2}}\right)$$
$$= -\tan^{-1}\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) - \sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right) + \frac{1}{2}\log(3+x) + \frac{1}{2}\log\left(\frac{3\sqrt{-1-x}}{}\right.$$

Mathematica [C] time = 0.42, size = 187, normalized size = 1.73

$$\frac{1}{4}\left(\log\left(2x^2+4x+3\right) + i\sqrt{1-2i\sqrt{2}}\tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - i\sqrt{1+2i\sqrt{2}}\tanh^{-1}\left(\frac{(2-i\sqrt{2}}{\sqrt{2+4i\sqrt{2}}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

```
[Out] (2*ArcSin[2 + x] - 2*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] + I*Sqrt[1 - (2*I)*Sqr
t[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + 2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[
2]]*Sqrt[-3 - 4*x - x^2])] - I*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*S
qrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])]
 + Log[3 + 4*x + 2*x^2])/4
```

fricas [B] time = 0.45, size = 187, normalized size = 1.73

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4}}{2(2x+3)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x +
3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/4*sqrt(2)*arctan(-1/2*(sqrt(
2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 -
4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/4*log(2*x^2 + 4*x + 3) - 1/8*log(-(2*
sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/8*log((2*sqrt(-x^2 - 4*x - 3)*x
- 4*x - 3)/x^2)
```

giac [B] time = 0.45, size = 197, normalized size = 1.82

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4}}{x+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*arcsin(x + 2) + 1/4*log(2*x^2
+ 4*x + 3) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 -
4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x +
2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

maple [B] time = 0.03, size = 370, normalized size = 3.43

$$\frac{\arcsin(x+2)}{2}-\frac{\sqrt{2}\,\arctan\left(\frac{(4x+4)\sqrt{2}}{4}\right)}{2}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)}{3\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}+\frac{\ln\left(2x^2+4x+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2)),x)
```

```
[Out] 1/2*arcsin(x+2)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*a
rctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/
(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2
-x)+1)+1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(
-3/2-x)+1)*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2
-12)^(1/2)*2^(1/2))-1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2
```

```
)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*x
^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-
3/2-x)+1)+1/4*ln(2*x^2+4*x+3)-1/2*2^(1/2)*arctan(1/4*(4+4*x)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x + sqrt(-x^2 - 4*x - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x+\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2)),x)
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2)),x)
[Out] Integral(1/(x + sqrt(-x**2 - 4*x - 3)), x)
```

3.760 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx$

Optimal. Leaf size=87

$$\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}+\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)+(1-(-1-x)^(1
/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {12, 638, 618, 204}

$$\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}+\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] (1 - Sqrt[-1 - x]/Sqrt[3 + x])/(1 - (3*(1 + x))/(3 + x) - (2*Sqrt[-1 - x])/
Sqrt[3 + x]) + ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]]/Sqrt[2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx &= 2\,\text{Subst}\left(\int -\frac{2x}{\left(1-2x+3x^2\right)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= -\left(4\,\text{Subst}\left(\int \frac{x}{\left(1-2x+3x^2\right)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} - \text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + 2\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+\frac{6\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + \frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 1.67, size = 881, normalized size = 10.13

$$\frac{1}{16}\left(\frac{8(x+3)}{2x^2+4x+3}+4\sqrt{2}\tan^{-1}\left(\sqrt{2}(x+1)\right)-\frac{2i\left(-2i+\sqrt{2}\right)\tan^{-1}\left(\frac{(x+2)\left(2(9\right.}{\left(8i+6\sqrt{2}\right)x^3+\left(-6\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}+8\sqrt{2}+36i\right)}\right.}{\sqrt{1+}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] ((8*(3 + x))/(3 + 4*x + 2*x^2) + (8*(3 + 2*x)*Sqrt[-3 - 4*x - x^2])/(3 + 4*
x + 2*x^2) + 4*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] - ((2*I)*(-2*I + Sqrt[2])*Ar
cTan[((2 + x)*(3*(5 + (4*I)*Sqrt[2]) + 16*(2 + I*Sqrt[2])*x + 2*(9 + (2*I)*
Sqrt[2])*x^2))/(12*I - 6*Sqrt[2] + (8*I + 6*Sqrt[2])*x^3 - 9*Sqrt[1 + (2*I)
*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x*(40*I - 5*Sqrt[2] - 12*Sqrt[1 + (2*I)*Sq
rt[2]]*Sqrt[-3 - 4*x - x^2]) + x^2*(36*I + 8*Sqrt[2] - 6*Sqrt[1 + (2*I)*Sqr
t[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]] + (2*(2*I + Sqrt[2])
*ArcTanh[((2 + x)*(3*(5*I + 4*Sqrt[2]) + 16*(2*I + Sqrt[2])*x + 2*(9*I + 2*
Sqrt[2])*x^2))/(-5*(8*I + Sqrt[2])*x + (-8*I + 6*Sqrt[2])*x^3 - 12*Sqrt[1 -
 (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2] + x^2*(-36*I + 8*Sqrt[2] - 6*Sqrt[1
- (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]) - 3*(4*I + 2*Sqrt[2] + 3*Sqrt[1 - (2
*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] - ((-2*I + Sq
rt[2])*Log[4*(3 + 4*x + 2*x^2)^2])/Sqrt[1 + (2*I)*Sqrt[2]] - ((2*I + Sqrt[2
])*Log[4*(3 + 4*x + 2*x^2)^2])/Sqrt[1 - (2*I)*Sqrt[2]] + ((2*I + Sqrt[2])*L
og[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + (2 + (2*I)*Sqrt[2])*x^2 - 2*Sqrt[
2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x*(4 + (8*I)*Sqrt[2] - 2*Sqrt[2 -
 (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] + ((-2*I +
 Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + (2 - (2*I)*Sqrt[2])*x^
2 - 2*Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] - 2*x*(-2 + (4*I)*Sqrt[2
] + Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]
])/16
```

fricas [A] time = 0.42, size = 121, normalized size = 1.39

$$\frac{2\sqrt{2}\left(2x^2+4x+3\right)\arctan\left(\sqrt{2}(x+1)\right)-\sqrt{2}\left(2x^2+4x+3\right)\arctan\left(\frac{\sqrt{2}\left(6x^2+20x+15\right)\sqrt{-x^2-4x-3}}{4\left(2x^3+11x^2+18x+9\right)}\right)+4\sqrt{-x^2-4x}}{8\left(2x^2+4x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(2)*(2*x^2 + 4*x + 3)*arctan(sqrt(2)*(x + 1)) - sqrt(2)*(2*x^2 +
 4*x + 3)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^
3 + 11*x^2 + 18*x + 9)) + 4*sqrt(-x^2 - 4*x - 3)*(2*x + 3) + 4*x + 12)/(2*x
^2 + 4*x + 3)
```

giac [B] time = 0.46, size = 263, normalized size = 3.02

$$\frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x}}{x+2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*(x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*(x + 3)/(2*x^2 + 4*x + 3) - 1/3
*(10*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 7*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x
 + 2)^2 - 2*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3)/(8*(sqrt(-x^2 - 4*x
 - 3) - 1)/(x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x
^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 +
 3)
```

maple [B] time = 0.09, size = 2407, normalized size = 27.67

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2))^2,x)
```

```
[Out] -3/8*(4*x+4)/(2*x^2+4*x+3)+1/4*2^(1/2)*arctan(1/4*(4*x+4)*2^(1/2))-1/2*(-4*
x-6)/(2*x^2+4*x+3)+1/72*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2
)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))*x^2/(-x-3/2)^2-8*arctanh(
3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x)*x^2/(-x-3/2)^2+2*2^(1/2)*arctan(1
/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-6*(3/(-x-3/2)^2*x^2-12)^(1/2)-16*ar
ctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(
-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)/(1/(-x-3/2)^2*x^2+2)+1/36*3^(1/2)*4^
(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(7*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-1
2)^(1/2)*2^(1/2))+4*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/(1/(
-x-3/2)*x+1)/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)-2/9*3^(1/2)*4^
(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(3*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-1
2)^(1/2)*2^(1/2))*x^6/(-x-3/2)^6+2*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/
2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))*x^6/(-x-3/2)^6-2*ln(((3/(-x-3/
2)^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))*x^6
/(-x-3/2)^6+4*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x)*x^6/(-x-3/2
)^6+(3/(-x-3/2)^2*x^2-12)^(1/2)*x^5/(-x-3/2)^5-(3/(-x-3/2)^2*x^2-12)^(3/2)*
```

```
x^2/(-x-3/2)^2+(3/(-x-3/2)^2*x^2-12)^(1/2)*x^4/(-x-3/2)^4-36*2^(1/2)*arctan
(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))*x^2/(-x-3/2)^2-2*(3/(-x-3/2)^2*x^
2-12)^(1/2)*x^3/(-x-3/2)^3-8*(3/(-x-3/2)^2*x^2-12)^(1/2)*x^2/(-x-3/2)^2-24*
ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^
2*x^2-4))*x^2/(-x-3/2)^2+24*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-
x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))*x^2/(-x-3/2)^2-48*arctanh(3/(-x-3/2)/
(3/(-x-3/2)^2*x^2-12)^(1/2)*x)*x^2/(-x-3/2)^2-48*2^(1/2)*arctan(1/6*(3/(-x-
3/2)^2*x^2-12)^(1/2)*2^(1/2))-8*(3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)+16*(
3/(-x-3/2)^2*x^2-12)^(1/2)-32*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)-1/
(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))+32*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*
x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))-64*arctanh(3/(-x-3/2)/
(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(
1/2)/(1/(-x-3/2)*x+1)/(1/(-x-3/2)^2*x^2+2)/((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(
-x-3/2)-1/(-x-3/2)^2*x^2+4)/((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3
/2)^2*x^2-4)-2/9*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arcta
n(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))+arctanh(3/(-x-3/2)/(3/(-x-3/2)^2
*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3
/2)*x+1)+1/18*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(11*2^(1/2)*arcta
n(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))*x^6/(-x-3/2)^6+8*ln(((3/(-x-3/2)
^2*x^2-12)^(1/2)*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))*x^6/(
-x-3/2)^6-8*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/
(1/(-x-3/2)^2*x^2-4))*x^6/(-x-3/2)^6+24*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^
2-12)^(1/2)*x)*x^6/(-x-3/2)^6+4*(3/(-x-3/2)^2*x^2-12)^(1/2)*x^5/(-x-3/2)^5-
(3/(-x-3/2)^2*x^2-12)^(3/2)*x^2/(-x-3/2)^2+(3/(-x-3/2)^2*x^2-12)^(1/2)*x^4/
(-x-3/2)^4-132*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))*x^2/
(-x-3/2)^2-8*(3/(-x-3/2)^2*x^2-12)^(1/2)*x^3/(-x-3/2)^3-8*(3/(-x-3/2)^2*x^2
-12)^(1/2)*x^2/(-x-3/2)^2-96*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)-1/(
-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))*x^2/(-x-3/2)^2+96*ln(((3/(-x-3/2)^2*
x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))*x^2/(-x-
3/2)^2-288*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x)*x^2/(-x-3/2)^2
-176*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-32*(3/(-x-3/2)
^2*x^2-12)^(1/2)*x/(-x-3/2)+16*(3/(-x-3/2)^2*x^2-12)^(1/2)-128*ln(((3/(-x-3
/2)^2*x^2-12)^(1/2)*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))+12
8*ln(((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2
)^2*x^2-4))-384*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-
3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)/(1/(-x-3/2)^2*x^2+
2)/((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/((3/(-x-3/2)
^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 4\,x - 3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 4\,x - 3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2,x)
```

```
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**2,x)
[Out] Integral((x + sqrt(-x**2 - 4*x - 3))**(-2), x)
```

3.761 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx$

Optimal. Leaf size=149

$$-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}-\frac{2\left(2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}-\frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -3/4*arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)+1/18*(-13+2
7*(-1-x)^(1/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/2))-2/
9*(2-(-1-x)^(1/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/2))
^2
```

Rubi [A] time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {12, 1660, 638, 618, 204}

$$-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}-\frac{2\left(2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}-\frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] -(13 - (27*Sqrt[-1 - x])/Sqrt[3 + x])/(18*(1 - (3*(1 + x))/(3 + x) - (2*Sqr
t[-1 - x])/Sqrt[3 + x])) - (2*(2 - Sqrt[-1 - x]/Sqrt[3 + x]))/(9*(1 - (3*(1
 + x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x])^2) - (3*ArcTan[(1 - (3*Sqrt[
-1 - x])/Sqrt[3 + x])/Sqrt[2]])/(2*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx = 2\,\mathrm{Subst}\left(\int \frac{2x\left(1+x^2\right)}{\left(1-2x+3x^2\right)^3}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= 4\,\mathrm{Subst}\left(\int \frac{x\left(1+x^2\right)}{\left(1-2x+3x^2\right)^3}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= -\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2}+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{\frac{56}{9}+\frac{16x}{3}}{\left(1-2x+3x^2\right)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)}-\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2}+\frac{3}{2}\,\mathrm{Subst}\left(\int \frac{1}{1-2x+3x^2}\right.$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)}-\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2}-3\,\mathrm{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x\right.$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)}-\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2}-\frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Mathematica [C] time = 2.46, size = 914, normalized size = 6.13

$$\frac{1}{32}\left(\frac{8(2x-3)}{\left(2x^2+4x+3\right)^2}-\frac{8\sqrt{-x^2-4x-3}\left(8x^3+22x^2+26x+15\right)}{\left(2x^2+4x+3\right)^2}-12\sqrt{2}\tan^{-1}\left(\sqrt{2}(x+1)\right)+\frac{6\left(2+i\sqrt{2}\right)\tan^{-}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] ((8*(-3 + 2*x))/(3 + 4*x + 2*x^2)^2 - (8*(2 + 3*x))/(3 + 4*x + 2*x^2) - (8*
Sqrt[-3 - 4*x - x^2]*(15 + 26*x + 22*x^2 + 8*x^3))/(3 + 4*x + 2*x^2)^2 - 12
*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] + (6*(2 + I*Sqrt[2])*ArcTan[((2 + x)*(3*(5
 + (4*I)*Sqrt[2]) + 16*(2 + I*Sqrt[2])*x + 2*(9 + (2*I)*Sqrt[2])*x^2))/(12*
I - 6*Sqrt[2] + (8*I + 6*Sqrt[2])*x^3 - 9*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 -
```

```
4*x - x^2] + x*(40*I - 5*Sqrt[2] - 12*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*
x - x^2]) + x^2*(36*I + 8*Sqrt[2] - 6*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x
 - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]] - (6*(2*I + Sqrt[2])*ArcTanh[((2 + x)*(
3*(5*I + 4*Sqrt[2]) + 16*(2*I + Sqrt[2])*x + 2*(9*I + 2*Sqrt[2])*x^2))/(-5*
(8*I + Sqrt[2])*x + (-8*I + 6*Sqrt[2])*x^3 - 12*Sqrt[1 - (2*I)*Sqrt[2]]*x*S
qrt[-3 - 4*x - x^2] + x^2*(-36*I + 8*Sqrt[2] - 6*Sqrt[1 - (2*I)*Sqrt[2]]*Sq
rt[-3 - 4*x - x^2]) - 3*(4*I + 2*Sqrt[2] + 3*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-
3 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] + (3*(-2*I + Sqrt[2])*Log[4*(3 +
 4*x + 2*x^2)^2])/Sqrt[1 + (2*I)*Sqrt[2]] + (3*(2*I + Sqrt[2])*Log[4*(3 + 4
*x + 2*x^2)^2])/Sqrt[1 - (2*I)*Sqrt[2]] - (3*(2*I + Sqrt[2])*Log[(3 + 4*x +
 2*x^2)*(3 + (6*I)*Sqrt[2] + (2 + (2*I)*Sqrt[2])*x^2 - 2*Sqrt[2 - (4*I)*Sqr
t[2]]*Sqrt[-3 - 4*x - x^2] + x*(4 + (8*I)*Sqrt[2] - 2*Sqrt[2 - (4*I)*Sqrt[2
]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] - (3*(-2*I + Sqrt[2])*L
og[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + (2 - (2*I)*Sqrt[2])*x^2 - 2*Sqrt[
2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] - 2*x*(-2 + (4*I)*Sqrt[2] + Sqrt[2
+ (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]])/32
```

fricas [A] time = 0.41, size = 171, normalized size = 1.15

$$-\frac{24\,x^3+6\,\sqrt{2}\left(4\,x^4+16\,x^3+28\,x^2+24\,x+9\right)\arctan\left(\sqrt{2}\,(x+1)\right)-3\,\sqrt{2}\left(4\,x^4+16\,x^3+28\,x^2+24\,x+9\right)\mathrm{a}}{16\left(4\,x^4+16\,x^3+28\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*x^3 + 6*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(sqrt(
2)*(x + 1)) - 3*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(1/4*sqr
t(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^3 + 11*x^2 + 18*x + 9))
+ 64*x^2 + 4*(8*x^3 + 22*x^2 + 26*x + 15)*sqrt(-x^2 - 4*x - 3) + 60*x + 36)
/(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)
```

giac [B] time = 0.45, size = 367, normalized size = 2.46

$$-\frac{3}{8}\sqrt{2}\arctan\left(\sqrt{2}\,(x+1)\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/8*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*((s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*(6*x^3 + 16*x^2 + 15*x + 9)/(2
*x^2 + 4*x + 3)^2 + 1/18*(618*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1547*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 2362*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x
+ 2)^3 + 2223*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 1174*(sqrt(-x^2 - 4*
x - 3) - 1)^5/(x + 2)^5 + 377*(sqrt(-x^2 - 4*x - 3) - 1)^6/(x + 2)^6 + 6*(s
qrt(-x^2 - 4*x - 3) - 1)^7/(x + 2)^7 + 117)/(8*(sqrt(-x^2 - 4*x - 3) - 1)/(
x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x^2 - 4*x - 3
) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 3)^2
```

maple [B] time = 0.28, size = 14529, normalized size = 97.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 4x - 3}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 4x - 3}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3,x)
```

```
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**3,x)
```

```
[Out] Timed out
```

3.762 $\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}\left(3x^4+6x^3+3x^2+2\right)$$

```
[Out] -1/15*(-x^4-2*x^3-x^2+1)^(3/2)*(3*x^4+6*x^3+3*x^2+2)
```

Rubi [A] time = 0.22, antiderivative size = 59, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1680, 12, 1247, 692, 629}

$$-\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2-\frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 - 2*x^3 - x
^4)^(3/2))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = \text{Subst}\left(\int \frac{1}{128}x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)$$
$$= \frac{1}{128}\text{Subst}\left(\int x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)$$
$$= \frac{1}{256}\text{Subst}\left(\int (-1+4x)^3\sqrt{15+8x-16x^2}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)$$
$$= -\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2} + \frac{1}{40}\text{Subst}\left(\int (-1+4x)\sqrt{15+$$
$$= -\frac{2}{15}\left(1-x^2-2x^3-x^4\right)^{3/2} - \frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}$$

Mathematica [A] time = 0.37, size = 62, normalized size = 1.48

$$\frac{1}{15}\sqrt{-x^4-2x^3-x^2+1}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

```
[Out] (Sqrt[1 - x^2 - 2*x^3 - x^4]*(-2 - x^2 - 2*x^3 + 2*x^4 + 12*x^5 + 18*x^6 +
12*x^7 + 3*x^8))/15
```

fricas [A] time = 0.40, size = 58, normalized size = 1.38

$$\frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{-x^4-2x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^4
 - 2*x^3 - x^2 + 1)
```

giac [A] time = 0.33, size = 58, normalized size = 1.38

$$\frac{1}{5}\left(x^4+2x^3+x^2-1\right)^2\sqrt{-x^4-2x^3-x^2+1} - \frac{1}{3}\left(-x^4-2x^3-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2*
x^3 - x^2 + 1)^(3/2)
```

maple [A] time = 0.01, size = 51, normalized size = 1.21

$$\frac{\left(x^2+x+1\right)\left(x^2+x-1\right)\left(3x^4+6x^3+3x^2+2\right)\sqrt{-x^4-2x^3-x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x+1)^3*(2*x+1)*(-x^4-2*x^3-x^2+1)^(1/2),x)
```

```
[Out] 1/15*(x^2+x+1)*(x^2+x-1)*(3*x^4+6*x^3+3*x^2+2)*(-x^4-2*x^3-x^2+1)^(1/2)
```

maxima [A] time = 0.83, size = 59, normalized size = 1.40

$$\frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
+ x + 1)*sqrt(-x^2 - x + 1)
```

mupad [B] time = 0.18, size = 38, normalized size = 0.90

$$-\frac{\left(3x^4+6x^3+3x^2+2\right)\left(-x^4-2x^3-x^2+1\right)^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x + 1)*(x + 1)^3*(1 - 2*x^3 - x^4 - x^2)^(1/2),x)
```

```
[Out] -((3*x^2 + 6*x^3 + 3*x^4 + 2)*(1 - 2*x^3 - x^4 - x^2)^(3/2))/15
```

sympy [B] time = 0.70, size = 182, normalized size = 4.33

$$\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{2x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**3*(1+2*x)*(-x**4-2*x**3-x**2+1)**(1/2),x)
```

```
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
+ 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2*x
**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*sqrt
(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 -
2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

3.763 $\int (1+2x)\left(x+x^2\right)^3\sqrt{1-(x+x^2)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}\left(3x^4+6x^3+3x^2+2\right)$$

```
[Out] -1/15*(-x^4-2*x^3-x^2+1)^(3/2)*(3*x^4+6*x^3+3*x^2+2)
```

Rubi [A] time = 0.24, antiderivative size = 59, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1593, 1680, 12, 1247, 692, 629}

$$-\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2-\frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 - 2*x^3 - x
^4)^(3/2))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*
p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
```

```
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx &= \int x^3(1+x)^3(1+2x)\sqrt{1-\left(x+x^2\right)^2}\,dx\\ &= \text{Subst}\left(\int \frac{1}{128}x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\ &= \frac{1}{128}\text{Subst}\left(\int x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\ &= \frac{1}{256}\text{Subst}\left(\int (-1+4x)^3\sqrt{15+8x-16x^2}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)\\ &= -\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}+\frac{1}{40}\text{Subst}\left(\int (-1+4x)\sqrt{15+\right.\\ &= -\frac{2}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}-\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.37, size = 62, normalized size = 1.48

$$\frac{1}{15}\sqrt{-x^4-2x^3-x^2+1}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] (Sqrt[1 - x^2 - 2*x^3 - x^4]*(-2 - x^2 - 2*x^3 + 2*x^4 + 12*x^5 + 18*x^6 +
12*x^7 + 3*x^8))/15
```

fricas [A] time = 0.40, size = 58, normalized size = 1.38

$$\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{-x^4-2\,x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^4
 - 2*x^3 - x^2 + 1)
```

giac [A] time = 0.39, size = 58, normalized size = 1.38

$$\frac{1}{5}\left(x^4+2\,x^3+x^2-1\right)^2\sqrt{-x^4-2\,x^3-x^2+1}-\frac{1}{3}\left(-x^4-2\,x^3-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2*
x^3 - x^2 + 1)^(3/2)
```

maple [A] time = 0.01, size = 51, normalized size = 1.21

$$\frac{\left(x^{2}+x+1\right)\left(x^{2}+x-1\right)\left(3x^{4}+6x^{3}+3x^{2}+2\right)\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x)
```

```
[Out] 1/15*(x^2+x+1)*(x^2+x-1)*(3*x^4+6*x^3+3*x^2+2)*(-x^4-2*x^3-x^2+1)^(1/2)
```

maxima [A] time = 1.04, size = 59, normalized size = 1.40

$$\frac{1}{15}\left(3\,x^{8}+12\,x^{7}+18\,x^{6}+12\,x^{5}+2\,x^{4}-2\,x^{3}-x^{2}-2\right)\sqrt{x^{2}+x+1}\,\sqrt{-x^{2}-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
+ x + 1)*sqrt(-x^2 - x + 1)
```

mupad [B] time = 3.42, size = 51, normalized size = 1.21

$$\sqrt{1-\left(x^{2}+x\right)^{2}}\,\left(\frac{x^{8}}{5}+\frac{4\,x^{7}}{5}+\frac{6\,x^{6}}{5}+\frac{4\,x^{5}}{5}+\frac{2\,x^{4}}{15}-\frac{2\,x^{3}}{15}-\frac{x^{2}}{15}-\frac{2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(1 - (x + x^2)^2)^(1/2)*(x + x^2)^3,x)
```

```
[Out] (1 - (x + x^2)^2)^(1/2)*((2*x^4)/15 - (2*x^3)/15 - x^2/15 + (4*x^5)/5 + (6*
x^6)/5 + (4*x^7)/5 + x^8/5 - 2/15)
```

sympy [B] time = 10.18, size = 182, normalized size = 4.33

$$\frac{x^{8}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{4x^{7}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{6x^{6}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{4x^{5}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{5}+\frac{2x^{4}\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x**2+x)**3*(1-(x**2+x)**2)**(1/2),x)
```

```
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
+ 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2*x
**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*sqrt
(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 -
2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

3.764 $\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

Optimal. Leaf size=102

$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1\right.$$

```
[Out] 1/7*(3-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/5*EllipticE(-1+x,1/3*I*3^(1/2))
*3^(1/2)+176/35*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+2/35*(13-3*(-1+x)^2)*
(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1106, 1091, 1176, 1180, 524, 424, 419}

$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1\right.$$

Antiderivative was successfully verified.

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + (
(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3])/35
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
```

```
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx &= \text{Subst}\left(\int \left(3-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)\\ &= \frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{7}\text{Subst}\left(\int \left(6-2x^2\right)\sqrt{3-2x^2-x\right.\\ &= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1\right.\\ &= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1\right.\\ &= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1\right.\\ &= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1\right.\end{aligned}$$

Mathematica [C] time = 0.77, size = 278, normalized size = 2.73

$$\frac{5x^9-45x^8+206x^7-602x^6+1152x^5-1420x^4+848x^3+352x^2-304i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\sqrt{\frac{x^2-2x+4}{x^2}}\,x^2F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{}}}{\sqrt{}}\right.\right.}{35\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (896 - 1056*x + 352*x^2 + 848*x^3 - 1420*x^4 + 1152*x^5 - 602*x^6 + 206*x^7
 - 45*x^8 + 5*x^9 + ((112*I)*Sqrt[2]*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2]*E
llipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] - (304*I)*Sqrt[2
]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*El
lipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-I + Sqrt[3])])/(35*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
[Out] integral((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

maple [B] time = 0.15, size = 1050, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+8*x)^(3/2),x)
[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-
66/35*x^3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+14/5*x^2*(-x^4+4*x^3-8*x^2+8*x)^(1/2
)-32/35*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-4/7*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+32/
7*(-1-I*3^(1/2))*((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+
I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))
^(1/2)/(I*3^(1/2)-1)/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*Ellip
ticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/
2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+64/5*(-1-I*3^(1/2))*((I*3^(1/2)-1)*
x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^
(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(I*3^(1/2)-1)/(-x*(x-2)*(
x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^
(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2
)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^
(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/
2)))^(1/2)))-16/5*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((I*3
^(1/2)-1)*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2
))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(
1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(x-2))^(1/2),(
(1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+1/2*(I*3^(
1/2)-1)*EllipticE(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2)
)*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-4/(I*3^(1/2)-1)*Ellipt
icPi(((I*3^(1/2)-1)*x/(1+I*3^(1/2))/(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2
)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))))/(-x*
(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int((8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.765 $\int \sqrt{8x - 8x^2 + 4x^3 - x^4}\, dx$

Optimal. Leaf size=62

$$\frac{1}{3}\sqrt{-(x-1)^4 - 2(x-1)^2 + 3}\,(x-1) - \frac{4F\left(\sin^{-1}(1-x)\middle| -\frac{1}{3}\right)}{\sqrt{3}} + \frac{2E\left(\sin^{-1}(1-x)\middle| -\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -2/3*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(-1+x,1/3*I*3^(1/2)
)*3^(1/2)+1/3*(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1106, 1091, 1180, 524, 424, 419}

$$\frac{1}{3}\sqrt{-(x-1)^4 - 2(x-1)^2 + 3}\,(x-1) - \frac{4F\left(\sin^{-1}(1-x)\middle| -\frac{1}{3}\right)}{\sqrt{3}} + \frac{2E\left(\sin^{-1}(1-x)\middle| -\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[ArcSin[1 -
x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])/Sqrt[3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
```

```
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{8x-8x^2+4x^3-x^4}\,dx &= \text{Subst}\left(\int \sqrt{3-2x^2-x^4}\,dx, x, -1+x\right)\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{1}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right.\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{2}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -\right.\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)-\frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right)+8\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{4F\left(\sin^{-1}(1-x\right.}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.63, size = 256, normalized size = 4.13

$$-\frac{x^5-5x^4+14x^3-24x^2+8i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\sqrt{\frac{x^2-2x+4}{x^2}}\,x^2F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)-\frac{2i\sqrt{2}(x-2)\sqrt{\frac{x^2-2x+4}{x^2}}\,xE\left(\sin^{-1}\right.}{\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}}}{3\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -1/3*(-16 + 24*x - 24*x^2 + 14*x^3 - 5*x^4 + x^5 - ((2*I)*Sqrt[2]*(-2 + x)*
x*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(S
qrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I +
Sqrt[3])*x)] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*S
qrt[(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^
3))]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^4+4x^3-8x^2+8x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+4x^3-8x^2+8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

maple [B] time = 0.08, size = 946, normalized size = 15.26

$$\frac{\sqrt{-x^4+4x^3-8x^2+8x}\,x}{3}+\frac{8\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(i\sqrt{3}-1\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{(}{(1-}}\right.}{3\left(i\sqrt{3}-1\right)\sqrt{-(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+8*x)^(1/2),x)
```

```
[Out] 1/3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)*x-1/3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+8/3*(-1
-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(
1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2
)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*EllipticF(
((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(
I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+8/3*(-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3
^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*
((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^
(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*(2*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(
x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1
/2))-2*EllipticPi(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),(1+I*3^(1/2))
/(I*3^(1/2)-1),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(
1/2)))-2/3*((x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x+2*(-1-I*3^(1/2))*((I*3^(1/2)-
1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2
))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(
I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^
(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+1/2*(I*3^(1/2)-1)
*EllipticE(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I
*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-4/(I*3^(1/2)-1)*EllipticPi(((
I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+
I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))))/(-(x-2)*(x-
1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

```
[Out] int((8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4 + 4x^3 - 8x^2 + 8x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.766 $\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=17

$$-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1106, 1095, 419}

$$-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= -\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.15, size = 156, normalized size = 9.18

$$\frac{\sqrt{\frac{4i}{x}+\sqrt{3}-i}\sqrt{-\frac{i(x-2)}{(\sqrt{3}-i)x}}x\left(-i\sqrt{3}x+x-4\right)F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{2}\sqrt{-\frac{4i}{x}+\sqrt{3}+i}\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[-I + Sqrt[3] + (4*I)/x]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x*(-
4 + x - I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(Sqrt[2]*Sqrt[I + Sqrt[3] - (4*I)/x
]*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{x^4-4x^3+8x^2-8x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^4 - 4*x^3 + 8*x^2 - 8*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-x^4+4x^3-8x^2+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

maple [B] time = 0.07, size = 200, normalized size = 11.76

$$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{(i\sqrt{3}-1)x}{(1+i\sqrt{3})(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{(1-i\sqrt{3})(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{(1+i\sqrt{3})(x-2)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{(i\sqrt{3}-1)x}{(1+i\sqrt{3})(x-2)}},\sqrt{\frac{(1+i\sqrt{3})(-1-i\sqrt{3})}{(i\sqrt{3}-1)(1-i\sqrt{3})}}\right)}{\left(i\sqrt{3}-1\right)\sqrt{-(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x)
```

```
[Out] 2*(-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+
I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))
^(1/2)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*Ellip
ticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/
2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-x^4+4x^3-8x^2+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4 + 4x^3 - 8x^2 + 8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
[Out] Integral(1/sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.767 $\int \frac{1}{\left(8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{8\sqrt{3}}$$

```
[Out] -1/24*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/12*EllipticF(-1+x,1/3*I*3^(1/
2))*3^(1/2)+1/24*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1106, 1092, 1180, 524, 424, 419}

$$\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + Elli
pticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin[1 - x], -1/3]/(4*
Sqrt[3])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(3-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right) \\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{48}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right. \\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -\right. \\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right) + \frac{1}{2} \\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}} - \frac{F\left(\sin^{-1}(1-x)\middle|-\right.}{4\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.91, size = 261, normalized size = 3.58

$$\frac{\sqrt{-x\left(x^3-4x^2+8x-8\right)}\left(\frac{\sqrt{2}\left(\sqrt{3}-i\right)\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{\frac{x^2-2x+4}{x^2}}} - \frac{x^2-4i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\sqrt{\frac{x^2-2x+4}{x^2}}\,x^2F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{}}\right.\right.}{x^2-2x+4}\right.}{24(x-2)x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*((Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-
2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(S
qrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[(4 - 2*x + x^2)/x^2] -
(2 + x^2 - (4*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[
(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(4 - 2*x + x^2)))/(24*(-2 + x)*x)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 +
128*x^4 - 128*x^3 + 64*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

maple [B] time = 0.07, size = 963, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x)
```

```
[Out] 2*x*(1/24+1/192*x^2)/(-x*(x^3-4*x^2+8*x-8))^(1/2)-1/32*(-x^3+4*x^2-8*x+8)/(
x*(-x^3+4*x^2-8*x+8))^(1/2)+1/6*(-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))
/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I
*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^(1/2))*
(x-1-I*3^(1/2))*x)^(1/2)*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1
/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+1/6*(
-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3
^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1
/2)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*(2*Ellip
ticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/
2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)/(1+I*3^
(1/2))/(x-2)*x)^(1/2),(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-1-I*3^(1
/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2)))-1/24*((x-1+I*3^(1/2))*(x-1-I*3^(1
/2))*x+2*(-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2
*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))
/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)/(
1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-
I*3^(1/2)))^(1/2))+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/2)-1)/(1+I*3^(1/2))
/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^
(1/2))-4/(I*3^(1/2)-1)*EllipticPi(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/
2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)
/(1-I*3^(1/2)))^(1/2))))/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(-x^4+4\,x^3-8\,x^2+8\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

```
[Out] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

```
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

3.768 $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{(7(x-1)^2+26)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{((x-1)^2+5)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}$$

```
[Out] 1/72*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(3/2)-7/432*EllipticE(-1+x
,1/3*I*3^(1/2))*3^(1/2)+11/432*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/432*
(26+7*(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1106, 1092, 1178, 1180, 524, 424, 419}

$$\frac{(7(x-1)^2+26)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{((x-1)^2+5)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((
26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (7
*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (11*EllipticF[ArcSin[1 - x
], -1/3])/(144*Sqrt[3])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\left(8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(3-2x^2-x^4\right)^{5/2}}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{1}{144}\text{Subst}\left(\int \frac{-38-6x^2}{\left(3-2x^2-x^4\right)^{3/2}}\,dx, x, -\right.$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{\text{Subs}}{}$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{\text{Subs}}{}$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{7}{432}\text{S}$$

$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{7E\left(\text{s}\right.}{}$$

Mathematica [C] time = 1.08, size = 298, normalized size = 2.73

$$\frac{\frac{7i\sqrt{2}(x-2)\sqrt{\frac{x^2-2x+4}{x^2}}\,x^2E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}} + \frac{7x^6-37x^5+115x^4-226x^3+274x^2-19i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\sqrt{\frac{x^2-2x+4}{x^2}}\left(x^3-4x^2+8x-8\right)x^3F\left(\text{si}\right.}{x^3-4x^2+8x-8}}{432x\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] (((7*I)*Sqrt[2]*(-2 + x)*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqr
t[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/S
qrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] + (36 - 232*x + 274*x^2 - 226*x^3 +
 115*x^4 - 37*x^5 + 7*x^6 - (19*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt
[3])*x)]*x^3*Sqrt[(4 - 2*x + x^2)/x^2]*(-8 + 8*x - 4*x^2 + x^3)*EllipticF[A
rcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqr
t[3])])/(-8 + 8*x - 4*x^2 + x^3))/(432*x*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))
])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{x^{12}-12x^{11}+72x^{10}-280x^9+768x^8-1536x^7+2240x^6-2304x^5+1536x^4-512x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^12 - 12*x^11 + 72*x^10 - 280*
x^9 + 768*x^8 - 1536*x^7 + 2240*x^6 - 2304*x^5 + 1536*x^4 - 512*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+4x^3-8x^2+8x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

maple [B] time = 0.08, size = 1039, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x)
```

```
[Out] -1/768*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/x^2-1/96*(-x^3+4*x^2-8*x+8)/((-x^3+4*x^
2-8*x+8)*x)^(1/2)+(1/36+1/288*x^2-1/96*x)*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/(x^3
-4*x^2+8*x-8)^2+2*x*(53/3456+5/1728*x^2-19/4608*x)/(-(x^3-4*x^2+8*x-8)*x)^(
1/2)+5/216*(-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)
^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2
))/(x-2))^(1/2)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1
/2)*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-
1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+7/108*(-1-I*3^(1/2))*((I*3
^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2
))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(I*3^(1/2)-1)/(
-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*(2*EllipticF(((I*3^(1/2)-1)
/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(
1-I*3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/
2),(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/
(1-I*3^(1/2)))^(1/2)))-7/432*((x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x+2*(-1-I*3^(
1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/
(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2
```

```
*(6+2*I*3^(1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)
*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))
+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((
1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-4/(I*3^(1/2
)-1)*EllipticPi(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),(-1-I*3^(1/2))/
(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1
/2))))/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(-x^4+4\,x^3-8\,x^2+8\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{4}+4x^{3}-8x^{2}+8x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(5/2),x)
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

3.769 $\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx$

Optimal. Leaf size=102

$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-\right.$$

```
[Out] 1/7*(3-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/5*EllipticE(-1+x,1/3*I*3^(1/2))
*3^(1/2)+176/35*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+2/35*(13-3*(-1+x)^2)*
(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1106, 1091, 1176, 1180, 524, 424, 419}

$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + (
(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3])/35
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
```

```
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx = \text{Subst}\left(\int \left(3-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)$$

$$= \frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{7}\text{Subst}\left(\int \left(6-2x^2\right)\sqrt{3-2x}\right.$$

$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3-2(-1+x)^2\right.$$

$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3-2(-1+x)^2\right.$$

$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3-2(-1+x)^2\right.$$

$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3-2(-1+x)^2\right.$$

Mathematica [C] time = 1.03, size = 278, normalized size = 2.73

$$\frac{\sqrt{-x\left(x^3-4x^2+8x-8\right)}\left(\sqrt{\frac{x^2-2x+4}{x^2}}\left(-5x^7+35x^6-116x^5+230x^4-228x^3+44x^2+152x-224\right)+304i\sqrt{2}\right.}{35(x-2)x\sqrt{\frac{x^2}{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-224 + 152
*x + 44*x^2 - 228*x^3 + 230*x^4 - 116*x^5 + 35*x^6 - 5*x^7) + 112*Sqrt[2]*(
-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqr
t[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])] +
(304*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticF[ArcSin[S
qrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])
)/(35*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-x^{4}+4\,x^{3}-8\,x^{2}+8\,x\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
[Out] integral((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\left(x^{2}-2\,x+4\right)(x-2)x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

maple [B] time = 0.07, size = 1050, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2-x)*x*(x^2-2*x+4))^(3/2),x)
[Out] -1/7*(-x^4+4*x^3-8*x^2+8*x)^(1/2)*x^5+5/7*(-x^4+4*x^3-8*x^2+8*x)^(1/2)*x^4-
66/35*(-x^4+4*x^3-8*x^2+8*x)^(1/2)*x^3+14/5*(-x^4+4*x^3-8*x^2+8*x)^(1/2)*x^
2-32/35*(-x^4+4*x^3-8*x^2+8*x)^(1/2)*x-4/7*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+32/
7*(-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+
I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))
^(1/2)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*Ellip
ticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/
2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+64/5*(-1-I*3^(1/2))*((I*3^(1/2)-1)/
(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^
(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(I*3^(1/2)-1)/(-(x-2)*(x-
1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*(2*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1
/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2
)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),(1+I*3^
(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/
2)))^(1/2)))-16/5*((x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x+2*(-1-I*3^(1/2))*((I*3
^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2
))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(
1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),(
(1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+1/2*(I*3^(
1/2)-1)*EllipticE(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2)
)*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-4/(I*3^(1/2)-1)*Ellipt
icPi(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2
)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))))/(-(x
-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\left(x^{2}-2\,x+4\right)(x-2)x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(x - 2)*(x^2 - 2*x + 4))^(3/2),x)
[Out] int((-x*(x - 2)*(x^2 - 2*x + 4))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x\left(2-x\right)\left(x^{2}-2x+4\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x**2-2*x+4))**(3/2),x)
[Out] Integral((x*(2 - x)*(x**2 - 2*x + 4))**(3/2), x)
```

3.770 $\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx$

Optimal. Leaf size=62

$$\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}\,(x-1)-\frac{4F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -2/3*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(-1+x,1/3*I*3^(1/2)
)*3^(1/2)+1/3*(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1106, 1091, 1180, 524, 424, 419}

$$\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}\,(x-1)-\frac{4F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[ArcSin[1 -
x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])/Sqrt[3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
```

```
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx &= \text{Subst}\left(\int \sqrt{3-2x^2-x^4}\,dx, x, -1+x\right)\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{1}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -\right.\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{2}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx\right.\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)-\frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right.\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{4F\left(\sin^{-1}\right.}{}\end{aligned}$$

Mathematica [C] time = 0.84, size = 256, normalized size = 4.13

$$\frac{\sqrt{-x\left(x^3-4x^2+8x-8\right)}\left(\sqrt{\frac{x^2-2x+4}{x^2}}\left(x^3-3x^2+4x-4\right)+8i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)+2\cdots\right.}{3(x-2)x\sqrt{\frac{x^2-2x+4}{x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-4 + 4*x -
 3*x^2 + x^3) + 2*Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3
])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*
Sqrt[3])/(-I + Sqrt[3])] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3
])*x)]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*
Sqrt[3])/(-I + Sqrt[3])]))/(3*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^4+4x^3-8x^2+8x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(x^2-2x+4\right)(x-2)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

maple [B] time = 0.08, size = 946, normalized size = 15.26

$$\frac{\sqrt{-x^4+4x^3-8x^2+8x}\,x}{3}+\frac{8\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(i\sqrt{3}-1\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(i\sqrt{3}\right.}{\left(1+i\sqrt{\right.}}}\right.}{3\left(i\sqrt{3}-1\right)\sqrt{-(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2-x)*x*(x^2-2*x+4))^(1/2),x)
```

```
[Out] 1/3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)*x-1/3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+8/3*(-1
-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(
1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2
)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*EllipticF(
((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(
I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+8/3*(-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3
^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*
((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^
(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*(2*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(
x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1
/2))-2*EllipticPi(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),(1+I*3^(1/2))
/(I*3^(1/2)-1),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(
1/2)))-2/3*((x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x+2*(-1-I*3^(1/2))*((I*3^(1/2)-
1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2
))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(
I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^
(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+1/2*(I*3^(1/2)-1)
*EllipticE(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I
*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-4/(I*3^(1/2)-1)*EllipticPi(((
I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+
I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))))/(-(x-2)*(x-
1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(x^2-2x+4\right)(x-2)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-x\,(x-2)\,\left(x^2-2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(x - 2)*(x^2 - 2*x + 4))^(1/2),x)
```

```
[Out] int((-x*(x - 2)*(x^2 - 2*x + 4))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x\,(2-x)\left(x^2-2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

```
[Out] Integral(sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

3.771 $\int \frac{1}{\sqrt{(2-x)x\left(4-2x+x^2\right)}}\,dx$

Optimal. Leaf size=17

$$-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1106, 1095, 419}

$$-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{(2-x)x\left(4-2x+x^2\right)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= -\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.27, size = 100, normalized size = 5.88

$$-\frac{\sqrt[3]{-1}(x-2)^2\sqrt{\frac{x(x+i\sqrt{3}-1)}{(x-2)^2}}\sqrt{\frac{-\sqrt[3]{-1}x+x-2}{x-2}}F\left(\sin^{-1}\left(\sqrt{-\frac{(-1)^{2/3}x}{x-2}}\right)|(-1)^{2/3}\right)}{\sqrt{-x(x^3-4x^2+8x-8)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] -(((-1)^(1/3)*(-2 + x)^2*Sqrt[(x*(-1 + I*Sqrt[3] + x))/(-2 + x)^2]*Sqrt[(-2
 + x - (-1)^(1/3)*x)/(-2 + x)]*EllipticF[ArcSin[Sqrt[-(((-1)^(2/3)*x)/(-2 +
 x))]], (-1)^(2/3)])/Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{x^4-4x^3+8x^2-8x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^4 - 4*x^3 + 8*x^2 - 8*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x^2-2x+4)(x-2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

maple [B] time = 0.07, size = 200, normalized size = 11.76

$$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{(i\sqrt{3}-1)x}{(1+i\sqrt{3})(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{(1-i\sqrt{3})(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{(1+i\sqrt{3})(x-2)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{(i\sqrt{3}-1)x}{(1+i\sqrt{3})(x-2)}},\sqrt{\frac{(1+i\sqrt{3})(-1-i\sqrt{3})}{(i\sqrt{3}-1)(1-i\sqrt{3})}}\right)}{(i\sqrt{3}-1)\sqrt{-(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2-x)*x*(x^2-2*x+4))^(1/2),x)
```

```
[Out] 2*(-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+
I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))
^(1/2)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*Ellip
ticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/
2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x^2-2x+4)(x-2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{-x\,(x-2)\,\left(x^2-2\,x+4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(1/2),x)
[Out] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\,(2-x)\left(x^2-2x+4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(1/2),x)
[Out] Integral(1/sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

3.772 $\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{8\sqrt{3}}$$

```
[Out] -1/24*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/12*EllipticF(-1+x,1/3*I*3^(1/
2))*3^(1/2)+1/24*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1106, 1092, 1180, 524, 424, 419}

$$\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + Elli
pticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin[1 - x], -1/3]/(4*
Sqrt[3])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(3-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right)$$
$$= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{48}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)$$
$$= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1$$
$$= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right) + \frac{1}{2}$$
$$= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}} - \frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right.}{4\sqrt{3}}$$

Mathematica [C] time = 0.99, size = 298, normalized size = 4.08

$$\frac{(x-2)^2x\left(x^2-2x+4\right)\left(-\frac{3\left(x^2-2x+4\right)x}{x-2}-3\left(x^2-2x+4\right)-4(2-x)\sqrt{\frac{x^2-2x+4}{(x-2)^2}}\left(\sqrt{\frac{x^2-2x+4}{(x-2)^2}}\,x+4i\sqrt{2}\sqrt{\frac{ix}{\left(\sqrt{3}+i\right)(x-2)}}\,F\right.\right.}{96\left(-x\left(x^3-4x^2+8x\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

```
[Out] ((-2 + x)^2*x*(4 - 2*x + x^2)*(2*(-1 + x)*x - 3*(4 - 2*x + x^2) - (3*x*(4 -
 2*x + x^2))/(-2 + x) - 4*(2 - x)*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*(x*Sqrt[
(4 - 2*x + x^2)/(-2 + x)^2] - Sqrt[2]*(I + Sqrt[3])*Sqrt[(I*x)/((I + Sqrt[3
])*(-2 + x))]*EllipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])] + (4*I)*Sqrt[2]*Sqrt[(I*x)/((I + Sqr
t[3])*(-2 + x))]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])])))/(96*(-(x*(-8 + 8*x - 4*x^2 + x
^3)))^(3/2))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 +
128*x^4 - 128*x^3 + 64*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x^2-2x+4\right)(x-2)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

maple [B] time = 0.08, size = 963, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2-x)*x*(x^2-2*x+4))^(3/2),x)
```

```
[Out] 2*(1/192*x^2+1/24)/(-(x^3-4*x^2+8*x-8)*x)^(1/2)*x-1/32*(-x^3+4*x^2-8*x+8)/(
(-x^3+4*x^2-8*x+8)*x)^(1/2)+1/6*(-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))
/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I
*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^(1/2))*
(x-1-I*3^(1/2))*x)^(1/2)*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1
/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+1/6*(
-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3
^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1
/2)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*(2*Ellip
ticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/
2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)/(1+I*3^
(1/2))/(x-2)*x)^(1/2),(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-1-I*3^(1
/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2)))-1/24*((x-1+I*3^(1/2))*(x-1-I*3^(1
/2))*x+2*(-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2
*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))
/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)/(
1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-
I*3^(1/2)))^(1/2))+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/2)-1)/(1+I*3^(1/2))
/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^
(1/2))-4/(I*3^(1/2)-1)*EllipticPi(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/
2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)
/(1-I*3^(1/2)))^(1/2))))/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x^2-2x+4\right)(x-2)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(3/2),x)
```

```
[Out] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x\left(2-x\right)\left(x^{2}-2x+4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

```
[Out] Integral((x*(2 - x)*(x**2 - 2*x + 4))**(-3/2), x)
```

3.773 $\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{\left(7(x-1)^2+26\right)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|\cdots\right.}{144\sqrt{3}}$$

```
[Out] 1/72*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(3/2)-7/432*EllipticE(-1+x
,1/3*I*3^(1/2))*3^(1/2)+11/432*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/432*
(26+7*(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1106, 1092, 1178, 1180, 524, 424, 419}

$$\frac{\left(7(x-1)^2+26\right)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|\cdots\right.}{144\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((
26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (7
*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (11*EllipticF[ArcSin[1 - x
], -1/3])/(144*Sqrt[3])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(3-2x^2-x^4\right)^{5/2}}\,dx,x,-1+x\right)\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}}-\frac{1}{144}\text{Subst}\left(\int \frac{-38-6x^2}{\left(3-2x^2-x^4\right)^{3/2}}\,dx,x,-
\\
&= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}}+\frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}}+\frac{\text{Subst}}{}
\\
&= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}}+\frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}}+\frac{\text{Subst}}{}
\\
&= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}}+\frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}}-\frac{7}{432}\text{Su}
\\
&= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}}+\frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}}+\frac{7E\left(\text{si}\right.}{}
\end{aligned}
$$

Mathematica [C] time = 1.09, size = 327, normalized size = 3.00

$$
\frac{(x-2)^3x^2\left(x^2-2x+4\right)^2\left(-\frac{7x\left(x^2-2x+4\right)}{x-2}-19i\sqrt{2}(x-2)\sqrt{\frac{ix}{\left(\sqrt{3}+i\right)(x-2)}}\sqrt{\frac{x^2-2x+4}{(x-2)^2}}F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}-i-\frac{4i}{x-2}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{i+\sqrt{3}}\right)+\frac{7i\sqrt{}}{}\right.}{432\left(-x\left(x^3-4x^2+8x-8\right)\right)^{5/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

```
[Out] ((-2 + x)^3*x^2*(4 - 2*x + x^2)^2*((-7*x*(4 - 2*x + x^2))/(-2 + x) + (36 +
216*x - 622*x^2 + 670*x^3 - 445*x^4 + 187*x^5 - 49*x^6 + 7*x^7)/((-2 + x)^2
*x*(4 - 2*x + x^2)) + ((7*I)*Sqrt[2]*x*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*Ell
ipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(I + Sqrt[3])])/Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))] - (19*I)*Sqrt[2
]*(-2 + x)*Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))]*Sqrt[(4 - 2*x + x^2)/(-2 +
x)^2]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(I + Sqrt[3])]))/(432*(-(x*(-8 + 8*x - 4*x^2 + x^3)))^(5/2)
)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{x^{12}-12x^{11}+72x^{10}-280x^9+768x^8-1536x^7+2240x^6-2304x^5+1536x^4-512x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)/(x^12 - 12*x^11 + 72*x^10 - 280*
x^9 + 768*x^8 - 1536*x^7 + 2240*x^6 - 2304*x^5 + 1536*x^4 - 512*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x^2-2x+4\right)(x-2)x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

maple [B] time = 0.07, size = 1039, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2-x)*x*(x^2-2*x+4))^(5/2),x)
```

```
[Out] -1/768*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/x^2-1/96*(-x^3+4*x^2-8*x+8)/((-x^3+4*x^
2-8*x+8)*x)^(1/2)+(1/288*x^2-1/96*x+1/36)*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/(x^3
-4*x^2+8*x-8)^2+2*(5/1728*x^2-19/4608*x+53/3456)/(-(x^3-4*x^2+8*x-8)*x)^(1/
2)*x+5/216*(-1-I*3^(1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)
^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2
))/(x-2))^(1/2)/(I*3^(1/2)-1)/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1
/2)*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-
1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))+7/108*(-1-I*3^(1/2))*((I*3
^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2
))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(I*3^(1/2)-1)/(
-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)*(2*EllipticF(((I*3^(1/2)-1)
/(1+I*3^(1/2))/(x-2)*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(
1-I*3^(1/2)))^(1/2))-2*EllipticPi(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/
2),(1+I*3^(1/2))/(I*3^(1/2)-1),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/
(1-I*3^(1/2)))^(1/2)))-7/432*((x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x+2*(-1-I*3^(
1/2))*((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/
```

```
(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2
*(6+2*I*3^(1/2))/(I*3^(1/2)-1)*EllipticF(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)
*x)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))
+1/2*(I*3^(1/2)-1)*EllipticE(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),((
1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1/2))-4/(I*3^(1/2
)-1)*EllipticPi(((I*3^(1/2)-1)/(1+I*3^(1/2))/(x-2)*x)^(1/2),(-1-I*3^(1/2))/
(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(I*3^(1/2)-1)/(1-I*3^(1/2)))^(1
/2))))/(-(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x^2-2\,x+4\right)(x-2)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(5/2),x)
```

```
[Out] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x\left(2-x\right)\left(x^2-2x+4\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(5/2),x)
```

```
[Out] Integral((x*(2 - x)*(x**2 - 2*x + 4))**(-5/2), x)
```

3.774 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} dx$

Optimal. Leaf size=730

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}-\frac{16c^3\left(8ad^2+c^3\right)\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^2\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}+\frac{2c\left(\frac{c}{d}+x\right.}{}$$

```
[Out] 1/7*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2)+2/35*c*(c/d+x)*(7*c^3
+20*a*d^2-3*c*d^2*(c/d+x)^2)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d^2-
16/35*c^3*(8*a*d^2+c^3)*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d
^2/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))/(4*a*d^2+c^3)^(1/2)+16/35*c^
(13/4)*(4*a*d^2+c^3)^(3/4)*(8*a*d^2+c^3)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a
*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)
))*EllipticE(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^
(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2
))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d
+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/d^5/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)
^(1/2)+8/35*c^(7/4)*(4*a*d^2+c^3)^(3/4)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*
d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))
)*EllipticF(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(
3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2)
)*(-c^(3/2)*(8*a*d^2+c^3)+(5*a*d^2+c^3)*(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+
4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^
3)^(1/2))^2)^(1/2)/d^5/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1106, 1091, 1176, 1197, 1103, 1195}

$$-\frac{16c^3\left(8ad^2+c^3\right)\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^2\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}+\frac{2c\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\left(20ad^2+7\right.}{35d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
```

```
[Out] ((c/d + x)*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2))/7 + (2*c*(c/d +
 x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]*(7*c^3 + 20*a*d^2 - 3*c*d
^2*(c/d + x)^2))/(35*d^2) - (16*c^3*(c^3 + 8*a*d^2)*(c/d + x)*Sqrt[4*a*c +
4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(35*d^2*Sqrt[c^3 + 4*a*d^2]*(Sqrt[c] + (d
^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (16*c^(13/4)*(c^3 + 4*a*d^2)^(3/4)*
(c^3 + 8*a*d^2)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3
+ 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] +
 (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/
4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*S
qrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (8*c^(7/4)*(c^3 + 4*a*d^2)^
(3/4)*(Sqrt[c^3 + 4*a*d^2]*(c^3 + 5*a*d^2) - c^(3/2)*(c^3 + 8*a*d^2))*Sqrt[
(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] +
 (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sq
rt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1
```

```
/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*Sqrt[4*a*c + 4*c^2*x^2
 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} dx = \text{Subst}\left(\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^{3/2} dx, x, \frac{c}{d}+x\right)$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} + \frac{3}{7}\text{Subst}\left(\int \left(2c\left(4a+\right.\right.\right.$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} + \frac{2c(c+dx)\sqrt{4ac+4c^2x}}{}$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} + \frac{2c(c+dx)\sqrt{4ac+4c^2x}}{}$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} + \frac{2c(c+dx)\sqrt{4ac+4c^2x}}{}$$

Mathematica [C] time = 6.20, size = 10468, normalized size = 14.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas")
[Out] integral((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

maple [B] time = 0.20, size = 5229, normalized size = 7.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(4\,c^2\,x^2 + 4\,c\,d\,x^3 + 4\,a\,c + d^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2),x)
```

```
[Out] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

```
[Out] Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(3/2), x)
```

3.775 $\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal. Leaf size=622

$$\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}-\frac{2c^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}+\frac{c^{3/4}\sqrt[4]{4ad^2+c^3}\left(-c^{3/2}\sqrt{\cdots}\right.}{\cdots}$$

```
[Out] 1/3*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)-2/3*c^2*(c/d+x)*(d^2*
x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(
1/2))/(4*a*d^2+c^3)^(1/2)+2/3*c^(9/4)*(4*a*d^2+c^3)^(3/4)*(cos(2*arctan((d*
x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4
*a*d^2+c^3)^(1/4)))*EllipticE(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1
/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(
4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/
(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/d^3/(d^2*x^4+4*c*d*x^3
+4*c^2*x^2+4*a*c)^(1/2)+1/3*c^(3/4)*(4*a*d^2+c^3)^(1/4)*(cos(2*arctan((d*x+
c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a
*d^2+c^3)^(1/4)))*EllipticF(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4
))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*
a*d^2+c^3)^(1/2))*(c^3+4*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4
*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3
)^(1/2))^2)^(1/2)/d^3/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 622, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1106, 1091, 1197, 1103, 1195}

$$-\frac{2c^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}+\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}+\frac{c^{3/4}\sqrt[4]{4ad^2+c^3}\left(-c^{3/2}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] ((c/d + x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/3 - (2*c^2*(c/d +
 x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(3*Sqrt[c^3 + 4*a*d^2]*(
Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (2*c^(9/4)*(c^3 + 4*a*d
^2)^(3/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*
d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*
(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^
3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a
*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (c^(3/4)*(c^3 + 4*a*d^2)^(1/4)*(c^
3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4
*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3
 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Elliptic
F[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^
3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
```

```
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \text{Subst}\left(\int \sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4} + \frac{1}{3}\text{Subst}\left(\int \frac{2c\left(4a+\frac{c^3}{d^2}\right) - \cdots}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c\cdots}}\right.$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4} + \frac{\left(2c^{5/2}\sqrt{c^3+4ad^2}\right)\text{Subst}\left(\int \cdots\right.}{3\cdots}$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4} - \frac{2c^2(c+dx)\sqrt{4ac+4c^2x^2+4c\cdots}}{3d\sqrt{c^3+4ad^2}\left(\sqrt{c}+\frac{(c+\cdots}{\sqrt{c^3+\cdots}}\right.}$$

Mathematica [C] time = 6.09, size = 5218, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
[Out] Result too large to show
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

maple [B] time = 0.07, size = 4890, normalized size = 7.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x)
[Out] 1/3*x*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)+1/3*c/d*(d^2*x^4+4*c*d*x^3+
4*c^2*x^2+4*a*c)^(1/2)+16/3*a*c*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d)^2*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d))^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)/(d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(
x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-
(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((
-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(
```

c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/
2))-8/3*c^3/d*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2
d(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2
d(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2
)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d))^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d)/(d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d*Ellip
ticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(
1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(
1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)
+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)
+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+
c^2)^(1/2))/d))^(1/2))+((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^
(1/2)+c^2)^(1/2))/d)*EllipticPi(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(
c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+
(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)))-2/3*c^2*((x-(-c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d*(x+(c+(-2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)+((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(
-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+
(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d)^2*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d^2*(-c+(2
d(-a*c)^(1/2)+c^2)^(1/2))+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d^2*(-c+(2*d*
(-a*c)^(1/2)+c^2)^(1/2))+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d^2*(c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))^2/d^2)/(-(c+(-2*d*(-a
*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c
)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*EllipticF(((-(c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c
+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c
+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(
1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2)

```
)/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d))^(1/2))+((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)*EllipticE(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(
-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(
-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/
d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)+4*c/d/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(
1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*EllipticPi(((-(c+(-2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*
(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d),((-(c+(2*d*
(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*
(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(
-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(
-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))))/(d
^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{4\,c^2\,x^2 + 4\,c\,d\,x^3 + 4\,a\,c + d^2\,x^4}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2),x)
[Out] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
[Out] Integral(sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

3.776 $\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$

Optimal. Leaf size=227

$$\frac{\sqrt[4]{4ad^2+c^3}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{2\sqrt[4]{c}\,d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[Out] 1/2*(4*a*d^2+c^3)^(1/4)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))
^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticF(sin(
2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^
3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+
4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^
3)^(1/2))^2)^(1/2)/c^(1/4)/d/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1106, 1103}

$$\frac{\sqrt[4]{4ad^2+c^3}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{2\sqrt[4]{c}\,d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] ((c^3 + 4*a*d^2)^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))
/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sq
rt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)
/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(2
*c^(1/4)*d*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{\sqrt[4]{c^3+4ad^2}\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(c^3+4ad^2\right)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)F\left(2\tan^{-1}\left(\cdots\right.\right.}{2\sqrt[4]{c}\,d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Mathematica [C] time = 2.25, size = 822, normalized size = 3.62

$$\frac{2\left(-c-dx+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\right)\left(c+dx+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\right)\sqrt{-\frac{\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\left(c+dx-\sqrt{c^2+2i\sqrt{a}d\sqrt{c}}\right)}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\sqrt{a}d\sqrt{c}}\right)\left(-c-dx+\sqrt{c^2-2i\sqrt{a}\sqrt{c}\cdots}\right.}}\cdots}{d\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] (2*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)*(c + Sqrt[c^2 - (2*I)*S
qrt[a]*Sqrt[c]*d] + d*x)*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c - S
qrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[
c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]
*Sqrt[c]*d] - d*x)))]*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c + Sqrt
[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*
d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] - d*x)))]*EllipticF[ArcSin[Sqrt[((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]
*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*S
qrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x))]], (Sq
rt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])^2/
(Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])
^2])/(d*Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*Sqrt[((Sqrt[c^2 - (2*I)*Sqrt[a]
*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt[c^2 - (2*I)*Sq
rt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 +
 (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)
)]*Sqrt[4*a*c + x^2*(2*c + d*x)^2])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

maple [B] time = 0.07, size = 1056, normalized size = 4.65

$$\frac{2\left(\frac{-c+\sqrt{c^2+2\sqrt{-ac}\,d}}{d}+\frac{c+\sqrt{c^2-2\sqrt{-ac}\,d}}{d}\right)\sqrt{\frac{\left(-\frac{c+\sqrt{c^2-2\sqrt{-ac}\,d}}{d}+\frac{c+\sqrt{c^2+2\sqrt{-ac}\,d}}{d}\right)\left(x-\frac{-c+\sqrt{c^2+2\sqrt{-ac}\,d}}{d}\right)}{\left(-\frac{c+\sqrt{c^2-2\sqrt{-ac}\,d}}{d}-\frac{-c+\sqrt{c^2+2\sqrt{-ac}\,d}}{d}\right)\left(x+\frac{c+\sqrt{c^2+2\sqrt{-ac}\,d}}{d}\right)}}\left(x+\frac{c+\sqrt{c^2+2\sqrt{-ac}\,d}}{d}\right)^2\sqrt{\cdots}}{\left(-\frac{c+\sqrt{c^2-2\sqrt{-ac}\,d}}{d}+\frac{c+\sqrt{c^2+2\sqrt{-ac}\,\cdots}}{d}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x)
```

```
[Out] 2*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)
*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)
*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)
/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/
(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(
d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2),x)
```

```
[Out] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

3.777 $\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=674

$$\left(-c^{3/2}\sqrt{4ad^2+c^3}+\right.$$

$$-\frac{d^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\left(4ad^2+c^3\right)^{3/2}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}+$$

```
[Out] -1/8*(c/d+x)*(c^3-4*a*d^2-c*d^2*(c/d+x)^2)/a/c/(4*a*d^2+c^3)/(d^2*x^4+4*c*d
*x^3+4*c^2*x^2+4*a*c)^(1/2)-1/8*d^2*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*
a*c)^(1/2)/a/(4*a*d^2+c^3)^(3/2)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2)
)+1/8*c^(1/4)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/
cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticE(sin(2*arctan((
d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^
(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+
4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^
2)^(1/2)/a/d/(4*a*d^2+c^3)^(1/4)/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)+
1/16*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arc
tan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticF(sin(2*arctan((d*x+c)/c^
(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c
^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(c^3+4*a*d^2-c^(3/2)*(4*a*d^2+c^3
)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^
2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/a/c^(5/4)/d/(4*a*d^2+c^3)^(3/4)/(
d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1106, 1092, 1197, 1103, 1195}

$$\left(-c^{3/2}\sqrt{4ad^2+c^3}+\right.$$

$$-\frac{d^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\left(4ad^2+c^3\right)^{3/2}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```

```
[Out] -((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(8*a*c*(c^3 + 4*a*d^2)*Sqr
t[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) - (d^2*(c/d + x)*Sqrt[4*a*c + 4
*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(8*a*(c^3 + 4*a*d^2)^(3/2)*(Sqrt[c] + (d^2
*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (c^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2
 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt
[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Elli
pticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqr
t[c^3 + 4*a*d^2])/2])/(8*a*d*(c^3 + 4*a*d^2)^(1/4)*Sqrt[4*a*c + 4*c^2*x^2 +
 4*c*d*x^3 + d^2*x^4]) + ((c^3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Sqr
t[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c]
 + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/
Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^
(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(16*a*c^(5/4)*d*(c^3 + 4*a*d
^2)^(3/4)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^{3/2}}\,dx, x, \frac{c}{d}+x\right)$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} + \frac{\text{Subst}\left(\int \frac{2c\left(4a+\frac{c^3}{d^2}\right)d^{\cdots}}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2\cdots}}\cdots\right.}{16ac^2\left(c^{\cdots}\right.}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1-\cdots}{\sqrt{c\left(4a+\cdots\right.}}\cdots\right.}{8a\sqrt{\cdots}}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} - \frac{d(c+dx)\sqrt{4ac+4c^2\cdots}}{8a\left(c^3+4ad^2\right)^{3/2}\left(\cdots\right.}$$

Mathematica [C] time = 6.13, size = 5276, normalized size = 7.83

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}}{d^4x^8+8\,cd^3x^7+24\,c^2d^2x^6+32\,c^3dx^5+32\,ac^2dx^3+32\,ac^3x^2+8\left(2\,c^4+acd^2\right)x^4+16\,a^2c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)/(d^4*x^8 + 8*c*d^3*x
^7 + 24*c^2*d^2*x^6 + 32*c^3*d*x^5 + 32*a*c^2*d*x^3 + 32*a*c^3*x^2 + 8*(2*c
^4 + a*c*d^2)*x^4 + 16*a^2*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

maple [B] time = 0.08, size = 5024, normalized size = 7.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2),x)
```

```
[Out] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

```
[Out] Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(-3/2), x)
```

3.778 $\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

Optimal. Leaf size=663

$$\frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}-\frac{2d^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\sqrt{256ae^3+5d^4}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}+\frac{\sqrt[4]{256ae^3+5d^4}\left(-3d^2\sqrt{\ldots}\right.}{\ldots}$$

```
[Out] 1/3*(1/4*d/e+x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)-2*d^2*(1/4*d/e+
x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/(1+16*e^2*(1/4*d/e+x)^2/(256
*a*e^3+5*d^4)^(1/2))/(256*a*e^3+5*d^4)^(1/2)+1/16*d^2*(256*a*e^3+5*d^4)^(3/
4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))^2)^(1/2)/cos(2*arctan(
(4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticE(sin(2*arctan((4*e*x+d)/(256*a
*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2))*(1+16*e^2*
(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*
e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))^2)^
(1/2)/e^2*2^(1/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)+1/96*(256*a*e
^3+5*d^4)^(1/4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))^2)^(1/2)/
cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticF(sin(2*arctan((4*
e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2
))*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(5*d^4+256*a*e^3-3*d^2*
(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)/(256*a*e^
3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))^2)^(1/2)/e^2*2^(1
/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1106, 1091, 1197, 1103, 1195}

$$-\frac{2d^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\sqrt{256ae^3+5d^4}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}+\frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}+\frac{\sqrt[4]{256ae^3+5d^4}\left(-3d^2\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] ((d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/3 - (2*d^2*
(d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/(Sqrt[5*d^4
+ 256*a*e^3]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])) + (d^2
*(5*d^4 + 256*a*e^3)^(3/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*
e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticE[
2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 +
256*a*e^3])/2])/(8*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4]) + ((5*d^4 + 256*a*e^3)^(1/4)*(5*d^4 + 256*a*e^3 - 3*d^2*Sqrt[5*d^4 + 2
56*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 2
56*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (
16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*
e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/
(48*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx = \text{Subst}\left(\int \sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4} + \frac{1}{3}\text{Subst}\left(\int \frac{\frac{1}{16}\left(\frac{5d^4}{e}+\right.}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256\right.}}\right.$$

$$= \frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4} + \frac{\left(d^2\sqrt{5d^4+256ae^3}\right)\text{Subst}\left(\right.}{}$$

$$= \frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4} - \frac{d^2(d+4ex)\sqrt{8ae^2-d^3x+8}}{2e\sqrt{5d^4+256ae^3}\left(1+\right.}$$

Mathematica [B] time = 6.12, size = 7543, normalized size = 11.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
[Out] Result too large to show
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="fricas"
)
[Out] integral(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.27, size = 7887, normalized size = 11.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2),x)
```

```
[Out] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

```
[Out] Integral(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

3.779 $\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx$

Optimal. Leaf size=235

$$\frac{\sqrt[4]{256ae^3+5d^4}\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(256ae^3+5d^4\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{\sqrt{2}\,e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

```
[Out] 1/2*(256*a*e^3+5*d^4)^(1/4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)
))^2)^(1/2)/cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticF(sin(
2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)
^(1/2))^(1/2))*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x
^4+8*d*e^2*x^3-d^3*x+8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(25
6*a*e^3+5*d^4)^(1/2))^2)^(1/2)/e*2^(1/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e
^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1106, 1103}

$$\frac{\sqrt[4]{256ae^3+5d^4}\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(256ae^3+5d^4\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{\sqrt{2}\,e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] ((5*d^4 + 256*a*e^3)^(1/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*
e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[
2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 +
256*a*e^3])/2])/(Sqrt[2]*e*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e} + 256ae^2\right) - 3d^2ex^2 + 8e^3x^4}}\,dx, x, \frac{d}{4e} + x\right)$$

$$= \frac{\sqrt[4]{5d^4 + 256ae^3}\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(5d^4+256ae^3\right)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1 + \frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)F\left(2\tan\right.}{\sqrt{2}\,e\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x}}$$

Mathematica [B] time = 2.28, size = 1065, normalized size = 4.53

$$-\frac{\left(-d - 4ex + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}\right)\left(d + 4ex - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}\right)\sqrt{-\frac{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}\left(d+4ex-\right.}{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2+2\sqrt{d^4-64a}}\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] -1/2*((-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)*(d - Sqrt[3*d^2 +
 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x)*Sqrt[-((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^
3]]*(d + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x))/((Sqrt[3*d^2 - 2*Sq
rt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^
2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)))]*Sqrt[(3*d^2 - 2*Sqrt[d^4 - 64*a*e^3
] - Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3
]] + d*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64
*a*e^3]]) + 4*e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt
[d^4 - 64*a*e^3]])*x)/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 +
 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*
x))]*EllipticF[ArcSin[Sqrt[((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*
d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] +
4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 -
64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x))]], (Sqrt[3
*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])^2/(S
qrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])
^2])/(e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 6
4*a*e^3]])*Sqrt[(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*(-d + Sqrt[3*d^2 + 2*
Sqrt[d^4 - 64*a*e^3]] - 4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sq
rt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^
3]] - 4*e*x))]*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8\,e^{3}x^{4} + 8\,de^{2}x^{3} - d^{3}x + 8\,ae^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

maple [B] time = 0.06, size = 1704, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x)
```

```
[Out] 1/2*(1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+
(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*((-1/4*(d*e+(3*e^2*d^2-
2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e
^2)^(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e
^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d*
e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*(x+1/4*(d*e+(3
*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)^2*((-1/4*(d*e+(3*e^2*d^2+
2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d
^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*
e^2)^(1/2))/e^2)/(1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/
e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d
*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*((-1/4*(d*e+(
3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-
64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2
)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/
(x+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)/(-1/
4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*e^2*d
^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2+2*(-64*a*
e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)
*e^2)^(1/2))/e^2)*2^(1/2)/(e^3*(x-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1
/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1
/2))/e^2)*(x-1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*
(x+1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*Elli
pticF(((-1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d
*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*e^2*
d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2-2*(-64*a
*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2
)*e^2)^(1/2))/e^2)/(x+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2
))/e^2))^(1/2),((-1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e
^2-1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(1/4*(-d*e
+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*e^2*d^2-2*(
-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^
(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1
/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8\,e^{3}x^{4} + 8\,de^{2}x^{3} - d^{3}x + 8\,ae^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2),x)
```

```
[Out] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

3.780 $\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=748

$$\frac{4e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}+\frac{384d^2e^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\left(d^4-64ae^3\right)\left(256ae^3+5d^4\right)^{3/2}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}$$

```
[Out] 4*e*(1/4*d/e+x)*(13*d^4-256*a*e^3-48*d^2*e^2*(1/4*d/e+x)^2)/(-16384*a^2*e^6
-64*a*d^4*e^3+5*d^8)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)+384*d^2*e^
2*(1/4*d/e+x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/(-64*a*e^3+d^4)/(
256*a*e^3+5*d^4)^(3/2)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))-12*
d^2*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))^2)^(1/2)/cos(2*arctan
((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticE(sin(2*arctan((4*e*x+d)/(256*
a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2))*2^(1/2)*(
1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d
^3*x+8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(
1/2))^2)^(1/2)/(-64*a*e^3+d^4)/(256*a*e^3+5*d^4)^(1/4)/(8*e^3*x^4+8*d*e^2*x
^3-d^3*x+8*a*e^2)^(1/2)-2*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))
^2)^(1/2)/cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticF(sin(2*
arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(
1/2))^(1/2))*2^(1/2)*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(5*d^
4+256*a*e^3-3*d^2*(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+
8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))
^2)^(1/2)/(-64*a*e^3+d^4)/(256*a*e^3+5*d^4)^(3/4)/(8*e^3*x^4+8*d*e^2*x^3-d^
3*x+8*a*e^2)^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 748, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1106, 1092, 1197, 1103, 1195}

$$\frac{4e\left(\frac{d}{4e}+x\right)\left(-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2+13d^4\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}+\frac{384d^2e^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\left(d^4-64ae^3\right)\left(256ae^3+5d^4\right)^{3/2}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
```

```
[Out] (4*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d
^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e
^3*x^4]) + (384*d^2*e^2*(d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 +
8*e^3*x^4])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e^3)^(3/2)*(1 + (16*e^2*(d/(4*
e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])) - (12*Sqrt[2]*d^2*Sqrt[(e*(8*a*e^2 - d
^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e)
 + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*
d^4 + 256*a*e^3])*EllipticE[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)]
, (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e^3)*(5*d^4 + 256*
a*e^3)^(1/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4]) - (2*Sqrt[2]*
```

```
(5*d^4 + 256*a*e^3 - 3*d^2*Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*
x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) +
x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4
 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (
1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e
^3)^(3/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4\right)^{3/2}}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}} - \frac{8\,\text{Subst}}{}$$

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}} - \frac{\left(12d^2e\right)}{}$$

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}} + \frac{96d^2e}{\left(d^4-6\right.}$$

Mathematica [B] time = 6.15, size = 7629, normalized size = 10.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}}{64\,e^6x^8+128\,de^5x^7+64\,d^2e^4x^6-16\,d^3e^3x^5+128\,ade^4x^3+d^6x^2-16\,ad^3e^2x+64\,a^2e^4-16\left(d^4e^2-8\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)/(64*e^6*x^8 + 128*
d*e^5*x^7 + 64*d^2*e^4*x^6 - 16*d^3*e^3*x^5 + 128*a*d*e^4*x^3 + d^6*x^2 - 1
6*a*d^3*e^2*x + 64*a^2*e^4 - 16*(d^4*e^2 - 8*a*e^5)*x^4), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.08, size = 8103, normalized size = 10.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2),x)
```

```
[Out] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(3/2),x)
```

```
[Out] Integral((8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)**(-3/2), x)
```

3.781 $\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

Optimal. Leaf size=452

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{16(2a+7)}{35\sqrt{}}$$

```
[Out] 1/7*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/35*(7+2*a)*(-1+x)*(1+(-1+x)^2
/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/35*(13+
5*a-3*(-1+x)^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+4/35*(3+a)*(16+5*a)*
(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*E
llipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-
2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)
^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))
/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+16/35*(7+2*a)*(1/(1+(-1+x)^2/(1+(4+a)^
(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)
^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(
1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^
(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+
x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1106, 1091, 1176, 1202, 531, 418, 492, 411}

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{16(2a+7)}{35\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (-16*(7 + 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x
))/(35*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(13 + 5*a - 3*(-1 + x)
^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + ((3 + a - 2*(-1
+ x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sq
rt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1
 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt
[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt
[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[
1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 +
 x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2
*(-1 + x)^2 - (-1 + x)^4])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} dx = \text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^{3/2} dx, x, -1+x\right)$$
$$= \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{7}\text{Subst}\left(\int \left(2(3+a)-2x\right.\right.$$
$$= -\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3+a-\right.$$

$$= -\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3+a-\right.$$

$$= -\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3+a-\right.$$

$$= \frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}}-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3}$$

$$= \frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}}-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3}$$

Mathematica [B] time = 6.13, size = 6287, normalized size = 13.91

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-x^4+4x^3-8x^2+a+8x\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

[Out] Timed out

maple [B] time = 0.08, size = 2655, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1
/2)-66/35*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+14/5*x^2*(-x^4+4*x^3-8*x^2+a+8
*x)^(1/2)+(3/7*a-32/35)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-3/7*a-4/7)*(-x^4
+4*x^3-8*x^2+a+8*x)^(1/2)-(a^2-(3/7*a-32/35)*a+12/7*a+16/7)*((-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))
^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1
+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(
-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1
/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+
(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)
-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(64/35*a+32/5)*((-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1
-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(
x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(
a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2)
)^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*
(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1
/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*E
llipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^
(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-
1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2)))^(1/2)))+(-32/35*a-16/5)*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-
1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1
/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(
1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-
2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4
)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(
1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(
a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1
```

```
-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2
))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(
1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-
(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(
a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-
1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2
)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^
(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1
+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+
(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/
(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-
1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+
4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a - x^{4} + 4 x^{3} - 8 x^{2} + 8 x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.782 $\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=397

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+\right.}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x}}$$

```
[Out] -2/3*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-
1+x)^4)^(1/2)+1/3*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/3*(3+a)*(1/(1+(-
1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF
((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^
(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^
(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+
x)^2/(1+(4+a)^(1/2))))^(1/2)+2/3*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+
(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(
-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(
-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x
)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2
))))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1106, 1091, 1202, 531, 418, 492, 411}

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+\right.}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (-2*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)
^4]*(-1 + x))/3 + (2*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^
2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*
Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))
/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4
]) + (2*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*El
lipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[
4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + S
qrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) + \frac{1}{3}\text{Subst}\left(\int \frac{2(3+a)-2x^2}{\sqrt{3+a-2x^2-x}}\right.$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) + \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+}{-2+2\sqrt{}}}\right.}{3\sqrt{3}}$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) - \frac{\left(2\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1}{-2+2}}\right.}{3\sqrt{3}}$$

$$= \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-$$

$$= \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-$$

Mathematica [B] time = 6.06, size = 3470, normalized size = 8.74

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (-1/3 + x/3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4] + (2*((4*(-Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[
((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4
+ a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4
 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[
-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt
[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], (
(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]
])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4
+ a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x
^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sq
rt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(S
```

```
qrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Ellipt
icF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + S
qrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[
-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]]))]])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 -
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 +
 Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 +
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1
- Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1
+ Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
 - Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[A
rcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1
 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1
 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-
1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x
))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4
 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3
 - x^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]]
+ x)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
 x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
+ x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
 + x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSi
n[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + S
qrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 -
Sqrt[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - S
qrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
 - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*
(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^
2])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x
^2 + 4*x^3 - x^4]))/3
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

maple [B] time = 0.03, size = 2519, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
[Out] 1/3*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/
3*a+4/3)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1
+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))
^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2
))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
)^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(
1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(
a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-
(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1
/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))
/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-4/3*((-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2)
)^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2)
)^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-
1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-
(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(
1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1
+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(
1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x
```

```
-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-2/3*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x
-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(
-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+
4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^
(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+
(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(
1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^
(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)
-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+
(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(
-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))
^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^
(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-
1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1
+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))
/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(
-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a
+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

```
[Out] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.783 $\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=144

$$\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] (1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*E
llipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-
2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)
^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))
/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1106, 1104, 418}

$$\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt
[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1104

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b +
q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (
2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$= \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x\right.}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= -\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] time = 1.49, size = 540, normalized size = 3.75

$$\frac{2\left(\sqrt{-\sqrt{a+4}-1}-x+1\right)\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(\sqrt{\sqrt{a+4}-1}-x+1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(\sqrt{-\sqrt{a+4}-1}-x+1\right)}}\left(\sqrt{-\sqrt{a+4}-1}+x-1\right)\sqrt{\frac{\sqrt{-\sqrt{a+}}\cdots}{\left(\sqrt{\sqrt{a+4}-1}-\sqrt{\cdots}\right.}}}{\sqrt{-\sqrt{a+4}-1}\sqrt{\frac{\left(\sqrt{-\sqrt{a+4}-1}-\sqrt{\sqrt{a+4}-1}\right)\left(\sqrt{-\sqrt{a}}\cdots\right.}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(\sqrt{-\sqrt{a}}\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt[
-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqrt[(
Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Ellip
ticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + S
qrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(Sq
rt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + S
qrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8 + 8*x - 4*x^
2 + x^3)])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}{x^4-4\,x^3+8\,x^2-a-8\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^4 - 4*x^3 + 8*x^2 - a - 8
*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

maple [B] time = 0.03, size = 530, normalized size = 3.68

$$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^{2}\sqrt{-}}{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
[Out] -((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+
(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+
4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/
((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/
(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(
x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/
2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)

[Out] Integral(1/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.784 $\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}} \, dx$

Optimal. Leaf size=437

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x}{1-}\right.}{2(a+4)\sqrt{\ldots}}$$

```
[Out] 1/2*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2*
(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)/(3+a-2*(-1
+x)^2-(-1+x)^4)^(1/2)+1/2*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^
2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2
/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2
/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/
2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/2*(1
/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Ell
ipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*
(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(
1/2))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((
1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1106, 1092, 1202, 531, 418, 492, 411}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x}{1-}\right.}{2(a+4)\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)
^2 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(
-1 + x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1
- Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
 + a])])/(2*(3 + a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sq
rt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1
 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(4 +
a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]
))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\text{Subst}\left(\int \frac{-2(3+a)+2x^2}{\sqrt{3+a-2x^2-x^4}}\,d\right.}{4\left(12+7a+\right.}$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\right.}{4\left(12\right.}$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\right.}{2\left(12\right.}$$

$$= \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+\right.}{2\left(12+7a+a^2\right)\sqrt{3+a-}}$$

$$= \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+\right.}{2\left(12+7a+a^2\right)\sqrt{3+a-}}$$

Mathematica [B] time = 6.09, size = 3526, normalized size = 8.07

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((6 + a - 8*x - a*x + 3*x^2 - x^3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4])/(2*
(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)) + ((4*(-Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 +
a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
- Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 +
 a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4
+ a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-
Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 +
a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2
 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*
(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
```

```
1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a
]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt
[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqr
t[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Elliptic
F[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqr
t[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^
2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + S
qrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 -
Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 +
Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 -
 Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[Arc
Sin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 -
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 +
 Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1
- Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])
/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))
/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 +
 a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt
[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 -
 x^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] +
x)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x
))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt
[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
 x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[
Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqr
t[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sq
rt[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]
] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
 x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4
 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-
1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]
```

```
)/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2
 + 4*x^3 - x^4])/(2*(3 + a)*(4 + a))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{x^8-8x^7+32x^6-2(a-64)x^4-80x^5+8(a-16)x^3-16(a-4)x^2+a^2+16ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^8 - 8*x^7 + 32*x^6 - 2*(a
- 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+4x^3-8x^2+a+8x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

maple [B] time = 0.03, size = 2601, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] 2*(1/4/(a^2+7*a+12)*x^3-3/4/(a^2+7*a+12)*x^2+1/4*(a+8)/(a^2+7*a+12)*x-1/4*(
a+6)/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-((a+5)/(a^2+7*a+12)-1/2*(
a+8)/(a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^
(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+
(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))
^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/
2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4
)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*
(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(
((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/
2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^
(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/(a^2+7
*a+12)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(
-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(
1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))
^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^
(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/
2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+
4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(
```

```
1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+
(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(
-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))
^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-
1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))-1/2/(a^2+7*a+12
)*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)
^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^
(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4
)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/
2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^
(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-
(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(
1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1
/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/
2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1
/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2)
)^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(
-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(
1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4 + 4x^3 - 8x^2 + a + 8x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

```
[Out] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

3.785 $\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=517

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{(2a+7)}{3(a+3)^2(}$$

```
[Out] 1/6*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/12
*(104+47*a+5*a^2+4*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-
(-1+x)^4)^(1/2)-1/3*(7+2*a)*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1
/2))/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(16+5*a)*(1/(1+(-1
+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF(
(-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(
1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(
1/2)/(3+a)/(4+a)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1
/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/3*(7+2*a)*(1/(1+(-1+x)^2/(1+(4+
a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4
+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a
)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2
))^(1/2)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+
a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1106, 1092, 1178, 1202, 531, 418, 492, 411}

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{(2a+7)}{3(a+3)^2(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)^(3/2)) + ((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 +
x))/(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((7
+ 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3
 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1
- Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
 + a])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1
+ (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
 ((16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellip
ticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 +
 a])])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (
-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^{5/2}}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{4+2(3+a)\cdots}{(3+a\cdots}\right.}{12\left(\cdots\right.}$$

$$= -\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a\cdots\right.}{6\left(12+7a+a^2\right)\left(3\cdots\right.}$$

$$= -\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a\cdots\right.}{6\left(12+7a+a^2\right)\left(3\cdots\right.}$$

$$= -\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a\cdots\right.}{6\left(12+7a+a^2\right)\left(3\cdots\right.}$$

$$= -\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(7+2a)\left(1-\sqrt{4\cdots}\right.}{3\left(12+7a+a^2\right)^2\sqrt{\cdots}}$$

$$= -\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(7+2a)\left(1-\sqrt{4\cdots}\right.}{3\left(12+7a+a^2\right)^2\sqrt{\cdots}}$$

Mathematica [B] time = 6.22, size = 6386, normalized size = 12.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+4x^3-8x^2+a\cdots}}{x^{12}-12x^{11}+72x^{10}-3(a-256)x^8-280x^9+24(a-64)x^7-32(3a-70)x^6+48(5a-48)x^5+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^12 - 12*x^11 + 72*x^10 -
3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5*a -
```

```
48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3 - 2
4*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 2757, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)
```

```
[Out] (1/6/(a^2+7*a+12)*x^3-1/2/(a^2+7*a+12)*x^2+1/6*(a+8)/(a^2+7*a+12)*x-1/6*(a+
6)/(a^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2
*(1/6*(2*a+7)/(a^2+7*a+12)^2*x^3-1/2*(2*a+7)/(a^2+7*a+12)^2*x^2+1/24*(5*a^2
+71*a+188)/(a^2+7*a+12)^2*x-1/24*(5*a^2+55*a+132)/(a^2+7*a+12)^2)/(-x^4+4*x
^3-8*x^2+a+8*x)^(1/2)-(1/6*(5*a^2+47*a+104)/(a^2+7*a+12)^2-1/12*(5*a^2+71*a
+188)/(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1
-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(
-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(
a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2
))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*Ellipti
cF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^
(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2
))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^
(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-2/3*(
2*a+7)/(a^2+7*a+12)^2*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1
-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(
-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(
a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2
))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1
+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2)
)^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)
^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^
(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1
```

```
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))-
1/3*(2*a+7)/(a^2+7*a+12)^2*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/
2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(
1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+
4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+
(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/
2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*
(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2
))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1
+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/
2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2
))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/
2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-
(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x
-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2)
)^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))
^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(
((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/
2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^
(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4
)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticP
i(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2)
)^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^
(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2)
)*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

3.786 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

Optimal. Leaf size=558

$$\frac{3}{16}(a+4)\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{8}\left((x-1)^2+1\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{7}(x-1$$

```
[Out] 1/8*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/7*(3+a-2*(-1+x)^2-(-1+x)
^4)^(3/2)*(-1+x)+3/16*(4+a)^2*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)
^(1/2))-16/35*(7+2*a)*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(
3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+3/16*(4+a)*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+
x)^4)^(1/2)+2/35*(13+5*a-3*(-1+x)^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)
+4/35*(3+a)*(16+5*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+
(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(
4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(
4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+
x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+16/35*(7+2*a)*(1/
(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Elli
pticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(
4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1
/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-
(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1680, 1673, 1091, 1176, 1202, 531, 418, 492, 411, 1107, 612, 621, 204}

$$\frac{3}{16}(a+4)\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{8}\left((x-1)^2+1\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{7}(x-1$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/16 + (
(1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2))/8 - (16*(7 + 2*
a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(35*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(13 + 5*a - 3*(-1 + x)^2)*Sqrt[3 +
 a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + ((3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)^(3/2)*(-1 + x))/7 + (3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 +
a - 2*(-1 + x)^2 - (-1 + x)^4]])/16 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sqrt[
1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1
 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt[4
+ a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 +
 Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 + x)
^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-
1 + x)^2 - (-1 + x)^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}dx = \text{Subst}\left(\int (1+x)\left(3+a-2x^2-x^4\right)^{3/2}dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^{3/2}dx, x, -1+x\right) + \text{Subst}\left(\int x\left(3+a-2x^2 -\right.\right.$$
$$= \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{3}{7}\text{Subst}\left(\int \left(2(3+a)-2\right.\right.$$
$$= \frac{1}{8}\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}\left(1+(-1+x)^2\right) - \frac{2}{35}\left(13+5a-3(1-x\right.$$
$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{8}\left(3+a-2(1-\right.$$
$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{8}\left(3+a-2(1-\right.$$
$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{8}\left(3+a-2(1-\right.$$
$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{8}\left(3+a-2(1-\right.$$

Mathematica [B] time = 6.14, size = 7235, normalized size = 12.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(x^5-4x^4+8x^3-ax-8x^2\right)\sqrt{-x^4+4x^3-8x^2+a+8x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(x^5 - 4*x^4 + 8*x^3 - a*x - 8*x^2)*sqrt(-x^4 + 4*x^3 - 8*x^2 + a
 + 8*x), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 2694, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] -1/8*x^6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+17/28*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2
)*x^5-43/28*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)*x^4+74/35*(-x^4+4*x^3-8*x^2+a+8*
x)^(1/2)*x^3+(5/16*a-9/20)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-11/56*a-29/
70)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(11/56*a+13/14)*(-x^4+4*x^3-8*x^2+a+8*
x)^(1/2)-(-(-11/56*a-29/70)*a-11/14*a-26/7)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+
4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1
+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+
(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)
^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4
)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1
/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+
4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1
+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))
^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*
(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2
))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^
(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2)
)^(1/2)))^(1/2))-(a^2-2*(5/16*a-9/20)*a+55/14*a+62/5)*((-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2
*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)
^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(
1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a
+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(
1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+
(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(
-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))
/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(
-1+(a+4)^(1/2))^(1/2)))^(1/2)))+(-32/35*a-16/5)*((x-1-(-1+(a+4)^(1/2))^(1/2
))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2)
)^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2)
)^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-
1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-
```

```
(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(
a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1
+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/
2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)
-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1
-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(
x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2
))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1
/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x
-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2
))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(
-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(
1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4+4x^3-8x^2+a+8x\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(-x^4+4x^3-8x^2+8x+a\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral(x*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.787 $\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=466

$$\frac{1}{4}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\right.}{3\sqrt{a-(x-1)^4-2(x-}}$$

```
[Out] 1/4*(4+a)*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))-2/3*(-1+x)*(
1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)
+1/4*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/3*(-1+x)*(3+a-2*(-1+x)^
2-(-1+x)^4)^(1/2)+2/3*(3+a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x
)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)
^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)
^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/(
(1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+2/3*(1/(1+
(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Ellipti
cE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a
)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2)
)*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+
a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1680, 1673, 1091, 1202, 531, 418, 492, 411, 1107, 612, 621, 204}

$$\frac{1}{4}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\right.}{3\sqrt{a-(x-1)^4-2(x-}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/4 - (2*(1 - Sqrt
[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*Sqrt[3 + a - 2*(-1
 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))
/3 + ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4]])/4 + (2*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 +
a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1
 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(
3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a
]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int (1+x)\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int x\sqrt{3+a-2x^2-$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) + \frac{1}{3}\text{Subst}\left(\int \frac{2(3+a)-2x}{\sqrt{3+a-2x^2-}}\right.$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\,\left(1+(-1+x)^2\right) + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\,\left(1+(-1+x)^2\right) + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\,\left(1+(-1+x)^2\right) + \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-}{1-\sqrt{}}\right.}{3\sqrt{3+a-2(1-x)^2-}}$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\,\left(1+(-1+x)^2\right) + \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-}{1-\sqrt{}}\right.}{3\sqrt{3+a-2(1-x)^2-}}$$

Mathematica [B] time = 6.09, size = 4389, normalized size = 9.42

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (1/6 - x/6 + x^2/4)*Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)] + (Sqrt[a - x*(-8
+ 8*x - 4*x^2 + x^3)]*((-8*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]
])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(S
qrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(
Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Ellip
ticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 +
Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt
[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
+ x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[
-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt
[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])
*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqr
t[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]
] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - S
qrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
 Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[
-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a
 + 8*x - 8*x^2 + 4*x^3 - x^4]) + (40*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]
*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))
]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)
)]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (6*a*(-Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sq
rt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[
-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt
[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqr
t[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[
Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqr
```

```
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqr
t[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sq
rt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
 - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4
]) - (4*((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x
)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[S
qrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt
[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqr
t[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]]
 - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt
[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 -
 Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])
/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))))/Sqrt[a + 8*x - 8*x^2
 + 4*x^3 - x^4]))/(6*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

maple [B] time = 0.03, size = 2551, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^(1/2)*x,x)
```

```
[Out] 1/4*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)*x^2-1/6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)*x
+1/6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(1/6*a-2/3)*((-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1
-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(
x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(
a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2)
)^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*
(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1
/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(
1/2))^(1/2)))^(1/2))-(1/2*a+10/3)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^
(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1
/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(
1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(
-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-
1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^
(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)
^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(
1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1
+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/
(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2
)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))
^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2
)))^(1/2)))-2/3*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*
(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2)
)^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(
x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a
+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1
+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^
(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-
(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2
)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*E
llipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(
1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4
```

```
)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))
-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(
a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/
2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2
)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/
((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1
/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1
-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(
1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/
2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-
(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
[Out] Integral(x*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.788 $\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=179

$$\frac{1}{2}\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] 1/2*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+(1/(1+(-1+x)^2/(1+
(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1
+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(
4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a
-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4
+a)^(1/2))))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1680, 1673, 1104, 418, 1107, 621, 204}

$$\frac{1}{2}\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]]/2 + (Sqrt[
1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 +
a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1104

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b +
q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (
2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \text{Subst}\left(\int \frac{1+x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{\sqrt{3+a-2x^2-x^4}} \cdots\right.$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x-x^2}}\,dx, x, (-1+x)^2\right) + \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\,\sqrt{1-\cdots}\right.}{\cdots}$$

$$= -\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\,\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \text{Subst}\left(\int \cdots\right.$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)\cdots}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\,\sqrt{3+a\cdots}}$$

Mathematica [B] time = 2.93, size = 813, normalized size = 4.54

$$\frac{2\left(-x+\sqrt{-\sqrt{a+4}-1}+1\right)\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(-x+\sqrt{\sqrt{a+4}-1}+1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(-x+\sqrt{-\sqrt{a+4}-1}+1\right)}}\left(x+\sqrt{-\sqrt{a+4}-1}-1\right)\sqrt{\frac{\sqrt{-\sqrt{a+}}\cdots}{\left(\sqrt{\sqrt{a+4}-1}-\sqrt{\cdots}\right.}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt[
-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqrt[(
Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*((1 +
 Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])^2] - 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]]
 + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8
+ 8*x - 4*x^2 + x^3)])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{x^4-4x^3+8x^2-a-8x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^4 - 4*x^3 + 8*x^2 - a -
 8*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^4+4x^3-8x^2+a+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

maple [B] time = 0.03, size = 788, normalized size = 4.40

$$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^{2}\ \sqrt{\ }}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] -((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+
(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+
4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/
((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/
(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(
x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/
2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*
EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^
(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*(
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)
)+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(
1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4
)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),
((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

```
[Out] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.789 $\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=474

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-}{2(a+3)(a+4)\sqrt{a-(}}$$

```
[Out] 1/2*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/2*(5+a+(-1+x)^2)*(
-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2*(-1+x)*(1+(-1+x)^2/(
1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2
)+1/2*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(
1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1
/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1
+(4+a)^(1/2))^(1/2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(
4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/2*(1/(1+(-1+x)^2/(1+(4+a
)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+
a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)
^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2)
)^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^
(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1680, 1673, 1092, 1202, 531, 418, 492, 411, 1107, 613}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-}{2(a+3)(a+4)\sqrt{a-(}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((5
+ a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 +
 x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1 - Sq
rt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipti
cE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a
])])/(2*(3 + a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1
 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x
)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(4 + a)*S
qrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*
Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
```

```
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1+x}{\left(3+a-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{\left(3+a-2x^2\right.}\right.$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{1}{2}\text{Subst}\left(\int \frac{}{\left(3+a\right.}\right.$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+x)^2\right)(}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1}}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+x)^2\right)(}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1}}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right.}{2\left(12+7a+a^2\right)\sqrt{3+a-2(1-}}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right.}{2\left(12+7a+a^2\right)\sqrt{3+a-2(1-}}$$

Mathematica [B] time = 6.08, size = 3593, normalized size = 7.58

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] ((-a - 2*x + a*x - a*x^2 - x^3)*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2)/(2*(3 +
a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8 + 8*x - 4*x^2 + x^3)
)^(3/2)) + ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((4*(-Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]
] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a
```

```
]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
 Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 +
a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
- Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]
]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 +
 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-
1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]]
 + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-
1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]
] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[
-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[
ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[
-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
 Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1
 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*
Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqr
t[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sq
rt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sq
rt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - S
qrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSi
n[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + S
qrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 -
Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/(
(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a
]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x
^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)
*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))
]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sq
rt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[
-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt
[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]]
```

```
- Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x
))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
+ Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/
(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2 +
 4*x^3 - x^4]))/(2*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^(3/2))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,x}{x^8-8\,x^7+32\,x^6-2\,(a-64)x^4-80\,x^5+8\,(a-16)x^3-16\,(a-4)x^2+a^2+16\,ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^8 - 8*x^7 + 32*x^6 - 2*(
a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

maple [B] time = 0.03, size = 2616, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] 2*(1/4/(a^2+7*a+12)*x^3+1/4*a/(a^2+7*a+12)*x^2-1/4*(a-2)/(a^2+7*a+12)*x+1/4
/(a^2+7*a+12)*a)/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)+1/2*(a-2)/(
a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*
(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1
/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2)
)/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1
/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2
))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(
-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
)^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))
```

```
^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-((1+a)/(a^2+7*
a+12)-1/(a^2+7*a+12)*a)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))
/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(
-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1
+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(
1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4
)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1
+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1
/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(
-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-
1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),
(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))
)-1/2/(a^2+7*a+12)*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2
))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2)
)^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1
/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2
)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1
-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+
(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+
4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(
-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2)
)^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2
)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(
a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/
2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^
(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))
^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
)^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(
1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2
)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^
(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(
-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.790 $\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=591

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{}{3(a+4)^2\sqrt{a}}$$

```
[Out] 1/6*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/6*(5+a+(-1+x)^2)*(
-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/3*(1+(-1+x)^2)/(4+a)^2
/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(104+47*a+5*a^2+4*(7+2*a)*(-1+x)^2)*(
-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/3*(7+2*a)*(-1+x)*(1+
(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-
1+x)^4)^(1/2)+1/12*(16+5*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x
)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)
^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)
^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a)/(4+a)^2/(3+a-2*(-1+x)^2-(-1
+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1
/2)+1/3*(7+2*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)
^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^
(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^
(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^
2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))
))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1680, 1673, 1092, 1178, 1202, 531, 418, 492, 411, 1107, 614, 613}

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{}{3(a+4)^2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(6*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (1
 + (-1 + x)^2)/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((5
+ a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)^(3/2)) + ((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/
(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((7 + 2*
a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3 + a
)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1 - Sq
rt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipti
cE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a
])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1
6 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF
[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])
])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
 x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
```

```
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1+x}{\left(3+a-2x^2-x^4\right)^{5/2}}\,dx,x,-1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^{5/2}}\,dx,x,-1+x\right) + \text{Subst}\left(\int \frac{x}{\left(3+a-2x^2\right.}\right.$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{1}{2}\text{Subst}\left(\int \frac{}{(3+}\right.$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} - \frac{\left(104+47a+5a^2+4(7+2\right.}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} + \frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} + \frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} + \frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} + \frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2}}$$

Mathematica [B] time = 6.13, size = 6452, normalized size = 10.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a}}{x^{12}-12\,x^{11}+72\,x^{10}-3\,(a-256)x^8-280\,x^9+24\,(a-64)x^7-32\,(3\,a-70)x^6+48\,(5\,a-48)x^5+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^12 - 12*x^11 + 72*x^10
- 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5*a
 - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3 -
 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 2777, normalized size = 4.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)
```

```
[Out] (1/6/(a^2+7*a+12)*x^3+1/6/(a^2+7*a+12)*a*x^2-1/6*(a-2)/(a^2+7*a+12)*x+1/6/(
a^2+7*a+12)*a)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2*(
1/6*(2*a+7)/(a^2+7*a+12)^2*x^3+1/6*(a^2-12)/(a^2+7*a+12)^2*x^2-1/24*(3*a^2-
23*a-116)/(a^2+7*a+12)^2*x+1/24*(3*a^2-7*a-60)/(a^2+7*a+12)^2)/(-x^4+4*x^3-
8*x^2+a+8*x)^(1/2)-(1/6*(a^2+23*a+68)/(a^2+7*a+12)^2+1/12*(3*a^2-23*a-116)/
(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(
a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/
2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)
^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))
^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(
1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-
1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))
/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(2/3*(a^2+2
*a-5)/(a^2+7*a+12)^2-2/3*(a^2-12)/(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x
-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1
+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-
1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/
2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(
1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2)
)*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^
(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^
(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-
(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)
*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4
```

```
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(
a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/
(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-
(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2)))^(1/2)))-1/3*(2*a+7)/(a^2+7*a+12)^2*((x-1-(-1+(a+4)^(1/2
))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2)
)^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)
*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))
^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((
1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1
/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/
2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^
(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2
))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(
1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))
^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+
4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(
1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))
*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(
1/2))^(1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x + a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

3.791 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

Optimal. Leaf size=585

$$\frac{4\left(21a^2+111a+140\right)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{315\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{4\left(21a^2+111a+140\right)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\ \Big(}{315\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\ \sqrt{a-(x-}}$$

```
[Out] 1/4*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/63*(15+7*(-1+x)^2)*(3+a-
2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)+3/8*(4+a)^2*arctan((1+(-1+x)^2)/(3+a-2*(-
1+x)^2-(-1+x)^4)^(1/2))+4/315*(21*a^2+111*a+140)*(-1+x)*(1+(-1+x)^2/(1-(4+a
)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+3/8*(4+a)*(1+(-1+
x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/315*(160+54*a+3*(20+7*a)*(-1+x)^2)*
(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+4/315*(3+a)*(100+33*a)*(1/(1+(-1+x)^
2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+
x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)
/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)
/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/
(1+(4+a)^(1/2))))^(1/2)-4/315*(21*a^2+111*a+140)*(1/(1+(-1+x)^2/(1+(4+a)^(1
/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(
1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/
2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1
/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)
^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1680, 1673, 1176, 1202, 531, 418, 492, 411, 12, 1107, 612, 621, 204}

$$\frac{4\left(21a^2+111a+140\right)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{315\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{4\left(21a^2+111a+140\right)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\ \Big(}{315\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\ \sqrt{a-(x-}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/8 + ((
1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2))/4 + (4*(140 + 11
1*a + 21*a^2)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x)
)/(315*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(2*(80 + 27*a) + 3*(20
 + 7*a)*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/315 +
 ((15 + 7*(-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/6
3 + (3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 +
x)^4]])/8 - (4*(140 + 111*a + 21*a^2)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a
]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sq
rt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[(1 + (-1 + x)^2
/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]) + (4*(3 + a)*(100 + 33*a)*Sqrt[1 + Sqrt[4 + a]]*(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a
]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}dx = \text{Subst}\left(\int (1+x)^2\left(3+a-2x^2-x^4\right)^{3/2}dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int 2x\left(3+a-2x^2-x^4\right)^{3/2}dx, x, -1+x\right) + \text{Subst}\left(\int \left(1+x^2\right)(3$$

$$= \frac{1}{63}\left(15+7(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) - \frac{1}{21}\text{Su}$$

$$= -\frac{2}{315}\left(2(80+27a)+3(20+7a)(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1$$

$$= \frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2} - \frac{2}{315}\left(2(80+27a)\right.$$

$$= \frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{4}\left(1+(-1+x)^2\right.$$

$$= \frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{4}\left(1+(-1+x)^2\right.$$

$$= \frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{4}\left(1+(-1+x)^2\right.$$

Mathematica [B] time = 6.17, size = 8500, normalized size = 14.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(x^6-4\,x^5+8\,x^4-ax^2-8\,x^3\right)\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
[Out] integral(-(x^6 - 4*x^5 + 8*x^4 - a*x^2 - 8*x^3)*sqrt(-x^4 + 4*x^3 - 8*x^2 +
 a + 8*x), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 2733, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] -1/9*x^7*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+19/36*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2
)*x^6-163/126*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)*x^5+71/42*(-x^4+4*x^3-8*x^2+a+
8*x)^(1/2)*x^4+(11/45*a-16/63)*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-13/120*
a-5/18)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(9/140*a+23/63)*x*(-x^4+4*x^3-8*
x^2+a+8*x)^(1/2)+(107/252*a+101/63)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(-(9/140
*a+23/63)*a-107/63*a-404/63)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2)
)^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(
x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a
+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1
+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2
))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*
EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^
(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*(
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)
)-(-2*(-13/120*a-5/18)*a+827/315*a+76/9)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^
(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a
+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1
+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1
/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(
1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)
*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^
(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-
1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1
/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1
/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/
2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*Elliptic
Pi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^
(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2
))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)
^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2
))^(1/2)))^(1/2)))+(a^2-3*(11/45*a-16/63)*a+68/105*a+16/9)*((x-1-(-1+(a+4)^
(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(
-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(
```

```
1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1
-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(
1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/
2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2)
)^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)
^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^
(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(
1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-
1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^
(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2
))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1
+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4
)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2)
)^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1
-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+
4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1
/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+
4)^(1/2))^(1/2)))^(1/2)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(a-x^{4}+4x^{3}-8x^{2}+8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral(x**2*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.792 $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

<u>**Optimal**</u>. Leaf size=485

$$\frac{1}{2}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{2(3a+}{1}$$

```
[Out] 1/2*(4+a)*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+2/15*(8+3*a)
*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)
^4)^(1/2)+1/2*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/15*(7+3*(-1+x)
^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+8/15*(3+a)*(1/(1+(-1+x)^2/(1+(4+
a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4
+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a
)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*
(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)
^(1/2))))^(1/2)-2/15*(8+3*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+
x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x
)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x
)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-
(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))
^(1/2)
```

<u>**Rubi**</u> **[A]** time = 0.53, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1680, 1673, 1176, 1202, 531, 418, 492, 411, 12, 1107, 612, 621, 204}

$$\frac{1}{2}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{2(3a+}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/2 + (2*(8 + 3*a)
*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(15*Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]*(-1 + x))/15 + ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]])/2 - (2*(8 + 3*a)*(1 - Sqrt[4 + a])*Sqrt
[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 +
 x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqrt[(
1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (8*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 +
 (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 +
a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqrt[(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4])
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int (1+x)^2\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int 2x\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int \left(1+x^2\right)\sqrt{3+a-\right.$$
$$= \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) - \frac{1}{15}\,\text{Subst}\Big($$
$$= \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) - \frac{\left(\sqrt{1-\frac{2(-1}{-2-2}}\right.}{}$$
$$= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} + \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{}$$
$$= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} - \frac{2(8+3a)\left(1-\sqrt{4+a}\right.}{15\sqrt{3+a-2(1}}$$
$$= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} - \frac{2(8+3a)\left(1-\sqrt{4+a}\right.}{15\sqrt{3+a-2(1}}$$

Mathematica [B] time = 6.12, size = 5647, normalized size = 11.64

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
[Out] Result too large to show
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

maple [B] time = 0.03, size = 2582, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^(1/2)*x^2,x)
```

```
[Out] 1/5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)*x^3-1/10*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)*
x^2+1/15*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)*x+1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2
)-(-1/15*a-4/3)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(
a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/
2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)
^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))
^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(
1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-
1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))
/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(1/5*a+28/1
5)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2
)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(
1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(
a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2)
)/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(
-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(
1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2)
)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(
a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/
2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))+(2/5*a+16/15)*((x-1
-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))
^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))
^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/
2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+
(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^
(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2)
)^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))
^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-
(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(
1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)
```

```
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1
+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(
-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1
/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4
)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(
x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
[Out] Integral(x**2*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.793 $\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=388

$$\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)}}$$

```
[Out] arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+(-1+x)*(1+(-1+x)^2/(1-
(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+(1/(1+(-1+x)^
2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+
x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)
/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)
/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/
(1+(4+a)^(1/2))))^(1/2)-(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/
(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(
1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(
1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x
)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2
)
```

Rubi [A] time = 0.39, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1680, 1673, 1202, 531, 418, 492, 411, 12, 1107, 621, 204}

$$\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/Sqrt[3 + a
- 2*(-1 + x)^2 - (-1 + x)^4] + ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 +
 x)^2 - (-1 + x)^4]] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 +
x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (
-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])]])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a])
)/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4]) + (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a])
)]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{\sqrt{3+a-2x^2-x^4}}\right.$$

$$= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(\cdot}{-2+}}\right.}{}$$

$$= \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x\right.}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= -\frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-}\right.}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1}}$$

$$= -\frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)}}\right.$$

Mathematica [B] time = 6.06, size = 1247, normalized size = 3.21

$$2\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\sqrt{\frac{\left(\sqrt{-\sqrt{a+4}-1}-\sqrt{\sqrt{a+4}-1}\right)\left(x+\sqrt{-\sqrt{a+4}-1}-1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(-x+\sqrt{-\sqrt{a+4}-1}+1\right)}}\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(x-\sqrt{\sqrt{a+4}-1}-\right.}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(x-\sqrt{-\sqrt{}}\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 +
Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] -
Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt
```

```
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[
(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt
[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*(((
Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sqrt[((Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]]
 + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sq
rt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]
]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]] - Sqrt[
-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]]
 - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]]
- Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]]
- Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]),
 ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[
-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sq
rt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(-Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a - x*(-8 + 8*x - 4*x^2
+ x^3)]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^2}{x^4-4x^3+8x^2-a-8x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^4 - 4*x^3 + 8*x^2 - a
 - 8*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x^4+4x^3-8x^2+a+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

maple [B] time = 0.03, size = 1147, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] ((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(
1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*
```

```
(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1
/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2)
)/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1
/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1
-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/
2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1
/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))
^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^
(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2
))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2
))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^
(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1
+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/
2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
[Out] Integral(x**2/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.794 $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=311

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x}{1-}\right.}{2(a+3)\sqrt{a-(x-1)^4-2(}}$$

```
[Out] (1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/2*(4+a)*(2+(-1+x)^2)*(
-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2*(-1+x)*(1+(-1+x)^2/(
1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/2*(
1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*El
lipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2
*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^
(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+
x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1680, 1673, 1178, 12, 1140, 492, 411, 1107, 613}

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x}{1-}\right.}{2(a+3)\sqrt{a-(x-1)^4-2(}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (1 + (-1 + x)^2)/((4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((4 +
a)*(2 + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2
 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1
 + x))/(2*(3 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1 - Sqrt[4 +
 a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[Arc
Tan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(
2*(3 + a)*Sqrt[((1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt
[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
```

```
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1140

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/
(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[x^2/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqr
t[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && NegQ[c/a]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{\left(3+a-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{\left(3+a-2x^2-x^4\right)^{3/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{\left(3+a-2x^2-x\right.}\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + 2\,\text{Subst}\left(\int \frac{x}{\left(3+a-2\right.}\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{3+a-2x^2-x^4}}\right.}{2(3+a)}$$

$$= \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{(4+a)\left(2+(-1+x)^2\right)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1\right.}}$$

$$= \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1}{2(3+a)\sqrt{3+a-2(1-x)^2-(1}}$$

$$= \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1}{2(3+a)\sqrt{3+a-2(1-x)^2-(1}}$$

Mathematica [B] time = 6.12, size = 2941, normalized size = 9.46

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] ((-a - 8*x - a*x + 6*x^2 + a*x^2 - 4*x^3 - a*x^3)*(a + 8*x - 8*x^2 + 4*x^3
- x^4)^2)/(2*(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8 +
8*x - 4*x^2 + x^3))^(3/2)) - ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((2*(-S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]
] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqr
t[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sq
rt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + S
qrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/(
(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 +
a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 -
 Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqr
t[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt
[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - (4*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
```

```
Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))
]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x
))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + ((-1 + Sqrt[-1 - S
qrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 + Sqrt[-1 + Sqrt[4 +
 a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[
-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 +
 a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4
+ a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4
 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*(((Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 -
 Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]]
, (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]) + ((-((-1 - Sqr
t[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[
4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4
 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4
 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]]
+ x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqr
t[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(-Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]))/(2*(3 + a
)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^(3/2))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,x^2}{x^8-8\,x^7+32\,x^6-2\,(a-64)x^4-80\,x^5+8\,(a-16)x^3-16\,(a-4)x^2+a^2+16\,ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^8 - 8*x^7 + 32*x^6 - 2
*(a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

maple [B] time = 0.03, size = 2607, normalized size = 8.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] 2*(1/4/(a+3)*x^3-1/4*(a+6)/(a^2+7*a+12)*x^2+1/4*(a+8)/(a^2+7*a+12)*x+1/4/(a
^2+7*a+12)*a)/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)-1/2*(a+8)/(a^2
+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(
1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-
1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2)
)^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/
2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^
(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-
(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1
-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(
1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1
/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(-2/(a^2+7*a+12)+
(a+6)/(a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))
^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(
a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1
+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2)
)^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1
/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+
4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))
*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(
a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(
1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/
2/(a+3)*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1
-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1
```

```
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
)^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-
(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-
1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2
))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(
1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(
1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1
/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF
(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))
^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1
/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))
^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/
2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(
1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-
(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1
/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.795 $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=582

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right.}{12\left(a^2+7a+12\right)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] 1/3*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/6*(4+a)*(2+(-1+x)^
2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+2/3*(1+(-1+x)^2)/(4+
a)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(29+7*a+(13+3*a)*(-1+x)^2)*(-1+x)
/(3+a)^2/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/12*(13+3*a)*(-1+x)*(1+(-1+
x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a)^2/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^
4)^(1/2)+1/12*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(
1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1
/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1
/2)))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((
1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/12*(13+3*
a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2
)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)
,(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4
+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a)^2/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1
/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 582, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1680, 1673, 1178, 1202, 531, 418, 492, 411, 12, 1107, 614, 613}

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right.}{12\left(a^2+7a+12\right)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(3*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (2
*(1 + (-1 + x)^2))/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)
^2 - (-1 + x)^4)^(3/2)) + ((29 + 7*a + (13 + 3*a)*(-1 + x)^2)*(-1 + x))/(12
*(3 + a)^2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((13 + 3*a)*(
1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(12*(3 + a)^2
*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((13 + 3*a)*(1 - Sqrt[4
 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(12*(3 + a)^2*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x
)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1
+ Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)
/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(12*(12 + 7*a
 + a^2)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4
 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{\left(3+a-2x^2-x^4\right)^{5/2}}\,dx,x,-1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{\left(3+a-2x^2-x^4\right)^{5/2}}\,dx,x,-1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{\left(3+a-2x^2-x\right.}\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + 2\,\text{Subst}\left(\int \frac{}{\left(3+a-\right.}\right.$$

$$= -\frac{\left(29+7a+(13+3a)(1-x)^2\right)(1-x)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(4+a)\left(2+(-\right.}{6\left(12+7a+a^2\right)\left(3+a\right.}$$

$$= \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{\left(29+7a+(13+3a)(1\right.}{12(3+a)^2(4+a)\sqrt{3+a-2(}}$$

$$= \frac{2\left(1+(-1+x)^2\right)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-\right.}$$

$$= \frac{2\left(1+(-1+x)^2\right)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-\right.}$$

$$= \frac{2\left(1+(-1+x)^2\right)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-\right.}$$

Mathematica [B] time = 6.17, size = 5812, normalized size = 9.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+4x^3-8x^2+a+8}}{x^{12}-12x^{11}+72x^{10}-3(a-256)x^8-280x^9+24(a-64)x^7-32(3a-70)x^6+48(5a-48)x^5+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^12 - 12*x^11 + 72*x^1
0 - 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5
*a - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3
 - 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 2780, normalized size = 4.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)
```

```
[Out] (1/6/(a+3)*x^3-1/6*(a+6)/(a^2+7*a+12)*x^2+1/6*(a+8)/(a^2+7*a+12)*x+1/6/(a^2
+7*a+12)*a)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2*(1/2
4*(13+3*a)/(a+3)/(a^2+7*a+12)*x^3-1/24*(a^2+27*a+84)/(a^2+7*a+12)^2*x^2+1/6
*(9*a+32)/(a^2+7*a+12)^2*x+1/12*(3*a^2+7*a-12)/(a^2+7*a+12)^2)/(-x^4+4*x^3-
8*x^2+a+8*x)^(1/2)-(-1/6*(a^2-9*a-44)/(a^2+7*a+12)^2-1/3*(9*a+32)/(a^2+7*a+
12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(
-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(
1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))
^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^
(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/
2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+
4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(1/3*(a^2-a-16)/(a^2
+7*a+12)^2+1/6*(a^2+27*a+84)/(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-
1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1
+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4
)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(
1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))
^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-
1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2)
)^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-
(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)
^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*Elli
pticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
```

```
(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^
(1/2))^(1/2)))^(1/2)))-1/12*(13+3*a)/(a+3)/(a^2+7*a+12)*((x-1-(-1+(a+4)^(1/
2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+
4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2
))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*(
(1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(
1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1
/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+
(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)
^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(
1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2)
)^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^
(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(
1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2)
)*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^
(1/2))^(1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

3.796 $\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx$

Optimal. Leaf size=129

$$-\frac{x^2\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)F\left(2\tan^{-1}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8x^4-x^3+8x+8}}$$

```
[Out] -1/696*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))^2)^(1/2)/cos(2*arc
tan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))*EllipticF(sin(2*arctan(1/87*(4+x)*29^(3
/4)/x*3^(1/2))),1/58*(1682+58*29^(1/2))^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)*((
261-6*(1+4/x)^2+(1+4/x)^4)/(87+(4+x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(3/4)*3^(1
/2)/(8*x^4-x^3+8*x+8)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2069, 12, 6719, 1103}

$$-\frac{x^2\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)F\left(2\tan^{-1}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8x^4-x^3+8x+8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

```
[Out] -(x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/x
^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcTan[(4 + x)/(Sqrt[3]*
29^(1/4)*x)], (29 + Sqrt[29])/58])/(8*Sqrt[3]*29^(1/4)*Sqrt[8 + 8*x - x^3 +
 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
```

```
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{1}{2\sqrt{2}\,(8-32x)^2\sqrt{\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}}}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(\left(256\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(8-32x)^2\sqrt{\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}}}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1069056-3932}}\right.}{\sqrt{8+8x-x^3+8x^4}}$$

$$= -\frac{x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)F\left(2\tan^{-1}\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}\,x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8+8x-x^3+8x^4}}$$

Mathematica [C] time = 0.84, size = 927, normalized size = 7.19

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], ((Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0
])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2,
 0])^2*Sqrt[((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x -
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1,
 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 4, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1
- #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^
2*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 4, 0])^2)])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8\,x^4 - x^3 + 8\,x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

maple [C] time = 1.13, size = 965, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-x^3+8*x+8)^(1/2),x)
```

```
[Out] 1/2*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)
)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*
(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))
^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z
^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)*E
llipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=3))*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=1))/(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=4)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8\,x^4 - x^3 + 8\,x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{8\,x^4 - x^3 + 8\,x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8)^(1/2),x)
```

```
[Out] int(1/(8*x - x^3 + 8*x^4 + 8)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8x^4 - x^3 + 8x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(8*x**4 - x**3 + 8*x + 8), x)
```

3.797 $\int \frac{1}{\left(8+8x-x^3+8x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=431

$$-\frac{\left(66-\left(\frac{4}{x}+1\right)^2\right)x^2}{1008\sqrt{8x^4-x^3+8x+8}}+\frac{\left(216-7\left(\frac{4}{x}+1\right)^2\right)\left(\frac{4}{x}+1\right)x^2}{12528\sqrt{8x^4-x^3+8x+8}}+\frac{7\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)\left(\frac{4}{x}+1\right)x^2}{432\sqrt{29}\sqrt{8x^4-x^3+8x+8}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)}+\frac{(14-\cdots}{\cdots}$$

```
[Out] -1/1008*(66-(1+4/x)^2)*x^2/(8*x^4-x^3+8*x+8)^(1/2)+1/12528*(216-7*(1+4/x)^2
)*(1+4/x)*x^2/(8*x^4-x^3+8*x+8)^(1/2)+7/12528*(261-6*(1+4/x)^2+(1+4/x)^4)*(
1+4/x)*x^2*29^(1/2)/(87+(4+x)^2*29^(1/2)/x^2)/(8*x^4-x^3+8*x+8)^(1/2)-7/125
28*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))^2)^(1/2)/cos(2*arctan(
1/87*(4+x)*29^(3/4)/x*3^(1/2)))*EllipticE(sin(2*arctan(1/87*(4+x)*29^(3/4)/
x*3^(1/2))),1/58*(1682+58*29^(1/2))^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)*((261-
6*(1+4/x)^2+(1+4/x)^4)/(87+(4+x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(1/4)*3^(1/2)/
(8*x^4-x^3+8*x+8)^(1/2)+1/50112*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(
1/2)))^2)^(1/2)/cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))*EllipticF(sin(
2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2))),1/58*(1682+58*29^(1/2))^(1/2))*(14
-5*29^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)*((261-6*(1+4/x)^2+(1+4/x)^4)/(87+(4+
x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(1/4)*3^(1/2)/(8*x^4-x^3+8*x+8)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {2069, 12, 6719, 1673, 1678, 1197, 1103, 1195, 1247, 636}

$$-\frac{\left(66-\left(\frac{4}{x}+1\right)^2\right)x^2}{1008\sqrt{8x^4-x^3+8x+8}}+\frac{\left(216-7\left(\frac{4}{x}+1\right)^2\right)\left(\frac{4}{x}+1\right)x^2}{12528\sqrt{8x^4-x^3+8x+8}}+\frac{7\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)\left(\frac{4}{x}+1\right)x^2}{432\sqrt{29}\sqrt{8x^4-x^3+8x+8}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)}+\frac{(14-\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] -((66 - (1 + 4/x)^2)*x^2)/(1008*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((216 - 7*(1
 + 4/x)^2)*(1 + 4/x)*x^2)/(12528*Sqrt[8 + 8*x - x^3 + 8*x^4]) + (7*(261 - 6
*(1 + 4/x)^2 + (1 + 4/x)^4)*(1 + 4/x)*x^2)/(432*Sqrt[29]*Sqrt[8 + 8*x - x^3
 + 8*x^4]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)) - (7*x^2*Sqrt[(261 - 6*(1 + 4/x)
^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x
)^2)/x^2)*EllipticE[2*ArcTan[(4 + x)/(Sqrt[3]*29^(1/4)*x)], (29 + Sqrt[29])
/58])/(144*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((14 - 5*Sqrt[29
])*x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/
x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcTan[(4 + x)/(Sqrt[3]
*29^(1/4)*x)], (29 + Sqrt[29])/58])/(576*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^
3 + 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
```

```
+ c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
```

```
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\left(8+8x-x^3+8x^4\right)^{3/2}}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{1}{16\sqrt{2}\,(8-32x)^2\left(\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}\right)^{3/2}}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(\left(32\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(8-32x)^2\left(\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}\right)^{3/2}}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{(8-3}{(1069056-393216}\right.}{8\sqrt{8+8x-x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{x(-65536-}{(1069056-393216}\right.}{8\sqrt{8+8x-x^3+8x^4}}$$

$$= \frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} - \frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\right.\right.}{37026344336}$$

$$= -\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}} + \frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} - \frac{\left(7\sqrt{1069056-393\right.}{}$$

$$= -\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}} + \frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} + \frac{7\left(261-6\left(1+\frac{4}{x}\right.\right.}{432\sqrt{29}\sqrt{8+8x-}}$$

Mathematica [C] time = 6.05, size = 4865, normalized size = 11.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(21924*Sqrt[8 + 8*x - x^3 + 8*x^4]) + (
(28*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*(-(EllipticF[ArcSin[Sqr
t[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - R
```

```
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]
)*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 4, 0])))]*Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]) + EllipticPi[(-Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0
])/(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 4, 0]), ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1
, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]))*Sqrt[((-Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*
Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0
] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))])/(Sqrt[8 + 8*x - x^3 + 8*x^4
]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3
 + 8*#1^4 & , 4, 0])) + (842*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2
, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 4, 0]))]], ((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Ro
ot[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8
*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3
 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
0]))]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#
1^4 & , 4, 0])*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]
))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*Sqrt[((x - Ro
ot[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 4, 0]))])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(
-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 4, 0])) - (224*((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(x - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4
, 0]) + (x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((-Root[8 + 8*
```

```
#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x
- Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 3, 0]))]*Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1,
 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#
1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*Sq
rt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0]
+ Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*((EllipticE[ArcSin[S
qrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] -
 Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 -
#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 4, 0])))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 3, 0]))/(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]) + (EllipticF[ArcSin[Sqrt[((x - Root[8
+ 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]
- Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 -
#1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] -
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root[8 + 8*#1 - #1^3
+ 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#
1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*(
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]) - Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 4, 0])))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Ro
ot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])) - (EllipticPi[(-Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])/(
-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 4, 0]), ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0
]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8
+ 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])
*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
 , 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0]
 - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 3, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/(-Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))))/Sqrt[8 +
 8*x - x^3 + 8*x^4])/6264
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{8\,x^4 - x^3 + 8\,x + 8}}{64\,x^8 - 16\,x^7 + x^6 + 128\,x^5 + 112\,x^4 - 16\,x^3 + 64\,x^2 + 128\,x + 64}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(8*x^4 - x^3 + 8*x + 8)/(64*x^8 - 16*x^7 + x^6 + 128*x^5 + 112*x^4 - 16*x^3 + 64*x^2 + 128*x + 64), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8\,x^{4}-x^{3}+8\,x+8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="giac")

[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)

maple [C] time = 0.12, size = 4426, normalized size = 10.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x^4-x^3+8*x+8)^(3/2),x)

[Out] -16*(-17/10962-57/12992*x+191/58464*x^2-7/3132*x^3)/(8*x^4-x^3+8*x+8)^(1/2)+421/12528*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+7/6264*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+

8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=4)))^(1/2)*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*EllipticF(((RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*
_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(-Ro
otOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(-Root
Of(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+(-R
ootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*Ellip
ticPi(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=
2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=
2)))^(1/2),(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=2)),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3
))*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))
/(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/
2)))-7/783*((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))+(-RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*((RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_
Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-
_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*
_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)*RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)^2)/(RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*EllipticF(((RootOf(8*_Z
^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^
4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(-RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(-RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+(-RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*EllipticE((
(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-Roo
tOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1
/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)
)*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/
(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(R
ootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2
))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))-
1/8/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))
*EllipticPi(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=2)))^(1/2),(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+

```
8,index=4)),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=3))*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=1))/(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4
)))^(1/2))))*2^(1/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z
^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8\,x^4-x^3+8\,x+8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="maxima")
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(8\,x^4-x^3+8\,x+8\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8)^(3/2),x)
[Out] int(1/(8*x - x^3 + 8*x^4 + 8)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8x^4-x^3+8x+8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**(3/2),x)
[Out] Integral((8*x**4 - x**3 + 8*x + 8)**(-3/2), x)
```

3.798 $\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{4x^4+4x^2+4x+1}}$$

```
[Out] -1/10*x^2*(cos(2*arctan(1/5*(1+1/x)*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*(1+
1/x)*5^(3/4)))*EllipticF(sin(2*arctan(1/5*(1+1/x)*5^(3/4))),1/10*(50+10*5^(
1/2))^(1/2))*((1+1/x)^2+5^(1/2))*((5-2*(1+1/x)^2+(1+1/x)^4)/((1+1/x)^2+5^(1
/2))^2)^(1/2)*5^(3/4)/(4*x^4+4*x^2+4*x+1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2069, 6719, 1103}

$$-\frac{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{4x^4+4x^2+4x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

```
[Out] -((Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(
Sqrt[5] + (1 + x^(-1))^2)^2]*x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4)],
(5 + Sqrt[5])/10])/(2*5^(1/4)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = -\left(16\,\text{Subst}\left(\int \frac{1}{(4-4x)^2\sqrt{\frac{1280-512x^2+256x^4}{(4-4x)^4}}}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1280-512x^2+256x^4}}\,dx, x, 1\right.}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= -\frac{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}\,x^2 F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{1+4x+4x^2+4x^4}}$$

Mathematica [C] time = 0.59, size = 249, normalized size = 2.31

$$\frac{(2-i)\sqrt{-\frac{1}{10}+\frac{i}{5}}\sqrt{\frac{\left(2i+\sqrt{-1-2i}-\sqrt{-1+2i}\right)\left(-2x+\sqrt{-1-2i}-i\right)}{\left(-2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(2x+\sqrt{-1-2i}+i\right)}}\left(2ix^2+2x+1\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\left(2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(2x+\sqrt{-1+2i}-i\right)}{\sqrt{-1+2i}\left(2x+\sqrt{-1-2i}+i\right)}}}{\sqrt{2}}\right)|\frac{1}{2}\right.}{\sqrt{\frac{(1+2i)\left((-1+i)+\sqrt{-1-2i}\right)\left(2ix^2+2x+1\right)}{\left(2x+\sqrt{-1-2i}+i\right)^2}}\sqrt{4x^4+4x^2+4x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

```
[Out] ((2 - I)*Sqrt[-1/10 + I/5]*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqrt[-1 + 2*I])*(-
I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(I + S
qrt[-1 - 2*I] + 2*x))]*(1 + 2*x + (2*I)*x^2)*EllipticF[ArcSin[Sqrt[((2*I +
Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] + 2*x))/(Sqrt[-1 + 2*
I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])/2])/(Sqrt[((1 + 2*I
)*((-1 + I) + Sqrt[-1 - 2*I])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] +
2*x)^2]*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

maple [C] time = 0.85, size = 961, normalized size = 8.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^(1/2),x)
```

```
[Out] (RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))
*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2
))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=2)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*
_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/
2)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,
index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_
Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(x-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(4*_Z^4+4*_Z^
2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/
(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(1/2),x)
```

```
[Out] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4x^4+4x^2+4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*x**4 + 4*x**2 + 4*x + 1), x)
```

3.799 $\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx$

Optimal. Leaf size=367

$$-\frac{\left(3-\left(\frac{1}{x}+1\right)^2\right)x^2}{\sqrt{4x^4+4x^2+4x+1}}+\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)x^2}{10\sqrt{4x^4+4x^2+4x+1}}+\frac{9\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)\left(\frac{1}{x}+1\right)x^2}{10\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{4x^4+4x^2+4x+1}}+\frac{3\left(3-\sqrt{5}\right)}{}$$

```
[Out] -(3-(1+1/x)^2)*x^2/(4*x^4+4*x^2+4*x+1)^(1/2)+1/10*(13-9*(1+1/x)^2)*(1+1/x)*
x^2/(4*x^4+4*x^2+4*x+1)^(1/2)+9/10*(5-2*(1+1/x)^2+(1+1/x)^4)*(1+1/x)*x^2/((
1+1/x)^2+5^(1/2))/(4*x^4+4*x^2+4*x+1)^(1/2)-9/10*x^2*(cos(2*arctan(1/5*(1+1
/x)*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*(1+1/x)*5^(3/4)))*EllipticE(sin(2*a
rctan(1/5*(1+1/x)*5^(3/4))),1/10*(50+10*5^(1/2))^(1/2))*((1+1/x)^2+5^(1/2))
*((5-2*(1+1/x)^2+(1+1/x)^4)/((1+1/x)^2+5^(1/2))^2)^(1/2)*5^(1/4)/(4*x^4+4*x
^2+4*x+1)^(1/2)+3/20*x^2*(cos(2*arctan(1/5*(1+1/x)*5^(3/4)))^2)^(1/2)/cos(2
*arctan(1/5*(1+1/x)*5^(3/4)))*EllipticF(sin(2*arctan(1/5*(1+1/x)*5^(3/4))),
1/10*(50+10*5^(1/2))^(1/2))*(3-5^(1/2))*((1+1/x)^2+5^(1/2))*((5-2*(1+1/x)^2
+(1+1/x)^4)/((1+1/x)^2+5^(1/2))^2)^(1/2)*5^(1/4)/(4*x^4+4*x^2+4*x+1)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {2069, 6719, 1673, 1678, 1197, 1103, 1195, 1247, 636}

$$-\frac{\left(3-\left(\frac{1}{x}+1\right)^2\right)x^2}{\sqrt{4x^4+4x^2+4x+1}}+\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)x^2}{10\sqrt{4x^4+4x^2+4x+1}}+\frac{9\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)\left(\frac{1}{x}+1\right)x^2}{10\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{4x^4+4x^2+4x+1}}+\frac{3\left(3-\sqrt{5}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

```
[Out] -(((3 - (1 + x^(-1))^2)*x^2)/Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + ((13 - 9*(1 +
 x^(-1))^2)*(1 + x^(-1))*x^2)/(10*Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + (9*(5 -
2*(1 + x^(-1))^2 + (1 + x^(-1))^4)*(1 + x^(-1))*x^2)/(10*(Sqrt[5] + (1 + x^
(-1))^2)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) - (9*(Sqrt[5] + (1 + x^(-1))^2)*Sqr
t[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2
*EllipticE[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(2*5^(3/4)*Sq
rt[1 + 4*x + 4*x^2 + 4*x^4]) + (3*(3 - Sqrt[5])*(Sqrt[5] + (1 + x^(-1))^2)*
Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*
x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(4*5^(3/4)
*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
```

```
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
```

```
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^{3/2}}\,dx = -\left(16\,\text{Subst}\left(\int \frac{1}{(4-4x)^2\left(\frac{1280-512x^2+256x^4}{(4-4x)^4}\right)^{3/2}}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{(4-4x)^4}{\left(1280-512x^2+256x^4\right)^{3/2}}\,dx,\right.}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{x\left(-1024-1024x^2\right)}{\left(1280-512x^2+256x^4\right)^{3/2}}\,dx,\right.}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} - \frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}\,x^2\right)\text{Su}}{1342177280\sqrt{1+4}}$$

$$= -\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}} + \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} - \frac{\left(9\sqrt{1280-512\left(1+\right.}\right.}{}$$

$$= -\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}} + \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} + \frac{9\left(5-2\left(1+\frac{1}{x}\right)^2+\right.}{10\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)}$$

Mathematica [C] time = 4.76, size = 602, normalized size = 1.64

$$36x^3-16x^2+\frac{(6-3i)\sqrt{-\frac{2}{5}+\frac{4i}{5}}\sqrt{\frac{\left(2i+\sqrt{-1-2i}-\sqrt{-1+2i}\right)\left(-2x+\sqrt{-1-2i}-i\right)}{\left(-2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(2x+\sqrt{-1-2i}+i\right)}}\left(2ix^2+2x+1\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\left(2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(2x+\sqrt{-1+2i}-i\right)}{\sqrt{-1+2i}\left(2x+\sqrt{-1-2i}+i\right)}}}{\sqrt{2}}\right)\middle|\frac{1}{2}\left(5-\sqrt{5}\right)\right)}{\sqrt{\frac{(1+2i)\left((-1+i)+\sqrt{-1-2i}\right)\left(2ix^2+2x+1\right)}{\left(2x+\sqrt{-1-2i}+i\right)^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

```
[Out] (19 + 42*x - 16*x^2 + 36*x^3 + (9*(-I + Sqrt[-1 - 2*I] - 2*x)*(-I - Sqrt[-1
 + 2*I] + 2*x)*(-I + Sqrt[-1 + 2*I] + 2*x))/2 - ((9*I)*Sqrt[-2/5 + (4*I)/5]
*(-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(2*I + Sqrt[-1 - 2*I] + Sqrt[-1 +
 2*I])*((I + Sqrt[-1 - 2*I])/2 + x)^2*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqrt[-1
 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2
*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1 - 2*I
])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2]*EllipticE[ArcSin[Sq
rt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] + 2*x))/(S
qrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])/2])/((-1
 + I) + Sqrt[-1 - 2*I]) + ((6 - 3*I)*Sqrt[-2/5 + (4*I)/5]*Sqrt[((2*I + Sqrt
[-1 - 2*I] - Sqrt[-1 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1
```

```
- 2*I] + Sqrt[-1 + 2*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*(1 + 2*x + (2*I)*x^2)
*EllipticF[ArcSin[Sqrt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[
-1 + 2*I] + 2*x))/(Sqrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5
 - Sqrt[5])/2])/Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1 - 2*I])*(1 + 2*x + (2*I
)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2])/(10*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4x^4+4x^2+4x+1}}{16x^8+32x^6+32x^5+24x^4+32x^3+24x^2+8x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x^4 + 4*x^2 + 4*x + 1)/(16*x^8 + 32*x^6 + 32*x^5 + 24*x^4 +
 32*x^3 + 24*x^2 + 8*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4x^4+4x^2+4x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

maple [C] time = 0.11, size = 2564, normalized size = 6.99

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^(3/2),x)
```

```
[Out] -8*(-9/20*x^3+1/5*x^2-21/40*x-19/80)/(4*x^4+4*x^2+4*x+1)^(1/2)+3/5*(RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))*((RootO
f(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2
)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+4*_Z^2
+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_
Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*((Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(x-RootOf(4*
_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(RootOf(4*
_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2))
```

```
-9/5*((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))+(RootOf(4*_Z^4+4*_Z^2
+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))*((RootOf(4*_Z^4+4*_Z
^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+
4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*(x-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,i
ndex=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*((RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4
+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*(
(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)^2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2
)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*EllipticF(((RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2
+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^
2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf
(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(RootO
f(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(Root
Of(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/(Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1
/2))+(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,inde
x=3))*EllipticE(((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2
+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,
index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))))/((x-RootOf(4*_Z^4+4*_Z^2+4
*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4\,x^4+4\,x^2+4\,x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(4\,x^4+4\,x^2+4\,x+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(3/2),x)
```

```
[Out] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4x^4 + 4x^2 + 4x + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**(3/2),x)
```

```
[Out] Integral((4*x**4 + 4*x**2 + 4*x + 1)**(-3/2), x)
```

3.800 $\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx$

Optimal. Leaf size=126

$$-\frac{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}\,x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

```
[Out] -1/4136*x^2*(cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arctan
(1/517*(4+3*x)*517^(3/4)/x))*EllipticF(sin(2*arctan(1/517*(4+3*x)*517^(3/4)
/x)),1/1034*(534578+19646*517^(1/2))^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*
(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(3/4)/(8*x^4-15*x^3
+8*x^2+24*x+8)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2069, 12, 6719, 1103}

$$-\frac{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}\,x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

```
[Out] -((Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt
[517] + (3 + 4/x)^2)^2]*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (5
17 + 19*Sqrt[517])/1034])/(8*517^(1/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x
^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
```

```
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{1}{2\sqrt{2}\,(24-32x)^2\sqrt{\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}}}\,dx, x, \frac{3}{4}+\right.\right.$$

$$= -\left(\left(256\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(24-32x)^2\sqrt{\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}}}\,dx, x, \frac{3}{4}+\right.\right.$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{}{\sqrt{2117}}\right.}{\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{4+3x}{\sqrt[4]{517}\,x}\right)|\frac{517+19\sqrt{5}}{1034}\right.}{8\sqrt[4]{517}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

Mathematica [C] time = 0.98, size = 1148, normalized size = 9.11

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
& , 1, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 +
 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 24*#1 + 8*#1^
2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]], ((Ro
ot[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^
2 - 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))/((Root[
8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 -
 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 &
, 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]*(x - Root
[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((Root[8 + 24*#1 +
 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
8*#1^4 & , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0]
))/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24
*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1
^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1,
 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Root
[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 24*#1 + 8*#1^2
- 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 &
 , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])*(Root[
8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^2 -
 15*#1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#
1^4 & , 2, 0])^2*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Ro
ot[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])^2)])/(Sqrt[8 + 24*x + 8
```

```
*x^2 - 15*x^3 + 8*x^4]*(-Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1,
0] + Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24*#1
+ 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
 8*#1^4 & , 4, 0]))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

maple [C] time = 1.68, size = 1180, normalized size = 9.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x)
```

```
[Out] 1/2*(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1)-RootOf(8*_Z^4-15*_Z^3+8*
_Z^2+24*_Z+8,index=4))*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-Root
Of(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+
24*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_
Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+
8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))^2*((Ro
otOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24
*_Z+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))/(RootOf(8
*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,
index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2)*((RootOf
(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+
8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4))/(RootOf(8*_Z^
4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,inde
x=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z
^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,ind
ex=2))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3
+8*_Z^2+24*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,
index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^
4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,
index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-
RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^4-15*_Z^3+8*_
Z^2+24*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf
(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24
*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-Root
Of(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))*(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24
*_Z+8,index=1)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4))/(-RootOf(8*_Z
^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3)+RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,ind
```

```
ex=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3
+8*_Z^2+24*_Z+8,index=4)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(1/2),x)
[Out] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8x^4-15x^3+8x^2+24x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(1/2),x)
[Out] Integral(1/sqrt(8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8), x)
```

3.801 $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=434

$$-\frac{\left(172-7\left(\frac{4}{x}+3\right)^2\right)x^2}{208\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(50896-2455\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{322608\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{2455\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right.\right.}{322608\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4}}$$

```
[Out] -1/208*(172-7*(3+4/x)^2)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/322608*(50
896-2455*(3+4/x)^2)*(3+4/x)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+2455/3226
08*(517-38*(3+4/x)^2+(3+4/x)^4)*(3+4/x)*x^2/((3+4/x)^2+517^(1/2))/(8*x^4-15
*x^3+8*x^2+24*x+8)^(1/2)-2455/322608*x^2*(cos(2*arctan(1/517*(4+3*x)*517^(3
/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))*EllipticE(sin(2*a
rctan(1/517*(4+3*x)*517^(3/4)/x)),1/1034*(534578+19646*517^(1/2))^(1/2))*((
3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^
(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/1290432*x^2*(cos(2*arct
an(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)*517^(3/4
)/x))*EllipticF(sin(2*arctan(1/517*(4+3*x)*517^(3/4)/x)),1/1034*(534578+196
46*517^(1/2))^(1/2))*(4910-203*517^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*(3
+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8
*x^2+24*x+8)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2069, 12, 6719, 1673, 1678, 1197, 1103, 1195, 1247, 636}

$$-\frac{\left(172-7\left(\frac{4}{x}+3\right)^2\right)x^2}{208\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(50896-2455\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{322608\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{2455\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right.\right.}{322608\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] -((172 - 7*(3 + 4/x)^2)*x^2)/(208*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
+ ((50896 - 2455*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(322608*Sqrt[8 + 24*x + 8*x^2
- 15*x^3 + 8*x^4]) + (2455*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x
^2)/(322608*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^
4]) - (2455*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x
)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/
4)*x)], (517 + 19*Sqrt[517])/1034])/(624*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 -
15*x^3 + 8*x^4]) + ((4910 - 203*Sqrt[517])*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(
517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*Ellipt
icF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034])/(2496*51
7^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
```

```
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{3/2}}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{1}{16\sqrt{2}\,(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{3/2}}\,d\right.\right.$$

$$= -\left(\left(32\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{3/2}}\,dx,\right.\right.$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int\right.}{8\sqrt{8+24x+8x^2-15x^3+8}}$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int\right.}{8\sqrt{8+24x+8x^2-15x^3+8}}$$

$$= \frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\right.\right.}}{}$$

$$= -\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3\right.}{322608\sqrt{8+24x+8x^2-15x}}$$

$$= -\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3\right.}{322608\sqrt{8+24x+8x^2-15x}}$$

Mathematica [C] time = 6.08, size = 6019, normalized size = 13.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}{64\,x^8-240\,x^7+353\,x^6+144\,x^5-528\,x^4+144\,x^3+704\,x^2+384\,x+64},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(64*x^8 - 240*x^7 + 353*x^
6 + 144*x^5 - 528*x^4 + 144*x^3 + 704*x^2 + 384*x + 64), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

maple [C] time = 0.12, size = 5421, normalized size = 12.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(3/2),x)
```

```
[Out] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8x^4-15x^3+8x^2+24x+8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(3/2),x)
```

```
[Out] Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-3/2), x)
```

3.802 $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=577

$$-\frac{\left(124415-6308\left(\frac{4}{x}+3\right)^2\right)x^2}{97344\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(18932921731-1086525994\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{78056941248\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{543262997\,(\ldots}{39028470624\left(\left(\frac{4}{x}\ldots\right.\right.}$$

```
[Out] -1/97344*(124415-6308*(3+4/x)^2)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)-1/62
4*(64489-1399*(3+4/x)^2)*x^2/(517-38*(3+4/x)^2+(3+4/x)^4)/(8*x^4-15*x^3+8*x
^2+24*x+8)^(1/2)+1/78056941248*(18932921731-1086525994*(3+4/x)^2)*(3+4/x)*x
^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/483912*(11921698-359497*(3+4/x)^2)*(
3+4/x)*x^2/(517-38*(3+4/x)^2+(3+4/x)^4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+5
43262997/39028470624*(517-38*(3+4/x)^2+(3+4/x)^4)*(3+4/x)*x^2/((3+4/x)^2+51
7^(1/2))/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)-543262997/39028470624*x^2*(cos(2
*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)*517
^(3/4)/x))*EllipticE(sin(2*arctan(1/517*(4+3*x)*517^(3/4)/x)),1/1034*(53457
8+19646*517^(1/2))^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^2+(3+4/x)^
4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/
2)+1/624455529984*x^2*(cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/co
s(2*arctan(1/517*(4+3*x)*517^(3/4)/x))*EllipticF(sin(2*arctan(1/517*(4+3*x)
*517^(3/4)/x)),1/1034*(534578+19646*517^(1/2))^(1/2))*(4346103976-175318963
*517^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+
517^(1/2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {2069, 12, 6719, 1673, 1678, 1197, 1103, 1195, 1663, 1660, 636}

$$-\frac{\left(124415-6308\left(\frac{4}{x}+3\right)^2\right)x^2}{97344\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(18932921731-1086525994\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{78056941248\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{543262997\,(\ldots}{39028470624\left(\left(\frac{4}{x}\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

```
[Out] -((124415 - 6308*(3 + 4/x)^2)*x^2)/(97344*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 +
8*x^4]) - ((64489 - 1399*(3 + 4/x)^2)*x^2)/(624*(517 - 38*(3 + 4/x)^2 + (3
+ 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + ((18932921731 - 108652
5994*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(78056941248*Sqrt[8 + 24*x + 8*x^2 - 15*x^
3 + 8*x^4]) + ((11921698 - 359497*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(483912*(517
- 38*(3 + 4/x)^2 + (3 + 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) +
(543262997*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x^2)/(39028470624
*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) - (5432
62997*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(
Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)]
, (517 + 19*Sqrt[517])/1034])/(75490272*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 1
5*x^3 + 8*x^4]) + ((4346103976 - 175318963*Sqrt[517])*(Sqrt[517] + (3 + 4/x
)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]
*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034
])/(1207844352*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
```

```
&& !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{5/2}}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{1}{128\sqrt{2}\,(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{5/2}}\,dx,\right.\right.$$

$$= -\left(\left(4\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{5/2}}\,dx, x, \frac{3}{4}\right.\right.$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{}{(21}\right.}{64\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{x(-}{}\right.}{64\sqrt{8+24x+}}$$

$$= \frac{\left(11921698-359497\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{483912\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} -$$

$$= -\frac{\left(64489-1399\left(3+\frac{4}{x}\right)^2\right)x^2}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} + $$

$$= -\frac{\left(124415-6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(64489-1\right.}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\right.\right.}$$

$$= -\frac{\left(124415-6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(64489-1\right.}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\right.\right.}$$

Mathematica [C] time = 6.06, size = 6084, normalized size = 10.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}{512\,x^{12}-2880\,x^{11}+6936\,x^{10}-4527\,x^9-8808\,x^8+16776\,x^7+5528\,x^6-17856\,x^5-384\,x^4+20160\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(512*x^12 - 2880*x^11 + 69
36*x^10 - 4527*x^9 - 8808*x^8 + 16776*x^7 + 5528*x^6 - 17856*x^5 - 384*x^4
+ 20160*x^3 + 15360*x^2 + 4608*x + 512), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="giac")
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

maple [C] time = 0.14, size = 5477, normalized size = 9.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="maxima")
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(5/2),x)
[Out] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8x^{4}-15x^{3}+8x^{2}+24x+8\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(5/2),x)
[Out] Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-5/2), x)
```

3.803 $\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}\,x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

```
[Out] -1/7356*x^2*(cos(2*arctan(1/613*(6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan(1
/613*(6-x)*613^(3/4)/x))*EllipticF(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)),1
/1226*(751538+111566*613^(1/2))^(1/2))*((6-x)^2/x^2+613^(1/2))*((613-182*(1
-6/x)^2+(-1+6/x)^4)/((6-x)^2/x^2+613^(1/2))^2)^(1/2)*613^(3/4)/(3*x^4+15*x^
3-44*x^2-6*x+9)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2069, 12, 6719, 1096}

$$-\frac{\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}\,x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

```
[Out] -(Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/x^2)^2
]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 - x)/(613^(1/4)*x)]
, (613 + 91*Sqrt[613])/1226])/(12*613^(1/4)*Sqrt[9 - 6*x - 44*x^2 + 15*x^3
+ 3*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
```

```
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = -\left(1296\,\mathrm{Subst}\left(\int \frac{1}{3(-6-36x)^2\sqrt{\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}}}\,dx, x, -\frac{1}{6}+\right.\right.$$

$$= -\left(432\,\mathrm{Subst}\left(\int \frac{1}{(-6-36x)^2\sqrt{\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}}}\,dx, x, -\frac{1}{6}+\frac{1}{x}\right.\right.$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}\,x^2\right)\mathrm{Subst}\left(\int\right.}{\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$= -\frac{\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}\,x}\right)\middle|\frac{613+91}{122}\right.}{12\sqrt[4]{613}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

Mathematica [C] time = 0.12, size = 826, normalized size = 6.35

$$-\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{(x-\mathrm{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1])(\mathrm{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2]-\mathrm{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4])}{(x-\mathrm{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2])(\mathrm{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1]-\mathrm{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4])}}\right)\right|}{\sqrt{(3x^4+15x^3-44x^2-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
& , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 -
 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]], ((Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((Root[
9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 +
 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 &
, 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]*Sqrt[(x -
 Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])/(x - Root[9 - 6*#1 -
 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*(x - Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 2, 0])^2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#
1^4 & , 3, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*
Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/(x - Root[9
 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])])/Sqrt[(9 - 6*x - 44*x^2 +
15*x^3 + 3*x^4)*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2
 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
& , 4, 0])]
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^{4}+15\,x^{3}-44\,x^{2}-6\,x+9}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^{4}+15\,x^{3}-44\,x^{2}-6\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

maple [C] time = 0.50, size = 1182, normalized size = 9.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x)
[Out] 2/3*(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-4
4*_Z^2-6*_Z+9,index=1))*((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/
(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z
^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(-Roo
tOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*
_Z+9,index=2)))^(1/2)*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))^2*(
-(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))/(-RootOf(3*_Z^4+15*_Z^3-
44*_Z^2-6*_Z+9,index=3)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-R
ootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(RootOf(3*_Z^4+15*_Z^3-44*_Z^
2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1)))^(1/2)*(-(
x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4))/(-RootOf(3*_Z^4+15*_Z^3-44
*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-Roo
tOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1)))^(1/2)/(Root
Of(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_
Z+9,index=2))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+
15*_Z^3-44*_Z^2-6*_Z+9,index=1))*3^(1/2)/((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(x-RootO
f(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=4)))^(1/2)*EllipticF(((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,
index=1))/(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*
_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index
=2))*(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-
44*_Z^2-6*_Z+9,index=2)))^(1/2),((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,inde
x=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))*(-RootOf(3*_Z^4+15*_Z^3
-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(-R
ootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=1))/(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_
Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,x^{4}+15\,x^{3}-44\,x^{2}-6\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(1/2),x)
```

```
[Out] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+15x^3-44x^2-6x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9), x)
```

3.804 $\int \frac{1}{\left(9-6x-44x^2+15x^3+3x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=444

$$-\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{3722\left(\left(\frac{6}{x}-1\right)^4-182\left(1-\right.\right.}{31728267\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)\sqrt{3x}}$$

```
[Out] -1/51759*(176-23*(1-6/x)^2)*x^2/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+1/3172826
7*(45401-3722*(1-6/x)^2)*(1-6/x)*x^2/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+3722
/31728267*(613-182*(1-6/x)^2+(-1+6/x)^4)*(1-6/x)*x^2/((6-x)^2/x^2+613^(1/2)
)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+3722/31728267*x^2*(cos(2*arctan(1/613*(
6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/613*(6-x)*613^(3/4)/x))*Elliptic
E(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)),1/1226*(751538+111566*613^(1/2))^(
1/2))*((6-x)^2/x^2+613^(1/2))*((613-182*(1-6/x)^2+(-1+6/x)^4)/((6-x)^2/x^2+
613^(1/2))^2)^(1/2)*613^(1/4)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)-1/126913068
*x^2*(cos(2*arctan(1/613*(6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/613*(6
-x)*613^(3/4)/x))*EllipticF(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)),1/1226*(
751538+111566*613^(1/2))^(1/2))*(7444-145*613^(1/2))*((6-x)^2/x^2+613^(1/2)
)*((613-182*(1-6/x)^2+(-1+6/x)^4)/((6-x)^2/x^2+613^(1/2))^2)^(1/2)*613^(1/4
)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2069, 12, 6719, 1673, 1678, 1183, 1096, 1182, 1247, 636}

$$-\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{3722\left(\left(\frac{6}{x}-1\right)^4-182\left(1-\right.\right.}{31728267\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)\sqrt{3x}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

```
[Out] -((176 - 23*(1 - 6/x)^2)*x^2)/(51759*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4
]) + ((45401 - 3722*(1 - 6/x)^2)*(1 - 6/x)*x^2)/(31728267*Sqrt[9 - 6*x - 44
*x^2 + 15*x^3 + 3*x^4]) + (3722*(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)*(1 -
 6/x)*x^2)/(31728267*(Sqrt[613] + (6 - x)^2/x^2)*Sqrt[9 - 6*x - 44*x^2 + 15
*x^3 + 3*x^4]) + (3722*Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[61
3] + (6 - x)^2/x^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticE[2*ArcTan[(
6 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[613])/1226])/(51759*613^(3/4)*Sqrt[9
- 6*x - 44*x^2 + 15*x^3 + 3*x^4]) - ((7444 - 145*Sqrt[613])*Sqrt[(613 - 182
*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/x^2)^2]*(Sqrt[613] + (6
 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[
613])/1226])/(207036*613^(3/4)*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 -
4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c
/a, 0] && LtQ[b/a, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
```

```
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\left(9-6x-44x^2+15x^3+3x^4\right)^{3/2}}\,dx = -\left(1296\,\text{Subst}\left(\int \frac{1}{27(-6-36x)^2\left(\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}\right)^{3/2}}\,dx, x, -\frac{1}{6}\right.\right.$$

$$= -\left(48\,\text{Subst}\left(\int \frac{1}{(-6-36x)^2\left(\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}\right)^{3/2}}\,dx, x, -\frac{1}{6}+\frac{1}{x}\right.\right.$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int\right.}{9\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int\right.}{9\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$= \frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\right.}\right.}{}$$

$$= -\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\right.}{31728267\sqrt{9-6x-44x^2+15\right.}}$$

$$= -\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\right.}{31728267\sqrt{9-6x-44x^2+15\right.}}$$

Mathematica [C] time = 6.05, size = 5428, normalized size = 12.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}{9\,x^8+90\,x^7-39\,x^6-1356\,x^5+1810\,x^4+798\,x^3-756\,x^2-108\,x+81},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)/(9*x^8 + 90*x^7 - 39*x^6 -
 1356*x^5 + 1810*x^4 + 798*x^3 - 756*x^2 - 108*x + 81), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^4+15\,x^3-44\,x^2-6\,x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

maple [C] time = 0.07, size = 5427, normalized size = 12.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^4+15\,x^3-44\,x^2-6\,x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="maxima")
[Out] integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(3\,x^4+15\,x^3-44\,x^2-6\,x+9\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(3/2),x)
[Out] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^4+15x^3-44x^2-6x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(3/2),x)
[Out] Integral((3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9)**(-3/2), x)
```

3.805 $\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx$

Optimal. Leaf size=56

$$-4x+21\log(x)-9\log(x+1)+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)$$

```
[Out] -4*x-12*arcsin(-1/2+1/2*x)+21*ln(x)-9*ln(1+x)-24*arctanh(3^(1/2)*(1+x)^(1/2
)/(3-x)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6742, 36, 29, 31, 105, 53, 619, 216, 93, 207}

$$-4x+21\log(x)-9\log(x+1)+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4*x + 12*ArcSin[(1 - x)/2] - 24*Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[1 + x])/Sqrt
[3 - x]] + 21*Log[x] - 9*Log[1 + x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx &= \int \left(-4+\frac{12}{x}+\frac{9}{x(1+x)}+\frac{12\sqrt{3-x}}{x\sqrt{1+x}}\right)dx\\
&= -4x+12\log(x)+9\int \frac{1}{x(1+x)}\,dx+12\int \frac{\sqrt{3-x}}{x\sqrt{1+x}}\,dx\\
&= -4x+12\log(x)+9\int \frac{1}{x}\,dx-9\int \frac{1}{1+x}\,dx-12\int \frac{1}{\sqrt{3-x}\sqrt{1+x}}\,dx+36\int -\\
&= -4x+21\log(x)-9\log(1+x)-12\int \frac{1}{\sqrt{3+2x-x^2}}\,dx+72\,\mathrm{Subst}\left(\int \frac{1}{-1+3x^2}\right.\\
&= -4x-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{1+x}}{\sqrt{3-x}}\right)+21\log(x)-9\log(1+x)+3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-}}\right.\\
&= -4x+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{1+x}}{\sqrt{3-x}}\right)+21\log(x)-9\log(1+x)
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 57, normalized size = 1.02

$$
-4x+21\log(x)-9\log(x+1)+24\sin^{-1}\left(\frac{\sqrt{3-x}}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{1-\frac{x}{3}}}{\sqrt{x+1}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4*x + 24*ArcSin[Sqrt[3 - x]/2] - 24*Sqrt[3]*ArcTanh[Sqrt[1 - x/3]/Sqrt[1 +
 x]] + 21*Log[x] - 9*Log[1 + x]
```

fricas [A] time = 0.41, size = 81, normalized size = 1.45

$$6\sqrt{3}\log\left(-\frac{\sqrt{3}(x+3)\sqrt{x+1}\sqrt{-x+3}+x^2-6x-9}{x^2}\right)-4x+12\arctan\left(\frac{\sqrt{x+1}(x-1)\sqrt{-x+3}}{x^2-2x-3}\right)-9\log(x+1)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] 6*sqrt(3)*log(-(sqrt(3)*(x + 3)*sqrt(x + 1)*sqrt(-x + 3) + x^2 - 6*x - 9)/x
^2) - 4*x + 12*arctan(sqrt(x + 1)*(x - 1)*sqrt(-x + 3)/(x^2 - 2*x - 3)) - 9
*log(x + 1) + 21*log(x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-8,0,%%%{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}]
at parameters values [-91.616423693]Warning, choosing root of [1,0,-8,0,%%%
{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}] at parameters values [-15.88045570
86]-9*ln(abs(-x-1))+21*ln(abs(x))+4*(-x+3)+36*ln(abs(-4*sqrt(3)+6*(2*sqrt(-
x+3)/(-2*sqrt(x+1)+4)-1/2*(-2*sqrt(x+1)+4)/sqrt(-x+3)))/abs(4*sqrt(3)+6*(2*
sqrt(-x+3)/(-2*sqrt(x+1)+4)-1/2*(-2*sqrt(x+1)+4)/sqrt(-x+3))))/sqrt(3)-24*(
-1/2*pi-atan(sqrt(-x+3)*((-1/2*(-2*sqrt(x+1)+4)/sqrt(-x+3))^2-1)/(-2*sqrt(x
+1)+4)))
```

maple [A] time = 0.02, size = 76, normalized size = 1.36

$$-4x+21\ln(x)-9\ln(x+1)+\frac{12\sqrt{x+1}\sqrt{-x+3}\left(-\sqrt{3}\operatorname{arctanh}\left(\frac{(x+3)\sqrt{3}}{3\sqrt{-x^2+2x+3}}\right)-\arcsin\left(\frac{x}{2}-\frac{1}{2}\right)\right)}{\sqrt{-x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(3-x)^(1/2)+3/(x+1)^(1/2))^2/x,x)
```

```
[Out] -4*x+21*ln(x)+12*(x+1)^(1/2)*(3-x)^(1/2)/(-x^2+2*x+3)^(1/2)*(-arcsin(-1/2+1
/2*x)-3^(1/2)*arctanh(1/3*(x+3)*3^(1/2)/(-x^2+2*x+3)^(1/2)))-9*ln(x+1)
```

maxima [A] time = 0.96, size = 57, normalized size = 1.02

$$-12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+2x+3}}{|x|}+\frac{6}{|x|}+2\right)-4x+12\arcsin\left(-\frac{1}{2}x+\frac{1}{2}\right)-9\log(x+1)+21\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] -12*sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 2*x + 3)/abs(x) + 6/abs(x) + 2) - 4*x
 + 12*arcsin(-1/2*x + 1/2) - 9*log(x + 1) + 21*log(x)
```

mupad [B] time = 7.91, size = 158, normalized size = 2.82

$$48\,\mathrm{atan}\left(\frac{\sqrt{3-x}-4\sqrt{3}+3\sqrt{3}\sqrt{x+1}}{\sqrt{x+1}-3\sqrt{3}\sqrt{3-x}+8}\right)-9\ln(x+1)-4x+21\ln(x)+12\sqrt{3}\ln\left(\frac{6x-12\sqrt{x+1}+4\sqrt{3}\sqrt{3-x}}{3x+6\sqrt{3}\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3/(x + 1)^(1/2) + 2*(3 - x)^(1/2))^2/x,x)
```

```
[Out] 48*atan(((3 - x)^(1/2) - 4*3^(1/2) + 3*3^(1/2)*(x + 1)^(1/2))/((x + 1)^(1/2
) - 3*3^(1/2)*(3 - x)^(1/2) + 8)) - 9*log(x + 1) - 4*x + 21*log(x) + 12*3^(
1/2)*log((6*x - 12*(x + 1)^(1/2) + 4*3^(1/2)*(3 - x)^(1/2) + 2*3^(1/2)*(x +
 1)^(1/2)*(3 - x)^(1/2) - 6)/(3*x + 6*3^(1/2)*(3 - x)^(1/2) - 18)) - 12*3^(
1/2)*log(((x + 1)^(1/2) - 1)/(3^(1/2) - (3 - x)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\sqrt{3-x}\,\sqrt{x+1}\,+3\right)^2}{x\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)**(1/2)+3/(1+x)**(1/2))**2/x,x)
```

```
[Out] Integral((2*sqrt(3 - x)*sqrt(x + 1) + 3)**2/(x*(x + 1)), x)
```

3.806 $\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}\sqrt{x^2+1}\,x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] -1/x-x-1/2*arcsinh(x)-ln((x^2+1)^(1/2)+1)+(x^2+1)^(1/2)+(x^2+1)^(1/2)/x+1/2
*x*(x^2+1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6742, 277, 215, 1591, 190, 43, 195}

$$\frac{1}{2}\sqrt{x^2+1}\,x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcSi
nh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx &= \int \left(-\frac{1}{1+\sqrt{1+x^2}}+\frac{x}{1+\sqrt{1+x^2}}+\frac{x^2}{1+\sqrt{1+x^2}}\right)dx\\
&= -\int \frac{1}{1+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+\sqrt{1+x^2}}\,dx+\int \frac{x^2}{1+\sqrt{1+x^2}}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1+x^2\right)+\int \left(-1+\sqrt{1+x^2}\right)dx-\int \left(-\frac{1}{x^2}+\frac{\sqrt{1+x^2}}{x^2}\right)dx\\
&= -\frac{1}{x}-x+\int \sqrt{1+x^2}\,dx-\int \frac{\sqrt{1+x^2}}{x^2}\,dx+\operatorname{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1+x^2}\right)\\
&= -\frac{1}{x}-x+\frac{\sqrt{1+x^2}}{x}+\frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx-\int \frac{1}{\sqrt{1+x^2}}\,dx+\operatorname{Subst}\left(\int \left(1+\right.\right.\\
&= -\frac{1}{x}-x+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}+\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 65, normalized size = 1.00

$$\frac{1}{2}\sqrt{x^2+1}\,x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcSi
nh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

fricas [A] time = 0.40, size = 84, normalized size = 1.29

$$-\frac{2\,x^2+2\,x\log(x)+2\,x\log\left(-x+\sqrt{x^2+1}+1\right)-x\log\left(-x+\sqrt{x^2+1}\right)-2\,x\log\left(-x+\sqrt{x^2+1}-1\right)-\left(x^2+\right.}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^2 + 2*x*log(x) + 2*x*log(-x + sqrt(x^2 + 1) + 1) - x*log(-x + sqr
t(x^2 + 1)) - 2*x*log(-x + sqrt(x^2 + 1) - 1) - (x^2 + 2*x + 2)*sqrt(x^2 +
1) - 2*x + 2)/x
```

giac [A] time = 0.42, size = 89, normalized size = 1.37

$$\frac{1}{2}\sqrt{x^2+1}\,(x+2)-x-\frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1}-\frac{1}{x}+\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)-\log\left(|x|\right)-\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*(x + 2) - x - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + 1/2*l
og(-x + sqrt(x^2 + 1)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + l
og(abs(-x + sqrt(x^2 + 1) - 1))
```

maple [A] time = 0.01, size = 56, normalized size = 0.86

$$-x-\frac{\sqrt{x^{2}+1}\,x}{2}-\frac{\operatorname{arcsinh}(x)}{2}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{2}+1}}\right)-\ln(x)-\frac{1}{x}+\frac{\left(x^{2}+1\right)^{\frac{3}{2}}}{x}+\sqrt{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(1+(x^2+1)^(1/2)),x)
```

```
[Out] -x-1/x-1/2*(x^2+1)^(1/2)*x-1/2*arcsinh(x)+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(
1/2))-ln(x)+1/x*(x^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,x-5\,\arctan\left(\frac{1}{2}\,x\right)+\int\frac{x^{6}+x^{5}-x^{4}}{3\,x^{4}+16\,x^{2}+\left(x^{4}+8\,x^{2}+16\right)\sqrt{x^{2}+1}+16}\,dx+\log\left(x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="maxima")
```

```
[Out] 2*x - 5*arctan(1/2*x) + integrate((x^6 + x^5 - x^4)/(3*x^4 + 16*x^2 + (x^4
+ 8*x^2 + 16)*sqrt(x^2 + 1) + 16), x) + log(x^2 + 4)
```

mupad [B] time = 0.04, size = 55, normalized size = 0.85

$$\left(\frac{x}{2}+1\right)\sqrt{x^{2}+1}-\frac{\operatorname{asinh}(x)}{2}-\ln(x)-x+\frac{\sqrt{x^{2}+1}}{x}-\frac{1}{x}+\operatorname{atan}\left(\sqrt{x^{2}+1}\,1\mathrm{i}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 1)/((x^2 + 1)^(1/2) + 1),x)
```

```
[Out] atan((x^2 + 1)^(1/2)*1i)*1i - x - asinh(x)/2 - log(x) + (x/2 + 1)*(x^2 + 1)
^(1/2) + (x^2 + 1)^(1/2)/x - 1/x
```

sympy [A] time = 4.51, size = 63, normalized size = 0.97

$$\frac{x\sqrt{x^{2}+1}}{2}-x+\frac{x}{\sqrt{x^{2}+1}}+\sqrt{x^{2}+1}-\log\left(\sqrt{x^{2}+1}+1\right)-\frac{\operatorname{asinh}(x)}{2}-\frac{1}{x}+\frac{1}{x\sqrt{x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/((x**2+1)**(1/2)+1),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 - x + x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2
+ 1) + 1) - asinh(x)/2 - 1/x + 1/(x*sqrt(x**2 + 1))
```

3.807 $\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{12}\left(2x^3+6x^2+\left(-2x^2-3x+4\right)\sqrt{x^2+1}-6\log\left(\sqrt{x^2+1}+1\right)-3\sinh^{-1}(x)\right)$$

```
[Out] 1/2*x^2+1/6*x^3-1/4*arcsinh(x)-1/2*ln((x^2+1)^(1/2)+1)+1/12*(-2*x^2-3*x+4)*
(x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 101, normalized size of antiderivative = 1.91, number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6742, 2117, 893, 195, 215, 261}

$$\frac{x^3}{6}+\frac{x^2}{2}-\frac{1}{4}\sqrt{x^2+1}\,x-\frac{1}{6}\left(x^2+1\right)^{3/2}+\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-\log\left(\sqrt{x^2+1}+x+1\right)+\frac{x}{2}-\frac{1}{4}\sinh$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] x/2 + x^2/2 + x^3/6 - (x*Sqrt[1 + x^2])/4 - (1 + x^2)^(3/2)/6 + 1/(2*(x + S
qrt[1 + x^2])) - ArcSinh[x]/4 + Log[x + Sqrt[1 + x^2]]/2 - Log[1 + x + Sqrt
[1 + x^2]]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \int \left(-\frac{1}{1+x+\sqrt{1+x^2}} + \frac{x}{1+x+\sqrt{1+x^2}} + \frac{x^2}{1+x+\sqrt{1+x^2}}\right) dx$$
$$= -\int \frac{1}{1+x+\sqrt{1+x^2}}\,dx + \int \frac{x}{1+x+\sqrt{1+x^2}}\,dx + \int \frac{x^2}{1+x+\sqrt{1+x^2}}\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{2-2x+x^2}{(1-x)^2 x}\,dx, x, 1+x+\sqrt{1+x^2}\right)\right) + \int \left(\frac{1}{2} + \frac{x}{2} - \frac{\sqrt{1+x^2}}{2}\right) dx + \int$$
$$= \frac{x}{2} + \frac{x^2}{2} + \frac{x^3}{6} - \frac{1}{2}\int \sqrt{1+x^2}\,dx - \frac{1}{2}\int x\sqrt{1+x^2}\,dx - \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x} + \frac{1}{(-1+x)^2}\right.\right.$$
$$= \frac{x}{2} + \frac{x^2}{2} + \frac{x^3}{6} - \frac{1}{4}x\sqrt{1+x^2} - \frac{1}{6}\left(1+x^2\right)^{3/2} + \frac{1}{2\left(x+\sqrt{1+x^2}\right)} + \frac{1}{2}\log\left(x+\sqrt{1+x^2}\right) -$$
$$= \frac{x}{2} + \frac{x^2}{2} + \frac{x^3}{6} - \frac{1}{4}x\sqrt{1+x^2} - \frac{1}{6}\left(1+x^2\right)^{3/2} + \frac{1}{2\left(x+\sqrt{1+x^2}\right)} - \frac{1}{4}\sinh^{-1}(x) + \frac{1}{2}\log\left(x\right.$$

Mathematica [A] time = 0.15, size = 88, normalized size = 1.66

$$\frac{1}{12}\left(2x^3+6x^2-2\left(x^2+1\right)^{3/2}+6\left(\frac{1}{\sqrt{x^2+1}+x}+\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{x^2+1}+x+1\right)\right)-3\left(\sqrt{x^2+1}\,x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] (6*x + 6*x^2 + 2*x^3 - 2*(1 + x^2)^(3/2) - 3*(x*Sqrt[1 + x^2] + ArcSinh[x])
 + 6*((x + Sqrt[1 + x^2])^(-1) + Log[x + Sqrt[1 + x^2]] - 2*Log[1 + x + Sqr
t[1 + x^2]]))/12
```

fricas [A] time = 0.41, size = 78, normalized size = 1.47

$$\frac{1}{6}x^3+\frac{1}{2}x^2-\frac{1}{12}\left(2x^2+3x-4\right)\sqrt{x^2+1}-\frac{1}{2}\log(x)-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}+1\right)+\frac{1}{4}\log\left(-x+\sqrt{x^2+1}\right)+\frac{1}{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*(2*x^2 + 3*x - 4)*sqrt(x^2 + 1) - 1/2*log(x) - 1/2
*log(-x + sqrt(x^2 + 1) + 1) + 1/4*log(-x + sqrt(x^2 + 1)) + 1/2*log(-x + s
qrt(x^2 + 1) - 1)
```

giac [A] time = 0.44, size = 80, normalized size = 1.51

$$\frac{1}{6}x^3+\frac{1}{2}x^2-\frac{1}{12}\left((2x+3)x-4\right)\sqrt{x^2+1}+\frac{1}{4}\log\left(-x+\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(|x|\right)-\frac{1}{2}\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\frac{1}{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*((2*x + 3)*x - 4)*sqrt(x^2 + 1) + 1/4*log(-x + sqr
t(x^2 + 1)) - 1/2*log(abs(x)) - 1/2*log(abs(-x + sqrt(x^2 + 1) + 1)) + 1/2*
log(abs(-x + sqrt(x^2 + 1) - 1))
```

maple [A] time = 0.01, size = 58, normalized size = 1.09

$$\frac{x^3}{6}+\frac{x^2}{2}-\frac{\sqrt{x^2+1}\,x}{4}-\frac{\operatorname{arcsinh}(x)}{4}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{2}-\frac{\ln(x)}{2}+\frac{\sqrt{x^2+1}}{2}-\frac{\left(x^2+1\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x)
```

```
[Out] 1/2*x^2-1/2*ln(x)+1/6*x^3-1/4*(x^2+1)^(1/2)*x-1/4*arcsinh(x)+1/2*(x^2+1)^(1
/2)-1/2*arctanh(1/(x^2+1)^(1/2))-1/6*(x^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,x^2-\frac{3}{56}\,\sqrt{7}\,\arctan\left(\frac{1}{7}\,\sqrt{7}\,(4\,x+3)\right)+\frac{1}{4}\,x+\int\frac{x^4+x^3-x^2}{4\,x^5+12\,x^4+19\,x^3+19\,x^2+\left(4\,x^4+12\,x^3+17\,x^2+12\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 3/56*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 3)) + 1/4*x + integrate((x
^4 + x^3 - x^2)/(4*x^5 + 12*x^4 + 19*x^3 + 19*x^2 + (4*x^4 + 12*x^3 + 17*x^
2 + 12*x + 4)*sqrt(x^2 + 1) + 12*x + 4), x) - 7/16*log(2*x^2 + 3*x + 2)
```

mupad [B] time = 0.04, size = 52, normalized size = 0.98

$$\frac{x^2}{2}-\frac{\ln(x)}{2}-\sqrt{x^2+1}\,\left(\frac{x^2}{6}+\frac{x}{4}-\frac{1}{3}\right)-\frac{\operatorname{asinh}(x)}{4}+\frac{x^3}{6}+\frac{\operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 1)/(x + (x^2 + 1)^(1/2) + 1),x)
```

```
[Out] (atan((x^2 + 1)^(1/2)*1i)*1i)/2 - asinh(x)/4 - log(x)/2 - (x^2 + 1)^(1/2)*(
x/4 + x^2/6 - 1/3) + x^2/2 + x^3/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+x-1}{x+\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/(1+x+(x**2+1)**(1/2)),x)
```

```
[Out] Integral((x**2 + x - 1)/(x + sqrt(x**2 + 1) + 1), x)
```

3.808 $\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}\,x}\,dx$

Optimal. Leaf size=14

$$2\sqrt{x-1}+2\log(x)$$

[Out] 2*ln(x)+2*(-1+x)^(1/2)

Rubi [A] time = 0.12, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6688}

$$2\sqrt{x-1}+2\log(x)$$

Antiderivative was successfully verified.

[In] Int[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]

[Out] 2*Sqrt[-1 + x] + 2*Log[x]

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}\,x}\,dx &= \int \left(\frac{1}{\sqrt{-1+x}}+\frac{2}{x}\right)dx\\ &= 2\sqrt{-1+x}+2\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$2\sqrt{x-1}+2\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]

[Out] 2*Sqrt[-1 + x] + 2*Log[x]

fricas [A] time = 0.40, size = 12, normalized size = 0.86

$$2\sqrt{x-1}+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(x - 1) + 2*log(x)

giac [A] time = 0.40, size = 12, normalized size = 0.86

$$2\sqrt{x-1}+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="giac")

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$2\ln(x) + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2*(x-1)^(1/2))/x/(x-1)^(1/2),x)
[Out] 2*ln(x)+2*(x-1)^(1/2)
```

maxima [A] time = 0.98, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x - 1) + 2*log(x)
```

mupad [B] time = 3.39, size = 12, normalized size = 0.86

$$2\ln(x) + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2*(x - 1)^(1/2))/(x*(x - 1)^(1/2)),x)
[Out] 2*log(x) + 2*(x - 1)^(1/2)
```

sympy [A] time = 0.16, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)**(1/2))/x/(-1+x)**(1/2),x)
[Out] 2*sqrt(x - 1) + 2*log(x)
```

3.809 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2 dx$

Optimal. Leaf size=61

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

```
[Out] a^2*x+4/3*a*c*x^(3/2)+6/5*a*b*x^(5/3)+1/2*c^2*x^2+12/13*b*c*x^(13/6)+3/7*b^
2*x^(7/3)
```

Rubi [A] time = 0.17, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6741, 6742}

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12*b*c*x^(13
/6))/13 + (3*b^2*x^(7/3))/7
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2 dx &= 6\,\text{Subst}\left(\int x^5\left(a+x^3(c+bx)\right)^2 dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int x^5\left(a+cx^3+bx^4\right)^2 dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int \left(a^2x^5+2acx^8+2abx^9+c^2x^{11}+2bcx^{12}+b^2x^{13}\right) dx, x, \sqrt[6]{x}\right)\\
&= a^2x+\frac{4}{3}acx^{3/2}+\frac{6}{5}abx^{5/3}+\frac{c^2x^2}{2}+\frac{12}{13}bcx^{13/6}+\frac{3}{7}b^2x^{7/3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.00

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12*b*c*x^(13
/6))/13 + (3*b^2*x^(7/3))/7
```

fricas [A] time = 0.40, size = 43, normalized size = 0.70

$$\frac{3}{7}\,b^{2}x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^{2}x^{2} + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{4}{3}\,acx^{\frac{3}{2}} + a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*
a*c*x^(3/2) + a^2*x
```

giac [A] time = 0.43, size = 43, normalized size = 0.70

$$\frac{3}{7}\,b^{2}x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^{2}x^{2} + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{4}{3}\,acx^{\frac{3}{2}} + a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*
a*c*x^(3/2) + a^2*x
```

maple [A] time = 0.00, size = 46, normalized size = 0.75

$$\frac{3b^{2}x^{\frac{7}{3}}}{7} + \frac{c^{2}x^{2}}{2} + \frac{6ab\,x^{\frac{5}{3}}}{5} + a^{2}x + 2\left(\frac{6b\,x^{\frac{13}{6}}}{13} + \frac{2a\,x^{\frac{3}{2}}}{3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(2/3)+c*x^(1/2))^2,x)
```

```
[Out] 1/2*c^2*x^2+2*c*(6/13*b*x^(13/6)+2/3*a*x^(3/2))+a^2*x+3/7*b^2*x^(7/3)+6/5*a
*b*x^(5/3)
```

maxima [A] time = 0.44, size = 45, normalized size = 0.74

$$\frac{3}{7}\,b^{2}x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^{2}x^{2} + a^{2}x + \frac{2}{15}\left(9\,bx^{\frac{5}{3}} + 10\,cx^{\frac{3}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + a^2*x + 2/15*(9*b*x^(5
/3) + 10*c*x^(3/2))*a
```

mupad [B] time = 3.36, size = 43, normalized size = 0.70

$$a^{2}\,x + \frac{3\,b^{2}\,x^{7/3}}{7} + \frac{c^{2}\,x^{2}}{2} + \frac{6\,a\,b\,x^{5/3}}{5} + \frac{4\,a\,c\,x^{3/2}}{3} + \frac{12\,b\,c\,x^{13/6}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(2/3) + c*x^(1/2))^2,x)
```

```
[Out] a^2*x + (3*b^2*x^(7/3))/7 + (c^2*x^2)/2 + (6*a*b*x^(5/3))/5 + (4*a*c*x^(3/2
))/3 + (12*b*c*x^(13/6))/13
```

sympy [A] time = 2.42, size = 60, normalized size = 0.98

$$a^{2}x + \frac{6abx^{\frac{5}{3}}}{5} + \frac{4acx^{\frac{3}{2}}}{3} + \frac{3b^{2}x^{\frac{7}{3}}}{7} + \frac{12bcx^{\frac{13}{6}}}{13} + \frac{c^{2}x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(2/3)+c*x**(1/2))**2,x)
```

```
[Out] a**2*x + 6*a*b*x**(5/3)/5 + 4*a*c*x**(3/2)/3 + 3*b**2*x**(7/3)/7 + 12*b*c*x
**(13/6)/13 + c**2*x**2/2
```

3.810 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx$

Optimal. Leaf size=114

$$a^3x+\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{b^3x^3}{3}+\frac{18}{17}b^2cx^{17/6}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

```
[Out] a^3*x+2*a^2*c*x^(3/2)+9/5*a^2*b*x^(5/3)+3/2*a*c^2*x^2+36/13*a*b*c*x^(13/6)+
9/7*a*b^2*x^(7/3)+2/5*c^3*x^(5/2)+9/8*b*c^2*x^(8/3)+18/17*b^2*c*x^(17/6)+1/
3*b^3*x^3
```

Rubi [A] time = 0.19, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{6741, 6742\}$

$$\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+a^3x+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{18}{17}b^2cx^{17/6}+\frac{b^3x^3}{3}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 + (36*a*b*c
*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5 + (9*b*c^2*x^(8/3))
/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx &= 6\,\text{Subst}\left(\int x^5\left(a+x^3(c+bx)\right)^3\,dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int x^5\left(a+cx^3+bx^4\right)^3\,dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int \left(a^3x^5+3a^2cx^8+3a^2bx^9+3ac^2x^{11}+6abcx^{12}+3ab^2x^{13}+c^3x^{14}+3b\right.\right.\\
&= a^3x+2a^2cx^{3/2}+\frac{9}{5}a^2bx^{5/3}+\frac{3}{2}ac^2x^2+\frac{36}{13}abcx^{13/6}+\frac{9}{7}ab^2x^{7/3}+\frac{2}{5}c^3x^{5/2}+\frac{9}{8}bc^2x^8
\end{aligned}$$

Mathematica [A] time = 0.09, size = 114, normalized size = 1.00

$$a^3x+\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{b^3x^3}{3}+\frac{18}{17}b^2cx^{17/6}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 + (36*a*b*c
*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5 + (9*b*c^2*x^(8/3))
/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

fricas [A] time = 0.42, size = 91, normalized size = 0.80

$$\frac{1}{3}b^3x^3+\frac{18}{17}b^2cx^{\frac{17}{6}}+\frac{9}{7}ab^2x^{\frac{7}{3}}+\frac{36}{13}abcx^{\frac{13}{6}}+\frac{3}{2}ac^2x^2+a^3x+\frac{9}{40}\left(5\,bc^2x^2+8\,a^2bx\right)x^{\frac{2}{3}}+\frac{2}{5}\left(c^3x^2+5\,a^2cx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/
6) + 3/2*a*c^2*x^2 + a^3*x + 9/40*(5*b*c^2*x^2 + 8*a^2*b*x)*x^(2/3) + 2/5*(
c^3*x^2 + 5*a^2*c*x)*sqrt(x)
```

giac [A] time = 0.34, size = 84, normalized size = 0.74

$$\frac{1}{3}b^3x^3+\frac{18}{17}b^2cx^{\frac{17}{6}}+\frac{9}{8}bc^2x^{\frac{8}{3}}+\frac{2}{5}c^3x^{\frac{5}{2}}+\frac{9}{7}ab^2x^{\frac{7}{3}}+\frac{36}{13}abcx^{\frac{13}{6}}+\frac{3}{2}ac^2x^2+\frac{9}{5}a^2bx^{\frac{5}{3}}+2\,a^2cx^{\frac{3}{2}}+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/6) + 3/2*a*c^2*x^2 + 9/5*a^2*b*x^(5/3
) + 2*a^2*c*x^(3/2) + a^3*x
```

maple [A] time = 0.00, size = 86, normalized size = 0.75

$$\frac{b^3x^3}{3}+\frac{2c^3x^{\frac{5}{2}}}{5}+\frac{9a\,b^2x^{\frac{7}{3}}}{7}+\frac{9a^2b\,x^{\frac{5}{3}}}{5}+a^3x+3\left(\frac{3b\,x^{\frac{8}{3}}}{8}+\frac{a\,x^2}{2}\right)c^2+3\left(\frac{6b^2x^{\frac{17}{6}}}{17}+\frac{12ab\,x^{\frac{13}{6}}}{13}+\frac{2a^2x^{\frac{3}{2}}}{3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(2/3)+c*x^(1/2))^3,x)
```

```
[Out] 2/5*c^3*x^(5/2)+3*c^2*(3/8*b*x^(8/3)+1/2*a*x^2)+3*c*(6/17*b^2*x^(17/6)+12/1
3*a*b*x^(13/6)+2/3*a^2*x^(3/2))+a^3*x+1/3*b^3*x^3+9/5*a^2*b*x^(5/3)+9/7*a*b
^2*x^(7/3)
```

maxima [A] time = 0.44, size = 85, normalized size = 0.75

$$\frac{1}{3}b^3x^3+\frac{18}{17}b^2cx^{\frac{17}{6}}+\frac{9}{8}bc^2x^{\frac{8}{3}}+\frac{2}{5}c^3x^{\frac{5}{2}}+a^3x+\frac{1}{5}\left(9\,bx^{\frac{5}{3}}+10\,cx^{\frac{3}{2}}\right)a^2+\frac{3}{182}\left(78\,b^2x^{\frac{7}{3}}+168\,bcx^{\frac{13}{6}}+91\,c^2x^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
a^3*x + 1/5*(9*b*x^(5/3) + 10*c*x^(3/2))*a^2 + 3/182*(78*b^2*x^(7/3) + 168*
b*c*x^(13/6) + 91*c^2*x^2)*a
```

mupad [B] time = 0.06, size = 84, normalized size = 0.74

$$a^3\,x+\frac{b^3\,x^3}{3}+\frac{2\,c^3\,x^{5/2}}{5}+\frac{9\,a^2\,b\,x^{5/3}}{5}+\frac{9\,a\,b^2\,x^{7/3}}{7}+\frac{3\,a\,c^2\,x^2}{2}+2\,a^2\,c\,x^{3/2}+\frac{9\,b\,c^2\,x^{8/3}}{8}+\frac{18\,b^2\,c\,x^{17/6}}{17}+\frac{36\,a\,b\,c\,x^{13/6}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(2/3) + c*x^(1/2))^3,x)
```

```
[Out] a^3*x + (b^3*x^3)/3 + (2*c^3*x^(5/2))/5 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^
(7/3))/7 + (3*a*c^2*x^2)/2 + 2*a^2*c*x^(3/2) + (9*b*c^2*x^(8/3))/8 + (18*b^
2*c*x^(17/6))/17 + (36*a*b*c*x^(13/6))/13
```

sympy [A] time = 3.45, size = 116, normalized size = 1.02

$$a^3x + \frac{9a^2bx^{\frac{5}{3}}}{5} + 2a^2cx^{\frac{3}{2}} + \frac{9ab^2x^{\frac{7}{3}}}{7} + \frac{36abcx^{\frac{13}{6}}}{13} + \frac{3ac^2x^2}{2} + \frac{b^3x^3}{3} + \frac{18b^2cx^{\frac{17}{6}}}{17} + \frac{9bc^2x^{\frac{8}{3}}}{8} + \frac{2c^3x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(2/3)+c*x**(1/2))**3,x)
```

```
[Out] a**3*x + 9*a**2*b*x**(5/3)/5 + 2*a**2*c*x**(3/2) + 9*a*b**2*x**(7/3)/7 + 36
*a*b*c*x**(13/6)/13 + 3*a*c**2*x**2/2 + b**3*x**3/3 + 18*b**2*c*x**(17/6)/1
7 + 9*b*c**2*x**(8/3)/8 + 2*c**3*x**(5/2)/5
```

3.811 $\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\,x^3}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] arctanh((a-b*(1-1/x^2))^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)+(a-b*(1-1/x^2))^(1/2
)/b
```

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {514, 446, 80, 63, 208}

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\,x^3}\,dx &= \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}}\,x}\,dx\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}}\,dx,x,\frac{1}{x^2}\right)\\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}-\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b}\\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 100, normalized size = 1.72

$$\frac{\sqrt{a-b}\left(ax^2-bx^2+b\right)+bx\sqrt{ax^2-bx^2+b}\,\tanh^{-1}\left(\frac{x\sqrt{a-b}}{\sqrt{x^2(a-b)+b}}\right)}{bx^2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) + b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTanh[(Sqr
t[a - b]*x)/Sqrt[b + (a - b)*x^2]])/(Sqrt[a - b]*b*Sqrt[a + b*(-1 + x^(-2))
]*x^2)
```

fricas [A] time = 0.44, size = 180, normalized size = 3.10

$$\left[\frac{\sqrt{a-b}\,b\log\left(-2\,(a-b)x^2-2\sqrt{a-b}\,x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}-b\right)+2\,(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2\left(ab-b^2\right)},\ \frac{\sqrt{-a+b}\,b\arctan\left(-\frac{\sqrt{-a+b}\,x^2\sqrt{}}{(a-b)}\right)}{ab-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x^
2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (sqr
t(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b)*x
^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}
+%%%{2,[0,2,1]%%%}+%%%{-2,[0,1,1]%%%}+%%%{2,[0,0,1]%%%},0,%%%{1,[2,4,0]%%%}
+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{-2,[1,4,1]%%%}+%%%{2,[1,3,1]%%%}+
%%%{4,[1,2,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{-2,[1,0,1]%%%}+%%%{1,[0,4,2]%%%}+%
%%{-2,[0,3,2]%%%}+%%%{-1,[0,2,2]%%%}+%%%{2,[0,1,2]%%%}+%%%{1,[0,0,2]%%%}] a
t parameters values [86,-97,-82]Sign error (%%%{b,0%%%}+%%%{-2*sqrt(a-b)*sq
rt(b),1/2%%%}+%%%{2*(a-b),1%%%}+%%%{-(a*sqrt(a-b)*sqrt(b)-b*sqrt(a-b)*sqrt(
b))/b,3/2%%%}+%%%{-(-a^2*sqrt(a-b)*sqrt(b)+2*a*b*sqrt(a-b)*sqrt(b)-b^2*sqrt
(a-b)*sqrt(b))/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached o
r unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 102, normalized size = 1.76

$$\frac{\sqrt{a\,x^{2}-b\,x^{2}+b}\,\left(b x \ln\left(\sqrt{a-b}\,x+\sqrt{a\,x^{2}-b\,x^{2}+b}\right)+\sqrt{a\,x^{2}-b\,x^{2}+b}\,\sqrt{a-b}\right)}{\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\,\sqrt{a-b}\,b\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x)
```

```
[Out] (a*x^2-b*x^2+b)^(1/2)*(ln((a-b)^(1/2)*x+(a*x^2-b*x^2+b)^(1/2))*b*x+(a*x^2-b
*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/x^2/(a-b)^(1/2)/b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 4.08, size = 46, normalized size = 0.79

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a-b+\frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^3*(a - b + b/x^2)^(1/2)),x)
```

```
[Out] atanh((a - b + b/x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) + (a - b + b/x^2)^
(1/2)/b
```

sympy [A] time = 3.18, size = 70, normalized size = 1.21

$$-\frac{\begin{cases} -\frac{1}{\sqrt{a}\,x^2} & \text{for } b=0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2} - \frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a-b}}\,\sqrt{a-b+\frac{b}{x^2}}}\right)}{\sqrt{-\frac{1}{a-b}}\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x**3/(a-b+b/x**2)**(1/2),x)
```

```
[Out] -Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True)
)/2 - atan(1/(sqrt(-1/(a - b))*sqrt(a - b + b/x**2)))/(sqrt(-1/(a - b))*(a
- b))
```

3.812 $\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\,x^3}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] arctanh((a-b*(1-1/x^2))^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)+(a-b*(1-1/x^2))^(1/2
)/b
```

Rubi [A] time = 0.14, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1978, 514, 446, 80, 63, 208}

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1978

```
Int[(Pq_)*(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*Pq*Expand
ToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && PolyQ[Pq, x] && BinomialQ[u, x]
 &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\,x^3}\,dx &= \int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\,x^3}\,dx \\
&= \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}}\,x}\,dx \\
&= -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b} \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 100, normalized size = 1.72

$$\frac{\sqrt{a-b}\left(ax^2-bx^2+b\right)+bx\sqrt{ax^2-bx^2+b}\,\tanh^{-1}\left(\frac{x\sqrt{a-b}}{\sqrt{x^2(a-b)+b}}\right)}{bx^2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) + b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTanh[(Sqr
t[a - b]*x)/Sqrt[b + (a - b)*x^2]])/(Sqrt[a - b]*b*Sqrt[a + b*(-1 + x^(-2))
]*x^2)
```

fricas [A] time = 0.44, size = 180, normalized size = 3.10

$$\left[\frac{\sqrt{a-b}\,b\log\left(-2\,(a-b)x^2-2\,\sqrt{a-b}\,x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}-b\right)+2\,(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2\left(ab-b^2\right)}, \frac{\sqrt{-a+b}\,b\arctan\left(-\frac{\sqrt{-a+b}\,x^2\sqrt{\ldots}}{(a-b)x\ldots}\right.}{ab-\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x^
2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (sqr
t(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b)*x
^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}
+%%%{2,[0,2,1]%%%}+%%%{-2,[0,1,1]%%%}+%%%{2,[0,0,1]%%%},0,%%%{1,[2,4,0]%%%}
+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{-2,[1,4,1]%%%}+%%%{2,[1,3,1]%%%}+
%%%{4,[1,2,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{-2,[1,0,1]%%%}+%%%{1,[0,4,2]%%%}+%
%%{-2,[0,3,2]%%%}+%%%{-1,[0,2,2]%%%}+%%%{2,[0,1,2]%%%}+%%%{1,[0,0,2]%%%}] a
t parameters values [86,-97,-82]Sign error (%%%{b,0%%%}+%%%{-2*sqrt(a-b)*sq
rt(b),1/2%%%}+%%%{2*(a-b),1%%%}+%%%{-(a*sqrt(a-b)*sqrt(b)-b*sqrt(a-b)*sqrt(
b))/b,3/2%%%}+%%%{-(-a^2*sqrt(a-b)*sqrt(b)+2*a*b*sqrt(a-b)*sqrt(b)-b^2*sqrt
(a-b)*sqrt(b))/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached o
r unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 102, normalized size = 1.76

$$\frac{\sqrt{a\,x^{2}-b\,x^{2}+b}\,\left(b x \ln \left(\sqrt{a-b}\,x+\sqrt{a\,x^{2}-b\,x^{2}+b}\right)+\sqrt{a\,x^{2}-b\,x^{2}+b}\,\sqrt{a-b}\right)}{\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\,\sqrt{a-b}\,b\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x)
```

```
[Out] (a*x^2-b*x^2+b)^(1/2)*(b*x*ln((a-b)^(1/2)*x+(a*x^2-b*x^2+b)^(1/2))+(a*x^2-b
*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/(a-b)^(1/2)/b/x^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 details)Is 4*a-4*b positive or negative?
```

mupad [B] time = 4.04, size = 62, normalized size = 1.07

$$\frac{\sqrt{a+b\,\left(\frac{1}{x^2}-1\right)}}{b}+\frac{\ln\left(x^2\,\left(2\,a-2\,b+2\,\sqrt{a-b}\,\sqrt{a+b\,\left(\frac{1}{x^2}-1\right)}+\frac{b}{x^2}\right)\right)}{2\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^3*(a + b*(1/x^2 - 1))^(1/2)),x)
```

```
[Out] (a + b*(1/x^2 - 1))^(1/2)/b + log(x^2*(2*a - 2*b + 2*(a - b)^(1/2)*(a + b*(
1/x^2 - 1))^(1/2) + b/x^2))/(2*(a - b)^(1/2))
```

sympy [A] time = 7.03, size = 70, normalized size = 1.21

$$-\frac{\begin{cases} -\frac{1}{\sqrt{a}x^2} & \text{for } b = 0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2} - \frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a-b}}\sqrt{a-b+\frac{b}{x^2}}}\right)}{\sqrt{-\frac{1}{a-b}}\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x**3/(a+b*(-1+1/x**2))**(1/2),x)
```

```
[Out] -Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True)
)/2 - atan(1/(sqrt(-1/(a - b))*sqrt(a - b + b/x**2)))/(sqrt(-1/(a - b))*(a
- b))
```

3.813 $\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}\,x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] 1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*(x^2+9)^(1
/2)*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1010, 377, 203, 444, 63, 207}

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}\,x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[9 + x^2])]/(2*Sqrt[5]) - ArcTanh[Sqrt[9 + x^2]/S
qrt[5]]/Sqrt[5]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx &= \int \frac{1}{(4+x^2)\sqrt{9+x^2}}\,dx+\int \frac{x}{(4+x^2)\sqrt{9+x^2}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(4+x)\sqrt{9+x}}\,dx,x,x^2\right)+\operatorname{Subst}\left(\int \frac{1}{4+5x^2}\,dx,x,\frac{x}{\sqrt{9+x^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}}+\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{9+x^2}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 64, normalized size = 1.21

$$-\frac{(2+i)\tanh^{-1}\left(\frac{9-2ix}{\sqrt{5}\sqrt{x^2+9}}\right)+(2-i)\tanh^{-1}\left(\frac{9+2ix}{\sqrt{5}\sqrt{x^2+9}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] -1/4*((2 + I)*ArcTanh[(9 - (2*I)*x)/(Sqrt[5]*Sqrt[9 + x^2])] + (2 - I)*ArcT
anh[(9 + (2*I)*x)/(Sqrt[5]*Sqrt[9 + x^2])])/Sqrt[5]
```

fricas [B] time = 0.44, size = 182, normalized size = 3.43

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-\sqrt{x^2+9}\left(x+\sqrt{5}\right)+\sqrt{5}x+9}+\frac{1}{2}x+\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)-\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*arctan(1/2*sqrt(2)*sqrt(x^2 - sqrt(x^2 + 9)*(x + sqrt(5)) + sqr
t(5)*x + 9) + 1/2*x + 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) - 1/5*sqrt(5)*arctan
(1/2*sqrt(2)*sqrt(x^2 - sqrt(x^2 + 9)*(x - sqrt(5)) - sqrt(5)*x + 9) + 1/2*
x - 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) + 1/10*sqrt(5)*log(50*x^2 - 50*sqrt(x^
2 + 9)*(x + sqrt(5)) + 50*sqrt(5)*x + 450) - 1/10*sqrt(5)*log(50*x^2 - 50*s
qrt(x^2 + 9)*(x - sqrt(5)) - 50*sqrt(5)*x + 450)
```

giac [B] time = 0.44, size = 123, normalized size = 2.32

$$-\frac{1}{10}\sqrt{5}\arctan\left(\frac{1}{2}x-\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)-\frac{1}{10}\sqrt{5}\arctan\left(-\frac{1}{2}x-\frac{1}{2}\sqrt{5}+\frac{1}{2}\sqrt{x^2+9}\right)+\frac{1}{10}\sqrt{5}\log\left(\left(x-\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*arctan(1/2*x - 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) - 1/10*sqrt(5
)*arctan(-1/2*x - 1/2*sqrt(5) + 1/2*sqrt(x^2 + 9)) + 1/10*sqrt(5)*log((x -
sqrt(x^2 + 9))^2 + 2*sqrt(5)*(x - sqrt(x^2 + 9)) + 9) - 1/10*sqrt(5)*log((x
 - sqrt(x^2 + 9))^2 - 2*sqrt(5)*(x - sqrt(x^2 + 9)) + 9)
```

maple [A] time = 0.02, size = 39, normalized size = 0.74

$$-\frac{\sqrt{5}\, \operatorname{arctanh}\left(\frac{\sqrt{x^2+9}\, \sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5}\, \arctan\left(\frac{\sqrt{5}\, x}{2 \sqrt{x^2+9}}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^2+4)/(x^2+9)^(1/2),x)
```

```
[Out] 1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*(x^2+9)^(1
/2)*5^(1/2))*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^2+9}\,\left(x^2+4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 9)*(x^2 + 4)), x)
```

mupad [B] time = 3.59, size = 67, normalized size = 1.26

$$\sqrt{5}\,\left(\ln\left(x-2{}\mathrm{i}\right)-\ln\left(\sqrt{5}\,\sqrt{x^2+9}+9+x\,2{}\mathrm{i}\right)\right)\,\left(\frac{1}{10}-\frac{1}{20}{}\mathrm{i}\right)+\sqrt{5}\,\left(\ln\left(x+2{}\mathrm{i}\right)-\ln\left(\sqrt{5}\,\sqrt{x^2+9}+9-x\,2{}\mathrm{i}\right)\right)\,\left(\frac{1}{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^2 + 4)*(x^2 + 9)^(1/2)),x)
```

```
[Out] 5^(1/2)*(log(x - 2i) - log(x*2i + 5^(1/2)*(x^2 + 9)^(1/2) + 9))*(1/10 - 1i/
20) + 5^(1/2)*(log(x + 2i) - log(5^(1/2)*(x^2 + 9)^(1/2) - x*2i + 9))*(1/10
 + 1i/20)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^2+4\right)\sqrt{x^2+9}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+4)/(x**2+9)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x**2 + 4)*sqrt(x**2 + 9)), x)
```

3.814 $\int x\left(1+\sqrt{1-x^2}\right)\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {14, 261}

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + Sqrt[1 - x^2]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1+\sqrt{1-x^2}\right)\,dx &= \int \left(x+x\sqrt{1-x^2}\right)\,dx\\ &= \frac{x^2}{2}+\int x\sqrt{1-x^2}\,dx\\ &= \frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + Sqrt[1 - x^2]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

fricas [A] time = 0.41, size = 22, normalized size = 0.96

$$\frac{1}{2}\,x^2+\frac{1}{3}\left(x^2-1\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)
```

giac [A] time = 0.33, size = 18, normalized size = 0.78

$$\frac{1}{2}x^2 - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}} - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2) - 1/2
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{x^2}{2} - \frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(-x^2+1)^(1/2)),x)
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

maxima [A] time = 0.43, size = 17, normalized size = 0.74

$$\frac{1}{2}x^2 - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$\frac{x^2}{2} + \sqrt{1-x^2}\left(\frac{x^2}{3} - \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1 - x^2)^(1/2) + 1),x)
[Out] x^2/2 + (1 - x^2)^(1/2)*(x^2/3 - 1/3)
```

sympy [A] time = 0.19, size = 27, normalized size = 1.17

$$\frac{x^2\sqrt{1-x^2}}{3} + \frac{x^2}{2} - \frac{\sqrt{1-x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x**2+1)**(1/2)),x)
[Out] x**2*sqrt(1 - x**2)/3 + x**2/2 - sqrt(1 - x**2)/3
```

3.815 $\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 261}

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)dx&=\int\left(x+x\sqrt{1-x^2}\right)dx\\&=\frac{x^2}{2}+\int x\sqrt{1-x^2}\,dx\\&=\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

fricas [A] time = 0.41, size = 25, normalized size = 1.09

$$\frac{1}{2}x^2+\frac{1}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

giac [B] time = 0.45, size = 54, normalized size = 2.35

$$\frac{1}{2}(x+1)^2+\frac{1}{6}((2x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1}-x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(x + 1)^2 + 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*
sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - x - 1
```

maple [A] time = 0.00, size = 26, normalized size = 1.13

$$\frac{x^2}{2}+\frac{\sqrt{x+1}\,\sqrt{-x+1}\,\left(x^2-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(-x+1)^(1/2)*(x+1)^(1/2)),x)
```

```
[Out] 1/3*(x+1)^(1/2)*(-x+1)^(1/2)*(x^2-1)+1/2*x^2
```

maxima [A] time = 0.97, size = 17, normalized size = 0.74

$$\frac{1}{2}x^2-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

mupad [B] time = 3.69, size = 35, normalized size = 1.52

$$\frac{x^2}{2}-\frac{\sqrt{1-x}\,\left(-\frac{x^3}{3}-\frac{x^2}{3}+\frac{x}{3}+\frac{1}{3}\right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1 - x)^(1/2)*(x + 1)^(1/2) + 1),x)
```

```
[Out] x^2/2 - ((1 - x)^(1/2)*(x/3 - x^2/3 - x^3/3 + 1/3))/(x + 1)^(1/2)
```

sympy [A] time = 92.60, size = 105, normalized size = 4.57

$$-x+\frac{(x+1)^2}{2}-2\left(\left\{\frac{x\sqrt{1-x}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2}\quad\text{for } x\ge -1\wedge x<1\right.\right)+2\left(\left\{\frac{x\sqrt{1-x}\sqrt{x+1}}{4}-\frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)**(1/2)*(1+x)**(1/2)),x)
```

```
[Out] -x + (x + 1)**2/2 - 2*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)
*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) + 2*Piecewise((x*sqrt(1 - x)*sqrt(
x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x + 1)/2)/2,
 (x >= -1) & (x < 1))) - 1
```

3.816 $\int x\left(1+\frac{1}{\sqrt{2+x}\,\sqrt{3+x}}\right)dx$

Optimal. Leaf size=33

$$\frac{x^2}{2}+\sqrt{x+2}\,\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

```
[Out] 1/2*x^2-5*arcsinh((2+x)^(1/2))+(2+x)^(1/2)*(3+x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {14, 80, 54, 215}

$$\frac{x^2}{2}+\sqrt{x+2}\,\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
```

```
[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)dx &= \int \left(x+\frac{x}{\sqrt{2+x}\sqrt{3+x}}\right)dx\\ &= \frac{x^2}{2}+\int \frac{x}{\sqrt{2+x}\sqrt{3+x}}\,dx\\ &= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-\frac{5}{2}\int \frac{1}{\sqrt{2+x}\sqrt{3+x}}\,dx\\ &= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-5\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{2+x}\right)\\ &= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-5\sinh^{-1}\left(\sqrt{2+x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
```

```
[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

fricas [A] time = 0.40, size = 37, normalized size = 1.12

$$\frac{1}{2}x^2+\sqrt{x+3}\sqrt{x+2}+\frac{5}{2}\log\left(2\sqrt{x+3}\sqrt{x+2}-2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + sqrt(x + 3)*sqrt(x + 2) + 5/2*log(2*sqrt(x + 3)*sqrt(x + 2) - 2*x
 - 5)
```

giac [A] time = 0.38, size = 39, normalized size = 1.18

$$\frac{1}{2}(x+3)^2+\sqrt{x+3}\sqrt{x+2}-3x+5\log\left(\sqrt{x+3}-\sqrt{x+2}\right)-9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(x + 3)^2 + sqrt(x + 3)*sqrt(x + 2) - 3*x + 5*log(sqrt(x + 3) - sqrt(x
+ 2)) - 9
```

maple [B] time = 0.01, size = 58, normalized size = 1.76

$$\frac{x^2}{2}-\frac{\sqrt{x+2}\,\sqrt{x+3}\,\left(5\ln\left(x+\frac{5}{2}+\sqrt{x^2+5x+6}\right)-2\sqrt{x^2+5x+6}\right)}{2\sqrt{x^2+5x+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+1/(x+2)^(1/2)/(x+3)^(1/2)),x)
```

```
[Out] -1/2*(x+2)^(1/2)*(x+3)^(1/2)*(-2*(x^2+5*x+6)^(1/2)+5*ln(x+5/2+(x^2+5*x+6)^(
1/2)))/(x^2+5*x+6)^(1/2)+1/2*x^2
```

maxima [A] time = 0.43, size = 36, normalized size = 1.09

$$\frac{1}{2}x^2 + \sqrt{x^2 + 5x + 6} - \frac{5}{2}\log\left(2x + 2\sqrt{x^2 + 5x + 6} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + sqrt(x^2 + 5*x + 6) - 5/2*log(2*x + 2*sqrt(x^2 + 5*x + 6) + 5)
```

mupad [B] time = 7.56, size = 180, normalized size = 5.45

$$\frac{\frac{10\left(\sqrt{x+2}-\sqrt{2}\right)}{\sqrt{x+3}-\sqrt{3}}+\frac{10\left(\sqrt{x+2}-\sqrt{2}\right)^3}{\left(\sqrt{x+3}-\sqrt{3}\right)^3}-\frac{8\sqrt{6}\left(\sqrt{x+2}-\sqrt{2}\right)^2}{\left(\sqrt{x+3}-\sqrt{3}\right)^2}}{\frac{\left(\sqrt{x+2}-\sqrt{2}\right)^4}{\left(\sqrt{x+3}-\sqrt{3}\right)^4}-\frac{2\left(\sqrt{x+2}-\sqrt{2}\right)^2}{\left(\sqrt{x+3}-\sqrt{3}\right)^2}+1} - 10\,\mathrm{atanh}\left(\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+3}-\sqrt{3}}\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1/((x + 2)^(1/2)*(x + 3)^(1/2)) + 1),x)
```

```
[Out] ((10*((x + 2)^(1/2) - 2^(1/2)))/((x + 3)^(1/2) - 3^(1/2)) + (10*((x + 2)^(1
/2) - 2^(1/2))^3)/((x + 3)^(1/2) - 3^(1/2))^3 - (8*6^(1/2)*((x + 2)^(1/2) -
 2^(1/2))^2)/((x + 3)^(1/2) - 3^(1/2))^2)/((((x + 2)^(1/2) - 2^(1/2))^4/((x
+ 3)^(1/2) - 3^(1/2))^4 - (2*((x + 2)^(1/2) - 2^(1/2))^2)/((x + 3)^(1/2) -
3^(1/2))^2 + 1) - 10*atanh(((x + 2)^(1/2) - 2^(1/2))/((x + 3)^(1/2) - 3^(1/
2))) + x^2/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\sqrt{x+2}\,\sqrt{x+3}+1\right)}{\sqrt{x+2}\,\sqrt{x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)**(1/2)/(3+x)**(1/2)),x)
```

```
[Out] Integral(x*(sqrt(x + 2)*sqrt(x + 3) + 1)/(sqrt(x + 2)*sqrt(x + 3)), x)
```

3.817 $\int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A] time = 0.16, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx &= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x(-1+x^4)}\right)dx\\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x(-1+x^4)}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 27, normalized size = 0.60

$$\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

fricas [A] time = 0.41, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="fricas")
[Out] arctan(x)
```

giac [A] time = 0.40, size = 31, normalized size = 0.69

$$\frac{1}{2}\left(\mathrm{sgn}(x)+1\right)\arctan(x)-\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x+1|\right)+\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="giac")
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

maple [A] time = 0.01, size = 35, normalized size = 0.78

$$\frac{\mathrm{arctanh}(x)}{2}+\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\,\left(2\arctan(x)+\ln(x-1)-\ln(x+1)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^6)^(1/2))/x/(-x^4+1),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x))/x^3+1/2*arctanh(x)+1/2*arctan
(x)
```

maxima [A] time = 0.96, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x-\sqrt{x^6}}{x\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)),x)
```

```
[Out] int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)), x)
```

sympy [A] time = 0.11, size = 2, normalized size = 0.04

$$\operatorname{atan}{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**6)**(1/2))/x/(-x**4+1),x)
```

```
[Out] atan(x)
```

3.818 $\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4} \, dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx &= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx\\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.60

$$\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

fricas [A] time = 0.41, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="fricas")
[Out] arctan(x)
```

giac [A] time = 0.39, size = 31, normalized size = 0.69

$$\frac{1}{2}\left(\mathrm{sgn}(x)+1\right)\arctan(x)-\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x+1|\right)+\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="giac")
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

maple [A] time = 0.01, size = 35, normalized size = 0.78

$$\frac{\operatorname{arctanh}(x)}{2}+\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\left(2\arctan(x)+\ln\left(x-1\right)-\ln\left(x+1\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(x^6)^(1/2)/x)/(-x^4+1),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1))/x^3+1/2*arctanh(x)+1/2*arctan
(x)
```

maxima [A] time = 0.97, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\frac{\sqrt{x^6}}{x} - 1}{x^4 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6)^(1/2)/x - 1)/(x^4 - 1),x)
```

```
[Out] int(((x^6)^(1/2)/x - 1)/(x^4 - 1), x)
```

sympy [A] time = 0.10, size = 2, normalized size = 0.04

$$\operatorname{atan}{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x**6)**(1/2)/x)/(-x**4+1),x)
```

```
[Out] atan(x)
```

3.819 $\int \frac{x-\sqrt{x^6}}{x-x^5} \, dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A] time = 0.10, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[x^6])/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx &= \int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx \\
&= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx \\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx \\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.60

$$\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[x^6])/(x - x^5),x]
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

fricas [A] time = 0.40, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="fricas")
[Out] arctan(x)
```

giac [A] time = 0.36, size = 31, normalized size = 0.69

$$\frac{1}{2}\left(\operatorname{sgn}(x)+1\right)\arctan(x)-\frac{1}{4}\left(\operatorname{sgn}(x)-1\right)\log\left(|x+1|\right)+\frac{1}{4}\left(\operatorname{sgn}(x)-1\right)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="giac")
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

maple [A] time = 0.01, size = 35, normalized size = 0.78

$$\frac{\operatorname{arctanh}(x)}{2}+\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\,\left(2\arctan(x)+\ln\left(x-1\right)-\ln\left(x+1\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^6)^(1/2))/(-x^5+x),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1))/x^3+1/2*arctanh(x)+1/2*arctan
(x)
```

maxima [A] time = 0.98, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x-\sqrt{x^6}}{x-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (x^6)^(1/2))/(x - x^5),x)
```

```
[Out] int((x - (x^6)^(1/2))/(x - x^5), x)
```

sympy [A] time = 0.10, size = 2, normalized size = 0.04

$$\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**6)**(1/2))/(-x**5+x),x)
```

```
[Out] atan(x)
```

3.820 $\int \frac{x}{x+\sqrt{x^6}}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A] time = 0.13, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6729, 1584, 6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x + Sqrt[x^6]),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6729

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{x+\sqrt{x^6}}\,dx &= \int \frac{x\left(x-\sqrt{x^6}\right)}{x^2-x^6}\,dx\\
&= \int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx\\
&= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx\\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 0.60

$$\frac{1}{2}\left(\frac{\sqrt{x^6}\left(\tan^{-1}(x)-\tanh^{-1}(x)\right)}{x^3}+\tan^{-1}(x)+\tanh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x + Sqrt[x^6]),x]
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

fricas [A] time = 0.40, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="fricas")
[Out] arctan(x)
```

giac [A] time = 0.37, size = 12, normalized size = 0.27

$$\frac{\arctan\left(x\sqrt{\operatorname{sgn}(x)}\right)}{\sqrt{\operatorname{sgn}(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="giac")
[Out] arctan(x*sqrt(sgn(x)))/sqrt(sgn(x))
```

maple [A] time = 0.01, size = 27, normalized size = 0.60

$$\frac{\arctan\left(\sqrt{\frac{\sqrt{x^6}}{x^3}}\,x\right)}{\sqrt{\frac{\sqrt{x^6}}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+(x^6)^(1/2)),x)
[Out] 1/((x^6)^(1/2)/x^3)^(1/2)*arctan(((x^6)^(1/2)/x^3)^(1/2)*x)
```

maxima [A] time = 0.96, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="maxima")
[Out] arctan(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{x+\sqrt{x^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + (x^6)^(1/2)),x)
[Out] int(x/(x + (x^6)^(1/2)), x)
```

sympy [A] time = 0.10, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x**6)**(1/2)),x)
[Out] atan(x)
```

3.821 $\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))+arctan(x^(1/2))*(x^3)^(1/2)/x^(3/2)-arctan
h(x^(1/2))*(x^3)^(1/2)/x^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx &= \int \frac{\sqrt{x}-\sqrt{x^3}}{x\left(1-x^2\right)}\,dx\\
&= \int \left(-\frac{1}{\sqrt{x}\left(-1+x^2\right)}+\frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\right) dx\\
&= -\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx+\int \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\,dx\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{x}\right)\right)+\frac{\sqrt{x^3}\int \frac{\sqrt{x}}{-1+x^2}\,dx}{x^{3/2}}\\
&= \frac{\left(2\sqrt{x^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,
\\
&= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,
\\
&= \tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 0.94

$$\frac{\left(x^{3/2}+\sqrt{x^3}\right)\tan^{-1}\left(\sqrt{x}\right)+\left(x^{3/2}-\sqrt{x^3}\right)\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

```
[Out] ((x^(3/2) + Sqrt[x^3])*ArcTan[Sqrt[x]] + (x^(3/2) - Sqrt[x^3])*ArcTanh[Sqrt
[x]])/x^(3/2)
```

fricas [A] time = 0.41, size = 6, normalized size = 0.12

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

giac [A] time = 0.32, size = 6, normalized size = 0.12

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

maple [A] time = 0.01, size = 41, normalized size = 0.79

$$\operatorname{arctanh}\left(\sqrt{x}\right)+\arctan\left(\sqrt{x}\right)+\frac{\sqrt{x^{3}}\,\left(2\arctan\left(\sqrt{x}\right)+\ln\left(\sqrt{x}-1\right)-\ln\left(\sqrt{x}+1\right)\right)}{2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x)
```

```
[Out] arctanh(x^(1/2))+arctan(x^(1/2))+1/2*(x^3)^(1/2)*(ln(x^(1/2)-1)-ln(x^(1/2)+
1)+2*arctan(x^(1/2)))/x^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\arctan\left(\sqrt{x}\right)-\int\frac{\sqrt{x}}{2\,(x+1)}\,dx+\int\frac{1}{4\left(\sqrt{x}+1\right)}\,dx+\int\frac{1}{4\left(\sqrt{x}-1\right)}\,dx+\frac{1}{2}\,\log\left(\sqrt{x}+1\right)-\frac{1}{2}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) - integrate(1/2*sqrt(x)/(x + 1), x) + integrate(1/4/(sqrt(x
) + 1), x) + integrate(1/4/(sqrt(x) - 1), x) + 1/2*log(sqrt(x) + 1) - 1/2*l
og(sqrt(x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int\frac{\sqrt{x^{3}}-\sqrt{x}}{x-x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3)^(1/2) - x^(1/2))/(x - x^3),x)
```

```
[Out] -int(((x^3)^(1/2) - x^(1/2))/(x - x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{x}}{x^{3}-x}\,dx-\int\left(-\frac{\sqrt{x^{3}}}{x^{3}-x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/2)-(x**3)**(1/2))/(-x**3+x),x)
```

```
[Out] -Integral(sqrt(x)/(x**3 - x), x) - Integral(-sqrt(x**3)/(x**3 - x), x)
```

3.822 $\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{x^3}\,\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\,\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))+arctan(x^(1/2))*(x^3)^(1/2)/x^(3/2)-arctan
h(x^(1/2))*(x^3)^(1/2)/x^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6729, 1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^3}\,\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\,\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6729

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx &= \int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx \\
&= \int \frac{\sqrt{x}-\sqrt{x^3}}{x\left(1-x^2\right)}\,dx \\
&= \int \left(-\frac{1}{\sqrt{x}\left(-1+x^2\right)}+\frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\right)dx \\
&= -\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx+\int \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\,dx \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{x}\right)\right)+\frac{\sqrt{x^3}\int \frac{\sqrt{x}}{-1+x^2}\,dx}{x^{3/2}} \\
&= \frac{\left(2\sqrt{x^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{}\right. \\
&= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right.}{x^{3/2}} \\
&= \tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.94

$$
\frac{\left(x^{3/2}+\sqrt{x^3}\right)\tan^{-1}\left(\sqrt{x}\right)+\left(x^{3/2}-\sqrt{x^3}\right)\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

```
[Out] ((x^(3/2) + Sqrt[x^3])*ArcTan[Sqrt[x]] + (x^(3/2) - Sqrt[x^3])*ArcTanh[Sqrt
[x]])/x^(3/2)
```

fricas [A] time = 0.41, size = 6, normalized size = 0.12

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

giac [A] time = 0.32, size = 6, normalized size = 0.12

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

maple [A] time = 0.01, size = 30, normalized size = 0.58

$$\frac{2 \arctan\left(\sqrt{\frac{\sqrt{x^3}}{x^{\frac{3}{2}}}}\, \sqrt{x}\right)}{\sqrt{\frac{\sqrt{x^3}}{x^{\frac{3}{2}}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+(x^3)^(1/2)),x)
```

```
[Out] 2/((x^3)^(1/2)/x^(3/2))^(1/2)*arctan(x^(1/2)*((x^3)^(1/2)/x^(3/2))^(1/2))
```

maxima [A] time = 0.99, size = 6, normalized size = 0.12

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^3} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3)^(1/2) + x^(1/2)),x)
```

```
[Out] int(1/((x^3)^(1/2) + x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x} + \sqrt{x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(x**3)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(x) + sqrt(x**3)), x)
```

3.823 $\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{(x-1)^3}\tan^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tan^{-1}\left(\sqrt{x-1}\right)-\frac{\sqrt{(x-1)^3}\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tanh^{-1}\left(\sqrt{x-1}\right)$$

```
[Out] arctan((-1+x)^(1/2))+arctanh((-1+x)^(1/2))+arctan((-1+x)^(1/2))*((-1+x)^3)^
(1/2)/(-1+x)^(3/2)-arctanh((-1+x)^(1/2))*((-1+x)^3)^(1/2)/(-1+x)^(3/2)
```

Rubi [A] time = 0.16, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {6729, 1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{(x-1)^3}\tan^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tan^{-1}\left(\sqrt{x-1}\right)-\frac{\sqrt{(x-1)^3}\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tanh^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[-1 + x]] + (Sqrt[(-1 + x)^3]*ArcTan[Sqrt[-1 + x]])/(-1 + x)^(3/
2) + ArcTanh[Sqrt[-1 + x]] - (Sqrt[(-1 + x)^3]*ArcTanh[Sqrt[-1 + x]])/(-1 +
 x)^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6729

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx,x,-1+x\right)\\
&= \text{Subst}\left(\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx,x,-1+x\right)\\
&= \text{Subst}\left(\int \frac{\sqrt{x}-\sqrt{x^3}}{x\left(1-x^2\right)}\,dx,x,-1+x\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{x}\left(-1+x^2\right)}+\frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\right)dx,x,-1+x\right)\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx,x,-1+x\right)+\text{Subst}\left(\int \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\,dx,x,-1+\right.\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{-1+x}\right)\right)+\frac{\sqrt{(-1+x)^3}\;\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx,x,\right.}{(-1+x)^{3/2}}\\
&= \frac{\left(2\sqrt{(-1+x)^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{-1+x}\right)}{(-1+x)^{3/2}}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{-1}\right.\\
&= \tan^{-1}\left(\sqrt{-1+x}\right)+\tanh^{-1}\left(\sqrt{-1+x}\right)-\frac{\sqrt{(-1+x)^3}\;\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{-}\right.}{(-1+x)^{3/2}}\\
&= \tan^{-1}\left(\sqrt{-1+x}\right)+\frac{\sqrt{(-1+x)^3}\;\tan^{-1}\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}}+\tanh^{-1}\left(\sqrt{-1+x}\right)-\sqrt{(\cdot}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 64, normalized size = 0.94

$$\left(\frac{\sqrt{(x-1)^3}}{(x-1)^{3/2}}+1\right)\tan^{-1}\left(\sqrt{x-1}\right)+\frac{\left((x-1)^{3/2}-\sqrt{(x-1)^3}\right)\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] (1 + Sqrt[(-1 + x)^3]/(-1 + x)^(3/2))*ArcTan[Sqrt[-1 + x]] + (((-1 + x)^(3/
2) - Sqrt[(-1 + x)^3])*ArcTanh[Sqrt[-1 + x]])/(-1 + x)^(3/2)
```

fricas [A] time = 0.42, size = 8, normalized size = 0.12

$$2 \arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x - 1))
```

giac [A] time = 0.41, size = 8, normalized size = 0.12

$$2 \arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x - 1))
```

maple [A] time = 0.01, size = 40, normalized size = 0.59

$$\frac{2 \arctan\left(\sqrt{\frac{\sqrt{(x-1)^{3}}}{(x-1)^{\frac{3}{2}}}}\,\sqrt{x-1}\right)}{\sqrt{\frac{\sqrt{(x-1)^{3}}}{(x-1)^{\frac{3}{2}}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x-1)^(1/2)+((x-1)^3)^(1/2)),x)
```

```
[Out] 2/(((x-1)^3)^(1/2)/(x-1)^(3/2))^(1/2)*arctan((((x-1)^3)^(1/2)/(x-1)^(3/2))^
(1/2)*(x-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\sqrt{x-1} - \int \frac{\sqrt{x-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x - 1) - integrate(sqrt(x - 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{x-1} - \sqrt{(x-1)^{3}}}{(x-1)^{3} - x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^(1/2) + ((x - 1)^3)^(1/2)),x)
```

```
[Out] int(-((x - 1)^(1/2) - ((x - 1)^3)^(1/2))/((x - 1)^3 - x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1} + \sqrt{(x-1)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+((-1+x)**3)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(x - 1) + sqrt((x - 1)**3)), x)
```

3.824 $\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right)dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {803}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && N
eQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g,
 0]
```

Rubi steps

$$\begin{aligned}\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right)dx &= \frac{3}{5(4+5x)}-\int \frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\,dx\\ &= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}\end{aligned}$$

Mathematica [A] time = 0.19, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

fricas [A] time = 0.42, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

giac [C] time = 0.44, size = 54, normalized size = 1.74

$$\frac{\sqrt{\frac{8}{5x+4}+\frac{9}{(5x+4)^2}-1}}{5\,\mathrm{sgn}\left(\frac{1}{5x+4}\right)}+\frac{3}{5\,(5\,x+4)}-\frac{1}{5}i\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4) - 1/5*I*sgn(1/(5*x + 4))
```

maple [A] time = 0.01, size = 32, normalized size = 1.03

$$\frac{\sqrt{\frac{8x}{5}-\left(x+\frac{4}{5}\right)^{2}+\frac{41}{25}}}{5x+4}+\frac{3}{5\,(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(4+5*x)
```

maxima [A] time = 0.96, size = 27, normalized size = 0.87

$$\frac{\sqrt{-x^2+1}}{5\,x+4}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1)/(5*x + 4) + 3/5/(5*x + 4)
```

mupad [B] time = 3.33, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(- 3/(5*x + 4)^2 - (4*x + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int\frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int\frac{}{25x^2\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)**2+(-5-4*x)/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

3.825 $\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A] time = 0.29, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {6742, 731, 725, 206, 807}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = \int \left(-\frac{3}{(4+5x)^2}-\frac{5}{(4+5x)^2\sqrt{1-x^2}}-\frac{4x}{(4+5x)^2\sqrt{1-x^2}}\right)dx$$
$$=\frac{3}{5(4+5x)}-4\int \frac{x}{(4+5x)^2\sqrt{1-x^2}}\,dx-5\int \frac{1}{(4+5x)^2\sqrt{1-x^2}}\,dx$$
$$=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

Mathematica [A] time = 0.15, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

fricas [A] time = 0.42, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

giac [C] time = 0.59, size = 54, normalized size = 1.74

$$\frac{\sqrt{\frac{8}{5\,x+4}+\frac{9}{(5\,x+4)^2}-1}}{5\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)}+\frac{3}{5\,(5\,x+4)}-\frac{1}{5}i\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4)
- 1/5*I*sgn(1/(5*x + 4))
```

maple [A] time = 0.01, size = 32, normalized size = 1.03

$$\frac{\sqrt{\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}}}{5x+4}+\frac{3}{5\,(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5-4*x-3*(-x^2+1)^(1/2))/(5*x+4)^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(8/5*x-(x+4/5)^2+41/25)^(1/2)+3/5/(5*x+4)
```

maxima [A] time = 0.61, size = 25, normalized size = 0.81

$$\frac{5\sqrt{x+1}\sqrt{-x+1}+3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/5*(5*sqrt(x + 1)*sqrt(-x + 1) + 3)/(5*x + 4)
```

mupad [B] time = 0.04, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x + 3*(1 - x^2)^(1/2) + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int \frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int \frac{}{25x^2\sqrt{1-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x**2+1)**(1/2))/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x
**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1
 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40
*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

3.826 $\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A] time = 0.14, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6742, 665, 216, 733, 844, 725, 206, 735}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
```

```
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3\left(1-x^2\right)}\,dx = \int \left(-\frac{3}{(4+5x)^2}+\frac{\sqrt{1-x^2}}{18(-1+x)}-\frac{\sqrt{1-x^2}}{2(1+x)}-\frac{5\sqrt{1-x^2}}{(4+5x)^2}+\frac{20\sqrt{1-x^2}}{9(4+5x)}\right) dx$$
$$= \frac{3}{5(4+5x)}+\frac{1}{18}\int \frac{\sqrt{1-x^2}}{-1+x}\,dx-\frac{1}{2}\int \frac{\sqrt{1-x^2}}{1+x}\,dx+\frac{20}{9}\int \frac{\sqrt{1-x^2}}{4+5x}\,dx-$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{1}{18}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{4}{9}\int \frac{5+4x}{(4+5x)\sqrt{1-x^2}}\,dx-$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{5}{9}\sin^{-1}(x)+\frac{1}{5}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{16}{45}\int \frac{1}{\sqrt{1-x^2}}$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

Mathematica [A] time = 0.09, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

fricas [A] time = 0.39, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

giac [B] time = 0.50, size = 68, normalized size = 2.19

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

maple [B] time = 0.04, size = 81, normalized size = 2.61

$$\frac{5\sqrt{\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}}\,x}{9}+\frac{3}{5\,(5x+4)}+\frac{\sqrt{-2x-(x-1)^2+2}}{18}-\frac{\sqrt{2x-(x+1)^2+2}}{2}+\frac{5\left(\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}\right)^{\frac{3}{2}}}{9\left(x+\frac{4}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x)
```

```
[Out] 3/5/(5*x+4)+1/18*(-(x-1)^2-2*x+2)^(1/2)-1/2*(-(x+1)^2+2*x+2)^(1/2)+5/9/(x+4
/5)*(8/5*x-(x+4/5)^2+41/25)^(3/2)+5/9*x*(8/5*x-(x+4/5)^2+41/25)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3\,x^2+\sqrt{-x^2+1}\,(4\,x+5)-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/(3*x^2 + sqrt(-x^2 + 1)*(4*x + 5) - 3), x)
```

mupad [B] time = 0.07, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((4*x + 5)*(1 - x^2)^(1/2) + 3*x^2 - 3),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2+4x\sqrt{1-x^2}+5\sqrt{1-x^2}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+3+(-5-4*x)*(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

3.827 $\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A] time = 0.13, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6742, 665, 216, 733, 844, 725, 206, 735}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
```

```
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx = \int \left(-\frac{3}{(4+5x)^2}+\frac{\sqrt{1-x^2}}{18(-1+x)}-\frac{\sqrt{1-x^2}}{2(1+x)}-\frac{5\sqrt{1-x^2}}{(4+5x)^2}+\frac{20\sqrt{1-x^2}}{9(4+5x)}\right.$$
$$= \frac{3}{5(4+5x)}+\frac{1}{18}\int \frac{\sqrt{1-x^2}}{-1+x}\,dx-\frac{1}{2}\int \frac{\sqrt{1-x^2}}{1+x}\,dx+\frac{20}{9}\int \frac{\sqrt{1-x^2}}{4+5x}$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{1}{18}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{4}{9}\int \frac{5+4x}{(4+5x)\sqrt{1-x^2}}$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{5}{9}\sin^{-1}(x)+\frac{1}{5}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{16}{45}\int \frac{1}{\sqrt{1-}}$$
$$= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

Mathematica [A] time = 0.06, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

fricas [A] time = 0.40, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="fri
cas")
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

giac [B] time = 0.37, size = 68, normalized size = 2.19

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="gia
c")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

maple [B] time = 0.04, size = 81, normalized size = 2.61

$$\frac{5\sqrt{\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}}\,x}{9}+\frac{3}{5\,(5x+4)}+\frac{\sqrt{-2x-(x-1)^2+2}}{18}-\frac{\sqrt{2x-(x+1)^2+2}}{2}+\frac{5\left(\frac{8x}{5}-\left(x+\frac{4}{5}\right)^2+\frac{41}{25}\right)^{\frac{3}{2}}}{9\left(x+\frac{4}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*(-x^2+1)^(1/2)*x),x)
```

```
[Out] 5/9*(8/5*x-(x+4/5)^2+41/25)^(1/2)*x+3/5/(5*x+4)+1/18*(-2*x-(x-1)^2+2)^(1/2)
-1/2*(2*x-(x+1)^2+2)^(1/2)+5/9/(x+4/5)*(8/5*x-(x+4/5)^2+41/25)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3\,x^2+4\,\sqrt{-x^2+1}\,x+5\,\sqrt{-x^2+1}\,-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="max
ima")
```

```
[Out] -integrate(1/(3*x^2 + 4*sqrt(-x^2 + 1)*x + 5*sqrt(-x^2 + 1) - 3), x)
```

mupad [B] time = 0.05, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(4*x*(1 - x^2)^(1/2) + 3*x^2 + 5*(1 - x^2)^(1/2) - 3),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2+4x\sqrt{1-x^2}+5\sqrt{1-x^2}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x**2-5*(-x**2+1)**(1/2)-4*x*(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

3.828 $\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2} dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)

Rubi [A] time = 0.65, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 13, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.302, Rules used = {6742, 277, 216, 266, 50, 63, 206, 733, 844, 725, 735, 264, 731}

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] Int[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}dx = \int \left(\frac{1}{\left(-2-x+2\sqrt{1-x^2}\right)^2} - \frac{1}{\sqrt{1-x^2}\left(-2-x+2\sqrt{1-x^2}\right)^2}\right)dx$$

$$= \int \frac{1}{\left(-2-x+2\sqrt{1-x^2}\right)^2}dx - \int \frac{1}{\sqrt{1-x^2}\left(-2-x+2\sqrt{1-x^2}\right)^2}dx$$

$$= -\int \left(\frac{1}{2x^2} - \frac{1}{x} + \frac{15}{2(4+5x)^2} + \frac{5}{4+5x} + \frac{1}{2x^2\sqrt{1-x^2}} - \frac{1}{x\sqrt{1-x^2}} + \frac{}{2(4}\right.$$

$$= \frac{3}{5(4+5x)} - \frac{1}{2}\int \frac{1}{x^2\sqrt{1-x^2}}dx + \frac{1}{2}\int \frac{\sqrt{1-x^2}}{x^2}dx - \frac{9}{2}\int \frac{1}{(4+5x)^2\sqrt{1}}$$

$$= \frac{3}{5(4+5x)} + \sqrt{1-x^2} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx + \frac{1}{2}\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2}\sin^{-1}(x) + \frac{5}{3}\tanh^{-1}\left(\frac{5+4x}{3\sqrt{1-x^2}}\right) - \frac{3}{10}\int \frac{}{\sqrt{1}}$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} + \tanh^{-1}\left(\frac{5+4x}{3\sqrt{1-x^2}}\right) - \tanh^{-1}\left(\sqrt{1-x^2}\right) - \frac{6}{5}\text{Su}$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x}$$

Mathematica [A] time = 0.18, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),
x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

fricas [A] time = 0.41, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

giac [B] time = 0.62, size = 68, normalized size = 2.19

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="giac")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

maple [A] time = 0.01, size = 32, normalized size = 1.03

$$\frac{\sqrt{\frac{8x}{5} - \left(x + \frac{4}{5}\right)^2 + \frac{41}{25}}}{5x + 4} + \frac{3}{5\,(5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(8/5*x-(x+4/5)^2+41/25)^(1/2)+3/5/(5*x+4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{56}\sqrt{7}\log\left(\frac{3\,x - 2\,\sqrt{7} - 2}{3\,x + 2\,\sqrt{7} - 2}\right) - \int -\frac{}{8\,(21\,x^9 + 278\,x^8 + 283\,x^7 - 2022\,x^6 - 3632\,x^5 + 2256\,x^4 + 7424\,x^3 + 1536\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="maxima")
```

```
[Out] -1/56*sqrt(7)*log((3*x - 2*sqrt(7) - 2)/(3*x + 2*sqrt(7) - 2)) - integrate(
-1/8*(100*x^7 + 285*x^6 + 264*x^5 + 80*x^4)/(21*x^9 + 278*x^8 + 283*x^7 - 2
022*x^6 - 3632*x^5 + 2256*x^4 + 7424*x^3 + 1536*x^2 - 8*(9*x^8 + 12*x^7 - 1
01*x^6 - 172*x^5 + 284*x^4 + 672*x^3 + 64*x^2 - 512*x - 256)*sqrt(x + 1)*sq
rt(-x + 1) - 4096*x - 2048), x) - 1/24*log(x + 2) + 1/16*log(x + 1) - 1/48*
log(x - 1)
```

mupad [B] time = 3.32, size = 19, normalized size = 0.61

$$\frac{\sqrt{1 - x^2} + \frac{3}{5}}{5\,x + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^2)^(1/2) - 1)/((1 - x^2)^(1/2)*(x - 2*(1 - x^2)^(1/2) + 2)^2),x
)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1 - x^2} - 1}{\sqrt{-(x - 1)\,(x + 1)}\,\left(x - 2\sqrt{1 - x^2} + 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x**2+1)**(1/2))/(2+x-2*(-x**2+1)**(1/2))**2/(-x**2+1)**(1/2
),x)
```

```
[Out] Integral((sqrt(1 - x**2) - 1)/(sqrt(-(x - 1)*(x + 1))*(x - 2*sqrt(1 - x**2)
 + 2)**2), x)
```

3.829 $\int \frac{a+bx^{-1+n}}{cx+dx^n} \, dx$

Optimal. Leaf size=43

$$\frac{b\log(x)}{d} - \frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{cd(1-n)}$$

[Out] b*ln(x)/d-(-a*d+b*c)*ln(d+c*x^(1-n))/c/d/(1-n)

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1593, 514, 446, 72}

$$\frac{b\log(x)}{d} - \frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{cd(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^(-1 + n))/(c*x + d*x^n),x]

[Out] (b*Log[x])/d - ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*d*(1 - n))

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx &= \int \frac{x^{-n}\left(a+bx^{-1+n}\right)}{d+cx^{1-n}}\,dx \\
&= \int \frac{b+ax^{1-n}}{x\left(d+cx^{1-n}\right)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{b+ax}{x(d+cx)}\,dx, x, x^{1-n}\right)}{1-n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{b}{dx}+\frac{-bc+ad}{d(d+cx)}\right)dx, x, x^{1-n}\right)}{1-n} \\
&= \frac{b\log(x)}{d} - \frac{(bc-ad)\log\left(d+cx^{1-n}\right)}{cd(1-n)}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.88

$$
\frac{\frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{c(n-1)}+b\log(x)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
[Out] (b*Log[x] + ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*(-1 + n)))/d
```

fricas [A] time = 0.42, size = 44, normalized size = 1.02

$$
\frac{(bc-ad)\log\left(cx+dx^n\right)+(adn-bc)\log(x)}{cdn-cd}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="fricas")
[Out] ((b*c - a*d)*log(c*x + d*x^n) + (a*d*n - b*c)*log(x))/(c*d*n - c*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{bx^{n-1}+a}{cx+dx^n}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="giac")
[Out] integrate((b*x^(n - 1) + a)/(c*x + d*x^n), x)
```

maple [A] time = 0.02, size = 73, normalized size = 1.70

$$
\frac{an\ln(x)}{(n-1)\,c} - \frac{a\ln\left(cx+d\,e^{n\ln(x)}\right)}{(n-1)\,c} - \frac{b\ln(x)}{(n-1)\,d} + \frac{b\ln\left(cx+d\,e^{n\ln(x)}\right)}{(n-1)\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(n-1))/(c*x+d*x^n),x)
[Out] 1/c/(n-1)*ln(x)*a*n-1/d/(n-1)*ln(x)*b-1/c/(n-1)*ln(c*x+d*exp(n*ln(x)))*a+1/
d/(n-1)*ln(c*x+d*exp(n*ln(x)))*b
```

maxima [B] time = 0.45, size = 85, normalized size = 1.98

$$b\left(\frac{\log(x)}{d}-\frac{n\log(x)}{d(n-1)}+\frac{\log\left(\frac{cx+dx^n}{d}\right)}{d(n-1)}\right)+a\left(\frac{n\log(x)}{c(n-1)}-\frac{\log\left(\frac{cx+dx^n}{d}\right)}{c(n-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="maxima")
```

```
[Out] b*(log(x)/d - n*log(x)/(d*(n - 1)) + log((c*x + d*x^n)/d)/(d*(n - 1))) + a*
(n*log(x)/(c*(n - 1)) - log((c*x + d*x^n)/d)/(c*(n - 1)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,x^{n-1}}{d\,x^n+c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(n - 1))/(d*x^n + c*x),x)
```

```
[Out] int((a + b*x^(n - 1))/(d*x^n + c*x), x)
```

sympy [A] time = 10.18, size = 212, normalized size = 4.93

$$\begin{cases} \tilde{\infty}\,(a+b)\log(x) & \text{for}\ c=0\wedge d=0\wedge n=1 \\ \frac{-\frac{anx}{n^2x^n-nx^n}+\frac{bn^2x^n\log(x)}{n^2x^n-nx^n}-\frac{bnx^n\log(x)}{n^2x^n-nx^n}-\frac{bnx^n}{n^2x^n-nx^n}}{d} & \text{for}\ c=0 \\ \frac{\frac{anx\log(x)}{nx-x}-\frac{ax\log(x)}{nx-x}+\frac{bx^n}{nx-x}}{c} & \text{for}\ d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for}\ n=1 \\ \frac{adn\log(x)}{cdn-cd}-\frac{ad\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd}-\frac{bc\log(x)}{cdn-cd}+\frac{bc\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(-1+n))/(c*x+d*x**n),x)
```

```
[Out] Piecewise((zoo*(a + b)*log(x), Eq(c, 0) & Eq(d, 0) & Eq(n, 1)), ((-a*n*x/(n
**2*x**n - n*x**n) + b*n**2*x**n*log(x)/(n**2*x**n - n*x**n) - b*n*x**n*log
(x)/(n**2*x**n - n*x**n) - b*n*x**n/(n**2*x**n - n*x**n))/d, Eq(c, 0)), ((a
*n*x*log(x)/(n*x - x) - a*x*log(x)/(n*x - x) + b*x**n/(n*x - x))/c, Eq(d, 0
)), ((a + b)*log(x)/(c + d), Eq(n, 1)), (a*d*n*log(x)/(c*d*n - c*d) - a*d*l
og(x + d*x**n/c)/(c*d*n - c*d) - b*c*log(x)/(c*d*n - c*d) + b*c*log(x + d*x
**n/c)/(c*d*n - c*d), True))
```

3.830 $\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] -1/2/x+x-1/2*arcsinh(x*2^(1/2))*2^(1/2)+1/2*(2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.13, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6740, 6742, 277, 215}

$$\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

```
[Out] -1/(2*x) + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx &= \int \left(1+\frac{1}{-1-\sqrt{1+2x^2}}\right)dx\\ &= x+\int \frac{1}{-1-\sqrt{1+2x^2}}\,dx\\ &= x+\int \left(\frac{1}{2x^2}-\frac{\sqrt{1+2x^2}}{2x^2}\right)dx\\ &= -\frac{1}{2x}+x-\frac{1}{2}\int \frac{\sqrt{1+2x^2}}{x^2}\,dx\\ &= -\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\int \frac{1}{\sqrt{1+2x^2}}\,dx\\ &= -\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

```
[Out] -1/2*1/x + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

fricas [A] time = 0.41, size = 44, normalized size = 1.05

$$\frac{\sqrt{2}\,x\log\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)+2\,x^2+\sqrt{2\,x^2+1}-1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*x*log(sqrt(2)*x - sqrt(2*x^2 + 1)) + 2*x^2 + sqrt(2*x^2 + 1) -
 1)/x
```

giac [A] time = 0.46, size = 57, normalized size = 1.36

$$\frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}\,x+\sqrt{2\,x^2+1}\right)+x-\frac{\sqrt{2}}{\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2-1}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + x - sqrt(2)/((sqrt(2)*x - s
qrt(2*x^2 + 1))^2 - 1) - 1/2/x
```

maple [A] time = 0.01, size = 45, normalized size = 1.07

$$x-\sqrt{2x^2+1}\,x-\frac{\sqrt{2}\,\operatorname{arcsinh}\left(\sqrt{2}\,x\right)}{2}-\frac{1}{2x}+\frac{\left(2x^2+1\right)^{\frac{3}{2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x)
```

```
[Out] x-1/2/x+1/2/x*(2*x^2+1)^(3/2)-x*(2*x^2+1)^(1/2)-1/2*arcsinh(2^(1/2)*x)*2^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x - \int \frac{1}{\sqrt{2\,x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x - integrate(1/(sqrt(2*x^2 + 1) + 1), x)
```

mupad [B] time = 3.39, size = 31, normalized size = 0.74

$$x - \frac{\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)}{2} + \frac{\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2} - \frac{1}{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)^(1/2)/((2*x^2 + 1)^(1/2) + 1),x)
```

```
[Out] x - (2^(1/2)*asinh(2^(1/2)*x))/2 + ((2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1/2)/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{2x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)**(1/2)/(1+(2*x**2+1)**(1/2)),x)
```

```
[Out] Integral(sqrt(2*x**2 + 1)/(sqrt(2*x**2 + 1) + 1), x)
```

3.831 $\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{3}\sqrt{4x^2-1}+\frac{\tanh^{-1}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}}+\frac{4x}{3}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

```
[Out] 4/3*x-1/9*arctanh(x*3^(1/2))*3^(1/2)+1/9*arctanh(3^(1/2)*(4*x^2-1)^(1/2))*3
^(1/2)-1/3*(4*x^2-1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6742, 444, 50, 63, 207, 388}

$$-\frac{1}{3}\sqrt{4x^2-1}+\frac{\tanh^{-1}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}}+\frac{4x}{3}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (4*x)/3 - Sqrt[-1 + 4*x^2]/3 - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[Sqr
t[3]*Sqrt[-1 + 4*x^2]]/(3*Sqrt[3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx &= \int \left(-\frac{x\sqrt{-1+4x^2}}{-1+3x^2}+\frac{-1+4x^2}{-1+3x^2}\right)dx\\
&= -\int \frac{x\sqrt{-1+4x^2}}{-1+3x^2}\,dx+\int \frac{-1+4x^2}{-1+3x^2}\,dx\\
&= \frac{4x}{3}+\frac{1}{3}\int \frac{1}{-1+3x^2}\,dx-\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+4x}}{-1+3x}\,dx,x,x^2\right)\\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{(-1+3x)\sqrt{-1+4x}}\,dx,x,x^2\right)\\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{1}{12}\text{Subst}\left(\int \frac{1}{-\frac{1}{4}+\frac{3x^2}{4}}\,dx,x,\sqrt{-1+4x^2}\right)\\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}+\frac{\tanh^{-1}\left(\sqrt{3}\,\sqrt{-1+4x^2}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.83

$$\frac{1}{9}\left(-3\sqrt{4x^2-1}+\sqrt{3}\tanh^{-1}\left(\sqrt{12x^2-3}\right)+12x-\sqrt{3}\tanh^{-1}\left(\sqrt{3}\,x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (12*x - 3*Sqrt[-1 + 4*x^2] - Sqrt[3]*ArcTanh[Sqrt[3]*x] + Sqrt[3]*ArcTanh[S
qrt[-3 + 12*x^2]])/9
```

fricas [A] time = 0.41, size = 80, normalized size = 1.23

$$\frac{1}{18}\sqrt{3}\log\left(\frac{6x^2+\sqrt{3}\sqrt{4x^2-1}-1}{3x^2-1}\right)+\frac{1}{18}\sqrt{3}\log\left(\frac{3x^2-2\sqrt{3}x+1}{3x^2-1}\right)+\frac{4}{3}x-\frac{1}{3}\sqrt{4x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*log((6*x^2 + sqrt(3)*sqrt(4*x^2 - 1) - 1)/(3*x^2 - 1)) + 1/18*
sqrt(3)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + 4/3*x - 1/3*sqrt(4*x^2
 - 1)
```

giac [B] time = 0.57, size = 133, normalized size = 2.05

$$\frac{1}{18}\sqrt{3}\log\left(\frac{\left|6x-2\sqrt{3}\right|}{\left|6x+2\sqrt{3}\right|}\right)-\frac{1}{18}\sqrt{3}\log\left(-\frac{\left|-12x-4\sqrt{3}+6\sqrt{4x^2-1}+\frac{6}{2x-\sqrt{4x^2-1}}\right|}{2\left(6x-2\sqrt{3}-3\sqrt{4x^2-1}-\frac{3}{2x-\sqrt{4x^2-1}}\right)}\right)+\frac{4}{3}x-\frac{1}{3}\sqrt{4x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*
log(-1/2*abs(-12*x - 4*sqrt(3) + 6*sqrt(4*x^2 - 1) + 6/(2*x - sqrt(4*x^2 -
1)))/(6*x - 2*sqrt(3) - 3*sqrt(4*x^2 - 1) - 3/(2*x - sqrt(4*x^2 - 1)))) + 4
/3*x - 1/3*sqrt(4*x^2 - 1)
```

maple [B] time = 0.05, size = 262, normalized size = 4.03

$$\frac{4x}{3}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\sqrt{3}\,x\right)}{9}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{3\left(\frac{2}{3}+\frac{8\left(x-\frac{\sqrt{3}}{3}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2\sqrt{36\left(x-\frac{\sqrt{3}}{3}\right)^{2}+24\left(x-\frac{\sqrt{3}}{3}\right)\sqrt{3}+3}}\right)}{18}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{3\left(\frac{2}{3}-\frac{8\left(x+\frac{\sqrt{3}}{3}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2\sqrt{36\left(x+\frac{\sqrt{3}}{3}\right)^{2}-24\left(x+\frac{\sqrt{3}}{3}\right)\sqrt{3}}}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x)
```

```
[Out] 4/3*x-1/9*arctanh(3^(1/2)*x)*3^(1/2)-1/18*(36*(x-1/3*3^(1/2))^2+24*(x-1/3*3
^(1/2))*3^(1/2)+3)^(1/2)-1/18*3^(1/2)*ln(x*4^(1/2)+(4*(x-1/3*3^(1/2))^2+8/3
*(x-1/3*3^(1/2))*3^(1/2)+1/3)^(1/2))*4^(1/2)+1/18*3^(1/2)*arctanh(3/2*(2/3+
8/3*(x-1/3*3^(1/2))*3^(1/2))*3^(1/2)/(36*(x-1/3*3^(1/2))^2+24*(x-1/3*3^(1/2
))*3^(1/2)+3)^(1/2))-1/18*(36*(x+1/3*3^(1/2))^2-24*(x+1/3*3^(1/2))*3^(1/2)+
3)^(1/2)+1/18*3^(1/2)*ln(x*4^(1/2)+(4*(x+1/3*3^(1/2))^2-8/3*(x+1/3*3^(1/2))
*3^(1/2)+1/3)^(1/2))*4^(1/2)+1/18*3^(1/2)*arctanh(3/2*(2/3-8/3*(x+1/3*3^(1/
2))*3^(1/2))*3^(1/2)/(36*(x+1/3*3^(1/2))^2-24*(x+1/3*3^(1/2))*3^(1/2)+3)^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x-\int\frac{x}{\sqrt{2\,x+1}\,\sqrt{2\,x-1}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x - integrate(x/(sqrt(2*x + 1)*sqrt(2*x - 1) + x), x)
```

mupad [B] time = 3.44, size = 60, normalized size = 0.92

$$\frac{4\,x}{3}+\frac{\sqrt{3}\,\ln\left(x-\frac{\sqrt{3}}{3}\right)}{18}-\frac{\sqrt{3}\,\ln\left(x+\frac{\sqrt{3}}{3}\right)}{18}+\frac{\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\,\sqrt{4\,x^2-1}\right)}{9}-\frac{\sqrt{4\,x^2-1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 1)^(1/2)/(x + (4*x^2 - 1)^(1/2)),x)
```

```
[Out] (4*x)/3 + (3^(1/2)*log(x - 3^(1/2)/3))/18 - (3^(1/2)*log(x + 3^(1/2)/3))/18
 + (3^(1/2)*atanh(3^(1/2)*(4*x^2 - 1)^(1/2)))/9 - (4*x^2 - 1)^(1/2)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(2x-1)\,(2x+1)}}{x+\sqrt{4x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-1)**(1/2)/(x+(4*x**2-1)**(1/2)),x)
```

```
[Out] Integral(sqrt((2*x - 1)*(2*x + 1))/(x + sqrt(4*x**2 - 1)), x)
```

3.832 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=195

$$-\frac{\sqrt{x^2-1}\left(ae^2-bde+cd^2\right)}{2e\left(d^2-e^2\right)(d+ex)^2}-\frac{\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)}{2\left(d^2-e^2\right)^{5/2}}+\frac{\sqrt{x^2-1}\left(c\left(d^3-4de^2\right.\right.}{2e\left(d^2-\right.}$$

```
[Out] -1/2*(3*b*d*e-a*(2*d^2+e^2)-c*(d^2+2*e^2))*arctanh((d*x+e)/(d^2-e^2)^(1/2)/
(x^2-1)^(1/2))/(d^2-e^2)^(5/2)-1/2*(a*e^2-b*d*e+c*d^2)*(x^2-1)^(1/2)/e/(d^2
-e^2)/(e*x+d)^2+1/2*(c*(d^3-4*d*e^2)-e*(3*a*d*e-b*(d^2+2*e^2)))*(x^2-1)^(1/
2)/e/(d^2-e^2)^2/(e*x+d)
```

Rubi [A] time = 0.21, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1651, 807, 725, 206}

$$-\frac{\sqrt{x^2-1}\left(ae^2-bde+cd^2\right)}{2e\left(d^2-e^2\right)(d+ex)^2}+\frac{\sqrt{x^2-1}\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)}{2e\left(d^2-e^2\right)^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+\right.\right.}{2\left(d^2-e^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] -((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x^2])/(2*e*(d^2 - e^2)*(d + e*x)^2) + (
(c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*Sqrt[-1 + x^2])/(2*e*(d
^2 - e^2)^2*(d + e*x)) - ((3*b*d*e - a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*Arc
Tanh[(e + d*x)/(Sqrt[d^2 - e^2]*Sqrt[-1 + x^2])])/(2*(d^2 - e^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} - \frac{\int \frac{-2(ad+cd-be)-\left(bd+\frac{cd^2}{e}-ae-2ce\right)x}{(d+ex)^2\sqrt{-1+x^2}}\,dx}{2\left(d^2-e^2\right)}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} + \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)\sqrt{-1+x}}{2e\left(d^2-e^2\right)^2(d+ex)}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} + \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)\sqrt{-1+x}}{2e\left(d^2-e^2\right)^2(d+ex)}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2} + \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)\sqrt{-1+x}}{2e\left(d^2-e^2\right)^2(d+ex)}$$

Mathematica [A] time = 0.39, size = 240, normalized size = 1.23

$$\frac{1}{2}\left(-\frac{\log\left(-\sqrt{x^2-1}\sqrt{d^2-e^2}+dx+e\right)\left(a\left(2d^2+e^2\right)-3bde+c\left(d^2+2e^2\right)\right)}{(d-e)^2(d+e)^2\sqrt{d^2-e^2}} + \frac{\log(d+ex)\left(a\left(2d^2+e^2\right)-3bde\right.}{(d-e)^2(d+e)^2\sqrt{d^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] ((Sqrt[-1 + x^2]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d*e + d^2*x - 4*e^
2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/((d^2 - e^2)^2*(d + e*x)^2)
+ ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*Log[d + e*x])/((d - e)^2*
(d + e)^2*Sqrt[d^2 - e^2]) - ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2)
)*Log[e + d*x - Sqrt[d^2 - e^2]*Sqrt[-1 + x^2]])/((d - e)^2*(d + e)^2*Sqrt[
d^2 - e^2]))/2
```

fricas [B] time = 0.46, size = 1174, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*e
^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5
 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a
 + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 + 2*(
(2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(d^2 - e^2)*log((
d^2*x + d*e + sqrt(d^2 - e^2)*(d*x + e) + (d^2 - e^2 + sqrt(d^2 - e^2)*d)*s
qrt(x^2 - 1))/(e*x + d)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b
*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 - (4*a + 3*c)*
d^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 +
b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*
x)*sqrt(x^2 - 1))/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^2*e^8 + (d^6*e^4 - 3
*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e
```

```
^9)*x), 1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c
)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*
d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 - 2*((2*a + c)*d^4*e^2 - 3*b*d^3
*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*
x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(-d^2 +
e^2)*arctan(-(sqrt(-d^2 + e^2)*sqrt(x^2 - 1)*e - sqrt(-d^2 + e^2)*(e*x + d)
)/(d^2 - e^2)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b*d^3*e^4 +
 (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 - (4*a + 3*c)*d^4*e^3 -
b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b*d^4*e^3
- (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x)*sqrt(x^
2 - 1))/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^2*e^8 + (d^6*e^4 - 3*d^4*e^6 +
 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e^9)*x)]
```

giac [B] time = 0.49, size = 536, normalized size = 2.75

$$\frac{\left(2\,ad^2+cd^2-3\,bde+ae^2+2\,ce^2\right)\arctan\left(-\frac{\left(x-\sqrt{x^2-1}\right)e+d}{\sqrt{-d^2+e^2}}\right)}{\left(d^4-2\,d^2e^2+e^4\right)\sqrt{-d^2+e^2}}+\frac{2\,cd^4\left(x-\sqrt{x^2-1}\right)^3e+2\,cd^5\left(x-\sqrt{x^2-1}\right)^2+2\,bd^4(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(-((x - sqrt(x^2 - 1))*
e + d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(-d^2 + e^2)) + (2*c*
d^4*(x - sqrt(x^2 - 1))^3*e + 2*c*d^5*(x - sqrt(x^2 - 1))^2 + 2*b*d^4*(x -
sqrt(x^2 - 1))^2*e - 2*a*d^2*(x - sqrt(x^2 - 1))^3*e^3 - 5*c*d^2*(x - sqrt(
x^2 - 1))^3*e^3 - 6*a*d^3*(x - sqrt(x^2 - 1))^2*e^2 - 7*c*d^3*(x - sqrt(x^2
 - 1))^2*e^2 + 2*c*d^4*(x - sqrt(x^2 - 1))*e + 3*b*d*(x - sqrt(x^2 - 1))^3*
e^4 + 5*b*d^2*(x - sqrt(x^2 - 1))^2*e^3 + 4*b*d^3*(x - sqrt(x^2 - 1))*e^2 -
 a*(x - sqrt(x^2 - 1))^3*e^5 - 3*a*d*(x - sqrt(x^2 - 1))^2*e^4 - 4*c*d*(x -
 sqrt(x^2 - 1))^2*e^4 - 10*a*d^2*(x - sqrt(x^2 - 1))*e^3 - 11*c*d^2*(x - sq
rt(x^2 - 1))*e^3 + c*d^3*e^2 + 2*b*(x - sqrt(x^2 - 1))^2*e^5 + 5*b*d*(x - s
qrt(x^2 - 1))*e^4 + b*d^2*e^3 + a*(x - sqrt(x^2 - 1))*e^5 - 3*a*d*e^4 - 4*c
*d*e^4 + 2*b*e^5)/((d^4*e^2 - 2*d^2*e^4 + e^6)*((x - sqrt(x^2 - 1))^2*e + 2
*d*(x - sqrt(x^2 - 1)) + e)^2)
```

maple [B] time = 0.03, size = 1407, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x)
```

```
[Out] -1/2/e/(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*a+
1/2/e^2/(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*b
*d-1/2/e^3/(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2
)*c*d^2-3/2*d/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(
1/2)*a+3/2/e*d^2/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2
)^(1/2)*b-3/2/e^2*d^3/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2
)/e^2)^(1/2)*c-3/2/e*d^2/(d^2-e^2)^2/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/
e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2
)/e^2)^(1/2))/(x+d/e))*a+3/2/e^2*d^3/(d^2-e^2)^2/((d^2-e^2)/e^2)^(1/2)*ln((
2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d
/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*b-3/2/e^3*d^4/(d^2-e^2)^2/((d^2-e^2)/e^2
)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^
2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*c+1/2/e/(d^2-e^2)/((d^2-e^2)
/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d
```

```
/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*a-3/2/e^2/(d^2-e^2)/((d^
2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)
*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*b*d+5/2/e^3/(d^2-e
^2)/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^
2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*c*d^2-1/e/
(d^2-e^2)/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*b+2/e^2/(d^
2-e^2)/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*d*c-c/e^3/((d^
2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)
*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-d>0)', see `assume?` for more det
ails)Is e-d positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^2+b\,x+a}{\sqrt{x^2-1}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{\sqrt{(x-1)(x+1)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(x**2-1)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt((x - 1)*(x + 1))*(d + e*x)**3), x)
```

3.833 $\int \frac{1+2x^8}{x(1+x^8)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

```
[Out] -1/4*arctanh((x^8+1)^(1/2))-1/4/(x^8+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 78, 63, 207}

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
```

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = \frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{x(1+x)^{3/2}}\,dx, x, x^8\right)$$
$$= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right)$$
$$= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right)$$
$$= -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)$$

Mathematica [**A**] time = 0.02, size = 28, normalized size = 1.00

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

fricas [**B**] time = 0.41, size = 52, normalized size = 1.86

$$-\frac{\left(x^8+1\right)\log\left(\sqrt{x^8+1}+1\right) - \left(x^8+1\right)\log\left(\sqrt{x^8+1}-1\right) + 2\sqrt{x^8+1}}{8\left(x^8+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="fricas")
[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) +
 2*sqrt(x^8 + 1))/(x^8 + 1)
```

giac [**A**] time = 0.38, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="giac")
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

maple [**A**] time = 0.03, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{\sqrt{x^8}}\right)}{4} - \frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8+1)/x/(x^8+1)^(3/2),x)
[Out] -1/4/(x^8+1)^(1/2)+1/4*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))
```

maxima [A] time = 0.96, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

mupad [B] time = 3.85, size = 20, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\sqrt{x^8+1}\right)}{4}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8 + 1)/(x*(x^8 + 1)^(3/2)),x)
```

```
[Out] - atanh((x^8 + 1)^(1/2))/4 - 1/(4*(x^8 + 1)^(1/2))
```

sympy [A] time = 22.80, size = 37, normalized size = 1.32

$$\frac{\log\left(\sqrt{x^8+1}-1\right)}{8}-\frac{\log\left(\sqrt{x^8+1}+1\right)}{8}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8+1)/x/(x**8+1)**(3/2),x)
```

```
[Out] log(sqrt(x**8 + 1) - 1)/8 - log(sqrt(x**8 + 1) + 1)/8 - 1/(4*sqrt(x**8 + 1)
)
```

3.834 $\int \frac{\sqrt{1+x^8}\left(1+2x^8\right)}{x+2x^9+x^{17}}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

```
[Out] -1/4*arctanh((x^8+1)^(1/2))-1/4/(x^8+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1586, 1593, 446, 78, 63, 207}

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
```

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^8}\left(1+2x^8\right)}{x+2x^9+x^{17}}\,dx &= \int \frac{1+2x^8}{\sqrt{1+x^8}\left(x+x^9\right)}\,dx \\
&= \int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx \\
&= \frac{1}{8}\,\text{Subst}\left(\int \frac{1+2x}{x(1+x)^{3/2}}\,dx, x, x^8\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

fricas [B] time = 0.42, size = 52, normalized size = 1.86

$$-\frac{\left(x^8+1\right)\log\left(\sqrt{x^8+1}+1\right) - \left(x^8+1\right)\log\left(\sqrt{x^8+1}-1\right) + 2\sqrt{x^8+1}}{8\left(x^8+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="fricas")
[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) +
 2*sqrt(x^8 + 1))/(x^8 + 1)
```

giac [A] time = 0.50, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="giac")
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

maple [A] time = 0.03, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{\sqrt{x^8}}\right)}{4}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x)
[Out] 1/4*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))-1/4/(x^8+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^8+1\right)\sqrt{x^8+1}}{x^{17}+2\,x^9+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="maxima")
[Out] integrate((2*x^8 + 1)*sqrt(x^8 + 1)/(x^17 + 2*x^9 + x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x^8+1}\,\left(2\,x^8+1\right)}{x^{17}+2\,x^9+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17),x)
[Out] int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^8+1}{x\left(x^8+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8+1)*(x**8+1)**(1/2)/(x**17+2*x**9+x),x)
[Out] Integral((2*x**8 + 1)/(x*(x**8 + 1)**(3/2)), x)
```

3.835 $\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right) dx$

Optimal. Leaf size=22

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {261}

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Int[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right) dx &= x-3x^3+\int \frac{x}{\sqrt{1-9x^2}}\, dx\\ &= x-3x^3-\frac{1}{9}\sqrt{1-9x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

fricas [A] time = 0.41, size = 18, normalized size = 0.82

$$-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

giac [A] time = 0.34, size = 18, normalized size = 0.82

$$-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$-3x^3+x-\frac{\sqrt{-9x^2+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-9*x^2+x/(-9*x^2+1)^(1/2),x)
```

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

maxima [A] time = 0.58, size = 18, normalized size = 0.82

$$-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.82

$$x-3\,x^3-\frac{\sqrt{\frac{1}{9}-x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - 9*x^2)^(1/2) - 9*x^2 + 1,x)
```

```
[Out] x - 3*x^3 - (1/9 - x^2)^(1/2)/3
```

sympy [A] time = 0.15, size = 17, normalized size = 0.77

$$-3x^3+x-\frac{\sqrt{1-9x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x**2+x/(-9*x**2+1)**(1/2),x)
```

```
[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9
```

3.836 $\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx$

Optimal. Leaf size=22

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6742, 261}

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Int[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx &= \int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)dx\\
&= x-3x^3+\int \frac{x}{\sqrt{1-9x^2}}\,dx\\
&= x-3x^3-\frac{1}{9}\sqrt{1-9x^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

fricas [A] time = 0.40, size = 18, normalized size = 0.82

$$-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

giac [A] time = 0.57, size = 18, normalized size = 0.82

$$-3x^3 + x - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="giac")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$-3x^3 + x - \frac{\sqrt{-9x^2 + 1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x)
[Out] -3*x^3+x-1/9*(-9*x^2+1)^(1/2)
```

maxima [A] time = 0.82, size = 18, normalized size = 0.82

$$-3x^3 + x - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

mupad [B] time = 0.03, size = 18, normalized size = 0.82

$$x - 3x^3 - \frac{\sqrt{\frac{1}{9} - x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (1 - 9*x^2)^(3/2))/(1 - 9*x^2)^(1/2),x)
[Out] x - 3*x^3 - (1/9 - x^2)^(1/2)/3
```

sympy [A] time = 1.17, size = 17, normalized size = 0.77

$$-3x^3 + x - \frac{\sqrt{1 - 9x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x**2+1)**(3/2))/(-9*x**2+1)**(1/2),x)
[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9
```

3.837 $\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx$

Optimal. Leaf size=17

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

```
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

Rubi [A] time = 0.06, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2034, 629}

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx, x, \sqrt{x}\right)\\ &= \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

fricas [A] time = 0.46, size = 11, normalized size = 0.65

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

giac [A] time = 0.37, size = 11, normalized size = 0.65

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

maple [A] time = 0.01, size = 12, normalized size = 0.71

$$\frac{6\left(x-3\sqrt{x}\right)^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x)
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-3\sqrt{x}\right)^{\frac{2}{3}}\left(2\sqrt{x}-3\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] integrate((x - 3*sqrt(x))^(2/3)*(2*sqrt(x) - 3)/sqrt(x), x)
```

mupad [B] time = 3.70, size = 11, normalized size = 0.65

$$\frac{6\left(x-3\sqrt{x}\right)^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 3*x^(1/2))^(2/3)*(2*x^(1/2) - 3))/x^(1/2),x)
[Out] (6*(x - 3*x^(1/2))^(5/3))/5
```

sympy [B] time = 1.27, size = 36, normalized size = 2.12

$$-\frac{18\sqrt{x}\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}+\frac{6x\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x**(1/2))**(2/3)*(-3+2*x**(1/2))/x**(1/2),x)
[Out] -18*sqrt(x)*(-3*sqrt(x) + x)**(2/3)/5 + 6*x*(-3*sqrt(x) + x)**(2/3)/5
```

3.838 $\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx$

Optimal. Leaf size=17

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

```
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2043, 1631, 629}

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1631

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rule 2043

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{d =
 Denominator[n]}, Dist[d, Subst[Int[x^(d - 1)*(SubstFor[x^n, Pq, x] /. x ->
 x^(d*n))*(a*x^(d*j) + b*x^(d*n))^p, x], x, x^(1/d)], x]] /; FreeQ[{a, b, j
, n, p}, x] && PolyQ[Pq, x^n] &&  !IntegerQ[p] && NeQ[n, j] && RationalQ[j,
 n] && IntegerQ[j/n] && LtQ[-1, n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\left(9-9x+2x^2\right)}{\sqrt[3]{-3x+x^2}}\,dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx,x,\sqrt{x}\right)\\
&= \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 17, normalized size = 1.00

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

fricas [A] time = 0.46, size = 11, normalized size = 0.65

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="fricas")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

giac [A] time = 0.33, size = 11, normalized size = 0.65

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="giac")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

maple [C] time = 0.11, size = 125, normalized size = 7.35

$$\frac{4\,3^{\frac{2}{3}}\left(-\mathrm{signum}\left(\frac{\sqrt{x}}{3}-1\right)\right)^{\frac{1}{3}} x^{\frac{11}{6}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{11}{3}\right],\left[\frac{14}{3}\right],\frac{\sqrt{x}}{3}\right)}{11\mathrm{signum}\left(\frac{\sqrt{x}}{3}-1\right)^{\frac{1}{3}}}-\frac{9\,3^{\frac{2}{3}}\left(-\mathrm{signum}\left(\frac{\sqrt{x}}{3}-1\right)\right)^{\frac{1}{3}} x^{\frac{4}{3}}\,\mathrm{hypergeom}}{4\mathrm{signum}\left(\frac{\sqrt{x}}{3}-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x)
[Out] 18/5*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x
^(5/6)*hypergeom([1/3,5/3],[8/3],1/3*x^(1/2))+4/11*3^(2/3)/signum(-1+1/3*x^
(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(11/6)*hypergeom([1/3,11/3],
[14/3],1/3*x^(1/2))-9/4*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/
3*x^(1/2)))^(1/3)*x^(4/3)*hypergeom([1/3,8/3],[11/3],1/3*x^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-9\,\sqrt{x}+9}{\left(x-3\,\sqrt{x}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="maxima")
[Out] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{2\,x-9\,\sqrt{x}+9}{{\left(x-3\,\sqrt{x}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3),x)
```

```
[Out] int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-9\sqrt{x} + 2x + 9}{\sqrt[3]{-3\sqrt{x} + x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x**(1/2))/(x-3*x**(1/2))**(1/3),x)
```

```
[Out] Integral((-9*sqrt(x) + 2*x + 9)/(-3*sqrt(x) + x)**(1/3), x)
```

3.839 $\int \frac{1}{\sqrt{4-9x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {216}

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 - 9*x^2],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 - 9*x^2],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

fricas [B] time = 0.39, size = 19, normalized size = 1.90

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9x^2+4}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

giac [A] time = 0.36, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arcsin(3/2*x)
```

maple [**A**] time = 0.00, size = 7, normalized size = 0.70

$$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-9*x^2+4)^(1/2),x)
```

```
[Out] 1/3*arcsin(3/2*x)
```

maxima [**A**] time = 1.96, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

mupad [**B**] time = 0.01, size = 6, normalized size = 0.60

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4 - 9*x^2)^(1/2),x)
```

```
[Out] asin((3*x)/2)/3
```

sympy [**A**] time = 0.15, size = 7, normalized size = 0.70

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x**2+4)**(1/2),x)
```

```
[Out] asin(3*x/2)/3
```

3.840 $\int \frac{1}{\sqrt{2-3x}\,\sqrt{2+3x}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {41, 216}

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-3x}\,\sqrt{2+3x}}\,dx &= \int \frac{1}{\sqrt{4-9x^2}}\,dx\\ &= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

fricas [B] time = 0.45, size = 25, normalized size = 2.50

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{3\,x+2}\,\sqrt{-3\,x+2}-2}{3\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(3*x + 2)*sqrt(-3*x + 2) - 2)/x)
```

giac [A] time = 0.37, size = 12, normalized size = 1.20

$$\frac{2}{3}\arcsin\left(\frac{1}{2}\sqrt{3x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arcsin(1/2*sqrt(3*x + 2))
```

maple [B] time = 0.01, size = 34, normalized size = 3.40

$$\frac{\sqrt{(-3x+2)(3x+2)}\ \arcsin\left(\frac{3x}{2}\right)}{3\sqrt{-3x+2}\ \sqrt{3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*x)^(1/2)/(3*x+2)^(1/2),x)
```

```
[Out] 1/3*((2-3*x)*(3*x+2))^(1/2)/(2-3*x)^(1/2)/(3*x+2)^(1/2)*arcsin(3/2*x)
```

maxima [A] time = 1.93, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

mupad [B] time = 0.15, size = 32, normalized size = 3.20

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{2}-\sqrt{2-3x}}{\sqrt{2}-\sqrt{3x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(3*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((2^(1/2) - (2 - 3*x)^(1/2))/(2^(1/2) - (3*x + 2)^(1/2))))/3
```

sympy [B] time = 1.04, size = 51, normalized size = 5.10

$$\begin{cases} -\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{for}\ \frac{3\left|x+\frac{2}{3}\right|}{4} > 1 \\ \frac{2\,\mathrm{asin}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(3)*sqrt(x + 2/3)/2)/3, 3*Abs(x + 2/3)/4 > 1), (2
*asin(sqrt(3)*sqrt(x + 2/3)/2)/3, True))
```

3.841 $\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1972, 216}

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx &= \int \frac{1}{\sqrt{4-9x^2}}\,dx\\ &= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

fricas [B] time = 0.42, size = 19, normalized size = 1.90

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9\,x^2+4}-2}{3\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="fricas")
```

[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)

giac [A] time = 0.39, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="giac")

[Out] 1/3*arcsin(3/2*x)

maple [A] time = 0.01, size = 7, normalized size = 0.70

$$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-3*x+2)*(3*x+2))^(1/2),x)

[Out] 1/3*arcsin(3/2*x)

maxima [A] time = 1.96, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="maxima")

[Out] 1/3*arcsin(3/2*x)

mupad [B] time = 0.01, size = 6, normalized size = 0.60

$$\frac{\mathrm{asin}\left(\frac{3\,x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-(3*x - 2)*(3*x + 2))^(1/2),x)

[Out] asin((3*x)/2)/3

sympy [A] time = 1.39, size = 7, normalized size = 0.70

$$\frac{\mathrm{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2-3*x)*(2+3*x))**(1/2),x)

[Out] asin(3*x/2)/3

3.842 $\int \frac{1}{\sqrt{15-2x-x^2}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

```
[Out] arcsin(1/4+1/4*x)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[15 - 2*x - x^2],x]
[Out] -ArcSin[(-1 - x)/4]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{15-2x-x^2}}\,dx &= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)\\ &= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[15 - 2*x - x^2],x]
[Out] -ArcSin[(-1 - x)/4]
```

fricas [B] time = 0.44, size = 29, normalized size = 2.42

$$-\arctan\left(\frac{\sqrt{-x^2-2\,x+15}\,(x+1)}{x^2+2\,x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

giac [A] time = 0.42, size = 6, normalized size = 0.50

$$\arcsin\left(\frac{1}{4}\,x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="giac")
[Out] arcsin(1/4*x + 1/4)
```

maple [A] time = 0.00, size = 7, normalized size = 0.58

$$\arcsin\left(\frac{x}{4} + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-2*x+15)^(1/2),x)
[Out] arcsin(1/4+1/4*x)
```

maxima [A] time = 1.97, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{1}{4}\,x - \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/4*x - 1/4)
```

mupad [B] time = 3.12, size = 6, normalized size = 0.50

$$\mathrm{asin}\left(\frac{x}{4} + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15 - x^2 - 2*x)^(1/2),x)
[Out] asin(x/4 + 1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 - 2x + 15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-2*x+15)**(1/2),x)
[Out] Integral(1/sqrt(-x**2 - 2*x + 15), x)
```

3.843 $\int \frac{1}{\sqrt{3-x}\,\sqrt{5+x}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

```
[Out] arcsin(1/4+1/4*x)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {53, 619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-x}\,\sqrt{5+x}}\,dx &= \int \frac{1}{\sqrt{15-2x-x^2}}\,dx\\
&= -\left(\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)\\
&= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.75

$$-2\sin^{-1}\left(\frac{\sqrt{3-x}}{2\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
```

```
[Out] -2*ArcSin[Sqrt[3 - x]/(2*Sqrt[2])]
```

fricas [B] time = 0.41, size = 29, normalized size = 2.42

$$-\arctan\left(\frac{\sqrt{x+5}\,(x+1)\sqrt{-x+3}}{x^2+2\,x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(x + 5)*(x + 1)*sqrt(-x + 3)/(x^2 + 2*x - 15))
```

giac [B] time = 0.33, size = 13, normalized size = 1.08

$$2\,\arcsin\left(\frac{1}{4}\,\sqrt{2}\,\sqrt{x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/4*sqrt(2)*sqrt(x + 5))
```

maple [B] time = 0.01, size = 31, normalized size = 2.58

$$\frac{\sqrt{(-x+3)\,(x+5)}\;\arcsin\left(\frac{x}{4}+\frac{1}{4}\right)}{\sqrt{-x+3}\,\sqrt{x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+3)^(1/2)/(x+5)^(1/2),x)
[Out] ((-x+3)*(x+5))^(1/2)/(-x+3)^(1/2)/(x+5)^(1/2)*arcsin(1/4*x+1/4)
```

maxima [A] time = 1.96, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{1}{4}\,x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/4*x - 1/4)
```

mupad [B] time = 3.43, size = 30, normalized size = 2.50

$$4\,\mathrm{atan}\left(\frac{\sqrt{3}-\sqrt{3-x}}{\sqrt{x+5}-\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - x)^(1/2)*(x + 5)^(1/2)),x)
[Out] 4*atan((3^(1/2) - (3 - x)^(1/2))/((x + 5)^(1/2) - 5^(1/2)))
```

sympy [B] time = 1.02, size = 41, normalized size = 3.42

$$\begin{cases} -2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+5}}{4}\right) & \text{for}\ \frac{|x+5|}{8}>1 \\ 2\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+5}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(1/2)/(5+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/4), Abs(x + 5)/8 > 1), (2*asin(sq
rt(2)*sqrt(x + 5)/4), True))
```

3.844 $\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

```
[Out] arcsin(1/4+1/4*x)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1981, 619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(3 - x)*(5 + x)],x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] && !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx &= \int \frac{1}{\sqrt{15-2x-x^2}}\,dx\\
&= -\left(\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)\\
&= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.75

$$-2\sin^{-1}\left(\frac{\sqrt{3-x}}{2\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(3 - x)*(5 + x)],x]
```

```
[Out] -2*ArcSin[Sqrt[3 - x]/(2*Sqrt[2])]
```

fricas [B] time = 0.41, size = 29, normalized size = 2.42

$$-\arctan\left(\frac{\sqrt{-x^2-2\,x+15}\,(x+1)}{x^2+2\,x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

giac [A] time = 0.35, size = 6, normalized size = 0.50

$$\arcsin\left(\frac{1}{4}\,x+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="giac")
[Out] arcsin(1/4*x + 1/4)
```

maple [A] time = 0.01, size = 7, normalized size = 0.58

$$\arcsin\left(\frac{x}{4}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-x+3)*(x+5))^(1/2),x)
[Out] arcsin(1/4*x+1/4)
```

maxima [A] time = 1.97, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{1}{4}\,x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/4*x - 1/4)
```

mupad [B] time = 3.37, size = 6, normalized size = 0.50

$$\mathrm{asin}\left(\frac{x}{4}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x - 3)*(x + 5))^(1/2),x)
[Out] asin(x/4 + 1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(3-x)\,(x+5)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))**(1/2),x)
[Out] Integral(1/sqrt((3 - x)*(x + 5)), x)
```

3.845 $\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

```
[Out] arcsin(4+x)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-15 - 8*x - x^2],x]
```

```
[Out] ArcSin[4 + x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)\\ &= \sin^{-1}(4+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-15 - 8*x - x^2],x]
```

```
[Out] ArcSin[4 + x]
```

fricas [B] time = 0.41, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{-x^2-8x-15}\,(x+4)}{x^2+8x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

giac [A] time = 0.34, size = 4, normalized size = 1.00

$$\arcsin(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="giac")
[Out] arcsin(x + 4)
```

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$\arcsin(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-8*x-15)^(1/2),x)
[Out] arcsin(x+4)
```

maxima [A] time = 1.99, size = 8, normalized size = 2.00

$$-\arcsin(-x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-x - 4)
```

mupad [B] time = 3.18, size = 4, normalized size = 1.00

$$\operatorname{asin}(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 8*x - x^2 - 15)^(1/2),x)
[Out] asin(x + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2-8x-15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-8*x-15)**(1/2),x)
[Out] Integral(1/sqrt(-x**2 - 8*x - 15), x)
```

3.846 $\int \frac{1}{\sqrt{-3-x}\,\sqrt{5+x}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

```
[Out] arcsin(4+x)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {53, 619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
```

```
[Out] ArcSin[4 + x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3-x}\,\sqrt{5+x}}\,dx &= \int \frac{1}{\sqrt{-15-8x-x^2}}\,dx\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx,x,-8-2x\right)\right)\\ &= \sin^{-1}(4+x)\end{aligned}$$

Mathematica [B] time = 0.01, size = 18, normalized size = 4.50

$$-2\sin^{-1}\left(\frac{\sqrt{-x-3}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
```

```
[Out] -2*ArcSin[Sqrt[-3 - x]/Sqrt[2]]
```

fricas [B] time = 0.41, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{x+5}\,(x+4)\sqrt{-x-3}}{x^2+8\,x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(x + 5)*(x + 4)*sqrt(-x - 3)/(x^2 + 8*x + 15))
```

giac [B] time = 0.32, size = 13, normalized size = 3.25

$$2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(x + 5))
```

maple [B] time = 0.01, size = 29, normalized size = 7.25

$$\frac{\sqrt{(-x-3)\,(x+5)}\;\arcsin\left(x+4\right)}{\sqrt{-x-3}\;\sqrt{x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-x)^(1/2)/(x+5)^(1/2),x)
[Out] ((-3-x)*(x+5))^(1/2)/(-3-x)^(1/2)/(x+5)^(1/2)*arcsin(x+4)
```

maxima [A] time = 1.97, size = 8, normalized size = 2.00

$$-\arcsin\left(-x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-x - 4)
```

mupad [B] time = 0.08, size = 33, normalized size = 8.25

$$4\,\mathrm{atan}\left(\frac{-\sqrt{-x-3}+\sqrt{3}\,1\mathrm{i}}{\sqrt{x+5}-\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x - 3)^(1/2)*(x + 5)^(1/2)),x)
[Out] 4*atan((3^(1/2)*1i - (- x - 3)^(1/2))/((x + 5)^(1/2) - 5^(1/2)))
```

sympy [B] time = 1.02, size = 41, normalized size = 10.25

$$\begin{cases} -2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+5}}{2}\right) & \text{for}\ \frac{|x+5|}{2}>1 \\ 2\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+5}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)**(1/2)/(5+x)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/2), Abs(x + 5)/2 > 1), (2*asin(sq
rt(2)*sqrt(x + 5)/2), True))
```

3.847 $\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

```
[Out] arcsin(4+x)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1981, 619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-3 - x)*(5 + x)],x]
```

```
[Out] ArcSin[4 + x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] && !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx &= \int \frac{1}{\sqrt{-15-8x-x^2}}\,dx\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)\\
&= \sin^{-1}(4+x)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 18, normalized size = 4.50

$$-2\sin^{-1}\left(\frac{\sqrt{-x-3}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-3 - x)*(5 + x)],x]
```

```
[Out] -2*ArcSin[Sqrt[-3 - x]/Sqrt[2]]
```

fricas [B] time = 0.42, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{-x^2-8x-15}\,(x+4)}{x^2+8x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

giac [A] time = 0.39, size = 4, normalized size = 1.00

$$\arcsin(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="giac")
[Out] arcsin(x + 4)
```

maple [A] time = 0.01, size = 5, normalized size = 1.25

$$\arcsin(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-x-3)*(x+5))^(1/2),x)
[Out] arcsin(x+4)
```

maxima [A] time = 1.95, size = 8, normalized size = 2.00

$$-\arcsin(-x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="maxima")
[Out] -arcsin(-x - 4)
```

mupad [B] time = 3.36, size = 4, normalized size = 1.00

$$\mathrm{asin}(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x + 3)*(x + 5))^(1/2),x)
[Out] asin(x + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(-x-3)(x+5)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))**(1/2),x)
[Out] Integral(1/sqrt((-x - 3)*(x + 5)), x)
```

3.848 $\int \left(1-\sqrt{x}\right) dx$

Optimal. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

```
[Out] x-2/3*x^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1 - Sqrt[x],x]
```

```
[Out] x - (2*x^(3/2))/3
```

Rubi steps

$$\int \left(1-\sqrt{x}\right) dx = x-\frac{2x^{3/2}}{3}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - Sqrt[x],x]
```

```
[Out] x - (2*x^(3/2))/3
```

fricas [A] time = 0.41, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*x^(3/2) + x
```

giac [A] time = 0.36, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*x^(3/2) + x
```

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-x^(1/2),x)
```

```
[Out] x-2/3*x^(3/2)
```

maxima [A] time = 0.90, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*x^(3/2) + x
```

mupad [B] time = 0.03, size = 7, normalized size = 0.64

$$x - \frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - x^(1/2),x)
```

```
[Out] x - (2*x^(3/2))/3
```

sympy [A] time = 0.06, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/2),x)
```

```
[Out] -2*x**(3/2)/3 + x
```

3.849 $\int \frac{1-x}{1+\sqrt{x}}\,dx$

Optimal. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

```
[Out] x-2/3*x^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1398, 26, 43}

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 + Sqrt[x]),x]
```

```
[Out] x - (2*x^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x}{1+\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\left(1-x^2\right)}{1+x}\,dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int (1-x)x\,dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(x-x^2\right)\,dx,x,\sqrt{x}\right)\\
&= x-\frac{2x^{3/2}}{3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 + Sqrt[x]),x]
[Out] x - (2*x^(3/2))/3
```

fricas [A] time = 0.40, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="fricas")
[Out] -2/3*x^(3/2) + x
```

giac [A] time = 0.33, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="giac")
[Out] -2/3*x^(3/2) + x
```

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)/(x^(1/2)+1),x)
[Out] -2/3*x^(3/2)+x
```

maxima [A] time = 0.91, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="maxima")
[Out] -2/3*x^(3/2) + x
```

mupad [B] time = 0.03, size = 7, normalized size = 0.64

$$x - \frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x^(1/2) + 1),x)
[Out] x - (2*x^(3/2))/3
```

sympy [A] time = 0.15, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x**(1/2)),x)
```

```
[Out] -2*x**(3/2)/3 + x
```

3.850 $\int \sqrt{\frac{1}{1-x^2}}\,dx$

Optimal. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

[Out] arcsin(x)*(1/(-x^2+1))^(1/2)*(-x^2+1)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6720, 216}

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 - x^2)^(-1)],x]

[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1}{1-x^2}}\,dx &= \left(\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(1 - x^2)^(-1)],x]

[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]

fricas [A] time = 0.45, size = 26, normalized size = 0.96

$$2\arctan\left(\frac{(x^2-1)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

giac [A] time = 0.42, size = 10, normalized size = 0.37

$$-\arcsin(x)\operatorname{sgn}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(x)*sgn(x^2 - 1)
```

maple [A] time = 0.01, size = 30, normalized size = 1.11

$$\sqrt{-\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(-x^2+1))^(1/2),x)
```

```
[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\frac{1}{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-1/(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{-\frac{1}{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/(x^2 - 1))^(1/2),x)
```

```
[Out] int((-1/(x^2 - 1))^(1/2), x)
```

sympy [A] time = 1.01, size = 7, normalized size = 0.26

$$\begin{cases}\operatorname{asin}{\left(x\right)} & \text{for}\: x > -1 \wedge x < 1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x**2+1))**(1/2),x)
```

```
[Out] Piecewise((asin(x), (x > -1) & (x < 1)))
```

3.851 $\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx$

Optimal. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

```
[Out] arcsin(x)*(1/(-x^2+1))^(1/2)*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6688, 6720, 216}

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x^2)/(1 - x^4)],x]
```

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx &= \int \sqrt{\frac{1}{1-x^2}}\,dx\\ &= \left(\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x^2)/(1 - x^4)],x]
```

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

fricas [A] time = 0.41, size = 26, normalized size = 0.96

$$2\arctan\left(\frac{\left(x^2-1\right)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="fricas")
[Out] 2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

giac [A] time = 0.34, size = 10, normalized size = 0.37

$$-\arcsin(x)\mathrm{sgn}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="giac")
[Out] -arcsin(x)*sgn(x^2 - 1)
```

maple [A] time = 0.01, size = 30, normalized size = 1.11

$$\sqrt{-\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2+1)/(-x^4+1))^(1/2),x)
[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-(x^2 + 1)/(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{-\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x^2 + 1)/(x^4 - 1))^(1/2),x)
[Out] int((-(x^2 + 1)/(x^4 - 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x^2+1}{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)/(-x**4+1))**(1/2),x)
[Out] Integral(sqrt((x**2 + 1)/(1 - x**4)), x)
```

3.852 $\int \sqrt{\frac{1}{-1+x^2}} \, dx$

Optimal. Leaf size=25

$$\sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\sin^{-1}(x)$$

```
[Out] arcsin(x)*(-x^2+1)^(1/2)*(1/(x^2-1))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6720, 217, 206}

$$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x^2)^(-1)],x]
```

```
[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1}{-1+x^2}}\,dx &= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx\\
&= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)\\
&= \sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)
\end{aligned}$$

Mathematica [B] time = 0.03, size = 56, normalized size = 2.24

$$\frac{1}{2}\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^(-1)],x]
```

```
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 +
 x/Sqrt[-1 + x^2]]))/2
```

fricas [A] time = 0.41, size = 14, normalized size = 0.56

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1))
```

giac [A] time = 0.36, size = 15, normalized size = 0.60

$$-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 - 1)))
```

maple [A] time = 0.01, size = 28, normalized size = 1.12

$$\sqrt{\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x^2-1))^(1/2),x)
```

```
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

maxima [A] time = 0.88, size = 14, normalized size = 0.56

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{\frac{1}{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x^2 - 1))^(1/2),x)
```

```
[Out] int((1/(x^2 - 1))^(1/2), x)
```

sympy [A] time = 1.52, size = 15, normalized size = 0.60

$$\begin{cases}\log\left(x+\sqrt{x^2-1}\right) & \text{for}\ x>-1\wedge x<1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x**2-1))**(1/2),x)
```

```
[Out] Piecewise((log(x + sqrt(x**2 - 1)), (x > -1) & (x < 1)))
```

3.853 $\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx$

Optimal. Leaf size=25

$$\sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\,\sin^{-1}(x)$$

```
[Out] arcsin(x)*(-x^2+1)^(1/2)*(1/(x^2-1))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6688, 6720, 217, 206}

$$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\,\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x^2)/(-1 + x^4)],x]
```

```
[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx &= \int \sqrt{\frac{1}{-1+x^2}}\,dx\\
&= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx\\
&= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)\\
&= \sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)
\end{aligned}$$

Mathematica [B]   time = 0.00, size = 56, normalized size = 2.24

$$\frac{1}{2}\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x^2)/(-1 + x^4)],x]
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 +
 x/Sqrt[-1 + x^2]]))/2
```

fricas [A]   time = 0.41, size = 14, normalized size = 0.56

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 - 1))
```

giac [A]   time = 0.32, size = 21, normalized size = 0.84

$$-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sgn}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="giac")
[Out] -log(abs(-x + sqrt(x^2 - 1)))*sgn(x^2 - 1)
```

maple [A]   time = 0.01, size = 28, normalized size = 1.12

$$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2+1)/(x^4-1))^(1/2),x)
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((x^2 + 1)/(x^4 - 1)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)/(x^4 - 1))^(1/2),x)
```

```
[Out] int(((x^2 + 1)/(x^4 - 1))^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)/(x**4-1))**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)/(x**4 - 1)), x)
```

3.854 $\int \frac{1}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

```
[Out] -2*(1-x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 - x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 - x]
```

fricas [A] time = 0.40, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-x + 1)
```

giac [A] time = 0.33, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-x + 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(1/2),x)
[Out] -2*(-x+1)^(1/2)
```

maxima [A] time = 0.89, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-x + 1)
```

mupad [B] time = 0.19, size = 9, normalized size = 0.82

$$-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x)^(1/2),x)
[Out] -2*(1 - x)^(1/2)
```

sympy [A] time = 0.06, size = 8, normalized size = 0.73

$$-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2),x)
[Out] -2*sqrt(1 - x)
```

3.855 $\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

```
[Out] -2*(1-x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {26, 32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/Sqrt[1 - x^2],x]
[Out] -2*Sqrt[1 - x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx &= \int \frac{1}{\sqrt{1-x}}\,dx\\ &= -2\sqrt{1-x}\end{aligned}$$

Mathematica [B] time = 0.02, size = 23, normalized size = 2.09

$$\frac{2(x-1)\sqrt{x+1}}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x^2],x]
[Out] (2*(-1 + x)*Sqrt[1 + x])/Sqrt[1 - x^2]
```

fricas [C] time = 0.41, size = 16, normalized size = 1.45

$$-\frac{2\sqrt{-x^2+1}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-x^2 + 1)/sqrt(x + 1)
```

giac [A] time = 0.32, size = 15, normalized size = 1.36

$$2\sqrt{2} - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2) - 2*sqrt(-x + 1)
```

maple [B] time = 0.00, size = 20, normalized size = 1.82

$$\frac{2(x-1)\sqrt{x+1}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x^2+1)^(1/2),x)
[Out] 2*(x-1)*(x+1)^(1/2)/(-x^2+1)^(1/2)
```

maxima [C] time = 0.92, size = 12, normalized size = 1.09

$$\frac{2(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 2*(x - 1)/sqrt(-x + 1)
```

mupad [B] time = 3.56, size = 16, normalized size = 1.45

$$-\frac{2\sqrt{1-x^2}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x^2)^(1/2),x)
[Out] -(2*(1 - x^2)^(1/2))/(x + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Integral(sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

3.856 $\int \frac{1}{\sqrt{1+x}} \, dx$

Optimal. Leaf size=9

$$2\sqrt{x+1}$$

[Out] 2*(1+x)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + x],x]

[Out] 2*Sqrt[1 + x]

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x}} \, dx = 2\sqrt{1+x}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 + x],x]

[Out] 2*Sqrt[1 + x]

fricas [A] time = 0.40, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(x + 1)

giac [A] time = 0.34, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(1/2),x, algorithm="giac")

[Out] 2*sqrt(x + 1)

maple [**A**] time = 0.00, size = 8, normalized size = 0.89

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)^(1/2),x)
```

```
[Out] 2*(x+1)^(1/2)
```

maxima [**A**] time = 0.88, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1)
```

mupad [**B**] time = 0.09, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + 1)^(1/2),x)
```

```
[Out] 2*(x + 1)^(1/2)
```

sympy [**A**] time = 0.06, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2),x)
```

```
[Out] 2*sqrt(x + 1)
```

3.857 $\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=9

$$2\sqrt{x+1}$$

```
[Out] 2*(1+x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {26, 32}

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/Sqrt[1 - x^2],x]
[Out] 2*Sqrt[1 + x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx &= \int \frac{1}{\sqrt{1+x}}\,dx \\ &= 2\sqrt{1+x} \end{aligned}$$

Mathematica [B] time = 0.02, size = 25, normalized size = 2.78

$$\frac{2\sqrt{1-x}\,(x+1)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/Sqrt[1 - x^2],x]
[Out] (2*Sqrt[1 - x]*(1 + x))/Sqrt[1 - x^2]
```

fricas [C] time = 0.41, size = 23, normalized size = 2.56

$$-\frac{2\,\sqrt{-x^2+1}\,\sqrt{-x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

giac [A] time = 0.43, size = 13, normalized size = 1.44

$$-2\sqrt{2} + 2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(2) + 2*sqrt(x + 1)
```

maple [B] time = 0.00, size = 22, normalized size = 2.44

$$\frac{2(x+1)\sqrt{-x+1}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(-x^2+1)^(1/2),x)
[Out] 2*(x+1)*(-x+1)^(1/2)/(-x^2+1)^(1/2)
```

maxima [A] time = 0.90, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x + 1)
```

mupad [B] time = 3.64, size = 18, normalized size = 2.00

$$\frac{2\sqrt{1-x^2}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(1 - x^2)^(1/2),x)
[Out] (2*(1 - x^2)^(1/2))/(1 - x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Integral(sqrt(1 - x)/sqrt(-(x - 1)*(x + 1)), x)
```

3.858 $\int \sqrt{1-x}\,dx$

Optimal. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

```
[Out] -2/3*(1-x)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x],x]
[Out] (-2*(1 - x)^(3/2))/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{1-x}\,dx=-\frac{2}{3}(1-x)^{3/2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x],x]
[Out] (-2*(1 - x)^(3/2))/3
```

fricas [A] time = 0.40, size = 12, normalized size = 0.92

$$\frac{2}{3}\,(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

giac [A] time = 0.35, size = 9, normalized size = 0.69

$$-\frac{2}{3}\,(-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(-x + 1)^(3/2)
```

maple [A]   time = 0.00, size = 10, normalized size = 0.77

$$-\frac{2\left(-x+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2),x)
[Out] -2/3*(-x+1)^(3/2)
```

maxima [A]   time = 0.81, size = 9, normalized size = 0.69

$$-\frac{2}{3}\left(-x+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="maxima")
[Out] -2/3*(-x + 1)^(3/2)
```

mupad [B]   time = 3.51, size = 9, normalized size = 0.69

$$-\frac{2\left(1-x\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2),x)
[Out] -(2*(1 - x)^(3/2))/3
```

sympy [A]   time = 0.06, size = 10, normalized size = 0.77

$$-\frac{2\left(1-x\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2),x)
[Out] -2*(1 - x)**(3/2)/3
```

3.859 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

```
[Out] -2/3*(1-x)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {26, 32}

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 + x],x]
[Out] (-2*(1 - x)^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx &= \int \sqrt{1-x}\,dx \\ &= -\frac{2}{3}(1-x)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.92

$$\frac{2(x-1)\sqrt{1-x^2}}{3\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 + x],x]
[Out] (2*(-1 + x)*Sqrt[1 - x^2])/(3*Sqrt[1 + x])
```

fricas [B] time = 0.41, size = 19, normalized size = 1.46

$$\frac{2\sqrt{-x^2+1}\,(x-1)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-x^2 + 1)*(x - 1)/sqrt(x + 1)
```

giac [A] time = 0.33, size = 15, normalized size = 1.15

$$-\frac{2}{3}(-x+1)^{\frac{3}{2}}+\frac{4}{3}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -2/3*(-x + 1)^(3/2) + 4/3*sqrt(2)
```

maple [B] time = 0.00, size = 20, normalized size = 1.54

$$\frac{2(x-1)\sqrt{-x^2+1}}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(x+1)^(1/2),x)
[Out] 2/3*(x-1)*(-x^2+1)^(1/2)/(x+1)^(1/2)
```

maxima [A] time = 0.83, size = 12, normalized size = 0.92

$$\frac{2}{3}(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

mupad [B] time = 3.52, size = 20, normalized size = 1.54

$$\frac{\left(\frac{2x}{3}-\frac{2}{3}\right)\sqrt{1-x^2}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x + 1)^(1/2),x)
[Out] (((2*x)/3 - 2/3)*(1 - x^2)^(1/2))/(x + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1+x)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x + 1), x)
```

3.860 $\int \sqrt{1+x}\, dx$

Optimal. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

```
[Out] 2/3*(1+x)^(3/2)
```

Rubi **[A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{1+x}\, dx = \frac{2}{3}(1+x)^{3/2}$$

Mathematica **[A]** time = 0.00, size = 11, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

fricas **[A]** time = 0.40, size = 7, normalized size = 0.64

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

giac **[A]** time = 0.32, size = 7, normalized size = 0.64

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$\frac{2 (x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2),x)
[Out] 2/3*(x+1)^(3/2)
```

maxima [A] time = 0.88, size = 7, normalized size = 0.64

$$\frac{2}{3} (x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x + 1)^(3/2)
```

mupad [B] time = 3.42, size = 7, normalized size = 0.64

$$\frac{2 (x+1)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2),x)
[Out] (2*(x + 1)^(3/2))/3
```

sympy [A] time = 0.06, size = 8, normalized size = 0.73

$$\frac{2 (x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2),x)
[Out] 2*(x + 1)**(3/2)/3
```

3.861 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

```
[Out] 2/3*(1+x)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {26, 32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x],x]
[Out] (2*(1 + x)^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx &= \int \sqrt{1+x}\,dx\\ &= \frac{2}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [B] time = 0.02, size = 27, normalized size = 2.45

$$\frac{2(x+1)\sqrt{1-x^2}}{3\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x],x]
[Out] (2*(1 + x)*Sqrt[1 - x^2])/(3*Sqrt[1 - x])
```

fricas [B] time = 0.41, size = 26, normalized size = 2.36

$$-\frac{2\sqrt{-x^2+1}\,(x+1)\sqrt{-x+1}}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(-x^2 + 1)*(x + 1)*sqrt(-x + 1)/(x - 1)
```

giac [A] time = 0.33, size = 13, normalized size = 1.18

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{4}{3}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
[Out] 2/3*(x + 1)^(3/2) - 4/3*sqrt(2)
```

maple [B] time = 0.00, size = 22, normalized size = 2.00

$$\frac{2(x+1)\sqrt{-x^2+1}}{3\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(-x+1)^(1/2),x)
[Out] 2/3*(x+1)*(-x^2+1)^(1/2)/(-x+1)^(1/2)
```

maxima [A] time = 0.92, size = 7, normalized size = 0.64

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x + 1)^(3/2)
```

mupad [B] time = 3.49, size = 22, normalized size = 2.00

$$\frac{\left(\frac{2x}{3} + \frac{2}{3}\right)\sqrt{1-x^2}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(1 - x)^(1/2),x)
[Out] (((2*x)/3 + 2/3)*(1 - x^2)^(1/2))/(1 - x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1-x)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(1 - x), x)
```

3.862 $\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x+1}\,\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arcsinh((2+3*x)^(1/2))*3^(1/2)+(1+x)^(1/2)*(2+3*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 54, 215}

$$\sqrt{x+1}\,\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx &= \sqrt{1+x}\,\sqrt{2+3x}-\frac{1}{2}\int \frac{1}{\sqrt{1+x}\,\sqrt{2+3x}}\,dx\\ &= \sqrt{1+x}\,\sqrt{2+3x}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{2+3x}\right)}{\sqrt{3}}\\ &= \sqrt{1+x}\,\sqrt{2+3x}-\frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.40

$$\frac{3\sqrt{x+1}\,(3x+2)-\sqrt{9x+6}\,\sinh^{-1}\left(\sqrt{3x+2}\right)}{3\sqrt{3x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

```
[Out] (3*Sqrt[1 + x]*(2 + 3*x) - Sqrt[6 + 9*x]*ArcSinh[Sqrt[2 + 3*x]])/(3*Sqrt[2
+ 3*x])
```

fricas [A] time = 0.42, size = 52, normalized size = 1.49

$$\frac{1}{12}\sqrt{3}\log\left(-4\sqrt{3}(6x+5)\sqrt{3x+2}\sqrt{x+1}+72x^2+120x+49\right)+\sqrt{3x+2}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-4*sqrt(3)*(6*x + 5)*sqrt(3*x + 2)*sqrt(x + 1) + 72*x^2 +
120*x + 49) + sqrt(3*x + 2)*sqrt(x + 1)
```

giac [A] time = 0.35, size = 39, normalized size = 1.11

$$\frac{1}{3}\sqrt{3}\left(\sqrt{3x+3}\sqrt{3x+2}+\log\left(\sqrt{3x+3}-\sqrt{3x+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3*x + 3)*sqrt(3*x + 2) + log(sqrt(3*x + 3) - sqrt(3*x + 2
)))
```

maple [B] time = 0.01, size = 67, normalized size = 1.91

$$-\frac{\sqrt{(x+1)(3x+2)}\,\sqrt{3}\,\ln\left(\frac{\left(3x+\frac{5}{2}\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)}{6\sqrt{3x+2}\,\sqrt{x+1}}+\sqrt{x+1}\,\sqrt{3x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+2)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] (x+1)^(1/2)*(3*x+2)^(1/2)-1/6*((x+1)*(3*x+2))^(1/2)/(3*x+2)^(1/2)/(x+1)^(1/
2)*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

maxima [A] time = 1.97, size = 41, normalized size = 1.17

$$-\frac{1}{6}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\sqrt{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5) + sqrt(3*x^2 +
5*x + 2)
```

mupad [B] time = 6.14, size = 172, normalized size = 4.91

$$\frac{2\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\left(\sqrt{2}-\sqrt{3x+2}\right)}{3\left(\sqrt{x+1}-1\right)}\right)}{3}-\frac{\frac{30\left(\sqrt{2}-\sqrt{3x+2}\right)}{\sqrt{x+1}-1}+\frac{10\left(\sqrt{2}-\sqrt{3x+2}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{24\sqrt{2}\left(\sqrt{2}-\sqrt{3x+2}\right)^2}{\left(\sqrt{x+1}-1\right)^2}}{\frac{\left(\sqrt{2}-\sqrt{3x+2}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{6\left(\sqrt{2}-\sqrt{3x+2}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^(1/2)/(x + 1)^(1/2),x)
```

```
[Out] (2*3^(1/2)*atanh((3^(1/2)*(2^(1/2) - (3*x + 2)^(1/2)))/(3*((x + 1)^(1/2) -
1))))/3 - ((30*(2^(1/2) - (3*x + 2)^(1/2)))/((x + 1)^(1/2) - 1) + (10*(2^(1
/2) - (3*x + 2)^(1/2))^3)/((x + 1)^(1/2) - 1)^3 + (24*2^(1/2)*(2^(1/2) - (3
*x + 2)^(1/2))^2)/((x + 1)^(1/2) - 1)^2)/((2^(1/2) - (3*x + 2)^(1/2))^4/((x
 + 1)^(1/2) - 1)^4 - (6*(2^(1/2) - (3*x + 2)^(1/2))^2)/((x + 1)^(1/2) - 1)^
2 + 9)
```

sympy [A] time = 1.62, size = 97, normalized size = 2.77

$$\begin{cases} \frac{3(x+1)^{\frac{3}{2}}}{\sqrt{3x+2}} - \frac{\sqrt{x+1}}{\sqrt{3x+2}} - \frac{\sqrt{3}\,\mathrm{acosh}\left(\sqrt{3}\,\sqrt{x+1}\right)}{3} & \text{for } 3\,|x+1| > 1 \\ i\sqrt{-3x-2}\,\sqrt{x+1} + \frac{\sqrt{3}\,i\,\mathrm{asin}\left(\sqrt{3}\,\sqrt{x+1}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((3*(x + 1)**(3/2)/sqrt(3*x + 2) - sqrt(x + 1)/sqrt(3*x + 2) - sqr
t(3)*acosh(sqrt(3)*sqrt(x + 1))/3, 3*Abs(x + 1) > 1), (I*sqrt(-3*x - 2)*sqr
t(x + 1) + sqrt(3)*I*asin(sqrt(3)*sqrt(x + 1))/3, True))
```

3.863 $\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

[Out] -1/3*arcsinh((2+3*x)^(1/2))*3^(1/2)+(1+x)^(1/2)*(2+3*x)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {26, 50, 54, 215}

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]

[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx$$
$$= \sqrt{1+x}\sqrt{2+3x} - \frac{1}{2}\int \frac{1}{\sqrt{1+x}\sqrt{2+3x}}\,dx$$
$$= \sqrt{1+x}\sqrt{2+3x} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+3x}\right)}{\sqrt{3}}$$
$$= \sqrt{1+x}\sqrt{2+3x} - \frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.40

$$\frac{3\sqrt{x+1}\,(3x+2) - \sqrt{9x+6}\,\sinh^{-1}\left(\sqrt{3x+2}\right)}{3\sqrt{3x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
```

```
[Out] (3*Sqrt[1 + x]*(2 + 3*x) - Sqrt[6 + 9*x]*ArcSinh[Sqrt[2 + 3*x]])/(3*Sqrt[2
+ 3*x])
```

fricas [B] time = 0.45, size = 96, normalized size = 2.74

$$\frac{\sqrt{3}\,(x-1)\log\left(-\frac{72\,x^3+4\,\sqrt{3}\,\sqrt{-x^2+1}\,(6\,x+5)\sqrt{3\,x+2}\,\sqrt{-x+1}+48\,x^2-71\,x-49}{x-1}\right) - 12\,\sqrt{-x^2+1}\,\sqrt{3\,x+2}\,\sqrt{-x+1}}{12\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*(x - 1)*log(-(72*x^3 + 4*sqrt(3)*sqrt(-x^2 + 1)*(6*x + 5)*sqr
t(3*x + 2)*sqrt(-x + 1) + 48*x^2 - 71*x - 49)/(x - 1)) - 12*sqrt(-x^2 + 1)*
sqrt(3*x + 2)*sqrt(-x + 1))/(x - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3\,x+2}\,\sqrt{-x+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)
```

maple [B] time = 0.01, size = 86, normalized size = 2.46

$$\frac{\sqrt{-x+1}\,\sqrt{3x+2}\,\sqrt{-x^2+1}\,\left(\sqrt{3}\,\ln\left(\sqrt{3}\,x+\frac{5\sqrt{3}}{6}+\sqrt{3x^2+5x+2}\right)-6\sqrt{3x^2+5x+2}\right)}{6\,(x-1)\,\sqrt{3x^2+5x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)*(3*x+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/6*(-x+1)^(1/2)*(3*x+2)^(1/2)*(-x^2+1)^(1/2)*(ln(5/6*3^(1/2)+3^(1/2)*x+(3*
x^2+5*x+2)^(1/2))*3^(1/2)-6*(3*x^2+5*x+2)^(1/2))/(x-1)/(3*x^2+5*x+2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3\,x+2}\,\sqrt{-x+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{3\,x+2}\,\sqrt{1-x}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2),x)
[Out] int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}\,\sqrt{3x+2}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(2+3*x)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Integral(sqrt(1 - x)*sqrt(3*x + 2)/sqrt(-(x - 1)*(x + 1)), x)
```

3.864 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x} dx$

Optimal. Leaf size=43

$$\frac{4\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{x+1}\right)$$

```
[Out] -arcsin(x)-arctanh((1-x)^(1/2)*(1+x)^(1/2))+4*(1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {98, 21, 105, 41, 216, 92, 206}

$$\frac{4\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

```
[Out] (4*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x] - ArcTanh[Sqrt[1 - x]*Sqrt[1 + x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
```

```
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx &= \frac{4\sqrt{1+x}}{\sqrt{1-x}}-2\int \frac{-\frac{1}{2}+\frac{x}{2}}{\sqrt{1-x}\,x\sqrt{1+x}}\,dx\\ &= \frac{4\sqrt{1+x}}{\sqrt{1-x}}+\int \frac{\sqrt{1-x}}{x\sqrt{1+x}}\,dx\\ &= \frac{4\sqrt{1+x}}{\sqrt{1-x}}-\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx+\int \frac{1}{\sqrt{1-x}\,x\sqrt{1+x}}\,dx\\ &= \frac{4\sqrt{1+x}}{\sqrt{1-x}}-\int \frac{1}{\sqrt{1-x^2}}\,dx-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x}\sqrt{1+x}\right)\\ &= \frac{4\sqrt{1+x}}{\sqrt{1-x}}-\sin^{-1}(x)-\tanh^{-1}\left(\sqrt{1-x}\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 1.42

$$\frac{2\left(\sqrt{1-x^2}\,\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+2x+2\right)}{\sqrt{1-x^2}}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

```
[Out] (2*(2 + 2*x + Sqrt[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/Sqrt[1 - x^2] - A
rcTanh[Sqrt[1 - x^2]]
```

fricas [B] time = 0.41, size = 74, normalized size = 1.72

$$\frac{2\,(x-1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+(x-1)\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+4\,x-4\,\sqrt{x+1}\,\sqrt{-x+1}-4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (2*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + (x - 1)*log((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x) + 4*x - 4*sqrt(x + 1)*sqrt(-x + 1) - 4)/(x - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [-93.616
423693]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [-17.8804557086]2*(-1/2*pi-atan(sqrt(x+1)*((-1/2*(-2*sqrt(-x+1)+2*sqrt(
2))/sqrt(x+1))^2-1)/(-2*sqrt(-x+1)+2*sqrt(2))))-ln(abs(2*sqrt(x+1)/(-2*sqrt
(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))+ln(abs(2*sqrt
(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))
+4*sqrt(x+1)*sqrt(-x+1)/(-x+1)
```

maple [A] time = 0.02, size = 70, normalized size = 1.63

$$\frac{\left(-x \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)-x \arcsin(x)+\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)+\arcsin(x)-4\sqrt{-x^{2}+1}\right)\sqrt{-x+1}\,\sqrt{x+1}}{(x-1)\sqrt{-x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(3/2)/x,x)
```

```
[Out] (-arcsin(x)*x-arctanh(1/(-x^2+1)^(1/2))*x+arcsin(x)+arctanh(1/(-x^2+1)^(1/2
))-4*(-x^2+1)^(1/2))*(-x+1)^(1/2)*(x+1)^(1/2)/(x-1)/(-x^2+1)^(1/2)
```

maxima [A] time = 1.99, size = 53, normalized size = 1.23

$$\frac{4\,x}{\sqrt{-x^{2}+1}}+\frac{4}{\sqrt{-x^{2}+1}}-\arcsin(x)-\log\left(\frac{2\sqrt{-x^{2}+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/ab
s(x) + 2/abs(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{x\,(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(x*(1 - x)^(3/2)),x)
```

```
[Out] int((x + 1)^(3/2)/(x*(1 - x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{3}{2}}}{x\,(1-x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(3/2)/x,x)
```

```
[Out] Integral((x + 1)**(3/2)/(x*(1 - x)**(3/2)), x)
```

3.865 $\int \frac{(1+x)^3}{x(1-x^2)^{3/2}} dx$

Optimal. Leaf size=35

$$\frac{4(x+1)}{\sqrt{1-x^2}} - \tanh^{-1}\left(\sqrt{1-x^2}\right) - \sin^{-1}(x)$$

```
[Out] -arcsin(x)-arctanh((-x^2+1)^(1/2))+4*(1+x)/(-x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1805, 844, 216, 266, 63, 206}

$$\frac{4(x+1)}{\sqrt{1-x^2}} - \tanh^{-1}\left(\sqrt{1-x^2}\right) - \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] (4*(1 + x))/Sqrt[1 - x^2] - ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^3}{x\left(1-x^2\right)^{3/2}}\,dx &= \frac{4(1+x)}{\sqrt{1-x^2}} - \int \frac{-1+x}{x\sqrt{1-x^2}}\,dx\\ &= \frac{4(1+x)}{\sqrt{1-x^2}} - \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{1}{x\sqrt{1-x^2}}\,dx\\ &= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^2\right)\\ &= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right)\\ &= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 1.34

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-x^2\right) - \sqrt{1-x^2}\,\sin^{-1}(x) + 4x + 3}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] (3 + 4*x - Sqrt[1 - x^2]*ArcSin[x] + Hypergeometric2F1[-1/2, 1, 1/2, 1 - x^
2])/Sqrt[1 - x^2]
```

fricas [B] time = 0.41, size = 63, normalized size = 1.80

$$\frac{2\,(x-1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + (x-1)\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + 4\,x - 4\,\sqrt{-x^2+1} - 4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (2*(x - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) + (x - 1)*log((sqrt(-x^2 + 1) - 1
)/x) + 4*x - 4*sqrt(-x^2 + 1) - 4)/(x - 1)
```

giac [A] time = 0.40, size = 44, normalized size = 1.26

$$\frac{8}{\frac{\sqrt{-x^2+1}-1}{x}+1} - \arcsin(x) + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 8/((sqrt(-x^2 + 1) - 1)/x + 1) - arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(
x))
```

maple [A] time = 0.01, size = 41, normalized size = 1.17

$$\frac{4x}{\sqrt{-x^2+1}} - \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) - \arcsin(x) + \frac{4}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^3/x/(-x^2+1)^(3/2),x)
```

```
[Out] 4*x/(-x^2+1)^(1/2)-arcsin(x)+4/(-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2))
```

maxima [A] time = 1.99, size = 53, normalized size = 1.51

$$\frac{4\,x}{\sqrt{-x^2+1}} + \frac{4}{\sqrt{-x^2+1}} - \arcsin(x) - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/ab
s(x) + 2/abs(x))
```

mupad [B] time = 3.20, size = 37, normalized size = 1.06

$$\ln\left(\sqrt{\frac{1}{x^2}-1} - \sqrt{\frac{1}{x^2}}\right) - \operatorname{asin}(x) - \frac{4\sqrt{1-x^2}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^3/(x*(1 - x^2)^(3/2)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - asin(x) - (4*(1 - x^2)^(1/2))/(x -
 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^3}{x\left(-(x-1)(x+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3/x/(-x**2+1)**(3/2),x)
```

```
[Out] Integral((x + 1)**3/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

3.866 $\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{4\sqrt{ax+1}}{\sqrt{1-ax}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)$$

```
[Out] -arcsin(a*x)-arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))+4*(a*x+1)^(1/2)/(-a*x+1)
^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {98, 21, 105, 41, 216, 92, 208}

$$\frac{4\sqrt{ax+1}}{\sqrt{1-ax}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[1 + a*x])/Sqrt[1 - a*x] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
```

```
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2\int \frac{-\frac{a}{2}+\frac{a^2x}{2}}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a}\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} + \int \frac{\sqrt{1-ax}}{x\sqrt{1+ax}}\,dx\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - a\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx + \int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx - a\,\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 72, normalized size = 1.41

$$\frac{2\left(\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)+2ax+2\right)}{\sqrt{1-a^2x^2}} - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (2*(2 + 2*a*x + Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/Sqrt[1 -
a^2*x^2] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

fricas [B] time = 0.42, size = 93, normalized size = 1.82

$$\frac{4\,ax + 2\,(ax-1)\arctan\left(\frac{\sqrt{ax+1}\,\sqrt{-ax+1}-1}{ax}\right) + (ax-1)\log\left(\frac{\sqrt{ax+1}\,\sqrt{-ax+1}-1}{x}\right) - 4\,\sqrt{ax+1}\,\sqrt{-ax+1} - 4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/(a*x)) + (a*
x - 1)*log((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/x) - 4*sqrt(a*x + 1)*sqrt(-a*
x + 1) - 4)/(a*x - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[42,56]2*(-1/2*pi-atan(sqrt(a*x+1)*((-1/2*(-2*sqrt(-a*x+1)+2*sqrt(2))/sqrt(
a*x+1))^2-1)/(-2*sqrt(-a*x+1)+2*sqrt(2))))-ln(abs(2*sqrt(a*x+1)/(-2*sqrt(-a
*x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-a*x+1)+2*sqrt(2))/sqrt(a*x+1)))+ln(abs(2*s
qrt(a*x+1)/(-2*sqrt(-a*x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-a*x+1)+2*sqrt(2))/sq
rt(a*x+1)))+4*sqrt(a*x+1)*sqrt(-a*x+1)/(-a*x+1)
```

maple [C] time = 0.04, size = 134, normalized size = 2.63

$$\frac{\left(-ax\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\operatorname{csgn}(a)-ax\arctan\left(\frac{ax\operatorname{csgn}(a)}{\sqrt{-(ax+1)(ax-1)}}\right)+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\operatorname{csgn}(a)+\arctan\left(\frac{ax\operatorname{csgn}}{\sqrt{-(ax+1)(}}\right.\right.}{(ax-1)\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x)
```

```
[Out] (-arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)*x*a-arctan(csgn(a)*a*x/(-(a*x+1)*(a
*x-1))^(1/2))*x*a+arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)-4*(-a^2*x^2+1)^(1/2
)*csgn(a)+arctan(csgn(a)*a*x/(-(a*x+1)*(a*x-1))^(1/2)))*csgn(a)*(-a*x+1)^(1
/2)*(a*x+1)^(1/2)/(a*x-1)/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 2.50, size = 65, normalized size = 1.27

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}}+\frac{4}{\sqrt{-a^2x^2+1}}-\arcsin\left(ax\right)-\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt(
-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a\,x+1\right)}^{3/2}}{x\,{\left(1-a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)),x)
```

```
[Out] int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+1\right)^{\frac{3}{2}}}{x\left(-ax+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(3/2)/x/(-a*x+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**(3/2)/(x*(-a*x + 1)**(3/2)), x)
```

3.867 $\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{4(ax+1)}{\sqrt{1-a^2x^2}}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

```
[Out] -arcsin(a*x)-arctanh((-a^2*x^2+1)^(1/2))+4*(a*x+1)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1805, 844, 216, 266, 63, 208}

$$\frac{4(ax+1)}{\sqrt{1-a^2x^2}}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (4*(1 + a*x))/Sqrt[1 - a^2*x^2] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
```

```
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx &= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\int \frac{-1+ax}{x\sqrt{1-a^2x^2}}\,dx \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\sin^{-1}(ax)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right) \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\sin^{-1}(ax)-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2} \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.04, size = 59, normalized size = 1.31

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-a^2x^2\right)-\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+4ax+3}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (3 + 4*a*x - Sqrt[1 - a^2*x^2]*ArcSin[a*x] + Hypergeometric2F1[-1/2, 1, 1/2
, 1 - a^2*x^2])/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.46, size = 82, normalized size = 1.82

$$\frac{4\,ax+2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-4\,\sqrt{-a^2x^2+1}-4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*x - 1)*log
((sqrt(-a^2*x^2 + 1) - 1)/x) - 4*sqrt(-a^2*x^2 + 1) - 4)/(a*x - 1)
```

giac [B] time = 0.54, size = 87, normalized size = 1.93

$$-\frac{a\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}-\frac{a\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{8\,a}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
 2*a)/(a^2*abs(x)))/abs(a) + 8*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x)
- 1)*abs(a))
```

maple [A] time = 0.01, size = 75, normalized size = 1.67

$$\frac{4ax}{\sqrt{-a^2x^2+1}}-\frac{a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{4}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 4*a*x/(-a^2*x^2+1)^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1
/2))+4/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 2.08, size = 65, normalized size = 1.44

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}}+\frac{4}{\sqrt{-a^2x^2+1}}-\arcsin\left(ax\right)-\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt(
-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 3.49, size = 82, normalized size = 1.82

$$\frac{4\,a\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{a\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (4*a*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (a*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - atanh((1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ax+1\right)^3}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.868 $\int \frac{1}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

```
[Out] arcsin(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - x^2],x]
[Out] ArcSin[x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - x^2],x]
[Out] ArcSin[x]
```

fricas [B] time = 0.44, size = 18, normalized size = 9.00

$$-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.34, size = 2, normalized size = 1.00

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] arcsin(x)
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2),x)
[Out] arcsin(x)
```

maxima [A] time = 1.68, size = 2, normalized size = 1.00

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsin(x)
```

mupad [B] time = 0.01, size = 2, normalized size = 1.00

$$\mathrm{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^2)^(1/2),x)
[Out] asin(x)
```

sympy [A] time = 0.14, size = 2, normalized size = 1.00

$$\operatorname{asin}{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2),x)
[Out] asin(x)
```

3.869 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

```
[Out] arcsin(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {26, 216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx &= \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sin^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.03, size = 32, normalized size = 16.00

$$-\tan^{-1}\left(\frac{x\sqrt{x^2+1}\,\sqrt{1-x^4}}{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

```
[Out] -ArcTan[(x*Sqrt[1 + x^2]*Sqrt[1 - x^4])/(-1 + x^4)]
```

fricas [B] time = 0.43, size = 27, normalized size = 13.50

$$-\arctan\left(\frac{\sqrt{-x^4+1}\,\sqrt{x^2+1}}{x^3+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^4 + 1)*sqrt(x^2 + 1)/(x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

maple [B] time = 0.02, size = 29, normalized size = 14.50

$$\frac{\sqrt{-x^4+1}\,\arcsin(x)}{\sqrt{x^2+1}\,\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(-x^4+1)^(1/2),x)
[Out] 1/(x^2+1)^(1/2)*(-x^4+1)^(1/2)/(-x^2+1)^(1/2)*arcsin(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.50

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2),x)
[Out] int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
[Out] Integral(sqrt(x**2 + 1)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.870 $\int \frac{1}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^2],x]
[Out] ArcSinh[x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}} \, dx = \sinh^{-1}(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^2],x]
[Out] ArcSinh[x]
```

fricas [B] time = 0.42, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 + 1))
```

giac [B] time = 0.43, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
[Out] -log(-x + sqrt(x^2 + 1))
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\operatorname{arcsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x)
[Out] arcsinh(x)
```

maxima [A] time = 2.00, size = 2, normalized size = 1.00

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(x)
```

mupad [B] time = 0.03, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^(1/2),x)
[Out] asinh(x)
```

sympy [A] time = 0.14, size = 2, normalized size = 1.00

$$\operatorname{asinh}{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2),x)
[Out] asinh(x)
```

3.871 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {26, 215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

```
[Out] ArcSinh[x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx &= \int \frac{1}{\sqrt{1+x^2}}\,dx \\ &= \sinh^{-1}(x) \end{aligned}$$

Mathematica [B] time = 0.02, size = 42, normalized size = 21.00

$$\log\left(1-x^2\right)-\log\left(x^3+\sqrt{1-x^2}\,\sqrt{1-x^4}-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

```
[Out] Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x^2]*Sqrt[1 - x^4]]
```

fricas [B] time = 0.44, size = 81, normalized size = 40.50

$$-\frac{1}{2}\log\left(\frac{x^3+\sqrt{-x^4+1}\,\sqrt{-x^2+1}-x}{x^3-x}\right)+\frac{1}{2}\log\left(-\frac{x^3-\sqrt{-x^4+1}\,\sqrt{-x^2+1}-x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log((x^3 + sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) + 1/2*log(-(x
^3 - sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

maple [B] time = 0.01, size = 29, normalized size = 14.50

$$\frac{\sqrt{-x^4+1}\ \operatorname{arcsinh}(x)}{\sqrt{-x^2+1}\ \sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x)
[Out] 1/(-x^2+1)^(1/2)/(x^2+1)^(1/2)*(-x^4+1)^(1/2)*arcsinh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.50

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(1 - x^4)^(1/2),x)
[Out] int((1 - x^2)^(1/2)/(1 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.872 $\int \sqrt{1-x^2}\, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}\,x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{1-x^2}\,x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2}\, dx &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\, dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{1}{2}\left(\sqrt{1-x^2}\,x+\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2] + ArcSin[x])/2
```

fricas [A] time = 0.45, size = 31, normalized size = 1.35

$$\frac{1}{2}\sqrt{-x^2+1}\,x-\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.36, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{\sqrt{-x^2+1}\;x}{2}+\frac{\arcsin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)+1/2*(-x^2+1)^(1/2)*x
```

maxima [A] time = 1.93, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.74

$$\frac{\mathrm{asin}(x)}{2}+\frac{x\,\sqrt{1-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2),x)
```

```
[Out] asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

sympy [A] time = 0.20, size = 15, normalized size = 0.65

$$\frac{x\sqrt{1-x^2}}{2}+\frac{\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 + asin(x)/2
```

3.873 $\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}\, x + \frac{1}{2}\sin^{-1}(x)$$

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {26, 195, 216}

$$\frac{1}{2}\sqrt{1-x^2}\, x + \frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\, dx &= \int \sqrt{1-x^2}\, dx \\
&= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\, dx \\
&= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.04, size = 50, normalized size = 2.17

$$\frac{1}{2}\left(\frac{\sqrt{1-x^4}\, x}{\sqrt{x^2+1}} + \tan^{-1}\left(\frac{x\sqrt{x^2+1}}{\sqrt{1-x^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 + x^2] + ArcTan[(x*Sqrt[1 + x^2])/Sqrt[1 - x^4]])
/2
```

fricas [B] time = 0.45, size = 60, normalized size = 2.61

$$\frac{\sqrt{-x^4+1}\sqrt{x^2+1}\,x-\left(x^2+1\right)\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2+1}}{x^3+x}\right)}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(sqrt(-x^4 + 1)*sqrt(x^2 + 1)*x - (x^2 + 1)*arctan(sqrt(-x^4 + 1)*sqrt(
x^2 + 1)/(x^3 + x)))/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

maple [B] time = 0.01, size = 42, normalized size = 1.83

$$\frac{\sqrt{-x^4+1}\,\left(\sqrt{-x^2+1}\;x+\arcsin(x)\right)}{2\sqrt{x^2+1}\;\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(x^2+1)^(1/2),x)
[Out] 1/2*(-x^4+1)^(1/2)/(x^2+1)^(1/2)*((-x^2+1)^(1/2)*x+arcsin(x))/(-x^2+1)^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\sqrt{1-x^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(x**2 + 1), x)
```

3.874 $\int \sqrt{1+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+x^2}\,dx &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{1}{2}\left(\sqrt{x^2+1}\,x+\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2] + ArcSinh[x])/2
```

fricas [A] time = 0.42, size = 25, normalized size = 1.19

$$\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

giac [A] time = 0.37, size = 25, normalized size = 1.19

$$\frac{1}{2}\sqrt{x^2+1}\,x - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{\sqrt{x^2+1}\,x}{2} + \frac{\operatorname{arcsinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2),x)
[Out] 1/2*arcsinh(x)+1/2*(x^2+1)^(1/2)*x
```

maxima [A] time = 1.95, size = 15, normalized size = 0.71

$$\frac{1}{2}\sqrt{x^2+1}\,x + \frac{1}{2}\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{\operatorname{asinh}(x)}{2} + \frac{x\sqrt{x^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2),x)
[Out] asinh(x)/2 + (x*(x^2 + 1)^(1/2))/2
```

sympy [A] time = 0.19, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^2+1}}{2} + \frac{\operatorname{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2),x)
[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

3.875 $\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {26, 195, 215}

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]

[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx &= \int \sqrt{1+x^2} \, dx\\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}} \, dx\\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.06, size = 70, normalized size = 3.33

$$\frac{1}{2}\left(\log\left(1-x^2\right)+\frac{\sqrt{1-x^4}\,x}{\sqrt{1-x^2}}-\log\left(x^3+\sqrt{1-x^2}\,\sqrt{1-x^4}-x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

```
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 - x^2] + Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x
^2]*Sqrt[1 - x^4]])/2
```

fricas [B] time = 0.43, size = 120, normalized size = 5.71

$$-\frac{2\sqrt{-x^4+1}\sqrt{-x^2+1}\,x+\left(x^2-1\right)\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)-\left(x^2-1\right)\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(-x^4 + 1)*sqrt(-x^2 + 1)*x + (x^2 - 1)*log((x^3 + sqrt(-x^4 +
1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) - (x^2 - 1)*log(-(x^3 - sqrt(-x^4 + 1)*sq
rt(-x^2 + 1) - x)/(x^3 - x)))/(x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

maple [B] time = 0.01, size = 47, normalized size = 2.24

$$-\frac{\sqrt{-x^4+1}\,\sqrt{-x^2+1}\,\left(\sqrt{x^2+1}\,x+\operatorname{arcsinh}(x)\right)}{2\left(x^2-1\right)\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-x^4+1)^(1/2)*(-x^2+1)^(1/2)*((x^2+1)^(1/2)*x+arcsinh(x))/(x^2-1)/(x^
2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(1/2)/(1 - x^2)^(1/2),x)
```

```
[Out] int((1 - x^4)^(1/2)/(1 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(-(x - 1)*(x + 1)), x)
```

3.876 $\int \left(\frac{a+b+cx^2}{d}\right)^m dx$

Optimal. Leaf size=49

$$\frac{dx\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^{m+1}{}_2F_1\left(1,m+\frac{3}{2};\frac{3}{2};-\frac{cx^2}{a+b}\right)}{a+b}$$

```
[Out] d*x*((a+b)/d+x^2*c/d)^(1+m)*hypergeom([1, 3/2+m],[3/2],-c*x^2/(a+b))/(a+b)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1972, 246, 245}

$$x\left(\frac{cx^2}{a+b}+1\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b + c*x^2)/d)^m,x]
```

```
[Out] (x*((a + b)/d + (c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c*x^2)/(a +
 b))])/(1 + (c*x^2)/(a + b))^m
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a+b+cx^2}{d}\right)^m dx &= \int \left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m dx\\ &= \left(\left(1+\frac{cx^2}{a+b}\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m\right)\int \left(1+\frac{cx^2}{a+b}\right)^m dx\\ &= x\left(1+\frac{cx^2}{a+b}\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.08

$$x\left(\frac{cx^2}{a+b}+1\right)^{-m}\left(\frac{a+b+cx^2}{d}\right)^m{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b + c*x^2)/d)^m,x]
```

```
[Out] (x*((a + b + c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c*x^2)/(a + b))
])/(1 + (c*x^2)/(a + b))^m
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{cx^2 + a + b}{d}\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="fricas")
```

```
[Out] integral(((c*x^2 + a + b)/d)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{cx^2 + a + b}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="giac")
```

```
[Out] integrate(((c*x^2 + a + b)/d)^m, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\frac{c\,x^2 + a + b}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2+a+b)/d)^m,x)
```

```
[Out] int(((c*x^2+a+b)/d)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{cx^2 + a + b}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^2 + a + b)/d)^m, x)
```

mupad [B] time = 4.67, size = 54, normalized size = 1.10

$$\frac{x\left(\frac{a+b}{d} + \frac{c\,x^2}{d}\right)^m {}_2\mathrm{F}_1\left(\frac{1}{2}, -m;\ \frac{3}{2};\ -\frac{c\,x^2}{a+b}\right)}{\left(\frac{c\,x^2}{a+b} + 1\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b + c*x^2)/d)^m,x)
```

```
[Out] (x*((a + b)/d + (c*x^2)/d)^m*hypergeom([1/2, -m], 3/2, -(c*x^2)/(a + b)))/(
(c*x^2)/(a + b) + 1)^m
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x**2+a+b)/d)**m,x)
```

```
[Out] Integral(((a + b + c*x**2)/d)**m, x)
```

3.877 $\int \frac{1}{x-\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=28

$$-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] -1/2*x^2-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2106, 30, 195, 215}

$$-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + x^2])^(-1),x]
```

```
[Out] -x^2/2 - (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x-\sqrt{1+x^2}}\,dx&=-\int x\,dx-\int \sqrt{1+x^2}\,dx\\&=-\frac{x^2}{2}-\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx\\&=-\frac{x^2}{2}-\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\sinh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.36

$$\frac{1}{2}\log\left(x-\sqrt{x^2+1}\right)-\frac{1}{4\left(x-\sqrt{x^2+1}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + x^2])^(-1),x]
[Out] -1/4*1/(x - Sqrt[1 + x^2])^2 + Log[x - Sqrt[1 + x^2]]/2
```

fricas [A] time = 0.42, size = 30, normalized size = 1.07

$$-\frac{1}{2}\,x^2-\frac{1}{2}\,\sqrt{x^2+1}\,x+\frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

giac [A] time = 0.34, size = 30, normalized size = 1.07

$$-\frac{1}{2}\,x^2-\frac{1}{2}\,\sqrt{x^2+1}\,x+\frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="giac")
[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{x^2}{2}-\frac{\sqrt{x^2+1}\,x}{2}-\frac{\operatorname{arcsinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(x^2+1)^(1/2)),x)
[Out] -1/2*x^2-1/2*arcsinh(x)-1/2*(x^2+1)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x-\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x - sqrt(x^2 + 1)), x)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.71

$$-\frac{\operatorname{asinh}(x)}{2}-\frac{x\,\sqrt{x^2+1}}{2}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x^2 + 1)^(1/2)),x)
```

```
[Out] - asinh(x)/2 - (x*(x^2 + 1)^(1/2))/2 - x^2/2
```

sympy [B] time = 0.35, size = 58, normalized size = 2.07

$$-\frac{x\operatorname{asinh}(x)}{2x-2\sqrt{x^2+1}}+\frac{x}{2x-2\sqrt{x^2+1}}+\frac{\sqrt{x^2+1}\operatorname{asinh}(x)}{2x-2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x**2+1)**(1/2)),x)
```

```
[Out] -x*asinh(x)/(2*x - 2*sqrt(x**2 + 1)) + x/(2*x - 2*sqrt(x**2 + 1)) + sqrt(x*
*2 + 1)*asinh(x)/(2*x - 2*sqrt(x**2 + 1))
```

3.878 $\int \frac{1}{x-\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

[Out] -1/2*arcsin(x)-1/2*arctanh(x/(-x^2+1)^(1/2))+1/4*ln(-2*x^2+1)

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 260, 402, 216, 377, 207}

$$\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[1 - x^2])^(-1),x]

[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = \int \left(\frac{x}{-1+2x^2}+\frac{\sqrt{1-x^2}}{-1+2x^2}\right)dx$$
$$= \int \frac{x}{-1+2x^2}\,dx + \int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx$$
$$= \frac{1}{4}\log\left(1-2x^2\right) - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\left(-1+2x^2\right)}\,dx$$
$$= -\frac{1}{2}\sin^{-1}(x) + \frac{1}{4}\log\left(1-2x^2\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)$$
$$= -\frac{1}{2}\sin^{-1}(x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) + \frac{1}{4}\log\left(1-2x^2\right)$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{1}{4}\log\left(1-2x^2\right) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) - \frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 - x^2])^(-1),x]
```

```
[Out] -1/2*ArcSin[x] - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

fricas [B] time = 0.44, size = 84, normalized size = 2.27

$$\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{1}{4}\log\left(2x^2-1\right)+\frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}(x+1)-x-1}{x^2}\right)-\frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}(\right.}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(2*x^2 - 1) + 1/4*log(-(x^2 + sqrt(
-x^2 + 1)*(x + 1) - x - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) +
x - 1)/x^2)
```

giac [B] time = 0.44, size = 140, normalized size = 3.78

$$-\frac{1}{4}\pi\text{sgn}(x)-\frac{1}{2}\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)+\frac{1}{4}\log\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right)+\frac{1}{4}\log\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*pi*sgn(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-
x^2 + 1) - 1)) + 1/4*log(abs(x + 1/2*sqrt(2))) + 1/4*log(abs(x - 1/2*sqrt(2
))) - 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x + 2)) +
1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x - 2))
```

maple [B] time = 0.05, size = 175, normalized size = 4.73

$$-\frac{\text{arctanh}\left(\frac{\left(1-\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}\right)\sqrt{2}}{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}+\frac{\text{arctanh}\left(\frac{\left(\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\sqrt{2}}{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}-\frac{\arcsin(x)}{2}+\frac{\ln\left(2x^2-1\right)}{4}-\frac{\sqrt{2}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(-x^2+1)^(1/2)),x)
```

```
[Out] 1/4*ln(2*x^2-1)-1/8*2^(1/2)*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)
+2)^(1/2)-1/2*arcsin(x)+1/4*arctanh(((x+1/2*2^(1/2))*2^(1/2)+1)*2^(1/2)/(-4
*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2))+1/8*2^(1/2)*(-4*(x-1
/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)-1/4*arctanh((1-(x-1/2*2^(1
/2))*2^(1/2))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x - \sqrt{-x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x - sqrt(-x^2 + 1)), x)
```

mupad [B] time = 0.14, size = 105, normalized size = 2.84

$$\frac{\ln\left(x-\frac{\sqrt{2}}{2}\right)}{4}+\frac{\ln\left(x+\frac{\sqrt{2}}{2}\right)}{4}-\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}-1\right)1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{4}+\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}+1\right)1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{4}-\frac{\mathrm{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (1 - x^2)^(1/2)),x)
```

```
[Out] log(x - 2^(1/2)/2)/4 + log(x + 2^(1/2)/2)/4 - log((2^(1/2)*((2^(1/2)*x)/2 -
 1)*1i - (1 - x^2)^(1/2)*1i)/(x - 2^(1/2)/2))/4 + log((2^(1/2)*((2^(1/2)*x)
/2 + 1)*1i + (1 - x^2)^(1/2)*1i)/(x + 2^(1/2)/2))/4 - asin(x)/2
```

sympy [A] time = 0.17, size = 17, normalized size = 0.46

$$\frac{\log\left(x-\sqrt{1-x^2}\right)}{2}-\frac{\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] log(x - sqrt(1 - x**2))/2 - asin(x)/2
```

3.879 $\int \frac{1}{x-\sqrt{1+2x^2}}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{2}\log\left(x^2+1\right)+\tanh^{-1}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)$$

```
[Out] arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 260, 402, 215, 377, 206}

$$-\frac{1}{2}\log\left(x^2+1\right)+\tanh^{-1}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] -(Sqrt[2]*ArcSinh[Sqrt[2]*x]) + ArcTanh[x/Sqrt[1 + 2*x^2]] - Log[1 + x^2]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x-\sqrt{1+2x^2}}\,dx &= \int \left(-\frac{x}{1+x^2}-\frac{\sqrt{1+2x^2}}{1+x^2}\right)dx \\
&= -\int \frac{x}{1+x^2}\,dx-\int \frac{\sqrt{1+2x^2}}{1+x^2}\,dx \\
&= -\frac{1}{2}\log\left(1+x^2\right)-2\int \frac{1}{\sqrt{1+2x^2}}\,dx+\int \frac{1}{\left(1+x^2\right)\sqrt{1+2x^2}}\,dx \\
&= -\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,x\right)-\frac{1}{2}\log\left(1+x^2\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{1+2x^2}}\right) \\
&= -\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,x\right)+\tanh^{-1}\left(\frac{x}{\sqrt{1+2x^2}}\right)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 1.85

$$\frac{1}{4}\left(-2\log\left(x^2+1\right)-\log\left(3x^2-2\sqrt{2x^2+1}\,x+1\right)+\log\left(3x^2+2\sqrt{2x^2+1}\,x+1\right)-4\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] (-4*Sqrt[2]*ArcSinh[Sqrt[2]*x] - 2*Log[1 + x^2] - Log[1 + 3*x^2 - 2*x*Sqrt[
1 + 2*x^2]] + Log[1 + 3*x^2 + 2*x*Sqrt[1 + 2*x^2]])/4
```

fricas [B] time = 0.44, size = 90, normalized size = 2.25

$$\sqrt{2}\,\log\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)-\frac{1}{2}\,\log\left(x^2+1\right)-\frac{1}{2}\,\log\left(\frac{2\,x^2-\sqrt{2\,x^2+1}\,(x+1)+x+1}{x^2}\right)+\frac{1}{2}\,\log\left(\frac{2\,x^2+\sqrt{2\,x^2+1}\,(}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] sqrt(2)*log(sqrt(2)*x - sqrt(2*x^2 + 1)) - 1/2*log(x^2 + 1) - 1/2*log((2*x^
2 - sqrt(2*x^2 + 1)*(x + 1) + x + 1)/x^2) + 1/2*log((2*x^2 + sqrt(2*x^2 + 1
)*(x - 1) - x + 1)/x^2)
```

giac [B] time = 0.41, size = 88, normalized size = 2.20

$$\sqrt{2}\,\log\left(-\sqrt{2}\,x+\sqrt{2\,x^2+1}\right)+\frac{1}{2}\,\log\left(\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2+2\,\sqrt{2}+3\right)-\frac{1}{2}\,\log\left(\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2-2\,\sqrt{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + 1/2*log((sqrt(2)*x - sqrt(2*x^2
 + 1))^2 + 2*sqrt(2) + 3) - 1/2*log((sqrt(2)*x - sqrt(2*x^2 + 1))^2 - 2*sqr
t(2) + 3) - 1/2*log(x^2 + 1)
```

maple [A] time = 0.01, size = 33, normalized size = 0.82

$$-\sqrt{2}\;\mathrm{arcsinh}\left(\sqrt{2}\;x\right)+\mathrm{arctanh}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(2*x^2+1)^(1/2)),x)
```

```
[Out] arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(2^(1/2)*x)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x - \sqrt{2\,x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x - sqrt(2*x^2 + 1)), x)
```

mupad [B] time = 3.50, size = 57, normalized size = 1.42

$$-\ln\left(x - \mathrm{i}\right) - \frac{\ln\left(x - \frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2} + \frac{1}{2}\mathrm{i}\right)}{2} + \frac{\ln\left(x + \frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2} - \frac{1}{2}\mathrm{i}\right)}{2} - \sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (2*x^2 + 1)^(1/2)),x)
```

```
[Out] log(x + (2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1i/2)/2 - log(x - (2^(1/2)*(x^2 + 1
/2)^(1/2))/2 + 1i/2)/2 - log(x - 1i) - 2^(1/2)*asinh(2^(1/2)*x)
```

sympy [A] time = 0.21, size = 27, normalized size = 0.68

$$-\log\left(x - \sqrt{2x^2 + 1}\right) - \sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x**2+1)**(1/2)),x)
```

```
[Out] -log(x - sqrt(2*x**2 + 1)) - sqrt(2)*asinh(sqrt(2)*x)
```

3.880 $\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx$

Optimal. Leaf size=54

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x+\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)$$

```
[Out] -1/4*x^2-1/2*arctanh(x/(-x^2+2)^(1/2))+1/4*ln(-x^2+1)+1/4*x*(-x^2+2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6725, 260, 266, 43, 478, 12, 377, 207}

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x+\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x^2]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx &= \int \left(\frac{x}{-1+x^2}-\frac{x^3}{2\left(-1+x^2\right)}+\frac{x^2\sqrt{2-x^2}}{2\left(-1+x^2\right)}\right)dx\\
&= -\left(\frac{1}{2}\int \frac{x^3}{-1+x^2}\,dx\right)+\frac{1}{2}\int \frac{x^2\sqrt{2-x^2}}{-1+x^2}\,dx+\int \frac{x}{-1+x^2}\,dx\\
&= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{2}\log\left(1-x^2\right)-\frac{1}{4}\int -\frac{2}{\sqrt{2-x^2}\left(-1+x^2\right)}\,dx-\frac{1}{4}\text{Subst}\left(\int \frac{x}{-1+\cdots}\right.\\
&= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{2}\log\left(1-x^2\right)-\frac{1}{4}\text{Subst}\left(\int \left(1+\frac{1}{-1+x}\right)dx,x,x^2\right)+\frac{1}{2}\int \frac{\cdots}{\sqrt{2-\cdots}}\\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\log\left(1-x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{2-x^2}}\right)\\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 1.43

$$\frac{1}{4}\left(-x^2+\sqrt{2-x^2}\,x+\log\left(1-x^2\right)-\log\left(\sqrt{2-x^2}-x+2\right)+\log\left(\sqrt{2-x^2}+x+2\right)+\log(1-x)-\log(x+1)\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2] - Log[2 -
x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4
```

fricas [A] time = 0.42, size = 67, normalized size = 1.24

$$-\frac{1}{4}\,x^2+\frac{1}{4}\,\sqrt{-x^2+2}\,x+\frac{1}{4}\,\log\left(x^2-1\right)-\frac{1}{8}\,\log\left(-\frac{\sqrt{-x^2+2}\,x+1}{x^2}\right)+\frac{1}{8}\,\log\left(\frac{\sqrt{-x^2+2}\,x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

giac [B] time = 0.44, size = 117, normalized size = 2.17

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(\left|x^2-1\right|\right)-\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(sq
rt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(abs(
x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

maple [B] time = 0.02, size = 111, normalized size = 2.06

$$-\frac{x^2}{4}+\frac{\sqrt{-x^2+2}\,x}{4}-\frac{\operatorname{arctanh}\left(\frac{-2x+4}{2\sqrt{-2x-(x-1)^2+3}}\right)}{4}+\frac{\operatorname{arctanh}\left(\frac{2x+4}{2\sqrt{2x-(x+1)^2+3}}\right)}{4}+\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}+\frac{\sqrt{-2x-(x-1)^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x)
```

```
[Out] 1/4*x*(-x^2+2)^(1/2)+1/4*(-(x-1)^2-2*x+3)^(1/2)-1/4*arctanh(1/2*(4-2*x)/(-(
x-1)^2-2*x+3)^(1/2))-1/4*(-(x+1)^2+2*x+3)^(1/2)+1/4*arctanh(1/2*(4+2*x)/(-(
x+1)^2+2*x+3)^(1/2))-1/4*x^2+1/4*ln(x-1)+1/4*ln(x+1)
```

maxima [B] time = 2.03, size = 94, normalized size = 1.74

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(x^2-1\right)+\frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x+2|}+\frac{2}{|2x+2|}+1\right)-\frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x-2|}+\frac{2}{|2x-2|}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) + 1/4*log(2*sqrt(-x^2 +
2)/abs(2*x + 2) + 2/abs(2*x + 2) + 1) - 1/4*log(2*sqrt(-x^2 + 2)/abs(2*x -
2) + 2/abs(2*x - 2) - 1)
```

mupad [B] time = 3.33, size = 86, normalized size = 1.59

$$\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}-\frac{\ln\left(\frac{-x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x-1}\right)}{4}+\frac{\ln\left(\frac{x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x+1}\right)}{4}+\frac{x\sqrt{2-x^2}}{4}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2*(2 - x^2)^(1/2) - x^3)/(2*x^2 - 2),x)
```

```
[Out] log(x - 1)/4 + log(x + 1)/4 - log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1))
/4 + log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2))/
4 - x^2/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{2x}{x^2-1}\right)dx+\int\frac{x^3}{x^2-1}\,dx+\int\left(-\frac{x^2\sqrt{2-x^2}}{x^2-1}\right)dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x**3+x**2*(-x**2+2)**(1/2))/(2*x**2-2),x)
```

```
[Out] -(Integral(-2*x/(x**2 - 1), x) + Integral(x**3/(x**2 - 1), x) + Integral(-x
**2*sqrt(2 - x**2)/(x**2 - 1), x))/2
```

3.881 $\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$

Optimal. Leaf size=60

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)$$

```
[Out] -1/4*x^2-1/2*arctanh(x/(-x^2+2)^(1/2))+1/4*ln(1-x)+1/4*ln(1+x)+1/4*x*(-x^2+
2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6742, 195, 216, 697, 402, 377, 207}

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x]/4 +
Log[1 + x]/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx &= \int \left(\frac{\sqrt{2-x^2}}{2}+\frac{2-x^2}{4(-1+x)}+\frac{2-x^2}{4(1+x)}+\frac{\sqrt{2-x^2}}{2\left(-1+x^2\right)}\right)dx\\ &= \frac{1}{4}\int \frac{2-x^2}{-1+x}\,dx+\frac{1}{4}\int \frac{2-x^2}{1+x}\,dx+\frac{1}{2}\int \sqrt{2-x^2}\,dx+\frac{1}{2}\int \frac{\sqrt{2-x^2}}{-1+x^2}\,dx\\ &= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\int \left(-1+\frac{1}{-1+x}-x\right)dx+\frac{1}{4}\int \left(1-x+\frac{1}{1+x}\right)dx+\frac{1}{2}\int \frac{1}{\sqrt{2-x^2}\,(-1\\ &= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{2-x^2}}\right)\\ &= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 1.28

$$\frac{1}{4}\left(-x^2+\sqrt{2-x^2}\,x+\log\left(1-x^2\right)-\log\left(\sqrt{2-x^2}-x+2\right)+\log\left(\sqrt{2-x^2}+x+2\right)+\log(1-x)-\log(x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2] - Log[2 -
x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4
```

fricas [A] time = 0.44, size = 67, normalized size = 1.12

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(x^2-1\right)-\frac{1}{8}\log\left(-\frac{\sqrt{-x^2+2}\,x+1}{x^2}\right)+\frac{1}{8}\log\left(\frac{\sqrt{-x^2+2}\,x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

giac [B] time = 0.51, size = 117, normalized size = 1.95

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(\left|x^2-1\right|\right)-\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(sq
rt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(abs(
x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

maple [B] time = 0.02, size = 111, normalized size = 1.85

$$-\frac{x^2}{4}+\frac{\sqrt{-x^2+2}\,x}{4}-\frac{\operatorname{arctanh}\left(\frac{-2x+4}{2\sqrt{-2x-(x-1)^2+3}}\right)}{4}+\frac{\operatorname{arctanh}\left(\frac{2x+4}{2\sqrt{2x-(x+1)^2+3}}\right)}{4}+\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}+\frac{\sqrt{-2x-(x-1)^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x)
```

```
[Out] 1/4*(-x^2+2)^(1/2)*x+1/4*(-2*x-(x-1)^2+3)^(1/2)-1/4*arctanh(1/2*(-2*x+4)/(-
2*x-(x-1)^2+3)^(1/2))-1/4*(2*x-(x+1)^2+3)^(1/2)+1/4*arctanh(1/2*(2*x+4)/(2*
x-(x+1)^2+3)^(1/2))-1/4*x^2+1/4*ln(x-1)+1/4*ln(x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,x^2-\int-\frac{x^2}{x-\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - integrate(-x^2/(x - sqrt(-x^2 + 2)), x)
```

mupad [B] time = 3.38, size = 86, normalized size = 1.43

$$\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}-\frac{\ln\left(\frac{-x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x-1}\right)}{4}+\frac{\ln\left(\frac{x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x+1}\right)}{4}+\frac{x\,\sqrt{2-x^2}}{4}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2 - x^2)^(1/2))/(x - (2 - x^2)^(1/2)),x)
```

```
[Out] log(x - 1)/4 + log(x + 1)/4 - log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1))
/4 + log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2))/
4 - x^2/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+2)**(1/2)/(x-(-x**2+2)**(1/2)),x)
```

```
[Out] Integral(x*sqrt(2 - x**2)/(x - sqrt(2 - x**2)), x)
```

3.882 $\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

```
[Out] -1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6742, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-x + Sqrt[2*x - x^2]),x]
```

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = \int \left(-\frac{1}{2}-\frac{1}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx$$
$$= -\frac{x}{2}-\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\,\mathrm{Subst}\left(\int \frac{1}{-4+4x^2}\,dx, x, \sqrt{2x-x^2}\right)$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)$$

Mathematica [A] time = 0.05, size = 39, normalized size = 0.76

$$\frac{1}{2}\left(-x-\sqrt{-((x-2)x)}-\log(1-x)+\tanh^{-1}\left(\sqrt{-((x-2)x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-x + Sqrt[2*x - x^2]),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[Sqrt[-((-2 + x)*x)]] - Log[1 - x])/2
```

fricas [A] time = 0.43, size = 66, normalized size = 1.29

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{-x^2+2\,x}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{-x^2+2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2
*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

giac [A] time = 0.43, size = 50, normalized size = 0.98

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)-\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
+ 2)) - 1/2*log(abs(x - 1))
```

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$-\frac{x}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-1)^2+1}}\right)}{2}-\frac{\ln\left(x-1\right)}{2}-\frac{\sqrt{-\left(x-1\right)^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x+(-x^2+2*x)^(1/2)),x)
```

```
[Out] -1/2*x-1/2*ln(x-1)-1/2*(-(x-1)^2+1)^(1/2)+1/2*arctanh(1/(-(x-1)^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x-\sqrt{-x^2+2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="maxima")
[Out] -integrate(x/(x - sqrt(-x^2 + 2*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x}{x-\sqrt{2\,x-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(x - (2*x - x^2)^(1/2)),x)
[Out] int(-x/(x - (2*x - x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x-\sqrt{-x^2+2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x**2+2*x)**(1/2)),x)
[Out] -Integral(x/(x - sqrt(-x**2 + 2*x)), x)
```

3.883 $\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

```
[Out] -1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6742, 43, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx &= \int \left(-\frac{x}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx\\ &= -\left(\frac{1}{2}\int \frac{x}{-1+x}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx\\ &= -\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\int \left(1+\frac{1}{-1+x}\right)dx+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx\\ &= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\,\text{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right)\\ &= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 0.76

$$\frac{1}{2}\left(-x-\sqrt{-((x-2)x)}-\log(1-x)+\tanh^{-1}\left(\sqrt{-((x-2)x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[Sqrt[-((-2 + x)*x)]] - Log[1 - x])/2
```

fricas [A] time = 0.47, size = 66, normalized size = 1.29

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{-x^2+2\,x}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{-x^2+2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2
*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

giac [A] time = 0.48, size = 50, normalized size = 0.98

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)-\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="giac")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
+ 2)) - 1/2*log(abs(x - 1))
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$-\frac{x}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-1)^2+1}}\right)}{2}-\frac{\ln\left(x-1\right)}{2}-\frac{\sqrt{-\left(x-1\right)^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(-x^2+2*x)^(1/2))/(2-2*x),x)
```

```
[Out] -1/2*x-1/2*ln(x-1)-1/2*(-(x-1)^2+1)^(1/2)+1/2*arctanh(1/(-(x-1)^2+1)^(1/2))
```

maxima [A] time = 1.98, size = 54, normalized size = 1.06

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{2\,\sqrt{-x^2+2\,x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x+\sqrt{2\,x-x^2}}{2\,x-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + (2*x - x^2)^(1/2))/(2*x - 2),x)
```

```
[Out] int(-(x + (2*x - x^2)^(1/2))/(2*x - 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x}{x-1}\,dx+\int \frac{\sqrt{-x^2+2x}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x**2+2*x)**(1/2))/(2-2*x),x)
```

```
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(-x**2 + 2*x)/(x - 1), x))/2
```

3.884 $\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

```
[Out] -1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6688, 2115, 6742, 43, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 2115

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*((g_.) + (h_.)*(x_))^(m_.
), x_Symbol] :> Int[(g + h*x)^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandT
oSum[v, x]])^n, x] /; FreeQ[{f, g, h, j, k, m, n}, x] && LinearQ[u, x] && Q
uadraticQ[v, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[v, x] && (EqQ[j
, 0] || EqQ[f, 1])) && EqQ[(Coefficient[u, x, 1]*g - h*(Coefficient[u, x, 0
] + f*j))^2 - f^2*k^2*(Coefficient[v, x, 2]*g^2 - Coefficient[v, x, 1]*g*h
```

```
+ Coefficient[v, x, 0]*h^2), 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx &= \int \frac{x+\sqrt{-(-2+x)x}}{2-2x}\,dx \\
&= \int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx \\
&= \int \left(-\frac{x}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx \\
&= -\left(\frac{1}{2}\int \frac{x}{-1+x}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx \\
&= -\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\int \left(1+\frac{1}{-1+x}\right)dx+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\,\mathrm{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right) \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.76

$$\frac{1}{2}\left(-x-\sqrt{-((x-2)x)}-\log(1-x)+\tanh^{-1}\left(\sqrt{-((x-2)x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[Sqrt[-((-2 + x)*x)]] - Log[1 - x])/2
```

fricas [A] time = 0.43, size = 64, normalized size = 1.25

$$-\frac{1}{2}x-\frac{1}{2}\sqrt{x}\sqrt{-x+2}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*s
qrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}]
at parameters values [-92.616423693]Warning, choosing root of [1,0,-4,0,%%%
{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}] at parameters values [-16.88045570
86]-1/2*(x+ln(abs(x-1))+sqrt(x)*sqrt(-x+2)-ln(abs(2*sqrt(x)/(-2*sqrt(-x+2)+
2*sqrt(2))+2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x)))+ln(abs(2*sqrt(x)/(-2*s
qrt(-x+2)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x))))
```

maple [A] time = 0.01, size = 51, normalized size = 1.00

$$-\frac{x}{2}-\frac{\ln\left(x-1\right)}{2}-\frac{\sqrt{-x+2}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-\left(x-2\right)x}}\right)+\sqrt{-\left(x-2\right)x}\right)\sqrt{x}}{2\sqrt{-\left(x-2\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x)
```

```
[Out] -1/2*(2-x)^(1/2)*x^(1/2)/(-x*(x-2))^(1/2)*((-x*(x-2))^(1/2)-arctanh(1/(-x*(
x-2))^(1/2)))-1/2*x-1/2*ln(x-1)
```

maxima [A] time = 1.97, size = 54, normalized size = 1.06

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^{2}+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{2\,\sqrt{-x^{2}+2\,x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

mupad [B] time = 4.78, size = 56, normalized size = 1.10

$$\operatorname{atanh}\left(\frac{\sqrt{x}\,\left(\sqrt{2}-\sqrt{2-x}\right)}{x+\sqrt{2}\,\sqrt{2-x}-2}\right)-\frac{\ln\left(x-1\right)}{2}-\frac{x}{2}-\frac{\sqrt{x}\,\sqrt{2-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^(1/2)*(2 - x)^(1/2))/(2*x - 2),x)
```

```
[Out] atanh((x^(1/2)*(2^(1/2) - (2 - x)^(1/2)))/(x + 2^(1/2)*(2 - x)^(1/2) - 2))
- log(x - 1)/2 - x/2 - (x^(1/2)*(2 - x)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{x}{x-1}\,dx+\int\frac{\sqrt{x}\,\sqrt{2-x}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)**(1/2)*x**(1/2))/(2-2*x),x)
```

```
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(x)*sqrt(2 - x)/(x - 1), x))/2
```

3.885 $\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx$

Optimal. Leaf size=54

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\sqrt{x}\right)$$

```
[Out] -1/2*x+1/2*arctanh((2-x)^(1/2)*x^(1/2))-1/2*ln(1-x)-1/2*(2-x)^(1/2)*x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2105, 101, 12, 92, 206, 43}

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] -(Sqrt[2 - x]*Sqrt[x])/2 - x/2 + ArcTanh[Sqrt[2 - x]*Sqrt[x]]/2 - Log[1 - x
]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2105

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[e, Int[(u*Sqrt[a + b*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^2
)*x), x], x] - Dist[f, Int[(u*Sqrt[c + d*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^
2)*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a*e^2 - c*f^2, 0] && N
eQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx &= \int \frac{\sqrt{2-x}\sqrt{x}}{2-2x}\,dx+\int \frac{x}{2-2x}\,dx\\ &= -\frac{1}{2}\sqrt{2-x}\sqrt{x}+\frac{1}{2}\int \frac{2}{(2-2x)\sqrt{2-x}\sqrt{x}}\,dx+\int \left(-\frac{1}{2}-\frac{1}{2(-1+x)}\right)dx\\ &= -\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{x}{2}-\frac{1}{2}\log(1-x)+\int \frac{1}{(2-2x)\sqrt{2-x}\sqrt{x}}\,dx\\ &= -\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{x}{2}-\frac{1}{2}\log(1-x)+2\,\text{Subst}\left(\int \frac{1}{4-4x^2}\,dx,x,\sqrt{2-x}\sqrt{x}\right)\\ &= -\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{x}{2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\sqrt{x}\right)-\frac{1}{2}\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 1.52

$$\frac{1}{2}\left(-x-\sqrt{-((x-2)x)}-\log\left(1-\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)+\tanh^{-1}\left(\frac{2-\sqrt{x}}{\sqrt{2-x}}\right)-\tanh^{-1}\left(\frac{\sqrt{x}+2}{\sqrt{2-x}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[(2 - Sqrt[x])/Sqrt[2 - x]] - ArcTanh[(2
 + Sqrt[x])/Sqrt[2 - x]] - Log[1 - Sqrt[x]] - Log[1 + Sqrt[x]])/2
```

fricas [A] time = 0.42, size = 64, normalized size = 1.19

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x}\,\sqrt{-x+2}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{x}\,\sqrt{-x+2}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{x}\,\sqrt{-x+2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*s
qrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}]
at parameters values [-92.616423693]Warning, choosing root of [1,0,-4,0,%%%
{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}] at parameters values [-16.88045570
86]2*(-x/4-1/4*ln(abs(x-1))-1/4*sqrt(x)*sqrt(-x+2)+1/4*ln(abs(2*sqrt(x)/(-2
```

```
*sqrt(-x+2)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x)))-1/4*ln(abs
(2*sqrt(x)/(-2*sqrt(-x+2)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x
))))
```

maple [A] time = 0.01, size = 51, normalized size = 0.94

$$-\frac{x}{2}-\frac{\ln(x-1)}{2}-\frac{\sqrt{-x+2}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-2)x}}\right)+\sqrt{-(x-2)\,x}\right)\sqrt{x}}{2\sqrt{-(x-2)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((-x+2)^(1/2)-x^(1/2)),x)
```

```
[Out] -1/2*(-x+2)^(1/2)*x^(1/2)/(-(x-2)*x)^(1/2)*((-(x-2)*x)^(1/2)-arctanh(1/(-(x
-2)*x)^(1/2)))-1/2*x-1/2*ln(x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{x}}{\sqrt{x}-\sqrt{-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x)/(sqrt(x) - sqrt(-x + 2)), x)
```

mupad [B] time = 0.06, size = 56, normalized size = 1.04

$$\operatorname{atanh}\left(\frac{\sqrt{x}\,\left(\sqrt{2}-\sqrt{2-x}\right)}{x+\sqrt{2}\,\sqrt{2-x}-2}\right)-\frac{\ln(x-1)}{2}-\frac{x}{2}-\frac{\sqrt{x}\,\sqrt{2-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((2 - x)^(1/2) - x^(1/2)),x)
```

```
[Out] atanh((x^(1/2)*(2^(1/2) - (2 - x)^(1/2)))/(x + 2^(1/2)*(2 - x)^(1/2) - 2))
- log(x - 1)/2 - x/2 - (x^(1/2)*(2 - x)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{-\sqrt{x}+\sqrt{2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/((2-x)**(1/2)-x**(1/2)),x)
```

```
[Out] Integral(sqrt(x)/(-sqrt(x) + sqrt(2 - x)), x)
```

3.886 $\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}}\,dx$

Optimal. Leaf size=27

$$-\frac{3\left(1-x^2\right)}{2\left(-\left((x+1)\left(1-x^2\right)\right)\right)^{2/3}}$$

```
[Out] -3/2*(-x^2+1)/(-(1+x)*(-x^2+1))^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2067, 2064, 37}

$$-\frac{3(1-x)(x+1)}{2\left(x^3+x^2-x-1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)*(-1 + x^2))^(-2/3),x]
```

```
[Out] (-3*(1 - x)*(1 + x))/(2*(-1 - x + x^2 + x^3)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(-\frac{16}{27}-\frac{4x}{3}+x^3\right)^{2/3}}\,dx, x, \frac{1}{3}+x\right)\\
&= \frac{\left(32\sqrt[3]{2}\,(-1-x)^{4/3}(-1+x)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{16}{9}-\frac{8x}{3}\right)^{4/3}\left(-\frac{16}{9}+\frac{4x}{3}\right)^{2/3}}\,dx, x, \frac{1}{3}+x\right)}{9\left(-1-x+x^2+x^3\right)^{2/3}}\\
&= -\frac{3(1-x)(1+x)}{2\left(-1-x+x^2+x^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.85

$$\frac{3(x-1)(x+1)}{2\left((x-1)(x+1)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)*(-1 + x^2))^(-2/3),x]
[Out] (3*(-1 + x)*(1 + x))/(2*((-1 + x)*(1 + x)^2)^(2/3))
```

fricas [A] time = 0.42, size = 20, normalized size = 0.74

$$\frac{3\left(x^3+x^2-x-1\right)^{\frac{1}{3}}}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="fricas")
[Out] 3/2*(x^3 + x^2 - x - 1)^(1/3)/(x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2-1\right)(x+1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="giac")
[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.74

$$\frac{3(x-1)(x+1)}{2\left((x+1)\left(x^2-1\right)\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x+1)*(x^2-1))^(2/3),x)
[Out] 3/2*(x-1)*(x+1)/((x+1)*(x^2-1))^(2/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2-1\right)(x+1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="maxima")
[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

mupad [B] time = 3.42, size = 20, normalized size = 0.74

$$\frac{3\left(\left(x^2-1\right)(x+1)\right)^{1/3}}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)*(x + 1))^(2/3),x)
[Out] (3*((x^2 - 1)*(x + 1))^(1/3))/(2*(x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x+1)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x**2-1))**(2/3),x)
[Out] Integral(((x + 1)*(x**2 - 1))**(-2/3), x)
```

3.887 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx$

Optimal. Leaf size=14

$$-\frac{2x}{\sqrt{x\left(x^2+1\right)}}$$

```
[Out] -2*x/(x*(x^2+1))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6719, 449}

$$-\frac{2x}{\sqrt{x\left(x^2+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
```

```
[Out] (-2*x)/Sqrt[x*(1 + x^2)]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{x\left(1+x^2\right)}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{x\left(1+x^2\right)}}\\
&= -\frac{2x}{\sqrt{x\left(1+x^2\right)}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 0.86

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
```

```
[Out] (-2*x)/Sqrt[x + x^3]
```

fricas [A] time = 0.42, size = 16, normalized size = 1.14

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{(x^2+1)x}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$-\frac{2x}{\sqrt{(x^2+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x)
[Out] -2*x/(x*(x^2+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{(x^2+1)x}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

mupad [B] time = 3.38, size = 138, normalized size = 9.86

$$-\frac{2x}{\sqrt{x^3+x}}-\frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{\frac{1}{2}+\frac{x\,1\mathrm{i}}{2}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{1-x\,1\mathrm{i}}\right)\middle|\frac{1}{2}\right)\sqrt{x\,1\mathrm{i}}\,2\mathrm{i}}{\sqrt{x^3+x}}+\frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{\frac{1}{2}+\frac{x\,1\mathrm{i}}{2}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{1-x\,1\mathrm{i}}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x*(x^2 + 1))^(1/2)*(x^2 + 1)),x)
[Out] ((1 - x*1i)^(1/2)*((x*1i)/2 + 1/2)^(1/2)*ellipticF(asin((1 - x*1i)^(1/2)),
1/2)*(x*1i)^(1/2)*2i)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*((x*1i)/2 + 1/2)^
(1/2)*ellipticE(asin((1 - x*1i)^(1/2)), 1/2)*(x*1i)^(1/2)*2i)/(x + x^3)^(1/
2) - (2*x)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)^(1/
2)*ellipticE(asin((-x*1i)^(1/2)), -1)*1i)/(x + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x*(x**2+1))**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

3.888 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}} \, dx$

Optimal. Leaf size=12

$$-\frac{2x}{\sqrt{x^3+x}}$$

```
[Out] -2*x/(x^3+x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2056, 449}

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (-2*x)/Sqrt[x + x^3]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}} \, dx &= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}} \, dx}{\sqrt{x+x^3}} \\ &= -\frac{2x}{\sqrt{x+x^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (-2*x)/Sqrt[x + x^3]
```

fricas [A] time = 0.42, size = 16, normalized size = 1.33

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 1}{\sqrt{x^3 + x}\left(x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-\frac{2x}{\sqrt{x^3 + x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x)
[Out] -2*x/(x^3+x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 1}{\sqrt{x^3 + x}\left(x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

mupad [B] time = 0.05, size = 10, normalized size = 0.83

$$-\frac{2\,x}{\sqrt{x^3 + x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x + x^3)^(1/2)),x)
[Out] -(2*x)/(x + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x - 1)(x + 1)}{\sqrt{x\left(x^2 + 1\right)}\left(x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**3+x)**(1/2),x)
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

3.889 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx$

Optimal. Leaf size=36

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

```
[Out] 2*x*((-x^2+1)^2/x/(x^2+1))^(1/2)/(-x^2+1)
```

Rubi [A] time = 0.14, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6718, 449}

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
```

```
[Out] (2*x*Sqrt[(1 - x^2)^2/(x*(1 + x^2))])/(1 - x^2)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx &= \frac{\left(\sqrt{x}\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}\,(1+x^2)^{3/2}}\,dx}{-1+x^2}\\ &= \frac{2x\sqrt{\frac{(1-x^2)^2}{x(1+x^2)}}}{1-x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.81

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

fricas [A] time = 0.42, size = 30, normalized size = 0.83

$$-\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{\left(x^2-1\right)^2}{\left(x^2+1\right)x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

maple [A] time = 0.01, size = 34, normalized size = 0.94

$$-\frac{2\sqrt{\frac{\left(x^2-1\right)^2}{\left(x^2+1\right)x}}\,x}{\left(x-1\right)\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x)
[Out] -2*x/(x-1)/(x+1)*((x^2-1)^2/x/(x^2+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{\left(x^2-1\right)^2}{\left(x^2+1\right)x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

mupad [B] time = 3.49, size = 48, normalized size = 1.33

$$-\frac{\left(2\,x^3+2\,x\right)\sqrt{\frac{1}{x^2+1}}\,\sqrt{{\left(x^2-1\right)}^2}\,\sqrt{\frac{1}{x}}}{\left(x^2-1\right)\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^2/(x*(x^2 + 1)))^(1/2)/(x^2 + 1),x)
```

```
[Out] -((2*x + 2*x^3)*(1/(x^2 + 1))^(1/2)*((x^2 - 1)^2)^(1/2)*(1/x)^(1/2))/((x^2
- 1)*(x^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2-1)**2/x/(x**2+1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

3.890 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx$

Optimal. Leaf size=33

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

```
[Out] 2*x*((-x^2+1)^2/(x^3+x))^(1/2)/(-x^2+1)
```

Rubi [A] time = 0.19, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6719, 2056, 449}

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
```

```
[Out] (2*x*Sqrt[(1 - x^2)^2/(x + x^3)])/(1 - x^2)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = \frac{\left(\sqrt{\frac{(-1+x^2)^2}{x+x^3}}\sqrt{x+x^3}\right)\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx}{-1+x^2}$$
$$= \frac{\left(\sqrt{x}\sqrt{1+x^2}\sqrt{\frac{(-1+x^2)^2}{x+x^3}}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}}\,dx}{-1+x^2}$$
$$= \frac{2x\sqrt{\frac{(1-x^2)^2}{x+x^3}}}{1-x^2}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.88

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

fricas [A] time = 0.41, size = 30, normalized size = 0.91

$$-\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

maple [A] time = 0.01, size = 34, normalized size = 1.03

$$-\frac{2\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}\;x}{(x-1)(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x)
[Out] -2/(x-1)/(x+1)*((x^2-1)^2/(x^2+1)/x)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{\left(x^2-1\right)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

mupad [B] time = 3.49, size = 43, normalized size = 1.30

$$-\frac{\sqrt{\frac{1}{x^3+x}}\,\left(2\,x^3+2\,x\right)\,\sqrt{{\left(x^2-1\right)}^2}}{\left(x^2-1\right)\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^2/(x + x^3))^(1/2)/(x^2 + 1),x)
```

```
[Out] -((1/(x + x^3))^(1/2)*(2*x + 2*x^3)*((x^2 - 1)^2)^(1/2))/((x^2 - 1)*(x^2 +
1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2-1)**2/(x**3+x))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

3.891 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\,x\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] arctanh(d^(1/2)*(a*x^2+b)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(a*x^2+b)^(1/2)/x/
a^(1/2)/d^(1/2)/(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {435, 444, 63, 217, 206}

$$\frac{\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\,x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{b+ax^2}\int \frac{x}{\sqrt{b+ax^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+\frac{b}{x^2}}\,x}$$
$$= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{b+ax}\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{a+\frac{b}{x^2}}\,x}$$
$$= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{bd}{a}+\frac{dx^2}{a}}}\,dx, x, \sqrt{b+ax^2}\right)}{a\sqrt{a+\frac{b}{x^2}}\,x}$$
$$= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{a}}\,dx, x, \frac{\sqrt{b+ax^2}}{\sqrt{c+dx^2}}\right)}{a\sqrt{a+\frac{b}{x^2}}\,x}$$
$$= \frac{\sqrt{b+ax^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{b+ax^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,x}$$

Mathematica [A] time = 0.13, size = 105, normalized size = 1.50

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{ac-bd}}\right)}{\sqrt{d}\sqrt{ax^2+b}\sqrt{ac-bd}\sqrt{\frac{a(c+dx^2)}{ac-bd}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b/x^2]*x*Sqrt[c + d*x^2]*ArcSinh[(Sqrt[d]*Sqrt[b + a*x^2])/Sqrt[a
*c - b*d]])/(Sqrt[d]*Sqrt[a*c - b*d]*Sqrt[b + a*x^2]*Sqrt[(a*(c + d*x^2))/(
a*c - b*d)])
```

fricas [A] time = 0.44, size = 208, normalized size = 2.97

$$\left[\frac{\sqrt{ad}\,\log\left(8\,a^2d^2x^4 + a^2c^2 + 6\,abcd + b^2d^2 + 8\left(a^2cd + abd^2\right)x^2 + 4\left(2\,adx^3 + (ac+bd)x\right)\sqrt{dx^2+c}\sqrt{ad}\sqrt{\frac{ax^2+b}{x^2}}\right)}{4\,ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*d)*log(8*a^2*d^2*x^4 + a^2*c^2 + 6*a*b*c*d + b^2*d^2 + 8*(a^2*c
*d + a*b*d^2)*x^2 + 4*(2*a*d*x^3 + (a*c + b*d)*x)*sqrt(d*x^2 + c)*sqrt(a*d)
```

```
*sqrt((a*x^2 + b)/x^2))/(a*d), -1/2*sqrt(-a*d)*arctan(1/2*(2*a*d*x^3 + (a*c
 + b*d)*x)*sqrt(d*x^2 + c)*sqrt(-a*d)*sqrt((a*x^2 + b)/x^2)/(a^2*d^2*x^4 +
a*b*c*d + (a^2*c*d + a*b*d^2)*x^2))/(a*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep)]Warning, choosin
g root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] a
t parameters values [-22,93,91]Warning, choosing root of [1,0,%%%{-2,[1,0,2
]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [31,-21,
88]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%
%%{1,[2,0,4]%%%}] at parameters values [76,-66,66]Warning, choosing root of
 [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at paramet
ers values [5,-23,79]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{
-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-88,9,6]Warning,
choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]
%%%}] at parameters values [-69,-8,31]Warning, choosing root of [1,0,%%%{-2
,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [
2,90.2102860468,-92]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-
4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-17,26.2119182013,
64]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%
%%{1,[2,0,4]%%%}] at parameters values [89,29.4664394325,-51]Warning, choos
ing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}]
 at parameters values [22,45.1969879479,76]Warning, choosing root of [1,0,%
%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters val
ues [-63,68.7323710029,13]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}
+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [43,85.975885
5961,12]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%
},0,%%%{1,[2,0,4]%%%}] at parameters values [72,95.2558838762,11]Warning, c
hoosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%
%%}] at parameters values [-9,91.3720739307,81]Warning, choosing root of [1
,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters
 values [15,23.9552401127,-50]Warning, choosing root of [1,0,%%%{-2,[1,0,2]
%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [93,60.82
46789905,-18]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,
0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-88,63.3562821955,93]Warn
ing, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2
,0,4]%%%}] at parameters values [-11,92.3620133325,-60]Warning, choosing ro
ot of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at pa
rameters values [-88,51.1034068516,72]Warning, choosing root of [1,0,%%%{-2
,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [
-90,71.1075269701,48]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{
-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-47,53.5483433446
,-60]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0
,%%%{1,[2,0,4]%%%}] at parameters values [70,89.9395644632,-32]Warning, cho
osing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%
}] at parameters values [-74,8.0407431256,-16]Warning, choosing root of [1,
0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters
values [85,16.2654368887,-14]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%
%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-77,83.07
05981795,31]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0
```

```
]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [76,41.5291932677,-48]Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [-47,25.6140411007,23]Warning, choosing root
 of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at para
meters values [32,76.6146142203,-62]Warning, choosing root of [1,0,%%%{-2,[
1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [66
,99.6590219955,-30]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4
,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-50,17.713730142,-4
4]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%
%{1,[2,0,4]%%%}] at parameters values [45,14.5515509131,62]Warning, choosin
g root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] a
t parameters values [44,88.8429926978,11]Warning, choosing root of [1,0,%%%
{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters value
s [-19,42.8964279308,-92]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+
%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [67,19.2648137
459,-22]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%
},0,%%%{1,[2,0,4]%%%}] at parameters values [9,87.7979063494,94]Warning, ch
oosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%
%}] at parameters values [95,98.9582961812,-92]Warning, choosing root of [1
,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters
 values [52,79.2538507222,22]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%
%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-43,16.76
38230952,21]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0
]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [71,61.8959259251,-83]Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [-30,23.6526960679,40]sym2poly/r2sym(const g
en & e,const index_m & i,const vecteur & l) Error: Bad Argument Valuesym2po
ly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argu
ment ValueWarning, choosing root of [1,0,%%%{-2,[1,0,0]%%%}+%%%{-2,[0,1,2]%
%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[2,0,0]%%%}+%%%{2,[1,1,2]%%%}+%%%{-2,[1,1,0]
%%%}+%%%{1,[0,2,4]%%%}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,2,0]%%%}] at parameters
values [0,56.2346625305,0]Warning, choosing root of [1,0,%%%{-2,[1,0,0]%%%}
+%%%{-2,[0,1,2]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[2,0,0]%%%}+%%%{2,[1,1,2]%%%
}+%%%{-2,[1,1,0]%%%}+%%%{1,[0,2,4]%%%}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,2,0]%%%}
] at parameters values [-43,9.82222589385,18]Warning, choosing root of [1,0
,%%%{-2,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[2,0,0]%%
%}+%%%{2,[1,1,2]%%%}+%%%{-2,[1,1,0]%%%}+%%%{1,[0,2,4]%%%}+%%%{-2,[0,2,2]%%%
}+%%%{1,[0,2,0]%%%}] at parameters values [95,6.65142845921,-60]Warning, ch
oosing root of [1,0,%%%{-2,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%}+%%%{-2,[0,1,0]%%%
},0,%%%{1,[2,0,0]%%%}+%%%{2,[1,1,2]%%%}+%%%{-2,[1,1,0]%%%}+%%%{1,[0,2,4]%%%
}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [65,77.8863785
956,-16]Warning, choosing root of [1,0,%%%{-2,[1,0,0]%%%}+%%%{-2,[0,1,2]%%%
}+%%%{-2,[0,1,0]%%%},0,%%%{1,[2,0,0]%%%}+%%%{2,[1,1,2]%%%}+%%%{-2,[1,1,0]%%
%}+%%%{1,[0,2,4]%%%}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,2,0]%%%}] at parameters va
lues [-81,85.3390313801,-19]gen.cc:simplify/tmp.type!=_EXT Error: Bad Argum
ent ValueEvaluation time: 21.47Done
```

maple [B] time = 0.05, size = 117, normalized size = 1.67

$$\frac{\left(a\,x^{2}+b\right)\sqrt{d\,x^{2}+c}\;\ln\left(\frac{2ad\,x^{2}+ac+bd+2\sqrt{ad\,x^{4}+ac\,x^{2}+bd\,x^{2}+bc}\;\sqrt{ad}}{2\sqrt{ad}}\right)}{2\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\;\sqrt{ad}\;\sqrt{ad\,x^{4}+ac\,x^{2}+bd\,x^{2}+bc}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)*ln(1/2*(2*a*d*x^2+2*(a*d*x^4+a*c*x^2+
b*d*x^2+b*c)^(1/2)*(a*d)^(1/2)+a*c+b*d)/(a*d)^(1/2))*(d*x^2+c)^(1/2)/(a*d)^
(1/2)/(a*d*x^4+a*c*x^2+b*d*x^2+b*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^2 + c}\sqrt{a + \frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + \frac{b}{x^2}}\,\sqrt{d\,x^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{x^2}}\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt(a + b/x**2)*sqrt(c + d*x**2)), x)
```

3.892 $\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{x^4-2x^2}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}-\frac{\sqrt{x^4-2x^2}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

```
[Out] 2/3*arctan(1/2*(x^2-2)^(1/2))*(x^4-2*x^2)^(1/2)/x/(x^2-2)^(1/2)-1/3*arctan(
(x^2-2)^(1/2))*(x^4-2*x^2)^(1/2)/x/(x^2-2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2056, 571, 83, 63, 203}

$$\frac{2\sqrt{x^4-2x^2}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}-\frac{\sqrt{x^4-2x^2}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

```
[Out] (2*Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]/2])/(3*x*Sqrt[-2 + x^2]) - (Sqr
t[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\sqrt{-2x^2+x^4}}{\left(-1+x^2\right)\left(2+x^2\right)}\,dx = \frac{\sqrt{-2x^2+x^4}\int \frac{x\sqrt{-2+x^2}}{\left(-1+x^2\right)\left(2+x^2\right)}\,dx}{x\sqrt{-2+x^2}}$$
$$= \frac{\sqrt{-2x^2+x^4}\ \text{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}}$$
$$= -\frac{\sqrt{-2x^2+x^4}\ \text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,(-1+x)}\,dx,x,x^2\right)}{6x\sqrt{-2+x^2}} + \frac{\left(2\sqrt{-2x^2+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,(2}\right.}{3x\sqrt{-2+x^2}}$$
$$= -\frac{\sqrt{-2x^2+x^4}\ \text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(4\sqrt{-2x^2+x^4}\right)\text{Subst}\left(\int \frac{1}{4+x^2}\,dx\right.}{3x\sqrt{-2+x^2}}$$
$$= \frac{2\sqrt{-2x^2+x^4}\ \tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} - \frac{\sqrt{-2x^2+x^4}\ \tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

Mathematica [A] time = 0.06, size = 52, normalized size = 0.63

$$-\frac{x\sqrt{x^2-2}\left(2\tan^{-1}\left(\frac{2}{\sqrt{x^2-2}}\right)+\tan^{-1}\left(\sqrt{x^2-2}\right)\right)}{3\sqrt{x^2\left(x^2-2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

```
[Out] -1/3*(x*Sqrt[-2 + x^2]*(2*ArcTan[2/Sqrt[-2 + x^2]] + ArcTan[Sqrt[-2 + x^2]]
))/Sqrt[x^2*(-2 + x^2)]
```

fricas [A] time = 0.43, size = 38, normalized size = 0.46

$$-\frac{1}{3}\arctan\left(\frac{\sqrt{x^4-2x^2}}{x}\right)+\frac{2}{3}\arctan\left(\frac{\sqrt{x^4-2x^2}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(sqrt(x^4 - 2*x^2)/x) + 2/3*arctan(1/2*sqrt(x^4 - 2*x^2)/x)
```

giac [C] time = 0.36, size = 46, normalized size = 0.55

$$\frac{1}{3}\left(\arctan\left(i\sqrt{2}\right)-2\arctan\left(\frac{1}{2}i\sqrt{2}\right)\right)\text{sgn}(x)+\frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\text{sgn}(x)-\frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/3*(arctan(I*sqrt(2)) - 2*arctan(1/2*I*sqrt(2)))*sgn(x) + 2/3*arctan(1/2*s
qrt(x^2 - 2))*sgn(x) - 1/3*arctan(sqrt(x^2 - 2))*sgn(x)
```

maple [A] time = 0.03, size = 63, normalized size = 0.76

$$-\frac{\sqrt{x^4-2x^2}\,\left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right)-4\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{6\sqrt{x^2-2}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x)
```

```
[Out] -1/6*(x^4-2*x^2)^(1/2)*(arctan((x-2)/(x^2-2)^(1/2))-arctan((x+2)/(x^2-2)^(1
/2))-4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4-2\,x^2}}{\left(x^2+2\right)\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - 2*x^2)/((x^2 + 2)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4-2\,x^2}}{\left(x^2-1\right)\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)),x)
```

```
[Out] int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(x^2-2\right)}}{\left(x-1\right)\left(x+1\right)\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**2)**(1/2)/(x**2-1)/(x**2+2),x)
```

```
[Out] Integral(sqrt(x**2*(x**2 - 2))/((x - 1)*(x + 1)*(x**2 + 2)), x)
```

3.893 $\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx$

Optimal. Leaf size=47

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}}$$

[Out] (-x^2+1)*arctan((x^2-2)^(1/2))*(1-1/(-x^2+1)^2)^(1/2)/x/(x^2-2)^(1/2)

Rubi [A] time = 0.46, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6722, 6725, 1990, 1146, 21, 261, 444, 50, 63, 203}

$$\frac{(1-x^2)\sqrt{x^4-2x^2}\sqrt{1-\frac{1}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}\sqrt{(x^2-1)^2-1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]

[Out] ((1 - x^2)*Sqrt[-2*x^2 + x^4]*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2 + x^2]*Sqrt[-1 + (-1 + x^2)^2])

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2}\,dx = \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-1+\left(-1+x^2\right)^2}}{\left(2-x^2\right)\left(-1+x^2\right)}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \left(\frac{\sqrt{-1+\left(-1+x^2\right)^2}}{2-x^2}+\frac{\sqrt{-1+\left(-1+x^2\right)^2}}{-1+x^2}\right)dx}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-1+\left(-1+x^2\right)^2}}{2-x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-1+(-1}}{-1+x}}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{2-x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{-1+x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{x\sqrt{-2+x^2}}{2-x^2}\,dx}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-}\right.}{x\sqrt{-2+x^2}\sqrt{-1+}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x}}{-1+x}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}} - \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2}\right.}{x\sqrt{-2}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,(-1+x)}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(1-x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(1-x^2\right)^2}}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

Mathematica [A] time = 0.12, size = 91, normalized size = 1.94

$$\frac{1}{2}\tan^{-1}\left(\frac{(x-1)(x+1)(x+2)\sqrt{\frac{x^2\left(x^2-2\right)}{\left(x^2-1\right)^2}}}{x\left(x^2-2\right)}\right)-\frac{1}{2}\tan^{-1}\left(\frac{(x-2)(x-1)(x+1)\sqrt{\frac{x^2\left(x^2-2\right)}{\left(x^2-1\right)^2}}}{x\left(x^2-2\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

```
[Out] -1/2*ArcTan[((-2 + x)*(-1 + x)*(1 + x)*Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2])
/(x*(-2 + x^2))] + ArcTan[((-1 + x)*(1 + x)*(2 + x)*Sqrt[(x^2*(-2 + x^2))/(
-1 + x^2)^2])/(x*(-2 + x^2))]/2
```

fricas [A] time = 0.42, size = 36, normalized size = 0.77

$$-\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2x^2}{x^4-2x^2+1}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] -arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

giac [A] time = 0.34, size = 18, normalized size = 0.38

$$-\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(x^2 - 2))*sgn(x^3 - x)
```

maple [A] time = 0.03, size = 63, normalized size = 1.34

$$-\frac{\sqrt{\frac{(x^2-2)x^2}{(x^2-1)^2}}\,(x^2-1)\left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right)\right)}{2\sqrt{x^2-2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x)
```

```
[Out] -1/2*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x-2)/(x^2-2)^(1/2))-arc
tan((x+2)/(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{-\frac{1}{(x^2-1)^2}+1}}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\sqrt{1-\frac{1}{(x^2-1)^2}}}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2),x)
```

```
[Out] int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{\frac{x^4}{x^4-2x^2+1} - \frac{2x^2}{x^4-2x^2+1}}}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x**2-1)**2)**(1/2)/(-x**2+2),x)
```

```
[Out] -Integral(sqrt(x**4/(x**4 - 2*x**2 + 1) - 2*x**2/(x**4 - 2*x**2 + 1))/(x**2
 - 2), x)
```

3.894 $\int \frac{\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}}{2+x^2}\,dx$

Optimal. Leaf size=123

$$\frac{\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{\left(1-x^2\right)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}-\frac{2\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{\left(1-x^2\right)^2}}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

```
[Out] -2/3*(-x^2+1)*arctan(1/2*(x^2-2)^(1/2))*((x^4-2*x^2)/(-x^2+1)^2)^(1/2)/x/(x
^2-2)^(1/2)+1/3*(-x^2+1)*arctan((x^2-2)^(1/2))*((x^4-2*x^2)/(-x^2+1)^2)^(1/
2)/x/(x^2-2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6719, 2056, 571, 83, 63, 203}

$$\frac{\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{\left(1-x^2\right)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}-\frac{2\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{\left(1-x^2\right)^2}}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (-2*(1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan[Sqrt[-2 + x^2]/2])/
(3*x*Sqrt[-2 + x^2]) + ((1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan
[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}}{2+x^2}\,dx = \frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{\left(-1+x^2\right)\left(2+x^2\right)}\,dx}{\sqrt{-2x^2+x^4}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\int \frac{x\sqrt{-2+x^2}}{\left(-1+x^2\right)\left(2+x^2\right)}\,dx}{x\sqrt{-2+x^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)}\,dx, x, x^2\right)}{2x\sqrt{-2+x^2}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,(-1+x)}\,dx, x, x^2\right)}{6x\sqrt{-2+x^2}} + \frac{\left(2\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\text{Subst}}{3x\sqrt{-2+}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(4\left(-1+x^2\right)\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}\right)\text{Subst}}{3x\sqrt{-2+}}$$

$$= -\frac{2\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{\left(1-x^2\right)^2}}\,\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{\left(1-x^2\right)^2}}\,\tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.57

$$\frac{\sqrt{\frac{x^2\left(x^2-2\right)}{\left(x^2-1\right)^2}}\left(x^2-1\right)\left(2\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)-\tan^{-1}\left(\sqrt{x^2-2}\right)\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2]*(-1 + x^2)*(2*ArcTan[Sqrt[-2 + x^2]/2]
 - ArcTan[Sqrt[-2 + x^2]]))/(3*x*Sqrt[-2 + x^2])
```

fricas [A] time = 0.42, size = 74, normalized size = 0.60

$$-\frac{1}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2x^2}{x^4-2x^2+1}}}{x}\right)+\frac{2}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2x^2}{x^4-2x^2+1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x) + 2/3*arctan
(1/2*(x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

giac [A] time = 0.41, size = 39, normalized size = 0.32

$$\frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)-\frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(1/2*sqrt(x^2 - 2))*sgn(x^3 - x) - 1/3*arctan(sqrt(x^2 - 2))*sgn(
x^3 - x)
```

maple [A] time = 0.03, size = 75, normalized size = 0.61

$$-\frac{\sqrt{\frac{(x^2-2)x^2}{(x^2-1)^2}}\,(x^2-1)\left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right)-4\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{6\sqrt{x^2-2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x)
```

```
[Out] -1/6*((x^2-2)/(x^2-1)^2*x^2)^(1/2)*(x^2-1)*(arctan((x-2)/(x^2-2)^(1/2))-arc
tan((x+2)/(x^2-2)^(1/2))-4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^4-2x^2}{(x^2-1)^2}}}{x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^4 - 2*x^2)/(x^2 - 1)^2)/(x^2 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-\frac{2x^2-x^4}{(x^2-1)^2}}}{x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2),x)
```

```
[Out] int((-(2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**4-2*x**2)/(x**2-1)**2)**(1/2)/(x**2+2),x)
```

```
[Out] Timed out
```

3.895 $\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx$

Optimal. Leaf size=133

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)^3}{3\left(x^2+1\right)}-\frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)-\frac{(3x+4)\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh}{x+1}$$

```
[Out] -4/3*(1-2*x)*(1+x)*(1+2*x/(x^2+1))^(1/2)-1/3*(1-x)*(1+x)^3*(1+2*x/(x^2+1))^
(1/2)/(x^2+1)-(4+3*x)*(x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+5*arcsinh(x)*(x^2
+1)^(1/2)*(1+2*x/(x^2+1))^(1/2)/(1+x)
```

Rubi [A] time = 0.07, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6723, 970, 739, 819, 780, 215}

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)^3}{3\left(x^2+1\right)}-\frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)-\frac{(3x+4)\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

```
[Out] (-4*(1 - 2*x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/3 - ((1 - x)*(1 + x)^3*Sqr
t[1 + (2*x)/(1 + x^2)])/(3*(1 + x^2)) - ((4 + 3*x)*(1 + x^2)*Sqrt[1 + (2*x)
/(1 + x^2)])/(1 + x) + (5*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x
])/(1 + x)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
```

```
!ILtQ[m + 2*p + 3, 0])
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\left(1+2x+x^2\right)^{5/2}}{\left(1+x^2\right)^{5/2}}dx}{\sqrt{1+2x+x^2}}$$

$$= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(2+2x)^5}{\left(1+x^2\right)^{5/2}}dx}{16(2+2x)}$$

$$= -\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} + \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(24-8x)(2+2x)^3}{\left(1+x^2\right)^{3/2}}dx}{48(2+2x)}$$

$$= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} + \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int}{48(2+}$$

$$= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} - \frac{(4+3x)\left(1+x^2\right)\sqrt{1+}}{1+x}$$

$$= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} - \frac{(4+3x)\left(1+x^2\right)\sqrt{1+}}{1+x}$$

Mathematica [A] time = 0.10, size = 64, normalized size = 0.48

$$\frac{(x+1)\left(3x^4-8x^3-18x^2+15\left(x^2+1\right)^{3/2}\sinh^{-1}(x)-12x-17\right)}{3\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

```
[Out] ((1 + x)*(-17 - 12*x - 18*x^2 - 8*x^3 + 3*x^4 + 15*(1 + x^2)^(3/2)*ArcSinh[
x]))/(3*Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2)^2)
```

fricas [A] time = 0.49, size = 117, normalized size = 0.88

$$-\frac{8\,x^3+8\,x^2+15\,\left(x^3+x^2+x+1\right)\log\left(-\frac{x^2-\left(x^2+1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-\left(3\,x^4-8\,x^3-18\,x^2-12\,x-17\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+8\,x}{3\,\left(x^3+x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*x^3 + 8*x^2 + 15*(x^3 + x^2 + x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^
2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) - (3*x^4 - 8*x^3 - 18*x^2 - 12*x - 17
)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 8*x + 8)/(x^3 + x^2 + x + 1)
```

giac [A] time = 0.47, size = 86, normalized size = 0.65

$$\left(\sqrt{2}+5\,\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}\left(x+1\right)-5\,\log\left(-x+\sqrt{x^2+1}\right)\mathrm{sgn}\left(x+1\right)+\frac{\left(\left(\left(3\,x\mathrm{sgn}\left(x+1\right)-8\,\mathrm{sgn}\left(x+1\right)\right)x-18\,\mathrm{s}\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] (sqrt(2) + 5*log(sqrt(2) + 1))*sgn(x + 1) - 5*log(-x + sqrt(x^2 + 1))*sgn(x
 + 1) + 1/3*((((3*x*sgn(x + 1) - 8*sgn(x + 1))*x - 18*sgn(x + 1))*x - 12*sg
n(x + 1))*x - 17*sgn(x + 1))/(x^2 + 1)^(3/2)
```

maple [A] time = 0.02, size = 62, normalized size = 0.47

$$\frac{\left(\frac{x^2+2x+1}{x^2+1}\right)^{\frac{5}{2}}\left(x^2+1\right)\left(3x^4-8x^3-18x^2-12x+15\left(x^2+1\right)^{\frac{3}{2}}\operatorname{arcsinh}(x)-17\right)}{3\left(x+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2/(x^2+1)*x)^(5/2),x)
```

```
[Out] 1/3*((x^2+2*x+1)/(x^2+1))^(5/2)/(x+1)^5*(x^2+1)*(15*arcsinh(x)*(x^2+1)^(3/2
)+3*x^4-8*x^3-18*x^2-12*x-17)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{2\,x}{x^2+1}+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(\frac{2\,x}{x^2+1}+1\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x)/(x^2 + 1) + 1)^(5/2),x)
```

```
[Out] int(((2*x)/(x^2 + 1) + 1)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{2x}{x^2+1}+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(5/2),x)
[Out] Integral((2*x/(x**2 + 1) + 1)**(5/2), x)
```

3.896 $\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx$

Optimal. Leaf size=90

$$-\left((1-x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)\right)-\frac{x\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{3\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\,\sinh^{-1}(x)}{x+1}$$

```
[Out] -(1-x)*(1+x)*(1+2*x/(x^2+1))^(1/2)-x*(x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+3*
arcsinh(x)*(x^2+1)^(1/2)*(1+2*x/(x^2+1))^(1/2)/(1+x)
```

Rubi [A] time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6723, 970, 739, 517, 388, 215}

$$-(1-x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)-\frac{x\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{3\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\,\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] -((1 - x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)]) - (x*(1 + x^2)*Sqrt[1 + (2*x)/
(1 + x^2)])/(1 + x) + (3*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x]
)/(1 + x)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
```

```
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx &= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\left(1+2x+x^2\right)^{3/2}}{\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{1+2x+x^2}}\\
&= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(2+2x)^3}{\left(1+x^2\right)^{3/2}}\,dx}{4(2+2x)}\\
&= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}+\frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(8-8x)(2+2x)}{\sqrt{1+x^2}}\,dx}{4(2+2x)}\\
&= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}+\frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{16-16x^2}{\sqrt{1+x^2}}\,dx}{4(2+2x)}\\
&= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{x\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\left(6\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{1}{\sqrt{1+x}}\cdots}{2+2x}\\
&= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{x\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{3\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\sinh^{-1}(x\cdots}{1+x}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.49

$$\frac{\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+3\sqrt{x^2+1}\sinh^{-1}(x)-2x-1\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(-1 - 2*x + x^2 + 3*Sqrt[1 + x^2]*ArcSinh[x]))/(
1 + x)
```

fricas [A] time = 0.41, size = 83, normalized size = 0.92

$$-\frac{3\,(x+1)\log\left(-\frac{x^2-\left(x^2+1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-\left(x^2-2\,x-1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+2\,x+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(3*(x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x +
 1)) - (x^2 - 2*x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 2*x + 2)/(x + 1)
```

giac [A] time = 0.37, size = 67, normalized size = 0.74

$$-\left(\sqrt{2}-3\,\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}\left(x+1\right)-3\,\log\left(-x+\sqrt{x^2+1}\right)\mathrm{sgn}\left(x+1\right)+\frac{\left(x\mathrm{sgn}\left(x+1\right)-2\,\mathrm{sgn}\left(x+1\right)\right)x-\mathrm{sgn}\left(x\right.}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2) - 3*log(sqrt(2) + 1))*sgn(x + 1) - 3*log(-x + sqrt(x^2 + 1))*sgn(
x + 1) + ((x*sgn(x + 1) - 2*sgn(x + 1))*x - sgn(x + 1))/sqrt(x^2 + 1)
```

maple [A] time = 0.02, size = 49, normalized size = 0.54

$$\frac{\left(\frac{x^2+2x+1}{x^2+1}\right)^{\frac{3}{2}}\left(x^2+1\right)\left(x^2-2x+3\sqrt{x^2+1}\,\operatorname{arcsinh}(x)-1\right)}{\left(x+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2/(x^2+1)*x)^(3/2),x)
```

```
[Out] ((x^2+2*x+1)/(x^2+1))^(3/2)/(x+1)^3*(x^2+1)*(3*arcsinh(x)*(x^2+1)^(1/2)+x^2
-2*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{2\,x}{x^2+1}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(\frac{2\,x}{x^2+1}+1\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x)/(x^2 + 1) + 1)^(3/2),x)
```

```
[Out] int(((2*x)/(x^2 + 1) + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(3/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(3/2), x)
```

3.897 $\int \sqrt{1+\frac{2x}{1+x^2}}\,dx$

Optimal. Leaf size=61

$$\frac{\sqrt{\frac{2x}{x^2+1}+1}\,\left(x^2+1\right)}{x+1}+\frac{\sqrt{\frac{2x}{x^2+1}+1}\,\sqrt{x^2+1}\,\sinh^{-1}(x)}{x+1}$$

```
[Out] (x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+arcsinh(x)*(x^2+1)^(1/2)*(1+2*x/(x^2+1)
)^(1/2)/(1+x)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6723, 970, 641, 215}

$$\frac{\sqrt{\frac{2x}{x^2+1}+1}\,\left(x^2+1\right)}{x+1}+\frac{\sqrt{\frac{2x}{x^2+1}+1}\,\sqrt{x^2+1}\,\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (Sqrt[1 + x^2]*Sqrt[1 + (2*
x)/(1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx = \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx}{\sqrt{1+2x+x^2}}$$
$$= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{2+2x}{\sqrt{1+x^2}}\,dx}{2+2x}$$
$$= \frac{(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{\left(2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2+2x}$$
$$= \frac{(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\sinh^{-1}(x)}{1+x}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.66

$$\frac{\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+\sqrt{x^2+1}\sinh^{-1}(x)+1\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)],x]
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2 + Sqrt[1 + x^2]*ArcSinh[x]))/(1 + x)
```

fricas [A] time = 0.41, size = 75, normalized size = 1.23

$$-\frac{(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x}{x+1}\right)-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")
[Out] -((x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1
)) - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

giac [A] time = 0.48, size = 49, normalized size = 0.80

$$-\left(\sqrt{2}-\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}\,(x+1)-\log\left(-x+\sqrt{x^2+1}\right)\mathrm{sgn}\,(x+1)+\sqrt{x^2+1}\,\mathrm{sgn}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="giac")
[Out] -(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x +
1) + sqrt(x^2 + 1)*sgn(x + 1)
```

maple [A] time = 0.01, size = 42, normalized size = 0.69

$$\frac{\sqrt{\frac{x^2+2x+1}{x^2+1}}\sqrt{x^2+1}\left(\mathrm{arcsinh}(x)+\sqrt{x^2+1}\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2/(x^2+1)*x)^(1/2),x)
```

```
[Out] ((x^2+2*x+1)/(x^2+1))^(1/2)/(x+1)*(x^2+1)^(1/2)*((x^2+1)^(1/2)+arcsinh(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{2\,x}{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(2*x/(x^2 + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{2\,x}{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x)/(x^2 + 1) + 1)^(1/2),x)
[Out] int(((2*x)/(x^2 + 1) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{2x}{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(1/2),x)
[Out] Integral(sqrt(2*x/(x**2 + 1) + 1), x)
```

3.898 $\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}} dx$

Optimal. Leaf size=109

$$\frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}}-\frac{(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{\sqrt{2}(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

```
[Out] (1+x)/(1+2*x/(x^2+1))^(1/2)-(1+x)*arcsinh(x)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))^
(1/2)-(1+x)*arctanh(1/2*(1-x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)/(x^2+1)^(1/2)/
(1+2*x/(x^2+1))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6723, 970, 735, 844, 215, 725, 206}

$$\frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}}-\frac{(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{\sqrt{2}(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] (1 + x)/Sqrt[1 + (2*x)/(1 + x^2)] - ((1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqr
t[1 + (2*x)/(1 + x^2)]) - (Sqrt[2]*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1
+ x^2])])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx &= \frac{\sqrt{1+2x+x^2}\int \frac{\sqrt{1+x^2}}{\sqrt{1+2x+x^2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}\\
&= \frac{(2+2x)\int \frac{\sqrt{1+x^2}}{2+2x}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}\\
&= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}}+\frac{(2+2x)\int \frac{2-2x}{(2+2x)\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}\\
&= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}}-\frac{(2+2x)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}+\frac{(2(2+2x))\int \frac{1}{(2+2x)\sqrt{1+x^2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}\\
&= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}}-\frac{(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}-\frac{(2(2+2x))\,\text{Subst}\left(\int \frac{1}{8-x^2}\,dx,x,\frac{2-2x}{\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}\\
&= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}}-\frac{(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}-\frac{\sqrt{2}(1+x)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.66

$$\frac{(x+1)\left(\sqrt{x^2+1}-\sqrt{2}\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)-\sinh^{-1}(x)\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x)*(Sqrt[1 + x^2] - ArcSinh[x] - Sqrt[2]*ArcTanh[(1 - x)/(Sqrt[2]*Sqr
t[1 + x^2])]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])
```

fricas [A] time = 0.42, size = 142, normalized size = 1.30

$$\frac{\sqrt{2}\,(x+1)\log\left(-\frac{x^2+\sqrt{2}\left(x^2-1\right)+\left(2\,x^2+\sqrt{2}\left(x^2+1\right)+2\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-1}{x^2+2\,x+1}\right)+(x+1)\log\left(-\frac{x^2-\left(x^2+1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)+\left(x^2+1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(x + 1)*log(-(x^2 + sqrt(2)*(x^2 - 1) + (2*x^2 + sqrt(2)*(x^2 + 1)
 + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(x^2 + 2*x + 1)) + (x + 1)*log(-
(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + (x^2 + 1)*
sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

giac [A] time = 0.57, size = 88, normalized size = 0.81

$$\frac{\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+1}-2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+1}-2\right|}\right)}{\operatorname{sgn}(x+1)}+\frac{\log\left(-x+\sqrt{x^2+1}\right)}{\operatorname{sgn}(x+1)}+\frac{\sqrt{x^2+1}}{\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2)/abs(-2*x + 2*sqrt(2
) + 2*sqrt(x^2 + 1) - 2))/sgn(x + 1) + log(-x + sqrt(x^2 + 1))/sgn(x + 1) +
 sqrt(x^2 + 1)/sgn(x + 1)
```

maple [A] time = 0.04, size = 79, normalized size = 0.72

$$\frac{x+1}{\sqrt{\frac{(x+1)^2}{x^2+1}}}+\frac{\left(-\operatorname{arcsinh}(x)-\sqrt{2}\;\operatorname{arctanh}\left(\frac{(-2x+2)\sqrt{2}}{4\sqrt{-2x+(x+1)^2}}\right)\right)(x+1)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\;\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2/(x^2+1)*x)^(1/2),x)
```

```
[Out] 1/((x+1)^2/(x^2+1))^(1/2)*(x+1)+(-arcsinh(x)-2^(1/2)*arctanh(1/4*(2-2*x)*2^
(1/2)/((x+1)^2-2*x)^(1/2)))/((x+1)^2/(x^2+1))^(1/2)/(x^2+1)^(1/2)*(x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{2\,x}{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x/(x^2 + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{2\,x}{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x)/(x^2 + 1) + 1)^(1/2),x)
```

```
[Out] int(1/((2*x)/(x^2 + 1) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x**2+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x/(x**2 + 1) + 1), x)
```

3.899 $\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}+1}}-\frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}+1}}-\frac{3(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{9(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

```
[Out] 3/2*(2+x)/(1+2*x/(x^2+1))^(1/2)+1/2*(-x^2-1)/(1+x)/(1+2*x/(x^2+1))^(1/2)-3*
(1+x)*arcsinh(x)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))^(1/2)-9/4*(1+x)*arctanh(1/2*
(1-x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6723, 970, 733, 813, 844, 215, 725, 206}

$$\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}+1}}-\frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}+1}}-\frac{3(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{9(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] (3*(2 + x))/(2*Sqrt[1 + (2*x)/(1 + x^2)]) - (1 + x^2)/(2*(1 + x)*Sqrt[1 + (
2*x)/(1 + x^2)]) - (3*(1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1
+ x^2)]) - (9*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1 + x^2])])/(2*Sqrt[2]*
Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1+2x+x^2}\int \frac{\left(1+x^2\right)^{3/2}}{\left(1+2x+x^2\right)^{3/2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{(4(2+2x))\int \frac{\left(1+x^2\right)^{3/2}}{(2+2x)^3}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= -\frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(3(2+2x))\int \frac{x\sqrt{1+x^2}}{(2+2x)^2}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(3(2+2x))\int \frac{-4+8x}{(2+2x)\sqrt{1+x^2}}\,dx}{8\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(3(2+2x))\int \frac{1}{\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(9(2+2x))\int \frac{1}{(2+2x)\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{3(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(9(2+2x))\,\mathrm{Subst}\left(\int \frac{1}{8-x^2}\,dx, x\right.}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{3(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{9(1+x)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{2\sqrt{2}\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

Mathematica [A] time = 0.12, size = 95, normalized size = 0.66

$$\frac{(x+1)\left(2\sqrt{x^2+1}\left(2x^2+9x+5\right)+9\sqrt{2}(x+1)^2\tanh^{-1}\left(\frac{x-1}{\sqrt{2}\sqrt{x^2+1}}\right)-12(x+1)^2\sinh^{-1}(x)\right)}{4\left(\frac{(x+1)^2}{x^2+1}\right)^{3/2}\left(x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] ((1 + x)*(2*Sqrt[1 + x^2]*(5 + 9*x + 2*x^2) - 12*(1 + x)^2*ArcSinh[x] + 9*S
qrt[2]*(1 + x)^2*ArcTanh[(-1 + x)/(Sqrt[2]*Sqrt[1 + x^2])]))/(4*((1 + x)^2/
(1 + x^2))^(3/2)*(1 + x^2)^(3/2))
```

fricas [A] time = 0.43, size = 205, normalized size = 1.42

$$\frac{10\,x^3+9\,\sqrt{2}\left(x^3+3\,x^2+3\,x+1\right)\log\left(-\frac{x^2+\sqrt{2}\left(x^2-1\right)+\left(2\,x^2+\sqrt{2}\left(x^2+1\right)+2\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-1}{x^2+2\,x+1}\right)+30\,x^2+12\left(x^3+3\,x^2+3\,x+\right.}{4\left(x^3+3\,x^2+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x^3 + 9*sqrt(2)*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 + sqrt(2)*(x^2 -
1) + (2*x^2 + sqrt(2)*(x^2 + 1) + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(
```

```
x^2 + 2*x + 1)) + 30*x^2 + 12*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 - (x^2 + 1)
*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + 2*(2*x^4 + 9*x^3 + 7*x^2 +
 9*x + 5)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 30*x + 10)/(x^3 + 3*x^2 + 3*x +
 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)
```

maple [A] time = 0.02, size = 218, normalized size = 1.51

$$(x+1)\left(-\left(x^2+1\right)^{\frac{3}{2}}x^3-6\sqrt{x^2+1}\,x^3-24x^2\operatorname{arcsinh}(x)+18\sqrt{2}\,x^2\operatorname{arctanh}\left(\frac{(x-1)\sqrt{2}}{2\sqrt{x^2+1}}\right)+\left(x^2+1\right)^{\frac{3}{2}}x^2+6\sqrt{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2/(x^2+1)*x)^(3/2),x)
```

```
[Out] 1/8/((x^2+2*x+1)/(x^2+1))^(3/2)*(x+1)*((x^2+1)^(5/2)*x-(x^2+1)^(3/2)*x^3-(x
^2+1)^(5/2)+(x^2+1)^(3/2)*x^2+18*2^(1/2)*arctanh(1/2*(x-1)*2^(1/2)/(x^2+1)^
(1/2))*x^2+5*x*(x^2+1)^(3/2)-6*(x^2+1)^(1/2)*x^3+36*2^(1/2)*arctanh(1/2*(x-
1)*2^(1/2)/(x^2+1)^(1/2))*x+3*(x^2+1)^(3/2)+6*(x^2+1)^(1/2)*x^2-24*arcsinh(
x)*x^2+18*2^(1/2)*arctanh(1/2*(x-1)*2^(1/2)/(x^2+1)^(1/2))+30*(x^2+1)^(1/2)
*x-48*arcsinh(x)*x+18*(x^2+1)^(1/2)-24*arcsinh(x))/(x^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x)/(x^2 + 1) + 1)^(3/2),x)
```

```
[Out] int(1/((2*x)/(x^2 + 1) + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x**2+1))**(3/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(-3/2), x)
```

3.900 $\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2} \, dx$

Optimal. Leaf size=28

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

[Out] -(1-x)*(1+2*x/(x^2+1))^(1/2)/(1+x)

Rubi [A] time = 0.12, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6723, 970, 637}

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] -(((1 - x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2} \, dx &= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\sqrt{1+2x+x^2}}{\left(1+x^2\right)^{3/2}} \, dx}{\sqrt{1+2x+x^2}} \\
&= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{2+2x}{\left(1+x^2\right)^{3/2}} \, dx}{2+2x} \\
&= -\frac{(1-x)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.93

$$\frac{(x-1)\sqrt{\frac{(x+1)^2}{x^2+1}}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
[Out] ((-1 + x)*Sqrt[(1 + x)^2/(1 + x^2)])/(1 + x)
```

fricas [A] time = 0.41, size = 31, normalized size = 1.11

$$\frac{(x-1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] ((x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x + 1)/(x + 1)
```

giac [A] time = 0.39, size = 30, normalized size = 1.07

$$\sqrt{2}\,\mathrm{sgn}(x+1)+\frac{x\mathrm{sgn}(x+1)-\mathrm{sgn}(x+1)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] sqrt(2)*sgn(x + 1) + (x*sgn(x + 1) - sgn(x + 1))/sqrt(x^2 + 1)
```

maple [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{(x-1)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2/(x^2+1)*x)^(1/2)/(x^2+1),x)
[Out] (x-1)/(x+1)*((x^2+2*x+1)/(x^2+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{2x}{x^2+1}+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt(2*x/(x^2 + 1) + 1)/(x^2 + 1), x)
```

mupad [B] time = 3.52, size = 23, normalized size = 0.82

$$\frac{\sqrt{\frac{2x}{x^2+1}+1}\,(x-1)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x)/(x^2 + 1) + 1)^(1/2)/(x^2 + 1),x)
[Out] (((2*x)/(x^2 + 1) + 1)^(1/2)*(x - 1))/(x + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x+1)^2}{x^2+1}}}{x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x + 1)**2/(x**2 + 1))/(x**2 + 1), x)
```

3.901 $\int \sqrt{x-x^2} F(x) \, dx$

Optimal. Leaf size=17

$$\text{Int}\left(\sqrt{x-x^2} F(x),x\right)$$

```
[Out] CannotIntegrate(F(x)*(-x^2+x)^(1/2),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{x-x^2} F(x) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[x - x^2]*F[x],x]
[Out] Defer[Int][Sqrt[x - x^2]*F[x], x]
```

Rubi steps

$$\int \sqrt{x-x^2} F(x) \, dx = \int \sqrt{x-x^2} F(x) \, dx$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \sqrt{x-x^2} F(x) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[x - x^2]*F[x],x]
[Out] Integrate[Sqrt[x - x^2]*F[x], x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^2+x} F(x),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^2 + x)*F(x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2+x} F(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + x)*F(x), x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2+x} \, F(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)*(-x^2+x)^(1/2),x)
[Out] int(F(x)*(-x^2+x)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2+x}\,F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + x)*F(x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int F(x)\,\sqrt{x-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)*(x - x^2)^(1/2),x)
[Out] int(F(x)*(x - x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x\,(x-1)}\,F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x**2+x)**(1/2),x)
[Out] Integral(sqrt(-x*(x - 1))*F(x), x)
```

3.902 $\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{F(x)}{\sqrt{x-x^2}},x\right)$$

```
[Out] CannotIntegrate(F(x)/(-x^2+x)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F[x]/Sqrt[x - x^2],x]
[Out] Defer[Int][F[x]/Sqrt[x - x^2], x]
```

Rubi steps

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Mathematica [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[x]/Sqrt[x - x^2],x]
[Out] Integrate[F[x]/Sqrt[x - x^2], x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^2+x}\,F(x)}{x^2-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^2 + x)*F(x)/(x^2 - x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="giac")
[Out] integrate(F(x)/sqrt(-x^2 + x), x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)/(-x^2+x)^(1/2),x)
[Out] int(F(x)/(-x^2+x)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="maxima")
[Out] integrate(F(x)/sqrt(-x^2 + x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)/(x - x^2)^(1/2),x)
[Out] int(F(x)/(x - x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{-x\,(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x**2+x)**(1/2),x)
[Out] Integral(F(x)/sqrt(-x*(x - 1)), x)
```

3.903 $\int \sqrt{1-x}\,\sqrt{x}\,F(x)\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\sqrt{x-x^2}\,F(x),x\right)$$

```
[Out] CannotIntegrate(F(x)*(-x^2+x)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{1-x}\,\sqrt{x}\,F(x)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - x]*Sqrt[x]*F[x],x]
[Out] Defer[Int][Sqrt[x - x^2]*F[x], x]
```

Rubi steps

$$\int \sqrt{1-x}\,\sqrt{x}\,F(x)\,dx=\int \sqrt{x-x^2}\,F(x)\,dx$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{1-x}\,\sqrt{x}\,F(x)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - x]*Sqrt[x]*F[x],x]
[Out] Integrate[Sqrt[1 - x]*Sqrt[x]*F[x], x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{x}\,\sqrt{-x+1}\,F(x),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\sqrt{-x+1}\,F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{-x+1}\,\sqrt{x}\,F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)*(-x+1)^(1/2)*x^(1/2),x)
```

```
[Out] int(F(x)*(-x+1)^(1/2)*x^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\sqrt{-x+1}\,F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \sqrt{x}\,F(x)\,\sqrt{1-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*F(x)*(1 - x)^(1/2),x)
```

```
[Out] int(x^(1/2)*F(x)*(1 - x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\sqrt{1-x}\,F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)**(1/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(1 - x)*F(x), x)
```

3.904 $\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{F(x)}{\sqrt{x-x^2}},x\right)$$

```
[Out] CannotIntegrate(F(x)/(-x^2+x)^(1/2),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]
[Out] Defer[Int][F[x]/Sqrt[x - x^2], x]
```

Rubi steps

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Mathematica [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]
[Out] Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]), x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{x}\sqrt{-x+1}\,F(x)}{x^2-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(x)*sqrt(-x + 1)*F(x)/(x^2 - x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

maple [A] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{-x+1}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)/(-x+1)^(1/2)/x^(1/2),x)
[Out] int(F(x)/(-x+1)^(1/2)/x^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{x}\,\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{F(x)}{\sqrt{x}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)/(x^(1/2)*(1 - x)^(1/2)),x)
[Out] int(F(x)/(x^(1/2)*(1 - x)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{x}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)**(1/2)/x**(1/2),x)
[Out] Integral(F(x)/(sqrt(x)*sqrt(1 - x)), x)
```

3.905 $\int F\left(\frac{a+bx}{x}\right) dx$

Optimal. Leaf size=11

$$\text{Int}\left(F\left(\frac{a}{x}+b\right),x\right)$$

```
[Out] CannotIntegrate(F(a/x+b),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{a+bx}{x}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[F[(a + b*x)/x],x]
[Out] Defer[Int][F[b + a/x], x]
```

Rubi steps

$$\int F\left(\frac{a+bx}{x}\right) dx = \int F\left(b+\frac{a}{x}\right) dx$$

Mathematica [A] time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{a+bx}{x}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[(a + b*x)/x],x]
[Out] Integrate[F[(a + b*x)/x], x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(F\left(\frac{bx+a}{x}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x, algorithm="fricas")
[Out] integral(F((b*x + a)/x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{bx+a}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x, algorithm="giac")
[Out] integrate(F((b*x + a)/x), x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int F\left(\frac{bx+a}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F((b*x+a)/x),x)
[Out] int(F((b*x+a)/x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{bx+a}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x, algorithm="maxima")
[Out] integrate(F((b*x + a)/x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.09

$$\int F\left(\frac{a+b\,x}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F((a + b*x)/x),x)
[Out] int(F((a + b*x)/x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{a+bx}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x)
[Out] Integral(F((a + b*x)/x), x)
```

3.906 $\int F\left(\frac{a+bx^2}{x^2}\right) dx$

Optimal. Leaf size=11

$$\text{Int}\left(F\left(\frac{a}{x^2}+b\right),x\right)$$

```
[Out] CannotIntegrate(F(b+a/x^2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[F[(a + b*x^2)/x^2],x]
```

```
[Out] Defer[Int][F[b + a/x^2], x]
```

Rubi steps

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx = \int F\left(b+\frac{a}{x^2}\right) dx$$

Mathematica [A] time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[(a + b*x^2)/x^2],x]
```

```
[Out] Integrate[F[(a + b*x^2)/x^2], x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(F\left(\frac{bx^2+a}{x^2}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="fricas")
```

```
[Out] integral(F((b*x^2 + a)/x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{bx^2+a}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="giac")
```

```
[Out] integrate(F((b*x^2 + a)/x^2), x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int F\left(\frac{b\,x^{2}+a}{x^{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F((b*x^2+a)/x^2),x)
[Out] int(F((b*x^2+a)/x^2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{bx^{2}+a}{x^{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="maxima")
[Out] integrate(F((b*x^2 + a)/x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.09

$$\int F\left(\frac{b\,x^{2}+a}{x^{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F((a + b*x^2)/x^2),x)
[Out] int(F((a + b*x^2)/x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{a+bx^{2}}{x^{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x**2+a)/x**2),x)
[Out] Integral(F((a + b*x**2)/x**2), x)
```

3.907 $\int F\left(\frac{x}{a+bx}\right) dx$

Optimal. Leaf size=13

$$\text{Int}\left(F\left(\frac{x}{a+bx}\right),x\right)$$

[Out] CannotIntegrate(F(x/(b*x+a)),x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{x}{a+bx}\right) dx$$

Verification is Not applicable to the result.

[In] Int[F[x/(a + b*x)],x]

[Out] Defer[Int][F[x/(a + b*x)], x]

Rubi steps

$$\int F\left(\frac{x}{a+bx}\right) dx = \int F\left(\frac{x}{a+bx}\right) dx$$

Mathematica [A] time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x}{a+bx}\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[F[x/(a + b*x)],x]

[Out] Integrate[F[x/(a + b*x)], x]

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(F\left(\frac{x}{bx+a}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F(x/(b*x+a)),x, algorithm="fricas")

[Out] integral(F(x/(b*x + a)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x}{bx+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F(x/(b*x+a)),x, algorithm="giac")

[Out] integrate(F(x/(b*x + a)), x)

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int F\left(\frac{x}{bx+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x/(b*x+a)),x)
```

```
[Out] int(F(x/(b*x+a)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x}{bx+a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(F(x/(b*x + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int F\left(\frac{x}{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x/(a + b*x)),x)
```

```
[Out] int(F(x/(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x}{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x+a)),x)
```

```
[Out] Integral(F(x/(a + b*x)), x)
```

3.908 $\int F\left(\frac{x^2}{a+bx^2}\right) dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(F\left(\frac{x^2}{a+bx^2}\right),x\right)$$

```
[Out] CannotIntegrate(F(x^2/(b*x^2+a)),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[F[x^2/(a + b*x^2)],x]
[Out] Defer[Int][F[x^2/(a + b*x^2)], x]
```

Rubi steps

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx = \int F\left(\frac{x^2}{a+bx^2}\right) dx$$

Mathematica [A] time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[x^2/(a + b*x^2)],x]
[Out] Integrate[F[x^2/(a + b*x^2)], x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(F\left(\frac{x^2}{bx^2+a}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="fricas")
[Out] integral(F(x^2/(b*x^2 + a)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{bx^2+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="giac")
[Out] integrate(F(x^2/(b*x^2 + a)), x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{b\,x^2+a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^2/(b*x^2+a)),x)
[Out] int(F(x^2/(b*x^2+a)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{bx^2+a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="maxima")
[Out] integrate(F(x^2/(b*x^2 + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int F\left(\frac{x^2}{b\,x^2+a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^2/(a + b*x^2)),x)
[Out] int(F(x^2/(a + b*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{a+bx^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x**2/(b*x**2+a)),x)
[Out] Integral(F(x**2/(a + b*x**2)), x)
```

3.909 $\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(F\left(\frac{x^2}{(a+bx)^2}\right),x\right)$$

```
[Out] CannotIntegrate(F(x^2/(b*x+a)^2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[F[x^2/(a + b*x)^2],x]
[Out] Defer[Int][F[x^2/(a + b*x)^2], x]
```

Rubi steps

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx = \int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Mathematica [A] time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[x^2/(a + b*x)^2],x]
[Out] Integrate[F[x^2/(a + b*x)^2], x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(F\left(\frac{x^2}{b^2x^2+2\,abx+a^2}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="fricas")
[Out] integral(F(x^2/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{(bx+a)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="giac")
[Out] integrate(F(x^2/(b*x + a)^2), x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{(bx+a)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^2/(b*x+a)^2),x)
[Out] int(F(x^2/(b*x+a)^2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{(bx+a)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="maxima")
[Out] integrate(F(x^2/(b*x + a)^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int F\left(\frac{x^2}{(a+b\,x)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^2/(a + b*x)^2),x)
[Out] int(F(x^2/(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{(a+bx)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x**2/(b*x+a)**2),x)
[Out] Integral(F(x**2/(a + b*x)**2), x)
```

3.910 $\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$

Optimal. Leaf size=17

$$\text{Int}\left(F\left(\frac{x^4}{(a+bx^2)^2}\right),x\right)$$

```
[Out] CannotIntegrate(F(x^4/(b*x^2+a)^2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[F[x^4/(a + b*x^2)^2],x]
[Out] Defer[Int][F[x^4/(a + b*x^2)^2], x]
```

Rubi steps

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx = \int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Mathematica [A] time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[x^4/(a + b*x^2)^2],x]
[Out] Integrate[F[x^4/(a + b*x^2)^2], x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(F\left(\frac{x^4}{b^2x^4+2\,abx^2+a^2}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="fricas")
[Out] integral(F(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^4}{(bx^2+a)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="giac")
```

```
[Out] integrate(F(x^4/(b*x^2 + a)^2), x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^4}{\left(b\,x^2+a\right)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^4/(b*x^2+a)^2),x)
```

```
[Out] int(F(x^4/(b*x^2+a)^2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^4}{\left(bx^2+a\right)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="maxima")
```

```
[Out] integrate(F(x^4/(b*x^2 + a)^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int F\left(\frac{x^4}{\left(b\,x^2+a\right)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^4/(a + b*x^2)^2),x)
```

```
[Out] int(F(x^4/(a + b*x^2)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^4}{\left(a+bx^2\right)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x**4/(b*x**2+a)**2),x)
```

```
[Out] Integral(F(x**4/(a + b*x**2)**2), x)
```

3.911 $\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)*b^(1/2)/(b*x^2+(b^2*x^4+a)^(1/2))^(1/2))*2^(1/2)/b^(1
/2)
```

Rubi [A] time = 0.11, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2132, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[
b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1-2bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[
b])
```

fricas [A] time = 1.81, size = 135, normalized size = 2.87

$$\left[\frac{\sqrt{2}\log\left(4\,b^2x^4+4\,\sqrt{b^2x^4+a}\,bx^2+2\left(\sqrt{2}\,b^{\frac{3}{2}}x^3+\sqrt{2}\,\sqrt{b^2x^4+a}\,\sqrt{b}\,x\right)\sqrt{bx^2+\sqrt{b^2x^4+a}}+a\right)}{4\,\sqrt{b}},-\frac{1}{2}\,\sqrt{2}\,\sqrt{-\frac{1}{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/4*sqrt(2)*log(4*b^2*x^4 + 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt(2)*b^(3/2)
*x^3 + sqrt(2)*sqrt(b^2*x^4 + a)*sqrt(b)*x)*sqrt(b*x^2 + sqrt(b^2*x^4 + a))
 + a)/sqrt(b), -1/2*sqrt(2)*sqrt(-1/b)*arctan(1/2*sqrt(2)*sqrt(b*x^2 + sqrt
(b^2*x^4 + a))*sqrt(-1/b)/x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

```
[Out] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{b^2\,x^4+a}+b\,x^2}}{\sqrt{b^2\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)
[Out] int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
[Out] Integral(sqrt(b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

3.912 $\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[Out] 1/2*arctan(x*2^(1/2)*b^(1/2)/(-b*x^2+(b^2*x^4+a)^(1/2))^(1/2))*2^(1/2)/b^(1
/2)
```

Rubi [A] time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2132, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqr
t[b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1+2bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqr
t[b])
```

fricas [A] time = 1.80, size = 146, normalized size = 3.04

$$\left[\frac{1}{4}\sqrt{2}\sqrt{-\frac{1}{b}}\log\left(4\,b^2x^4-4\,\sqrt{b^2x^4+a}\,bx^2+2\left(\sqrt{2}\,b^2x^3\sqrt{-\frac{1}{b}}-\sqrt{2}\,\sqrt{b^2x^4+a}\,bx\sqrt{-\frac{1}{b}}\right)\sqrt{-bx^2+\sqrt{b^2x^4+a}}+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/b)*log(4*b^2*x^4 - 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt
(2)*b^2*x^3*sqrt(-1/b) - sqrt(2)*sqrt(b^2*x^4 + a)*b*x*sqrt(-1/b))*sqrt(-b*
x^2 + sqrt(b^2*x^4 + a)) + a), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b*x^2
+ sqrt(b^2*x^4 + a))/(sqrt(b)*x))/sqrt(b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b\,x^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

```
[Out] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{b^2\,x^4+a}-b\,x^2}}{\sqrt{b^2\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)
[Out] int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
[Out] Integral(sqrt(-b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

3.913 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$

Optimal. Leaf size=169

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}\,d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}\,d^2+2ic^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}\,d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}\,d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}\,d^2+2ic^2}}$$

```
[Out] (1/2-1/2*I)*arctan((2*I*c*x+d*3^(1/2))/(-2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2-d^
2*3^(1/2))^(1/2))/(2*I*c^2-d^2*3^(1/2))^(1/2)-(1/2+1/2*I)*arctanh((-2*I*c*x
+d*3^(1/2))/(2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2+d^2*3^(1/2))^(1/2))/(2*I*c^2+d
^2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2133, 725, 204, 206}

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}\,d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}\,d^2+2ic^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}\,d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}\,d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}\,d^2+2ic^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

```
[Out] ((1/2 - I/2)*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]*
Sqrt[Sqrt[3] - (2*I)*x^2])])/Sqrt[(2*I)*c^2 - Sqrt[3]*d^2] - ((1/2 + I/2)*A
rcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3]
+ (2*I)*x^2])])/Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}-2ix^2}}\,dx + \left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}+2ix^2}}\,dx$$
$$= \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{2ic^2+\sqrt{3}\,d^2-x^2}\,dx, x, \frac{\sqrt{3}\,d-2icx}{\sqrt{\sqrt{3}+2ix^2}}\right) + \left(-\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int\right.$$
$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{2ic^2-\sqrt{3}\,d^2}\,\sqrt{\sqrt{3}-2ix^2}}\right)}{\sqrt{2ic^2-\sqrt{3}\,d^2}} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}\,d-2icx}{\sqrt{2ic^2+\sqrt{3}\,d^2}\,\sqrt{\sqrt{3}+2ix^2}}\right)}{\sqrt{2ic^2+\sqrt{3}\,d^2}}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
[Out] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="giac")
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(dx+c)\,\sqrt{4x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)),x)
```

```
[Out] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(c+dx)\sqrt{4x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)/(4*x**4+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)*sqrt(4*x**4 + 3)), x)
```

3.914 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$

Optimal. Leaf size=268

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}\,d^2+2ic^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}\,d^2+2ic^2\right)(c+dx)}+\frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}\,d^2+2ic^2}}\right)}{\left(-\sqrt{3}\,d^2+2ic^2\right)^{3/2}}+\frac{(1-i)c\tanh^{-1}\left(\ldots\right.}{\left(\sqrt{3}\,d^2+\ldots\right.}$$

```
[Out] (1+I)*c*arctan((2*I*c*x+d*3^(1/2))/(-2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2-d^2*3^
(1/2))^(1/2))/(2*I*c^2-d^2*3^(1/2))^(3/2)+(1-I)*c*arctanh((-2*I*c*x+d*3^(1/
2))/(2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2+d^2*3^(1/2))^(1/2))/(2*I*c^2+d^2*3^(1/
2))^(3/2)+(1/2-1/2*I)*d*(-2*I*x^2+3^(1/2))^(1/2)/(d*x+c)/(2*I*c^2-d^2*3^(1/
2))-(1/2+1/2*I)*d*(2*I*x^2+3^(1/2))^(1/2)/(d*x+c)/(2*I*c^2+d^2*3^(1/2))
```

Rubi [A] time = 0.31, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2133, 731, 725, 204, 206}

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}\,d^2+2ic^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}\,d^2+2ic^2\right)(c+dx)}+\frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}\,d^2+2ic^2}}\right)}{\left(-\sqrt{3}\,d^2+2ic^2\right)^{3/2}}+\frac{(1-i)c\tanh^{-1}\left(\ldots\right.}{\left(\sqrt{3}\,d^2+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
```

```
[Out] ((1/2 - I/2)*d*Sqrt[Sqrt[3] - (2*I)*x^2])/(((2*I)*c^2 - Sqrt[3]*d^2)*(c + d
*x)) - ((1/2 + I/2)*d*Sqrt[Sqrt[3] + (2*I)*x^2])/(((2*I)*c^2 + Sqrt[3]*d^2)
*(c + d*x)) + ((1 + I)*c*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - S
qrt[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^2])])/((2*I)*c^2 - Sqrt[3]*d^2)^(3/2) +
((1 - I)*c*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*S
qrt[Sqrt[3] + (2*I)*x^2])])/((2*I)*c^2 + Sqrt[3]*d^2)^(3/2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
```

```
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{\sqrt{3}-2ix^2}}\,dx + \left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{\sqrt{3}+2ix^2}}\,dx$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}\,d^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}\,d^2\right)(c+dx)} + \frac{((1+i)c)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}+2ix^2}}\,dx}{2c^2-i\sqrt{3}\,d^2} + \cdots$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}\,d^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}\,d^2\right)(c+dx)} + -\frac{((1+i)c)\,\mathrm{Subst}\left(\int \frac{1}{2ic^2+\sqrt{3}\,d^2-x^2} \cdots\right.}{2c^2-i\sqrt{3}\,d^2}$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}\,d^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}\,d^2\right)(c+dx)} + \frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{2ic^2-\sqrt{3}\,d^2}\sqrt{\sqrt{3}-2i\cdots}}\right.}{\left(2ic^2-\sqrt{3}\,d^2\right)^{3/2}}$$

Mathematica [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
[Out] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

maple **[F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(dx+c)^2\sqrt{4x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

```
[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2),x)
```

```
[Out] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(c+dx)^2\sqrt{4x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)**2/(4*x**4+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)**2*sqrt(4*x**4 + 3)), x
)
```

3.915 $\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

Optimal. Leaf size=41

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] -30*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+30*arctan(x^(1/6))+2*x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1840, 1620, 50, 63, 203}

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcTan[x^(1/6)
]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1840

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g =
Denominator[n]}, Dist[g, Subst[Int[x^(g*(m + 1) - 1)*(Pq /. x -> x^g)*(a +
b*x^(g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, b, m, p}, x] && PolyQ[Pq, x
] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{\sqrt{x}\left(-4+x^3\right)}{1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\sqrt{x}-x^{3/2}+x^{5/2}-\frac{5\sqrt{x}}{1+x}\right)dx, x, \sqrt[3]{x}\right)\\ &= 2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}-15\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+15\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x)}\,dx, x, \sqrt[3]{x}\right)\\ &= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\ &= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\tan^{-1}\left(\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcTan[x^(1/6)
]
```

fricas [A] time = 0.95, size = 25, normalized size = 0.61

$$\frac{6}{7}(x-35)x^{\frac{1}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}+30\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
[Out] 6/7*(x - 35)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 30*arctan(x^(1/6))
```

giac [A] time = 0.40, size = 27, normalized size = 0.66

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-30x^{\frac{1}{6}}+30\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

maple [A] time = 0.00, size = 28, normalized size = 0.68

$$\frac{6x^{\frac{7}{6}}}{7}+30\arctan\left(x^{\frac{1}{6}}\right)-\frac{6x^{\frac{5}{6}}}{5}+2\sqrt{x}-30x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-4)/(1+x^(1/3))/x^(1/2),x)
```

```
[Out] -30*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+30*arctan(x^(1/6))+2*x^(1/2)
```

maxima [A] time = 2.34, size = 27, normalized size = 0.66

$$\frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 30\,x^{\frac{1}{6}} + 30\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

mupad [B] time = 3.37, size = 27, normalized size = 0.66

$$30\,\mathrm{atan}\left(x^{1/6}\right) + 2\,\sqrt{x} - 30\,x^{1/6} - \frac{6\,x^{5/6}}{5} + \frac{6\,x^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 4)/(x^(1/2)*(x^(1/3) + 1)),x)
[Out] 30*atan(x^(1/6)) + 2*x^(1/2) - 30*x^(1/6) - (6*x^(5/6))/5 + (6*x^(7/6))/7
```

sympy [A] time = 11.87, size = 37, normalized size = 0.90

$$\frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 30\sqrt[6]{x} + 2\sqrt{x} + 30\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x**(1/3))/x**(1/2),x)
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 30*x**(1/6) + 2*sqrt(x) + 30*atan(x**(1/6))
```

3.916 $\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx$

Optimal. Leaf size=26

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] 3*x^(1/3)+6*arctan(x^(1/6))-3*ln(x^(1/3)+1)
```

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1593, 1819, 1810, 635, 203, 260}

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
```

```
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx &= \int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)x^{5/6}}\,dx \\
&= 6\,\text{Subst}\left(\int \frac{1+x^3}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\,\text{Subst}\left(\int \left(x+\frac{1-x}{1+x^2}\right)dx, x, \sqrt[6]{x}\right) \\
&= 3\sqrt[3]{x}+6\,\text{Subst}\left(\int \frac{1-x}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 3\sqrt[3]{x}+6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)-6\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 3\sqrt[3]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)-3\log\left(1+\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 38, normalized size = 1.46

$$3\sqrt[3]{x}+(-3-3i)\log\left(-\sqrt[6]{x}+i\right)-(3-3i)\log\left(\sqrt[6]{x}+i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
[Out] 3*x^(1/3) - (3 + 3*I)*Log[I - x^(1/6)] - (3 - 3*I)*Log[I + x^(1/6)]
```

fricas [A] time = 1.05, size = 20, normalized size = 0.77

$$3\,x^{\frac{1}{3}}+6\,\arctan\left(x^{\frac{1}{6}}\right)-3\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="fricas")
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

giac [A] time = 0.35, size = 20, normalized size = 0.77

$$3\,x^{\frac{1}{3}}+6\,\arctan\left(x^{\frac{1}{6}}\right)-3\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="giac")
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

maple [A] time = 0.01, size = 21, normalized size = 0.81

$$6\arctan\left(x^{\frac{1}{6}}\right)-3\ln\left(x^{\frac{1}{3}}+1\right)+3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)/(x^(5/6)+x^(7/6)),x)
[Out] 3*x^(1/3)+6*arctan(x^(1/6))-3*ln(1+x^(1/3))
```

maxima [A] time = 2.29, size = 20, normalized size = 0.77

$$3\,x^{\frac{1}{3}}+6\,\arctan\left(x^{\frac{1}{6}}\right)-3\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="maxima")
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

mupad [B] time = 3.36, size = 22, normalized size = 0.85

$$6\,\mathrm{atan}\left(x^{1/6}\right)-3\,\ln\left(36\,x^{1/3}+36\right)+3\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)/(x^(5/6) + x^(7/6)),x)
[Out] 6*atan(x^(1/6)) - 3*log(36*x^(1/3) + 36) + 3*x^(1/3)
```

sympy [A] time = 3.66, size = 24, normalized size = 0.92

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(x**(5/6)+x**(7/6)),x)
[Out] 3*x**(1/3) - 3*log(x**(1/3) + 1) + 6*atan(x**(1/6))
```

3.917 $\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

Optimal. Leaf size=42

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + 3\log\left(\sqrt[3]{x}+1\right) - 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] 6*x^(1/6)-3*x^(1/3)+3/2*x^(2/3)-6*arctan(x^(1/6))+3*ln(x^(1/3)+1)
```

Rubi [A] time = 0.15, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6688, 1593, 1802, 635, 203, 260}

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + 3\log\left(\sqrt[3]{x}+1\right) - 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^(1/
3)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \int \frac{1+\frac{1}{\sqrt{x}}}{1+\sqrt[3]{x}}\,dx$$
$$= 6\,\text{Subst}\left(\int \frac{x^2+x^5}{1+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\,\text{Subst}\left(\int \frac{x^2\left(1+x^3\right)}{1+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\,\text{Subst}\left(\int \left(1-x+x^3-\frac{1-x}{1+x^2}\right)dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\,\text{Subst}\left(\int \frac{1-x}{1+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)+6\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\tan^{-1}\left(\sqrt[6]{x}\right)+3\log\left(1+\sqrt[3]{x}\right)$$

Mathematica [C] time = 0.02, size = 54, normalized size = 1.29

$$\frac{3x^{2/3}}{2}-3\sqrt[3]{x}+6\sqrt[6]{x}+(3+3i)\log\left(-\sqrt[6]{x}+i\right)+(3-3i)\log\left(\sqrt[6]{x}+i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 + (3 + 3*I)*Log[I - x^(1/6)] + (3 - 3
*I)*Log[I + x^(1/6)]
```

fricas [A] time = 0.63, size = 30, normalized size = 0.71

$$\frac{3}{2}x^{\frac{2}{3}}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)+3\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

giac [A] time = 0.34, size = 30, normalized size = 0.71

$$\frac{3}{2}x^{\frac{2}{3}}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)+3\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

maple [A] time = 0.02, size = 48, normalized size = 1.14

$$-6\arctan\left(x^{\frac{1}{6}}\right)+\ln\left(x+1\right)+2\ln\left(x^{\frac{1}{3}}+1\right)-\ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)+\frac{3x^{\frac{2}{3}}}{2}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)/(x^(1/3)+1)/x^(1/2),x)
```

```
[Out] ln(x+1)+3/2*x^(2/3)-ln(x^(2/3)-x^(1/3)+1)+2*ln(x^(1/3)+1)-3*x^(1/3)+6*x^(1/
6)-6*arctan(x^(1/6))
```

maxima [A] time = 2.39, size = 30, normalized size = 0.71

$$\frac{3}{2}\,x^{\frac{2}{3}} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right) + 3\,\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

mupad [B] time = 0.03, size = 42, normalized size = 1.00

$$\frac{3\,x^{2/3}}{2} + 3\,\ln\left(\left(-6 + x^{1/6}\,6\mathrm{i}\right)\,\left(6 + x^{1/6}\,6\mathrm{i}\right)\right) - 3\,x^{1/3} - 6\,\mathrm{atan}\left(x^{1/6}\right) + 6\,x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)/(x^(1/2)*(x^(1/3) + 1)),x)
```

```
[Out] 3*log((x^(1/6)*6i - 6)*(x^(1/6)*6i + 6)) - 6*atan(x^(1/6)) - 3*x^(1/3) + (3
*x^(2/3))/2 + 6*x^(1/6)
```

sympy [A] time = 23.13, size = 39, normalized size = 0.93

$$6\sqrt[6]{x} + \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} + 3\log\left(\sqrt[3]{x} + 1\right) - 6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(1+x**(1/3))/x**(1/2),x)
```

```
[Out] 6*x**(1/6) + 3*x**(2/3)/2 - 3*x**(1/3) + 3*log(x**(1/3) + 1) - 6*atan(x**(1
/6))
```

3.918 $\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2} dx$

Optimal. Leaf size=20

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -arccsch(x*2^(1/2)/b^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {25, 335, 215}

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
```

```
[Out] -(ArcCsch[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2} dx &= \int \frac{1}{\sqrt{2+\frac{b}{x^2}}\,x^2} dx \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}} dx, x, \frac{1}{x}\right) \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 48, normalized size = 2.40

$$-\frac{x\sqrt{\frac{b}{x^2}+2}\,\tanh^{-1}\left(\frac{\sqrt{b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
```

```
[Out] -((Sqrt[2 + b/x^2]*x*ArcTanh[Sqrt[b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[b + 2*
x^2]))
```

fricas [B] time = 0.85, size = 75, normalized size = 3.75

$$\left[\frac{\log\left(-\frac{x^2-\sqrt{b}x\sqrt{\frac{2x^2+b}{x^2}}+b}{x^2}\right)}{2\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{2x^2+b}{x^2}}}{2x^2+b}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(x^2 - sqrt(b)*x*sqrt((2*x^2 + b)/x^2) + b)/x^2)/sqrt(b), sqrt(-b
)*arctan(sqrt(-b)*x*sqrt((2*x^2 + b)/x^2)/(2*x^2 + b))/b]
```

giac [B] time = 0.35, size = 44, normalized size = 2.20

$$\frac{\arctan\left(\frac{\sqrt{2x^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} - \frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - arctan(sqrt(b)/sqrt(-b))
*sgn(x)/sqrt(-b)
```

maple [B] time = 0.01, size = 50, normalized size = 2.50

$$-\frac{\sqrt{\frac{2x^2+b}{x^2}}\, x\ln\left(\frac{2b+2\sqrt{2x^2+b}\,\sqrt{b}}{x}\right)}{\sqrt{2x^2+b}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+b/x^2)^(1/2)/(2*x^2+b),x)
```

```
[Out] -((2*x^2+b)/x^2)^(1/2)*x/(2*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(2*x^2+b)^(1
/2)+b)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{b}{x^2}+2}}{2x^2+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b/x^2 + 2)/(2*x^2 + b), x)
```

mupad [B] time = 3.46, size = 17, normalized size = 0.85

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}}{2\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2 + 2)^(1/2)/(b + 2*x^2),x)
```

```
[Out] -asinh((2^(1/2)*b^(1/2))/(2*x))/b^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{b}{x^2}+2}}{b+2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x**2)**(1/2)/(2*x**2+b),x)
```

```
[Out] Integral(sqrt(b/x**2 + 2)/(b + 2*x**2), x)
```

3.919 $\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} \, dx$

Optimal. Leaf size=20

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -arccsc(x*2^(1/2)/b^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {25, 335, 216}

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

```
[Out] -(ArcCsc[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} \, dx &= \int \frac{1}{\sqrt{2-\frac{b}{x^2}}\,x^2} \, dx \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}} \, dx, x, \frac{1}{x}\right) \\
&= -\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 52, normalized size = 2.60

$$\frac{x\sqrt{2-\frac{b}{x^2}}\,\tan^{-1}\left(\frac{\sqrt{2x^2-b}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{2x^2-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

```
[Out] (Sqrt[2 - b/x^2]*x*ArcTan[Sqrt[-b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[-b + 2*x
^2])
```

fricas [B] time = 0.73, size = 84, normalized size = 4.20

$$\left[-\frac{\sqrt{-b}\log\left(-\frac{x^2-\sqrt{-b}\,x\sqrt{\frac{2\,x^2-b}{x^2}}-b}{x^2}\right)}{2\,b},\ -\frac{\arctan\left(\frac{\sqrt{b}\,x\sqrt{\frac{2\,x^2-b}{x^2}}}{2\,x^2-b}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-(x^2 - sqrt(-b)*x*sqrt((2*x^2 - b)/x^2) - b)/x^2)/b, -a
rctan(sqrt(b)*x*sqrt((2*x^2 - b)/x^2)/(2*x^2 - b))/sqrt(b)]
```

giac [B] time = 0.42, size = 40, normalized size = 2.00

$$\frac{\arctan\left(\frac{\sqrt{2\,x^2-b}}{\sqrt{b}}\right)\operatorname{sgn}(x)}{\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt{-b}}{\sqrt{b}}\right)\operatorname{sgn}(x)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 - b)/sqrt(b))*sgn(x)/sqrt(b) - arctan(sqrt(-b)/sqrt(b))*s
gn(x)/sqrt(b)
```

maple [B] time = 0.01, size = 62, normalized size = 3.10

$$-\frac{\sqrt{\frac{2x^2-b}{x^2}}\,x\ln\left(\frac{-2b+2\sqrt{-b}\,\sqrt{2x^2-b}}{x}\right)}{\sqrt{2x^2-b}\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-b/x^2)^(1/2)/(2*x^2-b),x)
```

```
[Out] -((2*x^2-b)/x^2)^(1/2)*x/(2*x^2-b)^(1/2)/(-b)^(1/2)*ln(2*((-b)^(1/2)*(2*x^2
-b)^(1/2)-b)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\frac{b}{x^2}+2}}{2\,x^2-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b/x^2 + 2)/(2*x^2 - b), x)
```

mupad [B] time = 3.47, size = 21, normalized size = 1.05

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{-b}}{2\,x}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2 - b/x^2)^(1/2)/(b - 2*x^2),x)
```

```
[Out] -asinh((2^(1/2)*(-b)^(1/2))/(2*x))/(-b)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{b}{x^2}+2}}{-b+2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x**2)**(1/2)/(2*x**2-b),x)
```

```
[Out] Integral(sqrt(-b/x**2 + 2)/(-b + 2*x**2), x)
```

3.920 $\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

```
[Out] arctanh((a+c/x^2)^(1/2)/a^(1/2))*a^(1/2)/e-arctanh(c^(1/2)/x/(a+c/x^2)^(1/2
))*c^(1/2)/d-arctanh((a*d-c*e/x)/(a*d^2+c*e^2)^(1/2)/(a+c/x^2)^(1/2))*(a*d^
2+c*e^2)^(1/2)/d/e
```

Rubi [A] time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1444, 1475, 896, 266, 63, 208, 844, 217, 206, 725}

$$-\frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + c/x^2]/Sqrt[a]])/e - (Sqrt[a*d^2 + c*e^2]*ArcTanh
[(a*d - (c*e)/x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[a + c/x^2])])/(d*e) - (Sqrt[c]*A
rcTanh[Sqrt[c]/(Sqrt[a + c/x^2]*x)])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 896

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f + a*e*g - c*(e
*f - d*g)*x, x]*(a + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx &= \int \frac{\sqrt{a+\frac{c}{x^2}}}{\left(e+\frac{d}{x}\right)x}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{\sqrt{a+cx^2}}{x(e+dx)}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\operatorname{Subst}\left(\int \frac{ad-cex}{(e+dx)\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} \\
&= -\frac{c\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, \frac{1}{x^2}\right)}{2e} + \left(\frac{ad}{e}+\frac{ce}{d}\right)\operatorname{Subst}\left(\int \frac{}{(e+dx)}\right. \\
&= -\frac{c\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{c}{x^2}}\,x}\right)}{d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+\frac{c}{x^2}}\right)}{ce} + \left(-\frac{ad}{e}-\frac{ce}{d}\right)\operatorname{Subst}\left(\right. \\
&= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e} - \frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{ad^2+ce^2}\sqrt{a+\frac{c}{x^2}}}\right)}{de} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a+\frac{c}{x^2}}\,x}\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 136, normalized size = 1.12

$$
\frac{x\sqrt{a+\frac{c}{x^2}}\left(\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ce-adx}{\sqrt{ax^2+c}\sqrt{ad^2+ce^2}}\right)+\sqrt{a}\,d\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+c}}\right)-\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{ax^2+c}}{\sqrt{c}}\right)\right)}{de\sqrt{ax^2+c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[a + c/x^2]*x*(Sqrt[a]*d*ArcTanh[(Sqrt[a]*x)/Sqrt[c + a*x^2]] + Sqrt[a
*d^2 + c*e^2]*ArcTanh[(c*e - a*d*x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[c + a*x^2])]
- Sqrt[c]*e*ArcTanh[Sqrt[c + a*x^2]/Sqrt[c]]))/(d*e*Sqrt[c + a*x^2])
```

fricas [A] time = 2.05, size = 1532, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + s
qrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + sqrt
(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e
^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e
^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqr
t((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt(
(a*x^2 + c)/x^2) + 2*c)/x^2) - sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d
^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2
 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(sq
rt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(c)*e
*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + 2*sqrt(-a*d^
2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x
^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), -1/2*(2*sqrt(-a)
*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-
```

```
(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) - 2*sqrt(-a*d^2 - c*
e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a
*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), 1/2*(2*sqrt(-c)*e*arct
an(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) + sqrt(a)*d*log(-2*a*x^2 -
 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(a*d^2 + c*e^2)*log((2*a*c*
d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*
x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(
d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 +
 c)) - 2*sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) -
sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a
*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2)
)/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-c)*e*arctan(sqrt(-c)*x*sq
rt((a*x^2 + c)/x^2)/(a*x^2 + c)) + sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*s
qrt((a*x^2 + c)/x^2) - c) + 2*sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)
*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 +
 a*c*e^2)*x^2)))/(d*e), -(sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x
^2)/(a*x^2 + c)) - sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^
2 + c)) - sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2
)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*
e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.03, size = 244, normalized size = 2.02

$$\frac{\sqrt{\frac{a x^2+c}{x^2}}\left(a d^2 \ln\left(\frac{-2adx+2ce+2\sqrt{a x^2+c}\,\sqrt{\frac{a d^2+c e^2}{e^2}}\,e}{ex+d}\right)+c e^2 \ln\left(\frac{-2adx+2ce+2\sqrt{a x^2+c}\,\sqrt{\frac{a d^2+c e^2}{e^2}}\,e}{ex+d}\right)+\sqrt{\frac{a d^2+c e^2}{e^2}}\,\sqrt{a}\,de\ln\left(ax+\cdots\right.\right.}{\sqrt{a x^2+c}\,\sqrt{\frac{a d^2+c e^2}{e^2}}\,d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2)^(1/2)/(e*x+d),x)
```

```
[Out] ((a*x^2+c)/x^2)^(1/2)*x*(a^(1/2)*d*ln(((a*x^2+c)^(1/2)*a^(1/2)+a*x)/a^(1/2)
)*e*((a*d^2+c*e^2)/e^2)^(1/2)-((a*d^2+c*e^2)/e^2)^(1/2)*c^(1/2)*ln(2*(c^(1/
2)*(a*x^2+c)^(1/2)+c)/x)*e^2+ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1/2
)*e-a*x*d+c*e)/(e*x+d))*a*d^2+ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1/
2)*e-a*x*d+c*e)/(e*x+d))*c*e^2)/(a*x^2+c)^(1/2)/d/e^2/((a*d^2+c*e^2)/e^2)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + c/x^2)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c/x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + c/x^2)^(1/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c/x**2)/(d + e*x), x)
```

3.921 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx$

Optimal. Leaf size=181

$$-\frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\frac{2ad+\frac{bd-2ce}{x}-be}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-e(bd-ce)}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

```
[Out] arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))*a^(1/2)/e-arctanh(1/2*(b
+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))*c^(1/2)/d-arctanh(1/2*(2*a*d-b*e+(b*d-
2*c*e)/x)/(a*d^2-e*(b*d-c*e))^(1/2)/(a+c/x^2+b/x)^(1/2))*(a*d^2-e*(b*d-c*e)
)^(1/2)/d/e
```

Rubi [A] time = 0.27, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1443, 1474, 895, 724, 206, 843, 621}

$$-\frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\frac{2ad+\frac{bd-2ce}{x}-be}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-e(bd-ce)}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] (Sqrt[a]*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/e - (Sqrt[
c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/d - (Sqrt[a*d^
2 - e*(b*d - c*e)]*ArcTanh[(2*a*d - b*e + (b*d - 2*c*e)/x)/(2*Sqrt[a*d^2 -
e*(b*d - c*e)]*Sqrt[a + c/x^2 + b/x])])/(d*e)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1443

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)
^(n2_.))^(p_.), x_Symbol] :> Int[((e + d*x^n)^q*(a + b*x^n + c*x^(2*n))^p)/
x^(n*q), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[n2, 2*n] && EqQ[mn, -
n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{\left(e+\frac{d}{x}\right)x}\,dx$$

$$= -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(e+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{ad-be-cex}{(e+dx)\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{c\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{d} + \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} + \left(-b+\frac{ad}{e}+\frac{c}{d}\right.$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} - \frac{(2c)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+\frac{2c}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d} + \left(2\left(b-\frac{ad}{e}-\frac{ce}{d}\right)\right)\mathrm{S}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d} - \frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\right.}{de}$$

Mathematica [A] time = 0.28, size = 189, normalized size = 1.04

$$-\frac{x\sqrt{a+\frac{bx+c}{x^2}}\left(\sqrt{ad^2-bde+ce^2}\tanh^{-1}\left(\frac{2adx+bd-bex-2ce}{2\sqrt{x(ax+b)+c}\sqrt{ad^2-bde+ce^2}}\right)-\sqrt{a}\,d\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)+\sqrt{c}\,e\tanh^{-1}\right.}{de\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] -((x*Sqrt[a + (c + b*x)/x^2]*(-(Sqrt[a]*d*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sq
rt[c + x*(b + a*x)])]) + Sqrt[c]*e*ArcTanh[(2*c + b*x)/(2*Sqrt[c]*Sqrt[c +
x*(b + a*x)])] + Sqrt[a*d^2 - b*d*e + c*e^2]*ArcTanh[(b*d - 2*c*e + 2*a*d*x
 - b*e*x)/(2*Sqrt[a*d^2 - b*d*e + c*e^2]*Sqrt[c + x*(b + a*x)])]))/(d*e*Sqr
t[c + x*(b + a*x)]))
```

fricas [A] time = 103.95, size = 2411, normalized size = 13.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*
sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4*a
*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^
2) + sqrt(a*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)
*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b
*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b
*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d*e*x
+ d^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt
((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - sqrt(c)*e*log(-(8*b*c*x
+ (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x +
 c)/x^2))/x^2) - sqrt(a*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (
b^2 + 4*a*c)*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a
*b*d^2 + 4*b*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)
*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^
2 + 2*d*e*x + d^2)))/(d*e), 1/2*(sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^2 -
 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(c)*e
*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt
((a*x^2 + b*x + c)/x^2))/x^2) - 2*sqrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*
sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*
x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c
*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b*c*e^2)*x)))/(d*e), -1/2*(2*sqrt(-a)*d*ar
ctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*
b*x + a*c)) - sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^
2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 2*sqrt(-a*d^2 + b*d*
e - c*e^2)*arctan(-1/2*sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a*d - b*e)*x^2 + (b
*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*d*e + c^2*e^2 + (
a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b*c*e^2)*x)))/(d*e)
, 1/2*(2*sqrt(-c)*e*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x +
 c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^
2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(a
*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a
^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3
*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x^2 + (
b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d
*e), -1/2*(2*sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b
*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 2*sqrt(-c)*e*arctan(1/2*(b*x^2 + 2*
c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - sqrt(a
*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a
^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3
*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x^2 + (
b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d
*e), 1/2*(2*sqrt(-c)*e*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*
x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x -
```

```
b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 2*s
qrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a*
d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*
d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b*
c*e^2)*x)))/(d*e), -(sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a
*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - sqrt(-c)*e*arctan(1/2*(b*x^
2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) +
sqrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a
*d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c
*d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b
*c*e^2)*x)))/(d*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.04, size = 397, normalized size = 2.19

$$-\frac{\sqrt{\frac{a x^2+bx+c}{x^2}}\left(-a^{\frac{3}{2}}d^2\ln\left(\frac{-2adx+bex-bd+2ce+2\sqrt{a x^2+bx+c}\sqrt{\frac{a d^2-bde+c e^2}{e^2}}\,e}{ex+d}\right)+\sqrt{a}\,bde\ln\left(\frac{-2adx+bex-bd+2ce+2\sqrt{a x^2+bx+c}\sqrt{\cdots}}{ex+d}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)/(e*x+d),x)
```

```
[Out] -((a*x^2+b*x+c)/x^2)^(1/2)*x*(((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*c^(1/2)*a^(1/
2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*e^2-((a*d^2-b*d*e+c*e^2)/e
^2)^(1/2)*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*d*e-a^(
3/2)*ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*x+x*
b*e-b*d+2*c*e)/(e*x+d))*d^2+a^(1/2)*ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e
+c*e^2)/e^2)^(1/2)*e-2*a*d*x+x*b*e-b*d+2*c*e)/(e*x+d))*b*d*e-a^(1/2)*ln((2*
(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*x+x*b*e-b*d+2*c
*e)/(e*x+d))*c*e^2)/(a*x^2+b*x+c)^(1/2)/d/e^2/a^(1/2)/((a*d^2-b*d*e+c*e^2)/
e^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x + c/x^2)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + b/x + c/x^2)^(1/2)/(d + e*x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + b/x + c/x**2)/(d + e*x), x)
```

3.922 $\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}} \, dx$

Optimal. Leaf size=26

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

[Out] 3/2*x^(2/3)+10/11*(x^3)^(1/5)*x^(1/2)

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {14, 15, 30}

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

Antiderivative was successfully verified.

[In] Int[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]

[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}} \, dx &= \int \left(\frac{1}{\sqrt[3]{x}}+\frac{\sqrt[5]{x^3}}{\sqrt{x}}\right) dx\\ &= \frac{3x^{2/3}}{2}+\int \frac{\sqrt[5]{x^3}}{\sqrt{x}} \, dx\\ &= \frac{3x^{2/3}}{2}+\frac{\sqrt[5]{x^3}\int \sqrt[10]{x} \, dx}{x^{3/5}}\\ &= \frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt{x}\sqrt[5]{x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

fricas [A] time = 0.85, size = 16, normalized size = 0.62

$$\frac{10}{11}\left(x^3\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="fricas")
[Out] 10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

giac [A] time = 0.51, size = 11, normalized size = 0.42

$$\frac{10}{11}\,x^{\frac{11}{10}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="giac")
[Out] 10/11*x^(11/10) + 3/2*x^(2/3)
```

maple [A] time = 0.00, size = 17, normalized size = 0.65

$$\frac{3x^{\frac{2}{3}}}{2}+\frac{10\left(x^3\right)^{\frac{1}{5}}\sqrt{x}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/6)+(x^3)^(1/5))/x^(1/2),x)
[Out] 3/2*x^(2/3)+10/11*(x^3)^(1/5)*x^(1/2)
```

maxima [A] time = 0.96, size = 16, normalized size = 0.62

$$\frac{10}{11}\left(x^3\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="maxima")
[Out] 10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

mupad [B] time = 3.54, size = 16, normalized size = 0.62

$$\frac{10\,\sqrt{x}\,{\left(x^3\right)}^{1/5}}{11}+\frac{3\,x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3)^(1/5) + x^(1/6))/x^(1/2),x)
[Out] (10*x^(1/2)*(x^3)^(1/5))/11 + (3*x^(2/3))/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/6)+(x**3)**(1/5))/x**(1/2),x)
[Out] Timed out
```

3.923 $\int \frac{2+x}{\sqrt{4x-x^2}} dx$

Optimal. Leaf size=26

$$-\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] 4*arcsin(-1+1/2*x)-(-x^2+4*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {640, 619, 216}

$$-\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/Sqrt[4*x - x^2],x]
```

```
[Out] -Sqrt[4*x - x^2] - 4*ArcSin[1 - x/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{\sqrt{4x-x^2}} \, dx &= -\sqrt{4x-x^2}+4 \int \frac{1}{\sqrt{4x-x^2}} \, dx\\
&= -\sqrt{4x-x^2}-\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}} \, dx,x,4-2x\right)\\
&= -\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 1.04

$$-\sqrt{-((x-4)x)}-8\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/Sqrt[4*x - x^2],x]
```

```
[Out] -Sqrt[-((-4 + x)*x)] - 8*ArcSin[Sqrt[1 - x/4]]
```

fricas [A] time = 0.75, size = 32, normalized size = 1.23

$$-\sqrt{-x^2 + 4x} - 8 \arctan\left(\frac{\sqrt{-x^2 + 4x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 4*x) - 8*arctan(sqrt(-x^2 + 4*x)/x)
```

giac [A] time = 0.61, size = 22, normalized size = 0.85

$$-\sqrt{-x^2 + 4x} + 4 \arcsin\left(\frac{1}{2}x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 4*x) + 4*arcsin(1/2*x - 1)
```

maple [A] time = 0.01, size = 23, normalized size = 0.88

$$4 \arcsin\left(\frac{x}{2} - 1\right) - \sqrt{-x^2 + 4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(-x^2+4*x)^(1/2),x)
[Out] 4*arcsin(1/2*x-1)-(-x^2+4*x)^(1/2)
```

maxima [A] time = 1.93, size = 22, normalized size = 0.85

$$-\sqrt{-x^2 + 4x} - 4 \arcsin\left(-\frac{1}{2}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 4*x) - 4*arcsin(-1/2*x + 1)
```

mupad [B] time = 3.53, size = 22, normalized size = 0.85

$$4 \operatorname{asin}\left(\frac{x}{2} - 1\right) - \sqrt{4x - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(4*x - x^2)^(1/2),x)
[Out] 4*asin(x/2 - 1) - (4*x - x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 2}{\sqrt{-x(x - 4)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x**2+4*x)**(1/2),x)
[Out] Integral((x + 2)/sqrt(-x*(x - 4)), x)
```

3.924 $\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{4}\left(x^2+6x\right)^{2/3}$$

[Out] 3/4*(x^2+6*x)^(2/3)

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {629}

$$\frac{3}{4}\left(x^2+6x\right)^{2/3}$$

Antiderivative was successfully verified.

[In] Int[(3 + x)/(6*x + x^2)^(1/3),x]

[Out] (3*(6*x + x^2)^(2/3))/4

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx=\frac{3}{4}\left(6x+x^2\right)^{2/3}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{4}(x(x+6))^{2/3}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + x)/(6*x + x^2)^(1/3),x]

[Out] (3*(x*(6 + x))^(2/3))/4

fricas [A] time = 0.65, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="fricas")

[Out] 3/4*(x^2 + 6*x)^(2/3)

giac [A] time = 0.43, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$\frac{3(x+6)x}{4\left(x^2+6x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+3)/(x^2+6*x)^(1/3),x)
```

```
[Out] 3/4*x*(x+6)/(x^2+6*x)^(1/3)
```

maxima [A] time = 0.86, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

mupad [B] time = 3.56, size = 9, normalized size = 0.60

$$\frac{3\left(x\ (x+6)\right)^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3)/(6*x + x^2)^(1/3),x)
```

```
[Out] (3*(x*(x + 6))^(2/3))/4
```

sympy [A] time = 0.16, size = 12, normalized size = 0.80

$$\frac{3\left(x^2+6x\right)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x**2+6*x)**(1/3),x)
```

```
[Out] 3*(x**2 + 6*x)**(2/3)/4
```

3.925 $\int \frac{4+x}{\left(6x-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

```
[Out] 1/9*(-12+7*x)/(-x^2+6*x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {636}

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x)/(6*x - x^2)^(3/2),x]
```

```
[Out] -(12 - 7*x)/(9*Sqrt[6*x - x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{4+x}{\left(6x-x^2\right)^{3/2}}\,dx=-\frac{12-7x}{9\sqrt{6x-x^2}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.86

$$\frac{7x-12}{9\sqrt{-((x-6)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x)/(6*x - x^2)^(3/2),x]
```

```
[Out] (-12 + 7*x)/(9*Sqrt[-((-6 + x)*x)])
```

fricas [A] time = 0.71, size = 27, normalized size = 1.23

$$-\frac{\sqrt{-x^2+6\,x}\,(7\,x-12)}{9\left(x^2-6\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

giac [**A**] time = 0.52, size = 27, normalized size = 1.23

$$-\frac{\sqrt{-x^2+6x}(7x-12)}{9\left(x^2-6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="giac")
[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

maple [**A**] time = 0.00, size = 23, normalized size = 1.05

$$-\frac{(x-6)(7x-12)x}{9\left(-x^2+6x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+4)/(-x^2+6*x)^(3/2),x)
[Out] -1/9*x*(x-6)*(-12+7*x)/(-x^2+6*x)^(3/2)
```

maxima [**A**] time = 0.88, size = 28, normalized size = 1.27

$$\frac{7x}{9\sqrt{-x^2+6x}}-\frac{4}{3\sqrt{-x^2+6x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="maxima")
[Out] 7/9*x/sqrt(-x^2 + 6*x) - 4/3/sqrt(-x^2 + 6*x)
```

mupad [**B**] time = 3.50, size = 18, normalized size = 0.82

$$\frac{7x-12}{9\sqrt{6x-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4)/(6*x - x^2)^(3/2),x)
[Out] (7*x - 12)/(9*(6*x - x^2)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+4}{(-x(x-6))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x**2+6*x)**(3/2),x)
[Out] Integral((x + 4)/(-x*(x - 6))**(3/2), x)
```

3.926 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

```
[Out] arctan((x^2+2*x)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {688, 203}

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sqrt{2x+x^2}\right)\\ &= \tan^{-1}\left(\sqrt{2x+x^2}\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+2}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+2}}\right)}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[2 + x]*ArcTan[Sqrt[x]/Sqrt[2 + x]])/Sqrt[x*(2 + x)]
```

fricas [A] time = 0.55, size = 17, normalized size = 1.42

$$2\,\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

giac [A] time = 0.59, size = 17, normalized size = 1.42

$$2\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

maple [A] time = 0.01, size = 13, normalized size = 1.08

$$-\arctan\left(\frac{1}{\sqrt{(x+1)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x^2+2*x)^(1/2),x)
[Out] -arctan(1/((x+1)^2-1)^(1/2))
```

maxima [A] time = 1.94, size = 9, normalized size = 0.75

$$-\arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(1/abs(x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{x^2+2\,x}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + x^2)^(1/2)*(x + 1)),x)
[Out] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\,(x+2)}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

3.927 $\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(2\sqrt{x^2+x}\right)$$

[Out] arctan(2*(x^2+x)^(1/2))

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {688, 203}

$$\tan^{-1}\left(2\sqrt{x^2+x}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + 2*x)*Sqrt[x + x^2]),x]

[Out] ArcTan[2*Sqrt[x + x^2]]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{2+8x^2}\,dx, x, \sqrt{x+x^2}\right)\\ &= \tan^{-1}\left(2\sqrt{x+x^2}\right)\end{aligned}$$

Mathematica [B] time = 0.02, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+1}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+1}}\right)}{\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + 2*x)*Sqrt[x + x^2]),x]

[Out] (2*Sqrt[x]*Sqrt[1 + x]*ArcTan[Sqrt[x]/Sqrt[1 + x]])/Sqrt[x*(1 + x)]

fricas [A] time = 0.65, size = 17, normalized size = 1.42

$$2\,\arctan\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

giac [A] time = 0.62, size = 17, normalized size = 1.42

$$2 \arctan\left(-2x + 2\sqrt{x^2 + x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

maple [A] time = 0.01, size = 15, normalized size = 1.25

$$-\arctan\left(\frac{1}{\sqrt{4\left(x+\frac{1}{2}\right)^2 - 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+1)/(x^2+x)^(1/2),x)
[Out] -arctan(1/(4*(x+1/2)^2-1)^(1/2))
```

maxima [A] time = 1.93, size = 11, normalized size = 0.92

$$-\arcsin\left(\frac{1}{|2x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(1/abs(2*x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{(2x+1)\sqrt{x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + 1)*(x + x^2)^(1/2)),x)
[Out] int(1/((2*x + 1)*(x + x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(x+1)}\,(2x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x**2+x)**(1/2),x)
[Out] Integral(1/(sqrt(x*(x + 1))*(2*x + 1)), x)
```

3.928 $\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx$

Optimal. Leaf size=15

$$-\sqrt{2x-x^2}$$

```
[Out] -(-x^2+2*x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {629}

$$-\sqrt{2x-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[2*x - x^2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{2x-x^2}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.80

$$-\sqrt{-((x-2)x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[-((-2 + x)*x)]
```

fricas [A] time = 0.61, size = 13, normalized size = 0.87

$$-\sqrt{-x^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

giac [A] time = 0.60, size = 13, normalized size = 0.87

$$-\sqrt{-x^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

maple [A] time = 0.00, size = 17, normalized size = 1.13

$$\frac{(x-2)\,x}{\sqrt{-x^2+2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(-x^2+2*x)^(1/2),x)
[Out] (x-2)*x/(-x^2+2*x)^(1/2)
```

maxima [A] time = 0.87, size = 13, normalized size = 0.87

$$-\sqrt{-x^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 2*x)
```

mupad [B] time = 3.64, size = 10, normalized size = 0.67

$$-\sqrt{-x\,\left(x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(2*x - x^2)^(1/2),x)
[Out] -(-x*(x - 2))^(1/2)
```

sympy [A] time = 0.14, size = 10, normalized size = 0.67

$$-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x**2+2*x)**(1/2),x)
[Out] -sqrt(-x**2 + 2*x)
```

3.929 $\int \frac{\sqrt{x-x^2}}{1+x} \, dx$

Optimal. Leaf size=54

$$\sqrt{x-x^2}+\sqrt{2}\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)-\frac{3}{2}\sin^{-1}(1-2x)$$

```
[Out] 3/2*arcsin(-1+2*x)+arctan(1/4*(1-3*x)*2^(1/2)/(-x^2+x)^(1/2))*2^(1/2)+(-x^2
+x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {734, 843, 619, 216, 724, 204}

$$\sqrt{x-x^2}+\sqrt{2}\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)-\frac{3}{2}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x - x^2]/(1 + x),x]
```

```
[Out] Sqrt[x - x^2] - (3*ArcSin[1 - 2*x])/2 + Sqrt[2]*ArcTan[(1 - 3*x)/(2*Sqrt[2]
*Sqrt[x - x^2])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x-x^2}}{1+x}\,dx &= \sqrt{x-x^2}-\frac{1}{2}\int \frac{1-3x}{(1+x)\sqrt{x-x^2}}\,dx\\
&= \sqrt{x-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{x-x^2}}\,dx-2\int \frac{1}{(1+x)\sqrt{x-x^2}}\,dx\\
&= \sqrt{x-x^2}-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)+4\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,\frac{-1+3x}{\sqrt{x-x^2}}\right)\\
&= \sqrt{x-x^2}-\frac{3}{2}\sin^{-1}(1-2x)+\sqrt{2}\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 95, normalized size = 1.76

$$
\sqrt{-((x-1)x)}-\frac{3\sqrt{-((x-1)x)}\,\sin^{-1}\left(\sqrt{1-x}\right)}{\sqrt{1-x}\,\sqrt{x}}+\frac{2\sqrt{2}\,\sqrt{-((x-1)x)}\,\tanh^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}\,\sqrt{x}}\right)}{\sqrt{x-1}\,\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x - x^2]/(1 + x),x]
```

```
[Out] Sqrt[-((-1 + x)*x)] - (3*Sqrt[-((-1 + x)*x)]*ArcSin[Sqrt[1 - x]])/(Sqrt[1 -
 x]*Sqrt[x]) + (2*Sqrt[2]*Sqrt[-((-1 + x)*x)]*ArcTanh[Sqrt[-1 + x]/(Sqrt[2]
*Sqrt[x])])/(Sqrt[-1 + x]*Sqrt[x])
```

fricas [A] time = 0.61, size = 49, normalized size = 0.91

$$
2\sqrt{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^2+x}}{2\,x}\right)+\sqrt{-x^2+x}-3\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + x)/x) + sqrt(-x^2 + x) - 3*arctan(
sqrt(-x^2 + x)/x)
```

giac [A] time = 0.50, size = 53, normalized size = 0.98

$$
2\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\frac{3\left(2\sqrt{-x^2+x}-1\right)}{2\,x-1}-1\right)\right)+\sqrt{-x^2+x}+\frac{3}{2}\arcsin\left(2\,x-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(-x^2 + x) - 1)/(2*x - 1) - 1)) + sq
rt(-x^2 + x) + 3/2*arcsin(2*x - 1)
```

maple [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{3\arcsin\left(2x-1\right)}{2}-\sqrt{2}\,\arctan\left(\frac{\left(3x-1\right)\sqrt{2}}{4\sqrt{3x-\left(x+1\right)^{2}+1}}\right)+\sqrt{3x-\left(x+1\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x)^(1/2)/(x+1),x)
```

```
[Out] (-(x+1)^2+3*x+1)^(1/2)+3/2*arcsin(2*x-1)-2^(1/2)*arctan(1/4*(3*x-1)*2^(1/2)
/(-(x+1)^2+3*x+1)^(1/2))
```

maxima [A] time = 1.91, size = 42, normalized size = 0.78

$$-\sqrt{2}\,\arcsin\left(\frac{3\,x}{|x+1|}-\frac{1}{|x+1|}\right)+\sqrt{-x^{2}+x}+\frac{3}{2}\,\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arcsin(3*x/abs(x + 1) - 1/abs(x + 1)) + sqrt(-x^2 + x) + 3/2*arcsi
n(2*x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x-x^2}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^2)^(1/2)/(x + 1),x)
```

```
[Out] int((x - x^2)^(1/2)/(x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x\,\left(x-1\right)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x)**(1/2)/(1+x),x)
```

```
[Out] Integral(sqrt(-x*(x - 1))/(x + 1), x)
```

3.930 $\int \sqrt{\sqrt[4]{x}+x}\,dx$

Optimal. Leaf size=59

$$\frac{2}{3}\sqrt{x+\sqrt[4]{x}}\,x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\,\sqrt[4]{x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)$$

```
[Out] -1/3*arctanh(x^(1/2)/(x^(1/4)+x)^(1/2))+1/3*x^(1/4)*(x^(1/4)+x)^(1/2)+2/3*x
*(x^(1/4)+x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2004, 2018, 2024, 2029, 206}

$$\frac{2}{3}\sqrt{x+\sqrt[4]{x}}\,x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\,\sqrt[4]{x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^(1/4) + x],x]
```

```
[Out] (x^(1/4)*Sqrt[x^(1/4) + x])/3 + (2*x*Sqrt[x^(1/4) + x])/3 - ArcTanh[Sqrt[x]
/Sqrt[x^(1/4) + x]]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sqrt[4]{x}+x}\,dx &= \frac{2}{3}x\sqrt{\sqrt[4]{x}+x}+\frac{1}{4}\int \frac{\sqrt[4]{x}}{\sqrt{\sqrt[4]{x}+x}}\,dx\\
&= \frac{2}{3}x\sqrt{\sqrt[4]{x}+x}+\text{Subst}\left(\int \frac{x^4}{\sqrt{x+x^4}}\,dx,x,\sqrt[4]{x}\right)\\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{x+x^4}}\,dx,x,\sqrt[4]{x}\right)\\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)\\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.97

$$\frac{3x^{5/4}-\sqrt{x^{3/4}+1}\,\sqrt[8]{x}\,\sinh^{-1}\left(x^{3/8}\right)+2x^2+\sqrt{x}}{3\sqrt{x+\sqrt[4]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^(1/4) + x],x]
[Out] (Sqrt[x] + 3*x^(5/4) + 2*x^2 - Sqrt[1 + x^(3/4)]*x^(1/8)*ArcSinh[x^(3/8)])/
(3*Sqrt[x^(1/4) + x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 4.57, size = 45, normalized size = 0.76

$$\frac{1}{3}\sqrt{x+x^{\frac{1}{4}}}\,x^{\frac{1}{4}}\left(2x^{\frac{3}{4}}+1\right)-\frac{1}{6}\log\left(\sqrt{\frac{1}{x^{\frac{3}{4}}}+1}+1\right)+\frac{1}{6}\log\left(\left|\sqrt{\frac{1}{x^{\frac{3}{4}}}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(x + x^(1/4))*x^(1/4)*(2*x^(3/4) + 1) - 1/6*log(sqrt(1/x^(3/4) + 1)
 + 1) + 1/6*log(abs(sqrt(1/x^(3/4) + 1) - 1))
```

maple [C] time = 0.10, size = 342, normalized size = 5.80

$$\frac{2\sqrt{x+x^{\frac{1}{4}}}\,x}{3}+\frac{\sqrt{x+x^{\frac{1}{4}}}\,x^{\frac{1}{4}}}{3}+\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x^{\frac{1}{4}}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x^{\frac{1}{4}}+1\right)}}\left(x^{\frac{1}{4}}+1\right)^{2}\sqrt{-\frac{x^{\frac{1}{4}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(x^{\frac{1}{4}}+1\right)}}\sqrt{-\frac{x^{\frac{1}{4}}-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x^{\frac{1}{4}}+1\right)}}\left(-\mathrm{Elli}\right.}{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\left(x^{\frac{1}{4}}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/4)+x)^(1/2),x)
```

```
[Out] 2/3*x*(x^(1/4)+x)^(1/2)+1/3*x^(1/4)*(x^(1/4)+x)^(1/2)+(-1/2-1/2*I*3^(1/2))*
((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I*3^(1/2))/(x^(1/4)+1))^(1/2)*(x^(1/4
)+1)^2*(-(x^(1/4)-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(x^(1/4)+1))^(1/2)
*(-(x^(1/4)-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(x^(1/4)+1))^(1/2)/(3/2+
1/2*I*3^(1/2))/(x^(1/4)*(x^(1/4)+1)*(x^(1/4)-1/2+1/2*I*3^(1/2))*(x^(1/4)-1/
2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I
*3^(1/2))/(x^(1/4)+1))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/
2))*x^(1/4)/(1/2+1/2*I*3^(1/2))/(x^(1/4)+1))^(1/2),(1/2+1/2*I*3^(1/2))/(3/2
+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+x^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + x^(1/4)), x)
```

mupad [B] time = 3.53, size = 27, normalized size = 0.46

$$\frac{8\,x\,\sqrt{x+x^{1/4}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{2};\ \frac{5}{2};\ -x^{3/4}\right)}{9\,\sqrt{x^{3/4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(1/4))^(1/2),x)
```

```
[Out] (8*x*(x + x^(1/4))^(1/2)*hypergeom([-1/2, 3/2], 5/2, -x^(3/4)))/(9*(x^(3/4)
 + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt[4]{x}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/4)+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**(1/4) + x), x)
```

3.931 $\int \sqrt{x + x^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{16\left(x^{3/2}+x\right)^{3/2}}{35x}+\frac{32\left(x^{3/2}+x\right)^{3/2}}{105x^{3/2}}$$

```
[Out] 32/105*(x+x^(3/2))^(3/2)/x^(3/2)-16/35*(x+x^(3/2))^(3/2)/x+4/7*(x+x^(3/2))^
(3/2)/x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2002, 2016, 2014}

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{16\left(x^{3/2}+x\right)^{3/2}}{35x}+\frac{32\left(x^{3/2}+x\right)^{3/2}}{105x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^(3/2)],x]
```

```
[Out] (32*(x + x^(3/2))^(3/2))/(105*x^(3/2)) - (16*(x + x^(3/2))^(3/2))/(35*x) +
(4*(x + x^(3/2))^(3/2))/(7*Sqrt[x])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \sqrt{x+x^{3/2}}\,dx = \frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}} - \frac{4}{7}\int \frac{\sqrt{x+x^{3/2}}}{\sqrt{x}}\,dx$$
$$= -\frac{16\left(x+x^{3/2}\right)^{3/2}}{35x} + \frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}} + \frac{8}{35}\int \frac{\sqrt{x+x^{3/2}}}{x}\,dx$$
$$= \frac{32\left(x+x^{3/2}\right)^{3/2}}{105x^{3/2}} - \frac{16\left(x+x^{3/2}\right)^{3/2}}{35x} + \frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{4\left(\sqrt{x}+1\right)\left(15x-12\sqrt{x}+8\right)\sqrt{x^{3/2}+x}}{105\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^(3/2)],x]
[Out] (4*(1 + Sqrt[x])*(8 - 12*Sqrt[x] + 15*x)*Sqrt[x + x^(3/2)])/(105*Sqrt[x])
```

fricas [A] time = 0.77, size = 30, normalized size = 0.51

$$\frac{4\left(15\,x^{2}+(3\,x+8)\sqrt{x}-4\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="fricas")
[Out] 4/105*(15*x^2 + (3*x + 8)*sqrt(x) - 4*x)*sqrt(x^(3/2) + x)/x
```

giac [A] time = 0.40, size = 33, normalized size = 0.56

$$\frac{4}{105}\left(15\left(\sqrt{x}+1\right)^{\frac{7}{2}}-42\left(\sqrt{x}+1\right)^{\frac{5}{2}}+35\left(\sqrt{x}+1\right)^{\frac{3}{2}}-8\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="giac")
[Out] 4/105*(15*(sqrt(x) + 1)^(7/2) - 42*(sqrt(x) + 1)^(5/2) + 35*(sqrt(x) + 1)^(
3/2) - 8)*sgn(x)
```

maple [A] time = 0.01, size = 28, normalized size = 0.47

$$\frac{4\sqrt{x^{\frac{3}{2}}+x}\,\left(\sqrt{x}+1\right)\left(15x-12\sqrt{x}+8\right)}{105\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^(3/2))^(1/2),x)
[Out] 4/105*(x+x^(3/2))^(1/2)*(x^(1/2)+1)*(15*x-12*x^(1/2)+8)/x^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^(3/2) + x), x)
```

mupad [B] time = 3.54, size = 27, normalized size = 0.46

$$\frac{2\,x\,\sqrt{x+x^{3/2}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},3;\;4;\;-\sqrt{x}\right)}{3\,\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(3/2))^(1/2),x)
```

```
[Out] (2*x*(x + x^(3/2))^(1/2)*hypergeom([-1/2, 3], 4, -x^(1/2)))/(3*(x^(1/2) + 1
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**(3/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**(3/2) + x), x)
```

3.932 $\int x\sqrt{x+x^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}+\frac{64\left(x^{3/2}+x\right)^{3/2}}{231\sqrt{x}}-\frac{256\left(x^{3/2}+x\right)^{3/2}}{1155x}+\frac{512\left(x^{3/2}+x\right)^{3/2}}{3465x^{3/2}}-\frac{32}{99}\left(x^{3/2}+x\right)^{3/2}$$

```
[Out] -32/99*(x+x^(3/2))^(3/2)+512/3465*(x+x^(3/2))^(3/2)/x^(3/2)-256/1155*(x+x^(
3/2))^(3/2)/x+64/231*(x+x^(3/2))^(3/2)/x^(1/2)+4/11*(x+x^(3/2))^(3/2)*x^(1/
2)
```

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2016, 2002, 2014}

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}+\frac{64\left(x^{3/2}+x\right)^{3/2}}{231\sqrt{x}}-\frac{256\left(x^{3/2}+x\right)^{3/2}}{1155x}+\frac{512\left(x^{3/2}+x\right)^{3/2}}{3465x^{3/2}}-\frac{32}{99}\left(x^{3/2}+x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[x + x^(3/2)],x]
```

```
[Out] (-32*(x + x^(3/2))^(3/2))/99 + (512*(x + x^(3/2))^(3/2))/(3465*x^(3/2)) - (
256*(x + x^(3/2))^(3/2))/(1155*x) + (64*(x + x^(3/2))^(3/2))/(231*Sqrt[x])
+ (4*Sqrt[x]*(x + x^(3/2))^(3/2))/11
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{x+x^{3/2}}\,dx &= \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2} - \frac{8}{11}\int \sqrt{x}\sqrt{x+x^{3/2}}\,dx \\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2} + \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2} + \frac{16}{33}\int \sqrt{x+x^{3/2}}\,dx \\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2} + \frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}} + \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2} - \frac{64}{231}\int \frac{\sqrt{x+x^{3/2}}}{\sqrt{x}}\,dx \\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2} - \frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x} + \frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}} + \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2} + \frac{128\int \frac{\sqrt{x+}}{x}}{1155} \\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2} + \frac{512\left(x+x^{3/2}\right)^{3/2}}{3465x^{3/2}} - \frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x} + \frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}} + \frac{4}{11}\sqrt{x}\,(x+x
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.54

$$\frac{4\left(\sqrt{x}+1\right)\sqrt{x^{3/2}+x}\left(-280x^{3/2}+315x^2+240x-192\sqrt{x}+128\right)}{3465\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[x + x^(3/2)],x]
[Out] (4*(1 + Sqrt[x])*Sqrt[x + x^(3/2)]*(128 - 192*Sqrt[x] + 240*x - 280*x^(3/2)
 + 315*x^2))/(3465*Sqrt[x])
```

fricas [A] time = 0.70, size = 40, normalized size = 0.43

$$\frac{4\left(315\,x^3-40\,x^2+\left(35\,x^2+48\,x+128\right)\sqrt{x}-64\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{3465\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="fricas")
[Out] 4/3465*(315*x^3 - 40*x^2 + (35*x^2 + 48*x + 128)*sqrt(x) - 64*x)*sqrt(x^(3/
2) + x)/x
```

giac [A] time = 0.38, size = 51, normalized size = 0.54

$$\frac{4}{3465}\left(315\left(\sqrt{x}+1\right)^{\frac{11}{2}}-1540\left(\sqrt{x}+1\right)^{\frac{9}{2}}+2970\left(\sqrt{x}+1\right)^{\frac{7}{2}}-2772\left(\sqrt{x}+1\right)^{\frac{5}{2}}+1155\left(\sqrt{x}+1\right)^{\frac{3}{2}}-128\right)\mathrm{sgn}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="giac")
[Out] 4/3465*(315*(sqrt(x) + 1)^(11/2) - 1540*(sqrt(x) + 1)^(9/2) + 2970*(sqrt(x)
 + 1)^(7/2) - 2772*(sqrt(x) + 1)^(5/2) + 1155*(sqrt(x) + 1)^(3/2) - 128)*sg
n(x)
```

maple [A] time = 0.00, size = 38, normalized size = 0.40

$$\frac{4\sqrt{x^{\frac{3}{2}}+x}\left(\sqrt{x}+1\right)\left(315x^2-280x^{\frac{3}{2}}+240x-192\sqrt{x}+128\right)}{3465\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^(3/2)+x)^(1/2),x)
```

```
[Out] 4/3465*(x^(3/2)+x)^(1/2)*(x^(1/2)+1)*(315*x^2-280*x^(3/2)+240*x-192*x^(1/2)
+128)/x^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{\frac{3}{2}} + x}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^(3/2) + x)*x, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{x + x^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + x^(3/2))^(1/2),x)
```

```
[Out] int(x*(x + x^(3/2))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{x^{\frac{3}{2}} + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x**(3/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**(3/2) + x), x)
```

3.933 $\int \left(1-x^2\right) \sqrt{\frac{1}{2-x^2}} \, dx$

Optimal. Leaf size=18

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

```
[Out] 1/2*x/(1/(-x^2+2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6720, 383}

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
```

```
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned} \int \left(1-x^2\right) \sqrt{\frac{1}{2-x^2}} \, dx &= \left(\sqrt{\frac{1}{2-x^2}} \sqrt{2-x^2}\right) \int \frac{1-x^2}{\sqrt{2-x^2}} \, dx\\ &= \frac{x}{2\sqrt{\frac{1}{2-x^2}}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
```

```
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

fricas [A] time = 0.79, size = 20, normalized size = 1.11

$$-\frac{1}{2} \left(x^3-2x\right) \sqrt{-\frac{1}{x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="fricas")
[Out] -1/2*(x^3 - 2*x)*sqrt(-1/(x^2 - 2))
```

giac [A] time = 0.48, size = 18, normalized size = 1.00

$$-\frac{1}{2}\sqrt{-x^2+2}\,x\mathrm{sgn}\left(x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 2)*x*sgn(x^2 - 2)
```

maple [A] time = 0.01, size = 20, normalized size = 1.11

$$-\frac{\left(x^2-2\right)\sqrt{-\frac{1}{x^2-2}}\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)*(1/(-x^2+2))^(1/2),x)
[Out] -1/2*(x^2-2)*x*(-1/(x^2-2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(x^2-1\right)\sqrt{-\frac{1}{x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)*sqrt(-1/(x^2 - 2)), x)
```

mupad [B] time = 3.50, size = 19, normalized size = 1.06

$$-\frac{x\,\left(x^2-2\right)\,\sqrt{-\frac{1}{x^2-2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)*(-1/(x^2 - 2))^(1/2),x)
[Out] -(x*(x^2 - 2)*(-1/(x^2 - 2))^(1/2))/2
```

sympy [B] time = 0.50, size = 26, normalized size = 1.44

$$-\frac{x^3\sqrt{\frac{1}{2-x^2}}}{2}+x\sqrt{\frac{1}{2-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)*(1/(-x**2+2))**(1/2),x)
[Out] -x**3*sqrt(1/(2 - x**2))/2 + x*sqrt(1/(2 - x**2))
```

3.934 $\int \sqrt{x^2+x^3-x^4}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{-x^4+x^3+x^2}\,(1-2x)}{8x}-\frac{\left(-x^2+x+1\right)\sqrt{-x^4+x^3+x^2}}{3x}-\frac{5\sqrt{-x^4+x^3+x^2}\,\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2+x+1}}$$

```
[Out] -1/8*(1-2*x)*(-x^4+x^3+x^2)^(1/2)/x-1/3*(-x^2+x+1)*(-x^4+x^3+x^2)^(1/2)/x-5
/16*arcsin(1/5*(1-2*x)*5^(1/2))*(-x^4+x^3+x^2)^(1/2)/x/(-x^2+x+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1903, 640, 612, 619, 216}

$$-\frac{\sqrt{-x^4+x^3+x^2}\,(1-2x)}{8x}-\frac{\left(-x^2+x+1\right)\sqrt{-x^4+x^3+x^2}}{3x}-\frac{5\sqrt{-x^4+x^3+x^2}\,\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + x^3 - x^4],x]
```

```
[Out] -((1 - 2*x)*Sqrt[x^2 + x^3 - x^4])/(8*x) - ((1 + x - x^2)*Sqrt[x^2 + x^3 -
x^4])/(3*x) - (5*Sqrt[x^2 + x^3 - x^4]*ArcSin[(1 - 2*x)/Sqrt[5]])/(16*x*Sqr
t[1 + x - x^2])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$\int \sqrt{x^2+x^3-x^4}\,dx = \frac{\sqrt{x^2+x^3-x^4}\int x\sqrt{1+x-x^2}\,dx}{x\sqrt{1+x-x^2}}$$
$$= -\frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} + \frac{\sqrt{x^2+x^3-x^4}\int \sqrt{1+x-x^2}\,dx}{2x\sqrt{1+x-x^2}}$$
$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} + \frac{\left(5\sqrt{x^2+x^3-x^4}\right)\int \frac{1}{\sqrt{1+x-x^2}}\,dx}{16x\sqrt{1+x-x^2}}$$
$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} - \frac{\left(\sqrt{5}\sqrt{x^2+x^3-x^4}\right)\text{Subst}\left(\int -\right.}{16x\sqrt{1+x-x^2}}$$
$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{\left(1+x-x^2\right)\sqrt{x^2+x^3-x^4}}{3x} - \frac{5\sqrt{x^2+x^3-x^4}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{1+x-x^2}}$$

Mathematica [A] time = 0.03, size = 84, normalized size = 0.79

$$\frac{\sqrt{-x^4+x^3+x^2}\left(2\sqrt{x^2-x-1}\left(8x^2-2x-11\right)-15\tanh^{-1}\left(\frac{2x-1}{2\sqrt{x^2-x-1}}\right)\right)}{48x\sqrt{x^2-x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + x^3 - x^4],x]
[Out] (Sqrt[x^2 + x^3 - x^4]*(2*Sqrt[-1 - x + x^2]*(-11 - 2*x + 8*x^2) - 15*ArcTa
nh[(-1 + 2*x)/(2*Sqrt[-1 - x + x^2])]))/(48*x*Sqrt[-1 - x + x^2])
```

fricas [A] time = 0.71, size = 62, normalized size = 0.58

$$-\frac{15\,x\arctan\left(-\frac{x-\sqrt{-x^4+x^3+x^2}}{x^2}\right)-\sqrt{-x^4+x^3+x^2}\left(8\,x^2-2\,x-11\right)+11\,x}{24\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="fricas")
[Out] -1/24*(15*x*arctan(-(x - sqrt(-x^4 + x^3 + x^2))/x^2) - sqrt(-x^4 + x^3 + x
^2)*(8*x^2 - 2*x - 11) + 11*x)/x
```

giac [A] time = 0.43, size = 60, normalized size = 0.56

$$\frac{1}{48}\left(15\arcsin\left(\frac{1}{5}\sqrt{5}\right)+22\right)\mathrm{sgn}(x)+\frac{5}{16}\arcsin\left(\frac{1}{5}\sqrt{5}\,(2\,x-1)\right)\mathrm{sgn}(x)+\frac{1}{24}\left(2\left(4\,x\mathrm{sgn}(x)-\mathrm{sgn}(x)\right)x-11\,\mathrm{sgn}(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="giac")
[Out] 1/48*(15*arcsin(1/5*sqrt(5)) + 22)*sgn(x) + 5/16*arcsin(1/5*sqrt(5)*(2*x -
1))*sgn(x) + 1/24*(2*(4*x*sgn(x) - sgn(x))*x - 11*sgn(x))*sqrt(-x^2 + x + 1
)
```

maple [A] time = 0.01, size = 81, normalized size = 0.76

$$-\frac{\sqrt{-x^4+x^3+x^2}\left(-12\sqrt{-x^2+x+1}\,x-15\arcsin\left(\frac{(2x-1)\sqrt{5}}{5}\right)+16\left(-x^2+x+1\right)^{\frac{3}{2}}+6\sqrt{-x^2+x+1}\right)}{48\sqrt{-x^2+x+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^3+x^2)^(1/2),x)
```

```
[Out] -1/48*(-x^4+x^3+x^2)^(1/2)*(16*(-x^2+x+1)^(3/2)-12*x*(-x^2+x+1)^(1/2)+6*(-x
^2+x+1)^(1/2)-15*arcsin(1/5*(2*x-1)*5^(1/2)))/x/(-x^2+x+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+x^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^3 + x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{-x^4+x^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 - x^4)^(1/2),x)
```

```
[Out] int((x^2 + x^3 - x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+x^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**3+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 + x**3 + x**2), x)
```

3.935 $\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx$

Optimal. Leaf size=25

$$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

```
[Out] x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6720, 191}

$$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a^2 + x^2)^3],x]
```

```
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx &= \frac{(a^2+x^2)^{3/2}\int \frac{1}{(a^2+x^2)^{3/2}}\,dx}{\sqrt{(a^2+x^2)^3}}\\ &= \frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a^2 + x^2)^3],x]
```

```
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

fricas [B] time = 0.73, size = 64, normalized size = 2.56

$$\frac{a^4 + 2\,a^2x^2 + x^4 + \sqrt{a^6 + 3\,a^4x^2 + 3\,a^2x^4 + x^6}\,x}{a^6 + 2\,a^4x^2 + a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^4 + 2*a^2*x^2 + x^4 + sqrt(a^6 + 3*a^4*x^2 + 3*a^2*x^4 + x^6)*x)/(a^6 +
2*a^4*x^2 + a^2*x^4)
```

giac [A] time = 0.42, size = 14, normalized size = 0.56

$$\frac{x}{\sqrt{a^2 + x^2}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

maple [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{\left(a^2 + x^2\right)x}{\sqrt{\left(a^2 + x^2\right)^3}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2+x^2)^3)^(1/2),x)
```

```
[Out] x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)
```

maxima [A] time = 0.90, size = 14, normalized size = 0.56

$$\frac{x}{\sqrt{a^2 + x^2}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

mupad [B] time = 3.51, size = 25, normalized size = 1.00

$$\frac{x\,\sqrt{\left(a^2 + x^2\right)^3}}{a^2\,\left(a^2 + x^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 + x^2)^3)^(1/2),x)
```

```
[Out] (x*((a^2 + x^2)^3)^(1/2))/(a^2*(a^2 + x^2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a^2 + x^2\right)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+x**2)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a**2 + x**2)**3), x)
```

3.936 $\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx$

Optimal. Leaf size=42

$$2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -ln(1+x+x^(1/2))-2/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1357, 703, 634, 618, 204, 628}

$$2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] +
 x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x}+2\,\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x}-\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt{x}\right)-\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x}-\log\left(1+\sqrt{x}+x\right)+2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt{x}\right)\\
&= 2\sqrt{x}-\frac{2\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1+\sqrt{x}+x\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] +
 x]
```

fricas [A] time = 0.94, size = 35, normalized size = 0.83

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + 2*sqrt(x) - log(x
+ sqrt(x) + 1)
```

giac [A] time = 0.39, size = 33, normalized size = 0.79

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt
(x) + 1)
```

maple [A] time = 0.00, size = 34, normalized size = 0.81

$$-\frac{2\sqrt{3}\ \arctan\left(\frac{\left(2\sqrt{x}+1\right)\sqrt{3}}{3}\right)}{3}-\ln\left(x+\sqrt{x}\ +1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x+x^(1/2)+1),x)
```

```
[Out] -ln(x+x^(1/2)+1)-2/3*arctan(1/3*(2*x^(1/2)+1)*3^(1/2))*3^(1/2)+2*x^(1/2)
```

maxima [A] time = 2.01, size = 33, normalized size = 0.79

$$-\frac{2}{3}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}\ +1\right)\right)+2\sqrt{x}\ -\log\left(x+\sqrt{x}\ +1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt
(x) + 1)
```

mupad [B] time = 3.45, size = 35, normalized size = 0.83

$$2\sqrt{x}-\frac{2\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sqrt{x}}{3}\right)}{3}-\ln\left(x+\sqrt{x}\ +1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + x^(1/2) + 1),x)
```

```
[Out] 2*x^(1/2) - (2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - log(x +
 x^(1/2) + 1)
```

sympy [A] time = 0.25, size = 49, normalized size = 1.17

$$2\sqrt{x}\ -\log\left(4\sqrt{x}\ +4x+4\right)-\frac{2\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) - log(4*sqrt(x) + 4*x + 4) - 2*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 +
 sqrt(3)/3)/3
```

3.937 $\int \frac{x}{1+\sqrt{x}+x}\,dx$

Optimal. Leaf size=32

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x+4/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1357, 701, 618, 204}

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + Sqrt[x] + x),x]
```

```
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+\sqrt{x}+x}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x+x^2}\right)dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}+x+2\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}+x-4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt{x}\right)\\ &= -2\sqrt{x}+x+\frac{4\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + Sqrt[x] + x),x]
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

fricas [A] time = 0.72, size = 27, normalized size = 0.84

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)+x-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="fricas")
[Out] 4/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + x - 2*sqrt(x)
```

giac [A] time = 0.36, size = 25, normalized size = 0.78

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+x-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="giac")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

maple [A] time = 0.00, size = 26, normalized size = 0.81

$$x+\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2\sqrt{x}+1\right)\sqrt{3}}{3}\right)}{3}-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+x^(1/2)+1),x)
[Out] x+4/3*3^(1/2)*arctan(1/3*(2*x^(1/2)+1)*3^(1/2))-2*x^(1/2)
```

maxima [A] time = 2.13, size = 25, normalized size = 0.78

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+x-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

mupad [B] time = 0.04, size = 27, normalized size = 0.84

$$x+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sqrt{x}}{3}\right)}{3}-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + x^(1/2) + 1),x)
```

```
[Out] x + (4*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - 2*x^(1/2)
```

sympy [A] time = 0.23, size = 37, normalized size = 1.16

$$-2\sqrt{x}+x+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x**(1/2)),x)
```

```
[Out] -2*sqrt(x) + x + 4*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 + sqrt(3)/3)/3
```

3.938 $\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx$

Optimal. Leaf size=76

$$\frac{512\left(2\sqrt{x}+1\right)}{405\sqrt{x+\sqrt{x}+1}}+\frac{64\left(2\sqrt{x}+1\right)}{135\left(x+\sqrt{x}+1\right)^{3/2}}+\frac{4\left(2\sqrt{x}+1\right)}{15\left(x+\sqrt{x}+1\right)^{5/2}}$$

```
[Out] 4/15*(1+2*x^(1/2))/(1+x+x^(1/2))^(5/2)+64/135*(1+2*x^(1/2))/(1+x+x^(1/2))^(
3/2)+512/405*(1+2*x^(1/2))/(1+x+x^(1/2))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1352, 614, 613}

$$\frac{512\left(2\sqrt{x}+1\right)}{405\sqrt{x+\sqrt{x}+1}}+\frac{64\left(2\sqrt{x}+1\right)}{135\left(x+\sqrt{x}+1\right)^{3/2}}+\frac{4\left(2\sqrt{x}+1\right)}{15\left(x+\sqrt{x}+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

```
[Out] (4*(1 + 2*Sqrt[x]))/(15*(1 + Sqrt[x] + x)^(5/2)) + (64*(1 + 2*Sqrt[x]))/(13
5*(1 + Sqrt[x] + x)^(3/2)) + (512*(1 + 2*Sqrt[x]))/(405*Sqrt[1 + Sqrt[x] +
x])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^{7/2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}}+\frac{32}{15}\,\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^{5/2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}}+\frac{64\left(1+2\sqrt{x}\right)}{135\left(1+\sqrt{x}+x\right)^{3/2}}+\frac{256}{135}\,\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^{3/2}}\,dx, x\right.\\
&= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}}+\frac{64\left(1+2\sqrt{x}\right)}{135\left(1+\sqrt{x}+x\right)^{3/2}}+\frac{512\left(1+2\sqrt{x}\right)}{405\sqrt{1+\sqrt{x}+x}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.64

$$
\frac{4\left(2\sqrt{x}+1\right)\left(256x^{3/2}+128x^2+432x+304\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

```
[Out] (4*(1 + 2*Sqrt[x])*(203 + 304*Sqrt[x] + 432*x + 256*x^(3/2) + 128*x^2))/(40
5*(1 + Sqrt[x] + x)^(5/2))
```

fricas [A] time = 0.82, size = 95, normalized size = 1.25

$$
-\frac{4\left(128\,x^5+272\,x^4+455\,x^3+232\,x^2-\left(256\,x^5+736\,x^4+1366\,x^3+1427\,x^2+839\,x+101\right)\sqrt{x}-128\,x-20\right.}{405\left(x^6+3\,x^5+6\,x^4+7\,x^3+6\,x^2+3\,x+1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/405*(128*x^5 + 272*x^4 + 455*x^3 + 232*x^2 - (256*x^5 + 736*x^4 + 1366*x
^3 + 1427*x^2 + 839*x + 101)*sqrt(x) - 128*x - 203)*sqrt(x + sqrt(x) + 1)/(
x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

giac [A] time = 0.43, size = 45, normalized size = 0.59

$$
\frac{4\left(2\left(8\left(2\left(4\sqrt{x}\left(2\sqrt{x}+5\right)+35\right)\sqrt{x}+65\right)\sqrt{x}+355\right)\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{\frac{5}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="giac")
```

```
[Out] 4/405*(2*(8*(2*(4*sqrt(x)*(2*sqrt(x) + 5) + 35)*sqrt(x) + 65)*sqrt(x) + 355
)*sqrt(x) + 203)/(x + sqrt(x) + 1)^(5/2)
```

maple [A] time = 0.00, size = 53, normalized size = 0.70

$$
\frac{\frac{8\sqrt{x}}{15}+\frac{4}{15}}{\left(x+\sqrt{x}+1\right)^{\frac{5}{2}}}+\frac{\frac{128\sqrt{x}}{135}+\frac{64}{135}}{\left(x+\sqrt{x}+1\right)^{\frac{3}{2}}}+\frac{\frac{1024\sqrt{x}}{405}+\frac{512}{405}}{\sqrt{x+\sqrt{x}+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x+x^(1/2)+1)^(7/2),x)
```

```
[Out] 4/15*(2*x^(1/2)+1)/(x+x^(1/2)+1)^(5/2)+64/135*(2*x^(1/2)+1)/(x+x^(1/2)+1)^(
3/2)+512/405*(2*x^(1/2)+1)/(x+x^(1/2)+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{x}+1\right)^{\frac{7}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x + sqrt(x) + 1)^(7/2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x}\,\left(x+\sqrt{x}+1\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)),x)
```

```
[Out] int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\left(\sqrt{x}+x+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x+x**(1/2))**(7/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(sqrt(x) + x + 1)**(7/2)), x)
```

3.939 $\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

```
[Out] -1/x-arcsinh(x)-ln((x^2+1)^(1/2)+1)+(x^2+1)^(1/2)+(x^2+1)^(1/2)/x
```

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 277, 215, 1591, 190, 43}

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 + x
^2]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx &= \int \left(-\frac{1}{1+\sqrt{1+x^2}}+\frac{x}{1+\sqrt{1+x^2}}\right)dx\\
&= -\int \frac{1}{1+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+\sqrt{1+x^2}}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1+x^2\right)-\int \left(-\frac{1}{x^2}+\frac{\sqrt{1+x^2}}{x^2}\right)dx\\
&= -\frac{1}{x}-\int \frac{\sqrt{1+x^2}}{x^2}\,dx+\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1+x^2}\right)\\
&= -\frac{1}{x}+\frac{\sqrt{1+x^2}}{x}-\int \frac{1}{\sqrt{1+x^2}}\,dx+\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx,x,\sqrt{1+x^2}\right)\\
&= -\frac{1}{x}+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}-\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 + x
^2]]
```

fricas [A] time = 0.69, size = 64, normalized size = 1.39

$$\frac{x\log\left(2x^2-\sqrt{x^2+1}(2x+1)+x+1\right)-x\log(x)-x\log\left(-x+\sqrt{x^2+1}+1\right)+\sqrt{x^2+1}(x+1)+x-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x^2+1)^(1/2)+1),x, algorithm="fricas")
```

```
[Out] (x*log(2*x^2 - sqrt(x^2 + 1)*(2*x + 1) + x + 1) - x*log(x) - x*log(-x + sqr
t(x^2 + 1) + 1) + sqrt(x^2 + 1)*(x + 1) + x - 1)/x
```

giac [A] time = 0.43, size = 79, normalized size = 1.72

$$\sqrt{x^2+1}-\frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1}-\frac{1}{x}+\log\left(-x+\sqrt{x^2+1}\right)-\log\left(|x|\right)-\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\log\left(\left|-x+\sqrt{x^2+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x^2+1)^(1/2)+1),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 1) - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + log(-x + sqrt(x^2 + 1
)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + log(abs(-x + sqrt(x^2
 + 1) - 1))
```

maple [A] time = 0.00, size = 53, normalized size = 1.15

$$-\sqrt{x^2+1}\,x-\operatorname{arcsinh}(x)-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)-\frac{1}{x}+\frac{(x^2+1)^{\frac{3}{2}}}{x}+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(1+(x^2+1)^(1/2)),x)
```

```
[Out] -1/x+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(1/2))-ln(x)+(x^2+1)^(3/2)/x-(x^2+1)^(
1/2)*x-arcsinh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^2-\frac{1}{2}x-\int\frac{x^3-x^2}{2\left(x^2+2\sqrt{x^2+1}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x^2+1)^(1/2)+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/2*x - integrate(1/2*(x^3 - x^2)/(x^2 + 2*sqrt(x^2 + 1) + 2), x)
```

mupad [B] time = 0.04, size = 46, normalized size = 1.00

$$\sqrt{x^2+1}-\ln(x)-\operatorname{asinh}(x)+\frac{\sqrt{x^2+1}}{x}-\frac{1}{x}+\operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^2 + 1)^(1/2) + 1),x)
```

```
[Out] atan((x^2 + 1)^(1/2)*1i)*1i - asinh(x) - log(x) + (x^2 + 1)^(1/2) + (x^2 +
1)^(1/2)/x - 1/x
```

sympy [A] time = 2.98, size = 48, normalized size = 1.04

$$\frac{x}{\sqrt{x^2+1}}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\operatorname{asinh}(x)-\frac{1}{x}+\frac{1}{x\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/((x**2+1)**(1/2)+1),x)
```

```
[Out] x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2 + 1) + 1) - asinh(x) - 1/
x + 1/(x*sqrt(x**2 + 1))
```

3.940 $\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}} dx$

Optimal. Leaf size=20

$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

[Out] 3/2*(x^2-1)^(1/3)/(1+x)^(2/3)

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {651}

$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]

[Out] (3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\, dx = \frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.15

$$\frac{3(x-1)\sqrt[3]{x+1}}{2\left(x^2-1\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]

[Out] (3*(-1 + x)*(1 + x)^(1/3))/(2*(-1 + x^2)^(2/3))

fricas [A] time = 0.46, size = 14, normalized size = 0.70

$$\frac{3\left(x^2-1\right)^{\frac{1}{3}}}{2\,(x+1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="fricas")

[Out] 3/2*(x^2 - 1)^(1/3)/(x + 1)^(2/3)

giac [A] time = 0.36, size = 13, normalized size = 0.65

$$\frac{3}{2}\left(-\frac{2}{x+1}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="giac")
[Out] 3/2*(-2/(x + 1) + 1)^(1/3)
```

maple [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{3(x-1)(x+1)^{\frac{1}{3}}}{2\left(x^2-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)^(2/3)/(x^2-1)^(2/3),x)
[Out] 3/2*(x-1)*(x+1)^(1/3)/(x^2-1)^(2/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-1\right)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\left(x^2-1\right)^{2/3}(x+1)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)),x)
[Out] int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((x-1)(x+1))^{\frac{2}{3}}(x+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(2/3)/(x**2-1)**(2/3),x)
[Out] Integral(1/(((x - 1)*(x + 1))**(2/3)*(x + 1)**(2/3)), x)
```

3.941 $\int \left(\left(1-x^6\right)^{2/3} + \frac{\left(1-x^6\right)^{2/3}}{x^6} \right) dx$

Optimal. Leaf size=35

$$\frac{1}{5}x\left(1-x^6\right)^{2/3} - \frac{\left(1-x^6\right)^{2/3}}{5x^5}$$

```
[Out] -1/5*(-x^6+1)^(2/3)/x^5+1/5*x*(-x^6+1)^(2/3)
```

Rubi [C] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {245, 364}

$$x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right) - \frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

```
[Out] -Hypergeometric2F1[-5/6, -2/3, 1/6, x^6]/(5*x^5) + x*Hypergeometric2F1[-2/3
, 1/6, 7/6, x^6]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(\left(1-x^6\right)^{2/3}+\frac{\left(1-x^6\right)^{2/3}}{x^6}\right) dx &= \int \left(1-x^6\right)^{2/3}\,dx+\int \frac{\left(1-x^6\right)^{2/3}}{x^6}\,dx\\
&= -\frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5}+x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.51

$$-\frac{\left(1-x^6\right)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

[Out] -1/5*(1 - x^6)^(5/3)/x^5

fricas [A] time = 0.51, size = 19, normalized size = 0.54

$$\frac{\left(x^{6}-1\right)\left(-x^{6}+1\right)^{\frac{2}{3}}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="fricas")

[Out] 1/5*(x^6 - 1)*(-x^6 + 1)^(2/3)/x^5

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^{6}+1\right)^{\frac{2}{3}}+\frac{\left(-x^{6}+1\right)^{\frac{2}{3}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="giac")

[Out] integrate((-x^6 + 1)^(2/3) + (-x^6 + 1)^(2/3)/x^6, x)

maple [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{\left(-x^{6}+1\right)^{\frac{2}{3}}\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)\left(x+1\right)\left(x-1\right)}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x)

[Out] 1/5*(-x^6+1)^(2/3)*(x^2-x+1)*(x^2+x+1)*(x+1)/x^5*(x-1)

maxima [A] time = 2.15, size = 38, normalized size = 1.09

$$\frac{\left(x^{6}-1\right)\left(x^{2}+x+1\right)^{\frac{2}{3}}\left(-x^{2}+x-1\right)^{\frac{2}{3}}\left(x+1\right)^{\frac{2}{3}}\left(x-1\right)^{\frac{2}{3}}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="maxima")

[Out] 1/5*(x^6 - 1)*(x^2 + x + 1)^(2/3)*(-x^2 + x - 1)^(2/3)*(x + 1)^(2/3)*(x - 1
)^(2/3)/x^5

mupad [B] time = 3.59, size = 14, normalized size = 0.40

$$-\frac{\left(1-x^{6}\right)^{5/3}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x^6)^(2/3)/x^6 + (1 - x^6)^(2/3),x)

[Out] -(1 - x^6)^(5/3)/(5*x^5)

sympy [C] time = 1.14, size = 68, normalized size = 1.94

$$\frac{x\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| x^{6}e^{2i\pi}\right)}{6\Gamma\left(\frac{7}{6}\right)} + \frac{\Gamma\left(-\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix} -\frac{5}{6}, -\frac{2}{3} \\ \frac{1}{6} \end{matrix}\middle| x^{6}e^{2i\pi}\right)}{6x^{5}\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**6+1)**(2/3)+(-x**6+1)**(2/3)/x**6,x)
```

```
[Out] x*gamma(1/6)*hyper((-2/3, 1/6), (7/6,), x**6*exp_polar(2*I*pi))/(6*gamma(7/
6)) + gamma(-5/6)*hyper((-5/6, -2/3), (1/6,), x**6*exp_polar(2*I*pi))/(6*x*
*5*gamma(1/6))
```

3.942 $\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^n}}$$

```
[Out] x^m/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {12, 449}

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x^n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}\left(2am+b(2m-n)x^n\right)}{2\left(a+bx^n\right)^{3/2}}\,dx &= \frac{1}{2}\int \frac{x^{-1+m}\left(2am+b(2m-n)x^n\right)}{\left(a+bx^n\right)^{3/2}}\,dx\\ &= \frac{x^m}{\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [C] time = 0.18, size = 111, normalized size = 7.40

$$\frac{x^m\sqrt{\frac{bx^n}{a}+1}\left(b(2m-n)x^n\,{}_2F_1\left(\frac{3}{2},\frac{m+n}{n};\frac{m}{n}+2;-\frac{bx^n}{a}\right)+2a(m+n)\,{}_2F_1\left(\frac{3}{2},\frac{m}{n};\frac{m+n}{n};-\frac{bx^n}{a}\right)\right)}{2a(m+n)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/n
, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m/n
, -((b*x^n)/a)]))/(2*a*(m + n)*Sqrt[a + b*x^n])
```

fricas [A] time = 0.67, size = 16, normalized size = 1.07

$$\frac{xx^{m-1}}{\sqrt{bx^n+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
fricas")
```

```
[Out] x*x^(m - 1)/sqrt(b*x^n + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b(2\,m-n)x^n + 2\,am)x^{m-1}}{2\,(bx^n + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(2am + (2m-n)\,b\,x^n)\,x^{m-1}}{2\,(b\,x^n + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(b*x^n+a)^(3/2),x)
```

```
[Out] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(b*x^n+a)^(3/2),x)
```

maxima [A] time = 1.13, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^n + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
maxima")
```

```
[Out] x^m/sqrt(b*x^n + a)
```

mupad [B] time = 3.68, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{a + b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*(2*a*m + b*x^n*(2*m - n)))/(2*(a + b*x^n)^(3/2)),x)
```

```
[Out] x^m/(a + b*x^n)^(1/2)
```

sympy [C] time = 98.05, size = 100, normalized size = 6.67

$$\frac{mx^m\Gamma\left(\frac{m}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{n}\\ \frac{m}{n}+1\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{a}\,n\Gamma\left(\frac{m}{n}+1\right)} + \frac{bx^mx^n\,(2m-n)\,\Gamma\left(\frac{m}{n}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{n}+1\\ \frac{m}{n}+2\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}n\Gamma\left(\frac{m}{n}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(2*m-n)*x**n)/(a+b*x**n)**(3/2),x)
```

```
[Out] m*x**m*gamma(m/n)*hyper((3/2, m/n), (m/n + 1,), b*x**n*exp_polar(I*pi)/a)/(
sqrt(a)*n*gamma(m/n + 1)) + b*x**m*x**n*(2*m - n)*gamma(m/n + 1)*hyper((3/2
, m/n + 1), (m/n + 2,), b*x**n*exp_polar(I*pi)/a)/(2*a**(3/2)*n*gamma(m/n +
 2))
```

3.943 $\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx$

Optimal. Leaf size=53

$$-\frac{4}{567}(3x+2)^{7/2}+\frac{8}{135}(3x+2)^{5/2}-\frac{10}{81}(3x+2)^{3/2}-\frac{4}{81}\sqrt{3x+2}$$

```
[Out] -10/81*(2+3*x)^(3/2)+8/135*(2+3*x)^(5/2)-4/567*(2+3*x)^(7/2)-4/81*(2+3*x)^(
1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 772}

$$-\frac{4}{567}(3x+2)^{7/2}+\frac{8}{135}(3x+2)^{5/2}-\frac{10}{81}(3x+2)^{3/2}-\frac{4}{81}\sqrt{3x+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

```
[Out] (-4*Sqrt[2 + 3*x])/81 - (10*(2 + 3*x)^(3/2))/81 + (8*(2 + 3*x)^(5/2))/135 -
 (4*(2 + 3*x)^(7/2))/567
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx &= \int \frac{x\left(1-2x^2\right)}{\sqrt{2+3x}} \, dx \\
&= \int \left(-\frac{2}{27\sqrt{2+3x}}-\frac{5}{9}\sqrt{2+3x}+\frac{4}{9}(2+3x)^{3/2}-\frac{2}{27}(2+3x)^{5/2}\right) dx \\
&= -\frac{4}{81}\sqrt{2+3x}-\frac{10}{81}(2+3x)^{3/2}+\frac{8}{135}(2+3x)^{5/2}-\frac{4}{567}(2+3x)^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.53

$$-\frac{2\sqrt{3x+2}\left(270x^3-216x^2-123x+164\right)}{2835}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

```
[Out] (-2*Sqrt[2 + 3*x]*(164 - 123*x - 216*x^2 + 270*x^3))/2835
```

fricas [A] time = 0.51, size = 24, normalized size = 0.45

$$-\frac{2}{2835}\left(270\,x^3-216\,x^2-123\,x+164\right)\sqrt{3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="fricas")
[Out] -2/2835*(270*x^3 - 216*x^2 - 123*x + 164)*sqrt(3*x + 2)
```

giac [A] time = 0.34, size = 37, normalized size = 0.70

$$-\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="giac")
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/
81*sqrt(3*x + 2)
```

maple [A] time = 0.00, size = 25, normalized size = 0.47

$$-\frac{2\left(270x^3-216x^2-123x+164\right)\sqrt{3x+2}}{2835}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^3+x)/(3*x+2)^(1/2),x)
[Out] -2/2835*(270*x^3-216*x^2-123*x+164)*(3*x+2)^(1/2)
```

maxima [A] time = 0.58, size = 37, normalized size = 0.70

$$-\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="maxima")
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/
81*sqrt(3*x + 2)
```

mupad [B] time = 0.04, size = 37, normalized size = 0.70

$$\frac{8\,(3\,x+2)^{5/2}}{135}-\frac{10\,(3\,x+2)^{3/2}}{81}-\frac{4\,\sqrt{3\,x+2}}{81}-\frac{4\,(3\,x+2)^{7/2}}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2*x^3)/(3*x + 2)^(1/2),x)
[Out] (8*(3*x + 2)^(5/2))/135 - (10*(3*x + 2)^(3/2))/81 - (4*(3*x + 2)^(1/2))/81
- (4*(3*x + 2)^(7/2))/567
```

sympy [A] time = 11.73, size = 46, normalized size = 0.87

$$-\frac{4\,(3x+2)^{\frac{7}{2}}}{567}+\frac{8\,(3x+2)^{\frac{5}{2}}}{135}-\frac{10\,(3x+2)^{\frac{3}{2}}}{81}-\frac{4\sqrt{3x+2}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**3+x)/(2+3*x)**(1/2),x)
[Out] -4*(3*x + 2)**(7/2)/567 + 8*(3*x + 2)**(5/2)/135 - 10*(3*x + 2)**(3/2)/81 -
 4*sqrt(3*x + 2)/81
```

3.944 $\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=31

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

```
[Out] -4*(1+x)^(1/4)+4*ln(1+(1+x)^(1/4))+2*(1+x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2012, 1593, 266, 43}

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
```

```
[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2012

```
Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] :> Dist[1/Coeffic
ient[u, x, 1], Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b, j
, n, p}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx, x, 1+x\right)\\ &= \text{Subst}\left(\int \frac{1}{\left(1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx, x, 1+x\right)\\ &= 4\,\text{Subst}\left(\int \frac{x^2}{1+x}\,dx, x, \sqrt[4]{1+x}\right)\\ &= 4\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)dx, x, \sqrt[4]{1+x}\right)\\ &= -4\sqrt[4]{1+x}+2\sqrt{1+x}+4\log\left(1+\sqrt[4]{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

fricas [A] time = 0.59, size = 25, normalized size = 0.81

$$2\sqrt{x+1}-4\,(x+1)^{\frac{1}{4}}+4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

giac [A] time = 0.32, size = 25, normalized size = 0.81

$$2\sqrt{x+1}-4\,(x+1)^{\frac{1}{4}}+4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

maple [A] time = 0.02, size = 26, normalized size = 0.84

$$4\ln\left(1+(x+1)^{\frac{1}{4}}\right)-4\,(x+1)^{\frac{1}{4}}+2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x+1)^(1/4)+(x+1)^(1/2)),x)
[Out] -4*(x+1)^(1/4)+4*ln(1+(x+1)^(1/4))+2*(x+1)^(1/2)
```

maxima [A] time = 0.65, size = 25, normalized size = 0.81

$$2\sqrt{x+1}-4\,(x+1)^{\frac{1}{4}}+4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

mupad [B] time = 0.07, size = 25, normalized size = 0.81

$$4\,\ln\left((x+1)^{1/4}+1\right)+2\,\sqrt{x+1}-4\,(x+1)^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + (x + 1)^(1/4)),x)
```

```
[Out] 4*log((x + 1)^(1/4) + 1) + 2*(x + 1)^(1/2) - 4*(x + 1)^(1/4)
```

sympy [A] time = 0.24, size = 27, normalized size = 0.87

$$-4\sqrt[4]{x+1}+2\sqrt{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)**(1/4)+(1+x)**(1/2)),x)
```

```
[Out] -4*(x + 1)**(1/4) + 2*sqrt(x + 1) + 4*log((x + 1)**(1/4) + 1)
```

3.945 $\int \frac{1+2x}{\sqrt{x+x^2}} dx$

Optimal. Leaf size=11

$$2\sqrt{x^2+x}$$

```
[Out] 2*(x^2+x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {629}

$$2\sqrt{x^2+x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/Sqrt[x + x^2],x]
[Out] 2*Sqrt[x + x^2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1+2x}{\sqrt{x+x^2}} dx = 2\sqrt{x+x^2}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$2\sqrt{x(x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/Sqrt[x + x^2],x]
[Out] 2*Sqrt[x*(1 + x)]
```

fricas [A] time = 0.53, size = 9, normalized size = 0.82

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x^2 + x)
```

giac [A] time = 0.36, size = 9, normalized size = 0.82

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x^2 + x)
```

maple [A] time = 0.00, size = 14, normalized size = 1.27

$$\frac{2\left(x+1\right)x}{\sqrt{x^{2}+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(x^2+x)^(1/2),x)
[Out] 2*(x+1)*x/(x^2+x)^(1/2)
```

maxima [A] time = 0.63, size = 9, normalized size = 0.82

$$2\sqrt{x^{2}+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x^2 + x)
```

mupad [B] time = 3.52, size = 9, normalized size = 0.82

$$2\sqrt{x\,\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(x + x^2)^(1/2),x)
[Out] 2*(x*(x + 1))^(1/2)
```

sympy [A] time = 0.14, size = 8, normalized size = 0.73

$$2\sqrt{x^{2}+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+x)**(1/2),x)
[Out] 2*sqrt(x**2 + x)
```

3.946 $\int \frac{1}{2\sqrt{x}(1+x)} dx$

Optimal. Leaf size=6

$$\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 63, 203}

$$\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2\sqrt{x}(1+x)} dx &= \frac{1}{2}\int \frac{1}{\sqrt{x}(1+x)} dx\\ &= \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{x}\right)\\ &= \tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]
```

fricas [A] time = 0.69, size = 4, normalized size = 0.67

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="fricas")
[Out] arctan(sqrt(x))
```

giac [A] time = 0.33, size = 4, normalized size = 0.67

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(x))
```

maple [A] time = 0.00, size = 5, normalized size = 0.83

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(x+1)/x^(1/2),x)
[Out] arctan(x^(1/2))
```

maxima [A] time = 1.39, size = 4, normalized size = 0.67

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="maxima")
[Out] arctan(sqrt(x))
```

mupad [B] time = 0.14, size = 4, normalized size = 0.67

$$\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)*(x + 1)),x)
[Out] atan(x^(1/2))
```

sympy [A] time = 0.21, size = 5, normalized size = 0.83

$$\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x**(1/2),x)
[Out] atan(sqrt(x))
```

3.947 $\int \frac{1}{x\sqrt{6x-x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\sqrt{6x-x^2}}{3x}$$

```
[Out] -1/3*(-x^2+6*x)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {650}

$$-\frac{\sqrt{6x-x^2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[6*x - x^2]),x]
```

```
[Out] -Sqrt[6*x - x^2]/(3*x)
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{6x-x^2}}{3x}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.85

$$\frac{x-6}{3\sqrt{-((x-6)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[6*x - x^2]),x]
```

```
[Out] (-6 + x)/(3*Sqrt[-((-6 + x)*x)])
```

fricas [A] time = 0.68, size = 16, normalized size = 0.80

$$-\frac{\sqrt{-x^2+6\,x}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

giac [A] time = 0.41, size = 25, normalized size = 1.25

$$\frac{2}{3\left(\frac{\sqrt{-x^2+6\,x}-3}{x-3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3/((sqrt(-x^2 + 6*x) - 3)/(x - 3) - 1)
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{x-6}{3\sqrt{-x^2+6x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+6*x)^(1/2),x)
```

```
[Out] 1/3*(x-6)/(-x^2+6*x)^(1/2)
```

maxima [A] time = 1.43, size = 16, normalized size = 0.80

$$-\frac{\sqrt{-x^2+6\,x}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

mupad [B] time = 3.51, size = 16, normalized size = 0.80

$$-\frac{\sqrt{6\,x-x^2}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x - x^2)^(1/2)),x)
```

```
[Out] -(6*x - x^2)^(1/2)/(3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-x\,(x-6)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+6*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-x*(x - 6))), x)
```

3.948 $\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx$

Optimal. Leaf size=17

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

```
[Out] 2/3*x^(3/2)+1/2*x^2
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{x^2}{2}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])*Sqrt[x],x]
[Out] (2*x^(3/2))/3 + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx &= \int \left(\sqrt{x}+x\right)dx\\ &= \frac{2x^{3/2}}{3}+\frac{x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])*Sqrt[x],x]
[Out] (2*x^(3/2))/3 + x^2/2
```

fricas [A] time = 0.57, size = 11, normalized size = 0.65

$$\frac{1}{2}\,x^2+\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

giac [A] time = 0.35, size = 11, normalized size = 0.65

$$\frac{1}{2}\,x^2+\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.71

$$\frac{x^2}{2}+\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)*x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)+1/2*x^2
```

maxima [B] time = 0.73, size = 26, normalized size = 1.53

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4-\frac{4}{3}\left(\sqrt{x}+1\right)^3+\left(\sqrt{x}+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2
```

mupad [B] time = 0.02, size = 11, normalized size = 0.65

$$\frac{x^2}{2}+\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2) + 1),x)
```

```
[Out] x^2/2 + (2*x^(3/2))/3
```

sympy [A] time = 0.14, size = 12, normalized size = 0.71

$$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x**(1/2)),x)
```

```
[Out] 2*x**(3/2)/3 + x**2/2
```

3.949 $\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=19

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

```
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x])/x^(1/3),x]
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx &= \int \left(\frac{1}{\sqrt[3]{x}}-\sqrt[6]{x}\right)dx\\ &= \frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x])/x^(1/3),x]
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

fricas [A] time = 0.63, size = 11, normalized size = 0.58

$$-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="fricas")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

giac [A] time = 0.37, size = 11, normalized size = 0.58

$$-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="giac")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

maple [A] time = 0.00, size = 12, normalized size = 0.63

$$-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^(1/2))/x^(1/3),x)
```

```
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

maxima [A] time = 0.48, size = 11, normalized size = 0.58

$$-\frac{6}{7}x^{\frac{7}{6}}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="maxima")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

mupad [B] time = 0.02, size = 12, normalized size = 0.63

$$-\frac{3\,x^{2/3}\,\left(4\,\sqrt{x}-7\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(1/2) - 1)/x^(1/3),x)
```

```
[Out] -(3*x^(2/3)*(4*x^(1/2) - 7))/14
```

sympy [A] time = 1.53, size = 15, normalized size = 0.79

$$-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**(1/2))/x**(1/3),x)
```

```
[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

3.950 $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}} \, dx$

Optimal. Leaf size=41

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] -6*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+6*arctan(x^(1/6))+2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {341, 50, 63, 203}

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^(1/3)),x]
```

```
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan[x^(1/6)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^{7/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{6x^{7/6}}{7} - 3\,\text{Subst}\left(\int \frac{x^{5/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3\,\text{Subst}\left(\int \frac{x^{3/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} - 3\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x)}\,dx, x, \sqrt[3]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6\tan^{-1}\left(\sqrt[6]{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^(1/3)),x]
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan[x^(1/6)]
```

fricas [A] time = 0.67, size = 25, normalized size = 0.61

$$\frac{6}{7}(x-7)x^{\frac{1}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} + 6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="fricas")
[Out] 6/7*(x - 7)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 6*arctan(x^(1/6))
```

giac [A] time = 0.38, size = 27, normalized size = 0.66

$$\frac{6}{7}x^{\frac{7}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 6x^{\frac{1}{6}} + 6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="giac")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))
```

maple [A] time = 0.00, size = 28, normalized size = 0.68

$$\frac{6x^{\frac{7}{6}}}{7} + 6\arctan\left(x^{\frac{1}{6}}\right) - \frac{6x^{\frac{5}{6}}}{5} + 2\sqrt{x} - 6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/3)+1),x)
```

```
[Out] -6*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+6*arctan(x^(1/6))+2*x^(1/2)
```

maxima [A] time = 1.63, size = 27, normalized size = 0.66

$$\frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 6\,x^{\frac{1}{6}} + 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="maxima")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))
```

mupad [B] time = 0.03, size = 27, normalized size = 0.66

$$6\,\mathrm{atan}\left(x^{1/6}\right) + 2\,\sqrt{x} - 6\,x^{1/6} - \frac{6\,x^{5/6}}{5} + \frac{6\,x^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/3) + 1),x)
[Out] 6*atan(x^(1/6)) + 2*x^(1/2) - 6*x^(1/6) - (6*x^(5/6))/5 + (6*x^(7/6))/7
```

sympy [A] time = 3.36, size = 37, normalized size = 0.90

$$\frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 6\sqrt[6]{x} + 2\sqrt{x} + 6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x**(1/3)),x)
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 2*sqrt(x) + 6*atan(x**(1/6))
```

3.951 $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x} dx$

Optimal. Leaf size=67

$$6\sqrt[3]{\sqrt{x}+1}+3\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log(x)}{2}-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

```
[Out] -1/2*ln(x)+3*ln(1-(1+x^(1/2))^(1/3))-2*arctan(1/3*(1+2*(1+x^(1/2))^(1/3))*3
^(1/2))*3^(1/2)+6*(1+x^(1/2))^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 57, 618, 204, 31}

$$6\sqrt[3]{\sqrt{x}+1}+3\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log(x)}{2}-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3
]] + 3*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x}\,dx, x, \sqrt{x}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}}\;+2\,\text{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx, x, \sqrt{x}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}}\;-\frac{\log(x)}{2}-3\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+\sqrt{x}}\right)-3\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x,\right.\\
&= 6\sqrt[3]{1+\sqrt{x}}\;+3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right)-\frac{\log(x)}{2}+6\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+\sqrt{x}}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}}\;-2\sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}}\right)+3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right)-\frac{\log(x)}{2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 1.31

$$6\sqrt[3]{\sqrt{x}+1}+2\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\log\left(\left(\sqrt{x}+1\right)^{2/3}+\sqrt[3]{\sqrt{x}+1}+1\right)-2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3
]] + 2*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[1 + (1 + Sqrt[x])^(1/3) + (1 + Sq
rt[x])^(2/3)]
```

fricas [A] time = 0.98, size = 65, normalized size = 0.97

$$-2\,\sqrt{3}\,\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\sqrt{x}+1\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\,\log\left(\left(\sqrt{x}+1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*arctan(2/3*sqrt(3)*(sqrt(x) + 1)^(1/3) + 1/3*sqrt(3)) + 6*(sqrt(
x) + 1)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log(
(sqrt(x) + 1)^(1/3) - 1)
```

giac [A] time = 0.63, size = 64, normalized size = 0.96

$$-2\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\,\log\left(\left|\left(\sqrt{x}+1\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="giac")
```

```
[Out] -2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(sqrt(x) + 1
)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log(abs((s
qrt(x) + 1)^(1/3) - 1))
```

maple [A] time = 0.01, size = 64, normalized size = 0.96

$$-2\sqrt{3}\ \arctan\left(\frac{\left(1+2\left(\sqrt{x}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)+2\ln\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)-\ln\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)^(1/3)/x,x)
```

```
[Out] 6*(x^(1/2)+1)^(1/3)+2*ln((x^(1/2)+1)^(1/3)-1)-ln((x^(1/2)+1)^(2/3)+(x^(1/2)
+1)^(1/3)+1)-2*arctan(1/3*(1+2*(x^(1/2)+1)^(1/3))*3^(1/2))*3^(1/2)
```

maxima [A] time = 1.72, size = 63, normalized size = 0.94

$$-2\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\ \log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(sqrt(x) + 1
)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log((sqrt(
x) + 1)^(1/3) - 1)
```

mupad [B] time = 3.83, size = 73, normalized size = 1.09

$$2\ \ln\left(\left(\sqrt{x}+1\right)^{1/3}-1\right)+6\left(\sqrt{x}+1\right)^{1/3}+\ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)-\ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}+\frac{\sqrt{3}\,1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)^(1/3)/x,x)
```

```
[Out] 2*log((x^(1/2) + 1)^(1/3) - 1) + 6*(x^(1/2) + 1)^(1/3) + log((x^(1/2) + 1)^
(1/3) - (3^(1/2)*1i)/2 + 1/2)*(3^(1/2)*1i - 1) - log((3^(1/2)*1i)/2 + (x^(1
/2) + 1)^(1/3) + 1/2)*(3^(1/2)*1i + 1)
```

sympy [C] time = 1.05, size = 39, normalized size = 0.58

$$-\frac{2\sqrt[6]{x}\,\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\right|\frac{e^{i\pi}}{\sqrt{x}}\right)}{\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))**(1/3)/x,x)
```

```
[Out] -2*x**(1/6)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*pi)/sqrt(x)
)/gamma(2/3)
```

3.952 $\int (1-\sqrt{x})\,dx$

Optimal. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

[Out] x-2/3*x^(3/2)

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[1 - Sqrt[x],x]

[Out] x - (2*x^(3/2))/3

Rubi steps

$$\int \left(1-\sqrt{x}\right)\,dx=x-\frac{2x^{3/2}}{3}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[1 - Sqrt[x],x]

[Out] x - (2*x^(3/2))/3

fricas [A] time = 0.67, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/2),x, algorithm="fricas")

[Out] -2/3*x^(3/2) + x

giac [A] time = 0.33, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/2),x, algorithm="giac")

[Out] -2/3*x^(3/2) + x

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-x^(1/2),x)
[Out] -2/3*x^(3/2)+x
```

maxima [A] time = 0.88, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="maxima")
[Out] -2/3*x^(3/2) + x
```

mupad [B] time = 0.00, size = 7, normalized size = 0.64

$$x - \frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - x^(1/2),x)
[Out] x - (2*x^(3/2))/3
```

sympy [A] time = 0.06, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/2),x)
[Out] -2*x**(3/2)/3 + x
```

3.953 $\int \left(1-\sqrt[4]{x}\right) dx$

Optimal. Leaf size=11

$$x-\frac{4x^{5/4}}{5}$$

[Out] x-4/5*x^(5/4)

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

[In] Int[1 - x^(1/4),x]

[Out] x - (4*x^(5/4))/5

Rubi steps

$$\int \left(1-\sqrt[4]{x}\right) dx = x-\frac{4x^{5/4}}{5}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

[In] Integrate[1 - x^(1/4),x]

[Out] x - (4*x^(5/4))/5

fricas [A] time = 1.28, size = 7, normalized size = 0.64

$$-\frac{4}{5}\,x^{\frac{5}{4}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/4),x, algorithm="fricas")

[Out] -4/5*x^(5/4) + x

giac [A] time = 0.35, size = 7, normalized size = 0.64

$$-\frac{4}{5}\,x^{\frac{5}{4}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/4),x, algorithm="giac")

[Out] -4/5*x^(5/4) + x

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-x^(1/4),x)
[Out] x-4/5*x^(5/4)
```

maxima [A] time = 0.62, size = 7, normalized size = 0.64

$$-\frac{4}{5}\,x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/4),x, algorithm="maxima")
[Out] -4/5*x^(5/4) + x
```

mupad [B] time = 0.02, size = 7, normalized size = 0.64

$$x - \frac{4\,x^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - x^(1/4),x)
[Out] x - (4*x^(5/4))/5
```

sympy [A] time = 0.06, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/4),x)
[Out] -4*x**(5/4)/5 + x
```

3.954 $\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} \, dx$

Optimal. Leaf size=11

$$x-\frac{4x^{5/4}}{5}$$

```
[Out] x-4/5*x^(5/4)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {26}

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x])/(1 + x^(1/4)),x]
```

```
[Out] x - (4*x^(5/4))/5
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} \, dx &= \int \left(1-\sqrt[4]{x}\right) \, dx \\ &= x-\frac{4x^{5/4}}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x])/(1 + x^(1/4)),x]
```

```
[Out] x - (4*x^(5/4))/5
```

fricas [A] time = 0.76, size = 7, normalized size = 0.64

$$-\frac{4}{5} \, x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="fricas")
```

```
[Out] -4/5*x^(5/4) + x
```

giac [A] time = 0.35, size = 7, normalized size = 0.64

$$-\frac{4}{5}\,x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="giac")
```

```
[Out] -4/5*x^(5/4) + x
```

maple [C] time = 0.01, size = 44, normalized size = 4.00

$$-\frac{4x^{\frac{5}{4}}}{5} + x - \ln(x-1) - \ln\left(\sqrt{x}-1\right) + \ln\left(\sqrt{x}+1\right) + 2\ln\left(x^{\frac{1}{4}}+1\right) + 2\ln\left(x^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^(1/2))/(x^(1/4)+1),x)
```

```
[Out] -4/5*x^(5/4)+x+2*ln(x^(1/4)+1)+2*ln(x^(1/4)-1)-ln(x-1)-ln(x^(1/2)-1)+ln(x^(
1/2)+1)
```

maxima [A] time = 0.61, size = 7, normalized size = 0.64

$$-\frac{4}{5}\,x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="maxima")
```

```
[Out] -4/5*x^(5/4) + x
```

mupad [B] time = 0.03, size = 7, normalized size = 0.64

$$x - \frac{4\,x^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(1/2) - 1)/(x^(1/4) + 1),x)
```

```
[Out] x - (4*x^(5/4))/5
```

sympy [A] time = 4.36, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**(1/2))/(1+x**(1/4)),x)
```

```
[Out] -4*x**(5/4)/5 + x
```

3.955 $\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{\tanh^{-1}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] arctanh(1/2*(2*b*d*x+a*d+b*c)/b^(1/2)/d^(1/2)/(a*c+(a*d+b*c)*x+b*d*x^2)^(1/
2))/b^(1/2)/d^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{1981, 621, 206\}$

$$\frac{\tanh^{-1}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] ArcTanh[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c + a*d)*x +
 b*d*x^2])]/(Sqrt[b]*Sqrt[d])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx &= \int \frac{1}{\sqrt{ac+(bc+ad)x+bdx^2}}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{4bd-x^2}\,dx, x, \frac{bc+ad+2bdx}{\sqrt{ac+(bc+ad)x+bdx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc+ad)x+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 95, normalized size = 1.56

$$\frac{2\sqrt{a+bx}\sqrt{bc-ad}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b\sqrt{d}\sqrt{(a+bx)(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] (2*Sqrt[b*c - a*d]*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(S
qrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)
])
```

fricas [A] time = 0.71, size = 192, normalized size = 3.15

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,\sqrt{bdx^2+ac+(bc+ad)x}\,(2\,bdx+bc+ad)\sqrt{bd}\,+8\left(b^2cd+abd^2\right)\right.}{2\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*sqrt(b
*d*x^2 + a*c + (b*c + a*d)*x)*(2*b*d*x + b*c + a*d)*sqrt(b*d) + 8*(b^2*c*d
+ a*b*d^2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*sqrt(b*d*x^2 + a*c + (b*c + a*d
)*x)*(2*b*d*x + b*c + a*d)*sqrt(-b*d)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a
*b*d^2)*x))/(b*d)]
```

giac [A] time = 0.72, size = 68, normalized size = 1.11

$$-\frac{\sqrt{bd}\log\left(\left|-2\left(\sqrt{bd}\,x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))*b
*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d)
```

maple [A] time = 0.01, size = 49, normalized size = 0.80

$$\frac{\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)*(d*x+c))^(1/2),x)
```

```
[Out] ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(a*c+(a*d+b*c)*x+b*d*x^2)^(1/2))/(b*
d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{(a+b\,x)\,(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x))^(1/2),x)
```

```
[Out] int(1/((a + b*x)*(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(a+bx)\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt((a + b*x)*(c + d*x)), x)
```

3.956 $\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx$

Optimal. Leaf size=65

$$-\frac{\tan^{-1}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] -arctan(1/2*(-2*b*d*x-a*d+b*c)/b^(1/2)/d^(1/2)/(a*c+(-a*d+b*c)*x-b*d*x^2)^(
1/2))/b^(1/2)/d^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1981, 621, 204}

$$-\frac{\tan^{-1}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] -(ArcTan[(b*c - a*d - 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c - a*d)*x
- b*d*x^2])]/(Sqrt[b]*Sqrt[d]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx &= \int \frac{1}{\sqrt{ac+(bc-ad)x-bdx^2}}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{-4bd-x^2}\,dx, x, \frac{bc-ad-2bdx}{\sqrt{ac+(bc-ad)x-bdx^2}}\right)\\ &= -\frac{\tan^{-1}\left(\frac{bc-ad-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc-ad)x-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 94, normalized size = 1.45

$$\frac{2\sqrt{a+bx}\sqrt{ad+bc}\sqrt{\frac{b(c-dx)}{ad+bc}}\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad+bc}}\right)}{b\sqrt{d}\sqrt{(a+bx)(c-dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] (2*Sqrt[b*c + a*d]*Sqrt[a + b*x]*Sqrt[(b*(c - d*x))/(b*c + a*d)]*ArcSin[(Sq
rt[d]*Sqrt[a + b*x])/Sqrt[b*c + a*d]])/(b*Sqrt[d]*Sqrt[(a + b*x)*(c - d*x)]
)
```

fricas [A] time = 0.60, size = 202, normalized size = 3.11

$$\left[-\frac{\sqrt{-bd}\log\left(8\,b^2d^2x^2 + b^2c^2 - 6\,abcd + a^2d^2 - 4\,\sqrt{-bdx^2 + ac + (bc - ad)x}\,(2\,bdx - bc + ad)\sqrt{-bd} - 8\left(b^2cd\right.\right.}{2\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*sqrt
(-b*d*x^2 + a*c + (b*c - a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(-b*d) - 8*(b^2*
c*d - a*b*d^2)*x)/(b*d), -sqrt(b*d)*arctan(1/2*sqrt(-b*d*x^2 + a*c + (b*c -
 a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(b*d)/(b^2*d^2*x^2 - a*b*c*d - (b^2*c*d
- a*b*d^2)*x))/(b*d)]
```

giac [A] time = 0.74, size = 59, normalized size = 0.91

$$-\frac{\log\left(\left|bc - ad + 2\,\sqrt{-bd}\left(\sqrt{-bd}\,x - \sqrt{-bdx^2 + bcx - adx + ac}\right)\right|\right)}{\sqrt{-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(b*c - a*d + 2*sqrt(-b*d)*(sqrt(-b*d)*x - sqrt(-b*d*x^2 + b*c*x - a
*d*x + a*c))))/sqrt(-b*d)
```

maple [A] time = 0.01, size = 55, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{bd}\left(x-\frac{-ad+bc}{2bd}\right)}{\sqrt{-bd\,x^2+ac+(-ad+bc)x}}\right)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)*(-d*x+c))^(1/2),x)
```

```
[Out] 1/(b*d)^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(-a*d+b*c)/b/d)/(a*c+(-a*d+b*c)*x-b
*d*x^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{(a+b\,x)\,(c-d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c - d*x))^(1/2),x)
[Out] int(1/((a + b*x)*(c - d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(a+bx)\,(c-dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))**(1/2),x)
[Out] Integral(1/sqrt((a + b*x)*(c - d*x)), x)
```

3.957 $\int \frac{1}{\sqrt{x}\left(1-x^2\right)}\,dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {329, 212, 206, 203}

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 - x^2)),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\left(1-x^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\sqrt{x}\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 - x^2)),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

fricas [B] time = 0.57, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="fricas")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

giac [B] time = 0.39, size = 22, normalized size = 1.69

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

maple [A] time = 0.01, size = 10, normalized size = 0.77

$$\operatorname{arctanh}\left(\sqrt{x}\right) + \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)/x^(1/2),x)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

maxima [B] time = 1.01, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="maxima")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

mupad [B] time = 3.34, size = 9, normalized size = 0.69

$$\operatorname{atan}\left(\sqrt{x}\right) + \operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/2)*(x^2 - 1)),x)
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

sympy [B] time = 0.39, size = 26, normalized size = 2.00

$$-\frac{\log\left(\sqrt{x}-1\right)}{2} + \frac{\log\left(\sqrt{x}+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)/x**(1/2),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

3.958 $\int \frac{\sqrt{x}}{x-x^3}\,dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1584, 329, 212, 206, 203}

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{x-x^3}\,dx = \int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$
$$= \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x - x^3),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

fricas [B] time = 0.58, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(\sqrt{x}+1\right) - \frac{1}{2}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="fricas")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

giac [B] time = 0.30, size = 22, normalized size = 1.69

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(\sqrt{x}+1\right) - \frac{1}{2}\,\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="giac")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

maple [A] time = 0.01, size = 10, normalized size = 0.77

$$\operatorname{arctanh}\left(\sqrt{x}\right) + \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-x^3+x),x)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

maxima [B] time = 1.01, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(\sqrt{x}+1\right) - \frac{1}{2}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.69

$$\operatorname{atan}\left(\sqrt{x}\right)+\operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x - x^3),x)
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

sympy [B] time = 0.54, size = 26, normalized size = 2.00

$$-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-x**3+x),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

3.959 $\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx$

Optimal. Leaf size=72

$$\frac{1}{2}\log\left(x^2+\left(1+\sqrt{3}\right)x-\sqrt{3}+2\right)+\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)$$

```
[Out] 1/2*ln(2+x^2-3^(1/2)+x*(1+3^(1/2)))+1/23*arctanh((1+2*x+3^(1/2))/(-4+6*3^(1
/2))^(1/2))*(299+184*3^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {634, 618, 206, 628}

$$\frac{1}{2}\log\left(x^2+\left(1+\sqrt{3}\right)x-\sqrt{3}+2\right)+\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] Sqrt[(13 + 8*Sqrt[3])/23]*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[2*(-2 + 3*Sqrt[3
])]] + Log[2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = \frac{1}{2}\int \frac{1+\sqrt{3}+2x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx + \frac{1}{2}\left(-1-\sqrt{3}\right)\int \frac{1}{2-\sqrt{3}+\left(1+\sqrt{3}\right)$$
$$= \frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2\right)+\left(1+\sqrt{3}\right)\mathrm{Subst}\left(\int \frac{1}{-2\left(2-3\sqrt{3}\right)-}\right.$$
$$= \sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{1+\sqrt{3}+2x}{\sqrt{2\left(-2+3\sqrt{3}\right)}}\right)+\frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right.\right.$$

Mathematica [A] time = 0.10, size = 72, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+\sqrt{3}x+x-\sqrt{3}+2\right)+\frac{\left(1+\sqrt{3}\right)\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{6\sqrt{3}-4}}\right)}{\sqrt{6\sqrt{3}-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] ((1 + Sqrt[3])*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[-4 + 6*Sqrt[3]]])/Sqrt[-4 +
 6*Sqrt[3]] + Log[2 - Sqrt[3] + x + Sqrt[3]*x + x^2]/2
```

fricas [A] time = 0.62, size = 100, normalized size = 1.39

$$\frac{1}{46}\sqrt{23}\sqrt{8\sqrt{3}+13}\log\left(-\frac{\sqrt{23}\sqrt{8\sqrt{3}+13}\left(5\sqrt{3}-11\right)-46x-23\sqrt{3}-23}{\sqrt{23}\sqrt{8\sqrt{3}+13}\left(5\sqrt{3}-11\right)+46x+23\sqrt{3}+23}\right)+\frac{1}{2}\log\left(x^2+x\left(\sqrt{3}+1\right)-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/46*sqrt(23)*sqrt(8*sqrt(3) + 13)*log(-(sqrt(23)*sqrt(8*sqrt(3) + 13)*(5*s
qrt(3) - 11) - 46*x - 23*sqrt(3) - 23)/(sqrt(23)*sqrt(8*sqrt(3) + 13)*(5*sq
rt(3) - 11) + 46*x + 23*sqrt(3) + 23)) + 1/2*log(x^2 + x*(sqrt(3) + 1) - sq
rt(3) + 2)
```

giac [A] time = 0.43, size = 80, normalized size = 1.11

$$-\frac{\left(\sqrt{3}+1\right)\log\left(\frac{\left|2x+\sqrt{3}-\sqrt{6\sqrt{3}-4}+1\right|}{\left|2x+\sqrt{3}+\sqrt{6\sqrt{3}-4}+1\right|}\right)}{2\sqrt{6\sqrt{3}-4}}+\frac{1}{2}\log\left(\left|x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(3) + 1)*log(abs(2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/abs(2*x
 + sqrt(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(abs(x^
2 + x*(sqrt(3) + 1) - sqrt(3) + 2))
```

maple [A] time = 0.02, size = 82, normalized size = 1.14

$$\frac{\operatorname{arctanh}\left(\frac{2x+1+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x+1+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}}+\frac{\ln\left(x^2+\sqrt{3}\,x+x-\sqrt{3}+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x)
```

```
[Out] 1/2*ln(3^(1/2)*x+x^2-3^(1/2)+x+2)+1/(-4+6*3^(1/2))^(1/2)*arctanh((1+2*x+3^(
1/2))/(-4+6*3^(1/2))^(1/2))+1/(-4+6*3^(1/2))^(1/2)*arctanh((1+2*x+3^(1/2))/
(-4+6*3^(1/2))^(1/2))*3^(1/2)
```

maxima [A] time = 1.00, size = 77, normalized size = 1.07

$$-\frac{\left(\sqrt{3}+1\right)\log\left(\frac{2x+\sqrt{3}-\sqrt{6\sqrt{3}-4}+1}{2x+\sqrt{3}+\sqrt{6\sqrt{3}-4}+1}\right)}{2\sqrt{6\sqrt{3}-4}}+\frac{1}{2}\log\left(x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(3) + 1)*log((2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/(2*x + sqr
t(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(x^2 + x*(sqr
t(3) + 1) - sqrt(3) + 2)
```

mupad [B] time = 4.23, size = 233, normalized size = 3.24

$$\ln\left(x-\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}+\frac{1}{2}\right)\left(2x+\sqrt{3}+1\right)\right)\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x*(3^(1/2) + 1) - 3^(1/2) + x^2 + 2),x)
```

```
[Out] log(x - ((((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(
3^(1/2) + 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) + 1/2)*(2*x + 3^(1/2)
 + 1))*((((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3
^(1/2) + 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) + 1/2) - log(x + ((((3
^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3^(1/2) + 7))
^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) - 1/2)*(2*x + 3^(1/2) + 1))*((((3^
(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3^(1/2) + 7))^
(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) - 1/2)
```

sympy [B] time = 1.55, size = 202, normalized size = 2.81

$$\left(\frac{1}{2}-\frac{\sqrt{11+64\sqrt{3}}}{2\left(-31+12\sqrt{3}\right)}\right)\log\left(x-\frac{287\sqrt{3}}{11+64\sqrt{3}}+\left(\frac{1}{2}-\frac{\sqrt{11+64\sqrt{3}}}{2\left(-31+12\sqrt{3}\right)}\right)\left(\frac{269}{214+139\sqrt{3}}+\frac{459\sqrt{3}}{214+139\sqrt{3}}\right)+\frac{521}{11+64\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x**2-3**(1/2)+x*(1+3**(1/2))),x)
```

```
[Out] (1/2 - sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))))*log(x - 287*sqrt(3)/(1
1 + 64*sqrt(3)) + (1/2 - sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))))*(269
/(214 + 139*sqrt(3)) + 459*sqrt(3)/(214 + 139*sqrt(3))) + 521/(11 + 64*sqrt
(3))) + (sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))) + 1/2)*log(x - 287*sq
rt(3)/(11 + 64*sqrt(3)) + (sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))) + 1
/2)*(269/(214 + 139*sqrt(3)) + 459*sqrt(3)/(214 + 139*sqrt(3))) + 521/(11 +
 64*sqrt(3)))
```

3.960 $\int \sqrt{x^2 + x^3}\, dx$

Optimal. Leaf size=37

$$\frac{2\left(x^3+x^2\right)^{3/2}}{5x^2}-\frac{4\left(x^3+x^2\right)^{3/2}}{15x^3}$$

```
[Out] -4/15*(x^3+x^2)^(3/2)/x^3+2/5*(x^3+x^2)^(3/2)/x^2
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2002, 2014}

$$\frac{2\left(x^3+x^2\right)^{3/2}}{5x^2}-\frac{4\left(x^3+x^2\right)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + x^3],x]
```

```
[Out] (-4*(x^2 + x^3)^(3/2))/(15*x^3) + (2*(x^2 + x^3)^(3/2))/(5*x^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x^2+x^3}\, dx &= \frac{2\left(x^2+x^3\right)^{3/2}}{5x^2}-\frac{2}{5}\int \frac{\sqrt{x^2+x^3}}{x}\, dx\\ &= -\frac{4\left(x^2+x^3\right)^{3/2}}{15x^3}+\frac{2\left(x^2+x^3\right)^{3/2}}{5x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.62

$$\frac{2\left(x^2(x+1)\right)^{3/2}(3x-2)}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + x^3],x]
```

```
[Out] (2*(x^2*(1 + x))^(3/2)*(-2 + 3*x))/(15*x^3)
```

fricas [A] time = 0.58, size = 22, normalized size = 0.59

$$\frac{2\sqrt{x^3+x^2}\left(3\,x^2+x-2\right)}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(x^3 + x^2)*(3*x^2 + x - 2)/x
```

giac [A] time = 0.38, size = 48, normalized size = 1.30

$$\frac{2}{15}\left(3(x+1)^{\frac{5}{2}}-10(x+1)^{\frac{3}{2}}+15\sqrt{x+1}\right)\operatorname{sgn}(x)+\frac{2}{3}\left((x+1)^{\frac{3}{2}}-3\sqrt{x+1}\right)\operatorname{sgn}(x)+\frac{4}{15}\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(x + 1)^(5/2) - 10*(x + 1)^(3/2) + 15*sqrt(x + 1))*sgn(x) + 2/3*((x
 + 1)^(3/2) - 3*sqrt(x + 1))*sgn(x) + 4/15*sgn(x)
```

maple [A] time = 0.00, size = 23, normalized size = 0.62

$$\frac{2(x+1)(3x-2)\sqrt{x^3+x^2}}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2)^(1/2),x)
```

```
[Out] 2/15*(x+1)*(3*x-2)*(x^3+x^2)^(1/2)/x
```

maxima [A] time = 0.45, size = 15, normalized size = 0.41

$$\frac{2}{15}\left(3x^2+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

mupad [B] time = 3.51, size = 22, normalized size = 0.59

$$\frac{2\sqrt{x^3+x^2}\left(3x^2+x-2\right)}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3)^(1/2),x)
```

```
[Out] (2*(x^2 + x^3)^(1/2)*(x + 3*x^2 - 2))/(15*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**3 + x**2), x)
```

3.961 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

[Out] arctan((x^2+2*x)^(1/2))

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {688, 203}

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x)*Sqrt[2*x + x^2]),x]

[Out] ArcTan[Sqrt[2*x + x^2]]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sqrt{2x+x^2}\right)\\ &= \tan^{-1}\left(\sqrt{2x+x^2}\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+2}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+2}}\right)}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]

[Out] (2*Sqrt[x]*Sqrt[2 + x]*ArcTan[Sqrt[x]/Sqrt[2 + x]])/Sqrt[x*(2 + x)]

fricas [A] time = 0.60, size = 17, normalized size = 1.42

$$2\,\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

giac [A] time = 0.46, size = 17, normalized size = 1.42

$$2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$-\arctan\left(\frac{1}{\sqrt{(x+1)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x^2+2*x)^(1/2),x)
[Out] -arctan(1/((x+1)^2-1)^(1/2))
```

maxima [A] time = 0.97, size = 9, normalized size = 0.75

$$-\arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(1/abs(x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{x^2+2x}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + x^2)^(1/2)*(x + 1)),x)
[Out] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(x+2)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

3.962 $\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx$

Optimal. Leaf size=95

$$-\frac{1}{2}\sqrt{x}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{5}{12}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{9}{32}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

```
[Out] 45/64*arcsin(1/5*(1+2*x^(1/2))*5^(1/2))+5/12*(1-x-x^(1/2))^(3/2)-1/2*(1-x-x
^(1/2))^(3/2)*x^(1/2)+9/32*(1+2*x^(1/2))*(1-x-x^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1357, 742, 640, 612, 619, 216}

$$-\frac{1}{2}\sqrt{x}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{5}{12}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{9}{32}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

```
[Out] (9*(1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/32 + (5*(1 - Sqrt[x] - x)^(3/2))/
12 - ((1 - Sqrt[x] - x)^(3/2)*Sqrt[x])/2 + (45*ArcSin[(1 + 2*Sqrt[x])/Sqrt[
5]])/64
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
```

```
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx = 2\,\text{Subst}\left(\int x^2\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x} - \frac{1}{2}\text{Subst}\left(\int\left(-1+\frac{5x}{2}\right)\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)$$
$$= \frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x} + \frac{9}{8}\text{Subst}\left(\int\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)$$
$$= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} + \frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x} + \frac{45}{64}\text{Subst}$$
$$= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} + \frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x} - \frac{1}{64}\left(9\sqrt{5}\right.$$
$$= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} + \frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x} + \frac{45}{64}\sin^{-1}\left(\right.$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.63

$$\frac{1}{96}\sqrt{-x-\sqrt{x}+1}\left(48x^{3/2}+8x-34\sqrt{x}+67\right)-\frac{45}{64}\sin^{-1}\left(\frac{-2\sqrt{x}-1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(67 - 34*Sqrt[x] + 8*x + 48*x^(3/2)))/96 - (45*ArcSi
n[(-1 - 2*Sqrt[x])/Sqrt[5]])/64
```

fricas [A] time = 2.32, size = 89, normalized size = 0.94

$$\frac{1}{96}\left(2\left(24x-17\right)\sqrt{x}+8x+67\right)\sqrt{-x-\sqrt{x}+1}-\frac{45}{128}\arctan\left(-\frac{\left(8x^2-\left(16x^2-38x+11\right)\sqrt{x}-9x+3\right)\sqrt{-x-}}{4\left(4x^3-13x^2+7x-1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 17)*sqrt(x) + 8*x + 67)*sqrt(-x - sqrt(x) + 1) - 45/128*arc
tan(-1/4*(8*x^2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x)
 + 1)/(4*x^3 - 13*x^2 + 7*x - 1))
```

giac [A] time = 0.39, size = 51, normalized size = 0.54

$$\frac{1}{96}\left(2\left(4\sqrt{x}\left(6\sqrt{x}+1\right)-17\right)\sqrt{x}+67\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\arcsin\left(\frac{1}{5}\sqrt{5}\left(2\sqrt{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 17)*sqrt(x) + 67)*sqrt(-x - sqrt(x) +
1) + 45/64*arcsin(1/5*sqrt(5)*(2*sqrt(x) + 1))
```

maple [A] time = 0.01, size = 67, normalized size = 0.71

$$\frac{45 \arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{64}-\frac{\left(-x-\sqrt{x}+1\right)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{5\left(-x-\sqrt{x}+1\right)^{\frac{3}{2}}}{12}-\frac{9\left(-2\sqrt{x}-1\right)\sqrt{-x-\sqrt{x}+1}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-x-x^(1/2)+1)^(1/2),x)
```

```
[Out] -1/2*(-x-x^(1/2)+1)^(3/2)*x^(1/2)+5/12*(-x-x^(1/2)+1)^(3/2)-9/32*(-2*x^(1/2
)-1)*(-x-x^(1/2)+1)^(1/2)+45/64*arcsin(2/5*5^(1/2)*(x^(1/2)+1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\sqrt{-x-\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*sqrt(-x - sqrt(x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\sqrt{1-\sqrt{x}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1 - x^(1/2) - x)^(1/2),x)
```

```
[Out] int(x^(1/2)*(1 - x^(1/2) - x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\sqrt{-\sqrt{x}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1-x-x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(-sqrt(x) - x + 1), x)
```

3.963 $\int \sqrt[3]{1+\sqrt{-3+x}}\,dx$

Optimal. Leaf size=35

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{7/3}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{4/3}$$

```
[Out] -3/2*(1+(-3+x)^(1/2))^(4/3)+6/7*(1+(-3+x)^(1/2))^(7/3)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 190, 43}

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{7/3}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[-3 + x])^(1/3),x]
```

```
[Out] (-3*(1 + Sqrt[-3 + x])^(4/3))/2 + (6*(1 + Sqrt[-3 + x])^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{1+\sqrt{-3+x}}\,dx &= \text{Subst}\left(\int \sqrt[3]{1+\sqrt{x}}\,dx, x, -3+x\right)\\
&= 2\,\text{Subst}\left(\int x\sqrt[3]{1+x}\,dx, x, \sqrt{-3+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-\sqrt[3]{1+x}+(1+x)^{4/3}\right)dx, x, \sqrt{-3+x}\right)\\
&= -\frac{3}{2}\left(1+\sqrt{-3+x}\right)^{4/3}+\frac{6}{7}\left(1+\sqrt{-3+x}\right)^{7/3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{3}{14}\left(\sqrt{x-3}+1\right)^{4/3}\left(4\sqrt{x-3}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[-3 + x])^(1/3),x]
[Out] (3*(1 + Sqrt[-3 + x])^(4/3)*(-3 + 4*Sqrt[-3 + x]))/14
```

fricas [A] time = 0.59, size = 21, normalized size = 0.60

$$\frac{3}{14}\left(4x+\sqrt{x-3}-15\right)\left(\sqrt{x-3}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="fricas")
[Out] 3/14*(4*x + sqrt(x - 3) - 15)*(sqrt(x - 3) + 1)^(1/3)
```

giac [A] time = 0.32, size = 23, normalized size = 0.66

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="giac")
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

maple [A] time = 0.00, size = 24, normalized size = 0.69

$$-\frac{3\left(1+\sqrt{x-3}\right)^{\frac{4}{3}}}{2}+\frac{6\left(1+\sqrt{x-3}\right)^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x-3)^(1/2))^(1/3),x)
[Out] -3/2*(1+(x-3)^(1/2))^(4/3)+6/7*(1+(x-3)^(1/2))^(7/3)
```

maxima [A] time = 0.43, size = 23, normalized size = 0.66

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="maxima")
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

mupad [B] time = 3.51, size = 16, normalized size = 0.46

$$(x-3)\ {}_2F_1\left(-\frac{1}{3},2;\ 3;\ -\sqrt{x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 3)^(1/2) + 1)^(1/3),x)
[Out] (x - 3)*hypergeom([-1/3, 2], 3, -(x - 3)^(1/2))
```

sympy [B] time = 1.17, size = 184, normalized size = 5.26

$$\frac{12(x-3)^{\frac{7}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}-\frac{6(x-3)^{\frac{5}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}+\frac{9(x-3)^{\frac{5}{2}}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}+\frac{15(x-3)^3\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}-\frac{9(}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)**(1/2))**(1/3),x)
```

```
[Out] 12*(x - 3)**(7/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)*
*2) - 6*(x - 3)**(5/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x
- 3)**2) + 9*(x - 3)**(5/2)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 15*(x - 3
)**3*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) - 9*(x -
3)**2*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 9*(x -
 3)**2/(14*(x - 3)**(5/2) + 14*(x - 3)**2)
```

3.964 $\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{3/2}-6\sqrt{\sqrt{2x-1}+3}$$

```
[Out] 2/3*(3+(-1+2*x)^(1/2))^(3/2)-6*(3+(-1+2*x)^(1/2))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 190, 43}

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{3/2}-6\sqrt{\sqrt{2x-1}+3}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
```

```
[Out] -6*Sqrt[3 + Sqrt[-1 + 2*x]] + (2*(3 + Sqrt[-1 + 2*x])^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+\sqrt{x}}}\,dx,x,-1+2x\right)\\
&= \text{Subst}\left(\int \frac{x}{\sqrt{3+x}}\,dx,x,\sqrt{-1+2x}\right)\\
&= \text{Subst}\left(\int \left(-\frac{3}{\sqrt{3+x}}+\sqrt{3+x}\right)dx,x,\sqrt{-1+2x}\right)\\
&= -6\sqrt{3+\sqrt{-1+2x}}+\frac{2}{3}\left(3+\sqrt{-1+2x}\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.81

$$\frac{2}{3}\left(\sqrt{2x-1}-6\right)\sqrt{\sqrt{2x-1}+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
[Out] (2*(-6 + Sqrt[-1 + 2*x])*Sqrt[3 + Sqrt[-1 + 2*x]])/3
```

fricas [A] time = 0.78, size = 22, normalized size = 0.59

$$\frac{2}{3}\sqrt{\sqrt{2x-1}+3}\left(\sqrt{2x-1}-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(sqrt(2*x - 1) + 3)*(sqrt(2*x - 1) - 6)
```

giac [A] time = 0.34, size = 27, normalized size = 0.73

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{\frac{3}{2}}-6\sqrt{\sqrt{2x-1}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

maple [A] time = 0.00, size = 28, normalized size = 0.76

$$\frac{2\left(3+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-6\sqrt{3+\sqrt{2x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+(2*x-1)^(1/2))^(1/2),x)
[Out] 2/3*(3+(2*x-1)^(1/2))^(3/2)-6*(3+(2*x-1)^(1/2))^(1/2)
```

maxima [A] time = 0.43, size = 27, normalized size = 0.73

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{\frac{3}{2}}-6\sqrt{\sqrt{2x-1}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

mupad [B] time = 3.58, size = 24, normalized size = 0.65

$$\frac{\sqrt{3}\,\left(2x-1\right)\,{}_2F_1\left(\frac{1}{2},2;\,3;\,-\frac{\sqrt{2x-1}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x - 1)^(1/2) + 3)^(1/2),x)
[Out] (3^(1/2)*(2*x - 1)*hypergeom([1/2, 2], 3, -(2*x - 1)^(1/2)/3))/6
```

sympy [B] time = 1.16, size = 265, normalized size = 7.16

$$-\frac{6\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{36\sqrt{2}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{4\sqrt{3}\left(x-\frac{1}{2}\right)^{3}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}-\frac{36\sqrt{3}}{3\sqrt{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] -6*sqrt(6)*(x - 1/2)**(5/2)*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x -
 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 36*sqrt(2)*(x - 1/2)**(5/2)/(3*sqr
t(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 4*sqrt(3)*(x - 1/2)**3*sq
rt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x -
1/2)**2) - 36*sqrt(3)*(x - 1/2)**2*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(
6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 108*(x - 1/2)**2/(3*sqrt(6)
*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2)
```

3.965 $\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx$

Optimal. Leaf size=29

$$-\sqrt{1-x}\left(2-\sqrt{x}\right)-\sin^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arcsin(x^(1/2))-(1-x)^(1/2)*(2-x^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1398, 785, 780, 216}

$$-\sqrt{1-x}\left(2-\sqrt{x}\right)-\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + Sqrt[x]),x]
```

```
[Out] -((2 - Sqrt[x])*Sqrt[1 - x]) - ArcSin[Sqrt[x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\sqrt{1-x^2}}{1+x}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\left(2-\sqrt{x}\right)\sqrt{1-x}-\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\left(2-\sqrt{x}\right)\sqrt{1-x}-\sin^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.90

$$\left(\sqrt{x}-2\right)\sqrt{1-x}-\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + Sqrt[x]),x]
[Out] (-2 + Sqrt[x])*Sqrt[1 - x] - ArcSin[Sqrt[x]]
```

fricas [A] time = 0.64, size = 33, normalized size = 1.14

$$\sqrt{x}\sqrt{-x+1}-2\sqrt{-x+1}+\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="fricas")
[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arctan(sqrt(-x + 1)/sqrt(x))
```

giac [A] time = 0.41, size = 29, normalized size = 1.00

$$\sqrt{x}\sqrt{-x+1}-2\sqrt{-x+1}+\arcsin\left(\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="giac")
[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arcsin(sqrt(-x + 1))
```

maple [B] time = 0.01, size = 48, normalized size = 1.66

$$-\frac{\sqrt{-x+1}\,\left(\arcsin\left(2x-1\right)-2\sqrt{-\left(x-1\right)x}\right)\sqrt{x}}{2\sqrt{-\left(x-1\right)x}}-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(x^(1/2)+1),x)
[Out] -1/2*(-x+1)^(1/2)*x^(1/2)*(-2*(-x*(x-1))^(1/2)+arcsin(2*x-1))/(-x*(x-1))^(1
/2)-2*(-x+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-x+1}}{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="maxima")
[Out] integrate(sqrt(-x + 1)/(sqrt(x) + 1), x)
```

mupad [B] time = 3.90, size = 39, normalized size = 1.34

$$\sqrt{x}\,\sqrt{1-x}-2\sqrt{1-x}-2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x^(1/2) + 1),x)
```

```
[Out] x^(1/2)*(1 - x)^(1/2) - 2*(1 - x)^(1/2) - 2*atan(x^(1/2)/((1 - x)^(1/2) - 1
))
```

sympy [C] time = 1.79, size = 32, normalized size = 1.10

$$i\sqrt{x}\sqrt{x-1} - 2i\sqrt{x-1} + i\operatorname{asinh}\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x**(1/2)),x)
```

```
[Out] I*sqrt(x)*sqrt(x - 1) - 2*I*sqrt(x - 1) + I*asinh(sqrt(x - 1))
```

3.966 $\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx$

Optimal. Leaf size=25

$$\sin^{-1}\left(\sqrt{x}\right)-\left(\sqrt{x}+2\right)\sqrt{1-x}$$

```
[Out] arcsin(x^(1/2))-(1-x)^(1/2)*(2+x^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1398, 785, 780, 216}

$$\sin^{-1}\left(\sqrt{x}\right)-\left(\sqrt{x}+2\right)\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 - Sqrt[x]),x]
```

```
[Out] -((2 + Sqrt[x])*Sqrt[1 - x]) + ArcSin[Sqrt[x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 785

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\sqrt{1-x^2}}{1-x}\,dx,x,\sqrt{x}\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{(-1-x)x}{\sqrt{1-x^2}}\,dx,x,\sqrt{x}\right)\right)\\
&= -\left(2+\sqrt{x}\right)\sqrt{1-x}+\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,\sqrt{x}\right)\\
&= -\left(2+\sqrt{x}\right)\sqrt{1-x}+\sin^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]   time = 0.03, size = 26, normalized size = 1.04

$$\sqrt{1-x}\left(-\sqrt{x}-2\right)+\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 - Sqrt[x]),x]
[Out] (-2 - Sqrt[x])*Sqrt[1 - x] + ArcSin[Sqrt[x]]
```

fricas [A]   time = 0.67, size = 36, normalized size = 1.44

$$-\sqrt{x}\sqrt{-x+1}-2\sqrt{-x+1}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="fricas")
[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))
```

giac [A]   time = 0.40, size = 32, normalized size = 1.28

$$-\sqrt{x}\sqrt{-x+1}-2\sqrt{-x+1}-\arcsin\left(\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="giac")
[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arcsin(sqrt(-x + 1))
```

maple [B]   time = 0.01, size = 48, normalized size = 1.92

$$\frac{\sqrt{-x+1}\left(\arcsin\left(2x-1\right)-2\sqrt{-\left(x-1\right)x}\right)\sqrt{x}}{2\sqrt{-\left(x-1\right)x}}-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(1-x^(1/2)),x)
[Out] -2*(-x+1)^(1/2)+1/2*(-x+1)^(1/2)*(arcsin(2*x-1)-2*(-(x-1)*x)^(1/2))/(-(x-1)
*x)^(1/2)*x^(1/2)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{-x+1}}{\sqrt{x}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="maxima")
[Out] -integrate(sqrt(-x + 1)/(sqrt(x) - 1), x)
```

mupad [B]   time = 3.65, size = 40, normalized size = 1.60

$$2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)-2\sqrt{1-x}-\sqrt{x}\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1 - x)^(1/2)/(x^(1/2) - 1),x)
```

```
[Out] 2*atan(x^(1/2)/((1 - x)^(1/2) - 1)) - 2*(1 - x)^(1/2) - x^(1/2)*(1 - x)^(1/
2)
```

sympy [A] time = 3.77, size = 87, normalized size = 3.48

$$2\left(\begin{cases} -\sqrt{1-x} + \frac{i\operatorname{acosh}\left(\sqrt{1-x}\right)}{2} - \frac{i(1-x)^{\frac{3}{2}}}{2\sqrt{-x}} + \frac{i\sqrt{1-x}}{2\sqrt{-x}} & \text{for } |x-1| > 1 \\ \frac{\sqrt{x}\sqrt{1-x}}{2} - \sqrt{1-x} + \frac{\operatorname{asin}\left(\sqrt{1-x}\right)}{2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1-x**(1/2)),x)
```

```
[Out] 2*Piecewise((-sqrt(1 - x) + I*acosh(sqrt(1 - x))/2 - I*(1 - x)**(3/2)/(2*sq
rt(-x)) + I*sqrt(1 - x)/(2*sqrt(-x)), Abs(x - 1) > 1), (sqrt(x)*sqrt(1 - x)
/2 - sqrt(1 - x) + asin(sqrt(1 - x))/2, True))
```

3.967 $\int \frac{x}{x-\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=21

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] -1/3*x^3-1/3*(x^2+1)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2106, 30, 261}

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 + x^2]),x]
```

```
[Out] -x^3/3 - (1 + x^2)^(3/2)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{x-\sqrt{1+x^2}}\,dx &= -\int x^2\,dx-\int x\sqrt{1+x^2}\,dx\\ &= -\frac{x^3}{3}-\frac{1}{3}\left(1+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 21, normalized size = 1.00

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 + x^2]),x]
```

```
[Out] -1/3*x^3 - (1 + x^2)^(3/2)/3
```

fricas [**A**] time = 0.62, size = 15, normalized size = 0.71

$$-\frac{1}{3}x^3 - \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

giac [**A**] time = 0.34, size = 15, normalized size = 0.71

$$-\frac{1}{3}x^3 - \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="giac")
[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

maple [**A**] time = 0.00, size = 16, normalized size = 0.76

$$-\frac{x^3}{3} - \frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(x^2+1)^(1/2)),x)
[Out] -1/3*x^3-1/3*(x^2+1)^(3/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x-\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(x/(x - sqrt(x^2 + 1)), x)
```

mupad [**B**] time = 0.04, size = 22, normalized size = 1.05

$$-\sqrt{x^2+1}\,\left(\frac{x^2}{3}+\frac{1}{3}\right)-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - (x^2 + 1)^(1/2)),x)
[Out] - (x^2 + 1)^(1/2)*(x^2/3 + 1/3) - x^3/3
```

sympy [**B**] time = 0.37, size = 56, normalized size = 2.67

$$\frac{2x^2}{3x-3\sqrt{x^2+1}} - \frac{x\sqrt{x^2+1}}{3x-3\sqrt{x^2+1}} + \frac{1}{3x-3\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x**2+1)**(1/2)),x)
[Out] 2*x**2/(3*x - 3*sqrt(x**2 + 1)) - x*sqrt(x**2 + 1)/(3*x - 3*sqrt(x**2 + 1))
 + 1/(3*x - 3*sqrt(x**2 + 1))
```

3.968 $\int \frac{x}{x-\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}+\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{2\sqrt{2}}$$

```
[Out] 1/2*x-1/4*arctanh(x*2^(1/2))*2^(1/2)-1/4*arctanh(2^(1/2)*(-x^2+1)^(1/2))*2^
(1/2)+1/2*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2107, 321, 206, 444, 50, 63, 207}

$$\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}+\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] x/2 + Sqrt[1 - x^2]/2 - ArcTanh[Sqrt[2]*x]/(2*Sqrt[2]) - ArcTanh[Sqrt[2]*Sq
rt[1 - x^2]]/(2*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2107

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> -Dist[d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{x-\sqrt{1-x^2}}\,dx &= -\int \frac{x^2}{1-2x^2}\,dx - \int \frac{x\sqrt{1-x^2}}{1-2x^2}\,dx \\
&= \frac{x}{2} - \frac{1}{2}\int \frac{1}{1-2x^2}\,dx - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{1-2x}\,dx, x, x^2\right) \\
&= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{(1-2x)\sqrt{1-x}}\,dx, x, x^2\right) \\
&= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.83

$$\frac{1}{4}\left(2\left(\sqrt{1-x^2}+x\right)-\sqrt{2}\tanh^{-1}\left(\sqrt{2-2x^2}\right)-\sqrt{2}\tanh^{-1}\left(\sqrt{2}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] (2*(x + Sqrt[1 - x^2]) - Sqrt[2]*ArcTanh[Sqrt[2]*x] - Sqrt[2]*ArcTanh[Sqrt[
2 - 2*x^2]])/4
```

fricas [B] time = 0.67, size = 97, normalized size = 1.49

$$\frac{1}{8}\sqrt{2}\log\left(\frac{6x^2-2\sqrt{2}\left(2x^2-3\right)+2\sqrt{-x^2+1}\left(3\sqrt{2}-4\right)-9}{2x^2-1}\right)+\frac{1}{8}\sqrt{2}\log\left(\frac{2x^2-2\sqrt{2}x+1}{2x^2-1}\right)+\frac{1}{2}x+\frac{1}{2}\sqrt{-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((6*x^2 - 2*sqrt(2)*(2*x^2 - 3) + 2*sqrt(-x^2 + 1)*(3*sqrt(2
) - 4) - 9)/(2*x^2 - 1)) + 1/8*sqrt(2)*log((2*x^2 - 2*sqrt(2)*x + 1)/(2*x^2
 - 1)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

giac [B] time = 0.57, size = 105, normalized size = 1.62

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\left|4x-2\sqrt{2}\right|}{\left|4x+2\sqrt{2}\right|}\right)-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}{\left|4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}\right)+\frac{1}{2}x+\frac{1}{2}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(4*x - 2*sqrt(2))/abs(4*x + 2*sqrt(2))) - 1/8*sqrt(2)*lo
g(abs(-4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)/abs(4*sqrt(2) + 2*(sqr
t(-x^2 + 1) - 1)^2/x^2 - 6)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

maple [B] time = 0.02, size = 175, normalized size = 2.69

$$\frac{x}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{2}\,x\right)}{4}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(1-\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}\right)\sqrt{2}}{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{8}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\sqrt{2}}{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(-x^2+1)^(1/2)),x)
```

```
[Out] 1/2*x-1/4*arctanh(2^(1/2)*x)*2^(1/2)+1/8*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(
1/2))*2^(1/2)+2)^(1/2)-1/8*2^(1/2)*arctanh(((x+1/2*2^(1/2))*2^(1/2)+1)*2^(1
/2)/(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2))+1/8*(-4*(x-1/
2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)-1/8*2^(1/2)*arctanh((1-(x-1
/2*2^(1/2))*2^(1/2))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2
)+2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x-\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(-x^2 + 1)), x)
```

mupad [B] time = 3.50, size = 127, normalized size = 1.95

$$\frac{x}{2}-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}-1\right)1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{8}-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}+1\right)1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{8}+\frac{\sqrt{2}\,\ln\left(x-\frac{\sqrt{2}}{2}\right)}{8}-\frac{\sqrt{2}\,\ln\left(x+\frac{\sqrt{2}}{2}\right)}{8}+\frac{\sqrt{1-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - (1 - x^2)^(1/2)),x)
```

```
[Out] x/2 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 - 1)*1i - (1 - x^2)^(1/2)*1i)/(x
 - 2^(1/2)/2)))/8 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 + 1)*1i + (1 - x^2
)^(1/2)*1i)/(x + 2^(1/2)/2)))/8 + (2^(1/2)*log(x - 2^(1/2)/2))/8 - (2^(1/2)
*log(x + 2^(1/2)/2))/8 + (1 - x^2)^(1/2)/2
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x - \sqrt{1 - x^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x - sqrt(1 - x**2)), x)
```

3.969 $\int \frac{x}{x-\sqrt{1+2x^2}}\,dx$

Optimal. Leaf size=31

$$-\sqrt{2x^2+1}+\tan^{-1}\left(\sqrt{2x^2+1}\right)-x+\tan^{-1}(x)$$

```
[Out] -x+arctan(x)+arctan((2*x^2+1)^(1/2))-(2*x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2107, 321, 203, 444, 50, 63}

$$-\sqrt{2x^2+1}+\tan^{-1}\left(\sqrt{2x^2+1}\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 + 2*x^2]),x]
```

```
[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2107

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> -Dist[d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{x-\sqrt{1+2x^2}}\,dx &= -\int \frac{x^2}{1+x^2}\,dx - \int \frac{x\sqrt{1+2x^2}}{1+x^2}\,dx\\
&= -x - \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{1+2x}}{1+x}\,dx, x, x^2\right) + \int \frac{1}{1+x^2}\,dx\\
&= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+2x}}\,dx, x, x^2\right)\\
&= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^2}{2}}\,dx, x, \sqrt{1+2x^2}\right)\\
&= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \tan^{-1}\left(\sqrt{1+2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.00

$$-\sqrt{2x^2+1} + \tan^{-1}\left(\sqrt{2x^2+1}\right) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 + 2*x^2]),x]
[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]
```

fricas [A] time = 0.76, size = 41, normalized size = 1.32

$$-x - \sqrt{2\,x^2+1} + \arctan(x) - \arctan\left(-\frac{x^2 - \sqrt{2\,x^2+1} + 1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -x - sqrt(2*x^2 + 1) + arctan(x) - arctan(-(x^2 - sqrt(2*x^2 + 1) + 1)/x^2)
```

giac [B] time = 0.46, size = 63, normalized size = 2.03

$$-\frac{1}{2}\pi - x - \sqrt{2\,x^2+1} + \arctan(x) + \arctan\left(-\frac{\left(\sqrt{2}\,x - \sqrt{2\,x^2+1}\right)^2 + 1}{2\left(\sqrt{2}\,x - \sqrt{2\,x^2+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
[Out] -1/2*pi - x - sqrt(2*x^2 + 1) + arctan(x) + arctan(-1/2*((sqrt(2)*x - sqrt(
2*x^2 + 1))^2 + 1)/(sqrt(2)*x - sqrt(2*x^2 + 1)))
```

maple [A] time = 0.01, size = 28, normalized size = 0.90

$$-x + \arctan(x) + \arctan\left(\sqrt{2x^2+1}\right) - \sqrt{2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(2*x^2+1)^(1/2)),x)
```

```
[Out] -x+arctan(x)+arctan((2*x^2+1)^(1/2))-(2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x-\sqrt{2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(2*x^2 + 1)), x)
```

mupad [B] time = 0.19, size = 64, normalized size = 2.06

$$-x-\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}-\ln\left(x-\mathrm{i}\right)\,1\mathrm{i}+\frac{\ln\left(x-\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}+\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}}{2}+\frac{\ln\left(x+\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - (2*x^2 + 1)^(1/2)),x)
```

```
[Out] (log(x - (2^(1/2)*(x^2 + 1/2)^(1/2))/2 + 1i/2)*1i)/2 - log(x - 1i)*1i - x +
 (log(x + (2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1i/2)*1i)/2 - 2^(1/2)*(x^2 + 1/2)
^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x-\sqrt{2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x - sqrt(2*x**2 + 1)), x)
```

3.970 $\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x}\right)^{3/2}+\frac{5}{32}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

```
[Out] -5/32*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))-5/12*(x+x^(1/2))^(3/2)+1/2*x^(1/2)
*(x+x^(1/2))^(3/2)+5/32*(1+2*x^(1/2))*(x+x^(1/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2018, 670, 640, 612, 620, 206}

$$\frac{1}{2}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x}\right)^{3/2}+\frac{5}{32}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (5*(1 + 2*Sqrt[x])*Sqrt[Sqrt[x] + x])/32 - (5*(Sqrt[x] + x)^(3/2))/12 + (Sq
rt[x]*(Sqrt[x] + x)^(3/2))/2 - (5*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
```

```
+ 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx &= 2\,\text{Subst}\left(\int x^2\sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2}-\frac{5}{4}\text{Subst}\left(\int x\sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{5}{12}\left(\sqrt{x}+x\right)^{3/2}+\frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2}+\frac{5}{8}\text{Subst}\left(\int \sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x}-\frac{5}{12}\left(\sqrt{x}+x\right)^{3/2}+\frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2}-\frac{5}{64}\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,d\right.\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x}-\frac{5}{12}\left(\sqrt{x}+x\right)^{3/2}+\frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2}-\frac{5}{32}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,\right.\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x}-\frac{5}{12}\left(\sqrt{x}+x\right)^{3/2}+\frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 58, normalized size = 0.71

$$
\frac{1}{96}\sqrt{x+\sqrt{x}}\left(48x^{3/2}+8x-10\sqrt{x}-\frac{15\sinh^{-1}\left(\sqrt[4]{x}\right)}{\sqrt{\sqrt{x}+1}\sqrt[4]{x}}+15\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (Sqrt[Sqrt[x] + x]*(15 - 10*Sqrt[x] + 8*x + 48*x^(3/2) - (15*ArcSinh[x^(1/4
)])/(Sqrt[1 + Sqrt[x]]*x^(1/4))))/96
```

fricas [A] time = 2.10, size = 54, normalized size = 0.66

$$
\frac{1}{96}\left(2\left(24x-5\right)\sqrt{x}+8x+15\right)\sqrt{x+\sqrt{x}}+\frac{5}{128}\log\left(4\sqrt{x+\sqrt{x}}\left(2\sqrt{x}+1\right)-8x-8\sqrt{x}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 5)*sqrt(x) + 8*x + 15)*sqrt(x + sqrt(x)) + 5/128*log(4*sqrt
(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 1)
```

giac [A] time = 0.44, size = 50, normalized size = 0.61

$$
\frac{1}{96}\left(2\left(4\sqrt{x}\left(6\sqrt{x}+1\right)-5\right)\sqrt{x}+15\right)\sqrt{x+\sqrt{x}}+\frac{5}{64}\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 5)*sqrt(x) + 15)*sqrt(x + sqrt(x)) + 5
/64*log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)
```

maple [A] time = 0.00, size = 54, normalized size = 0.66

$$-\frac{5\ln\left(\sqrt{x}+\frac{1}{2}+\sqrt{x+\sqrt{x}}\right)}{64}+\frac{\left(x+\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}}{2}-\frac{5\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{12}+\frac{5\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x+x^(1/2))^(1/2),x)
```

```
[Out] 1/2*x^(1/2)*(x+x^(1/2))^(3/2)-5/12*(x+x^(1/2))^(3/2)+5/32*(2*x^(1/2)+1)*(x+
x^(1/2))^(1/2)-5/64*ln(x^(1/2)+1/2+(x+x^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x}}\,\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x))*sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\sqrt{x+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x + x^(1/2))^(1/2),x)
```

```
[Out] int(x^(1/2)*(x + x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\sqrt{\sqrt{x}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(sqrt(x) + x), x)
```

3.971 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx$

Optimal. Leaf size=74

$$\frac{6x^{5/6}}{5}+2\sqrt{x}-3\sqrt[3]{x}-4\log\left(\sqrt[6]{x}+1\right)-\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

```
[Out] -3*x^(1/3)+6/5*x^(5/6)-4*ln(1+x^(1/6))-ln(1-x^(1/6)+x^(1/3))-2*arctan(1/3*(
1-2*x^(1/6))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1593, 1887, 1874, 31, 634, 618, 204, 628}

$$\frac{6x^{5/6}}{5}+2\sqrt{x}-3\sqrt[3]{x}-4\log\left(\sqrt[6]{x}+1\right)-\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx &= 6\,\text{Subst}\left(\int \frac{x^5+x^7}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int \frac{x^5\left(1+x^2\right)}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\,\text{Subst}\left(\int \left(-x+x^2+x^4+\frac{(1-x)x}{1+x^3}\right)dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}+6\,\text{Subst}\left(\int \frac{(1-x)x}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}+2\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right)-4\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-4\log\left(1+\sqrt[6]{x}\right)+3\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right)-\text{Subst}\left(\int \frac{-1}{1-\cdots}\right.\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)-6\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -\cdots\right.\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 74, normalized size = 1.00

$$\frac{6x^{5/6}}{5}+2\sqrt{x}-3\sqrt[3]{x}-4\log\left(\sqrt[6]{x}+1\right)-\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

fricas [A] time = 0.94, size = 57, normalized size = 0.77

$$2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\,x^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3\,x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/6) - 1/3*sqrt(3)) + 6/5*x^(5/6) + 2*sqrt(
x) - 3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

giac [A] time = 0.46, size = 55, normalized size = 0.74

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3
*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

maple [A] time = 0.01, size = 56, normalized size = 0.76

$$2\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{6}}-1\right)\sqrt{3}}{3}\right)-4\ln\left(x^{\frac{1}{6}}+1\right)-\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)+\frac{6x^{\frac{5}{6}}}{5}+2\sqrt{x}-3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3)+1)/(x^(1/2)+1),x)
```

```
[Out] 6/5*x^(5/6)+2*x^(1/2)-3*x^(1/3)-ln(x^(1/3)-x^(1/6)+1)+2*3^(1/2)*arctan(1/3*
(2*x^(1/6)-1)*3^(1/2))-4*ln(x^(1/6)+1)
```

maxima [A] time = 0.97, size = 55, normalized size = 0.74

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3
*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

mupad [B] time = 3.41, size = 95, normalized size = 1.28

$$2\sqrt{x}+\ln\left(\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(27+\sqrt{3}\,9\mathrm{i}\right)+36x^{1/6}+36\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)-\ln\left(\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(-27+\sqrt{3}\,9\mathrm{i}\right)+36x^{1/6}+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3) + 1)/(x^(1/2) + 1),x)
```

```
[Out] log((3^(1/2)*1i - 1)*(3^(1/2)*9i + 27) + 36*x^(1/6) + 36)*(3^(1/2)*1i - 1)
- 4*log(36*x^(1/6) + 36) - log((3^(1/2)*1i + 1)*(3^(1/2)*9i - 27) + 36*x^(1
/6) + 36)*(3^(1/2)*1i + 1) + 2*x^(1/2) - 3*x^(1/3) + (6*x^(5/6))/5
```

sympy [C] time = 3.78, size = 155, normalized size = 2.09

$$\frac{16x^{\frac{5}{6}}\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{11}{3}\right)}-\frac{8\sqrt[3]{x}\,\Gamma\left(\frac{8}{3}\right)}{\Gamma\left(\frac{11}{3}\right)}+2\sqrt{x}-2\log\left(\sqrt{x}+1\right)-\frac{16e^{-\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}\,e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16\log\left(-\sqrt[6]{x}\,e^{i\pi}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16e^{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/3))/(1+x**(1/2)),x)
```

```
[Out] 16*x**(5/6)*gamma(8/3)/(5*gamma(11/3)) - 8*x**(1/3)*gamma(8/3)/gamma(11/3)
+ 2*sqrt(x) - 2*log(sqrt(x) + 1) - 16*exp(-2*I*pi/3)*log(-x**(1/6)*exp_pola
r(I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*log(-x**(1/6)*exp_polar(I*pi
) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*exp(2*I*pi/3)*log(-x**(1/6)*exp_pola
r(5*I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3))
```

3.972 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx$

Optimal. Leaf size=115

$$\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}-8\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)+4\sqrt{3}\tan^{-1}$$

```
[Out] 12*x^(1/12)+4*x^(1/4)-3*x^(1/3)+12/7*x^(7/12)+4/3*x^(3/4)-6/5*x^(5/6)+12/13
*x^(13/12)-8*ln(1+x^(1/12))-2*ln(1-x^(1/12)+x^(1/6))+4*arctan(1/3*(1-2*x^(1
/12))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1593, 1836, 1887, 1874, 31, 634, 618, 204, 628}

$$\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}-8\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)+4\sqrt{3}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7 + (4*x^(3
/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*Sqrt[3]*ArcTan[(1 - 2*x^(1/1
2))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) + x^(1/6)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = 12\,\text{Subst}\left(\int \frac{x^{11}+x^{15}}{1+x^3}\,dx, x, \sqrt[12]{x}\right)$$
$$= 12\,\text{Subst}\left(\int \frac{x^{11}\left(1+x^4\right)}{1+x^3}\,dx, x, \sqrt[12]{x}\right)$$
$$= \frac{12x^{13/12}}{13} + \frac{12}{13}\text{Subst}\left(\int \frac{(13-13x)x^{11}}{1+x^3}\,dx, x, \sqrt[12]{x}\right)$$
$$= \frac{12x^{13/12}}{13} + \frac{12}{13}\text{Subst}\left(\int \left(13+13x^2-13x^3-13x^5+13x^6+13x^8-13x^9-\frac{13\left(1+x^2\right)}{1+x^3}\right)dx, x\right.$$
$$= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 12\,\text{Subst}\left(\int \frac{1+x^2}{1+x^3}\,dx,\right.$$
$$= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 4\,\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\right.$$
$$= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 8\log\left(1+\sqrt[12]{x}\right) - 2\,\text{Sub}$$
$$= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 8\log\left(1+\sqrt[12]{x}\right) - 2\log$$
$$= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} + 4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Mathematica [A] time = 0.09, size = 123, normalized size = 1.07

$$\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}+4\left(\sqrt[3]{-1}-1\right)\log\left(\sqrt[3]{-1}-\sqrt[12]{x}\right)-4\left(1+(-1)^{2/3}\right)\text{lo}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7 + (4*x^(3
/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*(-1 + (-1)^(1/3))*Log[(-1)^(
1/3) - x^(1/12)] - 4*(1 + (-1)^(2/3))*Log[-(-1)^(2/3) - x^(1/12)] - 8*Log[1
 + x^(1/12)]
```

fricas [A] time = 0.95, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{12}}-\frac{1}{3}\sqrt{3}\right)+\frac{12}{13}(x+13)x^{\frac{1}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="fricas")
```

```
[Out] -4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 12/13*(x + 13)*x^(1
/12) - 6/5*x^(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) +
4*x^(1/4) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

giac [A] time = 0.43, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="giac")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) + 12
*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

maple [A] time = 0.01, size = 81, normalized size = 0.70

$$\frac{12x^{\frac{13}{12}}}{13}-4\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{12}}-1\right)\sqrt{3}}{3}\right)-8\ln\left(x^{\frac{1}{12}}+1\right)-2\ln\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+\frac{12x^{\frac{7}{12}}}{7}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3)+1)/(x^(1/4)+1),x)
```

```
[Out] 12/13*x^(13/12)-6/5*x^(5/6)+4/3*x^(3/4)+12/7*x^(7/12)-2*x^(1/2)-3*x^(1/3)+4
*x^(1/4)+12*x^(1/12)-2*ln(x^(1/6)-x^(1/12)+1)-4*3^(1/2)*arctan(1/3*(2*x^(1/
12)-1)*3^(1/2))-8*ln(x^(1/12)+1)
```

maxima [A] time = 0.96, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="maxima")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) + 12
*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

mupad [B] time = 0.09, size = 130, normalized size = 1.13

$$4x^{1/4}+\ln\left(\left(-2+\sqrt{3}\,2i\right)\left(54-36x^{1/12}+\sqrt{3}\,18i\right)-144x^{1/12}+144\right)\left(-2+\sqrt{3}\,2i\right)-\ln\left(\left(2+\sqrt{3}\,2i\right)\left(36x^{1/12}-5\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3) + 1)/(x^(1/4) + 1),x)
```

```
[Out] log((3^(1/2)*2i - 2)*(3^(1/2)*18i - 36*x^(1/12) + 54) - 144*x^(1/12) + 144)
*(3^(1/2)*2i - 2) - 8*log(144*x^(1/12) + 144) - log((3^(1/2)*2i + 2)*(3^(1/
2)*18i + 36*x^(1/12) - 54) - 144*x^(1/12) + 144)*(3^(1/2)*2i + 2) - 2*x^(1/
2) - 3*x^(1/3) + 4*x^(1/4) + (4*x^(3/4))/3 - (6*x^(5/6))/5 + 12*x^(1/12) +
(12*x^(7/12))/7 + (12*x^(13/12))/13
```

sympy [C] time = 5.43, size = 221, normalized size = 1.92

$$\frac{64x^{\frac{13}{12}}\Gamma\left(\frac{16}{3}\right)}{13\Gamma\left(\frac{19}{3}\right)}+\frac{64x^{\frac{7}{12}}\Gamma\left(\frac{16}{3}\right)}{7\Gamma\left(\frac{19}{3}\right)}+\frac{64\sqrt[12]{x}\,\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)}-\frac{32x^{\frac{5}{6}}\Gamma\left(\frac{16}{3}\right)}{5\Gamma\left(\frac{19}{3}\right)}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{16\sqrt[3]{x}\,\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)}-2\sqrt{x}-4\log\left(\sqrt[4]{x}+1\right)+\frac{64}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/3))/(1+x**(1/4)),x)
```

```
[Out] 64*x**(13/12)*gamma(16/3)/(13*gamma(19/3)) + 64*x**(7/12)*gamma(16/3)/(7*ga
mma(19/3)) + 64*x**(1/12)*gamma(16/3)/gamma(19/3) - 32*x**(5/6)*gamma(16/3)
/(5*gamma(19/3)) + 4*x**(3/4)/3 + 4*x**(1/4) - 16*x**(1/3)*gamma(16/3)/gamm
a(19/3) - 2*sqrt(x) - 4*log(x**(1/4) + 1) + 64*exp(-I*pi/3)*log(-x**(1/12)*
exp_polar(I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3)) - 64*log(-x**(1/12)*exp_
polar(I*pi) + 1)*gamma(16/3)/(3*gamma(19/3)) + 64*exp(I*pi/3)*log(-x**(1/12
)*exp_polar(5*I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3))
```

3.973 $\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=4

$$x + \sin^{-1}(x)$$

```
[Out] x+arcsin(x)
```

Rubi [A] time = 0.04, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2156, 8, 216}

$$x + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] x + ArcSin[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2156

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{-1+x^2+\sqrt{1-x^2}} \, dx &= -\int -1 \, dx + \int \frac{1}{\sqrt{1-x^2}} \, dx \\ &= x + \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.06, size = 4, normalized size = 1.00

$$x + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] x + ArcSin[x]
```

fricas [B] time = 0.68, size = 20, normalized size = 5.00

$$x - 2 \arctan\left(\frac{\sqrt{-x^2+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.33, size = 4, normalized size = 1.00

$$x + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x + arcsin(x)
```

maple [B] time = 0.02, size = 51, normalized size = 12.75

$$x+\operatorname{arctanh}(x)+\arcsin(x)+\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}-\frac{\sqrt{-2x-(x-1)^2+2}}{2}+\frac{\sqrt{2x-(x+1)^2+2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-1+x^2+(-x^2+1)^(1/2)),x)
```

```
[Out] x+1/2*ln(x-1)-1/2*ln(x+1)+arctanh(x)-1/2*(-(x-1)^2-2*x+2)^(1/2)+arcsin(x)+1
/2*(-(x+1)^2+2*x+2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^2+\sqrt{-x^2+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^2 + sqrt(-x^2 + 1) - 1), x)
```

mupad [B] time = 0.03, size = 4, normalized size = 1.00

$$x + \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2 + (1 - x^2)^(1/2) - 1),x)
```

```
[Out] x + asin(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^2+\sqrt{1-x^2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-1+x**2+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x**2/(x**2 + sqrt(1 - x**2) - 1), x)
```

3.974 $\int \sqrt{\frac{1+x}{x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{\frac{1}{x}+1}\,x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

[Out] arctanh((1+1/x)^(1/2))+x*(1+1/x)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1972, 242, 47, 63, 207}

$$\sqrt{\frac{1}{x}+1}\,x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 + x)/x],x]

[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1+x}{x}}\,dx &= \int \sqrt{1+\frac{1}{x}}\,dx\\ &= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{1+\frac{1}{x}}\,x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{1+\frac{1}{x}}\,x - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)\\ &= \sqrt{1+\frac{1}{x}}\,x + \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 22, normalized size = 1.00

$$\sqrt{\frac{1}{x}+1}\,x + \tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x],x]
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

fricas [**B**] time = 0.89, size = 40, normalized size = 1.82

$$x\sqrt{\frac{x+1}{x}} + \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="fricas")
[Out] x*sqrt((x + 1)/x) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sqrt((x + 1)/x)
- 1)
```

giac [**A**] time = 0.39, size = 31, normalized size = 1.41

$$-\frac{1}{2}\log\left(\left|-2\,x + 2\,\sqrt{x^2+x} - 1\right|\right)\text{sgn}(x) + \sqrt{x^2+x}\,\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="giac")
[Out] -1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + sqrt(x^2 + x)*sgn(x)
```

maple [**B**] time = 0.00, size = 41, normalized size = 1.86

$$\frac{\sqrt{\frac{x+1}{x}}\,\left(\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)+2\sqrt{x^2+x}\right)x}{2\sqrt{(x+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x+1)/x)^(1/2),x)
```

```
[Out] 1/2*((x+1)/x)^(1/2)*x*(2*(x^2+x)^(1/2)+ln(x+1/2+(x^2+x)^(1/2)))/((x+1)*x)^(
1/2)
```

maxima [B] time = 0.44, size = 50, normalized size = 2.27

$$\frac{\sqrt{\frac{x+1}{x}}}{\frac{x+1}{x}-1}+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sq
rt((x + 1)/x) - 1)
```

mupad [B] time = 3.39, size = 18, normalized size = 0.82

$$\operatorname{atanh}\left(\sqrt{\frac{1}{x}+1}\right)+x\sqrt{\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/x)^(1/2),x)
```

```
[Out] atanh((1/x + 1)^(1/2)) + x*(1/x + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x+1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/x), x)
```

3.975 $\int \sqrt{\frac{1-x}{x}} \, dx$

Optimal. Leaf size=24

$$\sqrt{\frac{1}{x}-1}\,x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

```
[Out] -arctan((-1+1/x)^(1/2))+x*(-1+1/x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1972, 242, 47, 63, 203}

$$\sqrt{\frac{1}{x}-1}\,x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{1-x}{x}}\,dx &= \int \sqrt{-1+\frac{1}{x}}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}\,x - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}\,x - \operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x}}\right) \\
&= \sqrt{-1+\frac{1}{x}}\,x - \tan^{-1}\left(\sqrt{-1+\frac{1}{x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\sqrt{\frac{1}{x}-1}\,x - \tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/x],x]
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

fricas [A] time = 0.96, size = 26, normalized size = 1.08

$$x\sqrt{-\frac{x-1}{x}} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="fricas")
[Out] x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))
```

giac [A] time = 0.40, size = 28, normalized size = 1.17

$$\frac{1}{4}\pi\operatorname{sgn}(x) + \frac{1}{2}\arcsin(2x-1)\operatorname{sgn}(x) + \sqrt{-x^2+x}\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="giac")
[Out] 1/4*pi*sgn(x) + 1/2*arcsin(2*x - 1)*sgn(x) + sqrt(-x^2 + x)*sgn(x)
```

maple [A] time = 0.01, size = 40, normalized size = 1.67

$$\frac{\sqrt{-\frac{x-1}{x}}\left(\arcsin(2x-1) + 2\sqrt{-x^2+x}\right)x}{2\sqrt{-(x-1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+1)/x)^(1/2),x)
[Out] 1/2*(-(x-1)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-(x-1)*x)^(1/2)
```

maxima [A] time = 0.97, size = 37, normalized size = 1.54

$$-\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1}-\arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))
```

mupad [B] time = 0.04, size = 20, normalized size = 0.83

$$x\sqrt{\frac{1}{x}-1}-\operatorname{atan}\left(\sqrt{\frac{1}{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/x)^(1/2),x)
[Out] x*(1/x - 1)^(1/2) - atan((1/x - 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{1-x}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)**(1/2),x)
[Out] Integral(sqrt((1 - x)/x), x)
```

3.976 $\int \sqrt{\frac{-1+x}{x}}\, dx$

Optimal. Leaf size=24

$$\sqrt{x-1}\,\sqrt{x} - \sinh^{-1}\left(\sqrt{x-1}\right)$$

```
[Out] -arcsinh((-1+x)^(1/2))+(-1+x)^(1/2)*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1972, 242, 47, 63, 206}

$$\sqrt{\frac{x-1}{x}}\,x - \tanh^{-1}\left(\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/x],x]
```

```
[Out] Sqrt[(-1 + x)/x]*x - ArcTanh[Sqrt[(-1 + x)/x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{-1+x}{x}}\,dx &= \int \sqrt{1-\frac{1}{x}}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{\frac{-1+x}{x}}\,x + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{\frac{-1+x}{x}}\,x - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\frac{-1+x}{x}}\right) \\
&= \sqrt{\frac{-1+x}{x}}\,x - \tanh^{-1}\left(\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.58

$$
\frac{\sqrt{x}\,(x-1)+\sqrt{1-x}\,\sin^{-1}\left(\sqrt{1-x}\right)}{\sqrt{x-1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/x],x]
[Out] ((-1 + x)*Sqrt[x] + Sqrt[1 - x]*ArcSin[Sqrt[1 - x]])/Sqrt[-1 + x]
```

fricas [B] time = 0.67, size = 40, normalized size = 1.67

$$
x\sqrt{\frac{x-1}{x}} - \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="fricas")
[Out] x*sqrt((x - 1)/x) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(sqrt((x - 1)/x)
- 1)
```

giac [A] time = 0.38, size = 35, normalized size = 1.46

$$
\frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)\mathrm{sgn}(x)+\sqrt{x^2-x}\,\mathrm{sgn}(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="giac")
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))*sgn(x) + sqrt(x^2 - x)*sgn(x)
```

maple [B] time = 0.01, size = 45, normalized size = 1.88

$$
-\frac{\sqrt{\frac{x-1}{x}}\,\left(\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)-2\sqrt{x^2-x}\right)x}{2\sqrt{(x-1)\,x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-1)/x)^(1/2),x)
```

```
[Out] -1/2*((x-1)/x)^(1/2)*x*(-2*(x^2-x)^(1/2)+ln(x-1/2+(x^2-x)^(1/2)))/((x-1)*x)
^(1/2)
```

maxima [B] time = 0.43, size = 51, normalized size = 2.12

$$-\frac{\sqrt{\frac{x-1}{x}}}{\frac{x-1}{x}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt((x - 1)/x)/((x - 1)/x - 1) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(s
qrt((x - 1)/x) - 1)
```

mupad [B] time = 0.03, size = 24, normalized size = 1.00

$$x\sqrt{1-\frac{1}{x}}-\operatorname{atanh}\left(\sqrt{1-\frac{1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)/x)^(1/2),x)
```

```
[Out] x*(1 - 1/x)^(1/2) - atanh((1 - 1/x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x-1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)/x), x)
```

3.977 $\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx$

Optimal. Leaf size=24

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)-2\sqrt{\frac{1}{x}+1}$$

```
[Out] 2*arctanh((1+1/x)^(1/2))-2*(1+1/x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1973, 266, 50, 63, 207}

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)-2\sqrt{\frac{1}{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/x]/x,x]
```

```
[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\frac{1+x}{x}}}{x} dx &= \int \frac{\sqrt{1+\frac{1}{x}}}{x} dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x} dx, x, \frac{1}{x}\right) \\
&= -2\sqrt{1+\frac{1}{x}} - \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} dx, x, \frac{1}{x}\right) \\
&= -2\sqrt{1+\frac{1}{x}} - 2\,\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+\frac{1}{x}}\right) \\
&= -2\sqrt{1+\frac{1}{x}} + 2\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 24, normalized size = 1.00

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right) - 2\sqrt{\frac{1}{x}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x]/x,x]
[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

fricas [**A**] time = 0.42, size = 38, normalized size = 1.58

$$-2\sqrt{\frac{x+1}{x}} + \log\left(\sqrt{\frac{x+1}{x}}+1\right) - \log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="fricas")
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

giac [**A**] time = 0.38, size = 38, normalized size = 1.58

$$-\log\left(\left|-2\,x + 2\sqrt{x^2+x} - 1\right|\right)\text{sgn}(x) + \frac{2\,\text{sgn}(x)}{x - \sqrt{x^2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="giac")
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + 2*sgn(x)/(x - sqrt(x^2 + x))
```

maple [**B**] time = 0.01, size = 60, normalized size = 2.50

$$-\frac{\sqrt{\frac{x+1}{x}}\left(-x^2\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right) - 2\sqrt{x^2+x}\,x^2 + 2\left(x^2+x\right)^{\frac{3}{2}}\right)}{\sqrt{(x+1)\,x}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x+1)/x)^(1/2)/x,x)
```

```
[Out] -((x+1)/x)^(1/2)/x*(2*(x^2+x)^(3/2)-2*(x^2+x)^(1/2)*x^2-ln(x+1/2+(x^2+x)^(1
/2))*x^2)/((x+1)*x)^(1/2)
```

maxima [A] time = 0.44, size = 38, normalized size = 1.58

$$-2\sqrt{\frac{x+1}{x}} + \log\left(\sqrt{\frac{x+1}{x}} + 1\right) - \log\left(\sqrt{\frac{x+1}{x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="maxima")
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.83

$$2\,\mathrm{atanh}\left(\sqrt{\frac{1}{x}+1}\right) - 2\sqrt{\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/x)^(1/2)/x,x)
[Out] 2*atanh((1/x + 1)^(1/2)) - 2*(1/x + 1)^(1/2)
```

sympy [A] time = 3.23, size = 32, normalized size = 1.33

$$-2\sqrt{1+\frac{1}{x}} - \log\left(\sqrt{1+\frac{1}{x}} - 1\right) + \log\left(\sqrt{1+\frac{1}{x}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)**(1/2)/x,x)
[Out] -2*sqrt(1 + 1/x) - log(sqrt(1 + 1/x) - 1) + log(sqrt(1 + 1/x) + 1)
```

3.978 $\int \sqrt{\frac{x}{1+x}}\, dx$

Optimal. Leaf size=22

$$\sqrt{x}\,\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1958, 50, 54, 215}

$$\sqrt{x}\,\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{x}{1+x}}\, dx &= \int \frac{\sqrt{x}}{\sqrt{1+x}}\, dx \\
&= \sqrt{x}\,\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\,\sqrt{1+x}}\, dx \\
&= \sqrt{x}\,\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\, dx, x, \sqrt{x}\right) \\
&= \sqrt{x}\,\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1)-\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]
```

fricas [B] time = 0.43, size = 42, normalized size = 1.91

$$(x+1)\sqrt{\frac{x}{x+1}}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
+ 1)) - 1)
```

giac [B] time = 0.39, size = 35, normalized size = 1.59

$$\frac{1}{2}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)+\sqrt{x^2+x}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x +
1)
```

maple [B] time = 0.01, size = 43, normalized size = 1.95

$$-\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\left(\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)-2\sqrt{x^2+x}\right)}{2\sqrt{(x+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x+1)*x)^(1/2),x)
```

```
[Out] -1/2*(1/(x+1)*x)^(1/2)*(x+1)*(-2*(x^2+x)^(1/2)+ln(x+1/2+(x^2+x)^(1/2)))/((x
+1)*x)^(1/2)
```

maxima [B] time = 0.44, size = 51, normalized size = 2.32

$$-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(s
qrt(x/(x + 1)) - 1)
```

mupad [B] time = 0.04, size = 35, normalized size = 1.59

$$-\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right)-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(x + 1))^(1/2),x)
[Out] - atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2),x)
[Out] Integral(sqrt(x/(x + 1)), x)
```

3.979 $\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx$

Optimal. Leaf size=29

$$\tan^{-1}\left(\sqrt{-\frac{x+1}{x}}\right)-x\sqrt{-\frac{x+1}{x}}$$

```
[Out] arctan(((-1-x)/x)^(1/2))-x*((-1-x)/x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1972, 242, 51, 63, 204}

$$\tan^{-1}\left(\sqrt{-\frac{x+1}{x}}\right)-x\sqrt{-\frac{x+1}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-1 - x)/x],x]
```

```
[Out] -(x*Sqrt[-((1 + x)/x)]) + ArcTan[Sqrt[-((1 + x)/x)]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = \int \frac{1}{\sqrt{-1-\frac{1}{x}}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x^2}\,dx, x, \frac{1}{x}\right)$$
$$= -x\sqrt{-\frac{1+x}{x}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x}\,dx, x, \frac{1}{x}\right)$$
$$= -x\sqrt{-\frac{1+x}{x}} - \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-\frac{1+x}{x}}\right)$$
$$= -x\sqrt{-\frac{1+x}{x}} + \tan^{-1}\left(\sqrt{-\frac{1+x}{x}}\right)$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.48

$$\frac{\sqrt{x}\,(x+1) - \sqrt{x+1}\,\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}\,\sqrt{-\frac{x+1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-1 - x)/x],x]
[Out] (Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]])/(Sqrt[x]*Sqrt[-((1 + x)/x)
])
```

fricas [A] time = 0.44, size = 25, normalized size = 0.86

$$-x\sqrt{-\frac{x+1}{x}} + \arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="fricas")
[Out] -x*sqrt(-(x + 1)/x) + arctan(sqrt(-(x + 1)/x))
```

giac [A] time = 0.46, size = 35, normalized size = 1.21

$$\frac{1}{4}\,\pi\text{sgn}(x) - \frac{\arcsin\left(2\,x+1\right)}{2\,\text{sgn}(x)} - \frac{\sqrt{-x^2-x}}{\text{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="giac")
[Out] 1/4*pi*sgn(x) - 1/2*arcsin(2*x + 1)/sgn(x) - sqrt(-x^2 - x)/sgn(x)
```

maple [A] time = 0.01, size = 44, normalized size = 1.52

$$\frac{(x+1)\left(\arcsin\left(2x+1\right) + 2\sqrt{-x^2-x}\right)}{2\sqrt{-\frac{x+1}{x}}\,\sqrt{-\left(x+1\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1-x)/x)^(1/2),x)
```

```
[Out] 1/2*(x+1)*(2*(-x^2-x)^(1/2)+arcsin(2*x+1))/(-(x+1)/x)^(1/2)/(-(x+1)*x)^(1/2
)
```

maxima [A] time = 0.97, size = 35, normalized size = 1.21

$$-\frac{\sqrt{-\frac{x+1}{x}}}{\frac{x+1}{x}-1}+\arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-(x + 1)/x)/((x + 1)/x - 1) + arctan(sqrt(-(x + 1)/x))
```

mupad [B] time = 3.37, size = 23, normalized size = 0.79

$$\operatorname{atan}\left(\sqrt{-\frac{1}{x}-1}\right)-x\sqrt{-\frac{1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x + 1)/x)^(1/2),x)
```

```
[Out] atan((- 1/x - 1)^(1/2)) - x*(- 1/x - 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{-x-1}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((-x - 1)/x), x)
```

3.980 $\int \sqrt{(4-x)x}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{2}\sqrt{4x-x^2}\,(2-x)-2\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] 2*arcsin(-1+1/2*x)-1/2*(2-x)*(-x^2+4*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1979, 612, 619, 216}

$$-\frac{1}{2}\sqrt{4x-x^2}\,(2-x)-2\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(4 - x)*x],x]
```

```
[Out] -((2 - x)*Sqrt[4*x - x^2])/2 - 2*ArcSin[1 - x/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{(4-x)x}\,dx &= \int \sqrt{4x-x^2}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2}+2\int \frac{1}{\sqrt{4x-x^2}}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx,x,4-2x\right)\\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2}-2\sin^{-1}\left(1-\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 32, normalized size = 0.97

$$\frac{1}{2}(x-2)\sqrt{-((x-4)x)}-4\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(4 - x)*x],x]
[Out] ((-2 + x)*Sqrt[-((-4 + x)*x)])/2 - 4*ArcSin[Sqrt[1 - x/4]]
```

fricas [A] time = 0.44, size = 35, normalized size = 1.06

$$\frac{1}{2}\sqrt{-x^2+4x}(x-2)-4\arctan\left(\frac{\sqrt{-x^2+4x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) - 4*arctan(sqrt(-x^2 + 4*x)/x)
```

giac [A] time = 0.39, size = 25, normalized size = 0.76

$$\frac{1}{2}\sqrt{-x^2+4x}(x-2)+2\arcsin\left(\frac{1}{2}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) + 2*arcsin(1/2*x - 1)
```

maple [A] time = 0.01, size = 28, normalized size = 0.85

$$2\arcsin\left(\frac{x}{2}-1\right)-\frac{(-2x+4)\sqrt{-x^2+4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x+4)*x)^(1/2),x)
[Out] -1/4*(-2*x+4)*(-x^2+4*x)^(1/2)+2*arcsin(1/2*x-1)
```

maxima [A] time = 1.02, size = 36, normalized size = 1.09

$$\frac{1}{2}\sqrt{-x^2+4x}\,x-\sqrt{-x^2+4x}-2\arcsin\left(-\frac{1}{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 4*x)*x - sqrt(-x^2 + 4*x) - 2*arcsin(-1/2*x + 1)
```

mupad [B] time = 3.47, size = 26, normalized size = 0.79

$$2\,\mathrm{asin}\left(\frac{x}{2}-1\right)+\left(\frac{x}{2}-1\right)\sqrt{4x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(x - 4))^(1/2),x)
```

```
[Out] 2*asin(x/2 - 1) + (x/2 - 1)*(4*x - x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x(4-x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)**(1/2),x)
[Out] Integral(sqrt(x*(4 - x)), x)
```

3.981 $\int \frac{1}{\sqrt{(1-x)x}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

```
[Out] arcsin(-1+2*x)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1979, 619, 216}

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(1 - x)*x],x]
```

```
[Out] -ArcSin[1 - 2*x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(1-x)x}}\,dx &= \int \frac{1}{\sqrt{x-x^2}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right)\\ &= -\sin^{-1}(1-2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.50

$$-2\sin^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(1 - x)*x],x]
```

```
[Out] -2*ArcSin[Sqrt[1 - x]]
```

fricas [B] time = 0.45, size = 16, normalized size = 2.00

$$-2\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x^2 + x)/x)
```

giac [A] time = 0.39, size = 6, normalized size = 0.75

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(2*x - 1)
```

maple [A] time = 0.01, size = 7, normalized size = 0.88

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-x+1)*x)^(1/2),x)
```

```
[Out] arcsin(2*x-1)
```

maxima [A] time = 0.96, size = 6, normalized size = 0.75

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(2*x - 1)
```

mupad [B] time = 0.01, size = 6, normalized size = 0.75

$$\operatorname{asin}(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(x - 1))^(1/2),x)
```

```
[Out] asin(2*x - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(1-x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(1 - x)), x)
```

3.982 $\int \frac{x}{(x(2+x))^{3/2}}\,dx$

Optimal. Leaf size=13

$$\frac{x}{\sqrt{x^2+2x}}$$

```
[Out] x/(x^2+2*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1980, 636}

$$\frac{x}{\sqrt{x^2+2x}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x*(2 + x))^(3/2),x]
```

```
[Out] x/Sqrt[2*x + x^2]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !Generali
zedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(x(2+x))^{3/2}}\,dx &= \int \frac{x}{\left(2x+x^2\right)^{3/2}}\,dx\\ &= \frac{x}{\sqrt{2x+x^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{x}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x*(2 + x))^(3/2),x]
```

```
[Out] x/Sqrt[x*(2 + x)]
```

fricas [A] time = 0.43, size = 18, normalized size = 1.38

$$\frac{x+\sqrt{x^2+2\,x}+2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + 2*x) + 2)/(x + 2)
```

giac [A] time = 0.33, size = 16, normalized size = 1.23

$$\frac{2}{x-\sqrt{(x+2)x}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="giac")
[Out] 2/(x - sqrt((x + 2)*x) + 2)
```

maple [A] time = 0.00, size = 15, normalized size = 1.15

$$\frac{(x+2)\,x^2}{((x+2)\,x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x*(x+2))^(3/2),x)
[Out] x^2*(x+2)/(x*(x+2))^(3/2)
```

maxima [A] time = 0.43, size = 11, normalized size = 0.85

$$\frac{x}{\sqrt{x^2+2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="maxima")
[Out] x/sqrt(x^2 + 2*x)
```

mupad [B] time = 3.52, size = 13, normalized size = 1.00

$$\frac{\sqrt{x\,(x+2)}}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x*(x + 2))^(3/2),x)
[Out] (x*(x + 2))^(1/2)/(x + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x\,(x+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))**(3/2),x)
[Out] Integral(x/(x*(x + 2))**(3/2), x)
```

3.983 $\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx$

Optimal. Leaf size=22

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

```
[Out] arctanh(1/2*(1+1/x)^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1446, 1469, 627, 63, 207}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)]/(1 - x^2),x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1446

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx &= \int \frac{\sqrt{1+\frac{1}{x}}}{\left(-1+\frac{1}{x^2}\right)x^2}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{-1+x^2}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)\right) \\
&= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{x}}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)]/(1 - x^2),x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

fricas [A] time = 0.44, size = 33, normalized size = 1.50

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}x\sqrt{\frac{x+1}{x}}+3x+1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*x*sqrt((x + 1)/x) + 3*x + 1)/(x - 1))
```

giac [B] time = 0.53, size = 73, normalized size = 3.32

$$\frac{1}{2}\sqrt{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\text{sgn}(x)-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+x}+2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+x}+2\right|}\right)\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log((sqrt(2) - 1)/(sqrt(2) + 1))*sgn(x) - 1/2*sqrt(2)*log(abs(-
2*x - 2*sqrt(2) + 2*sqrt(x^2 + x) + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 +
x) + 2))*sgn(x)
```

maple [B] time = 0.02, size = 41, normalized size = 1.86

$$\frac{\sqrt{\frac{x+1}{x}}\,\sqrt{2}\,x\,\text{arctanh}\left(\frac{(3x+1)\sqrt{2}}{4\sqrt{x^2+x}}\right)}{2\sqrt{(x+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2)/(-x^2+1),x)
```

```
[Out] 1/2*((x+1)/x)^(1/2)*x/((x+1)*x)^(1/2)*2^(1/2)*arctanh(1/4*(3*x+1)*2^(1/2)/(
x^2+x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{\frac{1}{x}+1}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(1/x + 1)/(x^2 - 1), x)
```

mupad [B] time = 3.58, size = 17, normalized size = 0.77

$$\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}+1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/x + 1)^(1/2)/(x^2 - 1),x)
```

```
[Out] 2^(1/2)*atanh((2^(1/2)*(1/x + 1)^(1/2))/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{1+\frac{1}{x}}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(-x**2+1),x)
```

```
[Out] -Integral(sqrt(1 + 1/x)/(x**2 - 1), x)
```

3.984 $\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}\,x^2}\,dx$

Optimal. Leaf size=24

$$\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\,x\right)$$

```
[Out] 1/2*arctan(x*(1/2*5^(1/2)-1/2))
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6, 203}

$$\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[(3 - Sqrt[5])/2]*x]/2
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}\,x^2}\,dx &= \int \frac{1}{1+\sqrt{5}+\left(-1+\sqrt{5}\right)x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\,x\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 39, normalized size = 1.62

$$\frac{1}{4}i\log\left(-2ix+\sqrt{5}+1\right)-\frac{1}{4}i\log\left(2ix+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

```
[Out] (I/4)*Log[1 + Sqrt[5] - (2*I)*x] - (I/4)*Log[1 + Sqrt[5] + (2*I)*x]
```

fricas [A] time = 0.48, size = 11, normalized size = 0.46

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\left(\sqrt{5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(1/2*x*(sqrt(5) - 1))
```

giac [A] time = 0.36, size = 13, normalized size = 0.54

$$\frac{1}{2}\arctan\left(\frac{2x}{\sqrt{5}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(2*x/(sqrt(5) + 1))
```

maple [B] time = 0.03, size = 32, normalized size = 1.33

$$\frac{4\arctan\left(\frac{4x}{2+2\sqrt{5}}\right)}{\left(\sqrt{5}-1\right)\left(2+2\sqrt{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x^2+5^(1/2)+5^(1/2)*x^2),x)
```

```
[Out] 4/(5^(1/2)-1)/(2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))
```

maxima [A] time = 0.98, size = 11, normalized size = 0.46

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\left(\sqrt{5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(1/2*x*(sqrt(5) - 1))
```

mupad [B] time = 0.10, size = 45, normalized size = 1.88

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\left(\sqrt{5}+1\right)}{4\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\sqrt{\sqrt{5}+3}}\right)\left(\sqrt{5}+1\right)}{4\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5^(1/2) + 5^(1/2)*x^2 - x^2 + 1),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x*(5^(1/2) + 1))/(4*(5^(1/2)/4 + 1/4)*(5^(1/2) + 3)^
(1/2)))*(5^(1/2) + 1))/(4*(5^(1/2) + 3)^(1/2))
```

sympy [A] time = 0.58, size = 15, normalized size = 0.62

$$-\frac{\mathrm{atan}\left(x\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x**2+5**(1/2)+x**2*5**(1/2)),x)
```

```
[Out] -atan(x*(1/2 - sqrt(5)/2))/2
```

3.985 $\int \frac{1}{\sqrt{ax+bx^2}}\, dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x + b*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{ax+bx^2}}\, dx &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\, dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [B] time = 0.02, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + b*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

fricas [**A**] time = 0.46, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}},\, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

giac [**A**] time = 0.47, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

maple [**A**] time = 0.00, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a*x)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

maxima [**A**] time = 0.44, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

mupad [**B**] time = 3.47, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^2)^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**2), x)
```

3.986 $\int \frac{1}{\sqrt{x(a+bx)}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x*(a + b*x)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x(a+bx)}}\,dx &= \int \frac{1}{\sqrt{ax+bx^2}}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [B] time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(a + b*x)],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

fricas [A] time = 0.45, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx + a + 2\,\sqrt{bx^2 + ax}\,\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

giac [A] time = 0.41, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}\,x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}} + \sqrt{b\,x^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x+a))^(1/2),x)
```

```
[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [A] time = 0.43, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx + a + 2\,\sqrt{bx^2 + ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

mupad [B] time = 3.53, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}} + \sqrt{b\,x^2 + a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x))^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(a + b*x)), x)
```

3.987 $\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(b + a/x)*x^2],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx &= \int \frac{1}{\sqrt{ax+bx^2}}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b + a/x)*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

fricas [A] time = 0.47, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}},\,-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

giac [A] time = 0.42, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b+a/x)*x^2)^(1/2),x)
```

```
[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [A] time = 0.45, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

mupad [B] time = 3.59, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b + a/x))^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\left(\frac{a}{x}+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x+b)*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a/x + b)), x)
```

3.988 $\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx &= \int \frac{1}{\sqrt{ax+bx^2}}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

fricas [A] time = 0.46, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

giac [A] time = 0.50, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a/x^2+b/x)*x^3)^(1/2),x)
```

```
[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [A] time = 0.45, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

mupad [B] time = 3.53, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a/x^2 + b/x))^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3\left(\frac{a}{x^2}+\frac{b}{x}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x**2+b/x)*x**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**3*(a/x**2 + b/x)), x)
```

3.989 $\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx &= \int \frac{1}{\sqrt{ax+bx^2}}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [B] time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

fricas [A] time = 0.47, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}},-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

giac [A] time = 0.56, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a*x^2)/x)^(1/2),x)
```

```
[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [A] time = 0.44, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

mupad [B] time = 3.52, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x^2 + b*x^3)/x)^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a*x**2)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x**2 + b*x**3)/x), x)
```

3.990 $\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}} \, dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}} \, dx &= \int \frac{1}{\sqrt{ax+bx^2}} \, dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{ax+bx^2}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x*(a + b*x)])
```

fricas [A] time = 0.48, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx + a + 2\,\sqrt{bx^2 + ax}\,\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

giac [A] time = 0.47, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}\,x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}} + \sqrt{b\,x^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^4+a*x^3)/x^2)^(1/2),x)
```

```
[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [A] time = 0.44, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx + a + 2\,\sqrt{bx^2 + ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

mupad [B] time = 3.52, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}} + \sqrt{b\,x^2 + a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x^3 + b*x^4)/x^2)^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**4+a*x**3)/x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x**3 + b*x**4)/x**2), x)
```

3.991 $\int \frac{1}{\sqrt{acx+bcx^2}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{acx+bcx^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

fricas [A] time = 0.47, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{bc},-\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\,\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

giac [A] time = 0.50, size = 50, normalized size = 1.25

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}\,x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}\,a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

maple [A] time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c*x^2+a*c*x)^(1/2),x)
```

```
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

maxima [A] time = 0.44, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

mupad [B] time = 3.90, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*x + b*c*x^2)^(1/2),x)
```

```
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{acx + bcx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x**2+a*c*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*c*x + b*c*x**2), x)
```

3.992 $\int \frac{1}{\sqrt{c(ax+bx^2)}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*(a*x + b*x^2)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c\left(ax+bx^2\right)}}\,dx &= \int \frac{1}{\sqrt{acx+bcx^2}}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*(a*x + b*x^2)],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

fricas [A] time = 0.46, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{bc},-\frac{2\,\sqrt{-bc}\,\arctan\left(\frac{\sqrt{bcx^2+acx}\,\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

giac [A] time = 0.57, size = 50, normalized size = 1.25

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}\,x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}\,a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

maple [A] time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(b*x^2+a*x))^(1/2),x)
```

```
[Out] 1/(b*c)^(1/2)*ln((b*c*x+1/2*a*c)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))
```

maxima [A] time = 0.44, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

mupad [B] time = 3.56, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a*x + b*x^2))^(1/2),x)
```

```
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(ax+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x**2+a*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*(a*x + b*x**2)), x)
```

3.993 $\int \frac{1}{\sqrt{cx(a+bx)}} \, dx$

Optimal. Leaf size=40

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{c} x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b} \sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1979, 620, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{c} x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b} \sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*x*(a + b*x)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{cx(a+bx)}} \, dx &= \int \frac{1}{\sqrt{acx+bcx^2}} \, dx \\
&= 2 \operatorname{Subst}\left(\int \frac{1}{1-bcx^2} \, dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right) \\
&= \frac{2 \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{c} x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b} \sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 58, normalized size = 1.45

$$\frac{2 \sqrt{a} \sqrt{x} \sqrt{\frac{bx}{a}+1} \sinh^{-1}\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b} \sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*x*(a + b*x)],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

fricas [A] time = 0.44, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{bc}, -\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\,\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

giac [A] time = 0.49, size = 50, normalized size = 1.25

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}\,x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}\,a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

maple [A] time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(b*x+a))^(1/2),x)
```

```
[Out] 1/(b*c)^(1/2)*ln((b*c*x+1/2*a*c)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))
```

maxima [A] time = 0.46, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

mupad [B] time = 3.47, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(a + b*x))^(1/2),x)
```

```
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*x*(a + b*x)), x)
```

3.994 $\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 620, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*(b + a/x)*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx &= \int \frac{1}{\sqrt{acx+bcx^2}}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*(b + a/x)*x^2],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[c*x*(a + b*x)])
```

fricas [A] time = 0.45, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{bc},\,-\frac{2\,\sqrt{-bc}\,\arctan\left(\frac{\sqrt{bcx^2+acx}\,\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

giac [A] time = 0.52, size = 50, normalized size = 1.25

$$-\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}\,x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}\,a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a
))/(b*c)
```

maple [A] time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(b+a/x)*x^2)^(1/2),x)
```

```
[Out] 1/(b*c)^(1/2)*ln((b*c*x+1/2*a*c)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))
```

maxima [A] time = 0.45, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

mupad [B] time = 3.65, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2*(b + a/x))^(1/2),x)
```

```
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2\left(\frac{a}{x}+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a/x+b)*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*x**2*(a/x + b)), x)
```

3.995 $\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{1}{4}\sqrt{-x^2+\sqrt{x^2-1}\,x+1}\left(\sqrt{x^2-1}+3x\right)+\frac{3\sin^{-1}\left(x-\sqrt{x^2-1}\right)}{4\sqrt{2}}$$

```
[Out] 3/8*arcsin(x-(x^2-1)^(1/2))*2^(1/2)+1/4*(3*x+(x^2-1)^(1/2))*(1-x^2+x*(x^2-1
)^(1/2))^(1/2)
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]
```

Rubi steps

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx=\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Mathematica [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]
```

fricas [A] time = 1.03, size = 68, normalized size = 1.08

$$\frac{1}{4}\sqrt{-x^2+\sqrt{x^2-1}\,x+1}\left(3\,x+\sqrt{x^2-1}\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^2-1}\,x+1}}{2\sqrt{x^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)*(3*x + sqrt(x^2 - 1)) + 3/8*sqrt(2)*ar
ctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)/sqrt(x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2+\sqrt{x^2-1}\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2 + \sqrt{x^2-1}\, x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^2+(x^2-1)^(1/2)*x)^(1/2),x)
[Out] int((1-x^2+(x^2-1)^(1/2)*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2 + \sqrt{x^2-1}\, x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x\, \sqrt{x^2-1} - x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2),x)
[Out] int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2 + x\sqrt{x^2-1} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**2+x*(x**2-1)**(1/2))**(1/2),x)
[Out] Integral(sqrt(-x**2 + x*sqrt(x**2 - 1) + 1), x)
```

3.996 $\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=66

$$\frac{1}{2}\left(\sqrt{x}+3\sqrt{x+1}\right)\sqrt{\sqrt{x}\sqrt{x+1}-x}-\frac{3\sin^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)}{2\sqrt{2}}$$

```
[Out] -3/4*arcsin(x^(1/2)-(1+x)^(1/2))*2^(1/2)+1/2*(x^(1/2)+3*(1+x)^(1/2))*(-x+x^
(1/2)*(1+x)^(1/2))^(1/2)
```

Rubi [F] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] 2*Defer[Subst][Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x], x, Sqrt[1 +
 x]]
```

Rubi steps

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = 2\,\text{Subst}\left(\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx, x, \sqrt{1+x}\right)$$

Mathematica [B] time = 0.53, size = 180, normalized size = 2.73

$$-\frac{(x+1)\left(2x-2\sqrt{x+1}\sqrt{x}+1\right)^2\left(2\sqrt{\sqrt{x}\sqrt{x+1}-x}\left(-2x+2\sqrt{x+1}\sqrt{x}-3\right)+3\sqrt{-4x+4\sqrt{x+1}\sqrt{x}-2}\log\right.}{4\left(\sqrt{x+1}-\sqrt{x}\right)^3\left(x-\sqrt{x+1}\sqrt{x}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] -1/4*((1 + x)*(1 + 2*x - 2*Sqrt[x]*Sqrt[1 + x])^2*(2*Sqrt[-x + Sqrt[x]*Sqrt
[1 + x]]*(-3 - 2*x + 2*Sqrt[x]*Sqrt[1 + x]) + 3*Sqrt[-2 - 4*x + 4*Sqrt[x]*S
qrt[1 + x]]*Log[2*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]] + Sqrt[-2 - 4*x + 4*Sqrt[x
]*Sqrt[1 + x]]]))/((-Sqrt[x] + Sqrt[1 + x])^3*(1 + x - Sqrt[x]*Sqrt[1 + x])
^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+1}\,\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x+\sqrt{x+1}\,\sqrt{x}}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+(x+1)^(1/2)*x^(1/2))^(1/2)/(x+1)^(1/2),x)
[Out] int((-x+(x+1)^(1/2)*x^(1/2))^(1/2)/(x+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+1}\,\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{x}\,\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(x + 1)^(1/2) - x)^(1/2)/(x + 1)^(1/2),x)
[Out] int((x^(1/2)*(x + 1)^(1/2) - x)^(1/2)/(x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}\,\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x**(1/2)*(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
[Out] Integral(sqrt(sqrt(x)*sqrt(x + 1) - x)/sqrt(x + 1), x)
```

3.997 $\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=78

$$\sqrt{2\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\sqrt{2+\sqrt{5}}\left(\sqrt{x^2+1}+x\right)\right)-\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\sqrt{5}-2}\left(\sqrt{x^2+1}+x\right)\right)$$

```
[Out] arctanh((x+(x^2+1)^(1/2))*(2+5^(1/2))^(1/2))*(-2+2*5^(1/2))^(1/2)-arctan((x
+(x^2+1)^(1/2))*(-2+5^(1/2))^(1/2))*(2+2*5^(1/2))^(1/2)
```

Rubi [B] time = 0.57, antiderivative size = 319, normalized size of antiderivative = 4.09, number of steps used = 25, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {6742, 261, 1130, 203, 207, 1251, 824, 707, 1093, 1247, 699, 1279}

$$-\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)-\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
```

```
[Out] -2*Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[(1 + Sq
rt[5])/10]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[2/(5*(-1 + Sqrt[5]))]*Arc
Tan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcT
an[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - 2*Sqrt[2/(5*(-1 + Sqrt[5]))]*Arc
Tanh[Sqrt[2/(-1 + Sqrt[5])]*x] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(-1
 + Sqrt[5])]*x] - Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*S
qrt[1 + x^2]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqr
t[1 + x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 707

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = -\int\left(\frac{x}{x+x^3+\sqrt{1+x^2}}+\frac{2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\right)dx$$

$$= -\left(2\int\frac{\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx\right)-\int\frac{x}{x+x^3+\sqrt{1+x^2}}\,dx$$

$$= -\left(2\int\left(1+\frac{x\sqrt{1+x^2}}{-1+x^2+x^4}-\frac{x^2\left(1+x^2\right)}{-1+x^2+x^4}\right)dx\right)-\int\left(\frac{x}{\sqrt{1+x^2}}+\frac{x^2}{-1+x^2+x^4}-\frac{x^3}{-1}\right.$$

$$= -2x-2\int\frac{x\sqrt{1+x^2}}{-1+x^2+x^4}\,dx+2\int\frac{x^2\left(1+x^2\right)}{-1+x^2+x^4}\,dx-\int\frac{x}{\sqrt{1+x^2}}\,dx-\int\frac{x^2}{-1+x^2+}$$

$$= -\sqrt{1+x^2}+\frac{1}{2}\,\text{Subst}\left(\int\frac{x\sqrt{1+x}}{-1+x+x^2}\,dx,x,x^2\right)+2\int\frac{1}{-1+x^2+x^4}\,dx+\frac{1}{10}\left(-5+\right.$$

$$= -\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)+\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)+$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-2\sqrt{\frac{}{5}}$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\frac{2}{5}}\left(\right.$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\frac{}{5(}}$$

Mathematica [F] time = 0.43, size = 34, normalized size = 0.44

$$-\int\frac{2\sqrt{x^2+1}+x}{x^3+\sqrt{x^2+1}+x}\,dx$$

Antiderivative was successfully verified.

```
[In] Integrate[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
[Out] -Integrate[(x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2]), x]
```

fricas [B] time = 1.43, size = 383, normalized size = 4.91

$$\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{4}\sqrt{2}\sqrt{4x^4+4x^2+\sqrt{5}\left(2x^2+1\right)-2\left(2x^3+\sqrt{5}x+x\right)\sqrt{x^2+1}+1}\left(\sqrt{2}x+\sqrt{2}\sqrt{x^2+1}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/4*sqrt(2)*sqrt(4*x^4 + 4*x^2 + sqrt(5)*(
2*x^2 + 1) - 2*(2*x^3 + sqrt(5)*x + x)*sqrt(x^2 + 1) + 1)*(sqrt(2)*x + sqrt
(2)*sqrt(x^2 + 1))*sqrt(sqrt(5) + 1) - 1/2*sqrt(2)*sqrt(x^2 + 1)*sqrt(sqrt(
5) + 1)) + sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/8*sqrt(4*x^2 + 2*sqrt(5) + 2)
*(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 1) - 1/4*(sqrt(5)*sqrt(2)*x - s
qrt(2)*x)*sqrt(sqrt(5) + 1)) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(4*x^2 - 4*
sqrt(x^2 + 1)*x + (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*sqrt(2) + sqr
t(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*l
```

```
og(4*x^2 - 4*sqrt(x^2 + 1)*x - (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*
sqrt(2) + sqrt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) - 1/4*sqrt(2)*sqrt(s
qrt(5) - 1)*log(2*x + sqrt(2)*sqrt(sqrt(5) - 1)) + 1/4*sqrt(2)*sqrt(sqrt(5)
 - 1)*log(2*x - sqrt(2)*sqrt(sqrt(5) - 1))
```

giac [B] time = 0.86, size = 218, normalized size = 2.79

$$-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(-\frac{x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}}{\sqrt{2\sqrt{5}-2}}\right)-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(5) + 2)*arctan(-(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1))
)/sqrt(2*sqrt(5) - 2)) - 1/2*sqrt(2*sqrt(5) + 2)*arctan(x/sqrt(1/2*sqrt(5)
+ 1/2)) + 1/4*sqrt(2*sqrt(5) - 2)*log(-x + sqrt(x^2 + 1) + sqrt(2*sqrt(5) +
 2) - 1/(x - sqrt(x^2 + 1))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x + sqrt(1/2
*sqrt(5) - 1/2))) + 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x - sqrt(1/2*sqrt(5) -
1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(-x + sqrt(x^2 + 1) - sqrt(2*sqrt(5
) + 2) - 1/(x - sqrt(x^2 + 1))))
```

maple [B] time = 0.20, size = 438, normalized size = 5.62

$$-\frac{x}{2}+\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{-2+\sqrt{5}}}\right)}{10\sqrt{-2+\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{-2+\sqrt{5}}}\right)}{2\sqrt{-2+\sqrt{5}}}+\frac{2\sqrt{5}\,\sqrt{-2+\sqrt{5}}\,\operatorname{arctanh}\left(\frac{-x+\sqrt{x^2+1}}{\sqrt{-2+\sqrt{5}}}\right)}{5}+\frac{\operatorname{arctanh}\left(\frac{-x+\sqrt{}}{\sqrt{2-}}\right.}{2\sqrt{2+\sqrt{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x)
```

```
[Out] -5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5^(1/2)
)^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan
h(2*x/(-2+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2
))^(1/2))-1/2*(x^2+1)^(1/2)-1/2*x-1/2/(-2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1
/2)-x)/(-2+5^(1/2))^(1/2))+1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*arctan(((x^2+1)^(
1/2)-x)/(-2+5^(1/2))^(1/2))+1/2/(2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)
/(2+5^(1/2))^(1/2))+1/2*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)
/(2+5^(1/2))^(1/2))+1/2/((x^2+1)^(1/2)-x)+3/10*5^(1/2)/(-2+5^(1/2))^(1/2)*a
rctanh(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))-1/2/(-2+5^(1/2))^(1/2)*arctanh
(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))+3/10*5^(1/2)/(2+5^(1/2))^(1/2)*arcta
n(((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))+1/2/(2+5^(1/2))^(1/2)*arctan(((x^2+1
)^(1/2)-x)/(2+5^(1/2))^(1/2))+2/5*5^(1/2)*(-2+5^(1/2))^(1/2)*arctanh(((x^2+
1)^(1/2)-x)/(-2+5^(1/2))^(1/2))-2/5*(2+5^(1/2))^(1/2)*5^(1/2)*arctan(((x^2+
1)^(1/2)-x)/(2+5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-x-\frac{1}{2}\arctan(x)+\int\frac{2\,x^6+3\,x^4-x^2-1}{2\left(x^6+2\,x^4+2\,x^2+2\left(x^3+x\right)\sqrt{x^2+1}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -x - 1/2*arctan(x) + integrate(1/2*(2*x^6 + 3*x^4 - x^2 - 1)/(x^6 + 2*x^4 +
 2*x^2 + 2*(x^3 + x)*sqrt(x^2 + 1) + 1), x)
```

mupad [B] time = 4.38, size = 649, normalized size = 8.32

$$\frac{\ln\left(x+\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}-\frac{\ln\left(x-\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}+\frac{\ln\left(x-\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}-\frac{\ln\left(x+\frac{\sqrt{2}\sqrt{-\sqrt{5}\cdots}}{2}\right.}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 2*(x^2 + 1)^(1/2))/(x + (x^2 + 1)^(1/2) + x^3),x)
```

```
[Out] (log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2
- 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2)) - (log(x - (2^(1/2)*(5^(1/2) - 1)
^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1
/2)^(3/2)) + (log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))
/(2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) - (log(x + (2^
(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))/(2*(- 5^(1/2)/2 - 1/2)^(
1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) - ((log(x - (2^(1/2)*(5^(1/2) - 1)^(1/2
))/2) - log((2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*(x^2 + 1)^(1/2)*(5
^(1/2) + 1)^(1/2))/2 + 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(
3/2)))/((2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2
+ 1/2)^(1/2)) - ((log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2) - log((2^(1/2)*(
x^2 + 1)^(1/2)*(5^(1/2) + 1)^(1/2))/2 - (2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2 +
 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(3/2)))/((2*(5^(1/2)/2
- 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 1/2)^(1/2)) + ((log(
(2^(1/2)*(x^2 + 1)^(1/2)*(1 - 5^(1/2))^(1/2))/2 - (2^(1/2)*x*(- 5^(1/2) - 1
)^(1/2))/2 + 1) - log(x + (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2))*((- 5^(1/2)/2
 - 1/2)^(1/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((2*(- 5^(1/2)/2 - 1/2)^(1/2)
 + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2)) + ((log((2^(1/2)*x
*(- 5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*(x^2 + 1)^(1/2)*(1 - 5^(1/2))^(1/2))/2
 + 1) - log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2))*((- 5^(1/2)/2 - 1/2)^(1
/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^
(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x**2+1)**(1/2))/(x+x**3+(x**2+1)**(1/2)),x)
```

```
[Out] Timed out
```

3.998 $\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx$

Optimal. Leaf size=126

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{x^2+2x+2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{x^2+2x+2}}\right)$$

```
[Out] -1/2*arctanh((x*(5-5^(1/2))+2*5^(1/2))/(x^2+2*x+2)^(1/2)/(-10+10*5^(1/2))^(
1/2))*(-2+2*5^(1/2))^(1/2)-1/2*arctan((2*5^(1/2)-x*(5+5^(1/2)))/(x^2+2*x+2)
^(1/2)/(10+10*5^(1/2))^(1/2))*(2+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1036, 1030, 207, 203}

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{x^2+2x+2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{x^2+2x+2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[5])/2]*ArcTan[(2*Sqrt[5] - (5 + Sqrt[5])*x)/(Sqrt[10*(1 +
Sqrt[5])]*Sqrt[2 + 2*x + x^2])]) - Sqrt[(-1 + Sqrt[5])/2]*ArcTanh[(2*Sqrt[5
] + (5 - Sqrt[5])*x)/(Sqrt[10*(-1 + Sqrt[5])]*Sqrt[2 + 2*x + x^2])]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rubi steps

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = -\frac{\int \frac{-5-\sqrt{5}-2\sqrt{5}x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx}{2\sqrt{5}} + \frac{\int \frac{-5+\sqrt{5}+2\sqrt{5}x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx}{2\sqrt{5}}$$

$$= \left(2\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{20\left(1-\sqrt{5}\right)+2x^2}\,dx, x, \frac{2\sqrt{5}+\left(5-\sqrt{5}\right)x}{\sqrt{2+2x+x^2}}\right) + \left(2\left(5+\sqrt{\right.\right.$$

$$= -\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{2+2x+x^2}}\right) - \sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\right.$$

Mathematica [C] time = 0.04, size = 87, normalized size = 0.69

$$\frac{1}{2}i\left(\sqrt{1+2i}\tanh^{-1}\left(\frac{(1+i)x+(2+i)}{\sqrt{1+2i}\sqrt{x^2+2x+2}}\right)-\sqrt{1-2i}\tanh^{-1}\left(\frac{(2-2i)x+(4-2i)}{2\sqrt{1-2i}\sqrt{x^2+2x+2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

```
[Out] (I/2)*(Sqrt[1 + 2*I]*ArcTanh[((2 + I) + (1 + I)*x)/(Sqrt[1 + 2*I]*Sqrt[2 +
2*x + x^2])] - Sqrt[1 - 2*I]*ArcTanh[((4 - 2*I) + (2 - 2*I)*x)/(2*Sqrt[1 -
2*I]*Sqrt[2 + 2*x + x^2])])
```

fricas [B] time = 0.84, size = 770, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*5^(3/4)*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(1/200*sqrt(20*x^2 - 20*sqrt(x^
2 + 2*x + 2)*x - (2*5^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*
sqrt(2)*(2*x + 1) - 5*sqrt(2)))*sqrt(sqrt(5) + 5) + 20*x + 10*sqrt(5) + 30)
*(sqrt(10)*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2))*sqrt(s
qrt(5) + 5) + 10*sqrt(10)*(sqrt(5) + 3)) + 1/10*sqrt(5)*(sqrt(5)*(2*x + 1)
+ 5) + 1/2*sqrt(5)*x + 1/20*(5^(3/4)*(sqrt(5)*sqrt(2)*x - sqrt(2)*(x - 2))
- sqrt(x^2 + 2*x + 2)*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt
(2)) + 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) + 5*sqrt(2)))*sqrt(sqrt(5) + 5) -
 1/2*sqrt(x^2 + 2*x + 2)*(sqrt(5) + 3) + 1/2*x + 1) + 1/5*5^(3/4)*sqrt(2)*s
qrt(sqrt(5) + 5)*arctan(1/200*sqrt(20*x^2 - 20*sqrt(x^2 + 2*x + 2)*x + (2*5
^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) - 5
*sqrt(2)))*sqrt(sqrt(5) + 5) + 20*x + 10*sqrt(5) + 30)*(sqrt(10)*(5^(3/4)*(
sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2))*sqrt(sqrt(5) + 5) - 10*sqrt
(10)*(sqrt(5) + 3)) - 1/10*sqrt(5)*(sqrt(5)*(2*x + 1) + 5) - 1/2*sqrt(5)*x
+ 1/20*(5^(3/4)*(sqrt(5)*sqrt(2)*x - sqrt(2)*(x - 2)) - sqrt(x^2 + 2*x + 2)
*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2)) + 5^(1/4)*(sqrt(
5)*sqrt(2)*(2*x + 1) + 5*sqrt(2)))*sqrt(sqrt(5) + 5) + 1/2*sqrt(x^2 + 2*x +
 2)*(sqrt(5) + 3) - 1/2*x - 1) + 1/40*5^(1/4)*(sqrt(5)*sqrt(2) - 5*sqrt(2))
*sqrt(sqrt(5) + 5)*log(2*x^2 - 2*sqrt(x^2 + 2*x + 2)*x + 1/10*(2*5^(3/4)*sq
rt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) - 5*sqrt(2))
)*sqrt(sqrt(5) + 5) + 2*x + sqrt(5) + 3) - 1/40*5^(1/4)*(sqrt(5)*sqrt(2) -
5*sqrt(2))*sqrt(sqrt(5) + 5)*log(2*x^2 - 2*sqrt(x^2 + 2*x + 2)*x - 1/10*(2*
5^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) -
5*sqrt(2)))*sqrt(sqrt(5) + 5) + 2*x + sqrt(5) + 3)
```

giac [B] time = 0.72, size = 444, normalized size = 3.52

$$\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(256\left(\sqrt{5}\left(x-\sqrt{x^2+2x+2}\right)-2x+\sqrt{5}\sqrt{\sqrt{5}-2}+\sqrt{5}+2\sqrt{x^2+2x+2}-2\sqrt{\sqrt{5}-2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(5) - 2)*log(256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x +
sqrt(5)*sqrt(sqrt(5) - 2) + sqrt(5) + 2*sqrt(x^2 + 2*x + 2) - 2*sqrt(sqrt(5
) - 2) - 2)^2 + 256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5) + 2*
sqrt(x^2 + 2*x + 2) + sqrt(sqrt(5) - 2) + 2)^2) - 1/4*sqrt(2*sqrt(5) - 2)*l
og(256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5)*sqrt(sqrt(5) - 2)
 + sqrt(5) + 2*sqrt(x^2 + 2*x + 2) + 2*sqrt(sqrt(5) - 2) - 2)^2 + 256*(sqrt
(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5) + 2*sqrt(x^2 + 2*x + 2) - sqr
t(sqrt(5) - 2) + 2)^2) + 1/4*(pi + 4*arctan(1/2*(x - sqrt(x^2 + 2*x + 2))*(
2*sqrt(5)*sqrt(sqrt(5) - 2) + sqrt(5) + 4*sqrt(sqrt(5) - 2) + 3) + 3/2*sqrt
(5)*sqrt(sqrt(5) - 2) + 1/2*sqrt(5) + 7/2*sqrt(sqrt(5) - 2) + 3/2))*sqrt(2*
sqrt(5) - 2)/(sqrt(5) - 1) - 1/4*(pi + 4*arctan(-1/2*(x - sqrt(x^2 + 2*x +
2))*(2*sqrt(5)*sqrt(sqrt(5) - 2) - sqrt(5) + 4*sqrt(sqrt(5) - 2) - 3) - 3/2
*sqrt(5)*sqrt(sqrt(5) - 2) + 1/2*sqrt(5) - 7/2*sqrt(sqrt(5) - 2) + 3/2))*sq
rt(2*sqrt(5) - 2)/(sqrt(5) - 1)
```

maple [B] time = 0.11, size = 753, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(x^2+1)/(x^2+2*x+2)^(1/2),x)
```

```
[Out] -1/2*(10*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(-1/2*5^(1
/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+10+2*5^(1/2))^(1/2)*(5*arctan(1/80*(-22
+10*5^(1/2))^(1/2)*((5-5^(1/2))*(2*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2
-x)^2+5^(1/2)+3))^(1/2)*(11*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/
2-x)^2+25*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+4*5^(1/2)+10)*(-1/2
*5^(1/2)+1/2+x)/(-1/2*5^(1/2)-1/2-x)*(5^(1/2)-5)/((-1/2*5^(1/2)+1/2+x)^4/(-
1/2*5^(1/2)-1/2-x)^4+3*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+1))*(-
10+10*5^(1/2))^(1/2)*(-22+10*5^(1/2))^(1/2)+3*arctan(1/80*(-22+10*5^(1/2))^
(1/2)*((5-5^(1/2))*(2*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+5^(1/2)
+3))^(1/2)*(11*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+25*(-1
/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+4*5^(1/2)+10)*(-1/2*5^(1/2)+1/2+
x)/(-1/2*5^(1/2)-1/2-x)*(5^(1/2)-5)/((-1/2*5^(1/2)+1/2+x)^4/(-1/2*5^(1/2)-1
/2-x)^4+3*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+1))*(-10+10*5^(1/2)
)^(1/2)*5^(1/2)*(-22+10*5^(1/2))^(1/2)+20*arctanh((10*(-1/2*5^(1/2)+1/2+x)^
2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2
-x)^2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/2))^(1/2))*5^(1/2)-60*arctanh((10*(-
1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^
2/(-1/2*5^(1/2)-1/2-x)^2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/2))^(1/2)))/(-2*(
5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-5*(-1/2*5^(1/2)+1/2+x
)^2/(-1/2*5^(1/2)-1/2-x)^2-5^(1/2)-5)/((-1/2*5^(1/2)+1/2+x)/(-1/2*5^(1/2)-1
/2-x)+1)^2)^(1/2)/((-1/2*5^(1/2)+1/2+x)/(-1/2*5^(1/2)-1/2-x)+1)/(5^(1/2)-5)
/(-10+10*5^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{x^2+2x+2}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x+1}{\left(x^2+1\right)\,\sqrt{x^2+2\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)),x)
[Out] int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\left(x^2+1\right)\sqrt{x^2+2x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+1)/(x**2+2*x+2)**(1/2),x)
[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + 2*x + 2)), x)
```

3.999 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

```
[Out] arctan(x/(-x^2+(x^4+1)^(1/2))^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2128, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^4\right)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\end{aligned}$$

Mathematica [A] time = 1.06, size = 24, normalized size = 1.09

$$\cot^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcCot[Sqrt[-x^2 + Sqrt[1 + x^4]]/x]
```

fricas [B] time = 1.79, size = 62, normalized size = 2.82

$$-\frac{1}{4}\arctan\left(\frac{4\left(10\,x^{7}-6\,x^{3}+\left(7\,x^{5}-x\right)\sqrt{x^{4}+1}\right)\sqrt{-x^{2}+\sqrt{x^{4}+1}}}{17\,x^{8}-46\,x^{4}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] -1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqrt
(x^4 + 1))/(17*x^8 - 46*x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{4}+1\right)\sqrt{-x^{2}+\sqrt{x^{4}+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{4}+1\right)\sqrt{-x^{2}+\sqrt{x^{4}+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{4}+1\right)\sqrt{-x^{2}+\sqrt{x^{4}+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{\sqrt{x^{4}+1}-x^{2}}\,\left(x^{4}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)),x)
[Out] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

3.1000 $\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

[Out] arctanh(x*c^(1/2)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2))/a/c^(1/2)

Rubi [A] time = 0.14, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2128, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]

[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.49, size = 50, normalized size = 1.25

$$\frac{\sqrt{-\frac{1}{c}}\cot^{-1}\left(\frac{\sqrt{-\frac{1}{c}}\sqrt{d\sqrt{a+bx^4}+cx^2}}{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] (Sqrt[-c^(-1)]*ArcCot[(Sqrt[-c^(-1)]*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]])/x])/a
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4+a)\sqrt{cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^4+a)\sqrt{c\,x^2+\sqrt{b\,x^4+a}\;d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

```
[Out] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4+a)\sqrt{cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\,x^4+a)\,\sqrt{d\,\sqrt{b\,x^4+a}+c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)*sqrt(c*x**2 + d*sqrt(a + b*x**4))), x)
```

3.1001 $\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] arctan(x*c^(1/2)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2))/a/c^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2128, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(a+bx^4\right)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.56, size = 47, normalized size = 1.15

$$\frac{\sqrt{\frac{1}{c}}\cot^{-1}\left(\frac{\sqrt{\frac{1}{c}}\sqrt{d\sqrt{a+bx^4}-cx^2}}{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] (Sqrt[c^(-1)]*ArcCot[(Sqrt[c^(-1)]*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]])/x])/
a
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4+a)\sqrt{-cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^4+a)\sqrt{-c\,x^2+\sqrt{b\,x^4+a}\;d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(-c*x^2+(b*x^4+a)^(1/2)*d)^(1/2),x)
```

```
[Out] int(1/(b*x^4+a)/(-c*x^2+(b*x^4+a)^(1/2)*d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4+a)\sqrt{-cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\,x^4+a)\,\sqrt{d\,\sqrt{b\,x^4+a}-c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(-c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
[Out] Integral(1/((a + b*x**4)*sqrt(-c*x**2 + d*sqrt(a + b*x**4))), x)
```

3.1002 $\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$

Optimal. Leaf size=184

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}\,d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[Out] 1/2*arctanh(d^2*(c/d+x)^2*b^(1/2)/(a+b*d^4*(c/d+x)^4)^(1/2))/d^2/b^(1/2)-1/
2*c*(cos(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(
d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+d^2*(c/d+x)^2*b^(1/2))*((a+b*d^4*(c/d+x)^4)/(a^(1/2)+d^2*(c/d+
x)^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/d^2/(a+b*d^4*(c/d+x)^4)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1680, 1885, 220, 275, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}\,d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^
4*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*d^2*(c/d + x)^2)/Sqrt[a + b*d^4*(c/d + x)^4]]/(2*Sqrt[b]*d
^2) - (c*(Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)/(
Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x)
)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*d^2*Sqrt[a + b*d^4*(c/d + x)^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \text{Subst}\left(\int \frac{-\frac{c}{d}+x}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)$$
$$= \text{Subst}\left(\int \left(-\frac{c}{d\sqrt{a+bd^4x^4}}+\frac{x}{\sqrt{a+bd^4x^4}}\right)dx, x, \frac{c}{d}+x\right)$$
$$= -\frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)}{d} + \text{Subst}\left(\int \frac{x}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)$$
$$= -\frac{c\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)^2}}\,F\left(2\tan\ldots\right.}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+b(c+dx)^4}}$$
$$= -\frac{c\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)^2}}\,F\left(2\tan\ldots\right.}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+b(c+dx)^4}}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{b}\,d^2} - \frac{c\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\ldots}}{2\sqrt[4]{a}\,\sqrt[4]{b}\ldots}$$

Mathematica [C] time = 0.58, size = 330, normalized size = 1.79

$$\frac{\sqrt[4]{-1}\,\sqrt{2}\sqrt{-\frac{i\left(\sqrt[4]{-1}\,\sqrt[4]{a}+\sqrt[4]{b}\,(c+dx)\right)}{\sqrt[4]{-1}\,\sqrt[4]{a}-\sqrt[4]{b}\,(c+dx)}}\left(\sqrt{b}\,(c+dx)^2+i\sqrt{a}\right)\left(\left(\sqrt[4]{-1}\,\sqrt[4]{a}-\sqrt[4]{b}\,c\right)F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(\sqrt[4]{b}\,(c+dx)+\sqrt[4]{-1}\,\sqrt[4]{a}\right)}{\sqrt[4]{-1}\,\sqrt[4]{a}-\sqrt[4]{b}\,(c+dx)}}\right)\right|-\ldots}{\sqrt[4]{a}\,\sqrt{b}\,d^2\sqrt{\frac{\sqrt{b}\,(c+dx)^2+i\sqrt{a}}{\left(\sqrt[4]{-1}\,\sqrt[4]{a}-\sqrt[4]{b}\,(c+dx)\right)^2}}\,\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3
+ b*d^4*x^4],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[2]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((
-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*(I*Sqrt[a] + Sqrt[b]*(c + d*x)^2)*(
((-1)^(1/4)*a^(1/4) - b^(1/4)*c)*EllipticF[ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^
```

```
(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]
 - 2*(-1)^(1/4)*a^(1/4)*EllipticPi[-I, ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4
) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]))/(
a^(1/4)*Sqrt[b]*d^2*Sqrt[(I*Sqrt[a] + Sqrt[b]*(c + d*x)^2)/((-1)^(1/4)*a^(1
/4) - b^(1/4)*(c + d*x))^2]*Sqrt[a + b*(c + d*x)^4])
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x +
 b*c^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

maple [C] time = 0.42, size = 1528, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),x
)
```

```
[Out] 2*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(1/
4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b
^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2
)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(
1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-
a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^3)^(1/4
)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b
^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2
)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)
-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(b*d^4*(x-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(I/b
*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)
-c)/d))^(1/2)*((I/b*(-a*b^3)^(1/4)-c)/d*EllipticF((((-I/b*(-a*b^3)^(1/4)-c)
/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(
1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2),(((
I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*((1/b*(-a*b^3)^(1/4)-c)/
d-(-I/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)
-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))+((1/b*(
-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*EllipticPi((((-I/b*(-a*b^3)^(1
/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/
2),((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(1/
```

```
4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d),(((I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)
^(1/4)-c)/d)*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a
*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b
*(-a*b^3)^(1/4)-c)/d))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{b\,c^4 + 4\,b\,c^3\,d\,x + 6\,b\,c^2\,d^2\,x^2 + 4\,b\,c\,d^3\,x^3 + b\,d^4\,x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2),x)
```

```
[Out] int(x/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bc^4 + 4bc^3dx + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b*
c**4+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3
*x**3 + b*d**4*x**4), x)
```

3.1003 $\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$

Optimal. Leaf size=131

$$\frac{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[Out] 1/2*(cos(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(
d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+d^2*(c/d+x)^2*b^(1/2))*((a+b*d^4*(c/d+x)^4)/(a^(1/2)+d^2*(c/d+
x)^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/d/(a+b*d^4*(c/d+x)^4)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.041$, Rules used = {1106, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^
4*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)/(Sqrt[a]
+ Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4
)], 1/2])/(2*a^(1/4)*b^(1/4)*d*Sqrt[a + b*d^4*(c/d + x)^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\,F\left(2\tan^{-1}\right.}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

Mathematica [C] time = 0.06, size = 90, normalized size = 0.69

$$-\frac{i\sqrt{\frac{a+b(c+dx)^4}{a}}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}(c+dx)\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3
+ b*d^4*x^4],x]
```

```
[Out] ((-I)*Sqrt[(a + b*(c + d*x)^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt
[a]]*(c + d*x)], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a + b*(c + d*x)^4])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x +
 b*c^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

maple [C] time = 0.06, size = 1036, normalized size = 7.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),x
)
```

```
[Out] 2*((-c+(-a*b^3)^(1/4)/b)/d-(-c-I*(-a*b^3)^(1/4)/b)/d)*(((-c-I*(-a*b^3)^(1/4
)/b)/d-(-c+I*(-a*b^3)^(1/4)/b)/d)*(x-(-c+(-a*b^3)^(1/4)/b)/d)/((-c-I*(-a*b^
3)^(1/4)/b)/d-(-c+(-a*b^3)^(1/4)/b)/d)/(x-(-c+I*(-a*b^3)^(1/4)/b)/d))^(1/2)
*(x-(-c+I*(-a*b^3)^(1/4)/b)/d)^2*(((-c+I*(-a*b^3)^(1/4)/b)/d-(-c+(-a*b^3)^(
1/4)/b)/d)*(x-(-c-(-a*b^3)^(1/4)/b)/d)/((-c-(-a*b^3)^(1/4)/b)/d-(-c+(-a*b^3
)^(1/4)/b)/d)/(x-(-c+I*(-a*b^3)^(1/4)/b)/d))^(1/2)*(((-c+I*(-a*b^3)^(1/4)/b
)/d-(-c+(-a*b^3)^(1/4)/b)/d)*(x-(-c-I*(-a*b^3)^(1/4)/b)/d)/((-c-I*(-a*b^3)^
(1/4)/b)/d-(-c+(-a*b^3)^(1/4)/b)/d)/(x-(-c+I*(-a*b^3)^(1/4)/b)/d))^(1/2)/((
-c-I*(-a*b^3)^(1/4)/b)/d-(-c+I*(-a*b^3)^(1/4)/b)/d)/((-c+I*(-a*b^3)^(1/4)/b
)/d-(-c+(-a*b^3)^(1/4)/b)/d)/((x-(-c+(-a*b^3)^(1/4)/b)/d)*(x-(-c+I*(-a*b^3)
^(1/4)/b)/d)*(x-(-c-(-a*b^3)^(1/4)/b)/d)*(x-(-c-I*(-a*b^3)^(1/4)/b)/d)*b*d^
4)^(1/2)*EllipticF((((-c-I*(-a*b^3)^(1/4)/b)/d-(-c+I*(-a*b^3)^(1/4)/b)/d)*(
```

```
x-(-c+(-a*b^3)^(1/4)/b)/d)/((-c-I*(-a*b^3)^(1/4)/b)/d-(-c+(-a*b^3)^(1/4)/b)
/d)/(x-(-c+I*(-a*b^3)^(1/4)/b)/d))^(1/2),(((-c+I*(-a*b^3)^(1/4)/b)/d-(-c-(-
a*b^3)^(1/4)/b)/d)*((-c+(-a*b^3)^(1/4)/b)/d-(-c-I*(-a*b^3)^(1/4)/b)/d)/((-c
+(-a*b^3)^(1/4)/b)/d-(-c-(-a*b^3)^(1/4)/b)/d)/((-c+I*(-a*b^3)^(1/4)/b)/d-(-
c-I*(-a*b^3)^(1/4)/b)/d))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,c^4 + 4\,b\,c^3\,d\,x + 6\,b\,c^2\,d^2\,x^2 + 4\,b\,c\,d^3\,x^3 + b\,d^4\,x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2),x)
```

```
[Out] int(1/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bc^4 + 4bc^3dx + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b*
c**4+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3
*x**3 + b*d**4*x**4), x)
```

3.1004 $\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\left(ad+aex^2+cdx^4\right)}\,dx$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

```
[Out] arctanh(x*(-a*e+b*d)^(1/2)/d^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^(1/2)/(-a*e+b*d
)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2112, 208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[d]*Sqr
t[b*d - a*e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\left(ad+aex^2+cdx^4\right)}\,dx &= a\,\text{Subst}\left(\int \frac{1}{ad-\left(abd-a^2e\right)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{bd-ae}\,x}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}\end{aligned}$$

Mathematica [C] time = 1.96, size = 419, normalized size = 7.76

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{ae-\sqrt{a}\sqrt{ae^2-4cd^2}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(b+\sqrt{b^2-\right.}{ae+\sqrt{a}\sqrt{a}\right.}{\sqrt{2}d\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x
]
```

```
[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(a*e - Sqrt[a]*Sqrt[-4*c*d^2 + a*e
^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/
(a*e + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt
[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[a + b*x^2 + c*x^4])
```

fricas [A] time = 23.15, size = 305, normalized size = 5.65

$$\left[\frac{\log\left(-\frac{c^2d^2x^8+2\left(4\,bcd^2-3\,acde\right)x^6-\left(8\,abde-a^2e^2-2\left(4\,b^2+ac\right)d^2\right)x^4+a^2d^2+2\left(4\,abd^2-3\,a^2de\right)x^2+4\left(cdx^5+(2\,bd-ae)x^3+adx\right)\sqrt{cx^4+bx^2+a}\,\sqrt{bd^2-ade}}{c^2d^2x^8+2\,acdex^6+2\,a^2dex^2+\left(2\,acd^2+a^2e^2\right)x^4+a^2d^2}\right)}{4\sqrt{bd^2-ade}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*log(-(c^2*d^2*x^8 + 2*(4*b*c*d^2 - 3*a*c*d*e)*x^6 - (8*a*b*d*e - a^2*e
^2 - 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 + 2*(4*a*b*d^2 - 3*a^2*d*e)*x^2 + 4
*(c*d*x^5 + (2*b*d - a*e)*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(b*d^2 -
 a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*e*x^2 + (2*a*c*d^2 + a^2*e^
2)*x^4 + a^2*d^2))/sqrt(b*d^2 - a*d*e), -1/2*sqrt(-b*d^2 + a*d*e)*arctan(2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-b*d^2 + a*d*e)*x/(c*d*x^4 + (2*b*d - a*e)*x^2
 + a*d))/(b*d^2 - a*d*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{cx^4 - a}{\left(cdx^4 + aex^2 + ad\right)\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)),
 x)
```

maple [C] time = 0.09, size = 514, normalized size = 9.52

$$-\frac{a\left(-\operatorname{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2e-2d\right)\left(-\frac{\operatorname{arctanh}\left(\frac{2\operatorname{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2cx^2+b\operatorname{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2+bx^2+2a}{2\sqrt{\frac{(-ae+bd)\operatorname{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2}{d}}\sqrt{cx^4+bx^2+a}}\right)}{\sqrt{\frac{(-ae+bd)\operatorname{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2}{d}}}+\frac{\sqrt{2}\,(\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*a/d*sum((-_alpha^2*e-2*d)/_alpha/(2*_
alpha^2*c*d+a*e)*(-1/(_alpha^2/d*(-a*e+b*d))^(1/2)*arctanh(1/2*(2*_alpha^2*
c*x^2+_alpha^2*b+b*x^2+2*a)/(_alpha^2/d*(-a*e+b*d))^(1/2)/(c*x^4+b*x^2+a)^(
1/2))+1/a/d*2^(1/2)*_alpha*(_alpha^2*c*d+a*e)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(2+b*x^2/a-1/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(2+b*x^2/a+1/a*x^2*(-4*a*
c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(_alpha^2*(-4*a*c+b^2)^(1/2)*c*d+_alpha^2*b*c
*d+(-4*a*c+b^2)^(1/2)*a*e+a*b*e)/a/d/c,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))),_alpha=RootOf(_Z^4*c*d+_Z^2*a
*e+a*d))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^4 - a}{\left(cdx^4 + aex^2 + ad\right)\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)),
 x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a - c\,x^4}{\left(c\,d\,x^4 + a\,e\,x^2 + a\,d\right)\,\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{a}{ad\sqrt{a + bx^2 + cx^4} + aex^2\sqrt{a + bx^2 + cx^4} + cdx^4\sqrt{a + bx^2 + cx^4}}\right) dx - \int \frac{}{ad\sqrt{a + bx^2 + cx^4} + aex^2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*
x**4) + c*d*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt
(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*x**4*sqrt(
a + b*x**2 + c*x**4)), x)
```

3.1005 $\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\left(ad+aex^2+cdx^4\right)}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

```
[Out] arctan(x*(a*e+b*d)^(1/2)/d^(1/2)/(c*x^4-b*x^2+a)^(1/2))/d^(1/2)/(a*e+b*d)^(
1/2)
```

Rubi [A] time = 0.26, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2112, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[b*d + a*e]*x)/(Sqrt[d]*Sqrt[a - b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt
[b*d + a*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\left(ad+aex^2+cdx^4\right)}\,dx &= a\,\text{Subst}\left(\int \frac{1}{ad-\left(-abd-a^2e\right)x^2}\,dx, x, \frac{x}{\sqrt{a-bx^2+cx^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bd+ae}\,x}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd+ae}}\end{aligned}$$

Mathematica [C] time = 1.45, size = 416, normalized size = 7.85

$$\frac{i\sqrt{\frac{4cx^2}{\sqrt{b^2-4ac}-b}+2}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}\left(-\Pi\left(\frac{\left(b-\sqrt{b^2-4ac}\right)d}{\sqrt{a}\sqrt{ae^2-4cd^2}-ae};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}\,x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(\sqrt{b^2-4ac}-b\right)\cdots}{ae+\sqrt{a}\sqrt{ae^2-4c\cdots}}\cdots\right.\right.}{2d\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}\sqrt{a-bx^2+c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x
]
```

```
[Out] ((I/2)*Sqrt[2 + (4*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4
*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])] - EllipticPi[(
(b - Sqrt[b^2 - 4*a*c])*d)/(-(a*e) + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*Arc
Sinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(
b + Sqrt[b^2 - 4*a*c])] - EllipticPi[((-b + Sqrt[b^2 - 4*a*c])*d)/(a*e + Sq
rt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*
a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[c/(-b +
 Sqrt[b^2 - 4*a*c])]*d*Sqrt[a - b*x^2 + c*x^4])
```

fricas [A] time = 19.13, size = 304, normalized size = 5.74

$$\left[-\frac{\sqrt{-bd^2-ade}\,\log\left(-\frac{c^2d^2x^8-2\left(4\,bcd^2+3\,acde\right)x^6+\left(8\,abde+a^2e^2+2\left(4\,b^2+ac\right)d^2\right)x^4+a^2d^2-2\left(4\,abd^2+3\,a^2de\right)x^2+4\left(cdx^5-(2\,bd+ae)x^3+adx\right)}{c^2d^2x^8+2\,acdex^6+2\,a^2dex^2+\left(2\,acd^2+a^2e^2\right)x^4+a^2d^2}\right.}{4\left(bd^2+ade\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/4*sqrt(-b*d^2 - a*d*e)*log(-(c^2*d^2*x^8 - 2*(4*b*c*d^2 + 3*a*c*d*e)*x^
6 + (8*a*b*d*e + a^2*e^2 + 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 - 2*(4*a*b*d^
2 + 3*a^2*d*e)*x^2 + 4*(c*d*x^5 - (2*b*d + a*e)*x^3 + a*d*x)*sqrt(c*x^4 - b
*x^2 + a)*sqrt(-b*d^2 - a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*e*x^
2 + (2*a*c*d^2 + a^2*e^2)*x^4 + a^2*d^2))/(b*d^2 + a*d*e), 1/2*arctan(2*sqr
t(c*x^4 - b*x^2 + a)*sqrt(b*d^2 + a*d*e)*x/(c*d*x^4 - (2*b*d + a*e)*x^2 + a
*d))/sqrt(b*d^2 + a*d*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{cx^4-a}{\left(cdx^4+aex^2+ad\right)\sqrt{cx^4-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),
 x)
```

maple [C] time = 0.08, size = 517, normalized size = 9.75

$$-\frac{a\left(-\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2e-2d\right)\left(-\frac{\operatorname{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2cx^2-b\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2-bx^2+2a}{2\sqrt{-\frac{(ae+bd)\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2}{d}}\sqrt{cx^4-bx^2+a}}\right)}{\sqrt{-\frac{(ae+bd)\,\mathrm{RootOf}\left(cd\_Z^4+ae\_Z^2+ad\right)^2}{d}}}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4-b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(-b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*a/d*sum((-_alpha^2*e-2*d)/_alpha/(2*_a
lpha^2*c*d+a*e)*(-1/(-_alpha^2*(a*e+b*d)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*c
*x^2-_alpha^2*b-b*x^2+2*a)/(-_alpha^2*(a*e+b*d)/d)^(1/2)/(c*x^4-b*x^2+a)^(1
/2))+1/a/d*2^(1/2)*_alpha*(_alpha^2*c*d+a*e)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(2-1/a*b*x^2-(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)*(2-1/a*b*x^2+(-4*a*c+b^2)^(
1/2)/a*x^2)^(1/2)/(c*x^4-b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),-1/2*(-(-4*a*c+b^2)^(1/2)*_alpha^2*c*d+_alpha^2*b*c*
d-(-4*a*c+b^2)^(1/2)*a*e+a*b*e)/a/d/c,(-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2))),_alpha=RootOf(_Z^4*c*d+_Z^2*a*
e+a*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^4 - a}{\left(cdx^4 + aex^2 + ad\right)\sqrt{cx^4 - bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a - c\,x^4}{\left(c\,d\,x^4 + a\,e\,x^2 + a\,d\right)\,\sqrt{c\,x^4 - b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{a}{ad\sqrt{a - bx^2 + cx^4} + aex^2\sqrt{a - bx^2 + cx^4} + cdx^4\sqrt{a - bx^2 + cx^4}}\right) dx - \int \frac{cx^4}{ad\sqrt{a - bx^2 + cx^4} + aex^2\sqrt{a - bx^2 + cx^4} + cdx^4\sqrt{a - bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4-b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*
x**4) + c*d*x**4*sqrt(a - b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt
(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*x**4) + c*d*x**4*sqrt(
a - b*x**2 + c*x**4)), x)
```

3.1006 $\int \frac{1}{\sqrt{5-2x+x^2}\left(8+x^3\right)}\,dx$

Optimal. Leaf size=84

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2-2x+5}\right)$$

```
[Out] 1/12*arctanh((x^2-2*x+5)^(1/2))-1/12*arctan(1/3*(1-x)*3^(1/2)/(x^2-2*x+5)^(
1/2))*3^(1/2)-1/156*arctanh(1/13*(7-3*x)*13^(1/2)/(x^2-2*x+5)^(1/2))*13^(1/
2)
```

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2074, 724, 206, 1025, 982, 203, 1024, 207}

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2-2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] -ArcTan[(1 - x)/(Sqrt[3]*Sqrt[5 - 2*x + x^2])]/(4*Sqrt[3]) - ArcTanh[(7 - 3
*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 - 2*x +
x^2]]/12
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
```

```
]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{5-2x+x^2}\left(8+x^3\right)}\,dx &= \int \left(\frac{1}{12(2+x)\sqrt{5-2x+x^2}}+\frac{4-x}{12\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\right)dx\\
&= \frac{1}{12}\int \frac{1}{(2+x)\sqrt{5-2x+x^2}}\,dx+\frac{1}{12}\int \frac{4-x}{\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\,dx\\
&= -\left(\frac{1}{24}\int \frac{-2+2x}{\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\,dx\right)-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{14-6x}{\sqrt{5-2x+}}\right.\\
&= -\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx,x,\sqrt{5-2x+x^2}\right)+\text{Subst}\Bigg(\\
&= \frac{\tan^{-1}\left(\frac{-2+2x}{2\sqrt{3}\sqrt{5-2x+x^2}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{5-2x+x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.33, size = 160, normalized size = 1.90

$$\frac{1}{312}\left(-2\sqrt{13}\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)-13\left(\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{-2\sqrt[3]{-1}\,x+4x+5i\sqrt{3}+1}{\sqrt{2-2i\sqrt{3}}\sqrt{x^2-2x+5}}\right)+\left(\sqrt{3}-i\right)\tan^{-1}\left(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] (-13*((I + Sqrt[3])*ArcTan[(1 + (5*I)*Sqrt[3] + 4*x - 2*(-1)^(1/3)*x)/(Sqrt
[2 - (2*I)*Sqrt[3]]*Sqrt[5 - 2*x + x^2])] + (-I + Sqrt[3])*ArcTan[(1 - (5*I
)*Sqrt[3] + 2*(2 + (-1)^(2/3))*x)/(Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[5 - 2*x + x
^2])]) - 2*Sqrt[13]*ArcTanh[(7 - 3*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])])/312
```

fricas [B] time = 0.69, size = 154, normalized size = 1.83

$$\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x-2)+\frac{1}{3}\sqrt{3}\sqrt{x^2-2x+5}\right)-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2-2x+5}\right)+\frac{1}{156}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - 2) + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5))
 - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5)) +
1/156*sqrt(13)*log(-(sqrt(13)*(3*x - 7) + sqrt(x^2 - 2*x + 5)*(3*sqrt(13) +
 13) + 9*x - 21)/(x + 2)) + 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*(x - 2) - 3*
x + 6) - 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*x - x + 4)
```

giac [B] time = 0.59, size = 164, normalized size = 1.95

$$-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2-2x+5}\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2-2x+5}-2\right)\right)+\frac{1}{156}\sqrt{13}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5))) + 1/12*sqrt(3)
*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5) - 2)) + 1/156*sqrt(13)*log(ab
s(-2*x - 2*sqrt(13) + 2*sqrt(x^2 - 2*x + 5) - 4)/abs(-2*x + 2*sqrt(13) + 2*
sqrt(x^2 - 2*x + 5) - 4)) + 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 - 4*x + 4*
sqrt(x^2 - 2*x + 5) + 7) - 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 + 3)
```

maple [A] time = 0.03, size = 69, normalized size = 0.82

$$\frac{\operatorname{arctanh}\left(\sqrt{x^2-2x+5}\right)}{12}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{(-6x+14)\sqrt{13}}{26\sqrt{-6x+(x+2)^2+1}}\right)}{156}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2x-2)}{6\sqrt{x^2-2x+5}}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+8)/(x^2-2*x+5)^(1/2),x)
```

```
[Out] -1/156*13^(1/2)*arctanh(1/26*(14-6*x)*13^(1/2)/((x+2)^2-6*x+1)^(1/2))+1/12*
arctanh((x^2-2*x+5)^(1/2))+1/12*3^(1/2)*arctan(1/6*3^(1/2)/(x^2-2*x+5)^(1/2
)*(2*x-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+8\right)\sqrt{x^2-2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 8)*sqrt(x^2 - 2*x + 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3+8\right)\,\sqrt{x^2-2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)),x)
[Out] int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+2)\left(x^2-2x+4\right)\sqrt{x^2-2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+8)/(x**2-2*x+5)**(1/2),x)
[Out] Integral(1/((x + 2)*(x**2 - 2*x + 4)*sqrt(x**2 - 2*x + 5)), x)
```

3.1007 $\int \sqrt{\frac{x^2}{1+x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{\sqrt{x^2}\,\sqrt{x^2+1}}{x}$$

[Out] (x^2)^(1/2)*(x^2+1)^(1/2)/x

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1958, 15, 261}

$$\frac{\sqrt{x^2}\,\sqrt{x^2+1}}{x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2/(1 + x^2)],x]

[Out] (Sqrt[x^2]*Sqrt[1 + x^2])/x

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{x^2}{1+x^2}}\,dx &= \int \frac{\sqrt{x^2}}{\sqrt{1+x^2}}\,dx\\ &= \frac{\sqrt{x^2}\int \frac{x}{\sqrt{1+x^2}}\,dx}{x}\\ &= \frac{\sqrt{x^2}\,\sqrt{1+x^2}}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.85

$$\frac{x}{\sqrt{\frac{x^2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(1 + x^2)],x]
[Out] x/Sqrt[x^2/(1 + x^2)]
```

fricas [A] time = 0.66, size = 22, normalized size = 1.10

$$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="fricas")
[Out] (x^2 + 1)*sqrt(x^2/(x^2 + 1))/x
```

giac [A] time = 0.44, size = 15, normalized size = 0.75

$$\sqrt{x^2+1}\,\mathrm{sgn}(x)-\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + 1)*sgn(x) - sgn(x)
```

maple [A] time = 0.00, size = 23, normalized size = 1.15

$$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2+1))^(1/2),x)
[Out] (x^2+1)/x*(x^2/(x^2+1))^(1/2)
```

maxima [A] time = 1.08, size = 7, normalized size = 0.35

$$\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + 1)
```

mupad [B] time = 3.41, size = 13, normalized size = 0.65

$$\frac{\sqrt{x^4+x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2 + 1))^(1/2),x)
[Out] (x^2 + x^4)^(1/2)/x
```

sympy [B] time = 0.44, size = 36, normalized size = 1.80

$$x\sqrt{x^2}\sqrt{\frac{1}{x^2+1}}+\frac{\sqrt{x^2}\sqrt{\frac{1}{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(x**2+1))**(1/2),x)
[Out] x*sqrt(x**2)*sqrt(1/(x**2 + 1)) + sqrt(x**2)*sqrt(1/(x**2 + 1))/x
```

3.1008 $\int \sqrt{\frac{x^n}{1+x^n}}\,dx$

Optimal. Leaf size=46

$$\frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

```
[Out] 2*x*hypergeom([1/2, 1/2+1/n],[3/2+1/n],-x^n)*(x^n)^(1/2)/(2+n)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1958, 15, 364}

$$\frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^n/(1 + x^n)],x]
```

```
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -x^n])/(2 +
 n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{x^n}{1+x^n}}\,dx &= \int \frac{\sqrt{x^n}}{\sqrt{1+x^n}}\,dx\\ &= \left(x^{-n/2}\sqrt{x^n}\right)\int \frac{x^{n/2}}{\sqrt{1+x^n}}\,dx\\ &= \frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{2+n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.83

$$\frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^n/(1 + x^n)],x]
```

```
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])/(2
 + n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^n/(x^n + 1)), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^n/(x^n+1))^(1/2),x)
```

```
[Out] int((x^n/(x^n+1))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^n/(x^n + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^n/(x^n + 1))^(1/2),x)
```

```
[Out] int((x^n/(x^n + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**n/(1+x**n))**(1/2),x)
```

```
[Out] Integral(sqrt(x**n/(x**n + 1)), x)
```

3.1009 $\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=88

$$\frac{ef\tan^{-1}\left(\frac{x(4a^2-2ac+b^2)+abx^2+ab}{2a\sqrt{2a-c}\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

```
[Out] e*f*arctan(1/2*(a*b+(4*a^2-2*a*c+b^2)*x+a*b*x^2)/a/(2*a-c)^(1/2)/(a*x^4+b*x
^3+c*x^2+b*x+a)^(1/2))/a/d/(2*a-c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {2084}

$$\frac{ef\tan^{-1}\left(\frac{x(4a^2-2ac+b^2)+abx^2+ab}{2a\sqrt{2a-c}\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 +
 a*x^4]),x]
```

```
[Out] (e*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a - c]*Sq
rt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]*d)
```

Rule 2084

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[(a*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*Rt[a^2*(2*a - c),
 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(d*Rt[a^2*(2*a - c), 2]), x] /
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

Rubi steps

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = \frac{ef\tan^{-1}\left(\frac{ab+(4a^2+b^2-2ac)x+abx^2}{2a\sqrt{2a-c}\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)}{a\sqrt{2a-c}\,d}$$

Mathematica [C] time = 6.52, size = 13884, normalized size = 157.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b
*x^3 + a*x^4]),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 5.82, size = 324, normalized size = 3.68

$$\left[-\frac{\sqrt{-2a+c}\,ef\log\left(\frac{2ab^3x^3+2ab^3x-(8a^4-a^2b^2-4a^3c)x^4-8a^4+a^2b^2+4a^3c+(16a^4+10a^2b^2+b^4+8a^2c^2-4(6a^3+ab^2)c)x^2-4(a^2bx^2+a^2b+(4a^3\right.}{a^2x^4+2abx^3+2abx+(2a^2+b^2)x^2+a^2}}{2(2a^2-ac)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x - (8*a^4 - a^2*b^2 -
4*a^3*c)*x^4 - 8*a^4 + a^2*b^2 + 4*a^3*c + (16*a^4 + 10*a^2*b^2 + b^4 + 8*a
^2*c^2 - 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b + (4*a^3 + a*b^2 -
 2*a^2*c)*x)*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(-2*a + c))/(a^2*x^4
 + 2*a*b*x^3 + 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 - a*c)*d), -sqrt
(2*a - c)*e*f*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(2*a - c)*
a/(a*b*x^2 + a*b + (4*a^2 + b^2 - 2*a*c)*x))/((2*a^2 - a*c)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}\,\left(adx^2 + bdx + ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="giac")
```

```
[Out] integrate(-(e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
+ b*d*x + a*d)), x)
```

maple [C] time = 0.16, size = 242984, normalized size = 2761.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}\,\left(adx^2 + bdx + ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="maxima")
```

```
[Out] -integrate((e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
+ b*d*x + a*d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,f - e\,f\,x^2}{\left(a\,d\,x^2 + b\,d\,x + a\,d\right)\,\sqrt{a\,x^4 + b\,x^3 + c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*x
^2)^(1/2)),x)
```

```
[Out] int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*x
^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+a+bx^3+bx+cx^2}+bx\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx+\int\left(-\frac{1}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+a+bx^3+bx+}}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x**2+e*f)/(a*d*x**2+b*d*x+a*d)/(a*x**4+b*x**3+c*x**2+b*x+a)
**(1/2),x)
```

```
[Out] -e*f*(Integral(x**2/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + a*sq
rt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*x**3 + b*x
 + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x*
*2) + a*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*
x**3 + b*x + c*x**2)), x))/d
```

3.1010 $\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$

Optimal. Leaf size=88

$$\frac{ef\tanh^{-1}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

```
[Out] e*f*arctanh(1/2*(a*b-(4*a^2+2*a*c+b^2)*x+a*b*x^2)/a/(2*a+c)^(1/2)/(-a*x^4+b
*x^3+c*x^2+b*x-a)^(1/2))/a/d/(2*a+c)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = {2085}

$$\frac{ef\tanh^{-1}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*x
^3 - a*x^4]),x]
```

```
[Out] (e*f*ArcTanh[(a*b - (4*a^2 + b^2 + 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a + c]*S
qrt[-a + b*x + c*x^2 + b*x^3 - a*x^4]])])/(a*Sqrt[2*a + c]*d)
```

Rule 2085

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> -Sim
p[(a*f*ArcTanh[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*Rt[-(a^2*(2*a -
 c)), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]])])/(d*Rt[-(a^2*(2*a - c)), 2
]), x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g
, 0] && NegQ[a^2*(2*a - c)]
```

Rubi steps

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = \frac{ef\tanh^{-1}\left(\frac{ab-(4a^2+b^2+2ac)x+abx^2}{2a\sqrt{2a+c}\sqrt{-a+bx+cx^2+bx^3-ax^4}}\right)}{a\sqrt{2a+c}\,d}$$

Mathematica [C] time = 6.55, size = 15147, normalized size = 172.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2
 + b*x^3 - a*x^4]),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 6.03, size = 331, normalized size = 3.76

$$\left[\frac{\sqrt{2a+c}\,ef\log\left(\frac{2ab^3x^3+2ab^3x+(8a^4-a^2b^2+4a^3c)x^4+8a^4-a^2b^2+4a^3c-(16a^4+10a^2b^2+b^4+8a^2c^2+4(6a^3+ab^2)c)x^2-4(a^2bx^2+a^2b-(4a^3}{a^2x^4-2abx^3-2abx+(2a^2+b^2)x^2+a^2}\right.}{2(2a^2+ac)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x + (8*a^4 - a^2*b^2 + 4*
a^3*c)*x^4 + 8*a^4 - a^2*b^2 + 4*a^3*c - (16*a^4 + 10*a^2*b^2 + b^4 + 8*a^2
*c^2 + 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b - (4*a^3 + a*b^2 + 2
*a^2*c)*x)*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*sqrt(2*a + c))/(a^2*x^4 -
 2*a*b*x^3 - 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 + a*c)*d), -sqrt(-
2*a - c)*e*f*arctan(2*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*a*sqrt(-2*a -
c)/(a*b*x^2 + a*b - (4*a^2 + b^2 + 2*a*c)*x))/((2*a^2 + a*c)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}\left(adx^2 - bdx + ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="giac")
```

```
[Out] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
- b*d*x + a*d)), x)
```

maple [C] time = 0.16, size = 269221, normalized size = 3059.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}\left(adx^2 - bdx + ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
- b*d*x + a*d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{e\,f - e\,f\,x^2}{\left(a\,d\,x^2 - b\,d\,x + a\,d\right)\,\sqrt{-a\,x^4 + b\,x^3 + c\,x^2 + b\,x - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c*
x^2)^(1/2)),x)
```

```
[Out] int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c*
x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+cx^2}-bx\sqrt{-ax^4-a+bx^3+bx+cx^2}}\,dx+\int\left(-\frac{1}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x**2+e*f)/(-a*d*x**2+b*d*x-a*d)/(-a*x**4+b*x**3+c*x**2+b*x-
a)**(1/2),x)
```

```
[Out] e*f*(Integral(x**2/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) + a*sq
rt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a + b*x**3 + b
*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c
*x**2) + a*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a
 + b*x**3 + b*x + c*x**2)), x))/d
```

3.1011 $\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsinh((a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2130, 215}

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx &= \frac{\left(\sqrt{2}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}\\ &= \frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [B] time = 1.12, size = 148, normalized size = 3.22

$$\frac{\sqrt{2}\,x\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\tanh^{-1}\left(\frac{\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}{\sqrt{2}\,ax}\right)}{\sqrt{\frac{a(ax^2-1)}{b^2}}\left(x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*x*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-1 + a*x^2 + b*x
*Sqrt[(a*(-1 + a*x^2))/b^2])*ArcTanh[Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2)
)/b^2])]/(Sqrt[2]*a*x)])/(Sqrt[(a*(-1 + a*x^2))/b^2]*(x*(a*x + b*Sqrt[(a*(-
1 + a*x^2))/b^2]))^(3/2))
```

fricas [A] time = 16.05, size = 161, normalized size = 3.50

$$\left[\frac{\sqrt{2}\,b\log\left(-4\,ax^2-4\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-2\,\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(\sqrt{2}\,\sqrt{a}\,x+\frac{\sqrt{2}\,b\sqrt{\frac{a^2x^2-a}{b^2}}}{\sqrt{a}}\right)+1\right)}{2\,\sqrt{a}},-\sqrt{2}\,b\sqrt{-\frac{1}{a}}\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
+ b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2
 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*
sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,bx}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,bx}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

```
[Out] int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,bx}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^2 + b\,x\,\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}}{x\,\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)),x)
```

```
[Out] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax + b\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\right)}}{x\sqrt{\frac{a\left(ax^2-1\right)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(-a/b**2+a*
*2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
- 1)/b**2)), x)
```

3.1012 $\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsin((a*x-b*(a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2130, 216}

$$\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 + (a^2*x
^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx &= -\frac{\left(\sqrt{2}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}\\ &= \frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [B] time = 1.19, size = 161, normalized size = 3.50

$$\frac{\sqrt{2}b^2\sqrt{\frac{a(ax^2+1)}{b^2}}\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}\sqrt{x\left(b\sqrt{\frac{a(ax^2+1)}{b^2}}-ax\right)}\tanh^{-1}\left(\frac{\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}}{\sqrt{2}ax}\right)}{a^2\left(-bx^2\sqrt{\frac{a(ax^2+1)}{b^2}}+ax^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 +
(a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b^2*Sqrt[(a*(1 + a*x^2))/b^2]*Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2
))/b^2])]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTanh[Sqrt[a*x*(
a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)])/(a^2*(x + a*x^3 - b*x^2
*Sqrt[(a*(1 + a*x^2))/b^2]))
```

fricas [A] time = 12.88, size = 161, normalized size = 3.50

$$\left[\frac{1}{2}\sqrt{2}b\sqrt{-\frac{1}{a}}\log\left(4ax^2-4bx\sqrt{\frac{a^2x^2+a}{b^2}}+2\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}\left(\sqrt{2}ax\sqrt{-\frac{1}{a}}-\sqrt{2}b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^2x^2+a}{b^2}}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*s
qrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)
*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(2)*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-ax^2+\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,bx}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a\,x^2+\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,bx}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)
```

```
[Out] int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-ax^2 + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\, bx}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,x\,\sqrt{\frac{a}{b^2} + \frac{a^2\,x^2}{b^2}} - a\,x^2}}{x\,\sqrt{\frac{a}{b^2} + \frac{a^2\,x^2}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2)
/b^2)^(1/2)),x)
```

```
[Out] int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2)
/b^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x\left(ax - b\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\right)}}{x\sqrt{\frac{a\left(ax^2+1\right)}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**2+b*x*(a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(a/b**2+a**
2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**2
 + 1)/b**2)), x)
```

3.1013 $\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsinh((a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A] time = 1.17, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2131, 2130, 215}

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 2131

```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

$$=\frac{\left(\sqrt{2}\,b\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$=\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 0.14, size = 148, normalized size = 3.22

$$\frac{\sqrt{2}\,x\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\tanh^{-1}\left(\frac{\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}{\sqrt{2}\,ax}\right)}{\sqrt{\frac{a(ax^2-1)}{b^2}}\left(x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*x*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-1 + a*x^2 + b*x
*Sqrt[(a*(-1 + a*x^2))/b^2])*ArcTanh[Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2)
)/b^2])]/(Sqrt[2]*a*x)])/(Sqrt[(a*(-1 + a*x^2))/b^2]*(x*(a*x + b*Sqrt[(a*(-
1 + a*x^2))/b^2]))^(3/2))
```

fricas [A] time = 14.40, size = 161, normalized size = 3.50

$$\left[\frac{\sqrt{2}\,b\log\left(-4\,ax^2-4\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-2\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(\sqrt{2}\,\sqrt{a}\,x+\frac{\sqrt{2}\,b\sqrt{\frac{a^2x^2-a}{b^2}}}{\sqrt{a}}\right)+1\right)}{2\,\sqrt{a}},-\sqrt{2}\,b\sqrt{-\frac{1}{a}}\,\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
+ b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2
 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*
sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\, b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x+b*(a^2/b^2*x^2-a/b^2)^(1/2)))^(1/2)/x/(a^2/b^2*x^2-a/b^2)^(1/2)
,x)
```

```
[Out] int((x*(a*x+b*(a^2/b^2*x^2-a/b^2)^(1/2)))^(1/2)/x/(a^2/b^2*x^2-a/b^2)^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\, b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x\left(a\,x+b\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)),x)
```

```
[Out] int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a\left(ax^2-1\right)}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+(-a/b**2+a**2*x**2/b**2)**(1/2)*b))**(1/2)/x/(-a/b**2+a**
2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
- 1)/b**2)), x)
```

3.1014 $\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsin((a*x-b*(a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A] time = 1.17, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2131, 2130, 216}

$$\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 + (a^2*x
^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 2131

```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

$$=-\frac{\left(\sqrt{2}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx,x,-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$=\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 0.19, size = 161, normalized size = 3.50

$$\frac{\sqrt{2}\,b^2\sqrt{\frac{a(ax^2+1)}{b^2}}\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}\sqrt{x\left(b\sqrt{\frac{a(ax^2+1)}{b^2}}-ax\right)}\tanh^{-1}\left(\frac{\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}}{\sqrt{2}\,ax}\right)}{a^2\left(-bx^2\sqrt{\frac{a(ax^2+1)}{b^2}}+ax^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 +
(a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b^2*Sqrt[(a*(1 + a*x^2))/b^2]*Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2
))/b^2])]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTanh[Sqrt[a*x*(
a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)])/(a^2*(x + a*x^3 - b*x^2
*Sqrt[(a*(1 + a*x^2))/b^2]))
```

fricas [A] time = 12.96, size = 161, normalized size = 3.50

$$\left[\frac{1}{2}\sqrt{2}\,b\sqrt{-\frac{1}{a}}\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2+a}{b^2}}+2\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}\left(\sqrt{2}\,ax\sqrt{-\frac{1}{a}}-\sqrt{2}\,b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^2x^2+a}{b^2}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*s
qrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)
*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(2)*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax - \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\,b\right)x}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(-ax + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\,b\right)x}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*((a^2/b^2*x^2+a/b^2)^(1/2)*b-a*x))^(1/2)/x/(a^2/b^2*x^2+a/b^2)^(1/2)
,x)
```

```
[Out] int((x*((a^2/b^2*x^2+a/b^2)^(1/2)*b-a*x))^(1/2)/x/(a^2/b^2*x^2+a/b^2)^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax - \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\,b\right)x}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{-x\left(a\,x - b\,\sqrt{\frac{a}{b^2} + \frac{a^2\,x^2}{b^2}}\right)}}{x\,\sqrt{\frac{a}{b^2} + \frac{a^2\,x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2)^(1/2)))^(1/2)/(x*(a/b^2 + (a^2*x^2
)/b^2)^(1/2)),x)
```

```
[Out] int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2)^(1/2)))^(1/2)/(x*(a/b^2 + (a^2*x^2
)/b^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x\left(ax - b\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\right)}}{x\sqrt{\frac{a\left(ax^2+1\right)}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*((a/b**2+a**2*x**2/b**2)**(1/2)*b-a*x))**(1/2)/x/(a/b**2+a**2*
x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**2
 + 1)/b**2)), x)
```

3.1015 $\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}\,x+\sqrt{-1+x}\,x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx$

Optimal. Leaf size=19

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

```
[Out] 2*ln(1+(-4+x)^(1/2)+(-1+x)^(1/2))
```

Rubi [A] time = 0.56, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 66, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6688, 1586, 6684}

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x)/((1
+ Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] 2*Log[1 + Sqrt[-4 + x] + Sqrt[-1 + x]]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}\,x+\sqrt{-1+x}\,x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx &= \int \frac{\sqrt{-1+x}\left(-4+\sqrt{-4+x}\sqrt{-1+x}+x\right)}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx\\
&= \int \frac{-4+\sqrt{-4+x}\sqrt{-1+x}+x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(-4+x)\sqrt{-1+x}}\,dx\\
&= 2\log\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)
\end{aligned}$$

Mathematica [B] time = 1.38, size = 75, normalized size = 3.95

$$\frac{1}{2}\log\left(-5x-4\sqrt{x-4}\sqrt{x-1}+17\right)+\frac{1}{2}\log\left(-2x-2\sqrt{x-4}\sqrt{x-1}+5\right)-\tanh^{-1}\left(\sqrt{x-4}\right)+\tanh^{-1}\left(\frac{\sqrt{x-1}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x
)/((1 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] -ArcTanh[Sqrt[-4 + x]] + ArcTanh[Sqrt[-1 + x]/2] + Log[17 - 4*Sqrt[-4 + x]*
Sqrt[-1 + x] - 5*x]/2 + Log[5 - 2*Sqrt[-4 + x]*Sqrt[-1 + x] - 2*x]/2
```

fricas [B] time = 0.97, size = 96, normalized size = 5.05

$$-\frac{1}{2}\log\left(-(4x-11)\sqrt{x-1}\sqrt{x-4}+4x^2-21x+23\right)+\frac{1}{2}\log\left(\sqrt{x-1}\sqrt{x-4}-x+7\right)+\frac{1}{2}\log(x-5)+\frac{1}{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*log(-(4*x - 11)*sqrt(x - 1)*sqrt(x - 4) + 4*x^2 - 21*x + 23) + 1/2*log
(sqrt(x - 1)*sqrt(x - 4) - x + 7) + 1/2*log(x - 5) + 1/2*log(sqrt(x - 1) +
2) - 1/2*log(sqrt(x - 1) - 2) - 1/2*log(sqrt(x - 4) + 1) + 1/2*log(sqrt(x -
 4) - 1)
```

giac [B] time = 0.62, size = 58, normalized size = 3.05

$$-\log\left(\sqrt{x-1}-\sqrt{x-4}+1\right)-\log\left(\sqrt{x-1}-\sqrt{x-4}\right)+\log\left(\sqrt{x-1}+2\right)+\log\left(\left|-\sqrt{x-1}+\sqrt{x-4}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(sqrt(x - 1) - sqrt(x - 4) + 1) - log(sqrt(x - 1) - sqrt(x - 4)) + log(
sqrt(x - 1) + 2) + log(abs(-sqrt(x - 1) + sqrt(x - 4) - 3))
```

maple [B] time = 0.06, size = 147, normalized size = 7.74

$$\frac{7\sqrt{x-4}\,\sqrt{x-1}\,\operatorname{arctanh}\left(\frac{5x-17}{4\sqrt{x^2-5x+4}}\right)}{4\sqrt{x^2-5x+4}}+\frac{\ln(x-5)}{2}-\frac{\ln\left(-2+\sqrt{x-1}\right)}{2}+\frac{\ln\left(-1+\sqrt{x-4}\right)}{2}-\frac{\ln\left(1+\sqrt{x-4}\right)}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x-4)^(1/2)+x*(x-4)^(1/2)-4*(x-1)^(1/2)+x*(x-1)^(1/2))/(x^2-5*x+4)/(1
+(x-4)^(1/2)+(x-1)^(1/2)),x)
```

```
[Out] 1/2*ln(x-5)+1/2*ln(-1+(x-4)^(1/2))-1/2*ln(1+(x-4)^(1/2))-1/2*ln(-2+(x-1)^(1
/2))+1/2*ln((x-1)^(1/2)+2)+7/4*(x-4)^(1/2)*(x-1)^(1/2)/(x^2-5*x+4)^(1/2)*ar
ctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2))+1/4*(x-4)^(1/2)*(x-1)^(1/2)*(2*ln(-5
/2+x+(x^2-5*x+4)^(1/2))-5*arctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2)))/(x^2-5*
x+4)^(1/2)
```

maxima [B] time = 0.65, size = 94, normalized size = 4.95

$$\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{2x^2+2\left((x-1)\sqrt{x-4}+2x-6\right)\sqrt{x-1}+2(2x-3)\sqrt{x-4}-7x+3}{2\left((x-1)\sqrt{x-4}+2x-6\right)}\right)+\frac{1}{2}\log\left(\frac{(x-1}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*log(x - 1) + 1/2*log(1/2*(2*x^2 + 2*((x - 1)*sqrt(x - 4) + 2*x - 6)*sqr
t(x - 1) + 2*(2*x - 3)*sqrt(x - 4) - 7*x + 3)/((x - 1)*sqrt(x - 4) + 2*x -
6)) + 1/2*log(((x - 1)*sqrt(x - 4) + 2*x - 6)/(x - 1))
```

mupad [B] time = 6.10, size = 132, normalized size = 6.95

$$\frac{\ln(x-5)}{2}+2\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\sqrt{3}}{\sqrt{x-4}}\right)+\frac{7\,\mathrm{atanh}\left(\frac{4\left(\sqrt{x-1}-\sqrt{3}\right)}{\left(\frac{\left(\sqrt{x-1}-\sqrt{3}\right)^2}{x-4}+1\right)\sqrt{x-4}}\right)}{2}-\frac{5\,\mathrm{atanh}\left(\frac{194400\left(\sqrt{x-1}-\sqrt{3}\right)}{\left(\frac{48600\left(\sqrt{x-1}-\sqrt{3}\right)^2}{x-4}+48600\right)\sqrt{x-4}}\right)}{2}-\mathrm{atan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x - 1)^(1/2) + x*(x - 4)^(1/2) - 4*(x - 1)^(1/2) - (x - 4)^(1/2))/(
(x^2 - 5*x + 4)*((x - 1)^(1/2) + (x - 4)^(1/2) + 1)),x)
```

```
[Out] log(x - 5)/2 + 2*atanh(((x - 1)^(1/2) - 3^(1/2))/(x - 4)^(1/2)) + (7*atanh(
(4*((x - 1)^(1/2) - 3^(1/2)))/((((x - 1)^(1/2) - 3^(1/2))^2/(x - 4) + 1)*(x
 - 4)^(1/2))))/2 - (5*atanh((194400*((x - 1)^(1/2) - 3^(1/2)))/(((48600*((x
 - 1)^(1/2) - 3^(1/2))^2)/(x - 4) + 48600)*(x - 4)^(1/2))))/2 - atanh((x -
4)^(1/2)) + atanh((x - 1)^(1/2)/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)**(1/2)+x*(-4+x)**(1/2)-4*(-1+x)**(1/2)+x*(-1+x)**(1/2))/
(x**2-5*x+4)/(1+(-4+x)**(1/2)+(-1+x)**(1/2)),x)
```

```
[Out] Timed out
```

3.1016 $\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

Optimal. Leaf size=90

$$-\frac{\log\left(1-(x+1)^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}(x+1)-\sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}$$

```
[Out] -1/3*arctan(1/3*(1+2*3^(1/3)*(1+x)/(2+(1+x)^3)^(1/3))*3^(1/2))*3^(1/6)-1/18
*ln(1-(1+x)^3)*3^(2/3)+1/6*ln(3^(1/3)*(1+x)-(2+(1+x)^3)^(1/3))*3^(2/3)
```

Rubi [A] time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {433, 431, 377, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(\frac{3^{2/3}(x+1)^2}{\left((x+1)^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))]/3^(5/6))
+ Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(3*3^(1/3)) - Log[1 + (3
^(2/3)*(1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)
^(1/3)]/(6*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
```

```
x]
```

Rule 433

```
Int[(u_)^(p_.)*(v_)^(q_.)*(x_)^(m_.), x_Symbol] :> Int[NormalizePseudoBinom
ial[x^(m/p)*u, x]^p*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x]
 && IntegersQ[p, m/p] && PseudoBinomialPairQ[x^(m/p)*u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx &= \int \frac{1}{\left(-1+(1+x)^3\right)\sqrt[3]{2+(1+x)^3}}\,dx \\
&= \text{Subst}\left(\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{2+x^3}}\,dx, x, 1+x\right) \\
&= \text{Subst}\left(\int \frac{1}{-1+3x^3}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}\,x}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{-2}{1+\sqrt[3]{3}
\\
&= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{3}\,x+3^{2/3}x^2}\,dx, x, \frac{1+}{\sqrt[3]{2+(1}
\\
&= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{\log\left(1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}+\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{6\sqrt[3]{3}} + \frac{\text{Subst}\left(\right.}{}
\\
&= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}} + \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{\log\left(1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}\right.}{6\sqrt[3]{3}}
\end{aligned}
$$

Mathematica [A] time = 0.16, size = 120, normalized size = 1.33

$$\frac{\sqrt{3}\left(2\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)-\log\left(\frac{3^{2/3}(x+1)^2}{\left((x+1)^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)\right)-6\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{6\,3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] (-6*ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))] + Sqrt[
3]*(2*Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)] - Log[1 + (3^(2/3)*(
1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)])
)/(6*3^(5/6))
```

fricas [B] time = 10.82, size = 458, normalized size = 5.09

$$-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}\left(7\,x^4+28\,x^3+42\,x^2+30\,x+9\right)\left(x^3+3\,x^2+3\,x+3\right)^{\frac{2}{3}}+3^{\frac{1}{3}}\left(31\,x^6+186\,x^5+465\,x^4+666\right.}{x^6+6\,x^5+15\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3*
x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*
x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3 +
3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27*3^(
2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x + 3)
^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^3 + 3*
x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49*x^6 +
147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x + 3)^(2/3)
 - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5 + 22032*x
^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7 + 868*x^6
 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3 + 3*x^2 + 3
*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 34398*x^5 + 385
56*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^3+3\,x^2+3\,x+3\right)^{\frac{1}{3}}\left(x^2+3\,x+3\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

maple [C] time = 13.67, size = 2515, normalized size = 27.94

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x)
```

```
[Out] RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*ln((114324537294*RootO
f(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+4470714138*RootOf(_Z^3-9)+1
```

5589709631*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2+609642837*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2+97002637704*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3+291007913112*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2+291007913112*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+3793333208*RootOf(_Z^3-9)*x^3+11379999624*RootOf(_Z^3-9)*x^2+11379999624*RootOf(_Z^3-9)*x+15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)*x+5196569877*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+203214279*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3+2837496903*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)+8512490709*(x^3+3*x^2+3*x+3)^(2/3)*x+8512490709*(x^3+3*x^2+3*x+3)^(2/3)+45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x^2+91932017904*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x+15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)+2837496903*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2+5674993806*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x+45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)+609642837*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x+15589709631*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-36375989139*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2-1422499953*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3)/x/(x^2+3*x+3))-1/9*ln((-150700526433*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-10851288846*RootOf(_Z^3-9)+15589709631*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2+1122547122*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2-91806067827*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-275418203481*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2-275418203481*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x-6610555274*RootOf(_Z^3-9)*x^3-19831665822*RootOf(_Z^3-9)*x^2-19831665822*RootOf(_Z^3-9)*x-15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)*x+5196569877*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+374182374*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3-2269837425*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)-6809512275*(x^3+3*x^2+3*x+3)^(2/3)*x-6809512275*(x^3+3*x^2+3*x+3)^(2/3)-45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x^2-91932017904*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x-15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)-2269837425*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2-4539674850*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x-45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)+1122547122*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x+15589709631*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-36375989139*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2-2619276618*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3)/x/(x^2+3*x+3))*RootOf(_Z^3-9)-ln((-150700526433*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-10851288846*RootOf(_Z^3-9)+15589709631*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2+1122547122*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2-91806067827*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-275418203481*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2-275418203481*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x-6610555274*RootOf(_Z^3-9)*x^3-19831665822*RootOf(_Z^3-9)*x^2-19831665822*RootOf(_Z^3-9)*x-15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)*x+5196569877*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+374182374*RootOf(RootOf(_Z^3-9)^2+

```
9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3-2269837425*RootOf(_Z^3-9)
^2*(x^3+3*x^2+3*x+3)^(1/3)-6809512275*(x^3+3*x^2+3*x+3)^(2/3)*x-6809512275*
(x^3+3*x^2+3*x+3)^(2/3)-45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^
3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x^2-91932017904*RootOf
(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*
x+3)^(1/3)*x-15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^
2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)-2269837425*RootOf(_Z^3-9)^2*(x^
3+3*x^2+3*x+3)^(1/3)*x^2-4539674850*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3
)*x-45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf
(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)+1122547122*RootOf(RootOf(_Z^3-9)^2+9*_Z*Ro
otOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x+15589709631*RootOf(RootOf(_Z^3-9)^
2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-36375989139*RootOf(Root
Of(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2-2619276618*Roo
tOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3)/x/(x^2+
3*x+3))*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+3x^2+3x+3\right)^{\frac{1}{3}}\left(x^2+3x+3\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(x^2+3x+3\right)\left(x^3+3x^2+3x+3\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)),x)
[Out] int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x^2+3x+3\right)\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+3*x+3)/(x**3+3*x**2+3*x+3)**(1/3),x)
[Out] Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)
```

3.1017 $\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx$

Optimal. Leaf size=103

$$-\frac{\log\left(-x^3+2(1-x)^3+1\right)}{2\,2^{2/3}}+\frac{3\log\left(\sqrt[3]{1-x^3}+\sqrt[3]{2}\,(1-x)\right)}{2\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}$$

```
[Out] -1/4*ln(1+2*(1-x)^3-x^3)*2^(1/3)+3/4*ln(2^(1/3)*(1-x)+(-x^3+1)^(1/3))*2^(1/
3)+1/2*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(1/
3)
```

Rubi [F] time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] -(x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3]) - (1 + I*Sqrt[3])*Defer[Int][1/(
(-1 - I*Sqrt[3] + 2*x)*(1 - x^3)^(2/3)), x] - (1 - I*Sqrt[3])*Defer[Int][1/
((-1 + I*Sqrt[3] + 2*x)*(1 - x^3)^(2/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx &= \int \left(-\frac{1}{\left(1-x^3\right)^{2/3}}+\frac{2-x}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\right)dx\\
&= -\int \frac{1}{\left(1-x^3\right)^{2/3}}\,dx+\int \frac{2-x}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx\\
&= -x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\int \left(\frac{-1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}}+\frac{-1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\left(1-x\right.}\right.\\
&= -x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\left(-1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}}\,dx+\left(-1+i\sqrt{\right.}
\end{aligned}$$

Mathematica [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)), x]
```

fricas [B] time = 8.81, size = 289, normalized size = 2.81

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(2\cdot 4^{\frac{2}{3}}\left(x^5-x^4-3x^3+3x^2+x-1\right)\left(-x^3+1\right)^{\frac{1}{3}}+4\left(x^4-4x^3+5x^2-4x+1\right)\left(-x^3\right.\right.}{6\left(3x^6-9x^5+6x^4-x^3+6x^2-9x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(x^5 - x^4 - 3*x
^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 4*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-
x^3 + 1)^(2/3) + 4^(1/3)*(x^6 - 7*x^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))
/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x^2 - 9*x + 3)) - 1/24*4^(2/3)*log((2*4^(
1/3)*(-x^3 + 1)^(2/3)*(x^2 - 3*x + 1) - 4^(2/3)*(x^4 - 3*x^2 + 1) - 8*(-x^3
 + 1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/12*4^(2/3)*log(
-(4^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) - 4^(1/3)*(x^2 - x + 1) - 2*(-x^3 + 1)^(
2/3))/(x^2 - x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

maple [C] time = 8.03, size = 1026, normalized size = 9.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x)
```

```
[Out] 1/2*RootOf(_Z^3-2)*ln(-(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)*RootOf(_Z^3-2)^4*x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
^2*RootOf(_Z^3-2)^3*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(
_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2+RootOf(_Z^3-2)^2*x^2+2*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2-RootOf(_Z^3-2)^2*x-2*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-2*(-x^3+
1)^(1/3)*RootOf(_Z^3-2)*x-4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*(-x^3+1)^(1/3)*x+RootOf(_Z^3-2)^2+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)+4*RootOf(R
ootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*(-x^3+1)^(1/3))/(x^2-x+1))-1/2*
ln((2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4*
x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^3*
x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*Root
Of(_Z^3-2)^2-RootOf(_Z^3-2)^2*x^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+3*RootOf(_Z^3-2)^2*x+6*RootOf(RootOf(_Z^3-2
)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-2*(-x^3+1)^(1/3)*RootOf(_Z
^3-2)*x-RootOf(_Z^3-2)^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)*RootOf(_Z^3-2)+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)+2*(-x^3+1)^(2/3))/(x^2-x
+1))*RootOf(_Z^3-2)-ln((2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)*RootOf(_Z^3-2)^4*x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
^2*RootOf(_Z^3-2)^3*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(
```

```
_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-RootOf(_Z^3-2)^2*x^2-2*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+3*RootOf(_Z^3-2)^2*x+6*
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-2*(-x^
3+1)^(1/3)*RootOf(_Z^3-2)*x-RootOf(_Z^3-2)^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)+2*(-
x^3+1)^(2/3))/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2-1}{\left(1-x^3\right)^{2/3}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)),x)
```

```
[Out] -int((x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\left(1-x^3\right)^{\frac{2}{3}}-x\left(1-x^3\right)^{\frac{2}{3}}+\left(1-x^3\right)^{\frac{2}{3}}}\,dx-\int \left(-\frac{1}{x^2\left(1-x^3\right)^{\frac{2}{3}}-x\left(1-x^3\right)^{\frac{2}{3}}+\left(1-x^3\right)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2-x+1)/(-x**3+1)**(2/3),x)
```

```
[Out] -Integral(x**2/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1 - x**3)**
(2/3)), x) - Integral(-1/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1
 - x**3)**(2/3)), x)
```

3.1018 $\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{4}\tan^{-1}\left(\frac{x^2+1}{x\sqrt{x^4-1}}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{1-x^2}{x\sqrt{x^4-1}}\right)$$

```
[Out] -1/4*arctan((x^2+1)/x/(x^4-1)^(1/2))-1/4*arctanh((-x^2+1)/x/(x^4-1)^(1/2))
```

Rubi [C] time = 0.12, antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {490, 1211, 222, 1699, 206, 203}

$$\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

```
[Out] (-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 + I/8)*ArcTanh[((1 +
 I)*x)/Sqrt[-1 + x^4]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{-1+x^4}\left(1+x^4\right)}\,dx &= -\left(\frac{1}{2}\int \frac{1}{\left(i-x^2\right)\sqrt{-1+x^4}}\,dx\right)+\frac{1}{2}\int \frac{1}{\left(i+x^2\right)\sqrt{-1+x^4}}\,dx\\ &= -\left(\frac{1}{4}i\int \frac{i-x^2}{\left(i+x^2\right)\sqrt{-1+x^4}}\,dx\right)+\frac{1}{4}i\int \frac{i+x^2}{\left(i-x^2\right)\sqrt{-1+x^4}}\,dx\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{i-2x^2}\,dx,x,\frac{x}{\sqrt{-1+x^4}}\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{i+2x^2}\,dx,x,\frac{x}{\sqrt{-1+x^4}}\right)\\ &= \left(-\frac{1}{8}-\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)+\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.94

$$\frac{x^3\sqrt{1-x^4}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};x^4,-x^4\right)}{3\sqrt{x^4-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
[Out] (x^3*Sqrt[1 - x^4]*AppellF1[3/4, 1/2, 1, 7/4, x^4, -x^4])/(3*Sqrt[-1 + x^4]
)
```

fricas [A] time = 0.66, size = 51, normalized size = 1.04

$$\frac{1}{4}\arctan\left(\frac{\sqrt{x^4-1}\,x}{x^2+1}\right)+\frac{1}{8}\log\left(\frac{x^4+2\,x^2+2\,\sqrt{x^4-1}\,x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="fricas")
[Out] 1/4*arctan(sqrt(x^4 - 1)*x/(x^2 + 1)) + 1/8*log((x^4 + 2*x^2 + 2*sqrt(x^4 -
 1)*x - 1)/(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^4+1\right)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

maple [B] time = 0.03, size = 88, normalized size = 1.80

$$-\frac{\arctan\left(-\frac{\sqrt{x^4-1}}{x}+1\right)}{8}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}+1\right)}{8}+\frac{\ln\left(\frac{\frac{\sqrt{x^4-1}}{x}+\frac{x^4-1}{2x^2}+1}{-\frac{\sqrt{x^4-1}}{x}+\frac{x^4-1}{2x^2}+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+1)/(x^4-1)^(1/2),x)
```

```
[Out] 1/8*arctan((x^4-1)^(1/2)/x+1)-1/8*arctan(-(x^4-1)^(1/2)/x+1)+1/16*ln((1/2*(
x^4-1)/x^2+(x^4-1)^(1/2)/x+1)/(1/2*(x^4-1)/x^2-(x^4-1)^(1/2)/x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^4+1)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{x^4-1}\,(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)),x)
```

```
[Out] int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(x-1)(x+1)(x^2+1)}\,(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+1)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)), x)
```

3.1019 $\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

```
[Out] arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d
^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2112, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*
x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx &= a\,\text{Subst}\left(\int \frac{1}{ade-\left(abde-a\left(cd^2+ae^2\right)\right)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

Mathematica [C] time = 0.91, size = 383, normalized size = 4.79

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{2ae};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\right.\right.}{\sqrt{2}\,de\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])
,x]
```

```
[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(2*a*e), I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a
*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4
])
```

fricas [A] time = 67.60, size = 472, normalized size = 5.90

$$\left[-\frac{\sqrt{-cd^3e+bd^2e^2-ade^3}\,\log\left(-\frac{c^2d^2e^2x^8-2\left(3\,c^2d^3e-4\,bcd^2e^2+3\,acde^3\right)x^6+a^2d^2e^2+\left(c^2d^4-8\,bcd^3e-8\,abde^3+a^2e^4+4\left(2\,b^2+ac\right)d^2e^2\right)x^4-2\cdots}{c^2d^2e^2x^8+2\left(c^2d^3e+acde^3\right)x^6+a^2d^2e^2+\left(c^2\cdots\right.}\right.}{4\left(cd^3e-bd^2e^2+ade^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3)*log(-(c^2*d^2*e^2*x^8 - 2*(3*c^2
*d^3*e - 4*b*c*d^2*e^2 + 3*a*c*d*e^3)*x^6 + a^2*d^2*e^2 + (c^2*d^4 - 8*b*c*
d^3*e - 8*a*b*d*e^3 + a^2*e^4 + 4*(2*b^2 + a*c)*d^2*e^2)*x^4 - 2*(3*a*c*d^3
*e - 4*a*b*d^2*e^2 + 3*a^2*d*e^3)*x^2 + 4*(c*d*e*x^5 + a*d*e*x - (c*d^2 - 2
*b*d*e + a*e^2)*x^3)*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3)*sqrt(c*x^4 + b*x^
2 + a))/(c^2*d^2*e^2*x^8 + 2*(c^2*d^3*e + a*c*d*e^3)*x^6 + a^2*d^2*e^2 + (c
^2*d^4 + 4*a*c*d^2*e^2 + a^2*e^4)*x^4 + 2*(a*c*d^3*e + a^2*d*e^3)*x^2))/(c*
d^3*e - b*d^2*e^2 + a*d*e^3), 1/2*arctan(2*sqrt(c*d^3*e - b*d^2*e^2 + a*d*e
^3)*sqrt(c*x^4 + b*x^2 + a)*x/(c*d*e*x^4 + a*d*e - (c*d^2 - 2*b*d*e + a*e^2
)*x^2))/sqrt(c*d^3*e - b*d^2*e^2 + a*d*e^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{cx^4-a}{\sqrt{cx^4+bx^2+a}\left(cdx^2+ae\right)\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)
), x)
```

maple [C] time = 0.09, size = 555, normalized size = 6.94

$$-\frac{\sqrt{2}\,\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}+4}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{\frac{2\left(b+\sqrt{-4ac+b^2}\right)b}{ac}-4}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,de}+\frac{\sqrt{2}\,\sqrt{\frac{b}{2}\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/d/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2+4)^(1/2)*(2*(b+(-4*a*c+b^2)^(1/2))/a*x^2+4)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*(2*(b+
(-4*a*c+b^2)^(1/2))/a*b/c-4)^(1/2))+1/d/e*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^
(1/2))^(1/2)*(1+1/2/a*b*x^2-1/2*(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)*(1+1/2/a*b*
x^2+1/2*(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/
2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,-2/(-b+(-4*a*c+b^2)^(1/2))*c*
d/e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2))+1/e/d*2^(1/2)/(-1/a*b+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*b*x^
2-1/2*(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)*(1+1/2/a*b*x^2+1/2*(-4*a*c+b^2)^(1/2)
/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*x,-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^4 - a}{\sqrt{cx^4 + bx^2 + a}\left(cdx^2 + ae\right)\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a - c\,x^4}{\left(e\,x^2 + d\right)\,\left(c\,d\,x^2 + a\,e\right)\,\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{a}{ade\sqrt{a + bx^2 + cx^4} + ae^2x^2\sqrt{a + bx^2 + cx^4} + cd^2x^2\sqrt{a + bx^2 + cx^4} + cdex^4\sqrt{a + bx^2 + cx^4}}\right) dx - \int \frac{}{ade\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**2+a*e)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*e*sqrt(a + b*x**2 + c*x**4) + a*e**2*x**2*sqrt(a + b*x**2
 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*e*x**4*sqrt(a + b*
x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*e*sqrt(a + b*x**2 + c*x**4) + a*
e**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x**4)
 + c*d*e*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

3.1020 $\int \left(x+\frac{1-x^2}{1+x}\right) dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[x + (1 - x^2)/(1 + x),x]
```

```
[Out] x
```

Rubi steps

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[x + (1 - x^2)/(1 + x),x]
```

```
[Out] x
```

fricas [A] time = 1.54, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="fricas")
```

```
[Out] x
```

giac [A] time = 0.35, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="giac")
```

```
[Out] x
```

maple [A] time = 0.00, size = 2, normalized size = 2.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x+(-x^2+1)/(x+1),x)
```

```
[Out] x
```

maxima [**A**] time = 0.46, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="maxima")
[Out] x
```

mupad [**B**] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x - (x^2 - 1)/(x + 1),x)
[Out] x
```

sympy [**A**] time = 0.06, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x**2+1)/(1+x),x)
[Out] x
```

3.1021 $\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\sin^{-1}(x)$$

```
[Out] arcsin(x)-1/3*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/3*arctan(x/(-x^2+1)^
(1/2)/((-I+3^(1/2))/(3^(1/2)+I))^(1/2))*3^(1/2)-1/3*arctan(x*((-I+3^(1/2))/
(3^(1/2)+I))^(1/2)/(-x^2+1)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6742, 1107, 618, 204, 1293, 216, 1174, 377, 205}

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqr
t[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3]
)/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx &= \int \left(\frac{x}{1-x^2+x^4}-\frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\right)dx\\
&= \int \frac{x}{1-x^2+x^4}\,dx-\int \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)+\int \frac{1}{\sqrt{1-x^2}}\,dx-\int \frac{1}{\sqrt{1-x^2}\left(1-x^2+x^4\right)}\,dx\\
&= \sin^{-1}(x)+\frac{(2i)\int \frac{1}{\sqrt{1-x^2}\left(-1-i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}}-\frac{(2i)\int \frac{1}{\sqrt{1-x^2}\left(-1+i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}}-\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.\\
&= \sin^{-1}(x)+\frac{\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{(2i)\,\text{Subst}\left(\int \frac{1}{-1+i\sqrt{3}-\left(-1-i\sqrt{3}\right)x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{(2i)\,\text{Subst}\left(\int\right.}{}\\
&= \sin^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [B] time = 4.39, size = 1932, normalized size = 15.84

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] (24*ArcSin[x] - (2*(-I + Sqrt[3])*ArcTan[(x*(-7 - I*Sqrt[3] + 8*Sqrt[3]*x +
 I*(7*I + Sqrt[3])*x^2))/(-6 - (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*x^3 - 2*Sqr
t[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (2*I)*x^2*(9*I + Sqrt[3] + I*Sqrt[2 -
(2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)*Sqrt
[3]]*Sqrt[1 - x^2]))])/Sqrt[(1 - I*Sqrt[3])/6] + (2*(I + Sqrt[3])*ArcTan[(x
*(7 - I*Sqrt[3] - 8*Sqrt[3]*x + (7 + I*Sqrt[3])*x^2))/(-6 + (2*I)*Sqrt[3] +
 3*(I + Sqrt[3])*x^3 - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + x^2*(-18 -
 (2*I)*Sqrt[3] - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(-3*I + 11*Sq
rt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/Sqrt[(1 + I*Sqrt[3])/6]
 - (2*(I + Sqrt[3])*ArcTan[(x*(7 - I*Sqrt[3] + 8*Sqrt[3]*x + (7 + I*Sqrt[3]
)*x^2))/(6 - (2*I)*Sqrt[3] + 3*(I + Sqrt[3])*x^3 + 2*Sqrt[2 + (2*I)*Sqrt[3]
]*Sqrt[1 - x^2] + 2*x^2*(9 + I*Sqrt[3] + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x
^2]) + x*(-3*I + 11*Sqrt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/S
qrt[(1 + I*Sqrt[3])/6] + (2*(1 + I*Sqrt[3])*ArcTanh[(x*(7*I - Sqrt[3] + (8*
I)*Sqrt[3]*x + (7*I + Sqrt[3])*x^2))/(6 + (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*
x^3 + 2*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + 2*x^2*(9 - I*Sqrt[3] + Sqrt
[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)
*Sqrt[3]]*Sqrt[1 - x^2]))])/Sqrt[(1 - I*Sqrt[3])/6] - (4*I)*Sqrt[3]*Log[-1/
2 - (I/2)*Sqrt[3] + x^2] + (4*I)*Sqrt[3]*Log[(I/2)*(I + Sqrt[3]) + x^2] - (
I*(-I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[(1 - I*Sqrt[3])/6] +
 (I*(I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[(1 + I*Sqrt[3])/6]
+ ((1 - I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/Sqrt[(1 + I*Sqrt[3])/6
] + ((1 + I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/Sqrt[(1 - I*Sqrt[3])
/6] + ((1 + I*Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sqrt[3])*x^4 + (2*I)*Sqrt[
2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + Sqrt[2 - (2*I)*Sqrt[3]]*S
qrt[1 - x^2]) + x*(3 + (5*I)*Sqrt[3] + (3*I)*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1
 - x^2]) + I*x^3*(3*I + 3*Sqrt[3] + Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])]
)/Sqrt[(1 - I*Sqrt[3])/6] - (I*(-I + Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sqr
t[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + Sq
rt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*(3 - (3*I)*Sqrt[3] - I*Sqrt[6 -
(6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x*(-3*I + 5*Sqrt[3] + 3*Sqrt[6 - (6*I)*Sq
rt[3]]*Sqrt[1 - x^2])])/Sqrt[(1 - I*Sqrt[3])/6] + ((1 - I*Sqrt[3])*Log[-3*I
 + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2
] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3 - (5*I)*Sq
rt[3] - (3*I)*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x^3*(-3*I + 3*Sqrt
[3] + Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[(1 + I*Sqrt[3])/6] + (I
*(I + Sqrt[3])*Log[-3*I + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I
)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 -
x^2]) + x^3*(3 + (3*I)*Sqrt[3] + I*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) +
 I*x*(3*I + 5*Sqrt[3] + 3*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[(1
+ I*Sqrt[3])/6])/24
```

fricas [A] time = 0.65, size = 73, normalized size = 0.60

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-1\right)\sqrt{-x^2+1}}{3\left(x^3-x\right)}\right)-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [B] time = 0.42, size = 193, normalized size = 1.58

$$\frac{1}{2}\pi\operatorname{sgn}(x)-\frac{1}{6}\sqrt{3}\left(\pi\operatorname{sgn}(x)+2\arctan\left(-\frac{\sqrt{3}\,x\left(\frac{\sqrt{-x^2+1}-1}{x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\frac{1}{6}\sqrt{3}\left(\pi\operatorname{sgn}(x)+2\arctan\left(\frac{\sqrt{3}\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^
2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/
6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sq
rt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sq
rt(-x^2 + 1) - 1))
```

maple [B] time = 0.04, size = 234, normalized size = 1.92

$$-2\arctan\left(\frac{-1+\sqrt{-x^2+1}}{x}\right)+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{3}-\frac{i\sqrt{3}\,\ln\left(\frac{(-1-i\sqrt{3})(-1+\sqrt{-x^2+1})}{x}+\frac{\left(-1+\sqrt{-x^2+1}\right)^2}{x^2}-1\right)}{6}-\frac{i\sqrt{3}\,\ln\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x+(-x^2+1)^(1/2)),x)
```

```
[Out] 1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2))^2/x^2+(1+I*3^(1/2))*(-1+(-x^2+1)^(1/2)
)/x-1)-1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2))^2/x^2+(1-I*3^(1/2))*(-1+(-x^2+1
)^(1/2))/x-1)-2*arctan((-1+(-x^2+1)^(1/2))/x)+1/6*I*3^(1/2)*ln((-1+(-x^2+1)
^(1/2))^2/x^2+(I*3^(1/2)-1)*(-1+(-x^2+1)^(1/2))/x-1)-1/6*I*3^(1/2)*ln((-1+(
-x^2+1)^(1/2))^2/x^2+(-1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x-1)+1/3*3^(1/2)*ar
ctan(1/3*(2*x^2-1)*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+1}+\frac{1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1) + 1/x), x)
```

mupad [B] time = 3.92, size = 549, normalized size = 4.50

$$\operatorname{asin}(x)-\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{\frac{\sqrt{3}}{2}-x+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\,\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}+\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\,\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}-\frac{\cdots}{\sqrt{1-\left(\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x + (1 - x^2)^(1/2)),x)
```

```
[Out] asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(
1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)
^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2 -
1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^
3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 -
 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2))/((1 - (3^(1/2
)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (log(x - 3^
(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)
 - (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 +
 1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 + 1i/
2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2
+ 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (log(x - 3^(1/2
)/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i) + (
log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(
1/2) + 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x\sqrt{1-x^2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x*sqrt(1 - x**2) + 1), x)
```

3.1022 $\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\sin^{-1}(x)$$

```
[Out] arcsin(x)-1/3*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/3*arctan(x/(-x^2+1)^
(1/2)/((-I+3^(1/2))/(3^(1/2)+I))^(1/2))*3^(1/2)-1/3*arctan(x*((-I+3^(1/2))/
(3^(1/2)+I))^(1/2)/(-x^2+1)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 149, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {6742, 1293, 216, 1174, 377, 205, 1251, 773, 618, 204}

$$-\frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{1}{4}(1-x)^2+\frac{1}{4}(x+1)^2+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

```
[Out] (1 - x)^2/4 - x^2/2 + (1 + x)^2/4 + ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]
/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/S
qrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqr
t[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = \int \left(\frac{1}{2}(-1+x)+\frac{1+x}{2}-\frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}+\frac{x^3\left(1-x^2\right)}{1-x^2+x^4}\right)dx$$

$$= \frac{1}{4}(1-x)^2+\frac{1}{4}(1+x)^2-\int \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\,dx+\int \frac{x^3\left(1-x^2\right)}{1-x^2+x^4}\,dx$$

$$= \frac{1}{4}(1-x)^2+\frac{1}{4}(1+x)^2+\frac{1}{2}\operatorname{Subst}\left(\int \frac{(1-x)x}{1-x+x^2}\,dx,x,x^2\right)+\int \frac{1}{\sqrt{1-x^2}}\,dx-\int \frac{}{\sqrt{1}}$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)+\frac{(2i)\int}{}$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)-\frac{(2i)\operatorname{Subst}\left(\int \frac{1}{-1+i\sqrt{3}-\left(-1-i\sqrt{3}\right)x^2}\,dx,x,\frac{x}{\sqrt{1-x}}\right.}{\sqrt{3}}$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}$$

Mathematica [B] time = 1.37, size = 1910, normalized size = 15.66

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

```
[Out] ArcSin[x] - ((-I + Sqrt[3])*ArcTan[(x*(-7 - I*Sqrt[3] + 8*Sqrt[3]*x + I*(7*
I + Sqrt[3])*x^2))/(-6 - (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*x^3 - 2*Sqrt[2 -
(2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (2*I)*x^2*(9*I + Sqrt[3] + I*Sqrt[2 - (2*I)*
Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)*Sqrt[3]]*S
qrt[1 - x^2]))])/(2*Sqrt[6 - (6*I)*Sqrt[3]]) + ((I + Sqrt[3])*ArcTan[(x*(7
- I*Sqrt[3] - 8*Sqrt[3]*x + (7 + I*Sqrt[3])*x^2))/(-6 + (2*I)*Sqrt[3] + 3*(
I + Sqrt[3])*x^3 - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + x^2*(-18 - (2*
I)*Sqrt[3] - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(-3*I + 11*Sqrt[3
] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/(2*Sqrt[6 + (6*I)*Sqrt[3]])
 - ((I + Sqrt[3])*ArcTan[(x*(7 - I*Sqrt[3] + 8*Sqrt[3]*x + (7 + I*Sqrt[3])*
x^2))/(6 - (2*I)*Sqrt[3] + 3*(I + Sqrt[3])*x^3 + 2*Sqrt[2 + (2*I)*Sqrt[3]]*
Sqrt[1 - x^2] + 2*x^2*(9 + I*Sqrt[3] + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2
]) + x*(-3*I + 11*Sqrt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))])/(2*
Sqrt[6 + (6*I)*Sqrt[3]]) + ((1 + I*Sqrt[3])*ArcTanh[(x*(7*I - Sqrt[3] + (8*
I)*Sqrt[3]*x + (7*I + Sqrt[3])*x^2))/(6 + (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*
x^3 + 2*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + 2*x^2*(9 - I*Sqrt[3] + Sqrt
[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)
*Sqrt[3]]*Sqrt[1 - x^2]))])/(2*Sqrt[6 - (6*I)*Sqrt[3]]) - ((I/2)*Log[-1/2 -
 (I/2)*Sqrt[3] + x^2])/Sqrt[3] + ((I/2)*Log[(I/2)*(I + Sqrt[3]) + x^2])/Sqr
t[3] - ((I/4)*(-I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[6 - (6*I
)*Sqrt[3]] + ((I/4)*(I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[6 +
 (6*I)*Sqrt[3]] + ((1 + I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/(4*Sqr
t[6 - (6*I)*Sqrt[3]]) + ((1 - I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/
(4*Sqrt[6 + (6*I)*Sqrt[3]]) + ((1 + I*Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sq
rt[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + S
qrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3 + (5*I)*Sqrt[3] + (3*I)*Sqrt[6
 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) + I*x^3*(3*I + 3*Sqrt[3] + Sqrt[6 - (6*I)*
Sqrt[3]]*Sqrt[1 - x^2])])/(4*Sqrt[6 - (6*I)*Sqrt[3]]) - ((I/4)*(-I + Sqrt[3
])*Log[3*I + Sqrt[3] - (-I + Sqrt[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*S
qrt[1 - x^2] + (5*I)*x^2*(2 + Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*
(3 - (3*I)*Sqrt[3] - I*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x*(-3*I +
 5*Sqrt[3] + 3*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[6 - (6*I)*Sqrt
[3]] + ((1 - I*Sqrt[3])*Log[-3*I + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt
[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*
Sqrt[1 - x^2]) + x*(3 - (5*I)*Sqrt[3] - (3*I)*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[
1 - x^2]) - I*x^3*(-3*I + 3*Sqrt[3] + Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]
)])/(4*Sqrt[6 + (6*I)*Sqrt[3]]) + ((I/4)*(I + Sqrt[3])*Log[-3*I + Sqrt[3] -
 (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^
2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*(3 + (3*I)*Sqrt[3] + I*
Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) + I*x*(3*I + 5*Sqrt[3] + 3*Sqrt[6 +
(6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[6 + (6*I)*Sqrt[3]]
```

fricas [A] time = 0.61, size = 73, normalized size = 0.60

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-1\right)\sqrt{-x^2+1}}{3\left(x^3-x\right)}\right)-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [B] time = 0.58, size = 193, normalized size = 1.58

$$\frac{1}{2}\pi\mathrm{sgn}(x)-\frac{1}{6}\sqrt{3}\left(\pi\mathrm{sgn}(x)+2\arctan\left(-\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\frac{1}{6}\sqrt{3}\left(\pi\mathrm{sgn}(x)+2\arctan\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^
2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/
6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sq
rt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sq
rt(-x^2 + 1) - 1))
```

maple [B] time = 0.06, size = 234, normalized size = 1.92

$$-2\arctan\left(\frac{-1+\sqrt{-x^2+1}}{x}\right)+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{3}-\frac{i\sqrt{3}\,\ln\left(\frac{\left(-1-i\sqrt{3}\right)\left(-1+\sqrt{-x^2+1}\right)}{x}+\frac{\left(-1+\sqrt{-x^2+1}\right)^2}{x^2}-1\right)}{6}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x)
```

```
[Out] -2*arctan((-1+(-x^2+1)^(1/2))/x)+1/3*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-
1/6*I*3^(1/2)*ln((-1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x+(-1+(-x^2+1)^(1/2))^2
/x^2-1)-1/6*I*3^(1/2)*ln((1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x+(-1+(-x^2+1)^(
1/2))^2/x^2-1)+1/6*I*3^(1/2)*ln((1+I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x+(-1+(-x
^2+1)^(1/2))^2/x^2-1)+1/6*I*3^(1/2)*ln((I*3^(1/2)-1)*(-1+(-x^2+1)^(1/2))/x+
(-1+(-x^2+1)^(1/2))^2/x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2+\int -\frac{x^4-x^2}{x^3-x-\sqrt{x+1}\,\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate(-(x^4 - x^2)/(x^3 - x - sqrt(x + 1)*sqrt(-x + 1)), x)
```

mupad [B] time = 3.89, size = 549, normalized size = 4.50

$$\mathrm{asin}(x)-\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{\frac{\sqrt{3}}{2}-x+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\,\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}+\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\,\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^2)^(1/2))/(x - x^3 + (1 - x^2)^(1/2)),x)
```

```
[Out] asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(
1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)
^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2 -
1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^
3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 -
 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2))/((1 - (3^(1/2
)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (log(x - 3^
(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)
 - (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 +
 1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 + 1i/
2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2
+ 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (log(x - 3^(1/2
)/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i) + (
log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(
1/2) + 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{1-x^2}}{x^3-x-\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(x-x**3+(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(x*sqrt(1 - x**2)/(x**3 - x - sqrt(1 - x**2)), x)
```

3.1023 $\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$

Optimal. Leaf size=34

$$-\frac{(1-x)\left(x^3+x^2+x+1\right)^{-n}\left(1-x^4\right)^n}{n+1}$$

[Out] -(1-x)*(-x^4+1)^n/(1+n)/((x^3+x^2+x+1)^n)

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(1+x+x^2+x^3\right)^{-n}\left(1-x^4\right)^n \, dx$$

Verification is Not applicable to the result.

[In] Int[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]

[Out] Defer[Int][(1 - x^4)^n/(1 + x + x^2 + x^3)^n, x]

Rubi steps

$$\int \left(1+x+x^2+x^3\right)^{-n}\left(1-x^4\right)^n \, dx = \int \left(1+x+x^2+x^3\right)^{-n}\left(1-x^4\right)^n \, dx$$

Mathematica [A] time = 0.04, size = 31, normalized size = 0.91

$$\frac{(x-1)\left(x^3+x^2+x+1\right)^{-n}\left(1-x^4\right)^n}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]

[Out] ((-1 + x)*(1 - x^4)^n)/((1 + n)*(1 + x + x^2 + x^3)^n)

fricas [A] time = 0.62, size = 31, normalized size = 0.91

$$\frac{\left(-x^4+1\right)^n(x-1)}{\left(x^3+x^2+x+1\right)^n(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="fricas")

[Out] (-x^4 + 1)^n*(x - 1)/((x^3 + x^2 + x + 1)^n*(n + 1))

giac [B] time = 0.46, size = 81, normalized size = 2.38

$$\frac{\frac{xe^{\left(n\log\left(x^3+x^2+x+1\right)+n\log(-x+1)\right)}}{\left(x^3+x^2+x+1\right)^n}-\frac{e^{\left(n\log\left(x^3+x^2+x+1\right)+n\log(-x+1)\right)}}{\left(x^3+x^2+x+1\right)^n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="giac")

```
[Out] (x*e^(n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n - e^(
n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n)/(n + 1)
```

maple [A] time = 0.00, size = 32, normalized size = 0.94

$$\frac{(x-1)\left(-x^{4}+1\right)^{n}\left(x^{3}+x^{2}+x+1\right)^{-n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^n/((x^3+x^2+x+1)^n),x)
```

```
[Out] (x-1)/(n+1)*(-x^4+1)^n/((x^3+x^2+x+1)^n)
```

maxima [A] time = 1.09, size = 16, normalized size = 0.47

$$\frac{(x-1)(-x+1)^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="maxima")
```

```
[Out] (x - 1)*(-x + 1)^n/(n + 1)
```

mupad [B] time = 3.45, size = 31, normalized size = 0.91

$$\frac{\left(1-x^{4}\right)^{n}\,(x-1)}{(n+1)\,\left(x^{3}+x^{2}+x+1\right)^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^n/(x + x^2 + x^3 + 1)^n,x)
```

```
[Out] ((1 - x^4)^n*(x - 1))/((n + 1)*(x + x^2 + x^3 + 1)^n)
```

sympy [A] time = 72.39, size = 73, normalized size = 2.15

$$\begin{cases} \frac{x\left(1-x^{4}\right)^{n}}{n\left(x^{3}+x^{2}+x+1\right)^{n}+\left(x^{3}+x^{2}+x+1\right)^{n}} - \frac{\left(1-x^{4}\right)^{n}}{n\left(x^{3}+x^{2}+x+1\right)^{n}+\left(x^{3}+x^{2}+x+1\right)^{n}} & \text{for } n \neq -1 \\ -\log(x-1) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**n/((x**3+x**2+x+1)**n),x)
```

```
[Out] Piecewise((x*(1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x +
 1)**n) - (1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x + 1)
**n), Ne(n, -1)), (-log(x - 1), True))
```

3.1024 $\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$

Optimal. Leaf size=177

$$\log\left(20738073600000000b^8c^4+597005697024000000b^6c^6x^2+2583100705996800000b^5c^7x^3+9510507144\right.$$

```
[Out] 1/18432*ln(20738073600000000*b^8*c^4+597005697024000000*b^6*c^6*x^2+2583100
705996800000*b^5*c^7*x^3+9510507144806400000*b^4*c^8*x^4+2164168736951500800
0*b^3*c^9*x^5+32462531054272512000*b^2*c^10*x^6+14958734309808773529 6*c^12*
x^8+5308416*(12230590464*c^10*x^6+1990656000*b^2*c^8*x^4+1105920000*b^3*c^7
*x^3+38880000*b^4*c^6*x^2+79200000*b^5*c^5*x+12203125*b^6*c^4)*(5308416*c^4
*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2))/c^2
```

Rubi [A] time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2082}

$$\log\left(32462531054272512000b^2c^{10}x^6+21641687369515008000b^3c^9x^5+9510507144806400000b^4c^8x^4+2583\right.$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x
^4],x]
```

```
[Out] Log[20738073600000000*b^8*c^4 + 597005697024000000*b^6*c^6*x^2 + 2583100705
996800000*b^5*c^7*x^3 + 9510507144806400000*b^4*c^8*x^4 + 216416873695150080
00*b^3*c^9*x^5 + 32462531054272512000*b^2*c^10*x^6 + 149587343098087735296*
c^12*x^8 + 5308416*Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 +
5308416*c^4*x^4]*(12203125*b^6*c^4 + 79200000*b^5*c^5*x + 38880000*b^4*c^6*
x^2 + 1105920000*b^3*c^7*x^3 + 1990656000*b^2*c^8*x^4 + 12230590464*c^10*x^
6)]/(18432*c^2)
```

Rule 2082

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1*(33*b^2*c + 6*a*c^2 + 40*a^2*e))/320 - (22*a*c*e*x^2)/5 + (
22*b*c*e*x^3)/15 + (1*e*(5*c^2 + 4*a*e)*x^4)/4 + (4*b*e^2*x^5)/3 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px, x], x]*Sqrt[a
 + b*x + c*x^2 + e*x^4]])/(8*Rt[e, 2]), x]] /; FreeQ[{a, b, c, e}, x] && Eq
Q[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx=\frac{\log\left(20738073600000000b^8c^4+597005\right.}{}$$

Mathematica [C] time = 6.13, size = 1671, normalized size = 9.44

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416
*c^4*x^4],x]
```

```
[Out] (2*(x - (b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/
c)^2*(-((b*EllipticF[ArcSin[Sqrt[((c*x - b*Root[-44375 + 576000*#1 + 576000
*#1^2 + 5308416*#1^4 & , 1, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 53
08416*#1^4 & , 2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4
 & , 4, 0]))/((c*x - b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4
 & , 2, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]
 - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))]], -(((
Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0] - Root[-4437
5 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 3, 0])*(Root[-44375 + 576000
*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0] - Root[-44375 + 576000*#1 + 5760
00*#1^2 + 5308416*#1^4 & , 4, 0]))/((-Root[-44375 + 576000*#1 + 576000*#1^2
 + 5308416*#1^4 & , 1, 0] + Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416
*#1^4 & , 3, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & ,
2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])))]*
Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c) + (Ellip
ticPi[(-((b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])
/c) + (b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c)
/(-((b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c) +
 (b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c), Arc
Sin[Sqrt[((c*x - b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & ,
 1, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0] - R
oot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))/((c*x - b*R
oot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])*(Root[-44375
 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0] - Root[-44375 + 576000*
#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))]], -(((Root[-44375 + 576000*#1
+ 576000*#1^2 + 5308416*#1^4 & , 2, 0] - Root[-44375 + 576000*#1 + 576000*#
1^2 + 5308416*#1^4 & , 3, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308
416*#1^4 & , 1, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &
 , 4, 0]))/((-Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0
] + Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 3, 0])*(Root[-
44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0] - Root[-44375 + 57
6000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])))]*(-(b*Root[-44375 + 57600
0*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]) + b*Root[-44375 + 576000*#1 +
576000*#1^2 + 5308416*#1^4 & , 2, 0]))/c)*Sqrt[((-(b*Root[-44375 + 576000*#
1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]) + b*Root[-44375 + 576000*#1 + 576
000*#1^2 + 5308416*#1^4 & , 2, 0])*(x - (b*Root[-44375 + 576000*#1 + 576000
*#1^2 + 5308416*#1^4 & , 3, 0])/c))/(c*(x - (b*Root[-44375 + 576000*#1 + 57
6000*#1^2 + 5308416*#1^4 & , 2, 0])/c)*(-((b*Root[-44375 + 576000*#1 + 5760
00*#1^2 + 5308416*#1^4 & , 1, 0])/c) + (b*Root[-44375 + 576000*#1 + 576000*
#1^2 + 5308416*#1^4 & , 3, 0])/c))]*Sqrt[((c*x - b*Root[-44375 + 576000*#1
+ 576000*#1^2 + 5308416*#1^4 & , 1, 0])*(Root[-44375 + 576000*#1 + 576000*#
1^2 + 5308416*#1^4 & , 2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308
416*#1^4 & , 4, 0]))/((c*x - b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308
416*#1^4 & , 2, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &
 , 1, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0]))
]*((b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])/c - (
b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c)*Sqrt[(
(-(b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0]) + b*Ro
ot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])*(x - (b*Root[
-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c))/(c*(x - (b*R
oot[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c)*(-((b*Roo
t[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])/c) + (b*Root[-
44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c))]])/(Sqrt[-4437
5*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x^4]*(-((b*Root[-
44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 1, 0])/c) + (b*Root[-443
75 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c)*((b*Root[-44375 +
 576000*#1 + 576000*#1^2 + 5308416*#1^4 & , 2, 0])/c - (b*Root[-44375 + 576
000*#1 + 576000*#1^2 + 5308416*#1^4 & , 4, 0])/c))
```

fricas [A] time = 0.93, size = 164, normalized size = 0.93

$$\log \left(28179280429056\,c^8x^8+6115295232000\,b^2c^6x^6+4076863488000\,b^3c^5x^5+179159040000\,b^4c^4x^4+486\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="fricas")
```

```
[Out] 1/18432*log(28179280429056*c^8*x^8 + 6115295232000*b^2*c^6*x^6 + 4076863488
000*b^3*c^5*x^5 + 179159040000*b^4*c^4*x^4 + 486604800000*b^5*c^3*x^3 + 112
464000000*b^6*c^2*x^2 + 3906640625*b^8 + (12230590464*c^6*x^6 + 1990656000*
b^2*c^4*x^4 + 1105920000*b^3*c^3*x^3 + 38880000*b^4*c^2*x^2 + 79200000*b^5*
c*x + 12203125*b^6)*sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*
c*x - 44375*b^4))/c^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{5308416\,c^4x^4+576000\,b^2c^2x^2+576000\,b^3cx-44375\,b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44
375*b^4), x)
```

maple [C] time = 0.74, size = 1597, normalized size = 9.02

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2),x
)
```

```
[Out] 1/1152*(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*
_Z^2+96*_Z-71,index=4)*b/c)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/
c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+
96*_Z-71,index=2)*b/c))^(1/2)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)
*b/c)^2*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+1
0*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)
*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c-5/48*RootOf(_Z^4+10*_
Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/
c))^(1/2)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4
+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=
4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10
*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*
b/c))^(1/2)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^
4+10*_Z^2+96*_Z-71,index=2)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2
)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(c^4*(x-5/48*RootOf(_
Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,ind
ex=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(x-5/48*RootO
f(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2)*(5/48*RootOf(_Z^4+10*_Z^2+96*_
Z-71,index=2)*b/c*EllipticF(((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/
c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5
```

```
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+
96*_Z-71,index=2)*b/c))^(1/2),((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*
b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c)/(5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_
Z-71,index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*Roo
tOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2))+(5/48*RootOf(_Z^4+10*_Z^2+9
6*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*Ellipt
icPi(((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_
Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/
c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_Z^2
+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c))
^(1/2),(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*
_Z^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c
-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c),((5/48*RootOf(_Z^4+10*_Z^2
+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(5/4
8*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-
71,index=4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootO
f(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,in
dex=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{5308416\, c^4 x^4 + 576000\, b^2 c^2 x^2 + 576000\, b^3 c x - 44375\, b^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44
375*b^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-44375\, b^4 + 576000\, b^3\, c\, x + 576000\, b^2\, c^2\, x^2 + 5308416\, c^4\, x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^(
1/2),x)
```

```
[Out] int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^(
1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c**4*x**4+576000*b**2*c**2*x**2+576000*b**3*c*x-44375*
b**4)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-44375*b**4 + 576000*b**3*c*x + 576000*b**2*c**2*x**2 + 530
8416*c**4*x**4), x)
```

3.1025 $\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}} \, dx$

Optimal. Leaf size=100

$$\frac{1}{16}\log\left(4096x^8+8192x^7+12288x^6+19456x^5+17024x^4+13440x^3+9280x^2+\sqrt{64x^4+64x^3+64x^2+12\right.$$

```
[Out] 1/16*ln(921+2864*x+9280*x^2+13440*x^3+17024*x^4+19456*x^5+12288*x^6+8192*x^
7+4096*x^8+(512*x^6+768*x^5+960*x^4+1280*x^3+744*x^2+444*x+179)*(64*x^4+64*
x^3+64*x^2+120*x+9)^(1/2))
```

Rubi [B] time = 0.14, antiderivative size = 243, normalized size of antiderivative = 2.43, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2083, 2082}

$$\frac{1}{16}\log\left(4096x^8+8192x^7+512\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^6+12288x^6+768\sqrt{64x^4+64x^3+64x^2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

```
[Out] Log[921 + 2864*x + 9280*x^2 + 13440*x^3 + 17024*x^4 + 19456*x^5 + 12288*x^6
 + 8192*x^7 + 4096*x^8 + 179*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 4
44*x*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 744*x^2*Sqrt[9 + 120*x +
64*x^2 + 64*x^3 + 64*x^4] + 1280*x^3*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*
x^4] + 960*x^4*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 768*x^5*Sqrt[9
+ 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 512*x^6*Sqrt[9 + 120*x + 64*x^2 + 64*
x^3 + 64*x^4]]/16
```

Rule 2082

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1*(33*b^2*c + 6*a*c^2 + 40*a^2*e))/320 - (22*a*c*e*x^2)/5 + (
22*b*c*e*x^3)/15 + (1*e*(5*c^2 + 4*a*e)*x^4)/4 + (4*b*e^2*x^5)/3 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px, x], x]*Sqrt[a
 + b*x + c*x^2 + e*x^4]])/(8*Rt[e, 2]), x]] /; FreeQ[{a, b, c, e}, x] && Eq
Q[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rule 2083

```
Int[((A_) + (B_.)*(x_))/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^
3 + (e_.)*(x_)^4], x_Symbol] :> Dist[B, Subst[Int[x/Sqrt[(-3*d^4 + 16*c*d^2
*e - 64*b*d*e^2 + 256*a*e^3)/(256*e^3) + ((d^3 - 4*c*d*e + 8*b*e^2)*x)/(8*e
^2) - ((3*d^2 - 8*c*e)*x^2)/(8*e) + e*x^4], x], x, d/(4*e) + x], x] /; Free
Q[{a, b, c, d, e, A, B}, x] && EqQ[B*d - 4*A*e, 0] && EqQ[d*(141*d^3 - 752*
c*d*e - 400*b*e^2) + 16*e^2*(71*c^2 + 100*a*e), 0] && EqQ[144*(3*d^2 - 8*c*
e)^3 + 125*(d^3 - 4*c*d*e + 8*b*e^2)^2, 0]
```

Rubi steps

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}} \, dx = 4\,\text{Subst}\left(\int \frac{x}{\sqrt{-\frac{71}{4}+96x+40x^2+64x^4}} \, dx, x, \frac{1}{4}+x\right)$$

$$= \frac{1}{16}\log\left(921+2864x+9280x^2+13440x^3+17024x^4+19456x^5+\right.$$

Mathematica [C] time = 6.10, size = 2787, normalized size = 27.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

```
[Out] (8*(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])^2*(-(Ellip
ticF[ArcSin[Sqrt[((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1,
 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64*#
1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 6
4*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])
)]], -(((Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 +
 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2
 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#
1^4 & , 4, 0]))/((-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0]
+ Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1
 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#
1^3 + 64*#1^4 & , 4, 0])))]*Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 &
 , 2, 0]) + EllipticPi[(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & ,
1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])/(-Root[9 +
 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 + 64*#1^2
 + 64*#1^3 + 64*#1^4 & , 4, 0]), ArcSin[Sqrt[((x - Root[9 + 120*#1 + 64*#1^
2 + 64*#1^3 + 64*#1^4 & , 1, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*
#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/
((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 +
64*#1^3 + 64*#1^4 & , 4, 0]))]], -(((Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 +
64*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0]
)*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#
1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((-Root[9 + 120*#1 + 64*#1^2 +
64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4
 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root
[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])))]*(-Root[9 + 120*#1 +
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^
3 + 64*#1^4 & , 2, 0]))*Sqrt[(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*
#1^4 & , 3, 0])/((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2,
0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0]))]*(Root[9 + 120*#1 + 64*#1^2
+ 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1
^4 & , 4, 0])*Sqrt[((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & ,
1, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 +
120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64
*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 +
 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0
]))]*Sqrt[(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])/((x
 - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*
#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64
*#1^3 + 64*#1^4 & , 4, 0]))])/(Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4]*(
Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 120*#1 +
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])) + (2*EllipticF[ArcSin[Sqrt[((x - R
oot[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0])*(-Root[9 + 120*#1 +
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^
3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4
 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] + Roo
t[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))]], ((Root[9 + 120*#1
 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#
1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & ,
```

```
1, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((Root[9
 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - Root[9 + 120*#1 + 64*#1
^2 + 64*#1^3 + 64*#1^4 & , 3, 0])*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64
*#1^4 & , 2, 0] - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))
]*(x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])^2*Sqrt[(x -
 Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 3, 0])/((x - Root[9 + 12
0*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 +
 64*#1^3 + 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^
4 & , 3, 0]))]*(Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0] - R
oot[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])*Sqrt[(x - Root[9 +
120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0])/((x - Root[9 + 120*#1 + 64*
#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 +
 64*#1^4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0
]))]*Sqrt[((x - Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 1, 0])*(-
Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 +
 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))/((x - Root[9 + 120*#1 + 64*#1^2 +
64*#1^3 + 64*#1^4 & , 2, 0])*(-Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^
4 & , 1, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 & , 4, 0]))])/(
Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4]*(-Root[9 + 120*#1 + 64*#1^2 + 64
*#1^3 + 64*#1^4 & , 2, 0] + Root[9 + 120*#1 + 64*#1^2 + 64*#1^3 + 64*#1^4 &
 , 4, 0]))
```

fricas [A] time = 0.76, size = 97, normalized size = 0.97

$$\frac{1}{16}\log\left(-4096\,x^8-8192\,x^7-12288\,x^6-19456\,x^5-17024\,x^4-13440\,x^3-9280\,x^2-\left(512\,x^6+768\,x^5+960\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/16*log(-4096*x^8 - 8192*x^7 - 12288*x^6 - 19456*x^5 - 17024*x^4 - 13440*x
^3 - 9280*x^2 - (512*x^6 + 768*x^5 + 960*x^4 + 1280*x^3 + 744*x^2 + 444*x +
 179)*sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9) - 2864*x - 921)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

maple [C] time = 1.12, size = 2992, normalized size = 29.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+1)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x)
```

```
[Out] 1/4*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=4))*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,i
ndex=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*
_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)*(x-1/2*RootOf(4*_Z^4+8*_
Z^3+16*_Z^2+60*_Z+9,index=2))^2*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
```

```
index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4
*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60
*_Z+9,index=3)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*Ro
otOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)*((1/2*RootOf(4*_Z^4+8*_
Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index
=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_Z
^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9
,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)/(1/
2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16
*_Z^2+60*_Z+9,index=2))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-
1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/((x-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(x-1/2*Root
Of(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2)*EllipticF(((1/2*RootOf(4*
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2
+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(
1/2),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_
Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+
9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4
*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2))+(1/2*RootOf(4*_Z^4+8*_Z^3+16
*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))*(
(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3
+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,inde
x=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+
9,index=2)))^(1/2)*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))^2*
((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^
3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,ind
ex=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3)-1/2*RootOf(4*_Z^4
+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
+9,index=2)))^(1/2)*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2
*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3
+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=
4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+6
0*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))/(1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2
+60*_Z+9,index=1))/((x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(
x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8
*_Z^3+16*_Z^2+60*_Z+9,index=3))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9
,index=4)))^(1/2)*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)*Ellipt
icF(((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8
*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9
,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+6
0*_Z+9,index=2)))^(1/2),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)
-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+8*_Z
^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=
4))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,in
dex=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2))+(1/2*Root
Of(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+
60*_Z+9,index=2))*EllipticPi(((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,ind
ex=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z
^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
```

```
+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootO
f(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2),(1/2*RootOf(4*_Z^4+8*_Z^3+
16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))
/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^
3+16*_Z^2+60*_Z+9,index=2)),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,inde
x=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,in
dex=4))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^
4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+
9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2),x)
```

```
[Out] int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x+1}{\sqrt{64x^4+64x^3+64x^2+120x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x**4+64*x**3+64*x**2+120*x+9)**(1/2),x)
```

```
[Out] Integral((4*x + 1)/sqrt(64*x**4 + 64*x**3 + 64*x**2 + 120*x + 9), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```